Conventionalism in general relativity: Weatherall & Manchak's proof against theorem θ in context

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Abstract

Weatherall and Manchak (2014) show that Reichenbachean universal effects, constrained to a rank-2 tensor field representation in the geodesic equation, always exist in non-relativistic gravity but not so for relativistic spacetimes. Thus general relativity is less susceptible to underdetermination than its Newtonian predecessor. Dürr and Ben-Menahem (2022) argue these assumptions are exploitable as loopholes, effectively establishing a (rich) no-go theorem. I disambiguate between two targets of the proof, which have previously been conflated: the existence claim of at least one alternative geometry to a given one and Reichenbach's (in)famous "theorem theta", which amounts to a universality claim that any geometry can function as an alternative to any other. I show there is no (rich) no-go theorem to save theorem theta. I illustrate this by explicitly breaking one of the assumptions and generalising the proof to torsionful spacetimes. Finally, I suggest a programmatic attitude: rather than undermining the proof one can use it to systematically and rigorously articulate stronger propositions to be proved, thereby systematically exploring the space of alternative spacetime theories.

Table of contents

1. Introduction

Conventionality about space, or spacetime, is the view in the epistemology of geometry – loosely associated with ideas by Poincaré, Duhem, Schlick, Carnap, Reichenbach, and others – that ascertaining the geometry of the world requires in some way or another a conventional aspect. On the one hand, it is in the empiricist tradition to recognise that there is an empirical question to be asked about the *true* geometry of the world, as opposed to the abstract mathematical (hence analytically true) formalisms of Euclidean and non-Euclidean geometry: physical practice is seen to have a grip on revealing synthetic geometric facts about the world. On the other hand, geometric conventionalists hold that such a *physical geometry* is, at least in itself, beyond empirical reach: it can only be given empirical content by means of an anchoring point, a conventional choice that cannot itself be justified on empirical grounds.

Canonically, such geometric conventionalism is grounded in the idea that different spacetime theories can be empirically indistinguishable, despite being based on distinct geometric structures. This underdetermination arises when models with different geometries – models within the same spacetime theory or between *prima facie* distinct spacetime theories – can be made empirically equivalent by introducing 'universal forces' (or 'universal effects') that correct for their geometric differences in such a way that the empirically accessible trajectories of bodies remain unaffected. Hence the 'conventionality of geometry', one of the largest debates in the epistemology of spacetime since its conception.[1]

Ostensibly, this is the position that J.O. Weatherall and J.B. Manchak (2014, henceforth: W&M), set out to question rigorously in their "The Geometry of Conventionality":

> If one understands "force" in the standard way in the context of our best classical (i.e., non-quantum) theories of space and time, can one accommodate different choices of geometry by postulating some sort of "universal force field"? (W&M 2014, p. 234).

They present two proofs, one non-relativistic and one for general relativity (GR), restricting the universal force field to be represented by a rank-2 tensor field in the geodesic equation. The non-relativistic proof *confirms* that the conventionalist's claim that there are empirically adequate alternatives if one allows universal forces; the relativistic proof *falsifies* the claim that this can always be done for general relativistic spacetimes. This paper will focus on the relativistic proof, which shows that GR does not admit the same leeway for conventionalism as Newtonian gravity because there does not exist a reasonable "force" tensor that relates the geodesics of two conformally related metrics. That is, *under reasonable assumptions*.

Certainly, W&M showcase conventionalism in a novel way, mentioning much previous scholarship on the topic, but not obviously connecting to the questions posed in those works. An upshot is that they reformulate the problem in the full glory of modern spacetime theory, introducing much-needed rigour into a debate that has often relied on proofs of concept. As such, they kick off a fresh way to handle these questions. A downside is that at first sight their claims are remarkable, even shocking, given the numerous debates on this topic: has

[1] The modern literature on geometric conventionalism (let alone conventionalism generally) is too vast to review, but a (non-exhaustive) list from which I draw is (Sklar 1974; Glymour 1977; Dieks 1987; Ben-Menahem 2006; Acuña 2013; Ivanova 2015; Padovani 2017; Ivanova 2021; Dewar 2022; Dewar, Linnemann, and Read 2022; Dürr and Read 2024).

more than a century of scholarship missed a trick, perhaps because of the lack of a sufficiently rigorous analysis? In particular, a roadmap about whether we can now safely adopt a realist conception of general relativistic spacetimes, or what kind of future work would be required to establish such commitment, is left to the reader.

Given this unconventional anti-conventionality, controversy is to be expected. In fact, Patrick Dürr and Yemima Ben-Menahem (2022, henceforth: D&BM) write a response that makes no secret of their belief that the assumptions of the proof are overly restrictive, not sufficiently justified, open to counterexamples, and not in accordance with historical and philosophical scholarship on the topic. They present W&M's relativistic proof as an inconsistency proof, lining up its assumptions together with an existence claim of alternative geometries and pointing out how each of these assumptions can be rejected. As such, each rejection opens up possibilities for constructing empirically equivalent models.

One can feel at a loss about the overall gain of this debate, if any. If D&BM are correct, have W&M proven nothing of importance? In this paper, I clarify two different kinds of underdetermination that serve to underpin two very different positions, each of a conventionalist but nonetheless distinct flavour. Some conventionalists have in mind an existence claim: for any one given metric there exists *at least one* other metric which can do the same empirically adequate job, given the universal forces (what I will call (**UDT**-$\forall g \exists \tilde{g}$) in §3). Other conventionalists have in mind a universality claim (**UDT**-$\forall g \forall \tilde{g}$): *any* metric can replace *any* other metric, if suitably adjusted for by universal forces—a claim most prominently promulgated by Reichenbach, enshrined in his (in)famous "theorem θ":

> *Theorem* θ: "Given a geometry G' to which the measuring instruments conform, we can imagine a universal force F which affects the instruments in such a way that the actual geometry is an *arbitrary* geometry G, while the observed deviation from G is due to a universal deformation of the measuring instruments." (Reichenbach 1928, §8, p. 33, *emphasis added*)

This paper contextualises W&M's relativistic result by distinguishing between (restricted) existence and (restricted) universality claims. Expanding on a list of assumptions of the proof identified by D&BM, I discuss their loophole-based critique (§2). Then §3 clarifies the logical structure of the proof by distinguishing between claims of existence of model underdetermination or claims that model underdetermination is universal. I argue that the proof (a) severely restricts but does not undermine underdetermination *tout court*, leaving (an ever tightening) space for the existence of alternative models, and (b) theorem θ is nevertheless successfully dispelled—an observation none of the authors addressed clearly. I then generalise the proof to torsionful spacetimes (§4), showing that (i) for a given torsionful connection there exist some connections which cannot be related by a force field in the standard way, illustrating that violating an assumption does not serve as a loophole to save theorem θ in GR, and, more generally, that (ii) for a given torsionful connection there exists no conformally equivalent torsionful connection which can be so related. In §5 I discuss conventionalism more broadly and argue that the current debate concerns underdetermination, not conventionalism *per se*. Finally, §6 proposes reframing W&M's assumptions as part of a research programme to develop stronger theorems, eventually culminating in a systematic grip on the formal landscape of the space of (conceived and unconceived) relativistic spacetime theories.

2. Weatherall and Manchak's relativistic proof and its assumptions

To mathematically cash out the question whether in GR one can accommodate different geometries via the postulation of a universal force field, W&M pose it in terms of the affine-connection uniquely associated (given torsion-freeness and metric-compatibility) with the metric: for a relativistic spacetime on manifold M with metric g_{ab} with its associated Levi-Civita connection ∇, and a distinct metric $\widetilde{g}_{ab}$ (on the same M) with its associated connection $\widetilde{\nabla}$, and ξ^a the velocity vector (the unit-norm, timelike vector tangent to a particle's curve $\gamma(\tau)$, i.e., $\xi^a = \dot{\gamma}$).The question then reads: "Is there some rank-2 tensor field F_{ab} such that, given a curve γ, γ is a geodesic (up to reparametrisation) relative to ∇ just in case its acceleration relative to $\widetilde{\nabla}$ is given by $F^a{}_n\tilde{\xi}^n$, where $\tilde{\xi}^a$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$?" This gives sufficient ingredients to write down the following proposition[2]

Proposition 2 *[**The relativistic case**]—Let (M, g_{ab}) be a relativistic spacetime, let $\tilde{g}_{ab} = \Omega^2 g_{ab}$ be a metric conformally equivalent to g_{ab}, and let ∇ and $\widetilde{\nabla}$ be the Levi-Civita derivative operators compatible with g_{ab} and $\tilde{g}_{ab}$, respectively. Suppose Ω is non-constant. Then there is no tensor field F_{ab} such that an arbitrary curve γ is a geodesic relative to ∇ if and only if its acceleration relative to $\widetilde{\nabla}$ is given by $F^a{}_n\tilde{\xi}^n$, where $\tilde{\xi}^n$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$.* (Weatherall and Manchak 2014, p. 242)

That is, given any geometry, we cannot always write down a new geometry such that a rank-2 tensor relates their geodesics. Proposition 2 is then proven to be correct (in Appendix A one can follow the steps of this proof and the use of assumptions, in the context of the logically stronger proposition given in §4). Thus it is in general not possible to construct empirically-equivalent models by postulating such an F_{ab}.

All proofs make assumptions. Evaluating the importance of the W&M proof must involve an investigation of what exactly is assumed, and how weak or strong these assumptions are. Dürr & Ben-Menahem (2022) have listed some of the assumptions used in the proof, on which I will largely rely. Yet, I add to this list some topological assumptions and give a finer rendition of the assumption of Riemannian geometry. Also, since W&M explicitly restrict themselves to models of GR, I distinguish between assumptions within GR and assumptions that bring us beyond it. The list is:

- (**CONF**) The alternative metric is conformally related to the standard metric $\tilde{g}_{ab} = \Omega^2(x)g_{ab}$. (D&BM, p. 156)
- (**NORM**) $\tilde{\xi}^a$ is a unit-norm (with respect to the *new* geometry's metric) tangent vector to the particle's curve: $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = 1$. (*ibid.*, p. 156)
- (**FORCE**) The [geometrical] alternative to standard acceleration must take the 'standard' force-law form $\xi^b\nabla_b\xi^a = \tilde{\xi}^b\widetilde{\nabla}_b\tilde{\xi}^a + F^a{}_b\tilde{\xi}^b$ (*ibid.*, p. 156).

And here are assumptions that would take one beyond GR:

[2] Proposition 1 is the non-relativistic case, for which the verdict is that a suitable force tensor field *does* exist. Furthermore, I follow (Einstein 1921; Reichenbach 1928) in using G for geometry and F for universal effects, and F_{ab} for the rank-2 tensorial form of F. W&M use G_{ab} for the latter.

- (**RIEM**) Geometric alternatives must employ Riemannian geometry: being solely expressible in terms of a metric $\tilde{g}$ and associated Levi-Civita connection $\widetilde{\nabla}$. (*ibid.*, p. 156)

  Here I would add two assumptions constitutive of (**RIEM**):

  - (**RIEM-SYMM**) the affine-connection is torsion-free.
  - (**RIEM-COMP**) the affine-connection is fully metric compatible, so that the disformation vanishes identically.

- **(TOPO)** Geometric alternatives are constructed on the same manifold. In particular[3]:

  - (**DIM4**) the manifold in question is restricted to four dimensions.
  - (**HAUS**) points can be kept apart by open sets (for every pair of distinct points $p, q \in \mathcal{M}$ there exist neighbourhoods disjoint open sets $U, V \in \mathcal{M}$ such that $p \in U$, $q \in V$ and $U \cap V = \emptyset$). (Hausdorff condition)

These assumptions are indeed needed for the proof to come off the ground in the way that it does—see W&M's original and Appendix A for additional steps (in the context of a generalisation of Proposition 2).

Generally, (Dürr and Ben-Menahem 2022) is a *tour de force* of the conceptual history of geometric conventionalism, but begin their paper by scrutinizing W&M's proof by questioning its assumptions – without disputing its formal validity – by presenting the proof as a *no-go theorem*. The above list constitutes a set of mutually inconsistent premises when a particular conventionalist thesis is added:

- (**ALT-ACC**) Geometric alternatives for general-relativistic acceleration of a test-particle, $\xi^b \nabla_b \xi^a$ must exist [...]. (D&BM, p. 155)

As per usual with inconsistency, the proper response is to reject at least one premise, thereby restoring consistency. That is, the conjunction $\neg$((**ALT-ACC**) $\wedge$ (**CONF**) $\wedge$ (**NORM**) $\wedge$ (**RIEM**) $\wedge$ (**FORCE**) $\wedge$ (**TOPO**)) is equivalent to the disjunction of the negations of each conjunct: $\neg$(**ALT-ACC**) $\vee$ $\neg$(**CONF**) $\vee$ $\neg$(**NORM**) $\vee$ $\neg$(**RIEM**) $\vee$ $\neg$(**FORCE**) $\vee$ $\neg$(**TOPO**). In this way, framing a debate in terms of a set of mutually inconsistent premises is a natural way of clarifying and classifying different views in a complex debate, by identifying each view with the rejection of one of the premises. (I will articulate such classification in §6).

D&BM would reject one or all of the premises except (**ALT-ACC**), thereby retaining a conventionalist position; they attribute to W&M the position $\neg$(**ALT-ACC**), that a trade-off between geometries and universal forces is *never* possible. They (2022, pp. 156-157) speak of assumption (**NORM**) as "unwarranted", (**CONF**) as overly restrictive and thus "defeating its purpose", and of "by-passing" and "short-circuiting" the proof itself by rejecting such assumptions. For example, denying (**CONF**) opens up space for underdetermination of models, because two models are then allowed (interpreting conformal rescalings passively rather than as active transformations) to differ by 'running units', where the choice of numerical units trades off against different conformal scalings (*ibid.*, p. 157); one can do the same with

[3] As always in philosophy, some assumptions remain suppressed, e.g., smoothness (**SMOOTH**), paracompactness (**PARA**) and those that go 'even deeper down'. D&BM may very well have regarded (**TOPO**) as 'deeper down', so this is meant as supplementation, not correction. For reasons why see (§6).

volume elements (*ibid.*, p. 158). By denying (**NORM**), we can use reparametrisations of curves to create empirically equivalent models that differ with respect to lengths of vectors (*ibid.*, p. 156). By denying (**RIEM**), possibilities open up to modify the affine-connection so as to include non-Riemannian spacetimes (*ibid.*, pp. 159-160). Finally, by denying (**FORCE**), interpreting F more loosely as an 'interaction' or 'effect' rather than a strict 'force' such as the electromagnetic force represented by the Faraday tensor. Thus,

> Of course, W&M are aware of the fact that the strength of their theorems on which their reasoning rests depends on their assumptions. But the reader is led to believe that those assumptions are quite natural; to deny them would appear to exact a rebarbatively high price. It is the naturalness of those assumptions (not the validity of W&M's theorems) that we subsequently seek to question. (Dürr and Ben-Menahem 2022, p. 155)

3. Universality, not existence: most assumptions are not loopholes to theorem θ

Surely, W&M's paper is written in an anti-conventionalist tone. Reading the title of the paper and some of their commentary, they can easily be misread as implying that their result shows that GR does not allow for conventionalism at all. That is, as if trade-offs between metrics and universal forces/effects can never be made, or perhaps only in very exotic cases. Furthermore, some assumptions of §2 are not explicitly discussed, or only mentioned in passing. The one assumption that receives extensive attention is (**FORCE**). Take, for example, (**CONF**), the restriction to conformally equivalent spacetimes. In a footnote, they say:

> Note, though, that requiring conformal equivalence only strengthens our results. If the conventionalist cannot accommodate conformally equivalent metrics, then *a fortiori* one cannot accommodate arbitrary metrics; conversely, if Reichenbach's proposal fails even in the special case of conformally equivalent metrics, then it fails in the case of (arguably) greatest interest." (W&M 2014, fn. 13, p. 237)

A few pages later, the same is said about (**CONF**) in a footnote to Proposition 2 directly:

> Again, this restriction strengthens the result. If the proposal does not work even in this special case, it cannot work in general; moreover, the special case is arguably the most interesting. (W&M 2014, fn. 22, p. 242)

This may strike one as quaint, even plainly wrong: aiming to debunk conventionalism, discarding a whole class of spacetimes – *in casu* the conformally inequivalent spacetimes – does not strengthen but *weaken* the result. Right? If you take a subset of the full set of spacetimes, one excludes by fiat a whole range of candidate empirically indistinguishable spacetimes: why is one not allowed to look for empirically equivalent models outside of that subset?

I argue that both D&BM and W&M do not clearly distinguish between two conceivable positions that Proposition 2 can be taken to dispel, namely whether – with the help of universal forces – at least one model is equally capable of making the same predictions as a given model, or whether all of them are. The first is an existence claim, like (**ALT-ACC**). The second is a universality claim, like theorem θ. The crux of the matter is that there are different kinds of conventionalism—or different kinds of model underdetermination that give rise to different conventionalisms:

- **Existence** (**UDT**-$\forall g \exists \tilde{g}$): For each metric of an adequate model of some spacetime theory there exists at least one distinct metric of a model of that theory that is equally capable of predicting the same observable consequences, given suitable universal effects.

- **Universality** (**UDT**-$\forall g \forall \tilde{g}$): For each metric of an adequate model of some spacetime theory any other distinct metric of a model of that theory is equally capable of predicting the same observable consequences, given suitable universal effects.

Several remarks are in order. First, note that (**UDT**-$\forall g \forall \tilde{g}$) implies (**UDT**-$\forall g \exists \tilde{g}$). Then, (**UDT**-$\forall g \exists \tilde{g}$) is a rendering of theorem θ in terms of formal model underdetermination within a theory (excluding bona fide conventionalist worries about non-factual choices that enable measuring devices). Furthermore, in the context of GR, (**UDT**-$\forall g \exists \tilde{g}$) takes the form of (**ALT-ACC**). Finally, the clause 'suitable universal effects' can be broadly or narrowly fleshed out—W&M flesh it out narrowly as (**FORCE**).

From the formulation of (**ALT-ACC**) it is clear that D&BM assume that W&M's target is the existence claim of **UDT**-$\forall g \exists \tilde{g}$, since it states that W&M disprove the claim that geometric alternatives must *exist*. A closer reading reveals that W&M do not target (**ALT-ACC**)—even when all assumptions are fully granted. The question whether there in general *exist* particular pairs of Levi-Civita connections whose geodesics can be related by some F_{ab}, and hence whether (**UDT**-$\forall g \exists \tilde{g}$) is correct in GR or Newtonian gravity, is never posed by W&M. What they disprove instead, roughly, is the statement that the geodesics of any given spacetimes (M, g) and those of an alternative but *conformally equivalent* spacetime $(M, \tilde{g})$ can be related by a force field satisfying (**FORCE**). That is, we should consider these claims also with restricted quantifiers that only range over the conformally equivalent metric to a given metric:

- **Restricted Existence** (**UDT**-$\forall g \exists_{\text{conf}} \tilde{g}$): For each metric of an adequate model of some spacetime theory there exists at least one distinct conformally equivalent metric of a model of that theory that is equally capable of predicting the same observable consequences, given suitable universal effects.

- **Restricted Universality** (**UDT**-$\forall g \forall_{\text{conf}} \tilde{g}$): For each metric of an adequate model of some spacetime theory any other distinct conformally equivalent metric of a model of that theory is equally capable of predicting the same observable consequences, given suitable universal effects.

Again, (**UDT**-$\forall g \forall_{\text{conf}} \tilde{g}$) implies (**UDT**-$\forall g \exists_{\text{conf}} \tilde{g}$).

As I read it, W&M slide between two targets, i.e., (**UDT**-$\forall g \exists_{\text{conf}} \tilde{g}$) and (**UDT**-$\forall g \forall \tilde{g}$). On the one hand, "If the conventionalist cannot accommodate conformally equivalent metrics, then *a fortiori* one cannot accommodate arbitrary metrics", targets (**UDT**-$\forall g \forall \tilde{g}$). On the other hand, "[...] conversely, if Reichenbach's proposal fails even in the special case of conformally equivalent metrics, then it fails in the case of (arguably) greatest interest", has in mind (**UDT**-$\forall g \exists \tilde{g}$). Let us consider both.

D&BM (2022, p. 156) say that W&M "believe this restriction [i.e., (**CONF**)] doesn't diminish the argument's generality, since for Reichenbach *any arbitrary* geometry can be upheld [...]." The reason they attribute to W&M for this belief is their remark about the non-conventionality of causal structure, a point first made by Malament (1985): since Reichenbach famously took causal statements as non-conventional, he would presumably hold that conformal structure is factual because it derives directly from causal facts. Thus, on his own terms, he should be committed to the conformal part of the metric as non-conventional. On this reading, one would indeed hold that (**UDT**-$\forall g \exists_{\text{conf}} \tilde{g}$) implies (**UDT**-$\forall g \exists \tilde{g}$), since both classes exhaust the physically viable (or rather: factual) differences. Yet, few of us are committed to Reichenbach's causal theory of time—and besides, W&M themselves say they are not primarily concerned with Reichenbach's project. One could of course read this as a motivating remark why conformally equivalent spacetimes are *interesting*. The argument why (**UDT**-$\forall g \exists_{\text{conf}} \tilde{g} \rightarrow$ **UDT**-$\forall g \exists \tilde{g}$) remains unjustified, or at least implicit. This means that (**UDT**-$\forall g \exists \tilde{g}$) remains a viable option.[4]

Yet, despite D&BM's objections, denying (**CONF**) – or any of the assumptions for that matter, except (**FORCE**) – will not save theorem θ. For let us consider (**UDT**-$\forall g \forall \tilde{g}$) as an implicit target of the proof. First, Proposition 2 denies (**UDT**-$\forall g \exists_{\text{conf}} \tilde{g}$) in the context of GR. Then, the restricted universality claim implies the restricted existence claim (**UDT**-$\forall g \forall_{\text{conf}} \tilde{g} \rightarrow$ **UDT**-$\forall g \exists_{\text{conf}} \tilde{g}$), so we have (by *modus tollens*) that $\neg$(**UDT**-$\forall g \forall_{\text{conf}} \tilde{g}$). In turn, it is clear that (**UDT**-$\forall g \forall \tilde{g}$) implies (**UDT**-$\forall g \forall_{\text{conf}} \tilde{g}$), so we have $\neg$(**UDT**-$\forall g \forall \tilde{g}$). In this way we can interpret Proposition 2, roughly, as disproving the statement 'for any pair of (conformally equivalent) spacetimes (M, g) and $(M, \tilde{g})$, there is a force field satisfying (**FORCE**) that relates their geodesics', with the parentheses around 'conformally equivalent' highlighting the redundancy of this qualification.

Another way to see this is to consider the region of GR spacetimes, assuming (**TOPO**), that are conformally equivalent (**CONF**), Riemannian (**RIEM**), and renormalisable (**NORM**), as illustrated by the solid green region in Figure 1. W&M refute that any two spacetimes in this region can be related by a "standard" force tensor field F_{ab} satisfying (**FORCE**). Suppose we deny (**CONF**); this expands the space to include Riemannian, renormalisable spacetimes regardless of conformal equivalence. But this merely enlarges the region to that marked 'GR' in Figure 1b, with the original subset (Figure 1a) still contained within it. The same proof that rules out a force field in the narrower region applies to the wider one. Strictly analogous reasoning applies to the denial of (**RIEM**), (**NORM**) and (if one goes beyond GR) (**TOPO**): the nature of Proposition 2 as an argument against theorem θ is that it remains true under generalisation.

4. Generalising to torsionful connections

One way of generalising W&M's relativistic result in a controlled way is by explicitly rejecting (**RIEM**), or more specifically its constitutive premise (**RIEM-SYMM**), by allowing for torsionful connections, i.e. connections that generically (i.e., they are 'not-necessarily symmetric') have an anti-symmetric part:

[4] I am grateful to an anonymous reviewer for pointing me to a recent preprint by Roberts (202?), which putatively proves a strengthening of Proposition 2 without (**CONF**). However, the result does not straightforwardly imply Proposition 2 as claimed: a residual underdetermination remains when postulating spacetimes differing in physical magnitudes involving length. Despite $\widetilde{\nabla} = \nabla$ and $F^{ab} = 0$, a standard force is still conceived of as proportional to acceleration in the new geometry, $\tilde{\xi}^a \widetilde{\nabla} \tilde{\xi}^b = C^{abc} \tilde{\xi}^a \tilde{\xi}^b$ (on a ∇-geodesic). Roberts' force tensor thus fails to satisfy (**FORCE**) in the new geometry. See also §6.5.

Proposition 3 ***The relativistic torsionful case***—*Let (M, g_{ab}) be a relativistic spacetime, let $\tilde{g}_{ab} = \Omega^2 g_{ab}$ be a metric conformally equivalent to g_{ab}, and let ∇ and $\widetilde{\nabla}$ be not-necessarily-symmetric derivative operators compatible with g_{ab} and $\tilde{g}_{ab}$, respectively. Suppose Ω is non-constant. Then there is no tensor field F_{ab} such that an arbitrary curve γ is a geodesic relative to ∇ if and only if its acceleration relative to $\widetilde{\nabla}$ is given by $F^a{}_n\tilde{\xi}^n$, where $\tilde{\xi}^n$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$.*

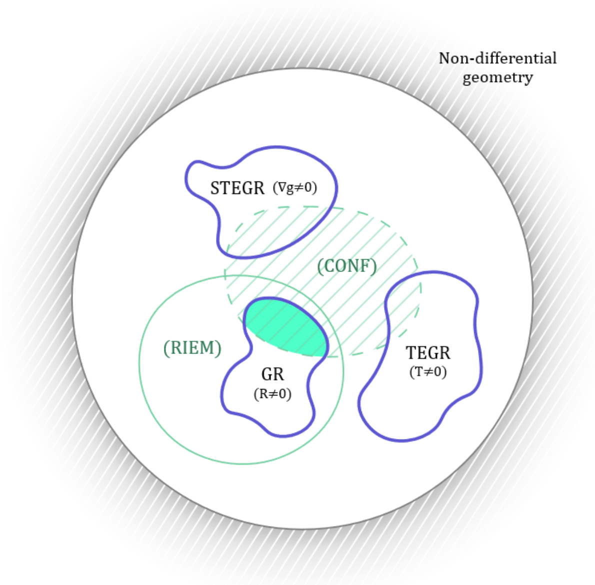
In light of the existence of empirically equivalent torsionful models, i.e., the so-called teleparallel equivalent of GR (cf. Hayashi and Shirafuji 1979), the truth or falsity of this proposition promises to be insightful. In Appendix A I show Proposition 3 to be true, starting from a more general difference tensor as given in (Jensen 2005). Because the additional torsionful terms do not depend on the metric, they are not affected by a conformal transformation, and so the proof goes through largely analogously to the proof of Proposition 2.

Again it is important to distinguish the target. If it is the universality claim (**UDT**-$\forall g \forall \tilde{g}$), then expanding the scope of a universal quantifier does not affect the falsity of the claim: if the universality claim fails in the original case, it trivially fails in the broader one too. The key point is that for a given torsionful connection, some others cannot be related by a force field F_{ab}—thus violating (**RIEM-SYMM**) but still refuting theorem θ under (**FORCE**). That does not mean this result tells us nothing new. Proposition 3 is a genuine generalisation in the sense that it proves that for any torsionful connection, there exists some (non-trivially conformally equivalent) torsionful connection that cannot be related by a force field F_{ab}.

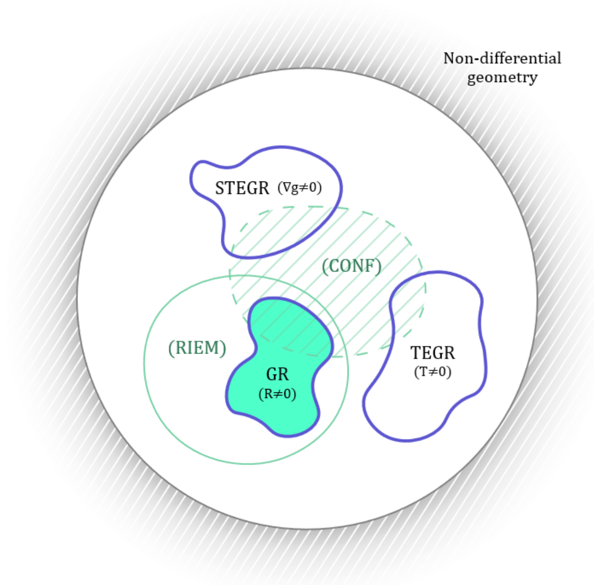
If the target is (**UDT**-$\forall g\ \nabla_{\text{torsionful}}; \exists \tilde{g}\ \widetilde{\nabla}_{\text{torsionful}}$) is taken, this existence claim fails within the class of conformally equivalent spacetimes, see Figure (1c). That is, it is a generalisation of Proposition 2 in the sense that for a given torsionful connection there exists no conformally equivalent torsionful connection that can be related by a force field F_{ab}.[5]

To pre-empt misreading: Proposition 3 does not rule out the existence of a teleparallel equivalent to GR. Logically speaking, a force field could still relate torsion-free and torsionful spacetimes, and this should motivate more stringent existence claims (cf. §6). However, the above framework does allow for a more direct assessment of whether torsion (in teleparallel gravity) counts as a *force*. The torsionful deviation from geodesic motion, governed by the Weitzenböck connection, does not appear to satisfy (**FORCE**): it enters the geodesic equation via the contorsion tensor contracted with two velocity vectors and cannot generally be cast as a conservative force represented by a rank-2 tensor. It would be more naturally interpreted as a geometric *effect*, albeit not solely through the metric alone: geometric facts would be represented by the metric *in tandem* with the connection and its associated contorsion. In this relativistic context, then, torsion is not a force.

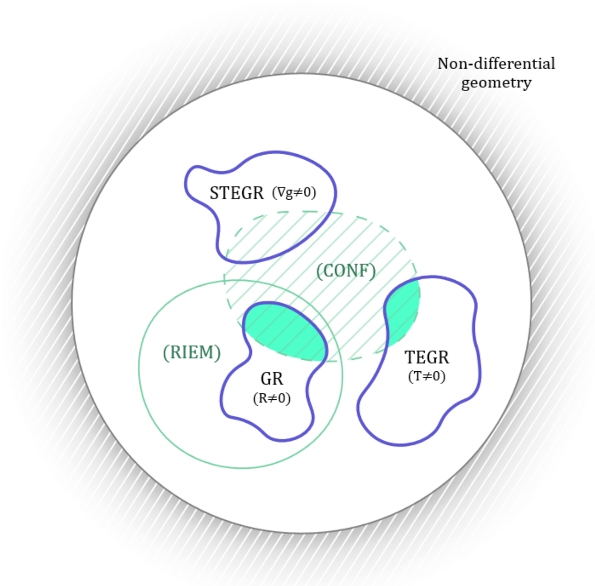
[5] I thank an anonymous reviewer for asking me to compare the proof of Proposition 2 with Theorem 1 in (Roberts 202?), which considers conformally *in*equivalent spacetimes. Just as Appendix A generalises Proposition 2, one might attempt to extend Roberts' Theorem 1 to not-necessarily-symmetric connections using Jensen's Eq.(A.1). However, Roberts' conclusion that $F^a{}_b = 0$ relies on the symmetry of the connecting field $C^a{}_{bc}$, which fails in the torsionful case. Following Theorem 1's proof (202?, p. 22, Eq. (6)), one sees that the torsion must cancel the symmetric part of the connection field identically: $C^a{}_{bc} = K^a{}_{(bc)}$. This condition is highly non-generic, so a generalisation of Theorem 1 along the lines of Proposition 3 does not straightforwardly carry over. Moreover, as noted in footnote 4, Theorem 1 is not strictly a generalisation of Proposition 2, nor is its torsionful counterpart a strict generalisation of Proposition 3.



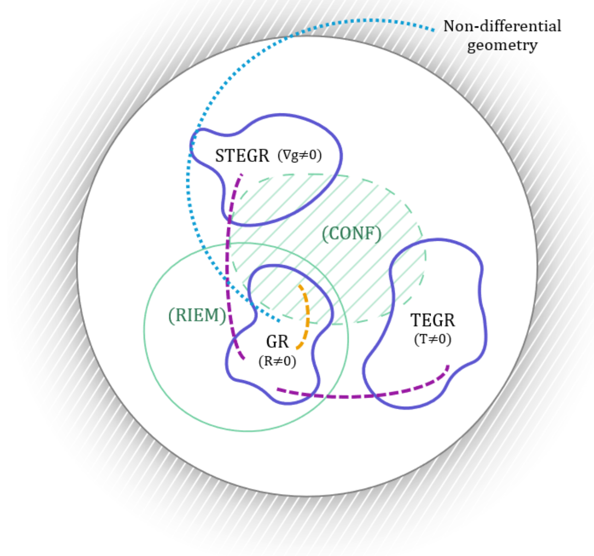
(a) For GR, W&M disprove the existence claim (**UDT**-$\forall g \exists \tilde{g}$) that there is a force tensor satisfying (**FORCE**) that can make two conformally equivalent spacetimes (highlighted region) empirically equivalent.



(b) Denying (**CONF**), one may still believe (**UDT**-$\forall g \exists \tilde{g}$) could be true in GR. Yet this does not undermine the disproof of the universality claim (**UDT**-$\forall g \forall \tilde{g}$), but generalises it: theorem θ remains false under (**FORCE**).



(c) Proposition 3 generalises the result in (a) by allowing torsionful connections (cf. §4 and Appendix A) by rejecting the assumption (**RIEM-SYMM**). This does not serve as a loophole to save theorem θ in GR; more generally, there exists no pair of conformally equivalent torsionful spacetimes related by a standard force field.



(d) To-be-proven existence claims: the yellow line indicating (**UDT**-$\forall g \exists \tilde{g}$), the purple lines the existence claims towards the other two nodes of the geometric trinity of gravity, and the blue line searching for the existence of models equivalent to GR that go beyond differential geometry (e.g. Einstein algebra formulations), all under (**FORCE**).

Figure 1: *The space of relativistic spacetimes and some inhabitants.*

5. Underdetermination, conventionalism and Conjecture θ

Note how all-encompassing theorem θ is: it states that for any arbitrary given metric, any other arbitrary metric is equally good, at least as far as observable facts go. What counts as a duration in one geometry may be a length in another; what is a continuous path in one may appear as a particle popping in and out of existence in the other. This is what makes theorem θ a version of the universality claim (**UDT**-$\forall g \forall \tilde{g}$), under a broad understanding of 'universal effects', which are particularly fleshed out under Helmholtzian measurability conditions and a fine-grained concept of physical geometry (an operationalist semantics also alluded by D&BM 2022, p. 161).

Reichenbach (1928, §8, p. 31–33) states theorem θ without proof. He extrapolates from Poincaré's (1891) intertranslatability of the three constant-curvature geometries, i.e., flat Euclidean, positively-curved Riemannian and negatively-curved Bolyai-Lobachevsky geometry, and adds: "No epistemological objection can be made against the correctness of theorem θ" (1928, p. 33). Would we see clearer when we think of it as 'Conjecture θ', in need of proof, as W&M (at least implicitly) seem to do? The answer involves some subtle interplay between the semantic and the epistemic components of realism, which will clarify the sliding between (**UDT**-$\forall g \forall \tilde{g}$) and (**UDT**-$\forall g \exists \tilde{g}$) discussed in §3.

On the one hand, starting from a conventionalist position that assumes that for a (physical) geometric fact to be *meaningful* we first need to stipulate a baseline to calibrate our measuring apparatuses (cf. Padovani 2017) – note Reichenbach explicit mention of "universal deformation of measuring instruments" – and with this stipulation itself not epistemically determinable, surely no further proof is required: Poincaré's one-to-one equivalence results *trivially* back it up.This amounts to a selective realism, arguing that to begin with there are no geometric facts independent of conventions that are not dictated by nature.

On the other hand, in that case the designation 'theorem' is also an exaggeration. Theorem θ would then amount to saying that geometric structure can always be semantically construed as non-geometric structure. Reichenbach's "no epistemological objection can be made" may express this, but then why not apply the same logic to *any* physical structure? Hypothetically any structure can be translated into some other structure in ways we cannot empirically distinguish, even in terms of structures we have not yet conceived?

It seems to me that Reichenbach, enamoured by Helmholtz' empirical approach to physical geometry and Einstein's successful application of curved spacetimes, had a too fine-grained conception of physical geometry. In particular, he takes the metric tensor as the unique vehicle to directly represents physical geometrical properties. In its wake, this enforces that F_{ab} cannot meaningfully perform that job and should be interpreted as something non-geometrical, e.g., a force. Thus, the metric was treated as exhaustively constitutive of 'physical geometry', stipulating that no other structure could meaningfully supplement or displace it.[6] On this fixed way of giving meaning to geometric terms, Reichenbach hides physical

[6] Essentially, this is an accusation of *essentialism*: just as a classical essentialist holds that a concept is defined by a fixed list of necessary properties, the conventionalist insists that geometrical properties consist exactly of the axioms or analytic stipulations they adopt. Putnam's "negative" essentialism (Putnam 1974) similarly responds to Quinean holism, by contending that reference should be secured by maximising internal coherence (e.g., simplicity, intuitive fit) and external coherence (basically: empirical confirmation) across the whole of science. Reichenbach shows clear concern with the practice of how our geometric terms actually

geometry behind an epistemic veil, because the metric tensor and the universal effect tensor are *presumptively* meaningfully distinct. In this sense, we need not enquire into (**UDT**-$\forall g \exists \tilde{g}$), for the existence of meaningfully distinct models is trivially secured: (**UDT**-$\forall g \forall \tilde{g}$) is already possible through mere substitution of the metric by another metric and a universal effect $g_{ab} = F_{ab} - \tilde{g}_{ab}$, without any need for (**FORCE**).

Another kind of 'conventionalist' (better: empiricist), emphasising the epistemic over the semantic, can still hold (**UDT**-$\forall g \exists \tilde{g}$). Such an anti-realist only needs *one* instance of underdetermination, and although W&M's Proposition 2 removes some, it does remove all underdetermination—unless each assumption is independently justified (which may well be possible). It seems this is the kind of geometric anti-realism W&M take as their foil, and D&BM implicitly endorse this by formulating (**ALT-ACC**). Thus W&M's project is meaningfully distinct from Reichenbach's.

The above thus explains why W&M – and this is criticised by D&BM (2022, pp. 161-162) – cite many conventionalist authors but do not engage with their key concepts: coordinative definitions, congruences, truth. Of course a good deal more is to be said about the nature of conventions and the conventionalist stance – indeed about a century-worth of literature fleshes out various 'conventional' choices – but little subtlety is needed to evaluate Proposition 2. For W&M only address the formal problem of the existence of model underdetermination without addressing conventionalism *per se*. Whether geometric statements are 'true-by-convention' are not central to their approach. That is, W&M adopt the same coordinative definitions commonly used to link the formalism of GR to the world.

6. A programme: systematically exploring the space of spacetime theories

Purely formally, the debate over geometric conventionalism partly stems from the observation that *prima facie* one can trade geometric structure for universal effects, at least mathematically. We saw that Reichenbach claimed this holds for *any* pair of metrics, and that W&M show that, in GR, under (**FORCE**), this is not so. Still, (**UDT**-$\forall g \exists \tilde{g}$) remains neither proven nor refuted, leaving open the possibility that *some* metrics can. Were (**UDT**-$\forall g \exists \tilde{g}$) proven correct for GR under (**FORCE**), there would definitively be no grounds for the conventionalist view (besides denying (**FORCE**)). Unfortunately, it is hard to get mathematical traction on such a general statement as (**UDT**-$\forall g \exists \tilde{g}$). Yet, the formal machinery of the proof provides intermediate positions, opening up possibilities for articulating – even generating – empirically equivalent models, bringing into focus those inhabitants of the formal space of alternative spacetime theories that are known to us. This section explores some options.

6.1. Logical structure of the 'go theorem' and its targets.

Rather than attempting to undermine the proof, I suggest a programmatic attitude: use the list of assumptions in §2 not just as inspiration but as formal tools to write down further propositions, quantified either universally or existentially. Tasdan and Thébault (2024) adopt a consonant constructive spirit. Citing D&BM, who deny Proposition 2 any merit, Tasdan and Thébault instead give the no-go theorem a more exploratory twist:

acquire determinate reference; yet, he disagrees with Putnam that theoretical virtues like simplicity, which underlie coherence, are truth-conducive. I would nevertheless argue that in any case he unjustifiably singles out geometric facts from other physical magnitudes, by treating their representation by the metric as fixed.

> Since the theorem contained in their Proposition 2 is both valid and non-trivial, we take there to be good cause to explore its implications as a 'go theorem' in the context of the negation of the various physical, mathematical and framework assumptions. (Tasdan and Thébault 2024, p. 492)

They then explore five rearticulations of spacetime conventionalism, "Spacetime Conventionalism 1–5".[7] Introducing these, Tasdan and Thébault appeal to (Dardashti 2021), which focusses on the heuristic value of no-go theorems for theory development in physics and highlights the fact that no-go results do not dictate *what* must be abandoned, only that *something* in the setup must give.[8] I wholeheartedly agree with Tasdan and Thébault's constructive attitude. However, Tasdan and Thébault do not find their five conventionalist theses via the negations of premises.[9] Rather, they affirmatively formulate their own without reference to the assumptions that underlie Proposition 2. Rather than following the logical structure of the theorem, they independently conceive of interesting conventionalist positions. In part, this is because they distinguish – following (Dardashti 2021) – between formal possibilities and physical interpretations, to be rejected independently. As such, they characterise (**RIEM**) as a formal assumption and (**CONF**) as a physical assumption.

Below I will proceed differently, staying closer to the no-go logic of the set of mutually inconsistent premisses on a formal level. This helps to explore the formal space of spacetime theories that can be generated given GR as a starting point, and actively using the assumptions in §2. This is reminiscent of D&BM's approach but gives their systematic approach a new direction along the lines of the constructive spirit of Tasdan and Thébault. It will also cover much additional territory, while keeping an eye on D&BM's requirement that alternatives should at least have a "modicum of *initial* plausibility" (2022, p. 170).

6.2. **¬(TOPO).** Assumption (**TOPO**) is relevant for W&M's proof in that it keeps the topological structure constant: the choice of different connections ∇ and $\widetilde{\nabla}$ is considered to

[7]The first three, which respectively focus on affine structure, inertial structure, and tidal effects, are ultimately rejected as either mathematically inconsistent, physically unmotivated, or ruled out by invariant geometric identities. (For further undermining of Conventionalism 3, see Theorem 2 in (Roberts 202?)). Spacetime Conventionalism 4 and 5, concerning the underdetermination of nomic structure and the possible non-uniqueness of the Bach tensor (i.e., the conformally invariant part of the Einstein tensor) as the *unique* conformally invariant rank-2 tensor in four dimensions (the *Bach conjecture*), remain potentially viable.

[8]Note that this method of identifying the structure of thesis rejection is a common formal method in philosophy, both heuristically and for the purposes of classifying by matching positions in an extant debate with the denial of each premise (see for example (Häggqvist 2009; Mulder and Muller 2023) for modal-logical no-go theorems that undermine destructive thought experiments).

[9]With one exception: for Spacetime Conventionalism 5, Tasdan and Thébault appeal to W&M's no-go theorem to claim that the universal effect tensor A^{ab}, i.e., the non-conformally invariant part of the Einstein tensor, is not a standard force apparently because it is not conformally invariant: "Clearly, by the theorem of Weatherall and Manchak (2014), A^{ab} will not be expressible as a Newtonian force" (*ibid.*, p. 503). The conclusion is correct but does not follow from the logic of the theorem: ¬(**CONF**) does not imply ¬(**FORCE**). Whether A^{ab} is a force depends on whether it enters the geodesic equation as a conservative rank-2 tensor term – not on its conformal behaviour. Because of the contracted Bianchi identity, $\nabla_a G^{ab} = 0$, we have $\nabla_a A^{ab} = \nabla_a B^{ab}$, and, because B^{ab} is interpreted to "characterise facts about geometric spacetime structure" (*ibid.*, p. 503), the divergence $\nabla_a A^{ab}$ is generally not a rank-2 tensor field in the geodesic equation. Moreover, if A^{ab} is interpreted as a tidal effect it fails to meet (**FORCE**) entirely for it does not act on a particle—it represents not the acceleration *of* curves but *between* curves).

take place on the same manifold, i.e., between $(\mathcal{M}, \nabla)$ and $(\mathcal{M}, \widetilde{\nabla})$, not between $(\mathcal{M}, \nabla)$ and $(\mathcal{N}, \widetilde{\nabla})$ (for $\mathcal{N} \neq \mathcal{M}$). This is a reasonable assumption, in line with the historical trend of the debate about conventionalism by focussing on local rather than global features. However, it is an open question whether some kind of topological conventionality relevantly interacts with traditional geometric conventionality.

Consider ¬(**DIM4**). In some theories, the effect of what is considered a force in GR can be modelled by a geometric structure in higher dimensions, such as the identification of electromagnetic charge with the value of momentum of particles in the fourth spatial dimension of Kaluza-Klein theory (Kaluza 2018, 1921). Roberts (202?) discusses how simply embedding GR in higher-dimensional spaces from which the four-dimensional spacetime curvature can be recovered generally reflects not a freedom of conventional choice, but creates incompleteness: such frameworks introduce fine-tuning problems and require unexplained features unless completed by additional physical laws. Kaluza-Klein theory succeeds *miraculously*, precisely because it posits such laws in five dimensions, eliminating arbitrariness.

Next, consider ¬(**HAUS**). That is, the denial that any two distinct points in the manifold can be separated by neighbourhoods. This means that there may be distinct spacetime events that may nevertheless be indistinguishable topologically: points of the manifold become 'glued together'.[10] Luc (2020) suggests that (**HAUS**) is not mandatory for GR but usually imposed as a deterministic demand: to rule out future evolutions otherwise compatible with the same initial data. J.B. Manchak (2013, pp. 47–48) argues that (non-Hausdorff) Misner spacetime – where causal regions and CTC regions are neatly separated – may be physically viable, too. This breakdown of standard GR topology also suggests an algebraic reformulation of spacetime in terms of Einstein algebras (Geroch 1972; cf. Müller 2013; Rosenstock, Barrett, and Weatherall 2015; Shi 202?), which avoids the situation that spacetime is defined by points and encodes motion algebraically: (**FORCE**) may not even be meaningful.

Finally, one can also consider upending causal properties by introducing divergent global structures. Reichenbach (1928, §12) already discusses underdetermination $G + F + A$ by postulating particular causal anomalies A such as singularities, closed timelike curves, or topological gluing—for an overview of suggestions towards concrete empirically equivalent models $G' + F' + A'$, see for example (Manchak 2013, Ch. 5) or (Arntzenius and Maudlin 2002; Mulder and Dieks 2017) for underdetermination of models in GR without closed timelike curves by models with them. More generally, Grimmer (202?) introduces a broad method – which he calls the ISE Method – to systematically generate topologically distinct but physically equivalent formulations of a spacetime theory. The Möbius–Euclid duality developed there illustrates how two theories with very different topologies (e.g., a point particle on a Möbius strip versus a line on the Euclidean plane) can nonetheless encode identical dynamics, thus demonstrating that topological structure can be redescribed without loss of empirical content. This provides ample leeway to construct initially plausible topologically distinct alternatives to models of GR.

[10] Because showing paracompactness in GR often relies on Hausdorffness, dropping (**HAUS**) can make (**PARA**) harder to establish for some equivalent model of GR. Yet there do exist non-Hausdorff paracompact spaces empirically equivalent to GR (cf. Wu and Weatherall forthcoming, Appendix, Lemma 1).

6.3. ¬**(RIEM).** It is certainly plausible to deny (**RIEM**). W&M pose the underdetermination question within the class of Levi-Civita connections. Yet, there is a plausible reason to consider a broader range of affine-connections to inform further propositions, namely the existence of metric-affine alternative theories to GR that do not centrally employ Levi-Civita connections. The main examples are the theories of the so-called 'geometric trinity of gravity'. In GR, gravitational effects are a manifestation of spacetime curvature, but it is by now well-known there is a theory *prima facie* distinct from GR, empirically equivalent to it, and in which gravitational effects are a manifestation of spacetime torsion (cf. Lyre and Eynck 2003; Knox 2011; Wolf and Read 2023; Mulder and Read 2024; Weatherall and Meskhidze 2024). This latter theory is known as the 'teleparallel equivalent of general relativity' (TEGR). Increasingly well-known known amongst philosophers (Chen and Read 2023; Wolf, Sanchioni, and Read 2024; Mulder 202?; Weatherall 202?) is a third theory (cf. Bahamonde et al. 2023; Heisenberg 2024), called the 'symmetric teleparallel equivalent of general relativity' (STEGR), in which gravitational effects are a manifestation of spacetime non-metricity, i.e., the non-compatibility of the connection with the metric.

Reichenbach had no scruples using an anti-symmetric affine-connection, as becomes clear in his geometrisation of electromagnetism. Giovanelli (Section 4, 2021) shows in what way Reichenbach was not naive about these matters, attempting (arguably successfully) to geometrise the electromagnetic field by decomposing the affine-connection into the Christoffel symbols as the product of a mixed anti-symmetrical tensor and a covariant vector.

Furthermore, the Appendix to *Raum und Zeit* makes clear that Reichenbach saw the geometrisation of gravity as one way of casting the physical content (that is, gravity) in a mathematical mould of geometry, which he regarded as a mere visual "shiny cloak"—see the recently translated (and long out of print) Appendix to Reichenbach's book analysed in (Giovanelli 2021). This inspires a search for what such a shiny cloak could be hiding, which (less metaphorically) means a 'force theory' or other causal theory in which the curvature is constant, and in particular flat:

- **UDT-**$\forall g \exists \eta$: for each metric featuring in an empirically adequate model of some spacetime theory, the Minkowski metric (or Euclidean, for non-relativistic theories) is capable of reproducing the same observable consequences, given suitable universal effects.

For GR and under (**FORCE**), this is proven *false* by Proposition 2, for W&M prove there are models for which this cannot be done, namely for conformally flat models, and hence there is no equivalent flat space standard force version of the theory of GR as a whole.

6.4. ¬**(FORCE).** In fact W&M (2014, pp. 235-237) are a bit more detailed than D&BM's formulation of (**FORCE**). I find three minimal specifications of their standard force: (**FORCE-a**) a force is some physical quantity acting on a massive body or point particle; (**FORCE-b**) forces are represented by rank-2 tensors at a point; (**FORCE-c**) the total force acting on a particle at a point must be proportional to the acceleration of the particle at that point, and vanishes just in case the acceleration vanishes.

One may deny **(FORCE-a)** by rejecting that universal effects must enter as force-like terms in the geodesic equation—after all, not all physical influences act by locally deflecting particles from geodesic motion. Tasdan and Thébault (2024) consider one such route via the geodesic deviation equation, which captures relative acceleration rather than particle-level forces. They find it promising but lacking concrete proposals.

One may deny **(FORCE-b)** by allowing universal effects to be encoded in higher-rank tensor fields, e.g. $C^a{}_{bc}\xi^b\xi^c$. Alternatively, one may even reject the universal effect to be fundamentally conservative. Alternatively, one might reject that universal effects must be conservative. While forces are typically derived from a potential, nothing *a priori* rules out universal effects of a "drag" type: velocity- or history-dependent interactions familiar from, for example, radiation theory. These violate integrability and energy conservation—serious drawbacks if deemed fundamental, but not obviously ruled out just by empirical adequacy.

To deny **(FORCE-c)**, one rejects the idea that universal effects must be proportional to acceleration. In other words, that it necessarily manifests as a deviation from inertial motion. In GR, famously, gravity causes no proper acceleration: free-falling particles follow geodesics. Likewise, one might imagine a universal effect that alters relative motion (e.g. via curvature or background fields) without pushing particles off their geodesics, and so without acting as a local force in the Newtonian sense.[11]

6.5. $\neg$**(CONF).** One may ask whether it is possible to drop the conformal restriction (**CONF**). Given that 'conformal structure', 'light-cone structure', and 'causal structure' are used interchangeably, one could investigate where such concepts come apart via different interpretations of identifying their factual or conventional underpinnings. For Reichenbach, the factual content of the theory of gravity is given by causal relations, as established via certain operational means such as the sending and receiving of light-signals. Before specifying one's method to establish them, lengths and durations are not distinguished.

But more to the formal point, under the coordinative definitions W&M adopt, one may drop (**CONF**) and subsequently attempt to prove the proposition that for any two spacetimes of GR there is no tensor field F_{ab} such that an arbitrary curve γ is a geodesic relative to ∇ iff its acceleration relative to $\widetilde{\nabla}$ is given by $F^a_n\tilde{\xi}^n$. Then introduce the connecting field (Eq. (A.1)) and write the $\tilde{g}$–acceleration of a g–geodesic, i.e., $\tilde{\xi}^b\widetilde{\nabla}_b\tilde{\xi}^a = \tilde{\xi}^b\nabla_b\tilde{\xi}^a - C^a{}_{bc}\tilde{\xi}^b\tilde{\xi}^c$, and ask whether the right hand side reduces to a linear map on the velocity vector field. But despite the (potentially simplifying) freedom to evaluate this on a g-geodesics, without imposing extra structure there is no straightforward route to relating ξ^a to $\tilde{\xi}^a$. Because covariant differentiation requires knowledge of how a field varies off the curve, there is thus no straightforward way to compute $\nabla_b\tilde{\xi}^a$.

In particular, there is no unique, linear cone-preserving map from g-timelike vectors to $\tilde{g}$-timelike vectors. This highlights the traction obtained via (**CONF**): if the metrics are conformally equivalent their light-cones coincide and thus admit of a one-to-one correspondence of time-like vectors (and *vice versa*). Without that, one is mixing some time-like vectors of one geometry with some space-like vectors of the other.

How could one proceed generally? I believe it beneficial to dive into some detail here, for it is worth emphasising how (**CONF**) and (**FORCE**) hang together in GR. In full generality, one may attempt the following. At each point p the timelike cones of two metrics

[11]Finding yourself in a Reichenbachean mood, hypothesise a non-constant scalar field $\tau(x)$ which alters the behaviour of rods and clocks without exerting any force on particles, e.g., shifting atomic frequencies. Test particles would still follow geodesics, but observers using τ-dependent measurement devices observe relative drifts or redshifts. Thus τ produces observable structure without deflecting motion, evading (**FORCE-c**).

g_{ab} and $\tilde{g}_{ab}$ appear as open double-cones in the tangent space T_pM. One may introduce a relative endomorphism $S^a{}_b = g^{ac}\tilde{g}_{cb}$. (Check the special conformal case: for $\tilde{g}_{ab} = \Omega^2 g_{ab}$ one finds $S^a{}_b = \Omega^2\delta^a{}_b$ and hence the g-cone and $\tilde{g}$-cone coincide again.) Generally, $S^a{}_b$ is self-adjoint with respect to g and thus diagonalizable with real eigenvalues $\{\lambda_i\}$, so that in a g-orthonormal basis we have $\tilde{g}_{ab} = \mathrm{diag}(-\lambda_0, \lambda_1, \lambda_2, \lambda_3)$. (Check again the conformal case: at a point, $\tilde{\xi}^a$ will not be parallel to ξ^a unless all these eigenvalues coincide.) In general, because the eigenvalues are positive – assuming the two metrics are non-degenerate and have the same signature – the spectral theorem tells us there is a unique smooth endomorphism $P^a{}_b = \sqrt{g^{-1}\tilde{g}}$ such that $S^a{}_b = P^a{}_n P^n{}_b$. We can now use this to carry back and forth vectors between the geometries: $\tilde{\xi}^a = (P^{-1})^a{}_b\xi^b \iff \xi^a = P^a{}_b\tilde{\xi}^b$. By construction $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = g_{ab}(P\tilde{\xi})^a(P\tilde{\xi})^b = g_{ab}\xi^a\xi^b$, and ξ^a is timelike with respect to the new metric $\tilde{g}$ iff $\tilde{g}_{ab}\xi^a\xi^b > 0$ and thus also $g_{ab}\xi^a S^b{}_c\xi^c > 0$. Thus, the sign of $\tilde{g}(\xi,\xi)$ is determined (pointwise) by the quadratic form $g(S\xi,\xi)$.

Now let us compute the acceleration of a ∇-geodesic in the new geometry,

$$\begin{aligned}\tilde{\xi}^b\tilde{\nabla}_b\tilde{\xi}^a &= \tilde{\xi}^b\nabla_b\big((P^{-1})^a{}_c\xi^c\big) + C^a{}_{bc}\tilde{\xi}^b\tilde{\xi}^c \\ &= \tilde{\xi}^b\big[\nabla_b(P^{-1})^a{}_c\xi^c + (P^{-1})^a{}_c\cancelto{0}{\nabla_b\xi^c}\big] + C^a{}_{bc}\tilde{\xi}^b\tilde{\xi}^c \\ &= -\tilde{\xi}^b\,\big((P^{-1})^a{}_d(\nabla_b P^d{}_e)(P^{-1})^e{}_c\big)\,\xi^c + C^a{}_{bc}\tilde{\xi}^b\tilde{\xi}^c,\end{aligned} \tag{6.1}$$

where the last step is an identity.[12] Conversely, let $C^a{}_{bc} = (P^{-1})^a{}_d\nabla_b P^d{}_c$ and check the acceleration of the curve in the original geometry,

$$\begin{aligned}\xi^b\nabla_b\xi^a &= \xi^b\nabla_b\big(P^a{}_d\tilde{\xi}^d\big) \\ &= \xi^b\nabla_b P^a{}_d\tilde{\xi}^d + P^a{}_d\xi^b\nabla_b\tilde{\xi}^d \\ &= \big(C^a{}_{bd}P^d{}_c\big)\xi^b\tilde{\xi}^c + P^a{}_d\big(-C^d{}_{bc}\xi^b\xi^c\big) \\ &= C^a{}_{bc}\xi^b\xi^c - C^a{}_{bc}\xi^b\xi^c = 0,\end{aligned} \tag{6.2}$$

recovering the original g–geodesic equation $\xi^b\nabla_b\xi^a = 0$.

Despite the relative triviality of the above derivation, making explicit the (1,1)-form P shows a rather general point about the prospects of denying $\neg$(**CONF**) on its own. The connecting field $C^a{}_{bc}$, which has tensor rank 3, of course measures precisely the deviation of a g-geodesic from a $\tilde{g}$-geodesic. As W&M observe, it does not resemble a standard force as demanded by (**FORCE**). The above, however, shows that it can generally be expressed as $C^a{}_{bc} = (P^{-1})^a{}_d\nabla_b P^d{}_c$, from which one sees that in general no simplification is possible: only when $P^a{}_b$ is a pure scalar multiple of the identity can you collapse that rank-3 object $C^a{}_{bc}$ down to a rank-2 tensor field. That is, if $P^a{}_b = \Lambda\delta^a{}_b$, then

$$C^a{}_{bc} = (P^{-1})^a{}_d\nabla_b P^d{}_c = \frac{1}{\Lambda}\delta^a{}_d\nabla_b\big(\Lambda\delta^d{}_c\big) = \delta^a{}_c\partial_b\ln\Lambda + \delta^a{}_b\partial_c\ln\Lambda, \tag{6.3}$$

[12] Starting from $P^a{}_d(P^{-1})^d{}_c = \delta^a{}_c$ and taking the derivative (with respect to ∇) on both sides gives $\nabla_b\big(P^a{}_d(P^{-1})^d{}_c\big) = (\nabla_b P^a{}_d)(P^{-1})^d{}_c + P^a{}_d\nabla_b(P^{-1})^d{}_c = 0$. Now solving for $\nabla_b(P^{-1})^d{}_c$ gives $\nabla_b(P^{-1})^d{}_c = -(P^{-1})^d{}_a(\nabla_b P^a{}_e)(P^{-1})^e{}_c$.

so one can write

$$C^a{}_{bc} = \delta^a{}_b \omega_c + \delta^a{}_c \omega_b, \qquad \omega_b = \partial_b \ln \Lambda, \tag{6.4}$$

such that one can write the geodesic equation

$$\tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a = F^a{}_b \tilde{\xi}^b, \qquad F^a{}_b = -2\Lambda^2(\omega_d \tilde{\xi}^d)\delta^a{}_b, \tag{6.5}$$

which is manifestly a rank-2 tensor acting on $\tilde{\xi}^b$. But this is precisely the conformal case that we have tried to deny! Thus it seems one cannot satisfy both $\neg$(**CONF**) and (**FORCE**).

6.6. $\neg$**(NORM).** In the previous subsection, no normalisation was imposed. Doing so simply gives $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = g_{ab}(P\tilde{\xi})^a(P\tilde{\xi})^b = g_{ab}\xi^a\xi^b = 1$. Technically: imposing normalisation does not shrink the space of allowable maps but simply picks out those vectors on which you evaluate $P^a{}_b$. Denying (**NORM**), i.e. both $g_{ab}\xi^a\xi^b \neq 1$ and $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b \neq 1$, but still setting $\tilde{\xi}^a = (P^{-1})^a{}_b\xi^b$, the geodesic equation obtains an additional reparametrization term (cf. Carroll 2003, p. 109): $\tilde{\xi}^b\tilde{\nabla}_b\tilde{\xi}^a = -C^a{}_{bc}\tilde{\xi}^b\tilde{\xi}^c + \tilde{\xi}^n\tilde{\nabla}_n \ln(|\tilde{g}(\tilde{\xi},\tilde{\xi})|^{1/2})\tilde{\xi}^a$. This is because $\tilde{\xi}^a$ no longer has constant unit-length in $\tilde{g}$, so one should correct for the failure of affinity in the $\tilde{g}$-geometry. This fact is used by D&BM (2022, p. 156), but I have here given the general case.

Within GR, (**NORM**) naturally goes hand in hand with (**CONF**): when the two spacetimes share the same light-cone structure, one can simply rescale the vectors since they are already pointing in the same direction. That is, having the conformal relation $\tilde{g}_{ab} = \Omega^2 g_{ab}$ gives a simple conformal rescaling for *normalised time-like* vectors: $\tilde{\xi}^a = \Omega^{-1}\xi^a$. Since a Levi–Civita connection determines its metric only up to an overall constant conformal factor, if one wants to satisfy (**FORCE**) one sees that the choice of norm is here not independent from the choice of scale: without a unit-norm condition one cannot match accelerations across geometries, and without a fixed conformal class one cannot compare norms.

7. Discussion: conceiving of alternatives through rigorous conventionalism

W&M's proof of Proposition 2 restricts the possibilities for model underdetermination in GR. Whether this restriction is significant is in the eye of the beholder. The fact remains that, under (**FORCE**), the proof refutes the universality claim that all metrics can be traded off against each other, but leaves open the existence claim that at least one alternative metric $\tilde{g}$ may exist for a given g. As such, D&BM's (**ALT-ACC**) can still be conjectured to hold, unless convincing justifications are given for each assumption listed in §2. Indeed, the responsibility to make things explicit often falls on those who deny underdetermination.

Yet, there is no rich no-go theorem to save (**UDT**-$\forall g \forall \tilde{g}$) in GR, for it requires only two premises: theorem θ and (**FORCE**). Even without a justification for any of the other assumptions, the only genuine loophole to save theorem θ is the restriction that the universal effect should act like a standard force field. The benefit of this constraint is that it affords mathematical tractability. But indeed, many conventionalists would reject it—by denying either that universal effects must enter the geodesic equation, or that they must be represented by rank-2 tensors. Reichenbach likely would have rejected the latter.

Ultimately, W&M's project is best viewed as distinct from the Reichenbachian one, or from (anti-)conventionalism proper. They consider the leeway possible between our leading theory of gravity and our leading concept of force, all the while keeping the usual coordinative definitions of GR intact. This is much in the spirit of David Malament:

> Philosophers of science have written at great length about the geometric structure of physical space. But they have devoted their attention primarily to the question of the epistemic status of our attributions of geometric structure. They have debated whether our attributions are *a priori* truths, empirical discoveries, or, in a special sense, matters of stipulation or convention. It is the goal of this chapter to explore a quite different issue — the role played by assumptions of spatial geometry *within physical theory* [...]. (Malament 1986, p. 405, *original emphasis*)

Seen in this light, I suggested continuing W&M's work by interpreting the relativistic proof of Prop. (2) as the starting point of a research programme, for example through extensions such as Proposition 3 (§4) and systematically exploring the space that I have begun to classify in §6. I have given only a handful of examples, but these can be multiplied by casting our net wider and wider over a space of alternatives, including those that move away from differential geometry, approaching the unconceived. Indeed, Stanford's (2010) so-called New Induction reminds us that our current theory space is unlikely to be exhausted. Constraining this space allows for a disciplined exploration (quite distinct from our stance on realism). Systematic rejections of the assumptions to W&M's proof provide such constraining and thus allow for a controlled exploration of the formal landscape underlying the titular "Theory of Spacetime Theories" of (Lehmkuhl, Schiemann, and Scholz 2016): with newly formulated propositions one can trace routes between charted regions of the space of spacetimes, forming a growing atlas of alternative relativistic formulations.

A. Proving Proposition 3: generalising to torsionful spacetimes

Proposition 3 in §4 is a genuine generalisation in the sense that it proves that for any torsionful connection, there exists some (non-trivially conformally equivalent) torsionful connection which cannot be related by a force field F_{ab}. To prove it, we take as a starting point Stuart Jensen's (2005) Eq. (3.1.28), which is the extension of the 'difference' tensor $C^a{}_{bc} = \frac{1}{2} g^{an} \left(\nabla_n g_{bc} - \nabla_b g_{nc} - \nabla_c g_{bn}\right)$, when both connections are allowed to be torsionful[13]

$$C^a{}_{bc} = \frac{1}{2} g^{an} \left(\nabla_n g_{bc} - \nabla_b g_{nc} - \nabla_c g_{bn} - \Delta T^a{}_{bd} - \Delta T_b{}^a{}_d + \Delta T_d{}^a{}_b\right), \tag{A.2}$$

for the torsion tensor $T^a{}_{bc}$, which measures the anti-symmetric part of the associated connection (in coordinate-language it is given by $T^\rho{}_{\mu\nu} := 2\Gamma^\rho{}_{[\mu\nu]}$), and ΔT_{abd} the difference between

[13]The difference tensor relates two connections to each other as they act on a smooth tensor α of arbitrary rank (Malament 2012, Proposition 1.7.3, p. 51) as:

$$\begin{aligned}\left(\widetilde{\nabla}_m - \nabla_m\right) \alpha^{a_1 \dots a_r}{}_{b1 \dots b_s} = {} & \alpha^{a_1 \dots a_r}{}_{n \dots b_s} C^n{}_{mb_1} + \dots + \alpha^{a_1 \dots a_r}{}_{b1 \dots n} C^n{}_{mb_s} \\ & - \alpha^{n \dots a_r}{}_{b1 \dots b_s} C^{a_1}{}_{mn} - \dots - \alpha^{a_1 \dots n}{}_{b1 \dots b_s} C^{a_r}{}_{mn}.\end{aligned} \tag{A.1}$$

The standard difference tensor in GR (cf. Malament 2012, p. 78, Eq. 1.9.6) holds only for a metric-compatible and torsion-free connection, hence note the use of (**RIEM-COMP**) and (**RIEM-SYMM**) in Proposition 2; here (**RIEM-SYMM**) is broken explicitly.

the torsion tensors associated with ∇ and $\widetilde{\nabla}$. Let us group the torsion terms together as a 'generalised contorsion tensor': $2K_{abc} := -\Delta T_{abd} - \Delta T_{bad} + \Delta T_{dab}$, which reduces to the usual contorsion tensor if just one of the connections is torsionful and not the other.

Then, taking (**CONF**), one now takes a non-trivial conformal factor Ω that relates a new metric $\tilde{g}_{ab} = \Omega^2 g_{ab}$ of a new geometry to the old one. One can now compute Eq. (A.2) for this new metric (for upper indices we have $\tilde{g}^{cd} = \Omega^2 \tilde{g}^{ca}\tilde{g}^{db} g_{ab} = \Omega^2\Omega^{-4}g^{cd} = \Omega^{-2}g^{cd}$.) we have

$$
\begin{aligned}
C^a{}_{bc} - K^a{}_{bc} &= \frac{1}{2\Omega^2} g^{an}\left[\nabla_n\left(\Omega^2 g_{bc}\right) - \nabla_b\left(\Omega^2 g_{nc}\right) - \nabla_c\left(\Omega^2 g_{bn}\right)\right] \\
&= \frac{1}{2\Omega^2} g^{an}\left(g_{bc}\nabla_n\Omega^2 + \Omega^2 \cancelto{0}{\nabla_n g_{bc}} - g_{nc}\nabla_b\Omega^2 - \Omega^2\cancelto{0}{\nabla_b g_{nc}} - g_{bn}\nabla_c\Omega^2 - \Omega^2\cancelto{0}{\nabla_c g_{bn}}\right) \\
&= \frac{1}{2\Omega^2}\left(g_{bc}g^{an}\nabla_n\Omega^2 - \delta_c{}^a\nabla_b\Omega^2 - \delta_b{}^a\nabla_c\Omega^2\right), \qquad \text{(A.3)}
\end{aligned}
$$

where the vanishing terms vanish due to metric compatibility (**RIEM-COMP**).

As in W&M's proof, given any smooth timelike curve γ, if ξ^a is the tangent field to γ with unit length relative to (the old) g_{ab}, then the tangent field to (the same curve) γ with unit length relative to (the new) metric $\tilde{g}_{ab}$ is given by

$$
\tilde{\xi}^a = \Omega^{-1}\xi^a, \qquad \text{(A.4)}
$$

which holds due to (**NORM**): if $g_{ab}\xi^a\xi^b = 1$ and $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = 1$ then $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = g_{ab}\Omega^2\tilde{\xi}^a\tilde{\xi}^b = 1$.

Then, from Eq. (A.1), we see that the difference tensor acts on a vector as $\widetilde{\nabla}_m\alpha^{a_1} = \nabla_m\alpha^{a_1} - \alpha^n C^{a_1}{}_{nm}$. Now take any *geodesic curve* of the old connection ∇_a, and compute the acceleration relative to (the new) $\widetilde{\nabla}_a$, which is

$$
\begin{aligned}
\tilde{a}^n := \tilde{\xi}^n\widetilde{\nabla}_n\tilde{\xi}^a = \tilde{\xi}^n\nabla_n\tilde{\xi}^a - C^a{}_{nm}\tilde{\xi}^n\tilde{\xi}^m &= \frac{\xi^n}{\Omega}\nabla_n\frac{\xi^m}{\Omega} - C^a{}_{nm}\frac{\xi^n}{\Omega}\frac{\xi^m}{\Omega} \\
&= \frac{\xi^n}{\Omega}\nabla_n\frac{\xi^m}{\Omega} + \left(\frac{1}{2\Omega^2}\left(\delta_m{}^a\nabla_n\Omega^2 + \delta_n{}^a\nabla_m\Omega^2 - g_{nm}g^{ar}\nabla_r\Omega^2\right) - K^a{}_{nm}\right)\frac{\xi^n}{\Omega}\frac{\xi^m}{\Omega},
\end{aligned}
$$

where Eq. (A.4) and Eq. (A.3) are used in the second and last line, respectively. Performing the derivatives and bringing terms together,

$$
\begin{aligned}
\tilde{\xi}^n\widetilde{\nabla}_n\tilde{\xi}^a + K^a{}_{nm}\frac{\xi^n\xi^m}{\Omega^2} &= \frac{\xi^n}{\Omega}\nabla_n\frac{\xi^m}{\Omega} + \frac{1}{2\Omega^2}\left(\delta_m{}^a\nabla_n\Omega^2 + \delta_n{}^a\nabla_m\Omega^2 - g_{nm}g^{ar}\nabla_r\Omega^2\right)\frac{\xi^n}{\Omega}\frac{\xi^m}{\Omega} \\
&= \frac{1}{\Omega^2}\cancelto{0}{\xi^n\nabla_n\xi^a} - \frac{1}{\Omega^3}\xi^n\xi^a\nabla_n\Omega + \frac{2\Omega}{2\Omega^4}\left(\delta_m{}^a\nabla_n\Omega + \delta_n{}^a\nabla_m\Omega - g_{nm}g^{ar}\nabla_r\Omega\right)\xi^n\xi^m \\
&= -\frac{1}{\Omega^3}\xi^n\xi^a\nabla_n\Omega + \frac{1}{\Omega^3}\left(\xi^n\xi^a\nabla_n\Omega + \xi^a\xi^m\nabla_m\Omega - \cancelto{1}{g_{nm}\xi^n\xi^m} g^{ar}\nabla_r\Omega\right) \\
&= \frac{1}{\Omega^3}\left(-\xi^a\xi^n + 2\xi^a\xi^n - g^{an}\right)\nabla_n\Omega \\
&= \frac{1}{\Omega^3}\left(\xi^a\xi^n - g^{an}\right)\nabla_n\Omega, \qquad \text{(A.5)}
\end{aligned}
$$

where the vanishing term in the second line is due to being on a geodesic of the old geometry, (**NORM**) is invoked in the third line, and indices are relabelled in the fourth.

Starting from the other side of Proposition 3, *assume* that a tensor field F_{ab} exists—with the properties as described in (**FORCE**). In that case, it would have to balance out the old geometry (in which there are no forces, since γ is a geodesic) relative to the acceleration of the new geometry, so that

$$F^a{}_m \tilde{\xi}^m = \tilde{\xi}^n \widetilde{\nabla}_n \tilde{\xi}^a. \tag{A.6}$$

The left-hand side can be rewritten as

$$F^a{}_m \tilde{\xi}^m = \frac{1}{\Omega} F^a{}_m \xi^m = \frac{1}{\Omega} \tilde{g}^{an} F_{nm} \xi^m, \tag{A.7}$$

while the right-hand side of Eq. (A.6) is known, for it is given by Eq. (A.5):

$$\tilde{g}^{an} F_{nm} \xi^m = \frac{1}{\Omega^2} \left(\xi^a \xi^n - g^{an}\right) \nabla_n \Omega + \frac{1}{\Omega} K^a{}_{nm} \xi^n \xi^m, \tag{A.8}$$

and this holds for any smooth timelike tangent vector ξ^a at any point p, for F_{ab} is a tensor.

Now a proof by contradiction. Pick any point p on the curve, and choose at that point two arbitrary distinct timelike vectors μ^a and η^a and their superposed vector $\zeta^a = \alpha\left(\mu^a + \eta^a\right)$, renormalised by the scalar α to be unit length relative to (the old) g_{ab}. The idea is that Eq. (A.8) applies to both μ^a and η^a individually as well as to ζ^a directly, after which the results can be set equal to each other. Thus,

$$\begin{aligned}
\tilde{g}^{an} F_{nm} \zeta^m &= \alpha\left(\tilde{g}^{an} F_{nm} \mu^m + \tilde{g}^{an} F_{nm} \eta^m\right) \\
&= \frac{\alpha}{\Omega^2}\left(\mu^a \mu^n - g^{an}\right) \nabla_n \Omega + \frac{\alpha}{\Omega^2}\left(\eta^a \eta^n - g^{an}\right) \nabla_n \Omega + \frac{\alpha}{\Omega}\left(\mu^n \mu^m + \eta^n \eta^m\right) K^a{}_{nm} \\
&= \frac{\alpha}{\Omega^2}\left(\mu^a \mu^n + \eta^a \eta^n - 2g^{an}\right) \nabla_n \Omega + \frac{\alpha}{\Omega}\left(\mu^n \mu^m + \eta^n \eta^m\right) K^a{}_{nm},
\end{aligned} \tag{A.9}$$

and applying Eq. (A.8) to ζ^a directly and then using its definition, we have

$$\begin{aligned}
\tilde{g}^{an} F_{nm} \zeta^m =& \frac{1}{\Omega^2}\left(\zeta^a \zeta^n - g^{an}\right) \nabla_n \Omega + \frac{\alpha^2}{\Omega} \zeta^n \zeta^m K^a{}_{nm} \\
=& \frac{\alpha^2}{\Omega^2}\left(\mu^a \mu^n + \mu^a \eta^n + \eta^a \mu^n + \eta^a \eta^n - \alpha^{-2} g^{an}\right) \nabla_n \Omega \\
&+ \frac{\alpha^2}{\Omega}\left(\mu^n \mu^m + \mu^n \eta^m + \eta^n \mu^m + \eta^n \eta^m\right) K^a{}_{nm}.
\end{aligned} \tag{A.10}$$

After equating Eqs. (A.9) and (A.10), and rearranging so as to isolate the metric, we finally obtain

$$\begin{aligned}
\left(\frac{2\alpha - 1}{\alpha}\right) g^{an} \nabla_n \Omega =& \left[(1-\alpha)\left(\mu^a \mu^n + \eta^a \eta^n\right) - \mu^a \eta^n - \eta^a \mu^n\right] \nabla_n \Omega \\
&+ \Omega\left[(1-\alpha)\left(\mu^n \mu^m + \eta^n \eta^m\right) - \mu^n \eta^m - \eta^n \mu^m\right] K^a{}_{nm}
\end{aligned} \tag{A.11}$$

But this is logically inconsistent: the left-hand side is a vector independent of μ^a and η^a, whereas μ^a and η^a are *arbitrary* vectors. Thus Proposition 3 is true: also in the torsionful case, there is no tensor field F_{ab} that can everywhere relate the geodesics of ∇ and $\widetilde{\nabla}$.

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