The PBR Theorem Requires No Preparation Independence

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

July 15, 2025

Abstract

The Pusey-Barrett-Rudolph (PBR) theorem proves that the joint wave function $\psi_1 \otimes \psi_2$ of a composite quantum system is ψ-ontic, representing the system's physical reality. We present a minimalist proof showing that this result, combined with the tensor product structure assigning ψ_1 to subsystem 1 and ψ_2 to subsystem 2, directly implies that ψ_1 and ψ_2 are ψ-ontic for their respective subsystems. This establishes ψ-ontology for single quantum systems without requiring preparation independence or other assumptions. Our proof challenges the widely held view that joint ψ-onticity permits subsystem ψ-epistemicity via correlations, providing a simpler, more direct understanding of the wave function's ontological status in quantum mechanics.

1 Introduction

The ontological status of the quantum wave function—whether it represents physical reality (ψ-ontic) or mere knowledge (ψ-epistemic)—remains a pivotal question in quantum mechanics [1]. The Pusey-Barrett-Rudolph (PBR) theorem proves that the joint wave function $\psi_1 \otimes \psi_2$ of a composite system is ψ-ontic, uniquely determining the system's physical state [2]. However, extending this result to single systems relies on the preparation independence postulate (PIP), which assumes uncorrelated ontic states for independently prepared subsystems. The widely held view suggests that relaxing PIP may allow subsystem wave functions to be ψ-epistemic, with distinct states sharing the same physical reality via correlations [3].

We present a minimalist proof that the PBR theorem's result for a composite system, combined with the tensor product structure assigning ψ_1 to subsystem 1 and ψ_2 to subsystem 2, directly implies that ψ_1 and ψ_2 are ψ-ontic for their respective subsystems. This establishes ψ-ontology for single systems without PIP or other assumptions, challenging the view that joint ψ-onticity permits subsystem ψ-epistemicity. Section 2 introduces the PBR theorem and PIP, Section 3 details our proof, and Section 4 addresses limitations of ψ-epistemic models, with implications discussed in Section 6.

2 The PBR Theorem: Joint ψ-Onticity and PIP

In ontological models of quantum mechanics, a physical system, which can be assigned to a wave function or pure state, is described by an ontic state $\lambda \in \Lambda$, which may include the wave function and additional hidden variables (e.g., particle positions in Bohmian mechanics). A wave function $|\psi\rangle$ is ψ-ontic if each ontic state λ corresponds to at most one quantum state, meaning the epistemic distributions $\mu_\psi(\lambda)$ and $\mu_\phi(\lambda)$ for distinct states $|\psi\rangle \neq |\phi\rangle$ have disjoint supports in the ontic state space Λ. This ensures that $|\psi\rangle$ is a physical property uniquely determined by λ. In contrast, $|\psi\rangle$ is ψ-epistemic if distinct states can share ontic states, allowing overlapping epistemic distributions [3]. In models with hidden variables, the ontic state may be $\lambda = (\lambda_\psi, \eta)$, where λ_ψ is the ψ-related part and η represents hidden variables. In a ψ-ontic model, the epistemic distribution takes the form:

$$\mu_\psi(\lambda) = \delta(\lambda_\psi - \psi)\nu_\psi(\eta), \tag{1}$$

where $\nu_\psi(\eta)$ is a distribution over hidden variables that may depend on $|\psi\rangle$, and distinct states $|\psi\rangle$ and $|\phi\rangle$ correspond to distinct λ_ψ, ensuring non-overlapping epistemic distributions.

The PBR theorem addresses the ontological status of the quantum wave function, specifically whether it is ψ-ontic or ψ-epistemic [2]. For a composite quantum system with Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$, the theorem considers product states $|\psi_1 \otimes \psi_2\rangle$, where $|\psi_1\rangle \in \mathcal{H}_1$ and $|\psi_2\rangle \in \mathcal{H}_2$ are prepared for subsystems 1 and 2, respectively. The PBR theorem proves that the joint wave function $|\psi_1 \otimes \psi_2\rangle$ is ψ-ontic, meaning the epistemic distributions $\mu_{\psi_1 \otimes \psi_2}(\lambda)$ and $\mu_{\phi_1 \otimes \phi_2}(\lambda)$ for distinct joint states $|\psi_1 \otimes \psi_2\rangle \neq |\phi_1 \otimes \phi_2\rangle$ have disjoint supports in the composite ontic state space Λ. For an ontic state $\lambda = (\lambda_\psi, \eta)$ with $\lambda_\psi = \psi_1 \otimes \psi_2$, the epistemic distribution is:

$$\mu_{\psi_1 \otimes \psi_2}(\lambda) = \delta(\lambda_\psi - \psi_1 \otimes \psi_2)\nu_{\psi_1 \otimes \psi_2}(\eta), \tag{2}$$

ensuring that each λ uniquely determines $|\psi_1 \otimes \psi_2\rangle$, making the joint wave function a physical property of the composite system.

The PBR theorem relies on PIP to extend ψ-onticity to single systems. PIP posits that when subsystems 1 and 2 are prepared independently, their ontic states are uncorrelated, such that the joint ontic state distribution factorizes:

$$\mu_{\psi_1 \otimes \psi_2}(\lambda_1, \lambda_2) = \mu_{\psi_1}(\lambda_1)\mu_{\psi_2}(\lambda_2), \tag{3}$$

where $\lambda_1 \in \Lambda_1$ and $\lambda_2 \in \Lambda_2$ are the ontic states of subsystems 1 and 2, respectively. The theorem tests ψ-epistemic models by considering multiple product states (e.g., $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$, $|+\rangle \otimes |0\rangle$, $|+\rangle \otimes |+\rangle$) measured in an entangled basis (e.g., $\frac{1}{\sqrt{2}}(|0\rangle|1\rangle - |1\rangle|0\rangle)$). In a ψ-epistemic model, distinct single-system states like $|0\rangle$ and $|+\rangle$ could share ontic states, leading to overlapping epistemic distributions $\mu_0(\lambda_1)$ and $\mu_+(\lambda_1)$. PIP ensures that the joint distributions $\mu_{0\otimes 0}(\lambda_1, \lambda_2) = \mu_0(\lambda_1)\mu_0(\lambda_2)$, $\mu_{0\otimes +}(\lambda_1, \lambda_2) = \mu_0(\lambda_1)\mu_+(\lambda_2)$, etc., also overlap if the single-system distributions do. This overlap predicts non-zero probabilities for measurement outcomes that quantum mechanics assigns zero probability, creating a contradiction unless the single-system states are ψ-ontic. Thus, PIP is critical to the PBR theorem's proof that $|\psi_1\rangle$ and $|\psi_2\rangle$ are ψ-ontic, as it prevents correlations between subsystems from allowing ψ-epistemicity [2].

The widely received view holds that the PBR theorem's joint ψ-onticity does not necessarily imply ψ-onticity for single systems if PIP is relaxed, allowing correlated ontic states across subsystems to permit ψ-epistemicity [3]. Models like that of Lewis et al. suggest that ψ-epistemicity for single systems is possible by introducing such correlations, though they did not show that their model fully reproduces quantum mechanics' entanglement measurement predictions [4]. Our minimalist proof, presented in the next section, challenges this view by demonstrating that ψ-onticity for single systems follows directly from the PBR theorem's result for composite systems and the tensor product structure, without requiring PIP.

3 A Direct Proof of Single-System ψ-Ontology

We prove that the subsystem wave functions $|\psi_1\rangle \in \mathcal{H}_1$ and $|\psi_2\rangle \in \mathcal{H}_2$ are ψ-ontic, using only the PBR theorem's result that the joint wave function $|\psi_1 \otimes \psi_2\rangle$ is ψ-ontic [2], and the tensor product structure of quantum mechanics, which ensures that $|\psi_1\rangle$ and $|\psi_2\rangle$ are associated with their respective subsystems in product states, without implying their ontological status (*i.e.*, whether they are ψ-ontic or ψ-epistemic for those subsystems).

The PBR theorem establishes that the joint state $|\psi_1 \otimes \psi_2\rangle$ is ψ-ontic, with the epistemic distribution:

$$\mu_{\psi_1\otimes\psi_2}(\lambda) = \delta(\lambda_\psi - \psi_1 \otimes \psi_2)\nu_{\psi_1\otimes\psi_2}(\eta), \tag{4}$$

in the composite ontic state space Λ, where $\lambda = (\lambda_\psi, \eta)$ and $\lambda_\psi = \psi_1 \otimes \psi_2$. This ensures that the physical reality of the composite system, described by λ, uniquely determines $|\psi_1 \otimes \psi_2\rangle$, with $\nu_{\psi_1\otimes\psi_2}(\eta)$ allowing variation in hidden variables. The tensor product structure of the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$ implies that the joint wave function $|\psi_1 \otimes \psi_2\rangle$ separates into $|\psi_1\rangle$ for subsystem 1 and $|\psi_2\rangle$ for subsystem 2. Correspondingly, the delta-distribution for the composite system can be decomposed as:

$$\delta(\lambda_\psi - \psi_1 \otimes \psi_2) = \delta(\lambda_{\psi_1} - \psi_1)\delta(\lambda_{\psi_2} - \psi_2), \tag{5}$$

where $\lambda_{\psi_1} = \psi_1$ and $\lambda_{\psi_2} = \psi_2$ are the ψ-related parts of the subsystem ontic states $\lambda_1 = (\lambda_{\psi_1}, \eta_1) \in \Lambda_1$ and $\lambda_2 = (\lambda_{\psi_2}, \eta_2) \in \Lambda_2$, respectively.

This decomposition reflects the separable nature of the product state and implies that the epistemic distribution for the composite system includes contributions from the ψ-related parts of each subsystem:

$$\mu_{\psi_1\otimes\psi_2}(\lambda) = \delta(\lambda_{\psi_1} - \psi_1)\delta(\lambda_{\psi_2} - \psi_2)\nu_{\psi_1\otimes\psi_2}(\eta). \tag{6}$$

For subsystem 1, we focus on the epistemic distribution over the ψ-related part λ_{ψ_1}, denoted $\mu_{\psi_1}(\lambda_{\psi_1})$, which is obtained by marginalizing over the hidden variables η:

$$\mu_{\psi_1}(\lambda_{\psi_1}) = \delta(\lambda_{\psi_1} - \psi_1). \tag{7}$$

Similarly, for subsystem 2:

$$\mu_{\psi_2}(\lambda_{\psi_2}) = \delta(\lambda_{\psi_2} - \psi_2). \tag{8}$$

The delta distributions $\delta(\lambda_{\psi_1} - \psi_1)$ and $\delta(\lambda_{\psi_2} - \psi_2)$ ensure that λ_{ψ_1} and λ_{ψ_2} are uniquely tied to $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. Thus, for distinct states $|\psi_1\rangle \neq |\phi_1\rangle$, the distributions $\mu_{\psi_1}(\lambda_{\psi_1})$ and $\mu_{\phi_1}(\lambda_{\psi_1})$ have disjoint supports in Λ_1, and similarly for subsystem 2, establishing that $|\psi_1\rangle$ and $|\psi_2\rangle$ are ψ-ontic.

This proof does not require assumptions about hidden variables or their correlations. Whether the ontic state is $\lambda = \lambda_\psi = \psi_1 \otimes \psi_2$ (no hidden variables) or $\lambda = (\psi_1 \otimes \psi_2, \eta)$ (with hidden variables), the PBR theorem's result ensures that $|\psi_1 \otimes \psi_2\rangle$ is uniquely determined by the composite system's physical reality via disjoint epistemic distributions. The decomposition $\delta(\lambda_\psi - \psi_1 \otimes \psi_2) = \delta(\lambda_{\psi_1} - \psi_1)\delta(\lambda_{\psi_2} - \psi_2)$ enforces that $|\psi_1\rangle$ and $|\psi_2\rangle$ are uniquely determined by their subsystems' ψ-related parts through $\mu_{\psi_1}(\lambda_{\psi_1}) = \delta(\lambda_{\psi_1} - \psi_1)$ and $\mu_{\psi_2}(\lambda_{\psi_2}) = \delta(\lambda_{\psi_2} - \psi_2)$, respectively. Hidden variables, if present, and their potential correlations are accounted for in $\nu_{\psi_1\otimes\psi_2}(\eta)$, but the disjointness of $\mu_{\psi_1}(\lambda_{\psi_1})$ and $\mu_{\phi_1}(\lambda_{\psi_1})$, and similarly for subsystem 2, depends only on the ψ-related parts λ_{ψ_1} and λ_{ψ_2}, ensuring ψ-onticity.

To summarize, ψ-ontology for single quantum systems can be established based on the PBR theorem's result about composite systems and the tensor product structure. The proof avoids reliance on PIP or assumptions about the independence of hidden variables. The conclusion that $|\psi_1\rangle$ and $|\psi_2\rangle$ are ψ-ontic holds regardless of correlations in hidden variables, as the decomposition of the ψ-related epistemic distribution and the tensor product structure enforce their role as physical properties of their respective subsystems, establishing ψ-ontology for a single quantum system in a direct and rigorous manner.

4 Failure of ψ-Epistemic Models

A notable attempt to construct a ψ-epistemic model, where distinct quantum states may share ontic states, is presented by Lewis et al. [4]. Their model is designed for single quantum systems and claims to reproduce quantum mechanics' Born rule for projective measurements. However, its limitations, particularly in addressing composite systems and entanglement measurements, mean it does not challenge our proof of subsystem ψ-ontology, which relies on the PBR theorem's ψ-onticity grounded in quantum mechanics' entanglement measurement predictions.

The Lewis et al. model defines an ontic state space $\Lambda = CP^{d-1} \times [0,1]$, where $|\lambda\rangle \in CP^{d-1}$ represents the ψ-related part (equivalent to the quantum state space, e.g., S^2 for qubits) and $x \in [0,1]$ is a hidden variable. For qubits, a preferred state $|0\rangle$ (north pole on the Bloch sphere) defines a hemisphere $\mathcal{R}_0$ ($\theta_\lambda < \pi/2$) and a subset $\mathcal{E}_0 = \{(\hat{\lambda}, x) : \hat{\lambda} \in \mathcal{R}_0, 0 \leq x < (1 - \sin\theta_\lambda)/2\}$, where θ_λ is the angle from $|0\rangle$. The epistemic state for a quantum state $|\psi\rangle \in \mathcal{R}_0$ is given by:

$$\mu_\psi(\hat{\lambda}, x) = \delta(\hat{\lambda} - \hat{\psi})\Theta\left(x - \frac{1 - \sin\theta_\psi}{2}\right) + \frac{1 - \sin\theta_\psi}{2}\mu_{\mathcal{E}_0}(\hat{\lambda}, x), \tag{9}$$

where $\mu_{\mathcal{E}_0}$ is a distribution over $\mathcal{E}_0$. This allows distinct states $|\psi\rangle, |\phi\rangle \in \mathcal{R}_0$ to share ontic states with different $\hat{\lambda} \in \mathcal{R}_0$, achieving ψ-epistemicity. The response function, $\xi_{\phi_k}(\hat{\lambda}, x) = \Theta\left[(|\langle\lambda|\phi_0\rangle|^2 - x)(-1)^k\right]$, ensures the Born rule for single-system projective measurements, where measurements are ordered relative to $|0\rangle$ (e.g., $|\langle\phi_0|0\rangle|^2 \geq |\langle\phi_1|0\rangle|^2$).

The model is explicitly constructed for single systems, reproducing quantum mechanics' Born rule for projective measurements. However, the PBR theorem, which our proof relies upon, leverages quantum mechanics' predictions for entanglement measurements in composite systems to establish ψ-onticity [2]. The Lewis et al. model does not provide a detailed framework for composite systems or specify response functions for joint measurements, particularly those involving entanglement, such as Bell-basis measurements critical to the PBR theorem.

The PBR theorem considers composite states like $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$, $|+\rangle \otimes |0\rangle$, and $|+\rangle \otimes |+\rangle$, measured in an entangled basis (e.g., $\frac{1}{\sqrt{2}}(|0\rangle|1\rangle - |1\rangle|0\rangle)$). If epistemic distributions overlap, a ψ-epistemic model predicts non-zero probabilities for outcomes quantum mechanics assigns zero probability, leading to a contradiction. In the Lewis et al. model, distinct states like $|0\rangle$ and $|+\rangle$ share ontic states in $\mathcal{E}_0$ with the same $\hat{\lambda} \in \mathcal{R}_0$. If extended to a composite system, the epistemic states for $|0\rangle \otimes |0\rangle$ and $|0\rangle \otimes |+\rangle$ could overlap, potentially predicting incorrect probabilities for entangled measurements. Lewis et al. suggest their model can be extended to composite systems by relaxing PIP, allowing correlated ontic states, but they provide no explicit construction demonstrating that it reproduces quantum mechanics' entanglement measurement predictions.

In addition, the model's reliance on a preferred state $|0\rangle$ to define $\mathcal{R}_0$ and $\mathcal{E}_0$ violates quantum mechanics' basis invariance, a principle requiring predictions to be consistent across all bases. Entanglement measurements, being basis-independent, may be disrupted by this preferred state, as response functions are ordered relative to $|0\rangle$. This further complicates the model's ability to address composite system predictions without redefinition for each basis, which is not provided.

Our proof relies on the PBR theorem's ψ-onticity for $\psi_1 \otimes \psi_2$, grounded in quantum mechanics' entanglement measurement predictions, and the tensor product structure to establish subsystem ψ-onticity. The Lewis et al. model's limitation lies in its failure to demonstrate that it can reproduce these predictions for composite systems. Without such a demonstration, it does not challenge the PBR theorem's conclusion or our proof's assertion that ψ_1 and ψ_2 are ψ-ontic for their subsystems.

5 Criticisms and Responses

5.1 Hidden Variables and Correlations

Criticism: The proof assumes that the decomposition of the delta distribution enforces ψ-onticity for subsystems, even in the presence of hidden variables η. However, if hidden variables introduce correlations between subsystems (as suggested by ψ-epistemic models like Lewis et al.'s), the disjointness of subsystem distributions might not hold. The paper dismisses this by claiming hidden variables do not affect the conclusion, but this is not rigorously justified. For example, if $\nu_{\psi_1\otimes\psi_2}(\eta)$ encodes correlations, the subsystem distributions $\mu_{\psi_1}(\lambda_1)$ and $\mu_{\psi_2}(\lambda_2)$ could overlap despite the delta distributions for λ_{ψ_1} and λ_{ψ_2}.

Response: This is incorrect. The PBR theorem establishes that $|\psi_1 \otimes \psi_2\rangle$ is ψ-ontic, with epistemic distribution $\mu_{\psi_1\otimes\psi_2}(\lambda) = \delta(\lambda_\psi - \psi_1 \otimes \psi_2)\nu_{\psi_1\otimes\psi_2}(\eta)$ (Section 3). The delta distribution decomposes as $\delta(\lambda_\psi - \psi_1 \otimes \psi_2) = \delta(\lambda_{\psi_1} - \psi_1)\delta(\lambda_{\psi_2} - \psi_2)$, fixing $\lambda_{\psi_1} = \psi_1$ and $\lambda_{\psi_2} = \psi_2$. Correlations in η, encoded in $\nu_{\psi_1\otimes\psi_2}(\eta)$, cannot alter λ_{ψ_1} or λ_{ψ_2}, as the delta functions enforce strict equality. Thus, for distinct states $|\psi_1\rangle \neq |\phi_1\rangle$, $\mu_{\psi_1}(\lambda_{\psi_1}) = \delta(\lambda_{\psi_1} - \psi_1)$ and $\mu_{\psi_2}(\lambda_{\psi_2}) = \delta(\lambda_{\psi_2} - \psi_2)$ have disjoint supports, ensuring ψ-onticity, as stated in Section 3. Hidden variables affect only η, not the ψ-related parts, preserving disjointness regardless of correlations.

5.2 Role of PIP

Criticism: The proof claims to avoid PIP, but the decomposition $\mu_{\psi_1\otimes\psi_2}(\lambda) = \delta(\lambda_{\psi_1} - \psi_1)\delta(\lambda_{\psi_2} - \psi_2)\nu_{\psi_1\otimes\psi_2}(\eta)$ implicitly assumes a form of independence or separability in the ontic states of the subsystems. If $\nu_{\psi_1\otimes\psi_2}(\eta)$ is not factorizable (i.e., if there are correlations), the proof's conclusion may not hold. The paper does not fully address how such correlations would be ruled out without PIP.

Response: The decomposition is a mathematical consequence of the tensor product structure of $\mathcal{H}_1 \otimes \mathcal{H}_2$, which assigns $|\psi_1\rangle$ to subsystem 1 and $|\psi_2\rangle$ to subsystem 2 for product states, and the PBR theorem's result that $|\psi_1 \otimes \psi_2\rangle$ is ψ-ontic (Section 3). The delta distribution $\delta(\lambda_\psi - \psi_1 \otimes \psi_2) = \delta(\lambda_{\psi_1} - \psi_1)\delta(\lambda_{\psi_2} - \psi_2)$ holds by definition for product states, requiring no assumption about the factorizability of $\nu_{\psi_1\otimes\psi_2}(\eta)$. Correlations in η do not affect the ψ-related parts λ_{ψ_1} and λ_{ψ_2}, which remain distinct for distinct states. The proof thus avoids PIP, relying solely on the tensor product and PBR result, as clarified in Section 3.

5.3 Lewis et al.'s Model

Criticism: The paper critiques Lewis et al.'s ψ-epistemic model for not addressing composite systems, but it does not construct a concrete counterexample where the model fails to reproduce the PBR theorem's predictions. A stronger rebuttal would require showing that any ψ-epistemic model for subsystems necessarily contradicts the PBR theorem's experimental predictions.

Response: The Lewis et al. model's structural limitations suffice (Section 4). Designed for single systems, it reproduces the Born rule for projective measurements but lacks response functions for composite-system entanglement measurements, such as Bell-basis measurements critical to the PBR theorem [2]. The model allows distinct states like $|0\rangle$ and $|+\rangle$ to share ontic states in $\Lambda = CP^{d-1} \times [0, 1]$, relying on correlations to violate PIP and permit ψ-epistemicity. However, it provides no framework for joint measurements, leaving unspecified how joint states like $|0 \otimes 0\rangle$ or $|0 \otimes +\rangle$ are represented or measured. Any extension to composite systems would require response functions that either reproduce quantum predictions (requiring ψ-onticity per the PBR theorem) or fail, contradicting quantum mechanics. The model's preferred state $|0\rangle$ violates basis invariance, further preventing compatibility with entanglement measurements, ensuring it cannot refute our proof's ψ-onticity claim.

6 Implications for ψ-Ontology

The widely held view posits that joint ψ-onticity may allow subsystem ψ-epistemicity through correlations between subsystem ontic states [3]. Our proof demonstrates that this is incorrect: the tensor product structure ensures that $|\psi_1\rangle$ and $|\psi_2\rangle$ are associated with subsystems 1 and 2, respectively, independently of their ontological status. Since $|\psi_1 \otimes \psi_2\rangle$ is ψ-ontic, with epistemic distribution $\mu_{\psi_1\otimes\psi_2}(\lambda) = \delta(\lambda_\psi - \psi_1 \otimes \psi_2)\nu_{\psi_1\otimes\psi_2}(\eta)$, the decomposition $\delta(\lambda_\psi - \psi_1 \otimes \psi_2) =$

$\delta(\lambda_{\psi_1} - \psi_1)\delta(\lambda_{\psi_2} - \psi_2)$ implies that $\mu_{\psi_1}(\lambda_{\psi_1})$ and $\mu_{\psi_2}(\lambda_{\psi_2})$ have disjoint supports for distinct states, establishing subsystem ψ-onticity without preparation independence.

The persistence of this view, 13 years after the PBR theorem [2], likely stems from a misconception that the epistemic distribution for $|\psi_1 \otimes \psi_2\rangle$ is not a delta distribution for λ_ψ due to mixing with hidden variables η. Since the full ontic state $\lambda = (\lambda_\psi, \eta)$ has a distribution $\nu_{\psi_1 \otimes \psi_2}(\eta)$ that is not a delta distribution, researchers may assume ψ_1 and ψ_2 could be ψ-epistemic. Our proof clarifies that the delta distribution $\delta(\lambda_\psi - \psi_1 \otimes \psi_2)$ ensures $\lambda_{\psi_1} = \psi_1$ and $\lambda_{\psi_2} = \psi_2$, enforcing ψ-onticity for subsystems regardless of hidden variables, thus resolving this misconception and reinforcing the wave function's physical reality.

7 Conclusion

We have demonstrated a minimalist proof that the PBR theorem's result—that the joint wave function $\psi_1 \otimes \psi_2$ is ψ-ontic—combined with the tensor product structure, directly establishes that ψ_1 and ψ_2 are ψ-ontic for their respective subsystems. This proves ψ-ontology for single quantum systems without preparation independence or other assumptions, correcting the widely held view that joint ψ-onticity permits subsystem ψ-epistemicity via correlations [3]. The Lewis et al. ψ-epistemic model, limited by its failure to reproduce quantum mechanics' entanglement measurement predictions, does not challenge our proof [4]. Our simpler proof strengthens the case for the wave function as a physical property of single systems, clarifying its ontological status in quantum mechanics.

References

[1] S. Gao, *The Meaning of the Wave Function: In Search of the Ontology of Quantum Mechanics*, Cambridge University Press, 2017.

[2] M. F. Pusey, J. Barrett, and T. Rudolph, "On the reality of the quantum state," *Nature Physics*, vol. 8, pp. 475–478, 2012.

[3] M. S. Leifer, "Is the quantum state real? An extended review of ψ-ontology theorems," *Quanta*, vol. 3, pp. 67–155, 2014.

[4] P. G. Lewis, D. Jennings, J. Barrett, and T. Rudolph, "Distinct quantum states can be compatible with a single state of reality," *Physical Review Letters*, vol. 109, p. 150404, 2012.