Randomness, Quantum Uncertainty, and Emergence: A Suggestion for Testing the Seemingly Untestable

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Abstract

The functioning of complex natural structures, such as living systems, has eluded a generally accepted theoretical basis and empirical verification for decades, partly because of a lack of meaningful experiments. We therefore propose a class of experiments designed to test whether hitherto unknown principles of order are at work in sufficiently complex natural dynamical systems that cannot be captured by known physical laws. Part of the underlying working hypothesis is that the quantum mechanical uncertainty principle leaves room for ordering phenomena in chaotic or nearly chaotic physical systems in the sense of a strong emergence principle, which would not be expected when treated according to conventional modelling approaches, as has already been formulated by several authors in various forms. Unlike some previous proposals which included coherent quantum-mechanical states, we do not require any macroscopic quantum coherence. The key idea behind testing this undoubtedly bold hypothesis is to compare two sufficiently complex, virtually identical setups, one of which is operating with deterministic pseudo-random number generators placed at certain key points that are sensitive to small changes, while the other is equipped with quantum-based physical random-number generators, the two setups being otherwise identical. Existing artificial neural networks are proposed as suitable test objects for this purpose, and the overall performance under identical training conditions could be used as a quantitative benchmark. As the working hypothesis used goes far beyond artificial networks, a successful outcome of such an experiment could have strong implications in many other branches of science.

Key words: Complex systems, neural networks, emergence, uncertainty principle, randomness

1 Introduction

In the exact sciences, which include disciplines related to mathematics and the natural sciences, a hypothesis put forward can be proven or refuted either by comprehensible logic or by reproducible experiments. This practice has been and continues to be extremely successful, but it is not applicable to all branches of science, or only to a limited extent. Direct counterparts to statements made by the exact sciences can be found, for example, in theological dogmas of many religions, which explicitly, and probably rightly, attempt to avoid such scrutiny altogether. However, there is also a grey area between these two extremes. This is where attempts are made to contextualize scientific results within a broader philosophical or even religious framework [1]. Prime examples are attempts to interpret cosmological findings on a metaphysical level, for example in relation to the creation and the meaning of the universe, or the discussion of the relevance of known physical laws for the occurrence of life or the nature of human intelligence [2]. Unfortunately, most corresponding scenarios that are circulating seem to be completely beyond the possibility of experimental verification.

In this short article we would like to revisit a question that has been raised by many authors before, namely whether quantum mechanics can be used to shed light on the fundamental principles of certain as yet poorly understood phenomena in complex structures that exhibit chaotic or nearly chaotic behaviour. For example, the much-discussed 'principle of emergence', according to which a kind of order can emerge from a multitude of individual building blocks which cannot be readily deduced from the properties of the individual parts [3], has often been cited in this context [4]. While such a presumed emergence can be derived with some effort from the known equations of quantum mechanics in comparatively simple, non-chaotic systems in condensed matter, with phenomena ranging from cystallization, magnetism, superfluidity and superconductivity [5], other, more complex ordering phenomena, such as those occurring in living systems, have so far defied a conclusive description by any known law of nature. While so-called 'downward causation' has been brought into play for biological systems (see, e.g., [6-8]) and even for digital computers [9], which could be seen as supporting the thesis of emergent principles at work, the topic is very contentious [10-13]. Many attempts have also been made to discuss quantum theory and chaos in relation to life in general, to the functioning of the human brain in particular, and to the controversial terms 'consciousness' and 'free will'. These physical and mathematical concepts have sometimes even been identified as causal factors (see, e.g., [14-22]). However, serious critical objections have been raised against the latter claims [23-26].

Unfortunately, the vast majority of these works are purely argumentative, and their conclusions generally do not lead to concrete proposals for experiments that could be verified with the rigour of a scientific proof. Only a very few publications are referring to actual or proposed experiments that could help to clarify these or related fundamental questions [24,27-32].

However, any scientific situation that is unclear ultimately requires a scientific clarification. Based on a particular hypothesis to be examined, a specific experiment should be designed to provide the most meaningful result possible regarding the validity of the underlying hypothesis. In the following, we formulate such a narrowly defined hypothesis and outline an experiment, the results of which could either prove the hypothesis, or can serve as a counterargument against it.

2 The hypothesis

The working hypothesis that we propose to test is based on the assumption that in every natural chaotic or just sufficiently complex physical system near chaotic behaviour [33], there is an as yet unknown principle of nature at work that is largely robust to environmental influences, provided that they do not destroy the integrity and functionality of the system. On the one hand, this principle must obey the known laws of physics, but on the other, it can allow for subtle, unnoticed changes that are still physically allowed by the uncertainty principle of quantum mechanics. Over time, however, such small changes in complex dynamical systems can have major effects [34], and may lead to a course of events that seems to run counter to purely statistical considerations. Scenarios invoking quantum uncertainty are not at all new and have been formulated by a number of scientists and philosophers, without [18,20,35] or with [36-40] a metaphysical or even theological context. Although they may seem a little hackneyed today, their justification as part of our working hypothesis lies in the simple fact that, despite decades of very intensive research, many phenomena occurring in complex natural dynamical structures have not found a satisfactory explanation, and therefore the introduction of unconventional ideas should at least be considered. While our proposal is based on the concept of indeterminism according to the view of the Copenhagen interpretation of quantum mechanics [41], it explicitly adds an additional ingredient, which can be interpeted as a yet unknown physical law or as an additional ordering principle of nature in the sense of the strong emergence principle [3], according to which it cannot be derived from the already known laws describing the constituents of the complex system and their microscopic mutual interactions. Whether this additional law or principle can ultimately be captured theoretically and mathematically, or whether it fundamentally eludes such analysis altogether, is left open here, as it is not decisive for the present proposal. We consider it necessary to include such an assumption in our working hypothesis, however, in order to specifically address the legitimate objections regarding the role of quantum coherence in natural extended complex dynamical systems at room temperature [23-26], while deliberately leaving open the possibility that quantum phenomena could nevertheless play a decisive role.

In a related experiment, we need to be able to distinguish between the behaviour over time of a complex system that is subject to quantum uncertainty, and another virtually identical system that is not. A potentially different behaviour must then be verifiable and quantifiable. The evaluation could include checking whether one of the systems performs certain tasks better than the other in a reproducible way, for example, in terms of speed or accuracy, thereby defining a certain quantitative benchmark.

3 The experiment

3.1 An implementation using current technology

A corresponding preliminary experiment could be carried out in a variant of existing computer-based neural networks, which are routinely used to find patterns in huge amounts of data and to generate predictions based on these data. Inspired by the internal structure of the brain, whose functioning has incidentally often been suggested to be on the verge of chaotic behaviour [42-45], simulated neuron-like entities are virtually connected, and mathematical parameters such as weights and biases, which determine the strength of connections between the simulated neurons and influence their activation, are adjusted during a learning process [46,47]. Introducing random noise at different stages of the learning process turned out to be very beneficial to approach an optimum learning performance [48-50], but randomness is mostly simulated by software-based deterministic pesudo-random number generators (PRNGs). The role of this randomness is usually interpreted as preventing the process from overfitting or getting stuck in so-called local minima, and driving it to seek better solutions [50]. However, such purely software-based architectures are inherently deterministic and predictable, since identical initial conditions must lead to identical results, and because even pseudo-random number generators are ultimately based on deterministic algorithms.

The key idea is to replace these PRNGs with quantum-random number generators (QRNGs) [51], making a sufficiently complex network intrinsically unpredictable, while those network components that are expected to obey more or less classical physical laws may remain unchanged. Such QRNGs could be based, for example, on the use of entangled photons [52], or on radioactive decay [53,54]. To apply the benchmark test, the overall performance of such a network should then be compared in a control experiment with an identical network using software-based PRNGs, with the same learning input and operating at the same clock-cycle rate equivalent, i.e., under otherwise identical conditions. If a fundamental difference in their properties were observed in favour of the QRNG version, this could be a first indication. Special care must be taken here to ensure that the accuracy during the simulation of the temporal behaviour of the complex system is not compromised by the digital discretization [55,56]. Quantum-based random number generators inherently have the potential to generate infinitely precise random numbers, whereas corresponding computer-generated random numbers always have finite precision.

For the time being, experiments with existing memristor-based networks may already provide a certain clue, as they have been reported to have the potential to outperform software-based networks in terms of accuracy [57] and speed [58]. Physical randomness is generated by the intrinsic instability of the memristor devices. However, it is unclear whether the statistical randomness of the respective many-body electronic system is equivalent to that realised in genuine QRNGs [51], although the electrons involved also ultimately obey the laws of quantum mechanics. Nevertheless, it would be of great interest to examine the performance of such already existing networks according to the benchmark test given above.

3.2 A thermodynamic consideration

Let us assume that we had implemented such a neural network with QRNGs whose performance outperformed that of its PRNS-based counterpart. If we were to statistically analyse the random-number streams generated in both cases during a single experiment, we would probably not find any significant difference between them, as their statistics would still conform to the statistical expectations for a single experiment according to the laws of known physics. However, over a longer period of time, certain differences or even patterns should emerge that deviate from these expectations, suggesting some kind of ordering phenomenon in the QRNGs network. This argument could, of course, be used to argue that the proposed experiment is doomed to failure from the outset because it seems to contradict the second law of thermodynamics. According to this law, the statistically most probable states must be assumed in the long term, which seems to be at odds with any principle of emerging order. However, this objection can be easily countered by the fact that local ordering phenomena, such as the formation of complex life forms, are permitted as long as the total entropy of the universe does not decrease. Since artificial neural networks do not emit any material metabolic products that enter their entropy balance, such an order would necessarily have to be accompanied by additional irreversible heat dissipation to the environment and thus increased energy consumption, which might even be measurable, and should ultimately lead to an increase of the total entropy.

3.3 Quantum coherence: not required here

If we really want to clearly establish a difference between the use of PRNGs and QRNGs beyond any doubt and without adding any further complications within our working hypothesis, we should design the experiment in such a way that the formation of a stable, macroscopically entangled coherent quantum-mechanical state involving the quantum nature of the QRNGs can be ruled out. The formation of such a state and its consequences would in themselves be very attractive topics for another, considerably more challenging experiment, but for the moment it conflicts with our working hypothesis of robustness, because such entangled states are inherently very fragile and prone to external perturbations. The existence of possible extended coherent states of quantum-mechanical origin has indeed been postulated to explain certain capabilities of the human brain, for example by Penrose and Hemroff [59-62], but it has also been widely and rightly questioned because of the problem of decoherence under the conditions prevailing in a living body at ambient temperature [23-26]. Similar arguments can be put forward for the present proposal. Although the concept described here relies heavily on consequences of the laws of quantum mechanics, it does not at all need a macroscopic quantum-mechanically coherent state, however. In our proposal we only require local quantum uncertainty but at many, possibly very distant key points in a complex structure. By simply keeping the distance between these points large enough, and using normal-conducting wires for the electrical interconnections between them, long-range quantum coherence over several of such points can be ruled out on the smallest time scales and at the temperature at which this structure is operating - a situation that is probably equivalent to that in living brains. The statistical behaviour due to local quantum uncertainty could then at first be simulated separately for each of the key points using suitable distribution functions in combination with conventional PRNGs. As there is *a priori* no reason to assume that simply replacing the PRNGs with their quantum cousins would make any difference under these circumstances, a positive result in favour of a QRNG-based system would therefore strongly suggest an unexpected natural tendency towards some kind of order.

3.4 Proposals for more tailored versions of the experiment

While current neural-network architectures are designed to perform rather specific tasks, other more open architectures, perhaps not yet implemented, may be more appropriate to test the current proposal. Existing versions of artificial neural networks are usually designed to first learn and then produce an output based on that training process, but only in response to an external query. These systems are often colloquially referred to as exhibiting a kind of 'artificial intelligence', perhaps because of the impressive results they sometimes obtain. However, an additional property that is usually ascribed to the term 'intelligence' is the ability not only to react, but also to reason in the absence of an external request. Many would agree that this can sometimes lead to the spontaneous generation of new, even revolutionary and disruptive ideas not directly related to recent learning activities. As far as we know, the ability to freely reflect in the absence of a related external query is not implemented in current models, but should (and certainly will) be considered as a challenge for future software architectures. It may be that this would be the ultimate way to reveal a clear distinction between deterministic and quantum-random systems and to provide either an ultimate evidence, or indication of the contrary.

As challenging as the proposed experiment may seem to implement, it may not at all be necessary to use very large-scale, close-to-nature systems for rapid testing of our proposal. At the very least, we would only need a sufficiently complex dynamical structure with a close connection to the 'physical' world to allow some kind of interaction with it, whose behaviour can be influenced at key points by natural physical random processes at an elementary level, and whose response to external stimuli can be compared with corresponding mathematical software simulations based on known physical laws. It is therefore also not at all necessary to limit oneself to digital neural architectures. In principle, every highly complex chaotic or near-chaotic physical system [33] could be considered, provided that its unpredictability can ultimately be traced back to quantum uncertainty.

4 Concluding remarks

At this point, it should be allowed to take the thread further and speculate about the consequences of a successful experiment (an unsuccessful experiment could, of course, be used as a counterargument against our working hypothesis). If no other explanations for a positive outcome could be found, an unknown law of nature or principle of order should be brought into play. According to this, only 'natural' intrinsically indeterministic complex physical systems subject to quantum uncertainty can exhibit certain ordering phenomenona, while corresponding deterministic replicas cannot. Whether or not this would prove to be the manifestation of a principle of 'strong emergence' [3] should then be seriously considered, as such a qualification seems appropriate. It would then even be conceivable that the same principle also applies to natural systems such as living organisms. However, if life were in fact driven by, or even the result of such a process in its origins, it would most probably not be possible to draw any further teleological conclusions, and questions such as whether life has a predetermined purpose for its existence would remain unanswered. Perhaps it would simply like to be part of natural systems and try to survive in them. These and related ethical considerations could remain safely sheltered in the realm of philosophy, individual faith, and religion.

At the very least, conducting experiments on this or related topics in any form, whether successful or not, can help to shed some light on an ongoing unsatisfactory situation and, in the best case scenario, help to free the principle of emergence from its niche, where it is sometimes considered a placeholder for our ignorance and a way to avoid the hard work of finding a detailed physical clarification [63].

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