Two quantum-mechanical arguments against the metaphysical equivalence between substratum and bundle theories of individuality

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Abstract: It is a widespread consensus among metaphysicians that the bundle and substratum theories are substantially different metaphysical theories of individuality. In a realist stance towards metaphysics, they cannot both track the truth when describing fundamental reality, thus they're rival metaphysical theories. Against that consensus, Jiri Benovsky has advanced a metametaphysical thesis that they are in fact metaphysically equivalent. This paper challenges Benovsky's equivalence thesis with two counter-arguments based on the metaphysics of quantum mechanics: quantum metaphysical indeterminacy and wavefunction realism. As we shall argue, while both substratum and bundle theories arguably fail in standard quantum mechanics, they fail in different ways. Hence, given Benovsky's own notion of metaphysical equivalence, they are not equivalent.

Keywords: Metaphysical equivalence; Metametaphysics; Metaphysics of quantum mechanics; Substratum and bundle theories of individuality; Metaphysical underdetermination

1. Introduction: Individuality in quantum mechanics

Let us assume, for the sake of argument, that it is reasonable to inquire about the individuality profile of quantum entities. More specifically, let us assume that in an object-oriented ontology for quantum mechanics, one could — or *should* — investigate whether such objects are individuals or non-individuals in the metaphysical sense of this term (see French 2018; Krause & Arenhart 2018; Arroyo & Arenhart 2024, for further distinctions). What can we say about that question, solely based on quantum mechanics?

The answer is well-known, but it is a bit frustrating: nothing very definitive. According to a well-established tradition in this debate, quantum particles may be understood as individuals, but they may also be understood as non-individuals. That is, one may plausibly attach a metaphysics of individuality to quantum objects, provided that such individuality is understood in terms of a 'transcendental individuality' (*i.e.* attributed through a haecceity or a bare particular), but not through a bundle theory of individuality requiring a robust version of the *Principle of Identity of Indiscernibles* (PII), which is said to fail in quantum mechanics, given the indiscernibility of quantum entities of the same kind (see Arenhart & Arroyo 2021 for the methodological discussion of connecting metaphysics with ontology and science). But quantum entities may also be seen as non-individuals, that is, as particulars failing to satisfy a principle of individuality, in some sense. The most famous incarnation of such a claim comes from the idea that quantum entities fail to enter into the relation of numerical identity, which deprives them of individuality too (for this debate, see the *locus classicus* French & Krause 2006, chap. 4, and also Arenhart 2017).

All of this is quite well established in the discussions about (non-)individuality in quantum mechanics. The result is the widely known *underdetermination of the metaphysics of individuality by the physics* of quantum mechanics. There is nothing in quantum mechanics suggesting that a 'non-individuals package' is better than the 'individuals package' or vice-versa; if one is to break the underdetermination, one has to appeal to extra-empirical factors such as continuity with classical theories (for a metaphysics of individuals) or to a suspicious over the intelligibility of transcendental principles of individuality (for a metaphysics of non-individuals; see Morganti 2015 for a general discussion).

The upshot of this debate is: through considerations originating in quantum mechanics, one often hears that bundle theories of individuality are not an option, but that still two major lines of approach may be freely considered, i) different forms of transcendental individuality and ii) different forms of non-individuality (see Arenhart 2017 for an overview of such options). This is, again, a re-statement of metaphysical underdetermination.

In the past few years, Jiri Benovsky (2008, 2016) has advanced a very intriguing thesis that has gone unnoticed by friends of the metaphysics of quantum mechanics: according to Benovsky, bundle theory and bare particulars (which is used in this paper as a synonym of *substratum*) are *metaphysically equivalent*. The sense in which such equivalence is argued for is based on the idea that there is no situation that one of the theories may explain, that the other cannot explain, by very similar lines of reasoning; also, there is no difficulty that one of the theories must face that is not also equally trouble for the other theory. So, in a sense, they are in the position that they succeed in the same places and fail in the same places alike.

All of that is in clear tension with the situation described in quantum mechanics, as mentioned before. Quantum mechanics, it is said, *distinguishes* between bare particulars and bundle theory by providing counterexamples to the latter and being wholly compatible with the former. What of metaphysical equivalence? If those theories are equivalent, both should be accepted, or both should be rejected, but they cannot really be distinguished by some situation. Surprisingly, the issue has not been brought to light so far in connection with quantum mechanics. Obviously, if the bundle theory is an option in quantum mechanics, along with bare particulars, then, the underdetermination just increases.

In this paper, we shall highlight what is really at issue in this apparent conflict. Both theses have a problem with standard quantum mechanics, we shall argue. On the one hand, the metaphysical underdetermination camp has been too quick to accept that substrata are compatible with quantum mechanics. As we shall claim, depending on the version of quantum theory adopted, bare particulars may have to go too. Now, that does not constitute evidence for the equivalence thesis. As we shall see, Benovsky's claim is based on a non-standard version of the bundle theory which may indeed be compatible with quantum mechanics, while his substratum theory is not. So, the equivalence thesis will have to go, again, provided that some specific interpretations of quantum theory are adopted. This, or so we shall argue, is particularly illuminating with regard to metametaphysical debates on the relationship between science and metaphysics.

The paper is structured as follows. In section 2 we revise the basics of the theories of individuality to be discussed in the paper. In section 3, we discuss the impact of quantum mechanics on some of these theories by invoking arguments stemming from quantum foundations. This includes the well-known failure of bundle theory due to the failure of the PII under the Permutation Symmetry principle, but also, we argue, the failure of substratum theories/bare particulars due to the quantum metaphysical indeterminacy and wave function realism. In section 4 we clarify the equivalence thesis by Benovsky. We indicate how the equivalence may be broken according to the discussion of section 3. We conclude in section 5 with some remarks on the methodology of metaphysics.

2. The basic concepts of individuality at play

For the purposes of our paper, a metaphysical theory of individuality has as its main goal determining *a principle of individuality*, which, as Lowe (2003, p.75, original emphasis) says, is for an individual "whatever it is that makes it the single object that it is — whatever it is that makes it *one* object, distinct from others, and the very object that it is as opposed to any other thing". That is, a principle of individuality has an explanatory power; it has to explain in clear terms what makes one thing exactly what it is and on what grounds the *numerical difference* of individuals is based. Notice that the characterization says nothing about *qualitatively distinguishing* objects; as far as individuality is concerned, a theory of individuality must account for the numerical difference of objects and for what they are; if a theory of individuality also offers principled ways to distinguish entities, that is a bonus (see Krause & Arenhart 2018, for the distinctions).

Here we shall be concerned with theories that attempt to *define* individuality in terms of more basic ingredients (see also French & Krause 2006, chap. 1). In a sense, these theories are typically understood as theories about the *composition* of particulars. The very idea of individuality is reduced or explained in terms of the more basic entities that are allowed to *constitute* an individual. As Demirli explains:

> In answering the internal constitution question, we may begin an inquiry about the various categories that go into the composition of individual substances and hope that at the end of this inquiry we will come up with a list of ingredients that constitute various individuals. Just as a certain recipe in a cook book provides us with a list of ingredients and instructions for mixing these ingredients together, we may maintain that the list or the recipe of individual substances — God's recipe book, so to say — will tell us what items from various categories are used, and how these items are combined. (Demirli 2010, p. 2)

So, what an individual is depends on the elements allowed to compose it. This composition will also have to grant uniqueness for individuals and account for their numerical difference. The ingredients involved need not be primitive, although they *may* be; in case they are not, then, they themselves must be defined in terms of even more basic ingredients.

The traditional bare particular approach advances the two basic, metaphysically primitive, ingredients for composing an individual: a primitive bare particular (which, as the name suggests, is a particular, not repeatable, or instantiable) and the properties of such an individual at a given time (properties are also to be taken as primitive ingredients here, for the sake of argument; we shall not enter in the details of whether these properties are universals or tropes, or how identity over time is to be treated). Socrates, for instance, is a kind of composition of a unique substratum, plus the properties of Socrates:[1]

$\text{Socrates}_{(BP)}$ = [Socrates' substratum, P_1, P_2, … P_k]

According to this theory, the numerical distinction of any two individuals is grounded on the fact that they have, as their components, distinct bare particulars; bare particulars are primitive posits of the theory, their numerical difference being a basic datum, not explained in terms of anything else (see also Benovsky 2016, chap. 1).

In the context of this theory, notice that *numerically different* individuals may share all of their properties, without being the same; if such a case presents itself, the individuals involved are *qualitatively indiscernible but numerically different*. A typical example would be Max Black's famous case of two absolutely indistinguishable spheres (Black 1952): Black has considered a thought experiment, suggesting that there could be a world consisting only of two iron spheres, of identical shape, mass, color, and so on. Nothing distinguishes the spheres, and they are still *two*. What accounts for their numerical difference? 'Bare particulars' is one possible answer. Obviously, another example *would be* quantum particles; as we mentioned, particles of the same kind, like electrons, may share all of their properties, and still, not be the same (see French & Krause 2006, chap. 4). This is a point we shall return to in the next section.[2]

Besides dealing with situations involving qualitatively indiscernible individuals, the theory of bare particulars also has another relevant feature: it accounts easily for identity over time and for qualitative change of individuals. Once the ground for individuality is the bare particular, an entity may change some or all of its properties and still be the same individual due to the presence of the bare particular. Sure, that change typically happens over time, and one has reasons to ensure that an individual remains numerically the same, despite changing its qualities.

Of course, all of those benefits are reaped at a substantially high cost. The primitive bare particular may raise many suspicions that it is just an *ad hoc* posit. Those not happy with such mysterious entities may prefer some version of the bundle theory of individuality, which clearly presents a more economic scenario. It defines individuals as the bundle determined by all the qualities the individual has, and only those. So, Socrates would be just a package of Socrates' properties.

$\text{Socrates}_{(BT)}$ = [P_1, P_2, …, P_k]

The bundle theory also requires another metaphysical posit, typically understood as a relation of compresence; this is a relation keeping all of the properties of the bundle actually bundled together (see Benovsky 2016, chap. 1). Still, each bundle has its own compresence relation. Notice that according to this account, we are now discussing, the compresence relation is not counted as an ingredient inside the bundle; it somehow stays at a second level, gluing the first-order properties of the individual.

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[1] We use '$\text{Socrates}_{(BP)}$' for the account of Socrates in the bare particular approach. Similarly, $\text{Socrates}_{(BT)}$ denotes Socrates according to the bundle theory.

[2] For the sake of conceptual precision, it is important to note here that unlike classical objects, in quantum mechanics the "joint determination" (Wolff 2015, p. 385) of incompatible properties — such as position and momentum or spin along different axes — is unavailable. Therefore, even if particles of the same kind share the same set of properties, they may not possess the same set of *actualized* properties at a given time, contrary to the classical case. We thank an anonymous referee for pointing this out.

In the context of the bundle theory, the explanation for the individuality of Socrates, what makes him the one thing he is and different from every other object, will have to rely only on one category of primitive ingredients, the constituting properties. As a result, it must be granted that no other entity exists that is constituted by exactly the same properties of Socrates. This is achieved by requiring that the individuals obey the famous *Principle of the Identity of Indiscernibles* (PII):

PII: If A and B share every property, then A = B.
PII (contrapositive form): If A is numerically different from B, there is at least one property doing the difference.

Again, notice that the properties involved in the formulation of the PII are just those composing the bundles, the relations of compresence not being invoked at any time for qualitative discernibility. There are distinct versions of the PII, depending on how one understands the properties allowed in the scope of the quantifier (see French & Bigaj 2024 for the classification we are using):

PII1 — If A and B share every property and every relation, then A = B.
PII2 — If A and B share every property, except for spatio-temporal properties, then A = B.
PII3 — If A and B share every non-relational property, then A = B.

PII3 and PII2 are violated in classical mechanics already, given that classical particles may, at least in principle, share all of their non-spatial properties. Given the traditional assumption of the impenetrability of classical particles, PII1 is saved. However, it is agreed by almost everyone that PII fails — in its three versions — in quantum mechanics (see French & Krause 2006, chap. 4; French & Bigaj 2024). Let us discuss this very quickly before dealing with the bare particulars.

3. Enter quantum mechanics

The root of the failure of different versions of PII in quantum mechanics comes from the fact that the latter obeys *Permutation Symmetry*: once a state for a collection of particles is available, the probability of a result for a measurement of an observable on that state is the same as the probability of a measurement of the same observable in the permuted state, where the latter is obtained by permuting particle labels in the state description. As a result, no physical observable can distinguish particles that were permuted; permutations are not observable, they engender no new physical state (we are switching from particles' descriptions to the particles themselves for the sake of intuitive clarity; nothing of importance hang on this, for our purposes).

Consider, as an example, the entangled state for the two electrons, labeled 1 and 2, which may be found either in Box A, or else in Box B. We may distribute one electron in each box (they cannot be both in the same box, given the Pauli Exclusion Principle); the description for such a situation, using Dirac notation (and omitting normalization factors, for simplicity) is:

$$|A\rangle_1|B\rangle_2 - |A\rangle_2|B\rangle_1.$$

After a permutation of labels (really, just swapping them), we have:

$$|A\rangle_2|B\rangle_1 - |A\rangle_1|B\rangle_2,$$

which is the same as

$$-(|A\rangle_1|B\rangle_2 - |A\rangle_2|B\rangle_1),$$

that is the original state with a reversed sign. Given that probabilities are calculated by the Born rule, the initial state and the permuted state originate the same probabilities. That makes any kind of discernibility by property attribution impossible (see French & Krause 2006, chap. 4 for further details).

More recently, inspired by the work of Saunders (2003) and later by developments of Muller & Saunders (2008), weaker forms of the PII have been defended in quantum mechanics (see Huggett & Norton 2014 for an account). Relations like '*x* has opposite spin in a given direction to *y*' are seen to obey permutation symmetry (if *x* has spin opposite to *y*, then *y* also has spin opposite to *x*), but to be irreflexive: as Saunders (2003, p. 294) already remarked, "[...] no particle can have opposite value of S [spin] to itself"; so, if *x* and *y* are in this relation, they must not be numerically the same. That is also said to grant a kind of discernibility by such symmetric and irreflexive relations to quantum particles (the so-called weak discernibility). However, although it is clear that such weaker versions of PII can be granted in quantum mechanics, it is not obvious that they contribute to saving a form of PII that is usable to ground a version of bundle individuality, given that such relations cannot account for what is expected when the problem of individuality is being discussed (see this line of criticism in Lowe 2015). However one takes that issue, the fact is that the actually relevant forms of the principle — relevant for the purposes of a theory of individuality as we have been considering it — fail in standard quantum mechanics.[3] More specifically, for the purpose of the present argument, PII-based versions of bundle theory fail in quantum mechanics due to the Permutation Symmetry principle.

Having rejected those forms of the bundle theory, one may now ask: What of bare particulars? Well, one cannot just direct the same kind of argument against bare particulars. They were designed to account for situations involving qualitative indiscernibility; as a result, they may survive the indiscernibility tests by quantum mechanics. However, they cannot be seen as resisting all of the oddities of quantum mechanics. Let us see why.

3.1. Quantum metaphysical indeterminacy

Let us assume, as we have been doing, that the reader is familiar with the Hilbert space formulation of quantum mechanics. In such a formulation, pure physical states are represented by unitary vectors and physical observables are represented by Hermitian operators. Each such operator gives rise to a set of eigenvectors associated with the operator, and these are connected with their respective eigenvalues, which are the possible results of a physical measurement. It is a simple fact of the formalism that for each Hermitian operator O in a Hilbert space, one may find another such operator Q such that O and Q do not share their set of eigenvectors. Well, the eigenvectors are the states in which a system is when they certainly have the value of the property represented by its eigenvalue. That means that no state can be an eigenstate of every observable.

This might be translated to the metaphysical vocabulary of 'determinates' and 'determinables', with the assumption of the eigenstate–eigenvalue link (EEL), which is a fundamental principle of standard quantum mechanics ever since the first formulations of the theory (for a survey on the history of the EEL, see Gilton, 2016). It consists of the following assumption:

[3] As an anonymous referee pointed out, this does not mean, however, that the bundle theory of individuality is entirely incompatible with quantum mechanics, as it can be maintained within, *e.g.*, modal interpretations. We leave, however, considerations of that case and its implications for the metaphysics of quantum mechanics for another occasion.

> [EEL:] A system has a determinate value for a given determinable property if and only if its state is an eigenstate of the operator corresponding to the property, and the determinate value is the eigenvalue for that eigenstate. (Lewis 2016, p. 76)

While metaphysicians of science have traditionally turned their attention to no-go theorems for quantum metaphysical indeterminacy (QMI), such as the Koshen–Specker theorem (see Darby 2010; Skow 2010; Arenhart & Felippe Jr. 2020; Arroyo 2022), it has been recently argued that the EEL is the most fundamental — and simpler — source of QMI (see, *e.g.*, Calosi & Wilson 2021 and Fletcher & Taylor 2021). We'll hold on to that idea in this subsection's case study.[4]

QMI is often taken as a failure of the omnimode/complete determination principle (for a historical consideration of this principle in the modern history of philosophy, see Mittelstaedt 1994, and Lombardi & Dieks 2016). This principle is very dear to our intuitions based on classical physics. It says that physical objects always have well-defined values (determinates) for all the properties (determinables) they bear. Every object must possess a definite/determinate value for properties/determinables such as mass, spatial location, size, and color. Under this view, for instance, it wouldn't even make sense to claim that coffee mugs and cats have color without a specific value for that property (say, orange). As far as we can tell, that's how we perceive the world, and no one would deny it in their most commonsensical days. Thus, intuition safely says that the same should hold for quantum objects. As a matter of fact, however, it doesn't.

Below we present a more general case, one in which EEL entails gappy QMI, that is, the kind of QMI in which no determinate value is instantiated for the determinable/property. In our example, we consider the position determinable/property. Take the simplest case in which a quantum object described by the state $|\psi\rangle$ is described as a superposition of the property of being located at region 1 and being located at region 2, such that $|\psi\rangle$ = a$|\psi_1\rangle$ + b$|\psi_2\rangle$, being 'a' and 'b' arbitrary numerical factors satisfying well-known restrictions. Think of, *e.g.*, orthogonal paths in a Mach–Zehnder interferometer or a double slit setup. It is well-known that, under such regular/textbook physical circumstances the system $|\psi\rangle$ displays the phenomenon of interference. And, as a consequence of interference, it is plainly false to ascribe to the object a determinate value for the position determinable, *i.e.*, it is false to state that: i) it has the property of being located at region 1; ii) it has the property of being located at region 2; iii) it has the property of being located at region 1 and the property of being located at region 2; iv) it has neither the property of being located at region 1 nor the property of being located at region 2. Under circumstances like this, it would be a category mistake to ask for the position determinable (Albert 1992) which doesn't have any determinate instantiation — hence, a gappy QMI.[5]

So, a system that is not in an eigenstate of the position observable, does not have a well-determined position, at least according to standard quantum mechanics. The previous observation, connected with the EEL, indicates that no state is the state of a system actually having a well-determined value for *every* possible property.

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[4] As an anonymous referee rightly pointed out, it is crucial to note that indeterminacy can arise even in interpretations of quantum mechanics that reject EEL (or at least *modify* the strict link with a vague or fuzzy link, see Lewis 2016). This is ensured by contextuality, which might provide a more direct grounding for QMI, as no-go theorems such as Kochen–Specker's primarily ensure that the indeterminacy in question is metaphysical rather than merely epistemic or semantic.

[5] Of course, this last claim is disputable, as there are authors who claim that the above example is a case of *glutty* QMI, *i.e.*, QMI in which *more than one* determinate is instantiated for the determinable. This is arguably a dissident voice. and this isn't the appropriate venus for the '*gappy* versus *glutty*' debate. For a defence of the glutty account against the gappy hegemony, see Calosi & Wilson (2021).

How does that impact the bare particular view? Well, bare particulars is a theory about *concrete particulars*. The fact is that a bare particular, being that which ties all of the properties of a particular, must always be located somewhere, and be in a specific position (even if unknown). The very idea of concrete objects is notoriously difficult to present in completely clear terms, and it comes with difficult questions concerning the nature of objects and the 'abstract/concrete' divide. All of those difficulties notwithstanding, it remains as a basic datum for metaphysicians that concrete particulars have space-time positions (see the general discussion in Retler & Bailey 2024).

Consider now our electron in a superposition of location between Boxes A and B, of the previous example. By the EEL, it is not in A, and also not in B. It is also not in both, and not in any other place. But then, the particle is nowhere to be found, and if that is also the case for the bare particular, what happens to the function that the bare particular should be doing, of instantiating the properties of the particle? Because, surely, the particle may not have a well-defined position, but it still has a well-defined mass, a well-defined electric charge, and so on, for all state independent properties, at least. How are those properties to be held together by the bare particular, if the bare particular is not in any place? A key question arises here: why is location in space considered a somehow privileged determinable for the bare particular's function? The answer is related to the theoretical role that the postulation of bare particulars is expected to play. While properties like mass and charge remain definite, the traditional expectation for a concrete bare particular is to anchor these properties in a specific spatial location, contributing to the unity of the object in a specific place and enabling it also to play the expected role in continuous identity through spacetime. The absence of a determinate position thus undermines a fundamental aspect of the bare particular's conceptual work, unlike the determinacy of other properties.

Also, a bare particular must persist through time, if it is to account for identity over time. But if the state of a system may evolve to a situation where the state describing it is a superposition of distinct locations, how can we account for the continuous trajectory of the particle? There is no way the bare particular can keep doing that theoretical work in quantum mechanics, provided we assume the EEL. As a result, there is a sense in which standard quantum mechanics with the EEL seems to be incompatible with the bare particulars.

But that is not the end of that metaphysical theory. As it is well known, not every formulation/interpretation of quantum mechanics accepts the EEL. While this doesn't exhausts all possibilities, let us briefly mention two cases in which QMI might be absent, the Bohmian mechanics and Everettian quantum mechanics (for a case in which versions of the GRW quantum mechanics being also free of QMI, see Calosi & Mariani 2021, pp. 6–7).

Bohmian mechanics introduces position as a hidden variable, so that a system is always in a given position, without it being the case that it has to be in a position eigenstate. In this sense, bare particulars could still be accepted if we change part of the formalism of quantum mechanics and reject the EEL. Bohmian mechanics is a typical go-to when eliminating QMI (see Skow 2010; Glick 2017; Chen 2022, but compare with Oldofredi 2024 for a dissident view), and it is fairly easy to see why such is a natural choice. After all, Bohmian mechanics modifies the EEL (see Lewis 2016, chap. 4) to make sure that particles *always* have definite positions. The question of whether we can find out about such determinates is a separate matter, as their initial conditions are fundamentally inaccessible to us (the so-called 'hidden variables'). So bare particulars *could*, at least in principle, survive as a lively metaphysical option for versions of Bohmian mechanics in which QMI doesn't hold (see also Pylkkänen, Hiley & Pättiniemi 2015).

The same holds for (some versions of) Everettian quantum mechanics. As Wilson (2020, p. 77) has it, here the strategy is to "replace indeterminacy with multiplicity". In Everettian quantum mechanics, everything that could happen indeed happens in some Everettian world somewhere in the Everettian multiverse, and each Everettian world is maximally complete in the omnimode determination sense. To achieve that, this strategy also modifies the EEL (see, again, Lewis 2016, chap. 4) to make sure that particles always have definite positions in each given Everettian world, hence there's arguably no QMI (but compare with Calosi & Wilson 2022, for an argument for the presence of QMI in Everettian quantum mechanics). That said, Everettian quantum mechanics might also be hospitable to (some form of) bare particulars (see also Conroy 2015).

3.2. Wavefunction realism

Wavefunction realism could potentially present a significant challenge to substratum theories. Wavefunctions are the kind of entity that is present in pretty much every 'realist' or 'ontic' version of quantum mechanics; for that reason, it has been claimed that one should naturally acquire ontological commitments towards it, as it would be an 'indispensable entity' for quantum mechanics (see Ney, 2012, 2021 for the full argument). Wavefunction realists are those who subscribe to that idea.

It has been argued by French (2013) that wavefunctions should not be understood within the ontological type of 'objects' (see Arroyo & Arenhart 2024, for the distinction between ontological types and ontological catalog). Wavefunction realism, instead, would be better suited for a metaphysical framework that aligns with an ontology of waves rather than discrete entities. If wavefunctions were to be considered objects, according to Brading & Skiles (2012) it would be mandatory to question what is their metaphysical profile of individuality. Yet, this issue seems absent from discussions of wavefunction realism. This might suggest that wavefunctions, whatever they might be, simply are not objects. Thus substratum theories, which rely on the ontological type of 'objects', may not be even applicable in this context.

However, Albert's (2013, 2023) version of wavefunction realism explicitly treats the wavefunction as a physical *object*, complicating the issue. Albert's approach involves distinguishing two kinds of spaces and relying on the fundamental–emergent duality to explain the nature of wavefunctions. Wavefunctions would inhabit a high-dimensional space (3N dimensions, where N is the number of particles), which is fundamental, and the three-dimensional space that we experience would be at best "emergent" (Albert 2023, p. 6) and at worst "flatly illusory" (Albert 1996, p. 277). This raises further concerns about how substratum theories could operate within such a high-dimensionality framework, especially given that the substratum, if applicable, would need to exist in a space distinct from ordinary three-dimensional physical reality. Wavefunctions, however, would exist in a high-dimensional space. The difficulty in conceptualizing the individuality of wavefunctions under this interpretation suggests that wavefunction realism, at least in Albert's formulation, may require a revision of traditional metaphysical assumptions regarding objects and their individuation.

These two issues point to a broader consideration: is an individuality profile a necessary condition for something to be classified as an object within a type-ontology framework? It doesn't seem so. If the wavefunction is an entity that does not require such a profile for its intelligibility, this challenges the assumption that individuation is always a prerequisite for objecthood. In fact, imposing such a criterion might obscure rather than clarify the nature of the wavefunction. Consequently, the presumption that a substratum is always available for any given physical object may be undermined by wavefunction realism.

4. Metaphysical equivalence

So, it seems that in standard quantum mechanics, where the eigenstate–eigenvalue link is adopted, bundle theories and bare particulars suffer the same fate. That does not apply to haecceities, and also not to non-individuals (although these views also are not devoid of their own difficulties; again, see Arenhart 2023). So, would both theories, bundles and bare particulars, have some kind of equivalence restored? Not so obviously. As we have seen, they fail in different aspects of the theory, and this somehow prevents their equivalence. But there is more. We have been treating the idea of metaphysical equivalence in very intuitive terms. Let us discuss that idea in more detail first.

Benovsky's claim of metaphysical equivalence between the theories of bundle and bare particulars relies heavily on his notion of *metaphysical equivalence*. In a nutshell, *equivalence* here is intended to mean that the theoretical posits of both theories do the same kind of explanatory work. Same kind of successful explanations, and also the same kind of explanatory difficulties. This is a kind of *functional approach* to the identity of the primitive posits in metaphysical theories.

> By its very nature, a primitive being primitive, it is non-analysible and we are not really given any information concerning its nature; we are told *what it does* rather than *what it is*. So, it is what it does that counts — after all, that's what any primitive is introduced for in a theory in the first place (otherwise there would be little justification for having it). Thus, primitives are individuated by what they do, what their functional role in a theory is, and, as a consequence, two primitives that do the same job just turn out to be equivalent for all theoretical purposes and metaphysically equivalent as well: they just are one and the same thing referred to in two different ways. (Benovsky 2016, p. 63, original emphasis)

The idea is that the distinct theories must perform identically when confronted with the same problems; otherwise, they do things differently and are no longer the same.[6] It is not enough that both bundle theories and bare particulars fail in quantum mechanics, they have to do so in the same kind of problem, to have the same kind of explanatory troubles. As we have seen, however, bundle theory fails to face the indistinguishability delivered by permutation symmetry, while bare particulars fail in front of the fact that position is not always well defined for quantum particles.

But that is actually not a problem for Benovsky's discussion. In fact, he acknowledges that the PII is not necessary for a bundle theory of individuality. Considering the case of Black's spheres, and the idea that they are qualitatively indiscernible, Benovsky comments that the PII is false:

> But this principle is false, for it is quite possible there to be two numerically distinct objects that have exactly the same properties (that are qualitative duplicates). The example of *two* spheres exactly alike in all of their properties is possible. (Benovsky 2016, p. 12, original emphasis)

Of course, as we mentioned, the bare particulars view survives the indiscernibility test; different substrata do the job of granting individuality to the spheres. However, as Benovsky notes, a similar strategy is open for the bundle theorist. Remember that distinct bundles are associated with distinct

[6] Here, we alternate between 'equivalent sets of primitives' and 'equivalent metaphysical theories' as synonymous expressions. This is allowed by Benovsky's characterization of the primitives in terms of their functional roles in the theories they are primitives of, and, of course, because, once one agrees that there is nothing more to the primitives except their behavior, then theories having equivalent sets of primitives are also equivalent (that is, they could not differ elsewhere).

compresence relations. Why don't we use such relations to account for what a bundle of properties is, and to account for the numerical difference of two such bundles? Concerning the strategy of appealing to the different bare particulars, he says that

> […] BTU [Bundle Theory with Universals] can use exactly the same strategy — remember that here we have different compresence relations, one per object, and so two objects, even qualitatively identical, will always be numerically distinct since the bundling relation that ties together their properties will be a different universal — exactly as in the case of STU [Substratum Theory with Universals] it will be a numerically different substratum. But then, as a *tu quoque,* one can ask: In virtue of what is a given compresence relation numerically distinct from another compresence relation? And there is no better answer to this question than to the same question about distinct substrata, the only option is primitive distinctness. (Benovsky 2016, p. 13, original emphasis)

So, the fact is that Benovsky has a distinct, more liberal, theory of bundles in mind than the ones we have been discussing here. While requiring a bundling relation is standard in bundle theories, what is quite unusual in Benovsky's approach is that he allows the bundling relations in each bundle to be a *different universal* playing a role in making what the constituted individual is. Instead of composing an individual only with its properties, *Benovsky defines the individual in terms of its properties and its compresence relation*. So, Socrates would be:

Socrates = [Compresence relation of Socrates, P_1, P_2, …, P_k]

Naturally, the compresence relation is not another ingredient among the properties of the individual, so it cannot be used to qualitatively distinguish individuals (and neither can it be used to save PII). However, it can be used to numerically distinguish distinct individuals. Individuals are distinct because they are tied differently, each one is tied by its respective compresence relation (for an earlier proposal along similar lines, see Rodriguez-Pereyra 2004; see Arenhart 2017 for a use of this theory in quantum mechanics).

In this new bundle theory, distinct bundles may instantiate the same qualities, because what makes them different is their compresence relation. As Benovsky remarked, this allows this bundle theory to face cases of qualitative indiscernibility just like the bare particulars. But is it now prey to the same kind of localization problem that affected the bare particular theory? Clearly not. A relation may be instantiated without having a specific location for it to be instantiated, as a kind of particular. It just happens to be there connecting the properties a particle happens to be instantiated in a particular moment, even if none of those properties is the particle's location. Consider the relation 'being a father of'. It is not in any specific place. The relata may change their positions, and still, the relation holds. Something similar holds for 'having spin opposite to' and other relations holding between quanta. So, a compresence relation may clearly be instantiated without it being in any particular position.

Now, that just grants that the new version of bundle theory is, at least *prima facie*, one available option for the individuality of quantum particles, while the substratum is available only for some interpretations breaking the EEL. That clearly distinguishes them, as theories of individuality. The fact is that, in Benovsky's terms, they do things in different ways. A bare particular must be located somewhere, while a compresence relation does not. That is also why one fails in standard quantum mechanics, and the other, at least at first sight, does not.

5. Conclusion

Let us pack things up. We have seen that according to the discussion connected to the problem of metaphysical underdetermination, the consensus is that bundle theory fails in quantum mechanics, while the substratum theory is wholly compatible with the theory. We have argued that both theories fail in standard quantum mechanics — it's important to emphasize that their failure is not absolute across all interpretations, but rather they cannot be implemented in every interpretational framework of quantum mechanics. A substratum, being a concrete particular, must always be occupying a particular position, something that orthodox quantum theory does not agree with. Certainly, one may achieve precisely this in different versions of quantum theory, like Bohmian or Everettian quantum mechanics. But that just indicates that the metaphysical view on individuality in this case is sensitive to the formulation of the theory. Substrata are not acceptable in quantum mechanics without further qualifications.

Furthermore, we have seen that a revised version of bundle theory is compatible with indiscernible entities. This makes room for a version of bundle theory independent of the PII and immune to its failure in quantum mechanics, but not for any other version of the substratum theory. That makes the bundle theory and the substratum theory inequivalent in quantum mechanics, in a quite strong sense: one of the theories fails (substratum), while the other is compatible with quantum mechanics (bundle). So, the equivalence of the theories breaks when certain versions of quantum mechanics are allowed. The theories are not on par.

What can we learn, as a moral, from all of this? One of the important things is that science, as already emphasized in the literature, may be used as a test field for metaphysical doctrines (Arenhart 2012, Morganti & Tahko 2017, Arenhart & Arroyo 2021a,b,c). Even if those scientific theories used to test metaphysics are not the final ones, and are not actually true, they are clearly delimiting the space of possibilities for the application of metaphysical doctrines, restricting such space for some metaphysical theories. Now, Benovsky dismisses conflicts between science and metaphysics as not so important for the epistemology of metaphysics:

> […] although it seems a reasonable and highly desirable thing to avoid contradictions with physics in order to gain support from it and to include metaphysics in a wider network of scientific research, the criterion seems to be a non-obligatory one, and one where we must proceed with care. (Benovsky 2016, p. 82)

He claims that precisely because we still do not have the final physics, because it seems that physics and metaphysics may be dealing with different kinds of objects, and because he does not see so far any conflict between the theories of metaphysics and the theories of physics. However, as we have discussed, a theory of individuality must account for the individuality of quantum entities too if they are thought of as individual objects, and some of these theories do indeed fail in some cases, conflicting with physics. So compatibility with physics may not bring any metaphysical theory to an open victory, given underdetermination and the provisional character of physics, but it may bring a metaphysical theory to being completely ruled out, due to incompatibility (see also McKenzie 2020, and Arroyo & Arenhart 2022 for more trouble for metaphysics in this connection).

Obviously, this kind of claim can only be achieved once the details of the connection between science and metaphysics are appropriately spelled out. It is easy to be led to believe that any physical object in quantum mechanics will just behave as any other object in our surroundings. But that is clearly not so. Not every metaphysical generalization will survive close scrutiny when taken to the quantum level. This connects with current complaints about the frailty of metaphysical theories detached from science, which forget to take into account the wide variance of the behavior of reality in different scales, including the quantum scale (see Humphreys 2013, sect. 5). Intuitive theories, those based on the behavior of normal-sized bodies, do not generalize obviously to the new scales with widely diverging behavior that are being discovered after the scientific revolution. We may not know how to extract metaphysical theories from our best current physics, so we may be far from knowing which metaphysical theory is true, if any. By connecting such theories with science, however, we may get to know which ones get *rejected* by our best current theories. As we have been suggesting elsewhere by means of a 'meta-Popperian' methodology in metametaphysics (Arenhart, 2012; Arenhart & Arroyo, 2021a,b,c, Arroyo & Arenhart, 2022b), this might be as far as we can go.

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