# Fundamentality and Disappearance of Spacetime: The Case of Quantum Gravity

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#### Abstract

Numerous theories of quantum gravity (QG) postulate non-spatiotemporal structures to describe physics at or beyond the Planck energy scale. This stands in stark contrast to the spatiotemporal framework provided by general relativity, which remains remarkably successful in low-energy regimes. The resulting tension gives rise to the so-called disappearance of spacetime (DST): the removal of spatiotemporal structures from the fundamental ontology of a theory and the corresponding challenge of reconciling this with the general relativistic picture. In this paper, I classify different instances of DST and highlight the necessary tradeoff between theory-specific features and general patterns across QG approaches. I argue that a precise formulation of the DST requires prior clarification of the relevant conception of fundamentality. In particular, I distinguish two forms of disappearance, corrisponding to intra-theoretic and inter-theoretic fundamentality relations. I argue that intra-theoretic analyses can yield meaningful results into the DST in QG only when supported by further justificatory arguments. To substantiate my claim, I examine the relationship between string theory, noncommutative geometry, and special relativity.

Keywords: Quantum gravity, Spacetime, Fundamentality, Disappearance of spacetime

# 1 Introduction

Quantum gravity (QG) denotes a family of research programmes aimed at elucidating the nature and dynamics of physical systems at extremely high energy scales (well beyond the reach of current experimental setups). The intended regime of applicability for theories of QG includes phenomena near or beyond the Planck scale. This speculative area of physical inquiry encompasses a wide range of alternative approaches that aim to both justify and, to certain extent, complement the achievements of general relativity (GR). The proliferation of alternatives can be attributed to a rapidly growing number of conjectures about the fundamental structure of reality in the quantum gravitational regime, all of which are typically constrained by the requirement that they reproduce GR in the low-energy limit.

In recent decades, the conjectural nature of research in QG has gathered the interest of numerous metaphysicians and philosophers of physics. Philosophical engagement with QG has taken two primary forms. First, philosophers have investigated the foundational and mathematical aspects of the new proposals, focusing on the construction of candidate theories of QG and their relationship to our current best physical theories. Second, they have explored QG as a case study in contemporary metaphysics. The physics of candidate QG theories at extremely short length scales has been used to motivate the proposal of novel ontological frameworks and new conceptions of the relationship between spacetime and matter. It is evident that there has been a considerable degree of overlap between the two approaches: formal developments in QG have often prompted new insights into traditional metaphysical debates, thereby necessitating revisions to existing accounts in order to remain consistent with contemporary physics.

A central topic that exemplifies the intersection of these philosophical approaches is the so-called *disappearance of spacetime* (henceforth abbreviated as DST). This term denotes a feature common to many leading approaches to QG, which postulate the existence of new non-spatiotemporal degrees of freedom and structures as fundamental constituents of the world. Specifically, these degrees of freedom are expected to dominate at or above the Planck energy scale. Consequently, a central aim of QG is to reconcile this novel conception of reality with the smooth, continuous spacetime described by GR.

The DST has been the subject of extensive and often divergent discussions across physics, philosophy and metaphysics. It raises a host of pressing research questions and generates a variety of conclusions that are not always easily reconciled. Specifically, the DST introduces deep epistemological concerns about the viability of theories of QG, as well as metaphysical conundrums regarding the nature of spacetime and the notion of fundamentality.

This paper investigates the DST in QG by examining its connection to the notion of fundamentality. Its primary aim is to distinguish between alternative formulations of the problem and to delineate the strategies deemed necessary for its resolution. I argue that a precise account of fundamentality is necessary to define the DST in concrete physical contexts, that is, in specific cases where theories of QG are said to exhibit this feature.

Furthermore, this paper challenges certain analysis of the DST in QG that conflate claims of fundamentality with ontological assertions about the status of spacetime. I contend that investigations into the DST must be divided into intra-theoretic and inter-theoretic approaches. These two types of investigations not only rely on different conceptions of fundamentality, but also address different research questions and require distinct methodological strategies. Identifying a specific investigative context, research question, and the relationship of the theory in question to neighbouring theories is essential for any claim about the DST in QG to be physically meaningful. This is, in fact, a necessary condition for the DST to inform and motivate research into potential solutions, including proposals for emergent spacetime.

To support these claims, this paper is structured as follows. Section 2 introduces the problem of the DST and its primary motivations within QG research. Section 3 examines how the DST is shaped by different conceptions of fundamentality. In particular, it distinguishes between intra-theoretic and inter-theoretic analyses and connects these to various ways of characterising what counts as fundamental. Finally, Section 4 investigates the relationship between these two types of analysis through a comparative case study of three theories: string theory, noncommutative geometry (NCG), and special relativity (SR).

# 2 Disappearance of Spacetime

Spacetime is often intuitively conceived as the set of all points at which physical events occur. These events include phenomena such as the flash of a light source or the scattering of particles. Spacetime theories aim to describe the structure of spacetime, that is, its geometry, while the ontological status of spacetime remains primarily a philosophical issue. In standard physical theories, spacetime geometry is often referenced either as a background structure for formulating relevant quantities (e.g., in QFT) or as a dynamical variable within the theory (e.g., in GR).

A natural conception that pervades much of the philosophical literature assigns a fundamental status to spacetime. Both space and time are viewed as ineliminable prerequisites for the existence and understanding of entities and phenomena. In this sense, they are regarded as fundamental structures. To illustrate this, (Sklar, 1983, 45) writes:

What could possibly constitute a more essential, a more ineliminable, component of our conceptual framework than that ordering of phenomena which places them in space and time? The spatiality and temporality of things is, we feel, the very condition of their existing at all and having other, less primordial, features. A world devoid of color, smell or taste we could, perhaps, imagine. Similarly a world stripped of what we take to be essential theoretical properties also seems conceivable to us. We could imagine a world without electric charge, without the atomic constitution of matter, perhaps without matter at all. But a world not in time? A world not spatial? Except to some Platonists, I suppose, such a world seems devoid of real being altogether.

This idea has been challenged in two key ways. First, the long-standing debate between substantivalists and relationists has raised questions about the fundamentality of spacetime versus that of material bodies. Substantivalists assert that spacetime is a fundamental structure, in that it is independent of material bodies. These bodies

may themselves be either as fundamental as spacetime or derivative structures, and thus non-fundamental, a view known as supersubstantivalism. This derivation relationship can be formalised using sophisticated approaches, such as mereology applied to location. In contrast, relationists argue that material bodies are fundamental, relegating spacetime to the derived ontology. According to this view, spacetime is not fundamental but must be reconstructed from the properties of material bodies

Second, contemporary physics has highlighted the breakdown of spatiotemporal notions in quantum gravitational regimes. It is generally agreed that GR, while providing an effective description of spacetime geometry at low energy, is not a fundamental theory.<sup>1</sup> This limitation implies that GR cannot account for, for example, the quantum corrections to the spacetime structure expected to arise at extremely high energies.

In contrast, presentations of QG often emphasise that QG theories are more fundamental than our current theories. By definition, a theory of QG is expected to offer a high-energy description of spacetime physics that complements GR in new regimes of applicability, while remaining compatible with GR in overlapping domains. This suggests that a novel theory of QG will describe a microscopic, high-energy structure near or beyond the Planck energy scale, which may differ radically from the geometry described by GR. However, this novel structure is expected to explain the success of GR as a low-energy theory.

Specifically, GR is understood to describe a spacetime structure that *approximates* the high-energy structure of QG. In other words, GR can be seen as a limiting case of the more fundamental theory of QG at low energies, and conversely, QG should reduce to GR under appropriate conditions. This implies that QG is *more fundamental* than GR in a certain sense, which needs to be further clarified. Similarly, the novel structure introduced by the theory of QG would be *more fundamental* than general relativistic spacetime. These conclusions raise important questions about the nature of this novel structure and the relationships between the respective fundamentality of GR and QG.

#### 2.1 DST: Whence and How?

In contrast to GR, a significant number of theories of QG reject the fundamentality of spacetime, thereby falling into the DST. This feature is motivated by deep expectations concerning the quantum gravitational regime, as well as considerations about the inter-theoretical relations between the theory under consideration and GR. In this regard, DST is primarily suggested by *definitional motivations* based on our expectations for the form that a theory of QG should take.<sup>2</sup> These motivations arise from the adoption of specific principles as heuristic guides for constructing the theory and addressing the problems it aims to solve. Some key definitional motivations include: (i) the formation of black holes, which prevent the sharp localisation of events (Bronstein, 1936/2012; Doplicher, Fredenhagen, & Roberts, 1995; Maresca, 2015); (ii) inconsistencies between the quantum and relativistic treatments of time (Isham, 1993); (iii)

<sup>&</sup>lt;sup>1</sup>For instance, GR is not UV-complete, therebt promoting the search for a new theory to complete it (see Crowther and Linnemann (2019)). Furthermore, GR cannot account for the quantum nature of matter, and therefore necessitates a complementary quantum theory. Finally, GR is arguably internally inconsistent, particularly in its failure to account for singularities. <sup>2</sup>I adapt the useful distinction between *definitional* and *external considerations* suggested by (Crowther,

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the emergence of uncertainty relations for core geometric quantities (Kiefer, 2007); (iv) violations of four-dimensionality, Lorentz signature, Riemannian metric, or background independence (Carlip, 2014; Girelli, Liberati, & Sindoni, 2009; Steinhaus & Thürigen, 2008); (v) analogies with condensed matter physics (Padmanabhan, 2014).<sup>3</sup> These motivations indicate that the new fundamental degrees of freedom in QG are expected to lack certain spatiotemporal features in order to describe specific domains of phenomena.

In addition to definitional motivations, there are *external motivations* for the DST in QG. These are unresolved issues or unexplained phenomena that arise in other physical theories and are expected to be addressed by a theory of QG. Some of these external motivations include: (i) the divergence problem (Hagar, 2014); (ii) singularities; (iii) black hole thermodynamics (Henson, 2009; Sorkin, 2005). While definitional motivations strongly suggest the fundamentality of a non-spatiotemporal structure, external motivations make a weaker case for non-spatiotemporality. This is because they inherit conceptual and foundational issues from their original theories, while also introducing new problems within the QG framework.

Definitional and external motivations point to contrasting conclusions. On the one hand, they suggest the elimination of spatiotemporal features from the fundamental domain of QG theories, supporting the view that most QG theories, despite their differences, may all exhibit DST.<sup>4</sup> On the other hand, the diversity of instances of DST challenges efforts to identify a common definition, pattern, or overarching problem across QG approaches (Jaksland & Salimkhani, 2023), thereby calling for a more nuanced classification. Specifically, DST can arise in three broad forms.

First, a theory may postulate a fundamental structure that does not satisfy one or more spatiotemporal features. These features include localisability, fourdimensionality, causal structure, etc. For example, in canonical QG, the background structure does not allow for the sharp localisation of events due to uncertainty relations between external and internal geometric structures (Kiefer, 2007, ch. 5). A similar argument applies to noncommutative approaches to spacetime, where the geometry depends on a noncommutative parameter, calling into question sharp localisation and causal structures (Lizzi, Manfredonia, Mercati, & Poulain, 2019).

Second, the theory may introduce fundamental structures that are ontologically different from spacetime itself. This is evidenced by theories that propose "atoms of spacetime" as fundamental structures. Such a discrete picture not only rejects the continuity of spacetime but also requires more advanced methods to recover GR in the appropriate limit. Techniques such as thermodynamical methods or the identification

<sup>&</sup>lt;sup>3</sup>Motivation (v), in particular, arises from the structural analogy between the relativistic description of gravity and the thermodynamic evolution of condensed matter systems. From a kinematical perspective, this analogy suggests that spacetime is a macroscopic approximation of a more fundamental, microscopic atomic structure. Physical microscopic effects are confined to a specific phase, with the system transitioning to the geometric, spatiotemporal phase under appropriate circumstances, which depend on the chosen approach. The heuristic contribution of this analogy to QG has been discussed in various contexts, including elasticity (Sakharov, 2000), thermodynamics (Jacobson, 1995), condensed matter physics (Bain, 2014; Oriti, 2014, 2022), and information theory (Verlinde, 2017).

<sup>&</sup>lt;sup>4</sup>This conclusion is contingent upon evaluating the argumentative strength of each motivation. Different approaches will likely disagree on the significance of these motivations, which could lead to theories of QG that retain spatiotemporality as a fundamental notion. However, such theories are typically limited compared to their non-spatiotemporal alternatives. Moreover, these theories is often limited to a perturbative description of the semi-classical regime.

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of relationships between graph and manifold structures are often employed (see, e.g., Carlip (2024); Oriti (2022)). This case is more radical than the first. While in the first case, fundamental structures fail to satisfy one or more spatiotemporal features, here, the structures fail to satisfy any spatiotemporal features at all, meaning they cannot be considered spatiotemporal in nature.

Third, the theory may explicitly exclude spacetime from the fundamental ontology. This scenario arises when a theory suggests that spacetime cannot even be included alongside non-spatiotemporal structures in the fundamental ontology. Typically, this is evidenced by the breakdown or indefinability of standard spatiotemporal structures at the relevant energy scale, signaling that spacetime itself is non-fundamental in this context.<sup>5</sup> This represents the most radical scenario, as it requires a comprehensive investigation into the entire fundamental ontology of the theory.

It is important to note that these three cases, while exemplified in theories of QG, are not exclusive to them. Certain reformulations of relativistic theories suggest that spatiotemporal features may be non-fundamental even in the classical context. Moreover, a theory may exhibit all three cases simultaneously. Specifically, the postulation of non-spatiotemporal structures implies a failure to satisfy a cluster of spatiotemporal features that are typically considered relevant for non-fundamental physics. In contrast, the rejection of spacetime is distinct from the introduction of non-spatiotemporal fundamental structures, making it a more radical scenario than the first two. The elimination of spacetime, or the absence of spatiotemporal structures in the fundamental ontology of the theory, is referred to as the *disappearance of spacetime*.

### 2.2 General vs Local Investigations of the DST

Instances of DST have been thoroughly addressed and discussed in recent philosophical literature (e.g. Callender and Huggett (2004); Carlip (2014); (Crowther, 2016, 13–15); Huggett and Wüthrich (2025); (Kiefer, 2007, Ch. 5); Oriti (2014); Padmanabhan (2014); (Rovelli, 2004, Ch. 10; 2009, 5–7); Wüthrich (2018)). These works share a general understanding of the DST and its implications. However, to the best of my knowledge, no precise formulation of DST has yet been provided.<sup>6</sup> This absence can be attributed to two primary reasons. First, the DST is often discussed alongside other distinct problems, such as the emergence of spacetime, empirical incoherence, or lack of physical salience. While these issues are interrelated, they are driven by different questions and require different solutions. Second, the DST has been examined from different philosophical perspectives, including both metaphysical and philosophy of physics inquiries. These investigations pursue different objectives and address various aspects of the theory or theories in question.

<sup>&</sup>lt;sup>5</sup>The literature on interpretations of QM highlights this issue as particularly prominent. For example, wave function realism excludes spacetime from the fundamental ontology, focusing instead on the wave function of the system and its configuration space. This is sometimes extended to the wavefunction of the entire universe. Consequently, wave function realism must address the challenge of recovering space as a derived entity to avoid empirical incoherence. Unlike QG, time is not included due to its complex role in QM.

QM. <sup>6</sup>The closest formulation of the DST is found in the classification of different levels of nonspatiotemporality in Oriti (2021) and Margoni and Oriti (in press). These works are valuable contributions to specifying the DST. However, they overlook a significant portion of the philosophical literature examining DST within isolated theories, often outside the scope of QG.

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Despite the diversity of approaches, there are three core aspects on which most discussions of DST converge. First, the DST raises profound foundational concerns for the theory in question. In typical cases, spacetime and its spatiotemporal features are essential for defining numerous structures. These include, for example, the spatiotemporal location of material bodies, the distinction between timelike and spacelike worldlines, or the evaluation of field quantities on spacetime regions. The removal of spacetime from the fundamental domain of the theory raises questions about the epistemological viability of that theory. Specifically, the challenge is how to reconcile a theory that posits non-spatiotemporal structures as fundamental with the preservation of the relevant features and structures that typically depend on a spatiotemporal background (the so-called *problem of the DST*). In other words, a theory without spacetime risks losing the definability of objects and concepts that, in the standard treatment, rely on a background spatiotemporal structure. Consequently, the new theory must "replace" the role of spacetime with new relations that are based on non-spatiotemporal structures.<sup>7</sup>

Second, the severity of the DST is a consequence of an unbridgeable discrepancy, or "gap," between spatiotemporal and non-spatiotemporal structures.<sup>8</sup> The two types of structures are often seen as incompatible and irreducible to one another.<sup>9</sup> However, I have emphasised that spatiotemporal features should ultimately be derivable from non-spatiotemporal structure, whether the theory is entirely classical (as in the case of relationism) or quantum (as in QG). This gap raises the crucial question of how to reconcile spatiotemporal and non-spatiotemporal structures in appropriate physical regimes. It is important to highlight that this issue may extend beyond the scope of physics into metaphysics, as it involves deep questions about the nature of spacetime itself.<sup>10</sup>

Third, another possibility is that the DST reflects an issue with the physical interpretation of the theory's mathematical formalism.<sup>11</sup> In the context of QG, this suggests

<sup>&</sup>lt;sup>7</sup>A similar problem already arises in standard theories such as QM. In that case, the classical phase space construction cannot be directly postulated due to the impossibility of defining pure states of both position and momentum. Within this context, the algebraic approach can be understood as an attempt to redefine the necessary objects and structures in the absence of a well-defined spatial framework.

<sup>&</sup>lt;sup>8</sup>For the permanence of spatiotemporal features in the quantum gravitational description of the highenergy regime, see Le Bihan and Linnemann (2019).

<sup>&</sup>lt;sup>9</sup>A possible interpretation of this "cognitive dissonance" involves identifying peculiar features that spacetime may possess in addition to its structure (see e.g. Le Bihan (2021)). These "spatiotemporal qualia" cannot be derived from non-spatiotemporal structures, making the dissonance unbridgeable. <sup>10</sup>Le Bihan (2021) distinguishes the scientific problem of recovering the general relativistic structure

<sup>&</sup>lt;sup>10</sup>Le Bihan (2021) distinguishes the scientific problem of recovering the general relativistic structure of spacetime from the ontological problem of identifying a suitable account of the spatiotemporal-nonspatiotemporal gap. He refers to the combination of these two aspects as the *hard problem of spacetime*. Specifically, metaphysical analysis is necessary to address the latter problem, though it is insufficient to solve the former. In particular, Le Bihan argues (but see also Baron and Le Bihan (2022b)) that a solution to the DST must remain neutral regarding the ontological status of spacetime. Furthermore, metaphysical intuitions can guide the investigation into the derivation of spatiotemporal structures, but they are insufficient to determine an appropriate solution.

<sup>&</sup>lt;sup>11</sup>To illustrate, consider non-relativistic QM, where wave function realism asserts that the wave function and its configuration space are fundamental, whereas spacetime is not. Consequently, spacetime must be recovered as a derived structure. In contrast, in Bohm's interpretation, spacetime is treated as a fundamental entity, so the problem of the DST does not arise. For further discussion, see Barrett (1996). This sensitivity to interpretation is also common in the philosophy of space and time. For example, while metric structures paradigmatically model spacetime, their chronogeometric meaning depends on preliminary interpretations. Different interpretations of mathematical structures entail distinct claims about spatiotemporality. For instance, the claim based on viable measuring rods and clocks (H.R. Brown, 2005) differs from one based on inertial reference frames (Knox, 2013).

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that the DST is not a direct consequence of the theory itself, but rather a limitation of the models we extrapolate from the formalism. Therefore, the DST might be contingent upon the specific interpretation chosen for the theory. Changing the interpretation could dissolve the problem. This raises the intriguing possibility that the DST is not an intrinsic feature of the theory but rather a result of our current interpretative framework.

However, it has been argued that the DST is robust under changes in interpretation (Wüthrich, 2018). In other words, modifying interpretations does not resolve the problem, since none of the potential interpretations can reinstate spatiotemporality. Therefore, the DST emerges as a genuine foundational feature of certain theories of QG. This distinguishes the DST in QG from similar challenges that may arise in GR and QM. The fact that this issue appears in numerous approaches to QG, despite their differences in technical apparatus, methodology, and interpretation, further underscores the distinctive nature of the problem.

On the other hand, it has been argued that the specific nature of instances of the DST precludes any meaningful examination of spatiotemporality in abstract terms. Jaksland and Salimkhani (2023), for example, criticise the abstract formulations and discussions of DST in the philosophical literature, contrasting them with those found in the physical literature. They argue that the DST is fundamentally tied to the problem of its emergence at low energy scales and that it encompasses a range of issues, each stemming from the absence of certain spatiotemporal features in the fundamental structure of the theory.

Discussions concerning the disappearance and emergence of spacetime in physics are highly context-dependent and embedded within specific theoretical frameworks. Therefore, a precise delineation of the pertinent spatiotemporal features is crucial for formulating the problem. On the one hand, this implies that abstract examinations of the DST fall short, as they rely on a vague notion: the DST marks a meaningful feature of the physical theories in question only if one can specify the exact list of spatiotemporal structures that are lost within particular investigative contexts. In this sense, context-dependence is a precondition for any formulation of the DST to be well-posed and physically significant.

On the other hand, this dependence reveals a methodological constraint: no privileged, "exceptional" formulation of the problem of the DST can be universally applicable or even formulated. Instead, the DST ultimately depends on the identification of the specific spatiotemporal features deemed problematic in the fundamental regime of a given theory. Consequently, generalised abstract formulations of the DST should be avoided.

One possible response to Jaksland and Salimkhani's critique is that the problem can still be analysed philosophically. A fruitful investigation would aim to uncover the conceptual substratum that is common to the various instances of DST, thereby enabling a more abstract discussion of the problem.

However, Jaksland and Salimkhani anticipate this suggestion and emphasise the ongoing disagreement regarding the precise set of features that define a structure as spatiotemporal. If this set cannot be definitively established at the outset, different accounts will select different features as spatiotemporal, leading to distinct claims

about the occurrence of DST. As a result, no single account can justify its particular set of features as the "correct" one. Ultimately, the problem of the DST is contingent upon the specification of the theory and conception in question, leading to different questions depending on the chosen set of features.

In conclusion, any proposed solution to the problem of the DST must clearly specify the set of properties deemed "spatiotemporal" and identify the specific theory under consideration. Without this specification, any claim of DST lacks epistemic or metaphysical content due to its vagueness. Specifically, it is of limited use as a motivation for solving the problem itself, including the identification of emergence processes for the spatiotemporal structures and features that were originally lost. Moreover, generalisations of the problem of the DST from specific theories and contexts to new ones may lead to incorrect conclusions, potentially complicating the efforts to reconcile spatiotemporal and non-spatiotemporal structures in QG.

# 3 Fundamentality and Spacetime

As illustrated, different instances of DST involve the elimination of spacetime or spatiotemporal features from the fundamental domain of the theory. The family of features considered spatiotemporal, and thus subject to elimination, must be specified from the outset. The recurring references to fundamentality in the examination of the DST underscore its central role in accommodating different instances. In other words, we expect fundamentality to play a significant role in explicating the DST in particular instances: spacetime must disappear from the fundamental domain and can only be admitted within the derived domain of the theory.<sup>12</sup> Consequently, an analysis of the notion of fundamentality promises to shed light on various cases of DST.

Despite the frequent use of "fundamental" in physics discussions and textbooks, the term is rarely specified in its technical use. This is in contrast with the growing centrality of fundamentality over the past decades in disputes ranging from metaphysics to the philosophy of science.<sup>13</sup> Fundamentality, in essence, is a comparative notion that relates two poles: its relata. It is usually associated to concepts such as primitivity, priority, independence, irreducibility, and terms like "unexplained explainer" or "all-God-has-to-do" (see, e.g., Tahko (2023)).

The relata of fundamentality can vary. For example, the fundamentality of entities differs from that of theories. The former may be fundamental because they are ontologically independent or undetermined, whereas the latter may be fundamental because they leave nothing unexplained within their regime of applicability.<sup>14</sup> This variety of uses creates significant ambiguity about whether fundamentality should be considered a unified or plural concept.

 $<sup>^{12}</sup>$ This does not apply if an eliminativist position is adopted, which would remove spacetime from the entire domain of the theory. Although theoretically feasible, this option is impractical from the perspective of physics due to the success of GR within its applicable regime. If pursued, this approach would require proof that spacetime is eliminated even in GR, without reference to the quantum gravitational regime.

<sup>&</sup>lt;sup>13</sup>In the philosophy of space and time, the long-standing debate between substantivalists and relationists has been re-conceptualised as a dispute over fundamentality rather than existence (Salimkhani, 2023). Likewise, Huggett and Wüthrich (2025) argue that the problem of the DST arises from many theories of QG negating the fundamentality of spacetime.

QG negating the fundamentality of spacetime. <sup>14</sup>Similarly, principles may be fundamental due to their indispensability, whereas explanations are fundamental when they offer a fine-grained *explanans*.

<sup>9</sup> 

Additionally, two distinct relations fall under the umbrella term "fundamentality." In the case of *absolute fundamentality*, an element is considered fundamental if it is complete, maximal, and independent. This is in contrast with non-fundamental elements and implies maximality, hence uniqueness. There can only be one absolutely fundamental element, the *fundamentalium*. Consequently, this relation induces a partition of the domain under consideration: the *fundamental domain* includes all fundamental elements, while its complement consists of *non-fundamental or derived elements*.

In contrast, relative fundamentality identifies one element as more fundamental than another.<sup>15</sup> This comparison can be repeated across elements within the domain of investigation. The resulting "chains of fundamentality relations" are directed from the more fundamental to the less fundamental and may not have a common root. Relative fundamentality also underpins claims about hierarchies or towers of stratified domains. In particular, this relation may indicate the presence of an absolutely fundamental element, if one exists. However, the reverse direction (from absolute to relative fundamentality) does not hold. Indeed, absolute fundamentality distinguishes between fundamental and non-fundamental elements, but is too coarse to specify the relative fundamentality among the derived entities themselves. Therefore, unless otherwise states, I will adopt the perspective of relative fundamentality in the following discussion.

Part of the literature treats fundamentality as a primitive notion. This is unhelpful in cases where disambiguating the different senses of "fundamental" is the primary concern. Instead, the relation of fundamentality can be specified by a range of related notions that define a precise partial ordering. In this context, a *conception of fundamentality* is a definition of the fundamentality relation in terms of a precise ordering relation. While the list of potential conceptions is vast, the principal contributions within the literature come from metaphysics and the philosophy of physics.

In general, conceptions of fundamentality are independent of specific instances of the DST. Nevertheless, they are necessary for framing the problem, which primarily concerns the replacement of spatiotemporal with non-spatiotemporal structures within the *fundamental* domain. Therefore, in the context of this paper, the goal of analysing fundamentality is to identify an appropriate ordering relation that enables the formulation of the DST in particular instances.

As demonstrated, instances of DST span across metaphysics, physics and philosophy of physics. Philosophers of physics may be concerned with its epistemological consequences, while physicists may interpret the problem as a prompt to seek strategies for recovering the spacetime structure at the appropriate scale. Metaphysicians, in turn, may investigate the DST both as a new scenario of intrinsic interest and as a challenge to certain conceptions of spacetime fundamentality. The diversity of approaches amplifies the concerns raised by Jaksland and Salimkhani regarding the multiplicity of irreducible problems of the DST.



 $<sup>^{15}\</sup>mathrm{A}$  formal presentation of relative fundamentality, emphasising the logical structure of the relation, can be found, e.g., in Correia (2021c).

Each of these approaches raises different questions and seeks different kinds of answers. Consequently, for any given theory, each approach will emphasise different spatiotemporal properties and structures that disappear from the fundamental domain, thereby producing different instances of DST. I contend that these instances depend on different and often incompatible conceptions of fundamentality that are assumed (often implicitly) by the investigator. In other words, the fundamentality of spacetime is ultimately contingent on specific choices of appropriate conception, which may conflict with one another. Such choices must take into account the specific context of application, namely, the problem being investigated and the expected epistemic outcomes. This context assigns a pivotal role to fundamentality and constrains which conceptions are appropriate. Compounded investigative questions that fail to distinguish the specific conceptions of fundamentality involved at each step can only yield misleading answers to the DST and should therefore be rejected as ill-posed.

### 3.1 Intra-theoretic

There are various approaches to the DST in the literature. One prominent approach investigates the DST as an intra-theoretic problem: I refer to this as *intra-theoretic* DST. This type of examination considers a single theory in isolation as its object of investigation. In its ordinary understanding, the theory ascribes a fundamental role to spacetime. However, upon further philosophical investigation and reformulation, spacetime is relinquished from its fundamental status and relegated to a derived structure. In other words, the spatiotemporal theory T is transformed into an empirically equivalent theory T' that rejects the fundamentality of spacetime.

In such cases, the DST does not arise from the introduction of new, nonspatiotemporal structures. Instead, it is a consequence of the reassignment of fundamentality relations within the ontology of the theory. The revised theory establishes new fundamentality relations that prioritise non-spatiotemporal structures over spacetime. However, it is crucial that this reformulation does not diminish the descriptive or predictive capabilities of the theory.

Metaphysical or interpretative considerations often motivate these intra-theoretic investigations. To illustrate this, consider the debate between subtantivalism and relationism as an example of intra-theoretic investigation. Substantivalism is compatible with spacetime fundamentalism.<sup>16</sup> Consequently, a substantivalist interpretation of GR may emphasise the dynamical role of spacetime models as solutions of the Einstein field equations, alongside with the possibility of empty spacetime. The substantivalist might argue that the dynamics of non-gravitating systems supervenes on the spacetime geometry, and thus is completely determined by it. Similarly, the substantivalist may argue that systems themselves can be ontologically reduced to the spatiotemporal regions they occupy. In this sense, substantivalism aligns with two conceptions of spacetime fundamentality: complete determination and ontological dependence.

In contrast, relationism assigns priority to matter fields over spacetime geometry. An interpretation of GR from a relationist perspective would emphasise the Machian

<sup>&</sup>lt;sup>16</sup>This may even be equivalent to spacetime fundamentalism. See e.g. (Salimkhani, 2023, 31).

<sup>11</sup> 

idea that it is the bodies that determine the geometry of spacetime.<sup>17</sup> Against spacetime fundamentalism, relationism implies that not only is the geometry of spacetime determined by the dynamics of matter fields, but spacetime itself is also ontologically reduced to these fields. In this view, the fundamentality relations identified by spacetime fundamentalism are reassigned, thereby relegating spacetime and spacetime geometry to derived structures. Therefore, in relationism, spacetime disappears from the fundamental ontology.<sup>18</sup>

It is important to note that this reassignment of fundamentality does not alter the theory of GR itself. In fact, GR remains invariant under reinterpretations motivated by metaphysical claims. In this context, the DST results from different assignments of fundamentality within the same theory's ontology. Specifically, both matter fields and spacetime belong to distinct ontological categories, both included in the ontology of GR. Consequently, the DST can be formulated by employing conceptions of fundamentality that emphasise inter-categorical relations (e.g., between spacetime and material systems) rather than inter-scale relations (e.g., between high- and low-energy structures).

Any intra-theoretic investigation of the DST necessitates the introduction of an appropriate conception of fundamentality. This provides a suitable framework to define which spatiotemporal features disappear and how they do so. A widely discussed proposal in metaphysics defines fundamentality in terms of *mereological dependence*: x is more fundamental than y if it is a proper part of y. Conversely, y is derived if it is constituted by, hence depends on, mereological simples or composites, i.e. it has proper parts.<sup>19</sup> In this context, spacetime can be fundamental as an uncomposed, simple structure. However, instances of DST imply that spacetime is composed of more fundamental parts, whether these are spatiotemporal regions or material bodies.

The mereological conception faces severe difficulties in the context of a classical spatiotemporal theory. If spacetime is fundamental, it can only be considered a "part" of bodies by an improper use of the word "part." More sophisticated approaches reverse the fundamentality relation and treat spacetime as a composite of spatiotemporal regions, which are identified as the locations occupied by more fundamental material objects. While this approach is intra-theoretic, it requires the justification of certain mereological principles, including harmony and inheritance. Furthermore, it cannot be applied to a theory of QG, as the introduction of non-spatiotemporal degrees of

<sup>&</sup>lt;sup>19</sup>This relation, in its extreme form, leads to *atomism*, which posits the existence of a fundamental set of simples. These "building blocks" possess suitable properties to constitute derived entities, often in terms of location or geometry. If this is the case, the building blocks "inherit" the property of being located within a certain region from the composite (*downward inheritance of location*). Furthermore, the inclusion relations between possible locations should align with the parthood relations between blocks and composite (*harmony*). See Baron and Le Bihan (2022a), as well as Gilmore, Calosi, and Costa (2024) for a formulation of these principles.



<sup>&</sup>lt;sup>17</sup>Mach specifically applied this insight to the determination of acceleration and rotation. See H. Brown and Lehmkuhl (2013) for a discussion on the relationship between Mach and GR.

<sup>&</sup>lt;sup>18</sup>Similarly, certain reinterpretations of the formalism of GR may imply intra-theoretic DST. As illustrated by Fletcher (2024), an analysis of fundamentality relations in GR based on mathematical determination and dependence raises the question as to whether the spatiotemporal models that solve Einstein's field equations can be considered fundamental. This contrasts with the relationism-substantivalism debate, where fundamentality relations cannot be "read off" the formalism. In this sense, mathematical dependence and determination guide the reconstruction of the fundamental domain of GR.

freedom makes traditional mereology inapplicable. Non-spatiotemporal parts violate the inheritance principles and cannot constitute spatiotemporal structures.<sup>20</sup>

Mereological fundamentality can be generalised to offer a second viable conception of fundamentality for intra-theoretic investigations: *ontological dependence*. According to this view, x is more fundamental than y if it is ontologically independent from y, whereas y ontologically depends on x. This relation highlights the directionality of dependence. It suggests a stratification of *quasi-autonomous domains*: entities at each level are ontologically dependent on those at a more fundamental level (e.g., due to constitution), but they also exhibit some degree of autonomy and novelty, which prevents the collapse of levels into one another.<sup>21</sup>

In this framework, spacetime is fundamental as a precondition or background for the existence of its occupiers, meaning that it is ontologically independent of bodies and matter. In contrast, the DST implies that spacetime is relegated to a derived entity and necessitates both the clarification of a more fundamental domain of structures and the characterisation of the dependence relation between them. In other words, this conception places spacetime in contrast to entities of a different ontological category. This is not an obstacle to intra-theoretic investigations, as both categories may be included within the same (two-sorted) ontology. For instance, GR is a theory of spacetime, but material bodies are also part of its ontology because of their contribution to the stress-energy tensor.

This conception also aligns with both traditional and supersubstantivalist views, though it has been criticised from a relationist perspective (see e.g. Salimkhani (2023)). This indicates that while ontological dependence may serve as a viable intra-theoretic conception of fundamentality, such relations cannot be inferred directly from the formalism. Any such inference requires specific translation principles that extrapolate ontological claims from formal relations, while also constraining the set of ontological claims to those compatible with the symmetries of the theory.

It is important to distinguish ontological dependence from a third conception: complete determination. This conception implies that x is more fundamental than y if y's properties are determined by x's properties, but not vice versa.<sup>22</sup> From an epistemological perspective, determination implies that x is sufficient to explain the properties of the derived entities, but x itself cannot be explained by anything else. It also implies that x is explanatorily indispensable: knowing the properties of the fundamentalium is necessary to know those of the derived entity. Consequently, spacetime is fundamental if it is necessary for knowing, understanding and explaining the properties of bodies and phenomena. In contrast, the DST implies that spatiotemporal features and structures are insufficient to determine, for example, the properties of matter fields.

 $<sup>^{22}</sup>$ Complete determination, as opposed to ontological dependence, arguably offers a conception of fundamentality that can be informed by scientific research. As argued by McKenzie (2019, 2022), scientific theories chart the determination relations between entities at different levels of fundamentality.



 $<sup>^{20}</sup>$ Le Bihan (2018) proposes a mereological bundle theory of space, viewing space as a partial structure. Spatial relations (the building blocks of space) are constituted by finer-grained, non-spatiotemporal structures. In contrast, Baron (2020, 2021), building on Lam and Wüthrich (2018) and Wüthrich (2018), argues that the mereological treatment of non-spatiotemporal fundamentalia is unwarranted. If viable, such an approach would first necessitate the introduction of a new, adequate mereological relation.

 $<sup>^{21}</sup>$ Ontological levels may include entities of the same or different categories. In the former case, fundamentality pertains to entities (or tokens) and is compatible with mereological composition. In the latter scenario, fundamentality relates categories (or types) and may violate the principles of mereological composition.

In metaphysical terms, determination has been understood in terms of grounding: x is more fundamental than y because y has its properties in virtue of x's properties.<sup>23</sup> The "in virtue of" relation signals a metaphysical (non-causal, synchronic) explanation of the properties of the non-fudamental. Grounding suggests the existence of an ungrounded fundamentum of objective facts, which is compatible with a stratified metaphysics that organises entities hierarchically.<sup>24</sup> As for spacetime, it is fundamental if it is necessary for grounding the properties of material bodies. In contrast, the DST inverts the grounding relation.

In summary, both ontological dependence and complete determination provide viable conceptions of fundamentality for intra-theoretic investigations of the DST. These conceptions offer precise definitions of fundamentality and specify how spacetime may fail to be fundamental upon reformulation of the theory.<sup>25</sup>

#### 3.2 Inter-theoretic

In contrast with the intra-theoretic case, conceptions of fundamentality that emphasise inter-scale relations over inter-categorical relations are appropriate for investigating the DST as an inter-theoretic problem: I refer to this scenario as *inter-theoretic DST*. This occurs when a new theory  $T_2$  is more fundamental than a previous spatiotemporal theory  $T_1$ , yet  $T_2$  denies the fundamentality of spacetime. In this context, the DST is a direct consequence of introducing new, non-spatiotemporal structures, rather than a re-examination or reassignment of fundamentality relations. Furthermore, the DST in this scenario arises from comparing the ontologies of *two* theories, with one being more fundamental than the other (in a sense of theory-fundamentality that must be appropriately specified).

While metaphysicians may prioritise intra-theoretic DST, I contend that it is the *inter-theoretic* DST that raises significant epistemological issues in the context of QG and can be informed by our physical perspective. Specifically, Section 2.1 emphasised the role of definitional and external motivations for the DST within the context of comparing two physical theories: GR and novel theories of QG. This indicates that a physically salient conception of fundamentality must take into account the scale-dependence of the structures under investigation, something precluded to intra-theoretic conceptions by definition.

This conclusion does not preclude the possibility that intra-theoretic and intertheoretic DST might coexist. However, when they do, it becomes important to

<sup>&</sup>lt;sup>23</sup>Grounding is defined as a relation between entities where the ground (the fundamental) is sufficient to metaphysically determine the grounded (the non-fundamental). This notion has been extensively discussed among metaphysicians in recent decades. For discussion, see e.g. Bliss and Trogdon (2021); Correia (2021a, 2021b); Correia and Schnieder (2012). For a naturalistic approach that accounts for the input of scientific theories, see Kortabarria and Giannotti (2024) and references therein.
<sup>24</sup>For indications on the long-standing problem of characterising grounding, see (McKenzie, 2022, 6–20)

<sup>&</sup>lt;sup>24</sup>For indications on the long-standing problem of characterising grounding, see (McKenzie, 2022, 6–20) and references therein. In particular, McKenzie argues that the relations of fundamentality decouple when moving from metaphysical research to actual scientific theories.

<sup>&</sup>lt;sup>25</sup>However, note that these conceptions may also lead to circularity. Specifically, ontological dependence necessitates a specification of the dependence relation between new fundamental non-spatiotemporal structure and non-fundamental spacetime. This specification is preliminary for this conception of fundamentality to be informative, providing an adequate background for the investigation. However, it also becomes a result of the investigation itself. Similarly, complete determination raises the question of how the *fundamentalium* determines the non-fundamental; otherwise, it remains a vague conception. Nevertheless, this specification emerges from an intra-theoretic investigation of the DST, rather than being a presupposition.

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distinguish the distinct conceptions of fundamentality involved in each phase of the investigation. Salimkhani (2023) offers an insightful example of the interplay between inter-theoretic and intra-theoretic DST. He defines the fundamentality of entities in terms of ontological priority, which is exemplified by the substantivalism-relationism debate. This debate raises the question of the ontological priority of spacetime over material structures: an intra-theoretic issue. In addressing this, Salimkhani challenges the prevailing spacetime fundamentalism suggested by substantivalism. To this end, he proposes spin-2 theory as a case of reduction of spacetime to non-spatiotemporal structures.

Salimkhani contends that, while the mathematical derivation of the metric g from the Minkowski metric  $\eta$  and the matter field h cannot directly indicate the fundamentality relations holding between the three (in terms of ontological dependence), a dynamical interpretation of spin-2 theory supports the view that h is the only ontologically independent structure. This intra-theoretic analysis concludes that h is more fundamental than both g and  $\eta$ , provided that dynamicism is accepted.<sup>26</sup> This interpretation of spin-2 theory brings the fundamentality relations to the limelight, while accounting for the demonstrated equivalence between the theory and GR. While this strategy aims to challenge spacetime fundamentalism, Salimkhani acknowledges that it does not establish spacetime non-fundamentalism as a definitive position but rather as an alternative: the specific dynamicist interpretation of  $\eta$  requires further support, which cannot be obtained solely within the theory.

In contrast, spacetime non-fundamentalism may prevail from an inter-theoretic perspective. QG, with its introduction of new non-spatiotemporal structures, already challenges spacetime fundamentalism through inter-theoretic DST. Salimkhani further contends that quantum spin-2 theory supports the fundamentality of non-spatiotemporal fields over the metric field, thus leading to the rejection of spacetime fundamentalism. He concludes that the reducibility of GR to quantum spin-2 theory implies the failure of spacetime fundamentalism in GR.

It is important to note that the final inference compounds intra- with intertheoretic DST. Indeed, the metric field may be non-fundamental in quantum spin-2 theory in terms of ontological dependence and determination, this does not entail that classical spin-2 theory will exhibit DST, nor that GR will exhibit DST by equivalence. The correct inference is that both quantum and classical spin-2 theory agree in rejecting spacetime within their respective regimes. In other words, intra-theoretic DST at high energies may not necessarily imply full inter-theoretic DST. In fact, QG points to the opposite conclusion when we consider the role of spacetime in GR. Therefore,

 $<sup>^{26}</sup>$ Salimkhani indicates the elimination of "miracles," the explanation of the chronogeometricity of g, and ontological parsimony as meta-theoretic motivations for a dynamicist understanding of classical spin-2 theory.

the inference from intra- to inter-theoretic DST requires additional argumentation to be fully supported.<sup>27</sup>

In the context of an inter-theoretic investigation, appropriate conceptions of fundamentality must account for the comparison between the structures described by a less fundamental theory, such as GR, and those introduced by a more fundamental theory, such as a theory of QG. One possible approach is to define fundamentality in terms of *energy scales*. By definition, x is more fundamental than y if x corresponds to a higher energy scale than y.<sup>28</sup> This definition is well-suited to physical practice, as each physical theory is associated with a specific energy scale that characterises the phenomena within its domain of applicability. These phenomena only emerge once a certain energy threshold is surpassed. In this framework, the new structures posed by a theory of QG are considered more fundamental than spacetime, as they are defined at an energy scale much higher than the general relativistic spacetime. Consequently, spacetime is considered non-fundamental.<sup>29</sup>

This energy-scale conception of fundamentality induces a partial ordering of theories according to their respective regimes of applicability. In general, it is important to keep conceptions of theory-fundamentality distinct from those of entity-fundamentality, as the two may not necessarily align. For example, it makes no sense to claim that a theory is a mereological part of another, and thus more fundamental. Energy scales, however, provide an notable exception: a theory  $T_1$  can be more fundamental than a theory  $T_2$  if the domain of  $T_1$  includes higher-energy phenomena and structures than  $T_2$ 's domain. This organisation allows for a hierarchy of related theories based on matching conditions.<sup>30</sup> For instance, a theory of QG is more fundamental than GR, as it postulates higher-energy structures than relativistic spacetime.

This hierarchy of theories can be further developed using a conception of theoryfundamentality based on *theory reduction*. Here, a theory  $T_1$  is considered more fundamental than  $T_2$  if it provides a more basic description of a system or phenomenon than  $T_2$  does. This implies that the successful parts of  $T_2$  can be derived, in principle, from  $T_1$  within the relevant domain, under appropriate conditions and approximation

<sup>&</sup>lt;sup>27</sup>Salimkhani is well aware of this difficulty and addresses it by first examining the functions of unification and continuity between theories in determining ontological commitments. In 2023, Ch. 6, he argues that metaphysical commitments should be selected to satisfy continuity conditions between relevant theories. In this context, unification extends the criteria of continuity to compatibility with all related theories. Consequently, metaphysical commitments, especially the analysis of fundamentality relations, should also account for the interpretation of neighbouring theories. He concludes that spacetime fundamentalism can be rejected based on meta-induction from the ontological commitments of well-confirmed theories, as it fails to satisfy the continuity criteria.

to satisfy the continuity criteria. <sup>28</sup>This conception is inspired by the use of renormalisation group methods in the effective field theory (EFT) approach to physical theories. The EFT approach views physical theories as effective descriptions of reality, each associated with a certain energy level. When the theory reaches the boundaries of this energy level, it breaks down due to non-negligible higher-energy effects. Note that this approach is designed specifically for quantum field theories that satisfy a decoupling condition: phenomena at different energy scales must not influence each other. It is debated whether theories of QG generally satisfy this condition, and thus whether they can be treated as EFTs. Nevertheless, "informal" or "practical" applications of the approach are still possible (see e.g. Crowther (2016)). <sup>29</sup>A hierarchical tower of structures can be built based on the energy scales associated to each structure.

<sup>&</sup>lt;sup>29</sup>A hierarchical tower of structures can be built based on the energy scales associated to each structure. This tower is neutral regarding the existence of a final, absolutely fundamental structure (Crowther, 2016, 78). The relevant conception of fundamentality here is one of relative fundamentality. <sup>30</sup>In the EFT approach, renormalisation group methods are employed to describe how the parameters of a structure of the structure. <sup>29</sup>A hierarchical tower is neutral regarding the existence of a final, absolutely fundamental structure (Crowther, 2016, 78). The relevant conception of fundamentality here is one of relative fundamentality. <sup>30</sup>In the EFT approach, renormalisation group methods are employed to describe how the parameters of the structure. <sup>29</sup>A hierarchical tower is a structure of the structure. <sup>29</sup>A hierarchical tower is a structure of the structure. <sup>20</sup>A hierarchical tower is a structure. <sup>20</sup>A hierarchica

<sup>&</sup>lt;sup>30</sup>In the EFT approach, renormalisation group methods are employed to describe how the parameters of theories are transformed as one transitions across energy scales. It is important to note that, in contrast to ontological determination and dependence, these methods can only organise theories that describe systems within the same category. In other words, this fundamentality relation is intra-categorical, as opposed to inter-categorical.

<sup>16</sup> 

techniques. In other words, the theory  $T_1$  reduces to the theory  $T_2$  in the appropriate limit.<sup>31</sup> In this case, the novel theory of QG is more fundamental than GR as it recovers GR in the appropriate limit.

This conception of theory-fundamentality implies that the laws of GR depend asymmetrically on the quantum gravitational physics. Furthermore, the domain of the new theory of QG must include the domain of GR (domain subsumption), ensuring that the reducing and the reduced theories are compatible within overlapping domains. This makes the less fundamental theory an approximation of the more fundamental one.

Finally, a stronger conception of theory-fundamentality, relevant for inter-theoretic investigations, is provided by the notion of a *final theory*. This conception links fundamentality to a broader spectrum of features and inter-theoretic relations that an absolutely fundamental theory should satisfy, in addition to theory reduction. Crowther (2019) offers an in-deep analysis of this conception, emphasising that to be absolutely fundamental, a theory must be explanatorily complete. This means that a fundamental theory should provide a comprehensive description of its domain of applicability and offer a satisfactory explanation of the phenomena it describes. In Crowther's words, "a fundamental theory [must] not leave anything apparently in need of explanation" (128). Therefore, a theory that fails to meet this criterion is considered non-fundamental.

Crowther's analysis draws upon physicists' perspectives on fundamental research, suggesting nine conditions that may indicate a theory's fundamentality. A theory of QG is expected to meet these conditions, making it more fundamental than GR in a stronger sense than simple theory reduction. However, Crowther argues that while these criteria are compelling, they are insufficient by themselves to define absolute theory-fundamentality. Instead, they are jointly necessary only in absence of contrary evidence. Current theories have not yet attained this objective, thus motivating physicists to "keep digging" in their pursuit of more fundamental theories. *Contra* Crowther, it has also been argued that these criteria are limited in scope, especially in the case of relative theory-fundamentality (Morganti, 2020). Additionally, it can be argued that theories of QG may be less fundamental than a potential "final theory," should such a theory ever be found.

### 3.3 From Intra- to Inter-theoretic DST

The distinction between intra- and inter-theoretic investigation of the DST is crucial. Significant differences arise not only in the possible conceptions of fundamentality that can provide a backdrop for the examination of the DST, but also in the focus of the investigation itself: whether it concerns a single theory or the relation between two. I contend that each type of investigation may contribute differently to the conclusions one seeks to defend through this analysis.

<sup>&</sup>lt;sup>31</sup>Nickles (1973) defines this case as reduction<sub>2</sub>. It is distinct from reduction<sub>1</sub>, or Nagel-Shaffner reduction, where the less fundamental theory  $T_2$  reduces to  $T_1$  since it is derivable from it by means of appropriate bridge laws. Crowther (2018) argues that theory reduction can be seen as the endeavour to demonstrate the in-principle derivability of one theory from another. This establishes the dependence relation by showing how the less fundamental theory derives from the more fundamental one. Notably, theory reduction does not inherently conflict with the notion of energy scale as a conception of fundamentality. Indeed, there is considerable overlap between the two concepts: see e.g. Castellani (2002).

For instance, I have argued that the substantivalism-relationism debate pertains to intra-theoretic DST. This is due to the fact that relationists reject the fundamentality of spacetime, in terms of both ontological dependence and complete determination, whereas inter-theoretic DST is irrelevant to this problem. In other words, in the substantivalism-relationism debate does not involve a comparison with another theory; it is an issue confined to the internal structure of a single theory (in this case, GR). In contrast, QG introduces an inter-theoretic problem of DST. The core issue here is how to accommodate the elimination of spatiotemporal degrees of freedom from the fundamental ontology of the theory, while still recovering spacetime and spatiotemporal features at low energy and ensuring correspondence with GR.

While an intra-theoretic analysis of the DST for QG is viable, I emphasised that it does not directly inform the inter-theoretic problem unless further links can be established. Specifically, in order to extend intra-theoretic DST to an inter-theoretic context, it is first necessary to postulate appropriate mediating principles that connect the structures spanned by the fundamentality relations in the two theories. This requires that an intra-theoretic analysis of fundamentality and DST be sufficiently developed. This can be a highly nontrivial task. For instance, consider the case of GR and QG.

In the case of GR, successful intra-theoretic investigations of fundamentality relations have already been provided in the literature. For example, (Fletcher, 2024, 24–37) argues that any interpretation of GR should allow for a correspondence between the formal structures of the theory and the ontology associated to potential interpretations. In this context, the correspondence applies specifically to the dependence relations between elements of the formalism, which are intended to reflect fundamentality relations within the ontology of various relativistic models.

Fletcher contends that the metric tensor is absolutely fundamental (independent of any other element of the theory), yet it is insufficient to form a complete minimal basis for deriving the rest of the theory. To complete the fundamental basis, matter fields must be introduced to achieve a conception of complete determination. By contrast, adding the cosmological constant to the metric field produces a complete minimal basis, but the pair may no longer fundamental as independent of the other elements of the theory.<sup>32</sup>

In contrast, the determination of intra-theoretic fundamentality relations for a theory of QG is highly demanding. This analysis presupposes a suitable interpretation of the formalism, which is often either absent or incomplete. Not only may there be disagreement about the physical significance and implications of certain formal structures, leading to disagreements about the possible models of the theory, but the theory itself may also be underdeveloped at the time of the investigation. This means that the map of fundamentality relations may remain incomplete.

Still, an inter-theoretic investigation is necessary to address the aforementioned hard problem of spacetime. As emphasised by Le Bihan (2021), this issue does

 $<sup>^{32}</sup>$ This conclusion is contingent on possible reformulations of the Einstein field equations. Indeed, certain reformulation may eliminate the cosmological constant without changing its value, e.g. by sending it to zero. The legitimacy of these reformulations, however, is debated. For alternative intra-theoretic investigations of the structure of fundamentality relations of GR, see e.g. the constructive approach of EPS.

<sup>18</sup> 

not concern the mere classification of relevant degrees of freedom as spatiotemporal or non-spatiotemporal, but rather the identification of specific *links* between the two types of structures. In the context of QG, these links explicitly depend on the energy scale at which the relevant structures are defined. A mathematical derivation of general relativistic structures provides valuable insight into the relationship but remains insufficient to explain how loe-energy phenomena supervene on the underlying non-spatiotemporal physics. In this context, specifying an appropriate inter-theoretic conception of fundamentality can be expected to advance the investigation of this inter-categorical and inter-theoretic connection, complementing the mathematical derivation of GR from the QG theory under examination.

One might object that a significant portion of these kinds of investigation could be conducted from the point of view of GR. For instance, one could use the structure of fundamentality relations in GR to inform the investigation of the new theory of QG. This approach could be justified by the expectation that the two theories will agree within overlapping regimes. However, the intra-theoretical analysis of QG cannot rely heavily on that of GR. This is because the analysis of fundamentality relations for the novel theory of QG would be compromised from the outset. In other words, an intratheoretic investigation of GR can only serve as a heuristic tool for investigating QG, not as a solid foundation for its intra-theoretic analysis.

Furthermore, the comparison between GR and a theory of QG may still be hindered by the absence of isomorphic substructures.<sup>33</sup> In fact, the two sets of fundamentality relations may be incomparable. For example, the manifold structure is considered fundamental in most formulations of GR, but discrete theories of QG remove it from their ontologies, thereby severing the corresponding fundamentality relations. It may prove impossible to find a substitute for the manifold structure that preserves isomorphic fundamentality relations. Similarly, certain noncommutative spacetime models obstruct direct comparisons (or isomorphisms) with SR or GR unless supplementary mediating principles are provided (see Section 4.2).

In conclusion, the case of QG suggests that an analysis of intra-theoretic fundamentality may not offer reliable insights into inter-theoretic comparability. This is particularly significant in the context of inter-theoretic reduction, both between quantum gravitational and relativistic theories, and between different approaches to QG. To further clarify this claim, I will examine the relationship between string theory, noncommutative geometry, and SR as a case study in the next section.

# 4 A Case Study

In the preceding sections, I examined the relationship between fundamentality and DST from an abstract point of view. However, I also emphasised (in agreement with Jaksland and Salimkhani) the necessity of considering actual instances of DST from the physical literature. This is crucial for two reasons. First, as Jaksland and Salimkhani point out, different instances of DST lead to irreconcilable conceptions of DST. Second,

<sup>&</sup>lt;sup>33</sup>Although this may not be a problem for certain accounts, such as the functional reductionists. Indeed, spacetime functionalism must establish by fiat that such a substructure can be found, at the cost of otherwise failing to realise important microscopic functions at the intended low-energy scale. See, e.g., Butterfield and Gomes (2023); Lam and Wüthrich (2018).

<sup>19</sup> 

an analysis of various theories of QG reveals that fundamentality relations cannot be directly inferred from the formalism alone, which adds complexity to the philosophical investigation.

A key case study in this regard is the relationship between string theory, NCG and SR. From an intra-theoretic perspective, these three theories provide very different depictions of the fundamentality of spacetime. Consequently, they give rise to different interpretations of DST. Nevertheless, there are also clear inter-theoretic connections among them. Specifically, the derivability of a relativistic theory in the appropriate regime suggests that spacetime is expected to "reappear" under appropriate conditions. As will be illustrated below, these interconnections highlight the intricate relationship between intra- and inter-theoretic notions of fundamentality and DST.

### 4.1 From String Theory to NCG

String theory is currently one of the most successful and prominent approaches to QG. Its description of the dynamics of d-dimensional strings provides a unified framework that incorporates both the fundamental forces of the Standard Model and gravitational interaction. As such, it has attracted considerable interest from philosophers of physics, particularly regarding issues such as its historical development (Cappelli, Castellani, Colomo, & Di Vecchia, 2012), confirmability (Dawid, 2013), and the status of spacetime ((Huggett & Wüthrich, 2025, Chs. 7–10); Vistarini (2019)).

The status of spacetime in string theory is, broadly speaking, a subject of controversy. While string theory does indeed contain a notion of spacetime, this is not without complication. One of its key predictions is the derivation of a spin-2 particle, the graviton, which is associated with the gravitational interaction. Additionally, it has be demonstrated that Einstein field equations, the fundamental equations of GR, can be derived within the framework of string theory in the low-energy limit. However, it remains debated whether string dynamics can be understood as the evolution of strings embedded in a fixed background spacetime, thereby postulating spatiotemporal structure from the outset.

Some scholars argue that, despite containing spacetime, string theory does not treat spacetime as a fundamental structure. First, the graviton is not a primitive structure of the theory, but rather the result of specific vibrational modes of massless closed strings in a quantum setting. Second, the Einstein field equations can only be derived in a specific low-energy limit, suggesting that string theory reduces to GR under those conditions (in the sense of Section 3.2). Finally, while strings are embedded in a *target space*, this space does not coincide with spacetime (Huggett, 2017). The action of a symmetry called T-duality exchanges wave numbers on a space of radius R with winding numbers on a smaller space of radius  $l_P/R$  (where  $l_P$  is the string length). Therefore, spacetime can only be recovered as a derived structure from the interactions of strings within target space (Huggett and Wüthrich (2025)).

Alternatively, string theory can be formulated within the *metastring framework*, which seeks to avoid the ambiguity introduced by T-duality in understanding the nature of target space (Freidel, Leigh, & Minic, 2015, 2017). In metastring theory, target space is treated as a fundamental structure of the theory, endowed with a more

complex geometric structure known as *Born geometry*. This construction is inspired by the analogy with the phase space in QM, where the target space in metastring theory takes on the structure of phase space for string kinematics.<sup>34</sup>

Metastring theory relies on three key geometric structures. First, the *symplectic structure* governs the noncommutativity of coordinates in phase space. These coordinates are dimensionless quantities<sup>35</sup> that induce translations over phase space, forming the noncommutative structure of a Heisenberg-Weyl group, similar to QM.

Second, the *polarisation metric* allows the decomposition of phase space into two subspaces. These subspaces must satisfy specific conditions dictated by the polarization metric.<sup>36</sup> This construction mirrors the distinction between position and momentum subspaces in quantum mechanical phase space. One of these subspaces is identified as the spatiotemporal structure.

Finally, a quantum metric enables the computation of probabilities within the quantum description of metastring states.

It is important to note that the spacetime structure in metastring theory is determined by substructures in phase space called *modular cells* or *spacetime qubits*. These cells consist of sets of commuting operations (procedures that can be performed simultaneously without associated uncertainty) and exhibit superposition and entanglement.

From an intra-theoretic perspective, metastring theory defines its fundamental domain to include both the action for the metastring and the target space, along with the Born geometric structure. Notably, phase space differs from spacetime: spacetime is a special submanifold of phase space. As such, spacetime in metastring theory is a *derived structure*: not only is it defined by specific conditions on the polarisation metric, but its metric structure is also determined by restrictions of the quantum metric, governed by the information encoded in the modular cells. In this sense, spacetime fails to meet the conception of complete determination and is not ontologically independent. Specifically, spacetime is not a primitive element in the theory, but a derived feature of the geometric structure.

Spacetime it also not fundamental in the mereological sense. While it is a subspace of the target space, it is obtained only through restriction. Consequently, it would be incorrect to think of target space as the mereological composition of spacetime and its transverse dual. Rather, the perspective should be reversed: spacetime is derived by restricting or decomposing a more fundamental structure. Therefore, the mereological conception does not fully capture the relationship between spacetime and target space. Even if we accept a mereological interpretation, spacetime remains derived rather than fundamental.

In conclusion, while spacetime is included in both string and metastring theory, it is not fundamental in either frameworks. In both cases, spacetime "disappears" from the fundamental ontology, though in a very specific sense of "disappearance."

<sup>&</sup>lt;sup>34</sup>In particular, the Born geometry equips the target space with a suitable structure, so that T-duality acts linearly on string states.
<sup>35</sup>By definition, they are defined as the ratio between position and momentum coordinates with two

By definition, they are defined as the ratio between position and momentum coordinates with two fundamental scales, one of length and one of energy, respectively. These quantities are built into the theory as postulates. <sup>36</sup>In particular, the resulting subspaces are Lagrangian. This means that they possess maximal dimension

<sup>&</sup>lt;sup>30</sup>In particular, the resulting subspaces are Lagrangian. This means that they possess maximal dimension and vanishing symplectic structure.

<sup>21</sup> 

Both string and metastring theory are also closely related to a distinct theory, NCG, which adopts an algebraic approach to describing spacetime structure and field dynamics (see, e.g., Aschieri et al. (2005); Connes and Marcolli (2008); Szabo (2006)). In NCG, geometric information is encoded in a suitable algebra of operators, with a noncommutative parameter introduced at the product level. This means that for elements f and g of the algebra,  $fg \neq gf$ : the order of composition matters.

String theory derives noncommutative structures under specific circumstances. For example, Seiberg and Witten (1999) showed that open string theory reduces to a noncommutative Yang-Mills theory when a constant, nonzero B-field is introduced as a background.<sup>37</sup> This noncommutative theory emerges in the zero-slope limit, where the string length approaches zero. In contrast, metastring theory naturally incorporates noncommutative structures at the level of phase space coordinates, particularly through the Heisenberg group of translations, which reflects the concept of relative locality (one of the postulates of metastring theory). However, noncommutativity is removed when modular variables are used to describe the spatiotemporal substructure.

In both string and metastring theories, NCG intersects with the original theory. In string theory, it arises in an appropriate limit, whereas in metastring theory, it is intrinsic to the theory without the need for any approximation. Nonetheless, both frameworks reject the fundamentality of spacetime. In NCG, it is indeed debated whether spacetime retains its fundamental status. Some argue that the algebraic structure itself is fundamental, and that spacetime is only recovered under appropriate circumstances, such as at low energy. If the noncommutative parameter represents an energy scale, spacetime is excluded from the domain of NCG as an ill-defined structure and can only be recovered at lower energies.<sup>38</sup>

For example, in the case of noncommutative Yang-Mills theory, it remains unclear whether the noncommutativity arises from a genuine feature of the background structure (e.g., at the Planck energy) or from the composition of classical field quantities. In contrast, the Heisenberg-Weyl algebra of translation has no direct spatiotemporal representation: it acts on metastring states across the entire phase space, not just on the spacetime substructure.

### 4.2 From NCG to Relativistic Theories

As illustrated, NCG does not have a direct spatiotemporal interpretation, primarily because its fundamental objects are algebraic in nature. Furthermore, the classical duality between algebraic and geometric construction, known as Gelfand duality, which allows translation from one type of model to the other, fails in the noncommutative context. Nevertheless, NCG has resurfaced over the last thirty years as a candidate theory of QG, reintroducing the problem of the status of spacetime within a new algebraic framework.

 $<sup>^{37}\</sup>mathrm{The}$  B-field generalises the electromagnetic field in the framework of two-dimensional worldsheets.

<sup>&</sup>lt;sup>38</sup>See Huggett, Lizzi, and Menon (2021). It is important to note that this result relies on the use of *commutative* spatiotemporal notions. Consequently, Huggett, Lizzi and Menon are also called to prove that this is the only possible characterisation of the spatiotemporal structure, and that no more general characterisation can be identified that also encompasses NCG. A discussion of their conclusion is beyond the scope of this paper and is left for future work. For the reconstruction of the spacetime structure, see Section 4.2.

<sup>22</sup> 

According to the algebraic perspective, algebraic structures are fundamental (see, e.g., Menon (2019)). However, different noncommutative geometric approaches prioritise distinct structures. For instance, Connes' spectral triple approach emphasises algebras of operators,<sup>39</sup> while the quantum group approach focuses on the algebra of quantum symmetries as the fundamental structure of the theory.<sup>40</sup> Both approaches extend the geometries described by the noncommutative theory beyond the domain of classical relativistic theories. Consequently, they raise the question of whether the new geometries should be classified as spatiotemporal.

In particular, applications of NCG to QG typically translate the properties of the underlying spatiotemporal model into an algebraic framework, and then modify this structure by introducing a noncommutative parameter. It has been argued that the resulting structure fails to retain key spatiotemporal features (Huggett et al., 2021). For example, noncommutativity prevents the precise localisation of events in spacetime, introduces non-local characteristics at the field-theoretic level, and renders the classical notion of worldline ill-defined due to the induced fuzziness.<sup>41</sup>

On one hand, NCG suggests that algebraic structures are fundamental because they allow for the complete determination of the geometric content of the theory. Extreme interpretations may even advocate for algebraic substantivalism, treating these algebraic structures as "real," and thereby suggesting that they may possess ontological independence.<sup>42</sup>

On the other hand, the standard understanding or arbitrary localisability, locality, and the sharpness of worldlines as essential spatiotemporal features implies that NCG exhibits some form of DST. The degree of this disappearance, that is, whether it aligns with case one or case two in Section 2.1, remains unclear.

At this juncture, it is important to note that noncommutative geometric approaches also postulate that the classical, commutative picture be recovered once the noncommutative parameter tends to zero. This is referred to as the *commutative limit* of the noncommutative geometric theory. In the context of QG, it implies that the standard spatiotemporal structure (whether special or general relativistic) should be recovered in the appropriate low-energy regime. Various techniques have been proposed to achieve this. For example, the noncommutative structure can "trivialise" to

<sup>&</sup>lt;sup>39</sup>Specifically, the basic object is the spectral triple. This is a triple  $(\mathcal{A}, H, D)$ , constituted by a (possibly noncommutative) algebra  $\mathcal{A}$  with a realisation as operators on a Hilbert space H. The differential structure is induced by a differential operator D, called the Dirac operator. In Connes' approach, all the geometric information can be encoded algebraically by a spectral triple. A noncommutative spectral triple then extends the possible geometries that can be studied, compared to the case of differential geometry. See (Connes & Marcolli, 2008, ch. 1, § 10).

<sup>&</sup>lt;sup>40</sup>Quantum groups are deformations of specific algebraic structures that generalise the classical symmetries. Each geometric model can be constructed as the structure invariant under the action of a quantum group. Consequently, in this approach, the quantum group encodes the symmetries of the underlying geometric structure. See, e.g., Aschieri (2009).
<sup>41</sup>This is especially relevant for two reasons. In SR, timelike worldlines represent the possible histories

<sup>&</sup>lt;sup>11</sup>This is especially relevant for two reasons. In SR, timelike worldlines represent the possible histories of physical systems, whereas null worldlines identify the boundary between causal and non-causal interactions between different systems. Fuzziness implies not only that the trajectories of physical systems become uncertain in the far past and future; it also undermines the causal structure of the theory due to uncertain boundaries between causal and non-causal interactions (Ballesteros, Gubitosi, Gutierrez-Sagredo, & Mercati, 2021).

Mercati, 2021). <sup>42</sup>Note that mereological conceptions of fundamentality do not apply to NCG. These algebraic structures have no combinatorial features, hence they cannot be represented as simplexes and thus "compose" a geometric structure by gluing and matching conditions. For this reasons, a mereological interpretation of the relationship between noncommutative algebraic structures and geometric content, I believe, is not viable.

<sup>23</sup> 

relativistic spacetime (see, e.g., Doplicher et al. (1995)), or it can be recovered by expanding the noncommutative structure in terms of the noncommutative parameter and truncating higher-energy terms (contraction). Alternatively, one might smear the relevant algebraic quantities, recovering standard spacetime as the set of points approximated by this procedure (see, e.g., Huggett et al. (2021)).

These procedures are expected to restore the necessary spatiotemporal features that were originally lost with the introduction of the noncommutative parameter. However, the comparison between the resulting spatiotemporal theory and NCG is highly non-trivial from an inter-theoretic perspective. In particular, algebraic structures do not easily translate into geometric ones, but such a translation is essential for assessing their spatiotemporal nature.

For instance, quantum groups are not properly groups but more complicated algebraic structures called deformed Hopf algebras. To recover, for example, the Poincaré group of classical symmetries, several procedures must be applied. Consequently, the interpretation of quantum groups as symmetries of a noncommutative spatiotemporal algebra is partly motivated by analogy with their commutative counterparts. This implies that a direct comparison between the high- and low-energy theories is not straightforward; the low-energy theory cannot be trivially embedded in the high-energy framework. To emphasise, there is no trivial notion of symmetry in NCG that can be compared with the classical framework, unless the specifics of the construction of noncommutative theories is taken into account.<sup>43</sup> In other words, inter-theoretic analysis requires careful consideration of the approximation, deformation, and analogy involved in the construction of the noncommutative theory.

Nevertheless, if we are able to complete this inter-theoretic investigation, we can conclude that spacetime is non-fundamental in NCG based on a comparison of energy scales. The noncommutative algebra describes a non-spatiotemporal structure that is defined at higher energy than spacetime.<sup>44</sup> Simultaneously, the commutative limit ensures that, by definition, the noncommutative theory must recover a spatiotemporal theory in the appropriate regime, and with it, the features of arbitrary localisability, locality, and sharpness of worldlines.

If this is the case, then in the commutative limit, we face two possibilities. On one hand, spacetime could be fundamental in the resulting theory, such as in SR. In this scenario, spacetime is recovered as a low-energy structure through an inter-theoretic perspective. Moreover, it remains fundamental from an intra-theoretic viewpoint. For example, the noncommutative theory can be trivialised into a commutative theory by sending the noncommutative parameter to zero (or equivalently, by performing a lowenergy approximation). At this point, Gelfand duality can be applied to recover the standard geometric picture. This procedure is also considered physically salient, in the sense that it represents a genuine physical process rather than a mere mathematical

 $<sup>^{43}</sup>$ Such specifics will include, for instance, the definition of the noncommutative geometric structure for noncommutative spacetime as the coset of the corresponding quantum group. Similarly, geometric noncommutativity causes the Casimir operators to be deformed. This implies that the interpretation of one of them as the mass Casimir of the theory is mediated by analogy with the special relativistic mass Casimir operator, which should be obtained from the former in the commutative limit.

<sup>&</sup>lt;sup>44</sup>It is common to identify the noncommutative parameter of the theory with the relevant energy scale. Heuristic arguments indicate that noncommutativity should be expected around the Planck scale, although this prediction is controversial in phenomenological applications.

procedure. It is necessary to demonstrate how the relevant spatiotemporal features, such as localisability, emerge within the commutative limit.

On the other hand, spacetime may be non-fundamental in SR, particularly in relationist accounts. In this case, the commutative limit would need to illustrate how the noncommutative theory reduces to the fundamental objects of SR (e.g., to classical matter fields), while the derivation of spacetime within SR is regarded as an entirely relativistic issue. In the second case, the commutative limit does not need to explain how spatiotemporal features are recovered directly from the noncommutative theory. Instead, the challenge lies in how the noncommutative structure relates to the fundamental objects of SR.

### 4.3 Discussion: What Fundamentality Relations?

The above intra-theoretic investigation delineates three situations. In string and metastring theory, spacetime does not satisfy any of the aforementioned conceptions of fundamentality. Instead, it is a derived structure that arises either from the graviton or from certain features of string interactions. In metastring theory, spacetime is derived due to the restriction of the fundamental (Born) geometry. This constitutes a weak case of DST, in that the elimination of spacetime from the fundamental ontology and its recovery as a derived structure are both exhibited within the same theory.

A second instance of DST is presented by NCG. Here, the DST follows from the combination of two claims: the fundamental structures of the theory are algebraic, and the duality between an algebraic picture and the standard geometric picture is severed. Specifically, the structures described by NCG fail to satisfy core spatiotemporal features. Consequently, the DST in NCG hinges on whether these features are necessary for a structure to be genuinely spatiotemporal. If they are, then their absence would indicate that spacetime is non-fundamental in NCG. Moreover, it suggests that the theory must provide an intra- or inter-theoretic derivation procedure to satisfy the correspondence with standard (commutative) relativistic theories.

Finally, both SR and GR can exhibit DST according to the aforementioned conceptions. It is important to note that the formalism of both theories is not entirely perspicuous regarding the fundamentality of their objects. In particular, any analysis of the fundamentality relations that relies on mathematical derivations must address two challenges.

First, it must demonstrate that the analysis does not depend significantly on a specific choice of primitives, thus ensuring robustness under reformulations. Otherwise, it must justify why that formulation is privileged or preferable in the specific investigative context.

Second, such an analysis should explicitly outline the scope of features under study. Formal derivations are appropriate only for capturing a specific family of features, namely those that can be represented within the models of the theory. The presence and fundamentality of additional features cannot be analysed merely through formal derivations. For example, chronogeometricity is not a formal feature, because it depends on the ability of specific models to represent measurements of distances, angles and durations. Consequently, an analysis of chronogeometricity in, for instance, SR would require extending the scope of the intra-theoretic investigation beyond the mathematical formalism of the theory.

As for the inter-theoretic investigation, the case study shows that the fundamentality and reappearance of spacetime crucially depend on the definition of *spatiotemporal theory*. To illustrate, if a theory is spatiotemporal when it derives gravitons, then string theory does not exhibit inter-theoretic DST. This is because the graviton is merely a specific vibrational mode of the closed string, and it already appears at the same high-energy level. Similarly, metastring theory does not exhibit DST because the spacetime structure can be recovered from the phase space geometry without any change in energy scale. In contrast, if a theory is spatiotemporal when it produces solutions to the Einstein field equations, string theory does exhibit inter-theoretic DST. This is because these solutions can only be derived in a specific low-energy limit of the theory. Consequently, the recovery of the relativistic models requires the comparison of higher- and lower-energy scales.

Moreover, the inter-theoretic analysis is sensitive to the relations between the involved theories and the chosen limit. For instance, Einstein field equations are derived in a different limit from the zero-slope limit, which instead produces a noncommutative geometric theory. The NCG of Yang-Mills fields does not recover spatiotemporality but is intended as an "intermediate stage," or mesoscopic theory, between the stringy and the classical relativistic regimes. Similarly, the zero-slope limit is distinct from the commutative limit, but only the latter is suitable for recovering the spatiotemporal structure from a noncommutative geometric theory at the lowenergy scale. Consequently, claims of inter-theoretic non-fundamentality and recovery (emergence?) of spacetime are incomplete without specifying the relevant limit. It is this limit that inter-theoretic analyses take into account when comparing ontologies.

It is also important to emphasise that in the metastring case, NCG is a subtheory rather than the result of an approximation procedure. This distinction shields the noncommutative structure from the issues present in other noncommutative theories, meaning that the interpretation of the formalism should shift according to the broader physical theory. In other words, there is no inherent connection between noncommutativity and DST; the connection arises only in specific instances where noncommutativity directly interferes with spatiotemporal features, such as the sharp localisability. In contrast, the NCG in metastring theory *inherits* the DST as instance of the DST in the broader theory.

Finally, in the case of a noncommutative theory of spacetime, it is noteworthy that the commutative limit reinstates the duality between algebra and geometry. This implies that recovering an algebraic picture of spacetime for SR in the limit does not suggest that spacetime is intra-theoretically non-fundamental in the corresponding geometric picture. In fact, the duality eliminates the asymmetry between dual models. In a relativistic theory, spacetime can be considered intra-theoretically fundamental, in the sense of being completely determined if its algebraic structure is recovered in the commutative limit. In other words, the fact that the inter-theoretic procedure first derives an algebraic structure, followed by a geometric structure through duality, does not indicate an intra-theoretic priority of the former over the latter.

In contrast, ontological positions may motivate an asymmetry between the algebraic and geometric pictures. For example, consider manifold substantivalism versus algebraic substantivalism. This account prescribes an ontological realism towards the manifold structure, thus requiring spacetime to be identified with a differential geometric structure, which is more fundamental than its algebraic formulation. If one advocates manifold substantivalism, duality cannot extend to a symmetric relationship between the two pictures due to an underlying ontological asymmetry. In that case, spacetime geometry will be more fundamental than its algebraic structure under a conception of ontological dependence. This distinction is crucial for intra-theoretic analyses of fundamentality, as well as for the evaluation of the inter-theoretic derivation of spacetime from a non-spatiotemporal algebraic theory.

# 5 Conclusion

The problem of the DST is a recurring issue across numerous candidate theories of QG. It raises significant epistemological concerns about these theories, despite the wide variety of fundamental structures they posit and the differing ways in which they treat general relativistic spacetime. The pervasiveness of the DST in QG highlights the importance of clearly formulating the problem, understanding its origins, and exploring potential solutions.

In this paper, I examined the relationship between DST and various conceptions of fundamentality. These conceptions must be distinguished in two categories: those that concern the relative fundamentality of entire theories, and those that concern the relative fundamentality of entires within or across theories. Specifying these conceptions is crucial for articulating how spacetime is fundamental, the view most directly challenged by instances of DST in QG.

A proper definition of the DST must accommodate the diversity of cases and investigative contexts. Such a definition should address the underlying gap between the different kinds of structures involved, while also delineating a boundary between metaphysical inquiries and those of direct interest to the philosophy of physics. The identification of the specific spatiotemporal features and structures that are claimed to disappear from the fundamental domain is a necessary step towards resolving the problem. However, it is important to note that these features and structures may vary from theory to theory.

Finally, I distinguished between inter- and intra-theoretic approaches to the DST. I argued that any inference from intra- to inter-theoretic DST must be supported by additional justificatory arguments. Examples from the QG literature involving interrelated theories suggest that such extrapolations of inter-theoretic from intra-theoretic results require a clear definition of spatiotemporality and a careful account of the inter-theoretic relationship between the theories in question.

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# References

- Aschieri, P. (2009). Noncommutative symmetries and gravity. P. Aschieri, M. Dimitrijević, P. Kulish, F. Lizzi, & J. Wess (Eds.), Noncommutative Spaces. Symmetries in Noncommutative Geometry and Field Theory (pp. 133–162). Berlin Heidelberg: Springer-Verlag.
- Aschieri, P., Blohmann, C., Dimitrijević, M., Meyer, F., Schupp, P., Wess, J. (2005). A gravity theory on noncommutative spaces. *Classical and Quantum Gravity*, 22(17), 3511–3532, https://doi.org/10.1088/0264-9381/22/17/011
- Bain, J. (2014). Three principles of quantum gravity in the condensed matter approach. Studies in History and Philosophy of Modern Physics, 46, 154–163, https://doi.org/10.1016/j.shpsb.2013.09.007
- Ballesteros, A., Gubitosi, G., Gutierrez-Sagredo, I., Mercati, F. (2021). Fuzzy worldlines with κ-poincaré symmetries. Journal of High Energy Physics, 2021, 80, https://doi.org/10.1007/JHEP12(2021)080
- Baron, S. (2020). The curious case of spacetime emergence. Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition, 177(8), 2207– 2226, https://doi.org/10.1007/s11098-019-01306-z
- Baron, S. (2021). Empirical incoherence and double functionalism. Synthese, 199, 413–439, https://doi.org/10.1007/s11229-019-02462-9
- Baron, S., & Le Bihan, B. (2022a). Composing spacetime. Journal of Philosophy, 119(1), 33–54, https://doi.org/10.5840/jphil202211912
- Baron, S., & Le Bihan, B. (2022b). Spacetime quietism in quantum gravity. A. Vassallo (Ed.), *The Foundations of Spacetime Physics* (pp. 155–175). New York: Routledge.
- Barrett, J.A. (1996). Empirical adequacy and the availability of reliable records in quantum mechanics. *Philosophy of Science*, 63(1), 49–64, https://doi.org/ 10.1086/289893

- Bliss, R., & Trogdon, K. (2021). Metaphysical Grounding. E.N. Zalta (Ed.), Stanford Encyclopedia of Philosophy (SEP). Retrieved from https://plato.stanford.edu/entries/grounding/
- Bronstein, M. (1936/2012). Republication of: Quantum theory of weak gravitational field. General Relativity and Gravitation, 44, 367–383, https://doi.org/10.1007/ s10714-011-1285-4
- Brown, H., & Lehmkuhl, D. (2013). Einstein, the reality of space, and the actionreaction principle. Retrieved from https://arxiv.org/abs/1306.4902
- Brown, H.R. (2005). *Physical Relativity. Space-time Structure from a Dynamical Perspective.* Oxford: Clarendon Press.
- Butterfield, J., & Gomes, H. (2023). Functionalism as a species of reduction. C. Soto (Ed.), Current Debates in Philosophy of Science. In Honor of Roberto Torretti (pp. 123–200). Cham: Springer.
- Callender, C., & Huggett, N. (2004). Introduction. *Physics meets philosophy at the Planck scale. Contemporary theories in quantum gravity* (pp. 1–30). Cambridge: Cambridge University Press.
- Cappelli, A., Castellani, E., Colomo, F., Di Vecchia, P. (2012). *The Birth of String Theory*. Cambridge: Cambridge University Press.
- Carlip, S. (2014). Challenges for emergent gravity. Studies in History and Philosophy of Modern Physics, 46, 200–208, https://doi.org/10.1016/j.shpsb.2012.11.002
- Carlip, S. (2024). Causal sets and an emerging continuum. General Relativity and Gravitation, 56, 95, https://doi.org/10.1007/s10714-024-03281-1
- Castellani, E. (2002). Reductionism, emergence, and effective field theories. Studies in History and Philosophy of Modern Physics, 33, 251–267, https://doi.org/ 10.1016/S1355-2198(02)00003-5
- Connes, A., & Marcolli, M. (2008). Noncommutative Geometry, Quantum Fields and Motives. Providence: American Mathematical Society.
- Correia, F. (2021a). Fundamentality from grounding trees. Synthese, 199, 5965–5994, https://doi.org/10.1007/s11229-021-03054-2

- Correia, F. (2021b). A kind route from grounding to fundamentality. Synthese, 199, 8299–8315, https://doi.org/10.1007/s11229-021-03163-y
- Correia, F. (2021c). The logic of relative fundamentality. *Synthese*, 198(Suppl 6), S1279–S1301, https://doi.org/10.1007/s11229-018-1709-8
- Correia, F., & Schnieder, B. (2012). Metaphysical Grounding. Understanding the Structyre of Reality. Cambridge: Cambridge University Press.
- Crowther, K. (2016). Effective Spacetime. Understanding Emergence in Effective Field Theory and Quantum Gravity. Cham: Springer.
- Crowther, K. (2018). Inter-theory relations in quantum gravity: Correspondence, reduction, and emergence. Studies in History and Philosophy of Modern Physics, 63, 74–85, https://doi.org/10.1016/j.shpsb.2017.12.002
- Crowther, K. (2019). When do we stop digging? Conditions on a fundamental theory of physics. A. Aguirre, B. Foster, & Z. Merali (Eds.), What is Fundamental? (pp. 123–133). Cham: Springer Verlag.
- Crowther, K., & Linnemann, N. (2019). Renormalizability, fundamentality and a final theory: The role of UV-completion in the search for quantum gravity. *British Journal for the Philosophy of Science*, 70(2), 377–406, https://doi.org/10.1093/bjps/axx052
- Dawid, R. (2013). String Theory and the Scientific Method. Cambridge: Cambridge University Press.
- Doplicher, S., Fredenhagen, K., Roberts, J.E. (1995). The quantum structure of spacetime at the planck scale and quantum fields. *Communications in Mathematical Physics*, 172, 187–220, https://doi.org/10.1007/BF02104515
- Fletcher, S.C. (2024). Foundations of General Relativity. Cambridge: Cambridge University Press.
- Freidel, L., Leigh, R.G., Minic, D. (2015). Metastring theory and modular spacetime. Journal of High Energy Physics, 2015, 6, https://doi.org/10.1007/ JHEP06(2015)006
- Freidel, L., Leigh, R.G., Minic, D. (2017). Modular spacetime and metastring theory. Journal of Physics: Conference Series, 804, 012032, https://doi.org/10.1088/

1742-6596/804/1/012032

- Gilmore, C., Calosi, C., Costa, D. (2024). Location and Mereology. E.N. Zalta (Ed.), *Stanford Encyclopedia of Philosophy (SEP)*. Retrieved from https://plato.stanford.edu/entries/location-mereology/
- Girelli, F., Liberati, S., Sindoni, L. (2009). Emergence of Lorentzian signature and scalar gravity. *Physical Review D*, 79, 044019, https://doi.org/10.1103/ PhysRevD.79.044019
- Hagar, A. (2014). Discrete or Continuous? The Quest for Fundamental Length in Modern Physics. Cambridge University Press: Cambridge.
- Henson, J. (2009). The causal set approach to quantum gravity. D. Oriti (Ed.), Approaches to Quantum Gravity. Toward a New Understanding of Space, Time and Matter (pp. 393–413). Cambridge University Press.
- Huggett, N. (2017). Target space ≠ space. Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 59, 81–88, https://doi.org/10.1016/j.shpsb.2015.08.007
- Huggett, N., Lizzi, F., Menon, T. (2021). Missing the point in noncommutative geometry. *Synthese*, 199(1–2), 4695–4728, https://doi.org/10.1007/s11229-020 -02998-1
- Huggett, N., & Wüthrich, C. (2025). *Out of Nowhere*. New York: Oxford University Press.
- Isham, C. (1993). Canonical quantum gravity and the problem of time. L. Ibort & M. Rodriguez (Eds.), *Integrable systems, quantum groups, and quantum field* theories (pp. 157–288). Dordrecht: Kluwer.
- Jacobson, T. (1995). Thermodynamics of spacetime: The Einstein equation of state. *Physical Review Letters*, 75, 1260, https://doi.org/10.1103/PhysRevLett.75 .1260
- Jaksland, R., & Salimkhani, K. (2023). The many problems of spacetime emergence in quantum gravity. British Journal for the Philosophy of Science, https:// doi.org/10.1086/727052

- Kiefer, C. (2007). Quantum Gravity. Second Edition. New York: Oxford University Press.
- Knox, E. (2013). Effective spacetime geometry. Studies in History and Philosophy of Modern Physics, 44, 346–356, https://doi.org/10.1016/j.shpsb.2013.04.002
- Kortabarria, M., & Giannotti, J. (2024). Scientific explanation as a guide to ground. Synthese, 203, 73, https://doi.org/10.1007/s11229-024-04492-4
- Lam, V., & Wüthrich, C. (2018). Spacetime is as spacetime does. Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 64, 39–51, https://doi.org/10.1016/j.shpsb.2018.04.003
- Le Bihan, B. (2018). Space emergence in contemporary physics: Why we do not need fundamentality, layers of reality and emergence. *Disputatio*, 10(49), 71–95, https://doi.org/10.2478/disp-2018-0004
- Le Bihan, B. (2021). Spacetime emergence in quantum gravity: functionalism and the hard problem. Synthese, 199, 371–393, https://doi.org/10.1007/s11229-019 -02449-6
- Le Bihan, B., & Linnemann, N. (2019). Have we lost spacetime on the way? Narrowing the gap between general relativity and quantum gravity. *Studies in History and Philosophy of Modern Physics*, 65, 112–121, https://doi.org/10.1016/j.shpsb .2018.10.010
- Lizzi, F., Manfredonia, M., Mercati, F., Poulain, T. (2019). Localization and reference frames in κ-Minkowski spacetime. *Physical Review D*, 99(8), 085003, https:// doi.org/10.1103/PhysRevD.99.085003
- Maresca, E. (2015). Noncommutative geometry and spacetime: A historical reconstruction. Journal of Physics: Conference Series, 2948, 012011, https://doi.org/ 10.1088/1742-6596/2948/1/012011
- Margoni, E., & Oriti, D. (in press). The emergence of spacetime: what role for functionalism? British Journal for the Philosophy of Science, , https://doi.org/ 10.1086/735277

- McKenzie, K. (2019). Fundamentality. S. Gibb, R. Findlay Hendry, & T. Lancaster (Eds.), *The Routledge Handbook of Emergence* (pp. 54–64). New York: Routledge.
- McKenzie, K. (2022). Fundamentality and Grounding. Cambridge: Cambridge University Press.
- Menon, T. (2019). Algebraic fields and the dynamical approach to physical geometry. Philosophy of Science, 86(5), 1273–1283, https://doi.org/10.1086/705508
- Morganti, M. (2020). Fundamentality in metaphysics and the philosophy of physics. Part II: The philosophy of physics. *Philosophy Compass*, 15, e12703, https://doi.org/10.1111/phc3.12703
- Nickles, T. (1973). Two concepts of intertheoretic reduction. The Journal of Philosophy, 70(7), 181–201, https://doi.org/10.2307/2024906
- Oriti, D. (2014). Disappearance and emergence of space and time in quantum gravity. Studies in History and Philosophy of Modern Physics, 46, 186–199, https:// doi.org/10.1016/j.shpsb.2013.10.006
- Oriti, D. (2021). Levels of spacetime emergence in quantum gravity. C. Wüthrich, B.L. Bihan, & N. Huggett (Eds.), *Philosophy Beyond Spacetime: Implications* From Quantum Gravity (pp. 16–40). Oxford: Oxford University Press.
- Oriti, D. (2022). Tensorial group field theory condensate cosmology as an example of spacetime emergence in quantum gravity. Retrieved from https://arxiv.org/abs/2112.02585
- Padmanabhan, T. (2014). Gravity and the spacetime: An emergent perspective. A. Ashtekar & V. Petkov (Eds.), Springer Handbook of Spacetime (pp. 213–242). Berlin, Heidelberg: Springer-Verlag.
- Rovelli, C. (2004). Quantum Gravity. Cambridge: Cambridge University Press.
- Rovelli, C. (2009). Unfinished revolution. D. Oriti (Ed.), Approaches to Quantum Gravity. Toward a New Understanding of Space, Time and Matter (pp. 3–12). Cambridge: Cambridge University Press.
- Sakharov, A.D. (2000). Vacuum quantum fluctuations in curved spacetime and the theory of gravitation. General Relativity and Gravitation, 32(2), 365–367, https://doi.org/10.1070/PU1991v034n05ABEH002498

- Salimkhani, K. (2023). The Non-Fundamentality of Spacetime: General Relativity, Quantum Gravity, and Metaphysics. New York: Routledge.
- Seiberg, N., & Witten, E. (1999). String theory and noncommutative geometry. Journal of High Energy Physics, 1999(09), 032, https://doi.org/10.1088/1126 -6708/1999/09/032
- Sklar, L. (1983). Prospects for a causal theory of space-time. R. Swinburne (Ed.), Space, Time and Causality (pp. 45–62). Dordrecht, Boston, London: D. Reidel Publishing Company.
- Sorkin, R. (2005). Causal sets: Discrete gravity (notes for the Valdivia Summer School). D.M.A. Gomberoff (Ed.), Lectures on quantum gravity. Proceedings of the Valdivia Summer School, Valdivia, Chile, January 2002. Plenum. Retrieved from http://arxiv.org/abs/gr-qc/0309009. arXiv:gr-qc/0309009
- Steinhaus, S., & Thürigen, J. (2008). Emergence of spacetime in a restricted spin-foam model. *Physical Review D*, 98(2), 026013, https://doi.org/10.1103/PhysRevD .98.026013
- Szabo, R.J. (2006). Symmetry, gravity and noncommutativity. Classical and Quantum Gravity, 23(22), R199, https://doi.org/10.1088/0264-9381/23/22/R01
- Tahko, T.E. (2023). Fundamentality. E.N. Zalta (Ed.), *Stanford Encyclopedia of Philosophy (SEP)*. Retrieved from https://plato.stanford.edu/entries/fundamentality/
- Verlinde, E.P. (2017). Emergent Gravity and the Dark Universe. SciPost Physics, 2(3), 016, https://doi.org/10.21468/scipostphys.2.3.016
- Vistarini, T. (2019). The Emergence of Spacetime in String Theory. London: Routledge.
- Wüthrich, C. (2018). The emergence of space and time. S. Gibb, R. Findlay Hendry, & T. Lancaster (Eds.), *The Routledge Handbook of Philosophy of Emergence* (pp. 315–326). New York: Routledge.