Can Measurement be Entirely Quantum?

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Abstract

I will look at Bohr's contentious doctrine of classical concepts - the claim that measurement requires classical concepts to be understood - and argue that measurement theory supports a similar conclusion. I will argue that representing a property in terms of a metric scale, which marks a shift from the empirical process of measurement to the informational output, introduces the inherently classical assumption of definite states and precise values, thus fulfilling Bohr's doctrine. I examine how realism about metric scales implies that Bohr's doctrine is ontological, while more moderate coherentist or model-based approaches to realism render it epistemological. Regardless of one's stance towards measurement realism, however, measurement cannot be entirely quantum and quantum mechanics can model only the empirical side of measurement, not its informational output. Finally, I discuss how this might influence our understanding of the measurement problem.

Keywords: Bohr; classical concepts; measurement; metrology; uncertainty; calibration

1 Introduction

One of the central ideas of Bohr's philosophy of physics is that measurements cannot be modelled entirely in quantum mechanical terms; instead we must treat some aspects of the measurement set-up as classical. This has come to be called the doctrine of classical concepts. Bohr's ideas are hard to pin down, and this doctrine is now frequently disregarded despite its influence on early interpretations of quantum mechanics.

In this paper I will show that Bohr's doctrine *should* still be taken seriously, even if it is only as an epistemological concern, by comparing it to implicit assumptions about precise values and definite states that are made in measurement theory. Measurement theory has a long history and there are various competing views on what exactly a measurement is, but a dominant and long standing core of many of these views is that measurement is, at its heart, the representation and quantification of a property via a metric scale. This is what allows us to give numerical outcomes as measurement results and converts our

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experience of the empirical world into usable information. I will argue that representing a property of the world in this way introduces the assumption that the property has a definite, precise value; an assumption that encapsulates Bohr's doctrine.

This paper is not intended to be a historical analysis of Bohr's ideas. I will take Bohr's doctrine of classical concepts as a starting point and draw out ways in which the philosophy of measurement raises the same issue, even if it does not match Bohr's account to the letter. Since Bohr's work is notoriously vague, I will largely follow Howard's (1994) reconstruction of Bohr's doctrine of classical concepts (which he notes goes beyond Bohr in many places)[2], as well as Zinkernagel (2015), and draw from Saunders (2005) and Camilleri and Schlosshauer (2015) for wider aspects of Bohr's thought. Howard's reconstruction is limited and does not cover all aspects of Bohr's thought; I use it here as it effectively captures the comparison to measurement theory I am making, but I do not defend it as the best, or only, interpretation of Bohr's work.

The comparison between Bohr's doctrine and measurement theory translates Bohr's ideas into a new context, pinning them down more precisely in a way that relates to our current understanding of measurement. There are two main implications: First, Bohr's doctrine can be seen either as an epistemological point or an ontological one depending on what stance one takes towards measurement realism. A strong version of realism about numerical measurement outcomes creates a conflict between the classical assumptions embodied by metric scales and the quantum description of the property. Model-based or coherentist approaches, however, foreground the pragmatic and conventional elements of measurement – especially when it comes to calibration and uncertainty – and use this to posit a more moderate form of realism. The tensions between quantum mechanics and strong realism may be reason to prefer the model-based approaches, where the use of metric scales and the assumption of precise values on those scales can be viewed as a pragmatic tool to make sense of measurement (which is a process with the operational goal of producing numerical outcomes). This enables the epistemological reading of Bohr.

Second, we can give a definitive answer to the title question – can measurement be entirely quantum? By upholding Bohr's doctrine I answer the question in the negative: measurement requires us to make certain classical assumptions and cannot be modelled entirely within quantum mechanics. This does not mean that the *world* cannot be modelled entirely within quantum mechanics. Measurement theory distinguishes between the empirical side of measurement, modelling the interactions of physical systems, and the informational side where we apply metric scales and measurement units (see Section 3).

[2] Howard gives a reconstruction of Bohr that he takes to be a coherent way of making Bohr's ideas more precise, but notes that he does not himself agree with Bohr's claims.

It is on the latter side that we must go beyond a quantum model of the world, even if it is just on a pragmatic basis (as per the epistemological reading of Bohr's doctrine). This side of measurement is essential if we are to produce a numerical measurement outcome. While a measurement device *can* be treated the same as any other type of system, if we are to use it to perform a measurement we must make use of metric scales in addition to an ordinary quantum state description.

As is inevitable with any discussion of measurement in quantum mechanics, the measurement problem looms in the background. Although this issue extends beyond the scope of what can be considered in the space available here, I will offer some suggestions on how the comparison between Bohr's doctrine and measurement theory could, while not resolving the problem, offer insights into it.

I will start, in section 2, by laying out Bohr's doctrine of classical concepts, focusing on Howard's (1994) reconstruction of it. Then, in section 3, I will lay out the basic elements of measurement theory and look at how it relies on representing properties with a metric scale. In section 4, I will argue that a realist view of the metric scales used in measurement introduces the assumption of precise values and draw the comparison between this and Bohr's doctrine of classical concepts. I will also argue that the treatment of uncertainties during the calibration of measurement devices countermands arguments for realism and pushes us towards an epistemological interpretation of Bohr's doctrine, but also demonstrates its ineliminability. Finally, in section 5, I will answer the title question and look at implications for the measurement problem.

2 Bohr's Doctrine of Classical Concepts

Many different interpretations have been given for what exactly Bohr means when he claims that classical concepts are needed to understand quantum mechanics. This is also mixed up with *complementarity*, although the two ideas can be separated. In Bohr's own words the doctrine of classical concepts is this:

> "[H]owever far the phenomena transcend the scope of the classical physical explanation, the account of evidence must be expressed in classical terms. The argument is simply that by the word "experiment" we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics." (Bohr 1949, p. 209)

Roughly speaking what it means for a concept to be classical is not that is is part of the formal theory of classical mechanics, but just that it describes the world in the way we intuitively think of in the classical domain. Concerns could be raised about whether such a classical terminology exists and there is plenty of entirely reasonable doubt as to whether it could ever be said to be unambiguous. This confusion is often put aside in favour of focusing on one specific classical assumption: that objects possess definite states. What is meant by a definite state is a state that can be specified independently of any other systems (separability) and has a definite value of a given property such as position and momentum, as is familiar in the classical domain. This, while not impossible within quantum mechanics, is at odds with the ubiquitous existence of superposition states.[3]

In some cases, Bohr has been taken to posit an ontological distinction between a classical macroscopic objects and microscopic quantum ones. However, it is clear from the above quote that Bohr's interest in the use of classical concepts is focused on experimental evidence and how it is communicated, rather than on the ontology of the world, so we can reject this ontological framing. There is a growing consensus, along these lines, that his argument is epistemological rather than ontological (Howard 1994; Saunders 2005; Camilleri & Schlosshauer 2015). The epistemological reading implies that every system is in principle describable by quantum mechanics, but for the sake of pragmatically describing a measurement we have to give certain systems a classical description as a definite state. Section 4.3 will argue in support of the epistemological reading.

Many instead take Bohr's doctrine as dividing between a quantum system and a classical apparatus, commonly read as an epistemological divide. While more plausible – especially since Bohr emphasizes the classical description of the apparatus – I follow Howard's (1994) claim that a classical description must also apply to the target property of the quantum system. I take Bohr to imply this when insisting that both the account of the apparatus and the measurement *results* must be expressed classically.

Howard argues that a classical description of the target property is one that treats the system as if it is in a definite, but unknown, state, represented by a *statistical mixture* (in contrast to the pure states and improper mixtures used in quantum mechanics). More specifically, we should use a statistical mixture of all the eigenstates of the observable representing that property. These eigenstates represent the possible definite states in which the system could be found upon measurement. This goes beyond the actual text of what Bohr says and tries to present the doctrine in more formal terms. Howard chooses to

[3]The issue here is with superpositions in the values of *single* properties. As I will explain below, Bohr's classical descriptions apply to a single property at a time. I will take having simultaneously definite values for *different* properties to be a separate issue that relates to Bohr's ideas on complementarity.

maintain the use of quantum terminology such as eigenstates despite Bohr's emphasis on using the terminology of classical physics. Instead, what is classical about the description is that it is a statistical mixture and we act *as if* the system is in a definite eigenstate, but we don't know which. Eigenstates are apt to represent definite states as a measurement of an observable on an eigenstate of that observable produces a single definite outcome (the associated eigenvalue) for the value of that observable property.

Crucially, *only the property we are investigating* should be represented in this way and not the system as a whole; assigning this classical description to the entire system would give incorrect predictions of its behaviour in subsequent measurements of other properties. This classical description is localised to a specific measurement scenario (this relates to Bohr's ideas about complementarity, which is the idea that the description of the system must depend on the measurement context - see Howard (1994) for a discussion of how this relates to the doctrine of classical concepts). Furthermore, we treat this as a one-off, instantaneous description suitable for this measurement alone; we do not evolve the statistical mixture with the Schrödinger equation.

One thing that Howard's reconstruction does not make clear is what the function of such a description is when we know that, absent a hidden variable interpretation, the quantum state *does not* have a definite value prior to the measurement actually taking place. If we know that the description as a statistical mixture is strictly false, then what exactly is its use? The fact that we should not evolve the description under quantum dynamics also means it cannot be used to model the interaction between system and apparatus, which would be done by evolving the state with an interaction Hamiltonian. The comparison with measurement theory in this paper will show why this description is useful for measurement by arguing that it plays a similar role to the metric scales used to represent properties.

For the classical description of the apparatus we can do better than a mixture; in fact, one of the notable features of measurement (at least in the Bohrian tradition) is that we always assume that we have exact knowledge of the state of the measurement device. For example, we assume the position of a pointer is either here or there and that we can determine this simply by glancing at it. This can even be found in textbook presentations of quantum mechanics: "The classical nature of the apparatus appears in the fact that, at any given instant, we can say with certainty that it is in one of the known states, ϕ_n, with some definite value of the quantity g" (Landau & Liftshitz 2013, pg. 21)[4]. Zinkernagel (2015) draws something similar from Bohr's work, calling it the *reference system* argument: To

[4]Landau and Liftshitz's work follows in the footsteps of Bohr, as Landau was Bohr's student (see discussion in Bell (1990)).

measure a property of the quantum system we must always do so relative to a well-defined, fixed reference system, otherwise the result would be meaningless. Part of the reference system argument is that we need this fixed state to define many of the essential elements that are used in the background of measurement, whether this be the scales and clocks the measuring device consists of or definitions of the properties being measured.[5] Thus, for the apparatus, we describe the relevant property (or properties) of the device with a single definite state. Sections 3 and 4 will discuss how we define scientific units in measurement theory, which is directly related to the reference frame argument and constitutes a natural extension of Bohr's ideas.

To summarise, there are two aspects to Bohr's doctrine of classical concepts: 1) treat the target property as classical by representing it with a statistical mixture of definite states, and 2) treat the measurement apparatus as being in a single definite state.

3 Measurement Theory

During the 20$^{\text{th}}$ century, largely separate from quantum mechanics, a general analysis of measurement was taking place.[6] I will lay out a brief history of this, focusing on two views from measurement theory – first the Representational Theory of Measurement and then a more recent model-based approach. These views will form the basis for a comparison with Bohr's doctrine and their respective commitments to realism about measurement will determine how we interpret the doctrine.

Measurement theory dates back to Helmholtz in 1887 but comes into sharp focus with Stevens and Suppes in the 1940s/1950s onwards (see Díez (1997a, b) for a history). The central output of this project was a formal analysis of how our qualitative experience of physical properties could be quantified by representing the property with a metric scale. The basic idea was to start with magnitudes that can be qualitatively compared through certain physical operations (e.g. lengths can be arranged from greater to lesser or placed end-to-end to combine their lengths).[7] From these physical operations, we establish a set of axioms that specify the relationships between magnitudes (e.g. ordering, additivity).

[5]The latter relates to Bohr's ideas of complementarity. He particularly discusses how a fixed reference system is necessary for spatio-temporal coordination and the definitions of energy and momentum (see Saunders 2005). Here I take a broader interpretation of the argument.

[6]An interesting historical question is what influence the development of measurement theory had on ideas about measurement in quantum mechanics, and vice versa. I do not attempt to address this question of historical lineage.

[7]Here I gloss over the differences between (for example) magnitudes, attributes, quantities and objects that formed a large part of early debates about measurement. These debates culminated in the representational view, which I take to incorporate this history (see Michell 1997).

Finally, we find a metric scale that satisfies these axioms (and find a set of numbers, e.g. the positive reals, that reflect this scale). Once we have this scale, we can use it to quantify a specific magnitude (e.g. we determine that some specific object is $5m$ in length – I will say more about units later on). Different types of properties have different relations, which are not always straightforwardly captured by numerical representation, and therefore are characterised by different scales. For example, it makes sense to say that a $10m$ rod is twice the length of a $5m$ one – mirroring the numerical relation that 10 is twice 5 – but it isn't really meaningful to say that 10°C is twice as hot as 5°C.[8]

This developed into the Representational Theory of Measurement (RTM), grounded primarily in the work of Suppes (e.g. Suppes (1951) – see Díez (1997a, b) and Tal (2020) for overviews), who combined work on the classification of different types of scales with work on the empirical operations that determine suitable axioms. The transition between *empirical* operations, which deal with physical objects and qualitative comparisons, and the *numerical* representation of these properties is the core element of what a measurement is in the RTM. This transition will be essential to understanding where Bohr's doctrine comes into play.

However, the RTM has been criticized as being too simplistic to be a full account of measurement. Mari, Wilson and Maul (2023) see it as at most a study of *measurability* that is useful for characterising the necessary features that properties must have to be the subject of measurement and for distinguishing between different types of properties. They argue that a full account of measurement must be focused on the actual method of attaining data with a practical procedure that specifies the design and operation of the measurement apparatus, the methods for controlling error, and the management of uncertainties (given Bohr's emphasis on the apparatus this is useful for the comparison I will be making); representing a property by a metric scale is just one part of this. Mari, Wilson and Maul are a prominent example of a model-based, coherentist approach to measurement, wherein measurement is dependent on a coherent network of theories and models that deal with the different aspects of measurement and calibration. Tal (2017) is another prominent example of the coherentist approach, with many similarities to Mari, Wilson and Maul. I will primarily use Mari, Wilson and Maul's model throughout this paper as it is the most thorough and detailed account, but will also discuss arguments from Tal where relevant.

In their model, Mari, Wilson and Maul break measurement down into five main stages:

[8] This is because temperature (in Celsius or Fahrenheit) is an interval scale while length is a ratio scale, and the zero point on the Celsius scale is arbitrary (see Stevens 1946).

1. Coupling: the target property is coupled to a property of the measurement device. E.g. the temperature of an object is coupled to the volume of the mercury in a thermometer.
2. Matching: the property of the measurement device is matched to a chosen reference property. E.g. the top line of mercury in a thermometer is compared to lines etched on the glass. (Length (spatial position) is the most common reference property used in measurement).
3. Local Scale Application: A numerical output can be read off from the reference property by applying a metric scale to it. E.g. We read off a value such as '30' (± some uncertainty) from the thermometer scale. This is also mapped back to the target property and converted into a local value for this property (this will depend on understanding the coupling procedure).
4. Creating a Public Scale: A standardized, shared public scale is independently agreed upon. (E.g. The Celsius scale of temperature is defined based on the freezing and boiling points of water under certain conditions of pressure etc).
5. Calibration: The local scale is calibrated with the public scale, ensuring that measurements taken on the local device correspond to agreed-upon standards. (E.g. We understand a reading of '30' on the local thermometer scale to convey the information that the target system has a temperature of 30°C ± some uncertainty)

Whilst this model foregrounds the practical procedure of measurement and calibration, it shares the same essential feature as the RTM: the transition between the qualitative property in the world and the numerical representation of it. Mari, Wilson and Maul replace basic *numerical representation* with the idea of *information* to stress the context dependence of the numerical result of measurement and how, for the numerical result to actually be an informative measurement outcome, it must be situated in terms of publicly shared and calibrated reference scales (this is the difference between the local numerical value of '30' in step 3 and the informative outcome '30°C' in step 5).[9] The transition from empirical to informational starts to take place in step (3) of the above process, when a local scale is applied to the reference property. This is the first step in the process where we get a numerical output that quantifies a property of the empirical world. We

[9]This use of 'information' is not connected to information theory or concepts such as Shannon information. Mari, Wilson and Maul do not give a precise definition of the difference between empirical and informational but elucidate the distinction by saying that if we think of a verbal utterance, the empirical side is the wavelength of the sound wave, its duration and other physical facts, while the informational side is the number of words and letters in the message, the meaning of the words, what language it is spoken in and other facts about the content of the utterance.

are no longer simply treating the scenario as two systems interacting, but have a specific operational goal of extracting a quantitative value.

How we define and apply metric scales will be essential for the arguments of the next section, and is worth exploring in more depth. Mari, Wilson and Maul emphasise that applying a metric scale (both the local and the shared public one) always has a physical basis in a reference artefact that is used to establish the units of the scale. For example, in the case of a length scale we start with a particular object – the metre rod used in the Paris international standard for example – which is designated as 1 unit. Concatenating two identical rods (which requires a specified operation for concatenation and another operation for determining whether two magnitudes are identical) end-to-end gives 2 units, etc. Creating a reference scale in this way is the basis for representing a property with a metric scale (this is where Mari, Wilson and Maul's model incorporates the work done by the RTM on the representation of properties and the empirical operations – such as comparison and concatenation – that are required to build a scale). Of course, our current methods for creating fundamental units are now significantly more complex and in most cases we have shifted to using fundamental constants such as the speed of light to define a base unit rather than using a specific artefact (see the next section for more on this).

In the background of this analysis of measurement is the question of whether, or to what extent, we should be realists about measurement. Mari, Wilson and Maul are measurement realists but state that while they commit to a moderate realism about (at least some) properties they are not realist about the *numerical values* that measurement produces. Their focus on calibration models leads them to claim that the numerical measurement outcomes should only be understood in an operational context (see Mari, Wilson, & Maul 2023, chapter 4).

Extending Mari, Wilson, and Maul's view even further is Tal (2017; 2018) who, while not explicitly stating whether his view is realist or anti-realist, strongly advocates for recognising how much of creating scales comes down to convention. Both in obviously pragmatic cases such as fixing the zero point on the Celsius scale and in less obvious cases such as specifying conventions for establishing when two quantities are equal. Tal's coherentist view sees both properties and numerical values as entirely model dependent.

Others, however, take a much stronger realist stance that *is* realist about the numerical values and takes the structure of the metric scales to be directly instantiated in the world. Both Michell (2005; 1997) and Isaac (2019) argue for measurement realism of this kind on the basis of how metric scales are used in measurement and the explanatory value that they have. Michell in particular sees this as a necessary consequence of the RTM

(Michell 2005) I will explore these arguments further in the following section, particularly regarding the role of precision in their ideas.

4 Quantum Measurement

I will now make the comparison between Bohr's doctrine of classical concepts and measurement theory. I will argue that representing a property by a metric scale introduces an implicit assumption that effectively corresponds to Bohr's doctrine of classical concepts. The assumption is introduced because applying a reference scale to a property requires us to assume that the property instantiates a precise value on that scale, hence implying that the property has a definite state. In particular, I will focus on how units are defined using reference artefacts which are assumed to have precise states. While the use of metric scales is sometimes treated as a mathematical idealisation, they also play an important role in discussions about realism in measurement.

I will first, in Section 4.1, look at how precise definite values have featured generally in debates about measurement realism and then at a more specific argument about defining units. Then in Section 4.2, I will argue that this fulfils Bohr's doctrine. Finally, in Section 4.3, I will look at how the treatment of uncertainties and calibration countermands the arguments for realism and pushes us towards an epistemological or pragmatic reading of Bohr's doctrine, but also shows how deeply ingrained the use of precise values and metric scales is.

4.1 Precision

On a general level, the idea of precision and definite values has played an important role in debates about measurement realism. Within scientific communities it is almost universally assumed that measurement outcomes take the form of $x \pm uncertainty$ (with appropriate units). In other words, the outcome is a single definite value with some uncertainty introduced by practical considerations. Chang (2004) identifies this as the *principle of single values*, the almost universally accepted assumption that real objects can have no more than one definite value in any situation, and therefore we get single definite values as measurement outcomes. Chang notes that this holds even in quantum mechanics (although he does not address some of the interpretational questions around this).

It is also common to assume that future measurements will reduce uncertainty and give increasingly precise values. This is widely relied upon in discussions of measurement realism: Isaac (2019) takes the fact that improving precision is an important goal for experimentalists, and that this has been by and large historically possible to do, as a reason to be a realist about the outcomes of measurement and see them as increasingly good estimates of a precise *definite* value that is instantiated in the world. He argues that the instantiation of metric scales by empirical properties (following the RTM) is what grounds precision as a criterion of success. Taking this further, *amenability to precision* has been suggested as a criterion to judge whether new theoretical concepts (such as a constant introduced as part of a new theory) should be taken seriously as elements of reality, and the overall success of a research programme in delivering increasingly precise measurements has been seen as a indicator of quality (Smith & Seth 2020; Stan & Smeenk 2023).

Quantum mechanics has raised many questions about the *limits* to precision in measurements, so it is arguable that quantum mechanics calls this whole programme of using precision as a basis for realism into doubt. However, quantum measurements are not of a fundamentally different type to other measurements in physics, and involve many of the same basic properties such as position, momentum etc that have been used and defined in a classical context; so we would expect to be able to use the same criteria – including precision – to judge quantum measurements. Additionally, the metric scales that we apply to these properties in a quantum context are the same ones that have been defined and analysed independently of quantum mechanics, so the claims we make about these scales are carried over to the quantum context. The public scales for length and momentum, for example, were well understood in a classical context and imported into quantum measurements complete with the assumptions of single values and precision. Maintaining this continuity between quantum and classical properties is important for the practical process of measurement in the lab (which amplifies quantum effects to classical readouts).

But, beyond these general points about precision and realism, I will draw out one specific argument from this literature that will be especially crucial for the comparison with Bohr's doctrine in the next section. This argument comes from Michell (1997) and concerns the definition of units. As introduced in section 3: to define a unit we take a particular artefact (such as the metre rod held in Paris) to designate as 1 unit on the scale and build the rest of the scale from that unit. We assume that the artefact has a *precise*, definite value of the relevant property that perfectly corresponds to 1 on the scale. This introduces the precision of the scale and stipulates *by definition* that this precision has empirical meaning and is instantiated by the object. Of course, this is a somewhat outdated method of defining units, the metre is now defined as the distance travelled by light in a vacuum in a certain fraction of a second (see Tal (2018) for discussion of this change). This shifts the definitions of the scales away from an empirical artefact that can never be practically observed with absolute precision towards a theoretical definition where such precision is possible. In many ways this reinforces the assumption of precision by stipulating an exact, fixed value of the constants used in the definition, thereby increasing the prominence of the numerical scale representing that physical constant. Regardless, the definition of the metre in turn depends on the SI unit of the second, which is itself defined based on the hyperfine ground-state transition frequency of the caesium-133 atom under certain conditions. As a result our current definitions of the SI units rely on a reference *phenomena* that is taken to be extremely stable rather than a specific physical artefact, but the connection between precision in the numerical scale and in the empirical world is maintained by assuming that this frequency is precisely instantiated. Additionally, we must inevitably use *some* empirical reference objects, both to make local calibrations and to calibrate the local scale to the shared public scale. These reference objects are ineliminable in the process of creating a numerical value scale and getting a numerical result from a measurement, and the assumption that they have precise, definite states is tied into how we define the units of the scale.

Michell (1997; 2005) builds on this (and other points) to conclude that if we adopt the RTM and take metric scales to represent properties then we must also be *realist* about those scales and the numerical results they give.[10] Numerical results are ratios between a specific magnitude and the unit. For example, a result of $5m$ tells us the object we are measuring is exactly 5 times the magnitude of the artefact used to define the metre. This implies that both the unit and the measured magnitude must be precisely instantiated in the world. Crucially, Michell rejects the line of thought that the precision and continuity of metric scales is just a convenient mathematical idealisation (as argued in e.g. Pap 1959). The scale is not just an abstraction from the actual objects in the world (which only instantiate a finite subset of the possible values on the scale and have operational limits on how precisely we can determine their values) but represents the full range of possible values that any specific magnitude could instantiate and defines the relationships between magnitudes. According to Michell, treating metric scales and their mathematical structure as merely convenient idealisations means they cannot be used in explanatory statements such as claims like “the fact that rod a spans the linear concatenation of rods b and c is explained by the fact that these rods are of lengths, l_a , l_b and l_c , respectively, and $l_a = l_b + l_c$” (Michell 1997, pg. 268). Thus, Michell concludes that making the realist assumption – that the continuity and precision of the metric scales is instantiated directly

[10]Here Michell is arguing that metric scales are not only used to represent but are directly instantiated, a claim which is connected to foundationalist work on measurement that predates the RTM (see Michell 1997 for discussion).

in the empirical world – has direct explanatory value.

This certainly does not exhaust the arguments for and against measurement realism, or address the possible responses to Michell's arguments. In particular, I have not discussed the role of uncertainty as a limit on precision, which I will return to in Section 4.3. My aim is not to defend this sort of realism, but to show how it leads to classical assumptions of definite states - which I take Michell's arguments to do. I will show in the next section that this line of argument about precision encapsulates Bohr's doctrine.

4.2 Bohr and Measurement Theory Compared

As the previous section showed, the use of metric scales in measurement has been argued to imply that properties in the world have precise states, due to how we define units and more generally in how we take the structure of the scales to explain the behaviour of the empirical properties. There are two parts of Bohr's doctrine: 1) treat the target property as classical by representing it with a statistical mixture of definite states and 2) treat the relevant property of the measurement apparatus as being in a single definite state. I will address both of these aspects and show that in both cases we assign a metric scale that implies precise values.

Starting with part (2), concerning the measurement apparatus: As Zinkernagel (2015) points out, part (2) of Bohr's doctrine is that we must have a precise state of the measurement apparatus, which we can compare the property of the quantum system to and use to define any constants and scales used in the measurement (the reference system argument). We can find the analogue of this in measurement theory by applying a metric scale to the reference property of the measurement apparatus. Here I turn to Mari, Wilson, and Maul's model as it makes a clearer distinction between system and apparatus than the RTM does. Applying a metric scale to the measurement apparatus starts in step (3) of their model and continues into steps (4) and (5), where we define units and calibrate local and public scales.

Following Michell's arguments given in the previous section, defining the units of a scale inextricably involves the assumption of precision (Mari, Wilson, and Maul – although they do not espouse Michell's realist conclusion – independently make a similar analysis of how we define units that makes the same assumption of precision). Whatever reference object we use to define 1 unit (whether this be the metre rod or the transition frequency of a Caesium-133 atom used to define the second) is assumed to have such a property precisely. We must assume that these reference objects for the publically agreed upon standard that we use to calibrate our measurements have precise states with definite values of the relevant property. This also applies to the local reference objects (such as a thermometer casing with division markings inscribed on it) that we use to the define the local scale, which is then calibrated to the public scale. This embodies Bohr's doctrine by assuming that the relevant parts of the apparatus used to define units and scales have precise definite states, as is indicated by the reference system argument given in Section 2. The assumption of precision made in the definition of units ignores any entanglement the reference property will inevitably form with its environment or the system being measured. Furthermore, defining units and calibrating the local scale with the public scale is done separately from any actual measurement and we assume that the states of the reference properties involved are not affected when they are placed in different environments (within reasonable parameters such as standard temperature and pressure). Regardless of the practical question of whether it is true that any variation or uncertainty due to entanglement will be negligible and subsumed by other uncertainties in the device (see further discussion in the next section), we are still making a definitional claim that whatever magnitude the reference property has, this corresponds to a *precise* value on the scale.

The other aspect of Bohr's doctrine is (1): giving a classical description to the target property. This is not as clearly evident in Bohr's thought as the reference system argument is and is primarily argued for by Howard, who proposes using a statistical mixture of eigenstates as a formal way of expressing the classical description. I take the comparison to measurement theory made below to support Howard's claim that the classical description should be applied to the target property as well as the measurement apparatus.

While applying a local scale is first done to the reference property (step 3), it is also important that this is then extended backwards to work out the correspondence between the scale of the reference property and the scale used to represent the target property (this is part of steps 3-5). In some cases, the target property is directly measurable (for example measuring the length of an object), in others it is indirect (as in the case of temperature, which must be matched to something like volume which is then directly compared to the reference property). But in all cases when we report the outcomes of measurement we do so in terms of the public scale that is taken to represent the target property and not the reference property (we report temperature in terms of Celsius, Fahrenheit or Kelvin not in terms of the units of length applied to the markings on the thermometer). So on top of representing the reference property with a metric scale, we also represent the target property with its own metric scale.

In section 2, I raised the question of what the purpose of Howard's proposed way of formalising the classical description of the target property – writing the description of the property as a statistical mixture of eigenstates – is, when we know it to be strictly speaking a false representation (the quantum system does not actually have a definite value that we are ignorant of). The answer is to think of it as mediating between the metric scale that we take to represent a property such as a length – for which the scale determines a continuous range of possible values that any specific object could instantiate – and the restrictions of quantum mechanics. Quantum mechanics restricts the possible values of the metric scale that we could get as outcomes of measurements. The set of eigenstates of an observable specify the precise measurement outcomes that are possible in a specific quantum measurement, just as the metric scale does more generally. Because quantum mechanics introduces fundamental limits on precision and continuity we need this further specification of the possible values in addition to the metric scales that were developed prior to the advent of quantum mechanics; for example, in addition to a continuous scale representing energy, we must specify the discrete energy levels that a system could instantiate. The classical description – like metric scales in general – defines and characterises the measureable property, but it is not taken as an empirical model of the property that can be evolved under dynamical equations.

Whether we take this application of the metric scale to imply that the target property has a precise value depends on whether we are realists about measurement outcomes. Michell's arguments for realism, given in the previous section, state that we should understand a measurement outcome to be a ratio between the reference artefact defining the scale and the target property, and this ratio is instantiated in the world, implying that the target property *does* have a precise value. When it comes to quantum measurements, however, we tend to be far more aware of the limits on precision, and this influences how we interpret definite measurement outcomes. Take, for example, POVM measurements; POVM measurements do not use eigenstates and are used for imperfectly isolated systems. These are the measurements that are practically carried out in laboratories. We do still report the results of a POVM measurement in terms of a precise metric scale, giving a result in the familiar single value form – $x \pm uncertainty$; for example, a POVM measurement of position separates the length scale into bins of finite resolution and we get a result when the system is sufficiently localised to one bin. Even when the system in not in a precise eigenstate we report it as a precise value and put all further considerations into the uncertainty. But this case makes the question of how we treat uncertainties unavoidable. While we *do* report the outcomes of POVM measurements as single precise values and may use Howard's proposed mixture of eigenstates to characterise the property in question, this is mainly as a pragmatic choice for convenience. We are well aware of the limits on precision in this case and do not take the single value outcome too literally. This challenges the arguments for realism about measurement outcomes.

The next section will consider this, and what this means for how we interpret Bohr's doctrine, in more depth. But what is clear is that both aspects of Bohr's doctrine – treating the 1) target property and 2) the property of the measurement device as classical – are fulfilled by the way we assign metric scales and define units, and the assumption of precise, definite states that is built into this. This is stronger for the case of the measurement apparatus, where reference properties and the precision assumed in defining units are largely taken for granted. For the target property, although we do apply precise values, we are more likely to be aware of the pragmatic elements of this.

4.3 Uncertainty and Calibration

While the precision inherent in metric scales and the way that we define units has been used as an argument for realism about measurement, this largely overlooks how we think of uncertainty and the practical calibration procedures that are used to implement these scales. Model-based approaches to measurement, such as Mari, Wilson and Maul's, foreground the role of calibration and uncertainty and, as a result, tend to reject the strong realist conclusion in favour of a more moderate realism about properties but not specific numerical measurement outcomes. This would imply an epistemological rather than ontological reading of Bohr's doctrine.

Before looking at this in more depth, here is what the realist conclusion means for Bohr's doctrine: Measurement realism, in Michell's sense, is realism about metric scales used to represent properties (i.e. the belief that these scales are *instantiated*) and about the definite values we get as measurement outcomes. Issac (2019) also emphasises how the arguments from precision in metric scales lead to realism specifically about the numerical values produced by measurement. In the context of Bohr's doctrine, the realist conclusion therefore entails that we are realist about the definite states (or mixtures of definite states) we assign both to the reference properties and the target property. This is an ontological reading of Bohr's doctrine: in measurement contexts the relevant properties really have classical definite states. Given that the quantum state fails to specify such definite states, this suggests an incompatibility between measurement realism and quantum mechanics. If we take Bohr's doctrine as ontological then we seem to be positing an ontological difference between what systems are like in the context of measurement and what they are like more generally. While this ontological reading of Bohr is possible (for example, Zinkernagel (2015) argues in support of a nuanced ontological view based on Bohr's idea of complementarity), it has generally been argued against (Saunders 2005; Camilleri & Schlosshauer 2015).

The alternative is an epistemological reading in which the assignment of definite states and precise values is a pragmatic choice that facilitates the operational goal of measurement and enables us to extract usable information from the measurement scenario. On this reading, although the world may *be* entirely quantum, the demands of measurement and the limitations of our abilities as observers – and indeed as language users – require that we use classical descriptions with definite states (Camilleri & Schlosshauer 2015).

Given the nature of uncertainties in quantum mechanics, the epistemological reading appears more promising. In a POVM measurement of position we see the uncertainty in the measurement results as representing the range of values that the system is approximately localised to. The quantum system does *not* actually instantiate a precise value, this is merely an artifact of how we report measurement outcomes – it is more convenient to stick to the familiar format of $x \pm \mathit{uncertainty}$ than to switch to reporting ranges of values. But this gives us reason to reject realism about the precise value x.

Mari, Wilson and Maul reject realism about numerical measurement outcomes based on similar considerations about the pragmatic and conventional elements of how we treat uncertainty and calibration, though they focus on uncertainties more generally and not the specific ontological uncertainty of quantum mechanics. The stated uncertainty in a measurement result includes many uncertainties from the functioning of the measurement device. The most easily identifiable sources of uncertainty are direct operational factors such as the finite resolution of the measurement scale, human error in reading off the scale (e.g. due to parallax). There is also a basic uncertainty in the functioning of the measurement device itself and the conditions (such as temperature and pressure) that it is designed to operate at; these are worked out from calibration tests. Additionally, these uncertainties are also present in the procedures used to define and calibrate unit scales, despite the assumptions of precision built into the process (as described in the previous sections). Tal (2017) argues that once we acknowledge *how many* idealisations are made and how often conventional standards are applied in the calibration process we must also accept that measurement is a coherentist modelling procedure that balances all these factors. Mari, Wilson and Maul make similar claims.

This analysis of uncertainties and calibration gives us reason to be sceptical of a realist interpretation of measurement outcomes (even if we are still realist about other aspects of measurement such as the existence of properties). Correspondingly, it implies that Bohr's doctrine of classical concepts is pragmatic and epistemological. The definite states we assign are convenient descriptors that allow us to get a handle on the measurement set-up and the scales and units involved, but we acknowledge that there are idealisations and conventional aspects at play. In Bohr's own statements, he repeatedly emphasizes that the doctrine of classicality stems from the need to be able to give an account of scientific evidence in a way that can easily be used and shared by a scientific community (as evidenced by the quote given in section 2); this is an epistemological not an ontological claim.

However, even if we see the use of definite states and precise values as pragmatic idealisations, thinking about the complexity of calibration also proves how ineliminable the use of them is. The most basic form of calibration is to use the instrument to measure properties with known values to check that the device produces correct readings, this is a simplistic black-box model of calibration that treats the measurement device as an input-output function. More sophisticated models of calibration – white-box models – break down the device in more detail to establish the sources of uncertainty within the device (Tal 2017). White-box calibration breaks the device down into a series of individual modules; each of these can be analysed to establish factors such as how the parts might react to changes in temperature or pressure, the effects of friction between components, etc. Treating calibration and uncertainty in this modular way strengthens Bohr's reference system argument considerably and illustrates why classical descriptions are ineliminable from measurement, even if they are epistemological tools: Calculating the uncertainties from all the different components of the device requires that the device be broken down and metric scales applied to each component *individually* to quantify the associated uncertainty budgets (Tal (2017) gives examples of these sorts of uncertainty budgets given by manufacturers). This means that a model of a measurement device requires a vast number of supplementary measurements to establish how the device functions, calibrate it correctly, and calculate its uncertainties. To enable this, precise metric scales must be applied many times over in order to quantify all the necessary components, making the use of these scales, and the pragmatic assumption of precise values, effectively ineliminable from measurement.[11]

As such, an epistemological reading of Bohr's doctrine shouldn't be seen as a reason to dismiss it; it is important to recognise that classical assumptions are built into how we report measurement results and are essential to how we design measurement devices and define units. Even if the classical assumption is understood to be a pragmatic idealisation it is still a necessary part of measurement. The next section will consider the implications of Bohr's doctrine and suggest how this might influence our understanding of the measurement problem.

[11]We also universally ignore the possible effects of entanglement between components of the device and treat them as separable – which is a classical assumption in itself. We take any effects from this to be negligible compared to other factors.

5 Can Measurement be Entirely Quantum?

We are now in a position to answer the title question: can measurement be entirely quantum? The answer is no. By looking at detailed models of the measurement process it becomes clear that there is far more to measurement than modelling the systems involved in quantum mechanical terms. In common accounts of quantum measurement, such as the decoherence account (which is particularly applied in the Many Worlds interpretation), what is modelled is *at most* the empirical side of measurement where the target system becomes coupled to the measurement device and to a reference property (steps 1-2 of Mari, Wilson and Maul's model).[12]

This does not touch on the *informational* side of measurement where we apply metric scales, define units, and specify procedures of calibration with the operational goal of producing numerical measurement outcomes (steps 3-5 of Mari, Wilson and Maul's model). We cannot ignore this practical side of measurement when thinking about quantum mechanics; it is here that classical assumptions of precise values and definite states creep in, even if it is only on a pragmatic basis. Bohr's doctrine, and the connection to measurement theory made in this paper, make this clear. Using metric scales is unavoidable in how we design measurement devices and quantify uncertainties, and is baked into our definitions of units. These conditions are necessary for the epistemological practice of measurement and have important ramifications for how measurement is conceived of.

That *measurement* is not entirely quantum does not imply a strong ontological claim that classical concepts are prior to quantum ones or that the *world* cannot be modelled entirely within quantum mechanics. All systems can be given a quantum mechanical description if necessary, but without the additional steps of applying metric scales and making classical assumptions of definite states we would be unable to extract a usable measurement outcome.

This conclusion has implications for how we understand the measurement problem. The epistemological reading of Bohr's doctrine is generally not taken to provide a solution to the measurement problem and it must still be combined with some further interpretation (although the ontological reading *does* attempt a solution – Zinkernagel 2015). However, even as an epistemological doctrine, the fact the measurement is not entirely quantum provides important context to the measurement problem and suggests that aspects of it have been overlooked.

The measurement problem, following Maudlin's (1995) presentation, is an inconsistency

[12] Alternatives being the dynamical collapse models – which also involve decoherence, or how Bohmian Mechanics resolves the measurement problem by specifying determinate trajectories.

between these three principles:

> "1.A The wave-function of a system is complete, i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.
>
> 1.B The wave-function always evolves in accord with a linear dynamical equation (e.g. the [time dependent] Schrödinger equation).
>
> 1.C Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the measuring device is either in a state that indicates spin up (and not down) or spin down (and not up)" (Maudlin 1995, pg. 7)

This is presented as a problem of quantum dynamics and completeness. Yet it presupposes a concept of measurement that is prior to and independent of quantum mechanics. Measurement is a hybrid process that involves both the quantum dynamics of the system and a set of operational procedures, yet the latter does not enter into the standard formulation of the measurement problem.

Principle 1.A takes for granted that we have a preconceived idea of measurable properties. What is not included in the measurement problem is how these properties are defined and the way in which metric scales are used to characterise the measurability of properties in the first place. As mentioned in Section 4.1, this has largely been done outside of quantum mechanics and imported in along with assumptions about how we expect those properties to behave in a classical domain. For example, the axioms used to characterise a property such as length include that length is infinitely divisible (see Michell 1997; Díez 1997a, b) – which faces challenges when we get to the Planck scale. The impact of this is that, when it comes to quantum measurements, we apply familiar criteria of success – such as precision and the principle of single values (which Chang (2004) identifies – see Section 4.1) – to judge their quality and to guide our reasoning without independently assessing whether these criteria apply. Relatively little has been done in measurement theory on whether quantum mechanics changes our understanding of the scales we use to represent these properties. Further analysis of quantum properties that focuses specifically on metric scales and measurement theory could help to clarify how we should think about the quantum state and the way that it specifies properties. As I have suggested in Section 4.2, tools such as Howard's suggested classical descriptions could assist with bridging this gap.[13] How we define properties in quantum mechanics has been given some attention

[13] This would also allow for a deeper investigation of the arguments in support of realism about measurement outcomes and whether they can be maintained within quantum mechanics (although I have suggested here that they cannot).

in the literature, but is largely separated from discussions of the measurement problem. Measurement theory, with its reliance on the representation of properties by metric scales, makes it clear that this is a mistake and the two issues should be seen as closely connected. How the quantum states specifies the values of properties and what it means for those properties to be measureable is an undeniable background to the measurement problem that deserves more attention.

Likewise, principle 1.C takes for granted that measurement produces determinate outcomes without examining the infrastructure of metric scales, reference standards and operational procedures that make this intelligible. These form a set of epistemological requirements that are necessary if we are to design processes that we call measurement.

There are multiple avenues to explore the implications of this and how these aspects of measurement could be incorporated into the measurement problem. Inevitably some quantum interpretation that specifies a solution to the measurement problem is still needed to make sense of superpositions and explain basis selection, but the epistemological conditions of measurement could be incorporated in a number of ways. One option is to stay close to the existing interpretations of quantum mechanics and their solutions to the measurement problem, but to see the informational side of measurement as exposing an added dimension of it. For example, the Many Worlds approach (Wallace 2012) relies on decoherence to take us arbitrarily close to determinate values. (Camilleri & Schlosshauer (2015) discuss how Bohr's ideas are compatible with decoherence.) The formal result of decoherence is the effective diagonalization of the density matrix; the off-diagonal terms, which represent interference, decay until they are negligible and we can treat the system *as if* if is correctly described by a classical statistical mixture. This effectively produces the description of the property that Howard proposes – a statistical mixture of eigenstates.[14] This is a promising way of connecting up the use of metric scales to the quantum mechanical model. Connecting the Many World's decoherence account of measurement to measurement theory also helps to specify what the actual process of measurement is from the perspective of the observer within a single branch rather than putting measurement in abstract third-personal terms that cover multiple branches of the wavefunction (Mason (2025) explores this aspect of measurement and how it influences the way in which we interpret the quantum state). The observer's viewpoint is essential if we are to understand the operational procedures that we rely on to define metric scales and characterise measureable properties.

[14]This could also be a way to recover realism about measurement outcomes at an emergent level, based on the emergentist programme that Wallace supports. It also exposes a new context in which classical descriptions such as the statistical mixture of eigenstates (the effectively diagonalised density matrix) could have novel explanatory value – which is part of the justification for emergent classical ontologies.

More radical approaches are available, however, that place the epistemology behind measurement at the core of the measurement problem. An example of this would be recent attempts to use transcendentalist ideas from Kant and phenomenologists such as Husserl to inform or resolve the measurement problem (e.g. French 2023). The idea of this is that there are epistemological preconditions on perception and our experience of the world, and therefore on what concepts we use to formulate scientific theories; it is this that creates the paradox of measurement in quantum mechanics. French's ideas focus on the fact that when conscious agents reflect on their own internal state, they always find it to be determinate (an idea that has particular resonance with Bohr's reference system argument) and this is treated as the key to the measurement problem. There have been multiple studies that look at the transcendentalist ideas in Bohr's work (e.g. Wiltsche 2024; Bitbol 2017), and measurement theory similarly draws from transcendentalist thought and ideas about human perception (as discussed in Michell (1997) and Mari, Wilson and Maul (2023, pg. 114)). As such, measurement theory, and its connection to Bohr identified here, could be an avenue to formalise the application of transcendentalist thought to quantum mechanics and connect these more abstract philosophical ideas to scientific measurement more concretely.

This is just a brief sketch of some of the possible implications. What is undeniable is that there is more to measurement than a model within quantum mechanics, and looking at measurement theory has the potential to shape how we think about the measurement problem.

6 Conclusion

Our exact solution to the measurement problem will depend on what interpretation of quantum mechanics we adopt. How Bohr's doctrine, and the debate about measurement realism, have a bearing on the measurement problem will be different in each of the main interpretations. But, identifying how Bohr's doctrine is fulfilled by the assumption of precision that comes along with using metric scales to represent properties and the way that we define units, which is a central element of both the RTM and model-based approaches to measurement, opens up a way to move this discussion forwards and recognises how debates in measurement theory could assist our understanding of quantum mechanics.

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