Is the geometric trinity of gravity a problematic case of underdetermination?

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Abstract

It is a striking fact that the theories of the so-called geometric trinity of gravity can model the same gravitational effects with such diverse geometric tools such as curvature, torsion, and non-metricity. Building on (Wolf et al. 2024), this contribution offers a clarification and expansion of responses to this underdetermination emerging in recent years, namely discriminatory approaches such as Occamism and spacetime functionalism, or reinterpretational approaches such as the common core and overarching solutions. Despite appearances, the future of the metric-affine structure that accommodates the geometric trinity seems to lie not in these empirically equivalent formulations but in heuristic for theory construction.

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1. The geometric trinity of gravity

At first glance, the existence of three empirically equivalent theories collectively known as the "geometric trinity of gravity" – General Relativity (GR), the Teleparallel Equivalent of General Relativity (TEGR), and the Symmetric Teleparallel Equivalent of General Relativity (STEGR) – poses an obstacle to a literal realist view of curvature as an intrinsic property of spacetime, thereby challenging one of twentieth-century physics' most profound discoveries.[1] While GR describes gravitational interactions via non-vanishing Riemann curvature $R^{\rho}{}_{\sigma\mu\nu} \neq 0$ defined by the torsion-free, metric-compatible Levi-Civita connection $\mathring{\nabla}$, TEGR and STEGR do not use curvature. TEGR employs a flat, metric-compatible "Weitzenböck" connection $\widetilde{\nabla}$ with torsion $T^{\rho}{}_{\mu\nu} := \Gamma^{\rho}{}_{[\mu\nu]} \neq 0$; STEGR uses a flat, torsion-free, non-metric-compatible "purely inertial" connection $\overline{\nabla}$ with non-metricity $Q_{\rho\mu\nu} := \overline{\nabla}_{\rho} g_{\mu\nu} \neq 0$.[2] Though these theories thus use different affine and metric properties, they turn out to be dynamically equivalent. The teleparallel field equations of TEGR, resulting from varying the action functional $S_{\text{TEGR}} = \frac{1}{2}\int d^4x\sqrt{e}T$ (for e the determinant of the tetrad field), as well as the symmetric teleparallel field equations, from $S_{\text{STEGR}} = \frac{1}{2}\int d^4x\sqrt{g}Q$, produce the same equations of motion, up to boundary terms, as those that result from varying GR's Einstein-Hilbert action: $S_{\text{EH}} = -S_{\text{TEGR}} + \text{b.t.} = -S_{\text{STEGR}} + \text{b.t.}$, for the torsion scalar $T := \frac{1}{4}T_{\mu\nu}{}^{\rho}T^{\mu\nu}{}_{\rho} + \frac{1}{2}T_{\rho\nu}{}^{\mu}T_{\mu\nu}{}^{\rho} - T^{\rho}{}_{\mu\rho}T_{\nu}{}^{\nu\mu}$ and non-metricity scalar $Q := \frac{1}{4}Q_{\mu\nu\rho}Q^{\mu\nu\rho} - \frac{1}{4}Q_{\mu\nu\rho}Q^{\nu\mu\rho} - \frac{1}{2}Q_{\mu\alpha}{}^{\alpha}Q^{\mu}{}_{\beta}{}^{\beta} - \frac{1}{2}Q_{\mu\alpha}{}^{\alpha}Q^{\gamma}{}_{\gamma}{}^{\mu}$.[3]

Read literally,[4] cashing out explanations in terms of these spacetime properties leads to rather different pictures of gravitational effects. Curvature is the property of spacetime that when we parallel transport a vector (or tensors generally) around a closed loop the direction of that vector is not preserved; torsion is the antisymmetric part of a connection that quantifies the failure of (infinitesimal) parallelograms to close when vectors are parallel transported along each other's directions; non-metricity quantifies the failure of the connection to preserve the metric under parallel transport, meaning that lengths and angles of vectors depend on the path connecting them. See Figure 1. In all three theories of the geometric trinity, test

[1] See Hehl et al. 1995a; Jiménez, Heisenberg, and Koivisto 2019; Bahamonde et al. 2023; Heisenberg 2024 for in-depth explorations of the geometric trinity and the technical details below. In general the discussion avoids the mention of tetrads, for brevity, since both metric formulations and tetrad formulations of three theories exist (cf. Capozziello, Falco, and Ferrara 2022).

[2] A handy iconic mnemonic is to always write a symmetric connection with a symmetric symbol such as '∘' or '−' and an anti-symmetric connection with an anti-symmetric symbol such as '∼'.

[3] This dynamical equivalence is thus a local matter. One may inquire whether the empirical equivalence of the trinity can be challenged in ways that go beyond the equations of motion, but no worked-out cases are forthcoming, besides suggestions that one may expect empirical divergence globally, such as the absence of black hole solutions (cf. Hayashi and Shirafuji 1979), or seeing different boundary effects (Wolf and Read 2023) or using quantum probes to exploit teleparallel analogues of the Aharonov-Bohm effect (Mulder and Read 2024). Additionally, TEGR requires an additional topological property, namely that the manifold is parallelizable, which means it must admit a global tetrad field.

[4] It would be too far afield to philosophically substantiate the idea of (and disagreements about) "literal interpretation" here. The underlying idea is (roughly) that the only tools for interpretation lie in the internal semantic architecture of the theory itself, rather than being formulated in an externally imposed theoretical superstructure (De Haro and Butterfield 2021; Dewar 2023). In the current context, the examples are taken to show sufficiently clearly what a literal interpretation amounts to: if a model employs curvature, torsion, or non-metricity to account for gravitational phenomena in an empirically adequate manner, then one maps these properties of the formalism to spacetime properties of the physical world.

particles follow the exact same trajectories, but the explanation why those trajectories are followed are – even though all three theories explain gravity in geometric terms rather than in terms of forces – radically different. Consider the famous apple that falls from a tree. Within GR, one solves the Einstein equations with Earth as an energy-momentum source and the apple freely falling along an affine-geodesic of $\mathring{\nabla}$, which coincides with the metric-geodesic (a path of extremal proper time). Within TEGR, the apple falls because Earth's mass this time sources torsion, and this torsion causes the apple to accelerate downward, deviating from the affine-geodesics of $\widetilde{\nabla}$. Within STEGR, because the metric is not preserved under parallel transport, the length of the apple's velocity vector $||u^\mu||(x^\rho) = g_{\mu\nu}(x^\rho)u^\mu u^\nu$ increases, so that while the apple is moving along an affine-geodesic of $\overline{\nabla}$, the motion is accelerated, which will be picked up by clocks and rulers tied to the metric. These explanations are so different from the ontological commitment to curvature – internalized during late night hours perusing MTW's (1973) visualizations of curved spacetimes – that one begins to doubt that commitment. Is spacetime truly curved?

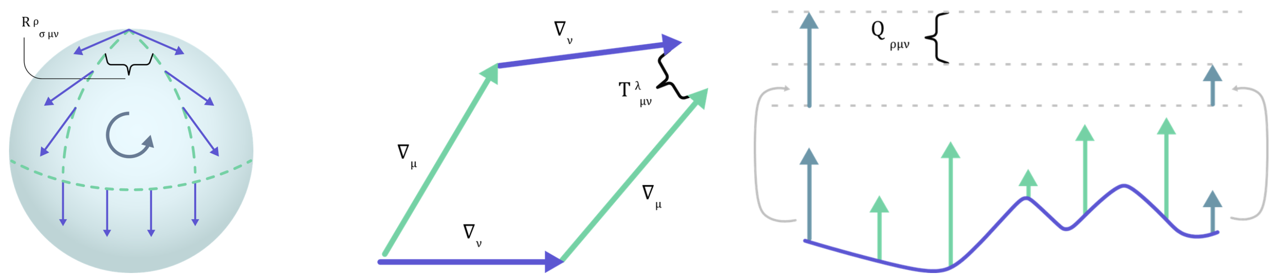

Figure 1: From left to right: (a) **Curvature**. The non-coincidence of vectors transported around a closed loop is measured by the Riemann tensor. (b) **Torsion**. The amount of non-closure of parallelograms formed by two vectors transported along each other is measured by the torsion tensor. (c) **Non-metricity**. The change of the length and angles of vectors due to non-preservation of the metric under parallel transport is measured by the non-metricity tensor. Figure reproduced on the basis of (Jiménez, Heisenberg, and Koivisto 2019, Fig 1).

The purpose of this article is to clarify and compare several recently emerging solutions to this apparent underdetermination in a briefly summarized but exact form. Relying extensively on Le Bihan and Read 2018; Wolf, Sanchioni, and Read 2024, it surveys both discriminatory responses, such as Ockhamism and functionalism, and reinterpretational strategies, including common core and overarching solutions.

2. Formal and problematic underdetermination and how to respond to it

Underdetermination is a word connoted in the mind of the philosopher with various arguments in debates about scientific realism, and strong opinions on it abide.[5] We speak of "transient underdetermination" or "weak underdetermination") when the data domain is restricted to the current domain that can technologically be probed. If the underdetermined theories make indistinguishable predictions about all data that can possibly be gathered, we speak

[5]The relevant literature is too vast to review, but here the following work is consulted: (Sklar 1975; van Fraassen 1980; Psillos 1999; Ladyman 2001; Chakravartty 2007; Stanford 2010; Azhar and Butterfield 2017; Le Bihan and Read 2018; De Haro and Butterfield 2021; Mulder 2024).

of "permanent underdetermination" (or "strong underdetermination").[6] If we grant that a theory has an observation base – i.e., the observable predictions can be isolated – then it is an objective fact whether two or more theories are indeed *formally* underdetermined by the data. That is,

> **Formal underdetermination of theories by data**. Multiple theories explain the same data by providing indistinguishable observable predictions, either transiently or permanently. The theories can therefore not be discerned on empirical grounds.

Formal underdetermination is ubiquitous in physics and usually does not justify something to worry about. More worrying is when formally underdetermined theories are suspected to be *ontologically divergent*, given plausible interpretations of the individual theories:

> **Problematic underdetermination of theories by data**. Multiple theories explain the same data by providing indistinguishable observable predictions *and* provide ontologically-divergent explanations, either transiently or permanently. The theories can therefore not be discerned on empirical grounds, which results in empirical support for multiple potentially inconsistent ontologies.

Thus, formal underdetermination of theory by data, sometimes but not always, brings the underdetermination of ontology in its wake.

Cases of problematic underdetermination require a philosophical response. One may of course assume a distinction between the observable and unobservable and argue *à la* constructive empiricism that we have an in-principle lack of empirical access to the latter, resulting in a (transitory or permanent) agnosticism. There are various ways to formulate more realist responses, here simplified into three main groups, cf. Fig. 2. First, one should not go to jail for resisting the monist intuition that only one theory can be correct, for a *pluralist* holds that perhaps we should not say there is a unique way the world is like, so that there are multiple overlapping ontologies, indexically organized as 'ontology$_1$' and 'ontology$_2$'. In fleshing out such a pluralism, one is not relieved from the responsibility to say whether the underdetermination is problematic or merely formal, since one needs to say *how many* ontologies there are.[7]

Second, one may appeal to supra-empirical criteria to choose one of the theories over the others on independent grounds, what is also called "discrimination" in (Le Bihan and Read 2018), such as choosing the simpler theory over the other, or the theory one deems more intelligible or more aesthetically pleasing. Or, one may draw on Kuhnian values such as consistency and fruitfulness (1973) or on technical criteria such as preferring a local or deterministic explanation. Such supra-empirical criteria for theory-choice are notoriously disputed because they need to be independently justified or otherwise argued for.

[6]These are not entirely synonymous. Strong underdetermination implies permanent underdetermination, but not *vice versa*, since there are cases of underdetermination that are permanent but apply to empirically *in*equivalent models that will always saturate the in-principle observable parameter space (Pitts 2010; cf. Wolf and Read 2025 for permanent underdetermination of empirically inequivalent dark energy models).

[7]Sophisticated pluralists recognize this responsibility of reading off the ontology from the theory (Feyerabend 1963; Chang 2012, Ch. 5.1).

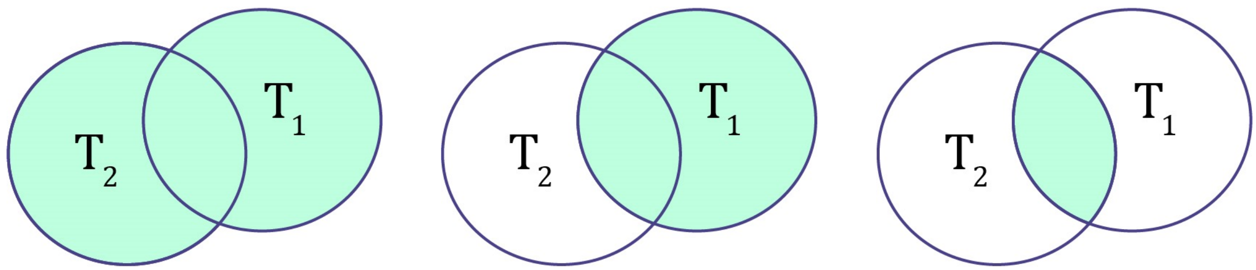

Figure 2: Theory-choice in a situation of problematic underdetermination. From left to right: (a) **Pluralism**. Commit to the ontology of all theories; (b) **Prefer on independent grounds**. Choose one of the theories on supra-empirical grounds (e.g. Ockhamism); (c) **Reinterpretation**. Ontologically commit to what the theories agree on in light of (c1) the **common core** as an individual viable theory, or (c2) an **overarching structure** that embeds the formalisms of all theories. Options (c1) and (c2) are presented in (Le Bihan and Read 2018)

Third, one may attempt to reinterpret the theories such that it turns from a case of problematic underdetermination into a case of (mere) formal underdetermination. Le Bihan and Read (2018) provide useful language for two options to achieve this. One of these is a *common core solution*, which aims to extract the shared structure from the theories to form a minimal theory that (i) preserves all empirical content of the original theories, (ii) avoids extra ontological commitments that differ across the full theories, and (iii) is ontologically viable in its own right, as a fully fledged physical theory in its own right. The other is an *overarching solution*, which is an ontological deflation across the underdetermined formalisms such that it treats the theories, so far distinctly interpreted, as merely variant representations of a common invariant ontology. That is, it retains all the formalisms but unifies them by embedding them in a more encompassing mathematical framework. Again, also such reinterpretative responses to problematic underdetermination ought to be independently argued for.[8]

Since the dynamical equivalence of the theories of the geometric trinity of gravity holds for all predictions of the theories, it forms a case of permanent underdetermination (see also Wolf, Sanchioni, and Read 2024). Additionally, the apparently physically divergent explanations of gravitational effects (cf. 1) seem to be a sufficient reason to regard this as a case of *problematic* underdetermination. In the remainder of this paper, the above possible responses to this permanent underdetermination are summarized and discussed.[9] Such overviews are foreshadowed in (Lyre and Eynck 2003; Mulder 2024; Mulder and Read 2024; Zhou 2025), and will most heavily rely on (Wolf, Sanchioni, and Read 2024).

[8]As another solution one may consider the judgment, relative to a convincing equivalence criterion, that the underdetermined theories are *theoretically equivalent* because they posit the same mathematical structure. This can be seen as a special of the common core solution, where theories together comprise the common core: they are all in fact the same theory and say the same things about the world. No wonder then that they make the same predictions. Examples are structural, functional, categorical, Morita, and definitional equivalence (cf. Glymour 1970; Barrett and Halvorson 2016; Weatherall 2019a,b; Dewar 2022, 2023; Knox and Wallace 2024).

[9]No pluralist accounts appear to be forthcoming in the literature of the geometric trinity.

3. Discrimination by implicit definability: surplus and superfluous structure

The three nodes of the geometric trinity of gravity do not appear to be mere reformulations of each other in the sense that they posit the same mathematical structure. One (preliminary) argument against theoretical equivalence of TEGR with GR and STEGR is that, because the parallelizability condition confines the solution space of TEGR to that of a subset of the possible solutions of GR, the cardinalities of the theories do not coincide. Despite the (local) empirical equivalence, TEGR allows for strictly fewer global solutions than GR. However, this argument is not convincing, for (i) all the candidate physical spacetimes, such as Minkowski, de Sitter, FLRW and black hole spacetimes, this requirement holds, and (ii) *locally* any differentiable manifold can be parallelized.

The more convincing argument is to show that the GR posits strictly less structure than the other two theories. This has been shown to be the case by (Weatherall and Meskhidze 2024, see also Knox 2011) in the case of GR and TEGR, and by (Golovnev 2024; Weatherall 2025) for the entire geometric trinity. The crux is that the Levi-Civita connection $\mathring{\nabla}$ is implicitly definable in each theory of the geometric trinity, because it is uniquely determined by the metric, whereas the teleparallel connections $\widetilde{\nabla}$ and $\overline{\nabla}$ are not.[10] Breaking the torsionfree condition allows many torsionful connections to be compatible with the metric and, similarly, breaking the compatibility condition introduces many flat torsionfree connections which are not distinguished by the metric. Another way to see this is that for the manifold M and the metric tensor g, the tuples $T_{\text{TEGR}} = (M, g, \widetilde{\nabla})$ and $T_{\text{STEGR}} = (M, g, \overline{\nabla})$ require explicit mention of the connection to specify all of the affine structure, while writing $T_{GR} = (M, g)$ suffices (and $T_{GR} = (M, g, \mathring{\nabla})$ would be redundant). This many-to-one mapping from TEGR/STEGR to GR, with the same empirical substructure, even up to isomorphism, means that the former have surplus structure.[11]

Weatherall and Meskhidze, as well as Golovnev, explicitly opt here for an Ockhamist discriminatory solution in favor of GR. This Ockhamist argument is straightforward: since GR and its teleparallel counterparts are empirically equivalent but GR requires only a metric tensor, it is the more parsimonious theory, which, by Ockham's razor, favoring theories that do not posit unnecessary structure, GR is preferable.

The argument that GR should be preferred for positing less structure has been foreshadowed in the case of TEGR in (Knox 2011), by arguing that conserved quantities are calculated by using the Levi-Civita connection "in disguise", mimicked by the Weitzenböck connection and the contorsion tensor. Although this mimicking by itself is symmetric between the theories, as pointed out in Mulder and Read 2024, in combination with the surplus structure argument this can be read as making the point that TEGR has "unnecessary" affine structure. Knox, however, uses this implicit observation to argue that TEGR is a *reformulation* of GR on a "relatively liberal attitude" towards ontology (Knox 2011, p. 274), i.e., her inertial frame spacetime functionalism (*avant la lettre*, given Knox 2013). Knox' po-

[10]A similar point is also made in (March, Wolf, and Read 2024; Wolf, Sanchioni, and Read 2024) in the context of the common core solution, see §4.

[11]For tetrad formulations of TEGR, in which one may fix the tetrad frame and define the Weitzenböck connection in terms of it, the additional "gauge degrees of freedom" (in the sense of surplus structure, cf. Weatherall 2015) reside in the tetrads $4 \times 4 = 16$ independent components, compared to the (symmetric) metric tensor's $\frac{4(4+1)}{2} = 10$.

sition appears to combine elements form the prefer-on-independent-grounds approach and the reinterpretative common core approach.

4. The common core of the geometric trinity is GR

On the premise that there exists an interesting common core capable of functioning as a distinct individual theory, one may search for what the three connections $\mathring{\nabla}$, $\widetilde{\nabla}$, and $\overline{\overline{\nabla}}$ have in common. Abstracting away from all the structure that is not in common between these, however, leaves one with an impoverished structure, not capable of functioning as an individual theory because it has not natural straight lines—what is called the "problem of missing inertial structure" in (Dürr and Read 2024).

Indeed, as argued in (Wolf, Sanchioni, and Read 2024, p. 28, who thank Adam Caulton and Oliver Pooley for discussion on this) and (March, Wolf, and Read 2024), no distinct dynamical common core exists for the relativistic trinity. In fact, the common core "just is" GR. The argument for this conclusion is essentially the implicit definability argument of the previous section (3): the minimal overlap between GR, TEGR and STEGR is the empirical substructure (M, g) itself, from which the inertial structure is implicitly uniquely defined by the metric structure: the Levi-Civita connection coefficient are $2\mathring{\Gamma}^{\rho}{}_{\mu\nu} = g^{\rho\tau}(\partial_{\mu}g_{\tau\nu} + \partial_{\nu}g_{\mu\tau} - \partial_{\tau}g_{\mu\nu})$.

Although the authors do not explicitly formulate the full solution, it is obvious that this works: since GR is our most successful theory of gravity, it is certainly a viable theory of its own, and a reinterpretation of GR, since it is the common core, is not required (or it is trivial). No reinterpretations of TEGR and STEGR are required, but one should be "purging what is not shared between" (Wolf, Sanchioni, and Read 2024, p. 27) the three theories and thus they are no longer in sight. Thus this does not just amount to the *same conclusion* as the Ockhamist argument, but functions as the *same argument*. Curiously, when the common core approach, which requires reinterpretations, identifies as the common core one of the original theories in full, this approach collapses into the Ockhamist approach, even though the latter does not require any reinterpretation.

In this light, it is not too clear whether the authors endorse this as a solution to the underdetermination of the geometric trinity or not. Indeed, (Wolf, Sanchioni, and Read 2024, pp. 26-28) actively resist it by arguing that such a solution would require arguments that TEGR and STEGR are somehow pathological—after all, a common core in general just adds to the number of theories or keeps the number equal, rather than subtract. Additionally, they resist Ockhamism by denying that the extra structure of (S)TEGR is totally superfluous because it may enhance our modeling capacities, e.g. by unifying gravity with particle physics.[12] However, it is not clear how this enhanced modeling capacity can help solve the *underdetermination* problem. Rather, it serves as a heuristic toward new theories, which would be more of an argument in favor of the overarching approach in which a larger mathematical embedding of the theories can do more than just their union.

[12] Weatherall (2025) argues that (S)TEGR is not like a Yang-Mills gauge theory because the Cartan connection is not a principal connection, and induces additional structures such as the tetrad, i.e., internal and external indices are mixed.

5. Overarching solutions: one MAG connection to rule them all?

An overarching solution to underdetermination among GR, TEGR, and STEGR would show that these are not genuinely distinct theories but rather different representations of a single gravitational theory formulated in varying terms of the affine-connection. Rather than applying Ockham's razor, this approach embraces surplus structure. Rather than purging of the excess structure of the common core account is a form of reductive interpretation (taking equivalence classes of symmetry-related models), the opposite "sophistication" move is made, in the sense of (Dewar 2019). Here the extra modeling capacities of the surplus structure is used to create different mathematical representations of the same phenomenon. More precisely: rather than reformulating the theory to include only quantities invariant under a symmetry, sophistication preserves TEGR and STEGR but alters their semantics by treating symmetry-related models as equivalent representations rather than distinct possibilities. Thus, sophistication changes interpretation rather than structure, making it suitable for a strategy to achieve the overarching solution.

One may naively consider (that is, it was a long-held belief of the author) that the knowledge we obtain about the world lies at the intersection of the concepts of curvature, torsion, and non-metricity. A natural candidate here is the concept of non-trivial path-dependence. Looking at just GR and TEGR – which is natural, for unlike curvature or torsion, non-metricity is not an *intrinsic* property of a connection, but rather a *relational* property between connection and metric – one generalizes to their common structure of a Riemann-Cartan manifold and assumes a general Cartan connection that is both curved *and* torsionful. Such a spacetime has local Lorentz symmetry *and* local translation symmetry (i.e., Cartan curvature has values in the Lie algebra of the Poincaré group $ISO(1,3)$), and theories constructed on this background are often called "Poincaré gauge theories" (PGT) (Baez and Wise 2015; Weatherall 2025). Then, a plausible candidate for what represents their commonality may be the Lie bracket of covariant derivatives [13]:

$$[\nabla_\mu, \nabla_\nu]\, V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\sigma{}_{\mu\nu} \nabla_\sigma V^\rho, \tag{5.2}$$

generating the torsion and Riemann tensors. This may be interpreted as that what torsion and curvature agree on, namely as a measure of the non-triviality of affine structure, representing how translations and rotations *jointly* fail to commute. Unfortunately, although this works to illustrate the conceptual interrelations, the Lie bracket is far from a mathematical invariant of the dynamical equations (nor a dynamical common core, for that matter), and so cannot serve as a central part of an overarching solution.

[13] That is,

$$\begin{aligned}
[\nabla_\mu, \nabla_\nu]\, V^\rho &= \partial_\mu \left(\nabla_\nu V^\rho\right) + \Gamma_{\mu\tau}{}^{\rho} \nabla_\nu V^\tau - \Gamma_{\mu\nu}{}^{\tau} \nabla_\tau V^\rho - \{\mu \leftrightarrow \nu\} \\
&= \partial_\mu \partial_\nu V^\rho + \partial_\mu \Gamma^\rho{}_{\nu\sigma} V^\sigma + \Gamma^\rho{}_{\nu\sigma} \partial_\mu V^\sigma + \Gamma^\rho{}_{\mu\sigma} \partial_\nu V^\sigma + \Gamma^\rho{}_{\mu\tau} \Gamma^\tau{}_{\nu\sigma} V^\sigma - \Gamma^\sigma{}_{\mu\nu} \partial_\sigma V^\rho - \Gamma^\tau{}_{\mu\nu} \Gamma^\rho{}_{\tau\sigma} V^\sigma - \{\mu \leftrightarrow \nu\} \\
&= \cancel{\partial_\mu \partial_\nu V^\rho} + \partial_\mu \Gamma^\rho{}_{\nu\sigma} V^\sigma + \cancel{\Gamma^\rho{}_{\nu\sigma} \partial_\mu V^\sigma} + \cancel{\Gamma^\rho{}_{\mu\sigma} \partial_\nu V^\sigma} + \Gamma^\rho{}_{\mu\tau} \Gamma^\tau{}_{\nu\sigma} V^\sigma - \Gamma^\sigma{}_{\mu\nu} \partial_\sigma V^\rho - \Gamma^\tau{}_{\mu\nu} \Gamma^\rho{}_{\tau\sigma} V^\sigma \\
&\quad - \cancel{\partial_\nu \partial_\mu V^\rho} - \partial_\nu \Gamma^\rho{}_{\mu\sigma} V^\sigma - \cancel{\Gamma^\rho{}_{\mu\sigma} \partial_\nu V^\sigma} - \cancel{\Gamma^\rho{}_{\nu\sigma} \partial_\mu V^\sigma} - \Gamma^\rho{}_{\nu\tau} \Gamma^\tau{}_{\mu\sigma} V^\sigma + \Gamma^\sigma{}_{\nu\mu} \partial_\sigma V^\rho + \Gamma^\tau{}_{\nu\mu} \Gamma^\rho{}_{\tau\sigma} V^\sigma \\
&= \left(\partial_\mu \Gamma^\rho{}_{\nu\sigma} - \partial_\nu \Gamma^\rho{}_{\mu\sigma} + \Gamma^\rho{}_{\mu\tau} \Gamma^\tau{}_{\nu\sigma} - \Gamma^\rho{}_{\nu\tau} \Gamma^\tau{}_{\mu\sigma}\right) V^\sigma - \left[\Gamma^\tau{}_{\mu\nu} - \Gamma^\tau{}_{\nu\mu}\right] \left(\partial_\tau V^\rho + \Gamma^\rho{}_{\tau\sigma} V^\sigma\right),
\end{aligned} \tag{5.1}$$

with in the last line the first four terms symmetric and the last two anti-symmetric under $\{\mu \leftrightarrow \nu\}$.

Recently, a sophisticated version of GR and TEGR was fully developed by (Chen, March, and Read 2025), along with various other formulations of teleparallel gravity. Following (Martens and Read 2020), they distinguish between *external* sophistication, i.e., the inserting more gauge transformations without changing the theory's mathematical structure, and *internal* sophistication, i.e., reformulating the theory so that symmetries become isomorphisms.[14] Then, they show that teleparallel gravity as a Cartan or higher gauge theory can be understood as internally sophisticated versions of more conventional formulations such as TEGR. (in this case by moving towards Cartan or higher gauge formulations of teleparallel gravity). Although not stated explicitly, the paper implements the spirit of the overarching solution to underdetermination, because it treats distinct teleparallel formulations as different representations of the same physical content.

In the standard formulation, models of teleparallel gravity are given either in terms of a triple $\langle M, g_{\mu\nu}, \nabla^{(W)}\rangle$ or a pair $\langle e^a{}_\mu, \omega^{ab}{}_\mu\rangle$. A more sophisticated interpretation recognizes that many such models are related by local Lorentz transformations of the tetrad, of the form $e^a{}_\mu \to \Lambda^a{}_b(x)\, e^b{}_\mu$, for $\Lambda^a{}_b(x)$ a local Lorentz transformation. In a naïve formulation, each such transformation yields a distinct model. However, if one expands the category of models to include local Lorentz gauge transformations as morphisms, these models become isomorphic, as Chen, March, and Read show. Therefore, the teleparallel equivalent of general relativity (TEGR) can be given a more sophisticated interpretation by enlarging the class of morphisms -— such as in a Cartan-geometric or higher gauge-theoretic framework – so that local frame rotations are treated as gauge redundancies rather than physically distinct configurations.

Moving on to incorporate STEGR, the mathematical structure should be further enriched and the common concept further weakened. In this case, one moves to a general metric-affine gravity (MAG), which allows for a general connection that can be curved, torsioned *and* non-metric. Its general affine connection is often written as[15]:

$$\Gamma^{\lambda}_{\mu\nu} = \mathring{\Gamma}^{\rho}{}_{\mu\nu} + K^{\rho}_{\mu\nu} + L^{\rho}_{\mu\nu}, \tag{5.3}$$

where $K^{\lambda}_{\mu\nu}$ is the contortion tensor (built from torsion) and $L^{\lambda}_{\mu\nu}$ the disformation tensor (built from non-metricity). The full symmetry group of MAG is the affine group: $\mathrm{Aff}(4,\mathbb{R}) = \mathrm{GL}(4,\mathbb{R}) \ltimes \mathbb{R}^4$, the semi-direct product of the general linear group and the group of spacetime translations, combining Lorentz transformations, scaling, and volume-preserving deformations with spacetime translations (Hehl et al. 1995b). One then has three independent fields, namely the tetrad, the connection and the metric.

An overarching solution in terms of MAG can then easily be formulated. One tolerates the surplus structure and recovers the theories of the geometric trinity by taking the appropriate limits, e.g. recovering GR by setting $K^{\lambda}_{\mu\nu} = 0$ and $L^{\lambda}_{\mu\nu} = 0$. Closest to formulating such a solution is (Zhou 2025).

[14] In category language, a sophisticated theory is one where more morphisms are added so that symmetries become isomorphisms, allowing you to identify equivalent models internally (see fn. 4).

[15] Although canonical texts like (Bahamonde et al. 2023; Heisenberg 2024) write it so, this may be too rough: the contorsion tensor cannot be defined as usual without the assumption of metric-compatibility. Nevertheless, since we only look at the three nodes of the geometric trinity, this is harmless in the current context.

Can we embed a sophisticated version of GR, TEGR and STEGR withing MAG? Each theory should then be interpretable in an internally sophisticated way by expanding the category of models to include gauge transformations such that the three formulations become equivalent as categories: their differences in torsion, curvature, or non-metricity would then reflect alternative presentations of the same underlying physical content. Such a treatment requires more space and time than currently available, but appears to be in sight, if modeled on (Chen, March, and Read 2025).

	Curvature	**Torsion**	**Non-Metricity**
General Relativity (GR)	yes	no	no
Teleparallel Gravity (TEGR)	no	yes	no
Symmetric Teleparallel Gravity (STEGR)	no	no	yes
Poincaré Gauge Theory (PGT)	yes	yes	no
Metric-Affine Gravity (MAG)	yes	yes	yes

Table 1: Presence of geometric structures in different (gauge) theories of gravity.

However, these sophisticated solutions achieve little more than a mathematical embedding. It is not clear whether the overarching theory offers a distinct ontological interpretation (cf. Le Bihan and Read 2018, p. 9). The justification of an overarching solution may very well lie in extensions of any one of the three nodes of the geometric trinity into empirically inequivalent domains, not in a solution to underdetermination.

Another overarching approach is the conventionalist solution in (Dürr and Read 2024), which resolves problematic underdetermination of the geometric trinity by treating the incompatible geometric structures of GR, TEGR, and STEGR not as competing claims about reality, but as formal choices that lack truth-values, i.e. conventions. Conventionalism denies there is a fact of the matter about which spacetime property, such as curvature, torsion, or non-metricity, is "true". Rather, each formalism can be chosen at convenience. Although this position is explicitly framed in (Wolf, Sanchioni, and Read 2024) as a "pluralist" option, it seems more natural to regard the stripping of truth-values of affine properties as a reinterpretation. Perhaps this is a semantic issue, and these properties can be regarded as having never had a truth-value. However, in that case, there was never a case of problematic underdetermination to begin with, and the formal underdetermination of the geometric trinity would be akin to any mathematical reformulation in physics. In addition, (Dürr and Read 2024) identifies their position explicitly as a *selective* realism, indicating that the three theories are not pluralistically accepted alongside each other.

6. Is there a future for the geometric trinity of gravity?

There is no question that, seen purely at the level of empirical equivalence, the theories of the geometric trinity are indeed formally underdetermined and, moreover, theoretically *in*equivalent. That gravitational effects can be modeled in such different ways is a highly non-trivial formal fact by itself. Literally read, this also amounts to a case of *problematic* underdetermination, if anything. But should we thereby be concerned about our ontological commitment to spacetime curvature, as represented by the Riemann tensor in terms of the Levi-Civita connection?

There are good *prima facie* reasons to resist such concerns. To date, the Ockhamist argument in (Weatherall and Meskhidze 2024; Weatherall 2025), and foreshadowed in (Knox 2011), appears to be the most worked out argument that achieves the goal of resolving problematic underdetermination, by discarding TEGR and STEGR on the basis of positing superfluous structure. Since parsimony arguments need to be justified by themselves, there may yet be more convincing alternative solutions. The common core solution formulated in (March, Wolf, and Read 2024; Wolf, Sanchioni, and Read 2024), is successful and favors GR itself as a common core, but this solution happens to coincide with the Ockhamist position (to the extent that it was offered as an alternative). The overarching solution of a metric affine $GL(4, \mathbb{R})$ structure, although *prima facie* promising, offers little more than an artificial gluing together trinity, as a trivial embedding without explanatory benefit. The sophisticated interpretation of GR and TEGR by (Chen, March, and Read 2025) can hopefully be extended meaningfully in this direction.

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Bibliography

Azhar, Feraz and Butterfield, Jeremy (2017). "Scientific Realism and Primordial Cosmology". *The Routledge Handbook of Scientific Realism*. Edited by Juha Saatsi. First edition. Routledge, pp. 304–320 (cit. on p. 3).

Baez, J. C. and Wise, D. K. (2015). "Teleparallel gravity as a higher gauge theory". *Communications in Mathematical Physics* 333.1, pp. 153–186 (cit. on p. 8).

Bahamonde, Sebastian et al. (2023). "Teleparallel gravity: from theory to cosmology". *Reports on Progress in Physics* 86.2, p. 026901 (cit. on pp. 2, 9).

Barrett, Thomas William and Halvorson, Hans (2016). "Glymour and Quine on Theoretical Equivalence". *Journal of Philosophical Logic* 45.5, pp. 467–483 (cit. on p. 5).

Capozziello, Salvatore, Falco, Vittorio De, and Ferrara, Carmen (2022). "Comparing Equivalent Gravities: common features and differences". *European Physical Journal* C 82.865 (cit. on p. 2).

Chakravartty, Anjan (2007). *A Metaphysics for Scientific Realism: Knowing the Unobservable*. Cambridge: Cambridge University Press (cit. on p. 3).

Chang, Hasok (2012). "Chapter 5: Pluralism in Science: A Call to Action". *Is Water H2O? Evidence, Realism and Pluralism*. Boston Studies in the Philosophy and History of Science, pp. 253–301 (cit. on p. 4).

Chen, Lu, March, Eleanor, and Read, James (2025). "Equivalence, reduction, and sophistication in teleparallel gravity". *Euro Jnl Phil Sci* 15.32 (cit. on pp. 9–11).

De Haro, Sebas and Butterfield, Jeremy (2021). "On symmetry and duality". *Synthese* 198, pp. 2973–3013 (cit. on pp. 2, 3).

Dewar, Neil (2019). "Sophistication About Symmetries". *British Journal for the Philosophy of Science* 70.2, pp. 485–521 (cit. on p. 8).

— (2022). *Structure and Equivalence*. Cambridge University Press (cit. on p. 5).

— (2023). "Interpretation and Equivalence; or, Equivalence and Interpretation". *Synthese* 201.4, pp. 1–24 (cit. on pp. 2, 5).

Dürr, Patrick M. and Read, James (2024). "An invitation to conventionalism: a philosophy for modern (space-)times". *Synthese* (cit. on pp. 7, 10).

Feyerabend, Paul (1963). "How to Be a Good Empiricist: A Plea for Tolerance in Matters Epistemological". *Philosophy of Science: The Delaware Seminar*. Vol. 2. Wiley Interscience, pp. 3–39 (cit. on p. 4).

Glymour, Clark (1970). "Theoretical Realism and Theoretical Equivalence". *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* 1970, pp. 275–288 (cit. on p. 5).

Golovnev, Alexey (2024). "Is there any Trinity of Gravity, to start with?" *Preprint* (cit. on p. 6).

Hayashi, Kenji and Shirafuji, Takeshi (1979). "New general relativity". *Physical Review D* 19 (12), pp. 3524–3553 (cit. on p. 2).

Hehl, Friedrich W. et al. (1995a). "Metric-affine gauge theory of gravity: field equations, Noether identities, world spinors, and breaking of dilation invariance". *Physics Reports* 258.1, pp. 1–171 (cit. on p. 2).

Hehl, Friedrich W. et al. (1995b). "Metric-affine gauge theory of gravity: field equations, Noether identities, world spinors, and breaking of dilation invariance". *Physics Reports* 258, pp. 1–171 (cit. on p. 9).

Heisenberg, Lavinia (2024). "Review on $f(Q)$ gravity". *Physics Reports* 1066, pp. 1–78 (cit. on pp. 2, 9).

Jiménez, Jose Beltrán, Heisenberg, Lavinia, and Koivisto, Tomi S. (2019). "The Geometrical Trinity of Gravity". *Universe* 5.7, p. 173. eprint: 1903.06830 (cit. on pp. 2, 3).

Knox, Eleanor (2011). "Newton-Cartan Theory and Teleparallel Gravity: The Force of a Formulation". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 42.4, pp. 264–275 (cit. on pp. 6, 11).

— (2013). "Effective Spacetime Geometry". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 44.3, pp. 346–356 (cit. on p. 6).

Knox, Eleanor and Wallace, David Mark (2024). "Functionalism Fit for Physics". Unpublished (cit. on p. 5).

Kuhn, Thomas (1973). "Objectivity, Value Judgment, and Theory Choice". *The Essential Tension (1977)*. Lecure delivered at Furman University, 30 November 1973. University of Chicago Press, p. 320 (cit. on p. 4).

Ladyman, James (2001). *Understanding Philosophy of Science*. New York: Routledge (cit. on p. 3).

Le Bihan, Baptiste and Read, James (2018). "Duality and Ontology". *Philosophy Compass* 13.12, e12555 (cit. on pp. 3–5, 10).

Lyre, Holger and Eynck, Tim Oliver (2003). "Curve It, Gauge It, or Leave It? Practical Underdetermination in Gravitational Theories". *Journal for General Philosophy of Science* 34.2, pp. 277–303 (cit. on p. 5).

March, Eleanor, Wolf, William J., and Read, James (2024). "On the Geometric Trinity of Gravity, Non-Relativistic Limits, and Maxwell Gravitation". *Philosophy of physics* 2.1, 15pp. (Cit. on pp. 6, 7, 11).

Martens, Niels C.M. and Read, James (2020). "Sophistry About Symmetries?" *Synthese* 199.1-2, pp. 315–344 (cit. on p. 9).

Misner, Charles W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation*. San Francisco: W. H. Freeman (cit. on p. 3).

Mulder, Ruward (2024). *Real commitment: a coarse-grained approach to reality via alternative theories in physics*. Supervized by Neil Dewar, Jeremy Butterfield, and Hasok Chang. Ph.D. Dissertation, Cambridge University (cit. on pp. 3, 5).

Mulder, Ruward and Read, James (2024). "Is spacetime curved? Assessing the underdetermination of general relativity and teleparallel gravity". *Synthese* 204.126 (cit. on pp. 2, 5, 6).

Pitts, J. Brian (2010). "Permanent Underdetermination From Approximate Empirical Equivalence in Field Theory: Massless and Massive Scalar Gravity, Neutrino, Electromagnetic, Yang?Mills and Gravitational Theories". *British Journal for the Philosophy of Science* 62.2, pp. 259–299 (cit. on p. 4).

Psillos, Stathis (1999). "Undeterdetermination Undermined". *Scientific Realism: How Science Tracks Truth*. New York: Routledge (cit. on p. 3).

Sklar, Lawrence (1975). "Methodological Conservatism". *Philosophical Review* 84.3, pp. 374–400 (cit. on p. 3).

Stanford, P. Kyle (2010). *Exceeding Our Grasp: Science, History, and the Problem of Unconceived Alternatives*. Oxford University Press USA (cit. on p. 3).

van Fraassen, Bas (1980). *The Scientific Image*. New York: Oxford University Press (cit. on p. 3).

Weatherall, James Owen (2015). "Understanding Gauge". *Philosophy of Science* 83.5, pp. 1039–1049 (cit. on p. 6).

— (2019a). "Part 1: Theoretical Equivalence in Physics". *Philosophy Compass* 14.5, e12592 (cit. on p. 5).

— (2019b). "Part 2: Theoretical Equivalence in Physics". *Philosophy Compass* 14.5, e12591 (cit. on p. 5).

— (2025). "On (Some) Gauge Theories of Gravity". *This volume* (cit. on pp. 6–8, 11).

Weatherall, James Owen and Meskhidze, Helen (2024). "Are General Relativity and Teleparallel Gravity Theoretically Equivalent?" *Unpublished* (cit. on pp. 6, 11).

Wolf, William J. and Read, James (2023). "Respecting Boundaries: Theoretical Equivalence and Structure Beyond Dynamics". *European Journal for Philosophy of Science* 13.4, pp. 1–28 (cit. on p. 2).

— (2025). "Navigating permanent underdetermination in dark energy and inflationary cosmology". *Unpublished* (cit. on p. 4).

Wolf, William J., Sanchioni, Marco, and Read, James (2024). "Underdetermination in Classic and Modern Tests of General Relativity". *Philosophy of the Natural Sciences* 14.57 (cit. on pp. 3, 5–7, 10, 11).

Zhou, Yitong (2025). "Are Geometrical Trinity of Gravity Underdetermined?" unpublished (cit. on pp. 5, 9).