Historical Announcement Logic: theory, semantics, tableaux and completeness

Abstract

This paper proposes a dynamic temporal logic that is appropriate for modeling the dynamics of scientific knowledge (especially in historical sciences, such as Archaeology, Paleontology and Geology). For this formalization of historical knowledge, the work is divided into two topics: firstly, we define a temporal branching structure and define the terms for application in Philosophy of Science; Finally, we define a logical system that consists of a variation of Public Announcement Logic in terms of temporal logic, with appropriate rules in a tableaux method.

Keywords: philosophy of science, branching temporal logic, dynamic logic, modal logic

1 Desiderata

In this paper we propose a Kripke frame to represent the different *factual possibilities of the past* or *probable pasts*, which branch backwards from the present. Probable pasts should not be confused with *counterfactual pasts*, this for a very simple reason: probable pasts are "eliminable", or better, the accessibility relation of these pasts can be eliminated with respect to present instant. Such property is strange in relation to counterfactual worlds or pasts, but it is expected when these "worlds" are, as in our interpretation, candidates for the real/actual past. More details about the different senses of "possibility" for temporal ramifications can be found in [4] (more specifically in the first chapter).

The Historical Announcement Logic (**HAL**) first assumes that, from an epistemic point of view, the ramifications of the past (as well as the ramifications of the future) can represent the different possible reconstructions of what occurred (or what will occur). The second fundamental assumption is that these different representations can be revised by subsequent knowledge.

Proposition 1 (vertical and horizontal evolution of historical knowledge)**.** *"Historical knowledge" can evolve in two directions, in depth and in accuracy:*

- *The "depth" of historical knowledge is directly proportional to the number of non-empty instants linearly linked from the present to the most remote past;*
- *The "accuracy" of historical knowledge is inversely proportional to the number of theories (or competing versions) for the sequence of past events.*

Proposition 2 (historical knowledge falsificationism)**.** *An instant of probable time can be eliminated by a historical announcement.*

Remark 1 (**HAL** and the historical sciences)**.** *This formal interpretation of the evolution of historical scientific knowledge does not necessarily assume realism or antirealism. Further studies can benefit from this interpretation to formalize different models for the historical sciences.[9][10]*

For example: When Renaissance artists were inspired by ancient sculpture, their preferred medium was pure white marble, but little did they know that Greco-Roman works were originally painted in dazzling and diverse tones. Thanks to analyzes of historical and archaeological sources, including studies using ultraviolet technology, we were able to revise these hypotheses[7].

Thus, our objective is to build a logical structure that can answer the question "what is historical knowledge?" in an inverse way, that is, "what is history while it is being known?". More specifically, in this article we will offer a logic that partially captures the intuition of what is a "historical as it is being known", such as the plurality of probable representations and the possibility of revision of historical knowledge. Our theoretical proposal in this paper can be seen as an alternative way to formally model scientific knowledge in historical sciences, rather than by quasi-truth theory.[1]

We will call **HAL** (Historical Announcement Logic) the temporal logic with announcement $[\cdot]$ (an operator typically used in Public Announcement Logic or **PAL**). In the diagrams below we provide a representation of how these revisions occur, in order to eliminate temporal ramifications. Just to introduce the concept in a simpler way, we represent the updates below with an implication and an exclamation mark.

Before properly defining **HAL** and its syntactic and semantic properties (final topic), we will present in the following topic more rigorously our epistemic interpretation of the historical ramifications.

2 Branches as theories

Definition 1 (chain). *A chain of time instants is a set of time instants ordered by a temporal precedence relation* $\prec$.

Definition 2 (succession). *A succession relation* $\succ$ *is the inverse relation of the precedence relation:* $(t_2 \succ t_1) \equiv (t_1 \prec t_2)$.

Definition 3 (history). *A history* $h(t, \mathcal{T}) \subseteq \mathcal{H}$ *is a chain of precedence* $t_1 \prec t_2 \prec t_3 \prec ... \prec t$ *that starts at an instant* $t \in \mathcal{T}$.

Definition 4 (destiny). *A destiny* $d(t, \mathcal{T}) \subseteq \mathcal{D}$ *is a chain of sucession* $t_3 \succ t_2 \succ t_1 \succ ... \succ t$ *that ends at an instant* $t \in \mathcal{T}$.

Definition 5 (branch). *A branch* $b(t, \mathcal{T}) \subseteq \mathcal{B}$ *is a history* $h(t, \mathcal{T})$ *or a destiny* $d(t, \mathcal{T})$*:* $\mathcal{H} \subseteq \mathcal{B}$*; and* $\mathcal{D} \subseteq \mathcal{B}$.

Definition 6 (node). *An instant of time* t *is a node when there are two histories* $h'(t, \mathcal{T})$ *and* $h''(t, \mathcal{T})$ *such that* $h' \neq h''$ *or when there are two destinies* $d'(t, \mathcal{T})$ *and* $d''(t, \mathcal{T})$ *such that* $d' \neq d''$.

Definition 7 (Temporal model). *A temporal model is a triple* $\mathcal{M} = \langle T, \prec, V \rangle$ *where* $\mathcal{T} = \langle T, \prec \rangle$ *is a tree or temporal frame,* T *is a non-empty set of time instants with a binary relation* $\prec$*, and* V *is a function-interpretation* $V : T \times PROP \to \{true, false\}$*, which assigns a truth value to each atomic proposition at each time instant in the temporal frame.*

Definition 8 (tree). *A tree is a Kripke structure* $\mathcal{T} = \langle T, \prec \rangle$ *where* T *is a non-empty set of time instants with a binary relation* $\prec$ *denoting precedence over time in* T.

Definition 9 (levels). l *is a level of a tree such that* $l = \{0, 1, 2, ..., n/n \in \mathbb{N}\}$.

Definition 10 (tree size). *A tree* $\mathcal{T}$ *is larger than a tree* $\mathcal{T}^*$ *if and only if the last level* l_n *of* $\mathcal{T}$ *is smaller than the last level* l_n *of* $\mathcal{T}^*$.

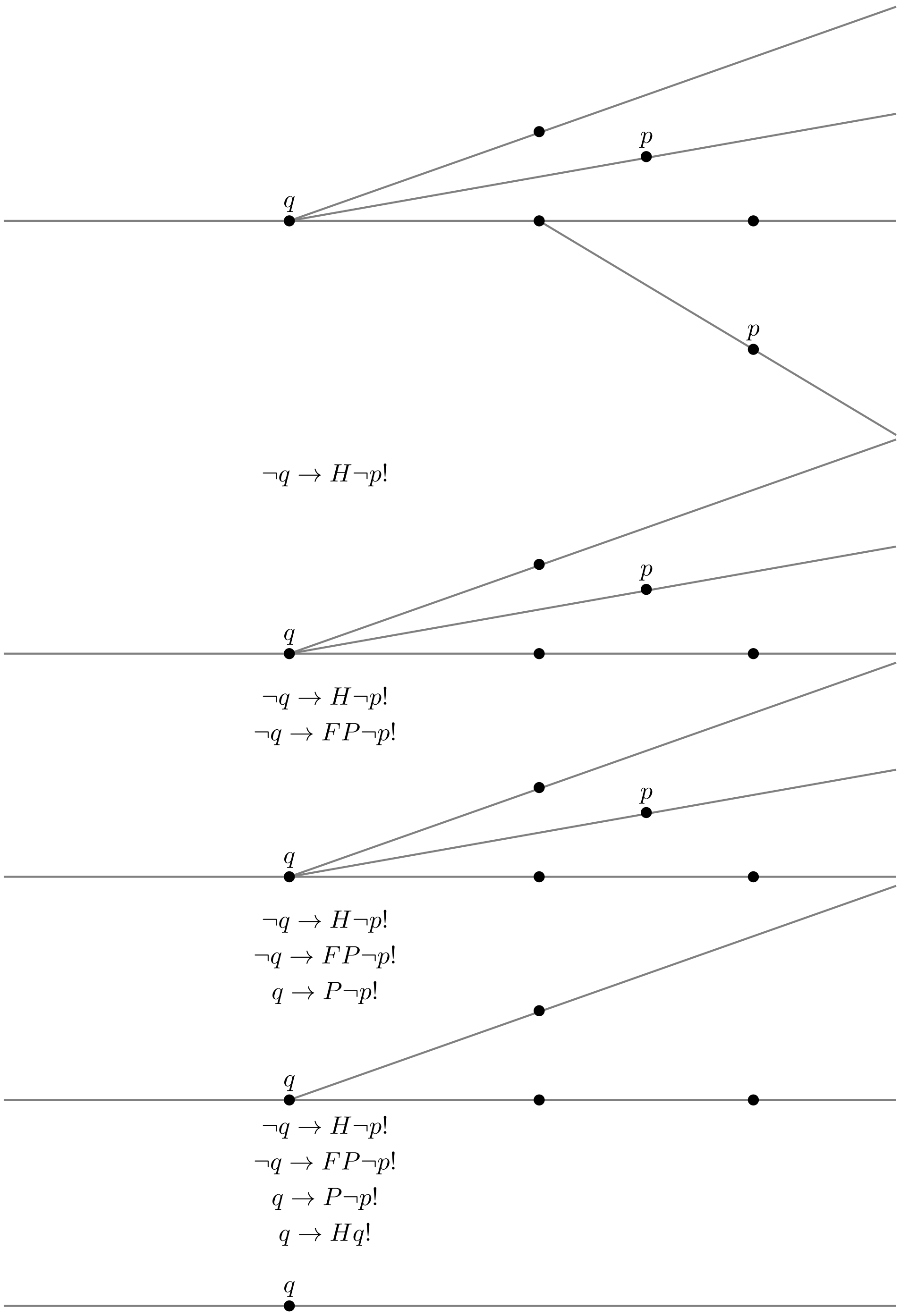


Fig. 1 A sequence of diagrams representing the update of time instants after historical announcements. On the right: past; on the left: future. The center diagram is an example of a structure that remains the same after a historical announcement (relative to the second diagram, from top to bottom).

For more details on the basis of temporal logic and branching temporal logics, see [15][16][8].

Now let's limit our definitions to terms for analyzing ramifications into the past, because our example is related to historical sciences. However, this approach can also be adapted for future ramifications.

Definition 11 (history level)**.** *$\mathcal{H}_l$ is the set of histories h at level l of a tree $\mathcal{T}$.*

Definition 12 (level of temporal instants)**.** *T_l is the set of instants t at level l of a tree $\mathcal{T}$.*

Instants are points that can serve as nodes to assemble a tree. Thus, based on the sequence of levels l of a tree, we can introduce a scalar term for the number of histories at each level:

Definition 13. *(number of histories in a level) $|\mathcal{H}_l|$ is the number of histories $h_1, h_2, ..., h_n$ at level l of a tree $\mathcal{T}$.*

Remark 2 (number of histories in an instant)**.** *When we want to specify that it is the number of histories h at a specific time t, we can use $|\mathcal{H}_l^t|$.*

Definition 14. *(number of instants in a level) $|\mathcal{T}_l|$ is the number of instants $t_1, t_2, ..., t_n$ at level l of a tree $\mathcal{T}$.*

In these terms, we can define the total set of histories h and the total set of instants t formally as follows:

$$\mathcal{H} \equiv \bigcup_{l=0}^{n} \mathcal{H}_l$$

$$T \equiv \bigcup_{l=0}^{n} T_l$$

Remark 3 (branching time)**.** *It is worth noting that the term "branching time" is not strictly the most appropriate, since time has a mathematical structure that evolves linearly. Some authors propose the term "branching histories" [3], but we do not use it in this paper because we give a more specific meaning to "histories".*

In historical sciences (such as Archaeology, History, Geology and Paleontology), generally only the different relevant epistemic possibilities of the past are of interest to scientists, so it can be useful to delimit the structure to an endpoint (which can be interpreted as the "present" instant). Another principle that may be interesting is that of connectivity, to make all moments connected to the same and unique end point.

Alternatively, we can assume a principle that we will call "pluperfect linearity"; This principle causes histories to be linear only after the second instant after the present. This is an interesting way of representing historical theories as total versions of what the real past might be like. Conversely, we have the principles of the beginning and the plufuture linearity.

- Transitivity: $\forall x \forall y \forall z (x \prec y \wedge y \prec z \rightarrow x \prec z)$;
- Irreflexivity: $\forall t \neg (t \prec t)$;
- Connectivity: $\forall t_1, \forall t_2, \exists t_3 ((t_3 \preceq t_1 \wedge t_3 \preceq t_2) \vee (t_1 \preceq t_3 \wedge t_2 \preceq t_3))$;
- Beginning: $\exists x \neg \exists y (y \prec x)$;
- End: $\exists x \neg \exists y (x \prec y)$;
- Pluperfect linearity: $\exists t_0 (\neg \exists t_n (t_0 \prec t_n) \wedge (\forall t_1 (t_1 \prec t_0) \rightarrow$ $\forall t_2 \forall t_3 ((t_2 \prec t_1 \wedge t_3 \prec t_1) \rightarrow (t_2 = t_3 \vee t_2 \prec t_3 \vee t_3 \prec t_2))))$;
- Plufuture linearity: $\exists t_0 (\neg \exists t_n (t_n \prec t_0) \wedge (\forall t_1 (t_0 \prec t_1) \rightarrow$ $\forall t_2 \forall t_3 ((t_1 \prec t_2 \wedge t_1 \prec t_3) \rightarrow (t_2 = t_3 \vee t_2 \prec t_3 \vee t_3 \prec t_2))))$.

In particular, the principle of connectivity does not have an equivalent axiom in a Kripke temporal structure. For more details about this and other properties in temporal logic structures, see [5].

Below we offer two examples of trees. In both cases we have a terminal time instant (present): the 0 point of the trees. In the second diagram, we have a tree with the principle of pluperfect linearity to translate the same historical theories that are in the tree in the first diagram.

Assuming temporal structures (in an epistemic interpretation) like those in the examples above, in the following section we will develop a dynamic temporal logic that allows us to review alternative theories for what may have occurred (or what could occur, if we consider the future).

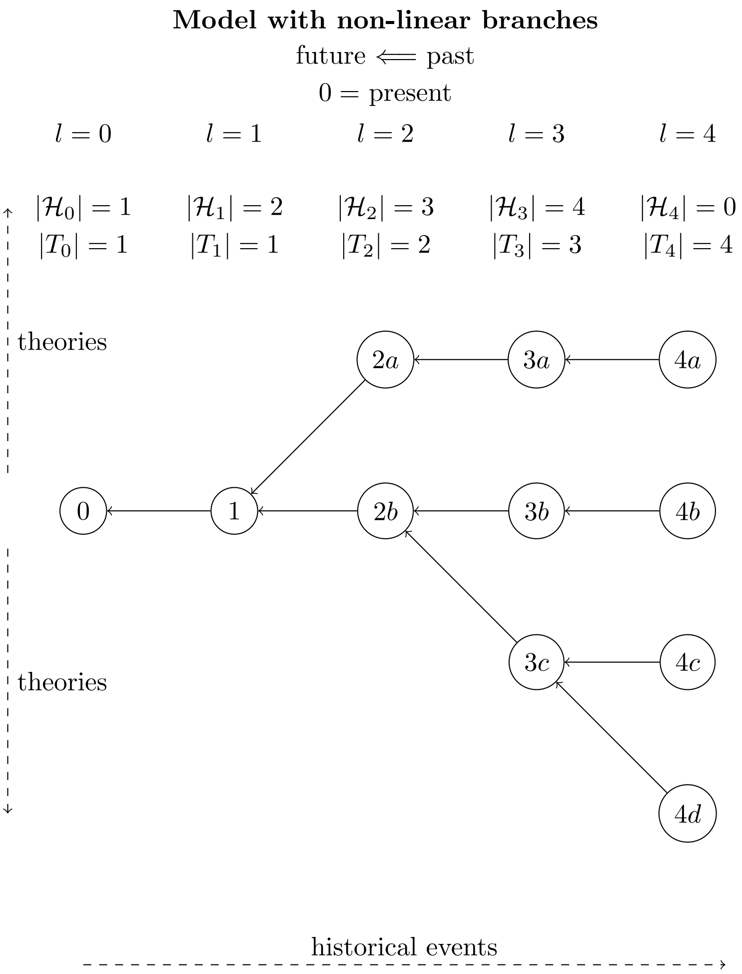


Fig. 2 Diagram representing a temporal semantic interpretation for philosophy of science in a model with non-linear branches.

3 Historical Announcement Logic

The language of **HAL** is the same as that of temporal logic, only with the addition of the public announcement logic operator. However, our interpretation gives temporal operators a subtle difference with the term “probable” (“probable” is understood as a relevant epistemic possibility in the historical sciences).

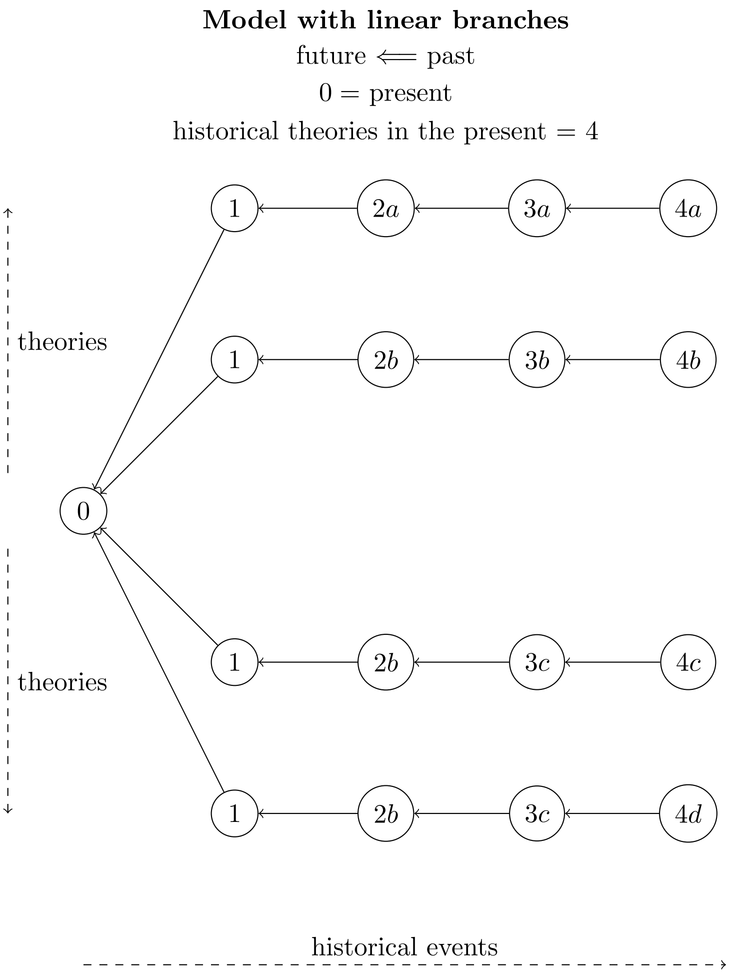


Fig. 3 Diagram representing a temporal semantic interpretation for philosophy of science in a model with linear branches.

$P\varphi$: "It is probably that it was the case that φ"

$F\varphi$: "It is probably that it will be the case that φ"

$H\varphi$: "It was necessarily the case that φ"

$G\varphi$: "It will necessarily be the case that φ"

$\langle\varphi\rangle\psi$: "after *some* historical announcement that φ is the case. ψ is the case"

$[\varphi]\psi$: "after *any* historical announcement that φ is the case, ψ is the case"

$$\varphi := p \in PROP \mid \bot \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid H\varphi \mid G\varphi \mid [\varphi]\psi.$$

$\mathcal{M}, t \models p$ iff $t \in V(p)$, for $p \in PROP$;

$\mathcal{M}, t \models \neg\varphi$ iff $\mathcal{M}, t \not\models \varphi$;

$\mathcal{M}, t \models \varphi \wedge \psi$ iff $\mathcal{M}, t \models \varphi$ and $\mathcal{M}, t \models \psi$;

$\mathcal{M}, t \models H\varphi$ iff $\mathcal{M}, t' \models \varphi$, for all time instant
$t' \in \mathcal{T}$ such that $t' \prec t$;

$\mathcal{M}, t \models G\varphi$ iff $\mathcal{M}, t' \models \varphi$, for all time instant
$t' \in \mathcal{T}$ such that $t \prec t'$;

$\mathcal{M}, t \models [\varphi]\psi$ iff if $\mathcal{M}, t \models \varphi$, then $\mathcal{M}|_{\varphi}, t \models \psi$.

The semantic condition for $[\varphi]\psi$ above is widely used, due to its simplicity, but it is actually not a precise definition, as some authors have noted[13][11]. A more rigorous definition is as follows:

$\mathcal{M}, t \models [\varphi]\psi$ iff for all $(\mathcal{M}^n, t_n)$,
if $\mathcal{M}^n = \mathcal{M}|_{\varphi}$ and $t_n = t$,
then $\mathcal{M}^n, t_n \models \psi$.

Operators by abbreviated definitions:

$$P\varphi \equiv \neg H\neg\varphi, H\varphi \equiv \neg P\neg\varphi, F\varphi \equiv \neg G\neg\varphi \text{ and } G\varphi \equiv \neg F\neg\varphi;$$

$$\langle\varphi\rangle\psi \equiv \neg[\varphi]\neg\psi \text{ and } [\varphi]\psi \equiv \neg\langle\varphi\rangle\neg\psi.$$

Definition 15 (Updated temporal model)**.** *Let any formula be* φ *from* **HAL***; a tree for* **HAL**, $\mathcal{T} = \langle T, \prec\rangle$*; a* V *valuation for atomic propositions,* $V : T \times PROP \rightarrow \{true, false\}$*; and* $\mathcal{M} = \langle \mathcal{T}, V\rangle$ *a temporal model. The update of* $\mathcal{M}$ *with respect to* φ *is a model*

$$\mathcal{M}|_{\varphi} = \langle T^!, \prec^!, V^!\rangle$$

where:

1. $T^! = ||\varphi||_{\mathcal{M}} = \{t \in T : \mathcal{M}, t \models \varphi\}$;
2. $\prec^! = \prec \cap (||\varphi||_{\mathcal{M}} \times ||\varphi||_{\mathcal{M}})$;
3. *for each* $t \in T^!$, $V^!(\varphi, t) = V(\varphi, t)$ *and* $V^!(\bot, t) = V(\bot, t)$.

Naturally, we are assuming the common assumptions of a branching temporal logic, such as the tautologies of classical propositional logic, transitivity and the axioms of the minimal $\mathbf{K}_t$ system. In addition to these axioms, we have the axioms for the principles of beginning, end, plufuture linearity and pluperfect linearity.

(PL)	All substitution instance of propositional tautologies
(GP)	$\varphi \to GP\varphi$
(HF)	$\varphi \to HF\varphi$
$(\mathbf{K}_G)$	$G(\varphi \to \psi) \to (G\varphi \to G\psi)$
$(\mathbf{K}_H)$	$H(\varphi \to \psi) \to (H\varphi \to H\psi)$
(TRAN^G)	$G\varphi \to GG\varphi$
(TRAN^H)	$H\varphi \to HH\varphi$
(BEG)	$H\bot \vee PH\bot$
(END)	$G\bot \vee FG\bot$
(LIN^{FF})	$PH\bot \to (PF\varphi \to (P\varphi \vee \varphi \vee F\varphi))$
(LIN^{PP})	$FG\bot \to (FP\varphi \to (P\varphi \vee \varphi \vee F\varphi))$

These axioms form the following systems:

$$\begin{aligned}
\mathbf{K}_t &= \mathbf{PL} + G + H \\
\mathbf{K}_b &= \mathbf{K}_t + (TRAN) + (IRREFL) \\
\mathbf{K}_{bF} &= \mathbf{K}_b + (BEG) \\
\mathbf{K}_{bP} &= \mathbf{K}_b + (END) \\
\mathbf{K}^*_{bF} &= \mathbf{K}_b + (CON^{BEG}) \\
\mathbf{K}^*_{bP} &= \mathbf{K}_b + (CON^{END}) \\
\mathbf{K}_{bFF} &= \mathbf{K}_b + (BEG) + (LIN^{FF}) \\
\mathbf{K}_{bPP} &= \mathbf{K}_b + (END) + (LIN^{PP})
\end{aligned}$$

Remark 4 (connectivity)**.** *The principle of connectivity introduced in the previous section encompasses the axiom of the beginning or the axiom of the end:* $(CON^{BEG})/(CON^{END})$.

$$\mathbf{HAL} = \mathbf{K}_t + [\cdot]$$

$$\mathbf{HAL}_b = \mathbf{K}_b + [\cdot]$$

$$\mathbf{HAL}_{bF} = \mathbf{K}_{bF} + [\cdot]$$

$$\mathbf{HAL}_{bP} = \mathbf{K}_{bP} + [\cdot]$$

$$\mathbf{HAL}^*_{bF} = \mathbf{K}^*_{bF} + [\cdot]$$

$$\mathbf{HAL}^*_{bP} = \mathbf{K}^*_{bP} + [\cdot]$$

$$\mathbf{HAL}_{bFF} = \mathbf{K}_{bFF} + [\cdot]$$

$$\mathbf{HAL}_{bPP} = \mathbf{K}_{bPP} + [\cdot]$$

Below are some important properties of **HAL**:

Theorem 3 (equivalent announcements)**.** *For any model:*

1. $\mathcal{M}, t \models \neg[\varphi]\neg\psi \leftrightarrow \langle\varphi\rangle\psi$;
2. $\mathcal{M}, t \models \neg\langle\varphi\rangle\neg\psi \leftrightarrow [\varphi]\psi$;
3. $\mathcal{M}, t \models [\varphi]\neg\psi \leftrightarrow \neg\langle\varphi\rangle\psi$;
4. $\mathcal{M}, t \models \neg[\varphi]\psi \leftrightarrow \langle\varphi\rangle\neg\psi$.

Proof. The first statement follows directly from the truth condition described for the operator. The remaining statements are easily proven by classical propositional logic. □

Theorem 4 (implication conversions)**.** *Make a temporal model* $\mathcal{M}$ *and let* t *be an instant in* $\mathcal{M}$*. For any* φ *and* ψ*, as formulas, as well as for any atomic formula* p *in* $\mathbf{HAL}^*$*:*

1. *If* $\mathcal{M}, t \models \langle\varphi\rangle\psi$*, then* $\mathcal{M}, t \models \varphi$;
2. *If* $\mathcal{M}, t \not\models [\varphi]\psi$*, then* $\mathcal{M}, t \models \varphi$;

3. If $\mathcal{M}, t \models \varphi$*, then* $\mathcal{M}, t \models \langle\varphi\rangle\top$*;*

4. If $\mathcal{M}, t \not\models \varphi$*, then* $\mathcal{M}, t \models [\varphi]\psi$*;*

5. If $\mathcal{M}|_{\varphi}, t \models p$*, then* $\mathcal{M}, t \models p$*;*

6. If $\mathcal{M}|_{\varphi}, t \not\models p$*, then* $\mathcal{M}, t \not\models p$*;*

7. $\mathcal{M}|_{\varphi}, t \models p$*, if and only if* $\mathcal{M}, t \models p$ *(if there is an instant* t*).*

Proof. 1. Suppose $(i)\mathcal{M}, t \models \langle\varphi\rangle\psi$, but $(ii)\mathcal{M}, t \not\models \varphi$. From (i), there must exist a $\mathcal{M}^n, t_n$ such that $\mathcal{M}^n = \mathcal{M}|_{\varphi}$ and $t_n = t$ and $\mathcal{M}^n, t_n \models \psi$. Of (ii), if there exists any $\mathcal{M}|\varphi$, certainly $t \notin T^!$, since $t \notin ||\varphi||_{\mathcal{M}}$. Thus, no t_n can be that t, which is a result incompatible with (i).

2. Suppose $\mathcal{M}, t \not\models [\varphi]\psi$. From the previous theorem, we know that this is equivalent to saying that $\mathcal{M}, t \models \langle\varphi\rangle\neg\psi$. From what was proved above, we conclude that $\mathcal{M}, t \models \varphi$.

3. Suppose $\mathcal{M}, t \not\models \langle\varphi\rangle\top$. According to the truth condition of historical announcements, there is no $\mathcal{M}^n, t_n$ such that: $\mathcal{M}^n = \mathcal{M}|_{\varphi}$ and $t_n = t$ and $\mathcal{M}^n, t_n \models \top$; in other words, for all $\mathcal{M}^n, t_n$: if $\mathcal{M}^n = \mathcal{M}|_{\varphi}$ and $t_n = t$, then $\mathcal{M}^n, t_n \not\models \top$. it is impossible that $\mathcal{M}^n, t_n \not\models \top$, so for all $\mathcal{M}^n, t_n$, or $\mathcal{M}^n \neq \mathcal{M}|_{\varphi}$ or $t_n \neq t$. That is, for all $\mathcal{M}^n, t_n$, if $\mathcal{M}^n = \mathcal{M}|_{\varphi}$, then $t_n \neq t$. If there is no $\mathcal{M}|_{\varphi}$, $\mathcal{M}, t \not\models \varphi$. But, if there is a model updated in this way, $\mathcal{M}|_{\varphi}$, we have that $t \notin ||\varphi||_{\mathcal{M}}$, and meanwhile $\mathcal{M}, t \not\models \varphi$.

4. Suppose $\mathcal{M}, t \not\models \varphi$. If we conceive some $\mathcal{M}|_{\varphi}$, there will be no instant of time, in what we have as $t_n \in T^!$, and that is t itself, since $t \notin ||\varphi||_{\mathcal{M}}$; however the semantic definition for $\mathcal{M}, t \models [\varphi]\psi$ will be vacuously satisfied. If there is no model $\mathcal{M}|_{\varphi}$, the same thing happens, because no $\mathcal{M}^n$ will correspond with $\mathcal{M}|_{\varphi}$.

5. The value of atomic formulas at an instant never changes in an updated model for all instants that remain after updating a model. Suppose that $(i)\mathcal{M}|_{\varphi}, t \models p$, but also that $(ii)\mathcal{M}, t \not\models p$. From (i), there is a $\mathcal{M}|_{\varphi}$ where $t \in T^!$ (however, $t \in T$). p is an atomic formula, so we have $\mathcal{M}|_{\varphi}, t \models p$, where, by definition, it is true in $V^!(P, t)$. By (ii), we know that this formula is not true in $V(P, t)$. It turns out that, due to the construction of $\mathcal{M}|_{\varphi}$, for each $t \in T^!$, $V^! = V$, which leads to a contradiction.

6. *Mutatis mutandis*, this can be proved by the same demonstration scheme above.

7. This follows directly from the last two items.

$\square$

Theorem 5 (announcement functionality)**.** *For any formulas* φ *and* ψ*:*

$$\langle\varphi\rangle\psi \to [\varphi]\psi$$

Proof. Suppose, for an arbitrary model $\mathcal{M}, t$, that $\mathcal{M}, t \not\models [\varphi]\psi$. By our semantic definition, this is the same as saying that it is not the case that, for all $\mathcal{M}^n, t_n$, if $\mathcal{M}^n = \ mathcalM|_\varphi$ and $t_n = t$, then $\mathcal{M}^n, t_n \models \psi$. Equivalently, there exists a $\mathcal{M}^n, t_n$ such that $\mathcal{M}^n = \mathcal{M}|_\varphi$ and $t_n = t$ and $\mathcal{M}^n, t_n \not\models \psi$. In other words, $\mathcal{M}, t \not\models \langle\varphi\rangle\psi$. $\square$

Theorem 6 (announcement partiality)**.** *For any formula* φ*:*

$$\not\models \langle\varphi\rangle\top$$

Proof. Suppose an arbitrary formula φ such that $\mathcal{M}, t \not\models \varphi$. Even if there is an updated model $\mathcal{M}|_\varphi$, there would not be an instant t such that $\mathcal{M}|_\varphi, t \models \top$, because $t \notin ||\varphi||_\mathcal{M}$. In other terms, $\mathcal{M}, t \not\models \langle\varphi\rangle$, since it cannot be that $\mathcal{M}^n, t_n$ such that $\mathcal{M}^n = \mathcal{M}|_\varphi$ and $t_n = t$ and $\mathcal{M}^n, t_n \models \top$; at least the second statement of the conjunction is false. $\square$

Theorem 7 (atomic formula preservation)**.** *For any atomic formula* p*:*

$$\models [p]p$$

Proof. This result can be easily demonstrated by reductio ad absurdum. Suppose $\not\models [p]p$. From the previous theorem, we know that this is equivalent to $\models \neg[p]p$, that is, $\models \langle p\rangle\neg p$. By definition, this means that there is an announcement of p at $\mathcal{M}, t$ and in an updated model $\mathcal{M}^n$ for the same instant t we have $\neg p$, therefore, for the same instant t, we have $\mathcal{M}^n, t \not\models p$, but we know that the domain of atomic propositions $PROP$ is the same for $\mathcal{M}$ and any updated model, then $\not\models_\mathcal{M} p$ and $\models_\mathcal{M} p$, which is absurd. $\square$

Theorem 8 (relations between announced formulas and other formulas)**.** *For any formula* φ, ψ, ξ *and for any atomic formula* p*:*

1. *Announcement and atomicity:* $\models [\varphi]p \leftrightarrow (\varphi \to p)$;
2. *Announcement and negation:* $\models [\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$;
3. *Announcement and implication:* $\models [\varphi](\psi \to \xi) \leftrightarrow ([\varphi]\psi \to [\varphi]\xi)$;
4. *Announcement and past:* $\models [\varphi]H\psi \leftrightarrow (\varphi \to H[\varphi]\psi)$;
5. *Announcement and future:* $\models [\varphi]G\psi \leftrightarrow (\varphi \to G[\varphi]\psi)$;
6. *Announcement composition:* $\models [\varphi][\psi]\xi \leftrightarrow [(\varphi \land [\varphi]\psi)]\xi$;
7. *Announcement RN: If* $\models \psi$, *then* $\models [\varphi]\psi$.

Proof. 1. $[\varphi]p \leftrightarrow (\varphi \to p)$

($\to$). Suppose $[\varphi]p$ and φ in $(\mathcal{M}, t)$. The fact that we have φ indicates that the formula in $[\varphi]p$ exists, so we have p in an updated model $\mathcal{M}|_{\varphi}$. By the previous theorem for the case of $\mathcal{M}|_{\varphi}, t \models p$, we obtain $\mathcal{M}, t \models p$.

($\leftarrow$). Now we will do an indirect proof. Suppose that $\varphi \to p$ in $(\mathcal{M}, t)$, but that $\neg[\varphi]p$. From what we proved in the previous theorem, we know that this is equivalent to $\langle\varphi\rangle\neg p$, which means that there is an announcement φ and that leaves us with $\neg p$ in $(\mathcal{M}|_{\varphi}, t)$ for the same instant t. Thus, from what we showed previously for atomic propositions, $\mathcal{M}, t \models \neg p$, but we also have at the same instant and model φ, which, by *modus ponens*, results in $\mathcal{M}, t \models p$.

Combining these results, we prove the biconditional.

2. $[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$

($\to$). We will do a proof by reduction to absurdity. Suppose that $[\varphi]\neg\psi$ and $\neg(\varphi \to \neg[\varphi]\psi)$ for a model $\mathcal{M}$ and any instant t. By classical logic, φ and $[\varphi]\psi$. By the definition of this last formula, we need to have ψ in $\mathcal{M}|_{\varphi}$ and in the same t, since the formula φ exists for the announcement. However, we also assume that $[\varphi]\neg\psi$, so we have $\neg\psi$ and ψ in $\mathcal{M}|_{\varphi}$.

($\leftarrow$). RAA. Let us assume $\varphi \to \neg[\varphi]\psi$ and $\neg[\varphi]\neg\psi$ for $\mathcal{M}$ and t. As we demonstrated in the previous theorem, the last formula is equivalent to $\langle\varphi\rangle\psi$, which means that φ, for $(\mathcal{M}, t)$, and ψ for $(\mathcal{M}|_{\varphi}, t)$, both in relation to the same instant. Thus, by modus ponens, $\neg[\varphi]\psi$, that is, $\langle\varphi\rangle\neg\psi$; by definition, $\neg\psi$ in $(\mathcal{M}|_{\varphi}, t)$, which marks a contradiction.

With the two conditionals, we arrive at the biconditional by classical logic.

3. $[\varphi](\psi \rightarrow \xi) \leftrightarrow ([\varphi]\psi \rightarrow [\varphi]\xi)$

   ($\rightarrow$). Reduction to absurdity. Assume that $[\varphi](\psi \rightarrow \xi)$ and $\neg([\varphi]\psi \rightarrow [\varphi]\xi)$ in $(\mathcal{M}, t)$. By $\neg \rightarrow$, $[\varphi]\psi$ and $\neg[\varphi]\xi$. We know that this last formula is equivalent to $\langle\varphi\rangle\neg\xi$, which means that φ and, for $(\mathcal{M}|_{\varphi}, t)$ in the same instant, $\neg\xi$. But since the announcement φ exists, then in t of $\mathcal{M}|_{\varphi}$ we also have $\psi \rightarrow \xi$. By modus tollens, $\neg\psi$. But the negation of the implication also entailed $[\varphi]\psi$, so ψ is the case in t of $\mathcal{M}|_{\varphi}$, which leaves us with a contradiction.

   ($\leftarrow$). Again, RAA. Assume that $[\varphi]\psi \rightarrow [\varphi]\xi$ and $\neg[\varphi](\psi \rightarrow \xi)$. By this negation, $\langle\varphi\rangle\neg(\psi \rightarrow \xi)$, which means that φ and $\not(\psi \rightarrow \xi)$ for an instant equal to t and an updated model $\mathcal{M}|_{\varphi}$. As we have a $\neg \rightarrow$, ψ and $\neg\xi)$ in $(\mathcal{M}|_{\varphi}, t)$. Since we have an announcement φ that leaves us ψ in t, then we know that $\langle\varphi\rangle\psi$ is the case in $(\mathcal{M}, t)$. How we prove the announcement functionality $(\langle\varphi\rangle\psi \rightarrow [\varphi]\psi)$, $[\varphi]\psi$. By modus ponens in the implication we assume, $[\varphi]\xi$, and as this announced formula exists, ξ in $(\mathcal{M}|_{\varphi}, t)$. But we verify $\neg\xi$ in this model and instant, so we have a contradiction.

   With these implications, we have the biconditional.

4. $[\varphi]H\psi \leftrightarrow (\varphi \rightarrow H[\varphi]\psi)$

   ($\rightarrow$). Another indirect proof. For $(\mathcal{M}, t)$, by hypothesis, $[\varphi]H\psi$ and $\neg(\varphi \rightarrow H[\varphi]\psi)$. By negating the implication: φ and $\neg H[\varphi]\psi$. We know that this last negation is equivalent to $P\neg[\varphi]\psi$, which means that there is a t_1 such that $t_1 \prec t$ and $\neg[\varphi]\psi$, which is equivalent to $\langle\varphi\rangle\neg\psi$, and means that there is an announcement φ and $\neg\psi$ follows in $\mathcal{M}|_{\varphi}$ and the same instant t_1. It turns out that we also assume $[\varphi]H\psi$, and the announcement and the updated model in t exist, so we have $H\psi$ in $\mathcal{M}|_{\varphi}, t$, therefore, in its predecessor t_1, we will have a contradiction, ψ and $\neg\psi$.

   ($\leftarrow$). RAA. Hypothetically, we have $\varphi \rightarrow H[\varphi]\psi$ and $\neg[\varphi]H\psi$ in $(\mathcal{M}, t)$. The equivalence of this last negation leaves us with $\langle\varphi\rangle\neg H\psi$. By definition, there is φ and updates the model $\mathcal{M}|_{\varphi}, t$, so that we have $\neg H\psi$, which is equivalent to $P\neg\psi$. Therefore, $t_1 \prec t$, and $\neg\psi$ in $\mathcal{M}|_{\varphi}, t_1$. But if the announcement exists in t, by modus ponens, $H[\varphi]\psi$. This results in $[\varphi]\psi$ in t_1, as this instant precedes t, and as the announcement exists at this instant also in the updated model, we have ψ and $\neg\psi$ in t_1 of $\mathcal{M}|_{\varphi}$.

Thus, we prove the biconditional.

5. $[\varphi]G\psi \leftrightarrow (\varphi \rightarrow G[\varphi]\psi)$

    ($\leftrightarrow$). This can be proven in the same way as in the demonstration numbered above, simply changing the direction of the precedence time relation, with $t \prec t_1$.

6. $[\varphi][\psi]\xi \leftrightarrow [(\varphi \wedge [\varphi]\psi)]\xi$

    For this proof, we need to show that an updated model of another updated model is equivalent to a conjunction of the updates announced in the first model, that is: $\mathcal{M}|_{\varphi|_\psi} = \mathcal{M}|_{\varphi\wedge[\varphi]\psi}$. As these models are fundamentally based on a set of instants $T^{!!}$ and $T^{!+!}$, respectively, we simply need to verify that, for any $t \in T$, $t \in T^{!!}$ if and only if $t \in T^{!+!}$.

    This can be demonstrated by reduction to absurdity. Suppose $\mathcal{M}|_{\varphi|_\psi} \neq \mathcal{M}|_{\varphi\wedge[\varphi]\psi}$. If so, then: either $T^{!!} \nsubseteq T^{!+!}$ or $T^{!+!} \nsubseteq T^{!!}$. In other words, there is some $t_!$ that is an element of one updated set of instants and not of the other.

    (case $T^{!!} \nsubseteq T^{!+!}$). Suppose $t_! \in T^{!!}$, but $t_! \notin T^{!+!}$. This last statement means that $t_! \notin \{t \in T : (\mathcal{M}, t) \models \varphi \Longrightarrow (\mathcal{M}|_\varphi, t) \models \psi\}$. However, from the first statement, we know that $t_! \in \{t \in T : (\mathcal{M}, t) \models \varphi \wedge [\varphi]\psi\}$. By the definition of the conjunction in our model, this is equivalent to saying that $t_! \in \{t \in T : (\mathcal{M}, t) \models \varphi\} \cap \{t \in T : (\mathcal{M}, t) \models [\varphi]\psi\}$. As $t_!$ is an element of the intersection, we know both that φ and $[\varphi]\psi$ in this model and instant. It turns out that the definition of this last formula designates precisely the set $\{t \in T : (\mathcal{M}, t) \models \varphi \Longrightarrow (\mathcal{M}|_\varphi, t) \models \psi\}$; from what we demonstrated for $t_!$, we have a contradiction.

    (case $T^{!+!} \nsubseteq T^{!!}$). Suppose, then, that $t_! \in T^{!+!}$, but $t_! \notin T^{!!}$. Conversely to the above reasoning, it follows that $t_! \in \{t \in T : (\mathcal{M}, t) \models \varphi \Longrightarrow (\mathcal{M}|_\varphi, t) \models \psi\}$, but $t_! \notin \{t \in T : (\mathcal{M}, t) \models \varphi\} \cap \{t \in T : (\mathcal{M}|_\varphi, t) \models [\varphi]\psi\}$. If $t_!$ does not belong to this intersection, then it does not belong to one of the two intersected sets, but in both cases it will contradict the previous statement.

    Therefore, $\mathcal{M}|_{\varphi|_\psi} = \mathcal{M}|_{\varphi\wedge[\varphi]\psi}$, which, by the truth conditions in our models, $[\varphi][\psi]\xi \leftrightarrow [(\varphi \wedge [\varphi]\psi)]\xi$.

7. $\psi \Longrightarrow [\varphi]\psi$

Since the necessitation rule does not preserve truth, but only validity, suppose that ψ is a valid formula, that is, $\mathcal{M}, t \models \psi$. In this model and any instant, given any formula φ, we have already proven that $\mathcal{M}, t \not\models \varphi \Longrightarrow \mathcal{M}, t \models [\varphi]\psi$. And if $\mathcal{M}, t \models \varphi$, then this can be put in announcement terms: $\mathcal{M}, t \models \langle\varphi\rangle\top$, which means that $\mathcal{M}|_{\varphi}, t \models \top$. Therefore, although the announcement may falsify ordinary formulas, it cannot introduce any formula that eliminates a formula with general validity in the system.

□

Remark 5 (RN)**.** *Item 7 of the theorem basically works as a modal logic necessitation rule (RN) analogous to the* RN_G *and* RN_H *in* $\mathbf{K}_t$*.*

Corollary 1. *Assume any formulas* φ, ψ, ξ*:*

1. *Announcement and conjunction:* $\models [\varphi](\psi \wedge \xi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\xi)$*;*
2. *Announcement and disjunction:* $\models [\varphi](\psi \vee \xi) \leftrightarrow ([\varphi]\psi \vee [\varphi]\xi)$*.*

Proof. By classical logic, we can translate the conjunction and disjunction operators in terms of negation and implication, therefore we can demonstrate both corollary items with the above theorem. □

These conversions are relevant because they can offer an easy way to prove the completeness of the system, since it is possible to find temporal static equivalences of any dynamic states in the system. As this is just a case of adaptation, a full presentation of completeness in the Historical Announcement Logic is not necessary. Just changing to temporal terminology, it works in exactly the same way as in Public Announcement Logic.[13][11][6]

$$[\varphi]Hp \Longleftrightarrow (\varphi \rightarrow H[\varphi]p) \Longleftrightarrow (\varphi \rightarrow H(\varphi \rightarrow p))$$

$$[\varphi]Gp \Longleftrightarrow (\varphi \rightarrow G[\varphi]p) \Longleftrightarrow (\varphi \rightarrow G(\varphi \rightarrow p))$$

Theorem 9 (completeness)**. HAL** *is complete with respect to the class of all* $\mathbf{K}_t$ *structures;* $\mathbf{HAL}_b$ *is complete with respect to the class of transitive (TRAN) and irreflexive (IRREFL) structures;* $\mathbf{HAL}_{bP}$ *and* $\mathbf{HAL}_{bF}$ *are complete with respect to the class of* *structures transitive (TRAN), irreflexive (IRREFL) and, respectively, with an end (END) or with a beginning (BEG); and* $\mathbf{HAL}_{bPP}/\mathbf{HAL}_{bFF}$ *is complete with respect to the class of transitive, irreflexive, with (END)/(BEG), and, respectively, with linear structures in histories (*LIN^{PP}*) or with linear structures in destinies (*LIN^{FF}*).*

Proof. As usual in completeness theorems, there are a series of steps to make it rigorous. To simplify this proof, we will just indicate a strategy for this demonstration, since completeness theorems for Public Announcement Logic (**PAL**) are already known, and Historical Announcement Logic can have analogous completeness theorems.[13][11][6]

Typically, completeness for $\mathbf{K}_t$ and its extensions can be demonstrated from canonical models and the Lindenbaum lemma.[12] A practical way to obtain completeness of **HAL** and its extensions is through a mapping function $f(m)$ that finds a static counterpart for any dynamic formula. This is possible because, as we demonstrated previously, these equivalences exist for all classical and temporal operators. In the Appendix we provide proof tableaux for all these formulas.

- $\mathbf{K}_t([\cdot]m) = \mathbf{HAL}$
- $\mathbf{K}_b([\cdot]m) = \mathbf{HAL}_b$
- $\mathbf{K}_{bF}/\mathbf{K}_{bP}([\cdot]m) = \mathbf{HAL}_{bF}/\mathbf{HAL}_{bP}$
- $\mathbf{K}_{bFF}/\mathbf{K}_{bPP}([\cdot]m) = \mathbf{HAL}_{bFF}/\mathbf{HAL}_{bPP}$

$$\begin{tikzcd}
\mathbf{K}_t([\cdot]m) \arrow[r] & \mathbf{K}_b([\cdot]m) \arrow[r] & \mathbf{K}_{bF}([\cdot]m) \arrow[r] & \mathbf{K}_{bFF}([\cdot]m) \\
\mathbf{K}_t(m) \arrow[r] \arrow[u] & \mathbf{K}_b(m) \arrow[r] \arrow[u] & \mathbf{K}_{bF}(m) \arrow[r] \arrow[u] & \mathbf{K}_{bFF}(m) \arrow[u]
\end{tikzcd}$$

$$\begin{tikzcd}
\mathbf{K}_t([\cdot]m) \arrow[r] & \mathbf{K}_b([\cdot]m) \arrow[r] & \mathbf{K}_{bP}([\cdot]m) \arrow[r] & \mathbf{K}_{bPP}([\cdot]m) \\
\mathbf{K}_t(m) \arrow[r] \arrow[u] & \mathbf{K}_b(m) \arrow[r] \arrow[u] & \mathbf{K}_{bP}(m) \arrow[r] \arrow[u] & \mathbf{K}_{bPP}(m) \arrow[u]
\end{tikzcd}$$

□

Finally, we will offer a tableaux method to prove these static equivalences.

4 HAL tableaux method

Classical propositional logic rules:

$$\begin{array}{c} \varphi \wedge \psi \\ \downarrow \\ \varphi \\ \psi \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\varphi \vee \psi} \\ / & \backslash \\ \varphi & \psi \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\varphi \to \psi} \\ / & \backslash \\ \neg\varphi & \psi \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\varphi \leftrightarrow \psi} \\ / & \backslash \\ \varphi \wedge \psi & \neg\varphi \wedge \neg\psi \end{array}$$

$$\begin{array}{c} \neg\neg\varphi \\ \downarrow \\ \varphi \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\varphi \wedge \psi)} \\ / & \backslash \\ \neg\varphi & \neg\psi \end{array} \qquad \begin{array}{c} \neg(\varphi \vee \psi) \\ \downarrow \\ \neg\varphi \\ \neg\psi \end{array} \qquad \begin{array}{c} \neg(\varphi \to \psi) \\ \downarrow \\ \varphi \\ \neg\psi \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{\neg(\varphi \leftrightarrow \psi)} \\ / & \backslash \\ \varphi \wedge \neg\psi & \neg\varphi \wedge \psi \end{array}$$

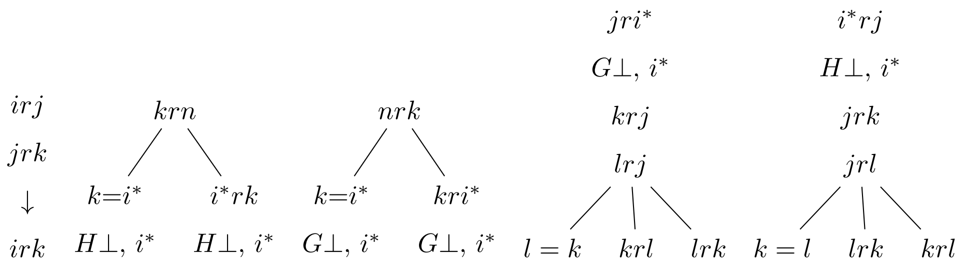
Temporal logic rules:

$$\begin{array}{cccc} G\varphi, i & F\varphi, i & H\varphi, i & P\varphi, i \\ irj & \downarrow & jri & \downarrow \\ \downarrow & irj & \downarrow & jri \\ \varphi, j & \varphi, j & \varphi, j & \varphi, j \end{array}$$

$$\begin{array}{cccc} \neg G\varphi, i & \neg F\varphi, i & \neg H\varphi, i & \neg P\varphi, i \\ \downarrow & \downarrow & \downarrow & \downarrow \\ F\neg\varphi, i & G\neg\varphi, i & P\neg\varphi, i & H\neg\varphi, i \end{array}$$

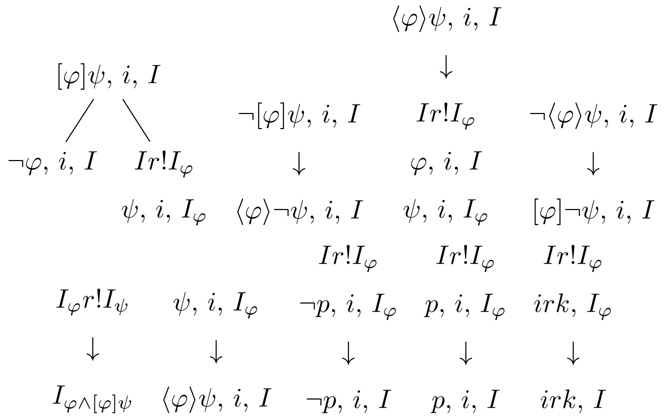
Remark 6 (r)**.** *Our temporal logic tableaux approach is based on Priest Graham trees[14]. r is a precedence relation in a proof tree. The irj or jri relations after the down arrow are new to the proof tree.*

Temporal logic extension rules (TRAN), (BEG), (END), (LIN^{PP}) and (LIN^{FF}):



Remark 7 (end point and starting point)**.** *The index i^* is the index that marks the beginning or end of a tree.*

Historical announcement rules:



Remark 8 (r!)**.** *The rules above consist of an adaptation of the tableaux method proposed by BALBIANI et. al.[2]. The r! relation adapt the temporal concept from the semantic notion of updating one model to another and preserving the validity of atomic formulas. According to these parameters, the symbol r! introduces a new kind of relation restricted to I structures.*

Remark 9 (PROP)**.** *p is an atomic proposition.*

Below we will provide proof trees for the end axiom, the pluperfect linearity axiom and the translation between past temporal modalities and historical announcements. There are analogous proofs for the beginning axiom, for the plufuture linearity axiom and for the translation between future temporal modalities and historical announcements.

$$G\bot \vee FG\bot \tag{1}$$

$$FG\bot \to (FP\varphi \to (P\varphi \vee \varphi \vee F\varphi)) \tag{2}$$

$$[\varphi]Hp \leftrightarrow (\varphi \to H[\varphi]p) \tag{3}$$

$\vdash_{\mathbf{K}_{bP}} G\bot \vee FG\bot$

1.	$\neg(G\bot \vee FG\bot)$, 1 ✓		RAA
2.	$\neg G\bot$, 1 ✓		1 $\neg\vee$
3.	$\neg FG\bot$, 1 ✓		1 $\neg\vee$
4.	$G\neg G\bot$, 1		3 $\neg F$
5.	$F\neg\bot$, 1 ✓		2 $\neg G$
6.	1r2		5 F
7.	$\neg\bot$, 2		5 F
8.	2=i*	2ri*	6 (END)
9.	$G\bot$, i*	$G\bot$, i*	6 (END)
10.	$\neg G\bot$, i* ✓	1ri*	$4, 6, 8$ G; $6, 8$ (TRAN)
11.	$F\neg\bot$, i* ✓	$\neg G\bot$, i* ✓	10 $\neg G$; $4, 10$ G
12.	i*r3	$F\neg\bot$, i* ✓	11 F; 11 $\neg G$
13.	$\neg\bot$, 3	i*r3	11 F; 12 F
14.	$\bot$, 3	$\neg\bot$, 3	9 G; 12 F
15.	$\underset{14, 13}{\otimes}$	$\bot$, 3	9 G
		$\underset{15, 14}{\otimes}$	

$\vdash_{\mathbf{K}^l_{bP}} FG\bot \to (FP\varphi \to (P\varphi \lor \varphi \lor F\varphi))$

<table>
<tr><td>1.</td><td colspan="3">$\neg(FG\bot \to (FP\varphi \to (P\varphi \lor \varphi \lor F\varphi)))$, 1 ✓</td><td>RAA</td></tr>
<tr><td>2.</td><td colspan="3">$FG\bot$, 1 ✓</td><td>$1 \neg \to$</td></tr>
<tr><td>3.</td><td colspan="3">$\neg(FP\varphi \to (P\varphi \lor \varphi \lor F\varphi))$, 1 ✓</td><td>$1 \neg \to$</td></tr>
<tr><td>4.</td><td colspan="3">$FP\varphi$, 1 ✓</td><td>$3 \neg \to$</td></tr>
<tr><td>5.</td><td colspan="3">$\neg(P\varphi \lor \varphi \lor F\varphi)$, 1 ✓</td><td>$3 \neg \to$</td></tr>
<tr><td>6.</td><td colspan="3">$\neg P\varphi$, 1 ✓</td><td>$5 \neg\lor$</td></tr>
<tr><td>7.</td><td colspan="3">$\neg\varphi$, 1</td><td>$5 \neg\lor$</td></tr>
<tr><td>8.</td><td colspan="3">$\neg F\varphi$, 1 ✓</td><td>$5 \neg\lor$</td></tr>
<tr><td>9.</td><td colspan="3">$H\neg\varphi$, 1</td><td>$6 \neg P$</td></tr>
<tr><td>10.</td><td colspan="3">$G\neg\varphi$, 1</td><td>$8 \neg F$</td></tr>
<tr><td>11.</td><td colspan="3">1r2</td><td>4 F</td></tr>
<tr><td>12.</td><td colspan="3">$P\varphi$, 2</td><td>4 F</td></tr>
<tr><td>13.</td><td colspan="3">kr2</td><td>12 P</td></tr>
<tr><td>14.</td><td colspan="3">φ, k</td><td>12 P</td></tr>
<tr><td>15.</td><td colspan="3">1ri*</td><td>2 F</td></tr>
<tr><td>16.</td><td colspan="3">$G\bot$, i*</td><td>2 F</td></tr>
<tr><td>17.</td><td>$1 = k$</td><td>1rk</td><td>kr1</td><td>$15, 16, 13, 11$ (LIN^{PP})</td></tr>
<tr><td>18.</td><td>$\otimes$
$14, 6$</td><td>$\neg\varphi$, k</td><td>$\neg\varphi$, k</td><td>$10, 17$ G; $9, 17$ H</td></tr>
<tr><td></td><td></td><td>$\otimes$
$18, 14$</td><td>$\otimes$
$18, 14$</td><td></td></tr>
</table>

$\vdash_{\mathbf{HAL}} [\varphi]Gp \to (\varphi \to G[\varphi]p)$

1.	$\neg([\varphi]Gp \to (\varphi \to G[\varphi]p))$, 1,I ✓		RAA
2.	$[\varphi]Gp$, 1,I ✓		$1 \neg \to$
3.	$\neg(\varphi \to G[\varphi]p)$, 1,I ✓		$1 \neg \to$
4.	φ, 1,I		$3 \neg \to$
5.	$\neg G[\varphi]p$, 1,I ✓		$3 \neg \to$
6.	$F\neg[\varphi]p$, 1,I ✓		$4 \neg G$
7.	1r2,I		$6\ F$
8.	$\neg[\varphi]p$, 2,I ✓		$6\ F$
9.	$\langle\varphi\rangle\neg p$, 2,I ✓		$8 \neg[\cdot]$
10.	φ, 2,I		$9\ \langle\cdot\rangle$
11.	Ir!I_{φ}		$9\ \langle\cdot\rangle$
12.	$\neg p$, 2,I_{φ}		$9\ \langle\cdot\rangle$
13.	$\neg\varphi$, 1,I	Gp, 1,I_{φ} ✓	$2\ [\cdot]$
14.	$\otimes$ $13, 4$	p, 2,I_{φ}	$13, 12\ G$
		$\otimes$ $14, 12$	

$\vdash_{\mathbf{HAL}} (\varphi \to G[\varphi]p) \to [\varphi]Gp$

1.	$\neg((\varphi \to G[\varphi]p) \to [\varphi]Gp)$, 1,I $\checkmark$			RAA
2.	$\varphi \to G[\varphi]p$, 1,I $\checkmark$			$1 \neg \to$
3.	$\neg[\varphi]Gp$, 1,I $\checkmark$			$1 \neg \to$
4.	$\langle\varphi\rangle\neg Gp$, 1,I $\checkmark$			$3 \neg[\cdot]$
5.	φ, 1,I			$4 \langle\cdot\rangle$
6.	Ir!I_φ			$4 \langle\cdot\rangle$
7.	$\neg Gp$, 1,I_φ			$4 \langle\cdot\rangle$
8.	$F\neg p$, 1,I_φ $\checkmark$			$7 \neg G$
9.	1r2,I_φ			$8\ F$
10.	$\neg p$, 2,I_φ			$9\ F$
11.	$\neg\varphi$, 1,I	$G[\varphi]p$, 1,I		$2 \to$
12.	$\otimes$ $11, 5$	1r2,I		9 r
13.		$[\varphi]p$, 2,I $\checkmark$		$11, 12\ G$
14.		$\neg\varphi$, 2,I	p, 2,I	$13\ [\cdot]$
15.		$\langle\varphi\rangle\neg p$, 2,I $\checkmark$	$\otimes$ $14, 10$	$10\ \mathrm{I}_\varphi$
16.		$\neg p$, 2,I_φ		$15, 6 \langle\cdot\rangle$
17.		φ, 2,I		$15 \langle\cdot\rangle$
		$\otimes$ $17, 14$		

5 Conclusion

In this paper we introduce a variation of the Public Announcement Logic (**PAL**) in temporal terms. This system of dynamic temporal logic allows us to represent history from an epistemologically defeasible point of view; different historical theories (or epistemic versions of the past) can be eliminated with historical announcements, which represent new factual information.

Historical Announcement Logic (**HAL**) can be extended in several ways in later logical studies, for example, with first-order logic and with Peircean and Ockhamist temporal branching logic[17]. In parallel, this system also allows several applications in Philosophy of Science (especially in Philosophy of Historical Sciences). This way we can compare the advantages of this model compared to others, such as Newton da Costa's Quasi-Truth Theory[1]. Further studies should detail the philosophical assumptions and show how models of the **HAL** system can be applied to represent descriptive and explanatory scientific knowledge.

References

[1] Arenhart, Jonas Rafael Becker; COSTA, Vítor Medeiros. (2021) Quasi-truth and incomplete information in historical sciences. *THEORIA. An International Journal for Theory, History and Foundations of Science*, 36 (1). pp. 113-137.

[2] Balbiani, P.; van Ditmarsch, H.; Herzig, A.; de Lima, T. (2007) A Tableau Method for Public Announcement Logics. *Automated Reasoning with Analytic Tableaux and Related Methods*, Lecture Notes in Computer Science, pp. 43–59.

[3] Belnap, N.; Perloff, M. ; Xu, M. (2001) *Facing the Future: Agents and Choices in Our Indeterminist World*, Oxford: Oxford University Press.

[4] Belnap, N.; Müller, T.; and Placek, T. (2022) *Branching Space-Times: Theory and Applications*, Oxford: Oxford University Press.

[5] van Benthem, Johan F. K. (1983) *The Logic of Time: A Model-Theoretic Investigation into the Varieties of Temporal Ontology and Temporal Discourse.* Dordrecht: D. Reidel Publishing Company.

[6] van Benthem, Johan. (2011) *Logical Dynamics of Information and Interaction.* Cambridge University Press.

[7] Brinkmann, Vinzenz; Dreyfus, Renee; KOCH-BRINKMANN, Ulrike. (2017) *Gods in Color: Polychromy in the Ancient World.* Prestel.

[8] Burgess Basic Tense Logic. Gabbay, D.; Guenthner. F. (eds.). (1984) *Handbook of Philosophical Logic. Volume II: Extensions of Classical Logic.* London: Kluwer Academic Publishers, pp. 89–133.

[9] Currie, Adrian. (2018) *Rock, Bone & Ruin: an optimist's guide to the historical sciences.* MIT University Press.

[10] Currie, Adrian. (2019) *Scientific Knowledge & the Deep Past: History Matters.* Cambridge University Press.

[11] van Ditmarsch, H.; van derHOEK, W.; KOOI, B. (2007) *Dynamic Epistemic Logic (Synthese Library).* [S.l.]: Springer.

[12] McArthur, Robert P. (1976) *Tense Logic.* Monograph. Dordrecht: D. Reidel Publishing Company.

[13] Pereira, Márcio. (2015) *Extensões de Primeira Ordem para a Lógica do Anúncio Público.* Doctoral thesis. Florianópolis: Universidade Federal de Santa Catarina.

[14] Priest, Graham. (2008) *An Introduction to Non-Classical Logic.* Cambridge: Cambridge University Press.

[15] Prior, Arthur N. (1967) *Past, Present and Future.* Oxford: Oxford University Press.

[16] Rescher, N.; Urquhart, A. (1971) *Temporal Logic.* Berlin: Springer.

[17] Reynolds, M. (2002) Axioms for Branching Time. *Journal of Logic and Computation*, 12(4): 679—697.

Appendix

Proof tableaux for the **HAL** theorems:

$\vdash_{\mathbf{HAL}} \langle\varphi\rangle\psi \to [\varphi]\psi$

1.	$\neg(\langle\varphi\rangle\psi \to [\varphi]\psi)$, 1,I ✓	RAA
2.	$\langle\varphi\rangle\psi$, 1,I ✓	$1 \neg \to$
3.	$\neg[\varphi]\psi$, 1,I ✓	$1 \neg \to$
4.	$\langle\varphi\rangle\neg\psi$, 1,I ✓	$3\ \neg[\cdot]$
5.	Ir!I_φ	$2\ \langle\cdot\rangle$
6.	φ, 1,I	$2\ \langle\cdot\rangle$
7.	ψ, 1,I_φ	$2\ \langle\cdot\rangle$
8.	$\neg\psi$, 1,I_φ	$4, 5, 6\ \langle\cdot\rangle$

$\underset{8,7}{\otimes}$

$\vdash_{\mathbf{HAL}} [p]p$

1.	$\neg[p]p$, 1,I ✓	RAA
2.	$\langle p\rangle\neg p$, 1,I ✓	$1\ \neg[\cdot]$
3.	Ir!I_φ	$2\ \langle\cdot\rangle$
4.	p, 1,I	$2\ \langle\cdot\rangle$
5.	$\neg p$, 1,I_φ	$2\ \langle\cdot\rangle$
6.	$\neg p$, 1,I	5 PROP

$\underset{6,4}{\otimes}$

$$\vdash_{\mathbf{HAL}} [\varphi]p \leftrightarrow (\varphi \to p)$$

$\vdash_{\mathbf{HAL}} [\varphi]p \to (\varphi \to p)$

1.	$\neg([\varphi]p \to (\varphi \to p))$, 1,I ✓		RAA
2.	$[\varphi]p$, 1,I ✓		1 $\neg \to$
3.	$\neg(\varphi \to p)$, 1,I ✓		1 $\neg \to$
4.	φ, 1,I		3 $\neg \to$
5.	$\neg p$, 1,I$_\varphi$		3 $\neg \to$
6.	$\neg\varphi$, 1,I	1,Ir!I$_p$	2 $[\cdot]$
7.	$\underset{6,4}{\otimes}$	p, 1,I$_p$	2 $[\cdot]$
8.		p, 1,I	7 PROP
		$\underset{8,5}{\otimes}$	

$\vdash_{\mathbf{HAL}} (\varphi \to p) \to [\varphi]p$

1.	$\neg((\varphi \to p) \to [\varphi]p)$, 1,I ✓		RAA
2.	$(\varphi \to p)$, 1,I ✓		1 $\neg \to$
3.	$\neg[\varphi]p$, 1,I ✓		1 $\neg \to$
4.	$\langle\varphi\rangle\neg p$, 1,I ✓		3 $\neg[\cdot]$
5.	φ, 1,I		4 $\langle\cdot\rangle$
6.	Ir!I$_\varphi$		4 $\langle\cdot\rangle$
7.	$\neg p$, 1,I$_\varphi$		4 $\langle\cdot\rangle$
8.	$\neg\varphi$, 1,I	p, 1,I	2 $\to$
9.	$\underset{8,5}{\otimes}$	$\neg p$, 1,I	7 PROP
		$\underset{9,8}{\otimes}$	

$\vdash_{\mathbf{HAL}} [\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$

$\vdash_{\mathbf{HAL}} [\varphi]\neg\psi \rightarrow (\varphi \rightarrow \neg[\varphi]\psi)$

1.	$\neg([\varphi]\neg\psi \rightarrow (\varphi \rightarrow \neg[\varphi]\psi))$, 1,I ✓		RAA
2.	$[\varphi]\neg\psi$, 1,I ✓		$1 \neg\rightarrow$
3.	$\neg(\varphi \rightarrow \neg[\varphi]\psi)$, 1,I ✓		$1 \neg\rightarrow$
4.	φ, 1,I		$3 \neg\rightarrow$
5.	$\neg\neg[\varphi]\psi$, 1,I ✓		$3 \neg\rightarrow$
6.	$[\varphi]\psi$, 1,I ✓		$5 \neg\neg$
7.	$\neg\varphi$, 1,I	Ir!I$_\varphi$	$6\ [\cdot]$
8.	$\underset{7,4}{\otimes}$	ψ, 1,I$_\varphi$	$6\ [\cdot]$
9.		$\neg\psi$, 1,I	$2,7\ [\cdot]$
		$\underset{9,8}{\otimes}$	

$\vdash_{\mathbf{HAL}} (\varphi \rightarrow \neg[\varphi]\psi) \rightarrow [\varphi]\neg\psi$

1.	$\neg((\varphi \rightarrow \neg[\varphi]\psi) \rightarrow [\varphi]\neg\psi)$, 1,I ✓		RAA
2.	$\varphi \rightarrow \neg[\varphi]\psi$, 1,I ✓		$1 \neg\rightarrow$
3.	$\neg[\varphi]\neg\psi$, 1,I ✓		$1 \neg\rightarrow$
4.	$\langle\varphi\rangle\neg\neg\psi$, 1,I ✓		$3 \neg[\cdot]$
5.	φ, 1,I		$4\ \langle\cdot\rangle$
6.	Ir!I$_\varphi$		$5\ \langle\cdot\rangle$
7.	$\neg\neg\psi$, 1,I$_\varphi$ ✓		$6\ \langle\cdot\rangle$
8.	ψ, 1,I$_\varphi$		$7 \neg\neg$
9.	$\neg\varphi$, 1,I	$\neg[\varphi]\psi$, 1,I ✓	$2 \rightarrow$
10.	$\underset{9,5}{\otimes}$	$\langle\varphi\rangle\neg\psi$, 1,I ✓	$9 \neg[\cdot]$
11.		$\neg\psi$, 1,I$_\varphi$	$10,5,6\ \langle\cdot\rangle$
		$\underset{11,8}{\otimes}$	

$\vdash_{\mathbf{HAL}} [\varphi](\psi \rightarrow \xi) \leftrightarrow ([\varphi]\psi \rightarrow [\varphi]\xi)$

$\vdash_{\mathbf{HAL}} [\varphi](\psi \to \xi) \to ([\varphi]\psi \to [\varphi]\xi)$

1.	$\neg([\varphi](\psi \to \xi) \to ([\varphi]\psi \to [\varphi]\xi))$, 1,I ✓	RAA
2.	$[\varphi](\psi \to \xi)$, 1,I ✓	$1 \neg \to$
3.	$\neg([\varphi]\psi \to [\varphi]\xi)$, 1,I ✓	$1 \neg \to$
4.	$[\varphi]\psi$, 1,I ✓	$3 \neg \to$
5.	$\neg[\varphi]\xi$, 1,I ✓	$3 \neg \to$
6.	$\langle\varphi\rangle\neg\xi$, 1,I ✓	$5 \neg[\cdot]$
7.	φ, 1,I	$6 \langle\cdot\rangle$
8.	Ir!I_φ	$6 \langle\cdot\rangle$
9.	$\neg\xi$, 1,I_φ	$6 \langle\cdot\rangle$
10.	$\neg\varphi$, 1,I ψ, 1,I_φ	$4\ [\cdot];\ 4, 8\ [\cdot]$
11.	$\otimes$ $10, 7$ $\psi \to \xi$, 1,I_φ ✓	$2, 10, 8\ [\cdot]$
12.	$\neg\psi$, 1,I_φ ξ, 1,I_φ	$11 \to$
	$\otimes$ $12, 10$ $\otimes$ $12, 9$	

$\vdash_{\mathbf{HAL}} ([\varphi]\psi \rightarrow [\varphi]\xi) \rightarrow [\varphi](\psi \rightarrow \xi)$

1.	$\neg(([\varphi]\psi \rightarrow [\varphi]\xi) \rightarrow [\varphi](\psi \rightarrow \xi))$, 1,I ✓			RAA
2.	$[\varphi]\psi \rightarrow [\varphi]\xi$, 1,I ✓			$1 \neg \rightarrow$
3.	$\neg[\varphi](\psi \rightarrow \xi)$, 1,I ✓			$1 \neg \rightarrow$
4.	$\langle\varphi\rangle\neg(\psi \rightarrow \xi)$, 1,I ✓			$3 \neg[\cdot]$
5.	φ, 1,I			$4 \langle\cdot\rangle$
6.	Ir!I_φ			$4 \langle\cdot\rangle$
7.	$\neg(\psi \rightarrow \xi)$, 1,$\mathrm{I}_\varphi$ ✓			$4 \langle\cdot\rangle$
8.	ψ, 1,I_φ			$7 \neg \rightarrow$
9.	$\neg\xi$, 1,I_φ			$7 \neg \rightarrow$
10.	$\neg[\varphi]\psi$, 1,I ✓	$[\varphi]\xi$, 1,I ✓		$2 \rightarrow$
11.		$\neg\varphi$, 1,I	ξ, 1,I_φ	$10\ [\cdot];\ 10, 6\ [\cdot]$
12.	$\langle\varphi\rangle\neg\psi$, 1,I ✓	$\otimes$ $11, 5$	$\otimes$ $11, 9$	$10 \neg[\cdot]$
13.	$\neg\psi$, 1,I_φ			$12, 5, 6 \langle\cdot\rangle$
	$\otimes$ $13, 8$			

$\vdash_{\mathbf{HAL}} [\varphi][\psi]\xi \leftrightarrow [(\varphi \wedge [\varphi]\psi)]\xi$

$$\vdash_{\mathbf{HAL}} [\varphi][\psi]\xi \rightarrow [(\varphi \wedge [\varphi]\psi)]\xi$$

1.	$\neg([\varphi][\psi]\xi \rightarrow [(\varphi \wedge [\varphi]\psi)]\xi)$, 1,I ✓	RAA
2.	$[\varphi][\psi]\xi$, 1,I ✓	$1 \neg \rightarrow$
3.	$\neg[(\varphi \wedge [\varphi]\psi)]\xi$, 1,I ✓	$1 \neg \rightarrow$
4.	$\langle(\varphi \wedge [\varphi]\psi)\rangle\neg\xi$, 1,I ✓	$3 \neg[\cdot]$
5.	$\varphi \wedge [\varphi]\psi$, 1,I ✓	$4 \langle\cdot\rangle$
6.	$\mathrm{Ir!I}_{\varphi\wedge[\varphi]\psi}$	$4 \langle\cdot\rangle$
7.	$\neg\xi$, 1,$\mathrm{I}_{\varphi\wedge[\varphi]\psi}$	$4 \langle\cdot\rangle$
8.	φ, 1,I	$5 \wedge$
9.	$[\varphi]\psi$, 1,I ✓	$5 \wedge$
10.	$\neg\varphi$, 1,I $\qquad$ $\mathrm{Ir!I}_\varphi$	$9 [\cdot]$
11.	$\underset{10,8}{\otimes}$ $\qquad$ ψ, 1,I_φ	$9 [\cdot]$
12.	$[\psi]\xi$, 1,I_φ ✓	$2, 10, 10 [\cdot]$
13.	$\neg\psi$, 1,I_φ $\qquad$ $\mathrm{I}_\varphi\mathrm{r!I}_\psi$	$12 [\cdot]$
14.	$\underset{13,11}{\otimes}$ $\qquad$ $\mathrm{I}_{\varphi\wedge[\varphi]\psi}$	13 r!
15.	ξ, 1,$\mathrm{I}_{\varphi\wedge[\varphi]\psi}$	$12, 14 [\cdot]$
	$\underset{15,7}{\otimes}$	

$\vdash_{\textbf{HAL}} [(\varphi \wedge [\varphi]\psi)]\xi \rightarrow [\varphi][\psi]\xi$

1.		$\neg([(\varphi \wedge [\varphi]\psi)]\xi \rightarrow [\varphi][\psi]\xi)$, 1,I ✓		RAA
2.		$[(\varphi \wedge [\varphi]\psi)]\xi$, 1,I ✓		$1 \neg \rightarrow$
3.		$\neg[\varphi][\psi]\xi$, 1,I ✓		$1 \neg \rightarrow$
4.		$\langle\varphi\rangle\neg[\psi]\xi$, 1,I ✓		$3 \neg[\cdot]$
5.		φ, 1,I ✓		$4 \langle\cdot\rangle$
6.		$\mathrm{Ir!I}_{\varphi}$		$4 \langle\cdot\rangle$
7.		$\neg[\psi]\xi$, 1,I_{φ} ✓		$4 \langle\cdot\rangle$
8.		$\langle\psi\rangle\neg\xi$, 1,$\mathrm{I}_{\varphi}$ ✓		$7 \neg[\psi]$
9.		ψ, 1,I_{φ}		$8 \langle\cdot\rangle$
10.		$\mathrm{I}_{\varphi}\mathrm{r!I}_{\psi}$		$8 \langle\cdot\rangle$
11.		$\mathrm{I}_{\varphi \wedge [\varphi]\psi}$		10 r!
12.		$\neg\xi$, 1,$\mathrm{I}_{\varphi \wedge [\varphi]\psi}$		$8, 11 \langle\cdot\rangle$
13.	$\neg(\varphi \wedge [\varphi]\psi)$, 1,I ✓		ξ, 1,$\mathrm{I}_{\varphi \wedge [\varphi]\psi}$	$2 [\cdot]$
14.	$\neg\varphi$, 1,I	$\neg[\varphi]\psi$, 1,I ✓	$\otimes$ 13, 12	$13 \neg\wedge$
15.	$\otimes$ 14, 5	$\langle\varphi\rangle\neg\psi$, 1,I ✓		$14 \neg[\cdot]$
16.		$\neg\psi$, 1,I_{φ}		$15, 5, 6 \langle\cdot\rangle$
		$\otimes$ 16, 9		

$$\vdash_{\textbf{HAL}} [\varphi]Hp \leftrightarrow (\varphi \rightarrow H[\varphi]p)$$

$\vdash_{\mathbf{HAL}} [\varphi]Hp \to (\varphi \to H[\varphi]p)$

1.	$\neg([\varphi]Hp \to (\varphi \to H[\varphi]p))$, 1,I ✓		RAA
2.	$[\varphi]Hp$, 1,I ✓		1 $\neg \to$
3.	$\neg(\varphi \to H[\varphi]p)$, 1,I ✓		1 $\neg \to$
4.	φ, 1,I		3 $\neg \to$
5.	$\neg H[\varphi]p$, 1,I ✓		3 $\neg \to$
6.	$P\neg[\varphi]p$, 1,I ✓		4 $\neg H$
7.	0r1,I		6 P
8.	$\neg[\varphi]p$, 0,I ✓		6 P
9.	$\langle\varphi\rangle\neg p$, 0,I ✓		8 $\neg[\cdot]$
10.	φ, 0,I		9 $\langle\cdot\rangle$
11.	Ir!I$_\varphi$		9 $\langle\cdot\rangle$
12.	$\neg p$, 0,I$_\varphi$		9 $\langle\cdot\rangle$
13.	$\neg\varphi$, 1,I	Hp, 1,I$_\varphi$ ✓	2 $[\cdot]$
14.	$\otimes$ $13, 4$	p, 0,I$_\varphi$	$13, 12$ H
		$\otimes$ $14, 12$	

$\vdash_{\mathbf{HAL}} (\varphi \to H[\varphi]p) \to [\varphi]Hp$

1.	$\neg((\varphi \to H[\varphi]p) \to [\varphi]Hp)$, 1,I ✓			RAA
2.	$\varphi \to H[\varphi]p$, 1,I ✓			1 $\neg \to$
3.	$\neg[\varphi]Hp$, 1,I ✓			1 $\neg \to$
4.	$\langle\varphi\rangle\neg Hp$, 1,I ✓			3 $\neg[\cdot]$
5.	φ, 1,I			4 $\langle\cdot\rangle$
6.	Ir!I$_\varphi$			4 $\langle\cdot\rangle$
7.	$\neg Hp$, 1,I$_\varphi$			4 $\langle\cdot\rangle$
8.	$P\neg p$, 1,I$_\varphi$ ✓			7 $\neg H$
9.	0r1,I$_\varphi$			8 P
10.	$\neg p$, 0,I$_\varphi$			9 P
11.	$\neg\varphi$, 1,I	$H[\varphi]p$, 1,I		2 $\to$
12.	$\otimes$ $11, 5$	0r1,I		9 r
13.		$[\varphi]p$, 0,I ✓		$11, 12$ H
14.		$\neg\varphi$, 0,I	p, 0,I	13 $[\cdot]$
15.		$\langle\varphi\rangle\neg p$, 0,I ✓	$\otimes$ $14, 10$	10 I$_\varphi$
16.		$\neg p$, 0,I$_\varphi$		$15, 6$ $\langle\cdot\rangle$
17.		φ, 0,I		15 $\langle\cdot\rangle$
		$\otimes$ $17, 14$		