Quantum ontology: a modal bundle-theorist relational proposal

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Abstract

Quantum mechanics poses several challenges in ontological elucidation. Contextuality threatens determinism and favors realism about *possibilia*. Indistinguishability challenges traditional identity criteria associated with individual objects. Entanglement favors holistic and relational approaches. These issues, in close connection with different interpretations of quantum mechanics, have given rise to various proposals for the ontology of quantum mechanics. There is a proposal that is realistic about *possibilia*, where quantum systems are seen as bundles of possible intrinsic properties. This proposal is developed in close connection with modal interpretations and addresses quantum contextuality straightforwardly. There are also proposals based on relations, associated with the relational quantum mechanics interpretation. In this paper, features of these proposals are combined to obtain a modal bundle-theorist relational proposal. Its aim is to consistently and straightforwardly address both contextuality and the holistic and relational aspects of quantum mechanics arising from quantum entanglement. The proposal, if some additional principles are assumed, may turn out to be both an instance of moderate structuralism and priority monism.

Keywords

Moderate Structural Realism; Ontology of properties; Priority Monism; Quantum holism; Quantum entanglement.

Introduction

On the one hand, the ontology of bundles of possible intrinsic properties associated with modal interpretations of quantum mechanics (da Costa, Lombardi and Lastiri, 2013) primarily addresses the quantum issues of contextuality and indistinguishability. It does not directly account the features of quantum mechanics (QM from now on) that seem to indicate that the properties of quantum systems are inherent relations in a sense that may lead to the endorsement of one form or another of quantum holism (Teller 1986, Howard 1989, Healey 1991). On the other hand, there are proposals for QM ontology associated with the relational quantum mechanics interpretation (RQM from now on, see Rovelli 1996), in which it is admitted that (at least some) quantum properties are inherent relations. One such proposal, due to Oldofredi (2021), employs a bundle theory to account for quantum systems, which is an instance of moderate structural realism. The other proposal, due to Candiotto (2017), neglects that quantum systems can be understand as objects in any sense (not even bundles) and defends a form of radical structural realism.

In this paper we intend to reformulate the ontology of bundles of possible properties, associated with modal interpretations, in relational terms. The proposal aims to combine the relational and modal character of quantum properties, in a manner that endorses a strong form of holism that supports priority monism. In this way, we could have an ontology compatible with modal interpretations that directly addresses the issue of quantum contextuality (as the already available ontology of bundles of possible intrinsic properties) but, at the same time, can also directly account for relational and holistic features.

To achieve this goal, we argue, following the lead of Esfeld (2004), that it is possible to state the equivalence between intrinsic properties of a composite system with inherent relational properties of the subsystems. Thus, while the composite system (which could reasonably represent the universe if certain assumptions are made) corresponds to a bundle of intrinsic properties, the subsystems (which could represent not only microscale physical systems but also ordinary objects) correspond to bundles of inherent relations. These relations would suffice to entail priority monism, as Schaffer (2010b) convincingly argues (as long as we accept the same mereological principles assumed by this author, such as the unrestricted composition principle). Since in this proposal there are not relations ‘all the way down’ but are aggregated into bundles, the present proposal would be compatible with moderate structural realism (in the spirit of Oldofredi's proposal).

In the first section we briefly introduce modal interpretations and the ontology of bundles of possible intrinsic properties associated with them. In the second section, we present RQM and the Oldofredi's and Candiotto's proposals. In the third section we describe the relational and holistic features that QM has due to quantum entanglement. In the fourth section we find the core proposal of this ontology of bundles of possible relations for QM. In the fifth and sixth sections it is defended that the present proposal is an instance of priority monism and of moderate structural realism, respectively. Sections 7-9 are corollaries meant to delve into some subtleties.

1. Modal interpretations and the ontology of bundles of possible intrinsic properties

Let us first briefly mention the main features of modal interpretations. They distinguish between a dynamical-state and a value-state. The dynamical-state determines which are the possible properties of a quantum system and assigns a probability to each of them. The value-state determines which are the actual properties of a system, i. e. which are the definite values that a certain set of commuting observables acquires. The eigenstate-eigenvalue link (EEL) is (at least) partially rejected in the sense that it is admissible for a system to have actual properties that do not correspond to observable eigenstates. Each particular modal interpretation is equipped with a specific property ascription or actualization rule that fixes which is the value-state at a given moment. Their aim is to reduce measurement interactions to ordinary interactions. To achieve that, quantum systems are supposed to have definite values in every circumstance, with independence of measurement scenarios. Projection postulate is discarded. As a result, quantum systems always evolve unitarily. As examples of modal interpretations we have the Biorthogonal-Decomposition Modal Interpretation (BDMI) (Kochen 1985, Dieks 1988, 1989), the Spectral-Decomposition Modal Interpretation (SDMI) (Vermaas and Dieks, 1995), the Perspectival Modal Interpretation (PMI) (Bene and Dieks, 2002) and the Modal-Hamiltonian Interpretation (MHI) (Lombardi and Castagnino, 2008). For a summary see the entry 'Modal Interpretations' in SEP (Lombardi and Dieks, 2021).

In correspondence with this family of interpretations, there is a proposal for an ontology of bundles of possible intrinsic properties (OPP for now on) due to da Costa, Lombardi and Lastiri (2013). The main postulates of this ontology are:

(1) The physical observables, mathematically represented by self-adjoint operators in the Hilbert space, are in the ontological domain *instances of universal type-properties*. For example, operator O corresponds with the universal type-property $[O]$ with instances $[O^i]$.

(2) The physical values, mathematically represented by the eigenvalues of the self-adjoint operators, are in the ontological domain *possible case-properties*. For example, to certain instance of the type-property $[O^i]$ corresponds the case-properties $[o^i_j]$ if it is the case that $O^i \left|o^i_j\right\rangle = o^i_j \left|o^i_j\right\rangle$.

(3) The physical systems, mathematically represented by an algebra of self-adjoint operators in the Hilbert space, are in the ontological domain *bundles of instances of type-properties with their corresponding possible case-properties*. For example, to system S^i corresponds the bundle of instances of type-properties $h^i = \{[A^i],[B^i],[C^i],...\}$.

In brief notation, we can associate system S^i to bundle $S^i(\mathcal{O}^i)$. Of course, it is possible to enrich the notation to include the corresponding case-properties, as Calosi (2022) does, for whom system S^i is the bundle constituted by the collection $< C = \{O^i \subset \mathcal{O}^i\}, \{o^i_j\} >$. The OPP is primarily designed to account for quantum contextuality. Due to this feature, quantum systems fail to satisfy the so-called ‘requisite determination’ (which holds in the classical domain). This metaphysical principle states that if an object x has a certain instance of a type-property O^i, then one of its possible case-properties o_j^i is actualized. As formally stated by Calosi (2022)

$$O^i(x) \rightarrow \exists o_j^i \left(o_j^i (x) \right) \tag{1}$$

So it is admissible (even necessary) that quantum systems have instances of type-properties that are not determined in a certain case-property. This physical fact is reinforced by Kochen and Specker theorem (1967) which entails that quantum modality does not have an epistemic but a metaphysical nature. The OPP addresses this QM feature by allowing quantum properties to have a modal character. As established in the above postulates, quantum systems are bundles of *possible* properties. Which of those possible properties are assumed to be actualized depends on the particular property ascription or actualization rule that is set by each particular modal interpretation. In this fashion, we have a clean and straightforward account of quantum contextuality and the failure of the ‘requisite determination’, at a price of endowing modality with metaphysical significance.

What about quantum indistinguishability? Despite the fact that OPP is an application of the bundle theory to quantum systems, the authors (in agreement with the so-called ‘received view’ on the quantum issue of indistinguishability) reject that quantum bundles are individuals (see da Costa and Lombardi, 2014). For them, in case of identical quantum bundles, the principle of identity of indiscernibles (PII) does not apply, contrary to what is expected for classical bundles. PII formally states

$$\forall x \forall y \forall P \left(Px \leftrightarrow Py \right) \rightarrow \left(x = y \right) \tag{2}$$

Let us delve a little into this point. There are three versions of PII, differing in the set of properties allowed. Ordered in increasing logical strength, they are (French and Krause 2006)

PII(1) If two objects have all their monadic and relational properties in common, they are identical.

PII(2) If two objects have all their monadic and relational properties in common, except the spatial and temporal ones, they are identical.

PII(3) If two objects have all their monadic properties in common, they are identical.

PII(1) is necessary but trivial. PII(2) and PII(3), logically stronger, are exposed to counterexamples (famously Black's spheres). The OPP focuses on PII(3), since in this approach the relations that quantum bundles may have reduce to their intrinsic properties (that is why it is an ontology of possible *intrinsic* properties). It is a fact, according to quantum statistics, that quantum systems of the same kind are indiscernible, i. e. they differ *solo numero*. If PII(3) holds, then quantum systems could not have such a numerical difference. Is it a matter that PII(3) fails in the quantum domain? According to the OPP authors, that would be the case if quantum systems were bundles of actual properties (as classical bundles are supposed to be). But since quantum systems are, according to this ontology, bundles of *possible* properties, PII(3) simply does not apply. Fortunately, the modal character of quantum properties seems to save the honor of PII. In OPP, modality is the touchstone that helps to solve the puzzle of indistinguishability. We return to this topic in Section 9.

Like every version of the bundle theory, OPP must face the question of what is the relation between properties that constitute proper bundles. *Compresence* is the equivalence relation (which must be assumed to be primitive) that constitute proper bundles if properties are tropes. Similarly, *coinstantiation* is the equivalence relation (which must be assumed to be primitive) that constitute proper bundles if properties are instances of universals. Since the latter is the case with OPP, it seems that OPP is committed to such a primitive metaphysical relation. Nevertheless, the rejection of PII entails that OPP is radical about the identity conditions of quantum bundles. As stressed above, they are not individuals. Even more, they do not have identity conditions at all. In a recent proposal due to one of OPP authors (Lombardi 2023) it is argued that in OPP quantum bundles are not even objects. In this scenario, we have that every aggregate of quantum properties could count as a proper quantum bundle. As a result, no bundling primitive relation is required. Of course, aggregates must be entailed by a partition of a total system that is one of its legitimate tensor product structures (otherwise talking about *systems* would be meaningless). This feature of OPP can make it appear parsimonious with respect to other quantum ontologies.

2. RQM and relational ontologies

Rovelli's (1996) relational quantum mechanics (RQM) is probably one of the most popular interpretations of QM today. It has inspired at least two proposals about QM ontology. Candiotto (2017) sketches a proposal in which quantum ontology is an instance of ontic structural realism (OSR from now on; for reference see Ladyman 2007, 2020). Oldofredi (2021) articulates a proposal in which quantum ontology is an instance of moderate structural realism (MSR from now on; for reference see Esfeld 2004, Esfeld and Lam 2010).

Some salient features of RQM are mentioned in this paragraph. RQM solves the measurement problem by reducing measurement interactions to ordinary interactions. According to this interpretation, every interaction entails the occurrence of an event. This means that each interaction entails the acquisition of definite values by a certain set of observables of an object-system. The distinctive feature of RQM (at least in its original formulation) is that the object-system acquires definite values by means of a projection of its state that is only relative to a witness-system. It should be emphasized that RQM neglects an ontological status to the quantum state; it is just a device for computing quantum probabilities. Another important feature of RQM that notably distinguishes it from modal and other relational interpretations is that quantum systems do not evolve in a purely unitary fashion, since the occurrence of events depends on (albeit relative) state projections. Furthermore, according to RQM there is no such thing as a 'quantum state' of the universe, since quantum states and quantum properties are always relative to a witness-system.

Usually, the RQM ontology is assumed to be one in which relational events are the basic elements (see Laudisa and Rovelli, 2021). In this vein, Candiotto (2017) stresses that the salient feature of RQM ontology is that properties of quantum systems are inherently relational. The point at issue is that Candiotto defends the ontological priority of relations. It is precisely in this sense that she considers the RQM ontology as an instance of OSR. In fact, this approach is realist only with respect to relations and the structures that they conform. When discussing the RQM ontology, she is extremely critical of ontologies in which objects have ontological priority over relations (e. g. ontologies in which properties inherit in a substratum or are intrinsic or in which fundamental relations are merely external). She avoids any realist commitment with respect to quantum objects and prefers to consider quantum systems as simple processes, that are supposed to take the place of the *relata* of such relations. However, according to her, the related processes are themselves relations, in a manner that all that there are in the world are 'interrelated relations', thus subscribing to the OSR slogan 'relations all the way down'.

Oldofredi (2021) seems to take a moderate stance regarding the RQM ontology. According to him, the RQM ontology is one of bundles of properties that include, but not exclusively, inherent relations. There are also certain state-independent properties (such as mass, charge, total spin) that ought to be considered intrinsic. Oldofredi employs mereological bundle theory (MBT) to articulate his RQM ontology. According to the author, quantum systems 'should be defined as mereological bundles of observer-dependent properties varying in virtue of interactions'. MBT (a proposal due to Paul 2017) takes *mereological composition* as the bundling relation that yields proper bundles. That is, the properties are related to the object they constitute as the parts are with respect to the whole. Mereological bundles are individuated by spatio-temporal relations that hold among them. Of course, this poses an issue when we come to quantum indistinguishability. Oldofredi has to assume 'ungrounded difference' as an additional metaphysical principle to allow numerical difference among indistinguishable bundles. Furthermore, in order to accommodate mereological bundles to quantum contextuality, Oldofredi has to accept that at a given time not every property is metaphysically determined.

There is a substantive difference between these two RQM ontology proposals. Candiotto is not realist with respect to objects, Oldofredi is so. According to the former, there are no objects, only relations. According to the latter, objects and relations are in ontological parity. Oldofredi considers that his RQM ontology is compatible with MSR, which is the moderate form of structural realism put forward by Esfeld. According to Esfeld (2004), OSR rejects both (1) things must have intrinsic properties over and above the relations in which they stand and (2) relations require relata, that is, things which stand in the relations. Esfeld draws our attention to the fact that, contrary to what OSR advocates hold, it is not necessary to reject (2) if (1) is rejected. MSR arises from the rejection of (1) but the acceptance of (2). Certainly, if the properties inhered in a substratum or if the objects were bundles of intrinsic properties, they would be constituted or even identified independently of their relations and would have ontological priority over them. However, it is possible to take objects as bundles constituted by inherent relations. If they were so, we would obtain relata that are constituted by the relations in which they stand, so they could not have priority over relations. Relations and objects would remain in ontological parity. It must be mentioned that MSR, in contrast with OSR, allow intrinsic properties corresponding to non-contextual, state-independent observables (such as mass, charge, etc.), as long as they do not underlie the relations nor make quantum systems individuals.

Two critical assessments of Oldofredi's proposal. First, how come mereological bundles are in ontological parity with properties and relations if properties and relations are the parts of the bundles? As to the extent that MBT does not explicitly endorse holism, the parts should be assumed to be prior to the whole. Consequently, it is not completely clear how an MBT-based proposal could be an instance of MSR. Second, MBT seems to lose its appeal when we move into the quantum domain. In fact, the rationale behind the use of bundles theories is to dispense with metaphysical assumptions that stand over and above properties themselves. But, as long as mereological bundles cannot be individuated in the quantum domain in virtue of spatio-temporal relations as in the classical domain, Oldofredi has to adopt an additional metaphysical principle, i. e. ungrounded difference, to account for the numerical difference between identical quantum bundles. With respect to Candiotto's proposal, it is possible to raise the same objection that is commonly raised against any form of radical OSR: relations are unintelligible without relata. It seems that Candiotto radically rejects the ontological fundamentality of objects because she has in mind a notion of object that necessarily involves a substratum or intrinsic properties. That is, for her and presumably for OSR advocates, the alternative to giving ontological priority to relations over objects is only to give priority to objects over relations.

3. Holistic features based on quantum entanglement

In the last decades, a group of authors has worked on the topic of the holistic features of quantum mechanics based on entangled states. Consider the simplest example. Let the half-integer spin particles S^1 and S^2, subsystems of S^U. Possible states of S^1 are $\left|\uparrow_1\right\rangle$ (spin up) or $\left|\downarrow_1\right\rangle$ (spin down), which are eigenstates of observable S_{1z} (spin in *z* direction) with eigenvalues (1) and (-1) respectively. Similarly, possible states of S^2 are $\left|\uparrow_2\right\rangle$ or $\left|\downarrow_2\right\rangle$, eigenstates of

S_{2z} with eigenvalues (1) and (-1). Assume that the state of the composite system S^U is the *singlet*

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle \otimes |\downarrow_2\rangle - |\downarrow_1\rangle \otimes |\uparrow_2\rangle\right) \tag{3}$$

The state $|\Psi\rangle$ is non-separable since it cannot be written as a tensor product of eigenstates of subsystems S^1 and S^2. Furthermore, it is a linear combination (with coefficients $1/\sqrt{2}$) of the product states $|\uparrow_1\rangle \otimes |\downarrow_2\rangle$ and $|\downarrow_1\rangle \otimes |\uparrow_2\rangle$. These product states are degenerate eigenstates of observable S_{Uz} with eigenvalue (0). Any linear combination of degenerate eigenstates is also an eigenstate of the same observable with the same eigenvalue. As a result, the state $|\Psi\rangle$ is also an eigenstate of S_{Uz} with eigenvalue (0). From the singlet $|\Psi\rangle$ the following correlations between S_{1z} and S_{2z} arise

$$\begin{aligned} S_{1z} : (1) &\leftrightarrow S_{2z} : (-1) \\ S_{1z} : (-1) &\leftrightarrow S_{2z} : (1) \end{aligned} \tag{4}$$

Of course, in any of these two mentioned cases the observable S_{Uz} of S^U has value (0)

$$\begin{aligned} S_{Uz} &= \left[(1) + (-1)\right]^2 = 0 \\ S_{Uz} &= \left[(-1) + (1)\right]^2 = 0 \end{aligned} \tag{5}$$

The state $|\Psi\rangle$ is a non-separable or entangled state with respect to the tensor product structure (TPS from now on) that partitions system S^U into subsystems S^1 and S^2 (entanglement is TPS-relative, see Earman 2015). This is the example considered in real or ideal experiments of the EPR-Bohm type.

It should be emphasized that the correlations due to quantum entanglement cannot be lost if the dynamics is purely unitary. This fact has led many authors to speculate about the possibility of an entangled quantum state of the universe, thus endorsing a form of holism. This possibility is not available for QM interpretations that include some sort of projection postulate. In fact, the projection of an entangled state into one in which each of the subsystems are in a particular eigenstate leads necessary to a product state. In particular, RQM does not allow the existence of massively entangled states, since according to its postulates, every ordinary interaction leads to a separable state of the composite of object- and witness-system. Of course, this possibility is indeed at our disposal if a modal interpretation of QM is embraced.

According to the advocates of quantum holism, the subsystems of the universe do not have properties by themselves but in conjunction with the rest of the subsystems of the universe. In the singlet state example above, subsystem S^1 does not have by itself the property $|\uparrow_1\rangle\langle\uparrow_1|$ nor $|\downarrow_1\rangle\langle\downarrow_1|$ (as long as EEL is assumed). The same happens *mutatis mutandis* with S^2. Nevertheless, it is indeed possible to assign to system S^U (assuming EEL) the determinate property $|\Psi\rangle\langle\Psi|$, which is precisely the property that S^U has when the correlations between S^1 and S^2 specified in the right-numbered expression (4) exist. In fact, $|\Psi\rangle\langle\Psi|$ is a determinate property of S^U because the state of S^U is precisely $|\Psi\rangle$ and it is an eigenstate of the global observable S_{Uz}. Otherwise, property $|\Psi\rangle\langle\Psi|$ could have been assigned only a certain probability. In modal parlance, property $|\Psi\rangle\langle\Psi|$ of system S^U is actual under the condition that the state of S^U is $|\Psi\rangle$, otherwise it would have been just a possible property. What has been exemplified here with S_{Uz} is also valid for other global observables such as total spin $(S_{1x}+S_{2x})^2+(S_{1y}+S_{2y})^2+(S_{1z}+S_{2z})^2$, total momentum (P_1+P_2) or relative distance (Q_1-Q_2). In fact, these two last global observables are the ones used by Einstein, Podolsky and Rosen in their original paper (1935).

Let us move on to a more precise characterization of quantum holism (QH for now on). We mention here three variants (following the summary of Esfeld 2001). For Teller (1986) QH is relational in the sense that the existence of inherent relations between subsystems of the universe entails holism. Inherent relations are defined as strongly non-supervenient relations. The author makes use of a notion previously introduced by Cleland (1984). She defines a weakly non-supervenient relation as the one that is not determined by the intrinsic properties of its relata, but it still requires that some intrinsic properties inherit in the relata. A relation is strongly non-supervenient if is not determined by the intrinsic properties of its relata and it is not required that some intrinsic properties inherit in the relata (for formal definitions see French and Krause 2006, Section 4.4). According to Teller, the relations that hold between entangled systems are a case of strongly non-supervenient relations. Continuing with the previous example, the author reduces the property $|\Psi\rangle\langle\Psi|$ of S^U to a strongly non-supervenient relation $R(S^1,S^2)$ between S^1 and S^2 that could not supervene on intrinsic properties $|\uparrow_1\rangle\langle\uparrow_1|$ or $|\downarrow_1\rangle\langle\downarrow_1|$ of either S^1 nor on $|\uparrow_2\rangle\langle\uparrow_2|$ or $|\downarrow_2\rangle\langle\downarrow_2|$ of S^2. Healey (1991) characterizes QH in terms of properties of the total system that do not supervene on the properties of its subsystems. In this QH variant, the supervenience failure occurs directly between the property $|\Psi\rangle\langle\Psi|$ of S^U and the properties belonging to S^1 and S^2. That is, the property $|\Psi\rangle\langle\Psi|$ cannot be decomposed into properties of the subsystems. Lastly, Howard (1989) characterizes QH not in terms of supervenience but in terms of separability failure. The separability principle established by this author is that (1) spatially separated systems possess their own, distinct physical state each and that (2) the joint state of two or more separated systems is wholly determined by their separated states. In our example, it is clear that if the state of S^U is the singlet $|\Psi\rangle$, then (1) it is not possible to assign to S^1 and S^2 vector states but only reduced states that are obtained by taking partial traces and (2) the state $|\Psi\rangle$ of S^U cannot be obtained as a product of the reduced states of S^1 and S^2. So, in case of entanglement, it is clear that Howard's separability principle fails on both (1) and (2). If separability fails, two or more entangled systems constitute a single system, which is equivalent to endorse QH.

Esfeld (2004) combines the proposals of Teller and Healey. According to him, QH entails that 'the non-supervenient relations of entanglement among the parts of a quantum whole amount to the whole having intrinsic properties that do not supervene on intrinsic properties of the parts' and that 'there are properties of the whole which indicate the manner in which the parts are related with each other with respect to some of the properties that make something a quantum system' (p. 611). As a result, we have that the intrinsic property $|\Psi\rangle\langle\Psi|$ of S^U amounts to the relation $R(S^1, S^2)$ between S^1 and S^2. Based on entanglement relations, Esfeld (2004) and Esfeld and Lam (2010) articulate a scientific ontology framed in MSR. More recently, Esfeld (2017) put forwards a minimalist ontology that, though still a form of MSR, may be far from QH. More about the latter proposal in Section 6.

We conclude this section by noting that the non-supervenient nature of entanglement relations is a crucial metaphysical aspect, enabling the formulation of an ontology for quantum mechanics that can be considered an instance of ontic structural realism (OSR) or moderate structural realism (MSR). Unlike object-oriented metaphysics, where non-external relations are always non-fundamental, OSR and MSR are metaphysical frameworks in which relations are taken as fundamental items. We spell out this claim in Section 6.

4. The ontology of bundles of possible relations

In this section, an ontology of bundles of possible relations (OPR) for QM is sketched. This proposal is expected to be compatible with most of the modal interpretations currently at disposal in the literature. The main inspiration for OPR is OPP. The latter focused mainly on contextuality and indistinguishability. Our aim is that in OPR not only contextuality and indistinguishability, but also QH based on entanglement relations, obtain a clear ontological account. The entanglement relations, following the lead of Esfeld (2004), will be considered as strongly non-supervenient relations and the relevant constituents of our bundles. Intrinsic properties, associated with non-contextual, state-independent observables, are allowed, since they do not suffice to create identity conditions and to make quantum systems individuals, in a manner compatible with MSR. The cornerstone of OPR will be its compatibility with priority monism (PM from now on), since this last feature is what will allow us to consider that bundles, though constituted by relations, are in an ontological parity with constituent relations, such as MSR demands.

Please have in mind postulates (1), (2) and (3) of OPP (Section 1). We now add the following definition of subsystems and their corresponding bundles:

(4) If a system $S^U(\mathcal{O}^U)$ supports a TPS such that for every $O^U \in \mathcal{O}^U$ it obtains that $O^U = O^1 \otimes I^2 + I^1 \otimes O^2$, the set $\mathcal{O}^1$ whose elements are the observables O^1 defines the subsystem $S^1(\mathcal{O}^1)$ and the instances of type-properties $[O^1]$ constitute its

corresponding bundle. The set $\mathcal{O}^2$ whose elements are the observables O^2 defines the subsystem $S^2\left(\mathcal{O}^2\right)$ and the instances of type-properties $\left[O^2\right]$ constitute its corresponding bundle. Let $\{TPS\}$ be the set of all the TPS supported by $S^U\left(\mathcal{O}^U\right)$ such that for every $O^U \in \mathcal{O}^U$ it obtains that $O^U = O^i \otimes I^{ii} + I^i \otimes O^{ii}$.

This definition is partially inspired in the so-called 'composite systems postulate' of MHI (see Lombardi and Castagnino 2008). In postulate (4) we have that the bundles corresponding to subsystems are still constituted by a set of instances of *intrinsic* type-properties. Let us explore whether it is possible to define them by means of a set of relations. We add this auxiliary trivial assumption

(5) Let the total system $S^U\left(\mathcal{O}^U\right)$ and its subsystems $S^1\left(\mathcal{O}^1\right)$ and $S^2\left(\mathcal{O}^2\right)$. Let $\left|o_j^1\right\rangle$ be the eigenstates of $O^1 \in \mathcal{O}^1$ corresponding to the possible case-properties $\left[o_j^1\right]$ and $\left|o_j^2\right\rangle$ the eigenstates of $O^2 \in \mathcal{O}^2$ corresponding to the possible case-properties $\left[o_j^2\right]$.

Now let us introduce the following assumption that allows for the existence of correlations between the possible case-properties of subsystems due to quantum entanglement

(6) The state $\left|\Psi_U\right\rangle$ of S^U is pure and non-separable such that $\left|\Psi_U\right\rangle = \sum_j c_j \left|o_j^1\right\rangle \otimes \left|o_j^2\right\rangle$ with two or more $c_j \neq 0$.

As stressed in Section 3, the assumption (6) is reasonable if unitary QM is embraced, as it is the case if a modal interpretation is adopted. Additionally, the supposition that the state $\left|\Psi_U\right\rangle$ is pure ensures that the are no correlations between S^U and any external system. This way, system S^U is a closed system that could reasonably refer to the entire universe. We are now in position to assign the following properties and relations to system S^U and subsystems S^1 and S^2, based on postulates (1-5) and assumption (6)

(7) S^U has the actual intrinsic case-property $\left|\Psi_U\right\rangle\left\langle\Psi_U\right|\left(S^U\right)$ and all the actual intrinsic case-properties that commute with it.

(8) The subsystems S^1 and S^2 have the actual non-supervenient type-relation $R_{\left|\Psi_U\right\rangle\left\langle\Psi_U\right|}\left(S^1, S^2\right)$.

(9) The subsystems S^1 and S^2 have possible case-relations $R_i\left(S^1, S^2\right)$. Each $R_i\left(S^1, S^2\right)$ corresponds with each possible correlated pair of case-properties $\left[o_j^1\right]$ and $\left[o_j^2\right]$.

As stressed in Section 3, according to Esfeld (2004) the property $\left|\Psi_U\right\rangle\left\langle\Psi_U\right|$ of S^U does not reduce to relation $R\left(S^1, S^2\right)$ nor otherwise, but they are equivalent: 'the non-supervenient relations of entanglement among the parts of a quantum whole amount to the whole having intrinsic properties that do not supervene on intrinsic properties of the parts' (p. 611). Assuming this principle, we have as a result that the intrinsic property of the total system defined in postulate (7) and the non-supervenient relation among the subsystems defined in postulate (8) are equivalent. Assuming also the modal perspective, it is possible to extend such equivalence to affirm that the actual non-supervenient type-relation defined in postulate (8) amounts to the set of possible case-relations defined in postulate (9). Formally:

$$|\Psi_U\rangle\langle\Psi_U|\left(S^U\right)\leftrightarrow R_{|\Psi_U\rangle\langle\Psi_U|}\left(S^1,S^2\right)\leftrightarrow R_i\left(S^1,S^2\right) \tag{6}$$

From postulates (7), (8) and (9) it is possible to partially re-state postulates (3) and (4) in relational terms

(3') A physical system in pure state is in the ontological domain a *bundle of instances of* intrinsic *type-properties with their corresponding possible* intrinsic *case-properties*. Among every possible intrinsic case-properties, the intrinsic case-property $|\Psi_U\rangle\langle\Psi_U|\left(S^U\right)$ together with all the intrinsic case-properties that commute with it are actual.

(4') Every pair of subsystems yielded by a TPS with respect to the state $|\Psi_U\rangle$ is non-separable is, in the ontological domain, a pair of bundles constituted by the *instance of type-relation* $R_{|\Psi_U\rangle\langle\Psi_U|}\left(S^1,S^2\right)$ and all those *instances of type-relations* that commute with it, together with their corresponding sets of *possible case-relations* $R_i\left(S^1,S^2\right)$.

These reformulated postulates continue to assign a bundle of intrinsic properties (as in OPP) but only to isolated systems in pure states (a condition that probably only the universe strictly satisfies). However, these postulates now assign bundles of possible case-relations to all those pairs of subsystems yielded by TPSs that leave the total system state non-separable (a feature of OPR that differs strikingly from OPP).

A series of clarifications are provided next.

(1st) given assumption (6), an intrinsic *case*-property of the total system amounts to a *type*-relation of its subsystems, since if the state $|\Psi_U\rangle$ is non-separable and determines a *case*-property of the total system, say $|\Psi_U\rangle\langle\Psi_U|\left(S^U\right)$, by the very same fact it is determined a relation among the parts that it is still open to further determination, say $R_{|\Psi_U\rangle\langle\Psi_U|}\left(S^1,S^2\right)$. For example, if the state $|\Psi_U\rangle$ determines that spin in *z* direction $S_{Uz}=\left(S_{1z}+S_{2z}\right)^2$ has definite value cero, i. e. $S_{Uz}:(0)$, in a composite systems of a pair of fermions, then the relation $R_{|\Psi_U\rangle\langle\Psi_U|}\left(S^1,S^2\right)$ is a *type*-relation with respect to the following two possible *case*-relations

$$\begin{gathered}S_{1z}:(1)\wedge S_{2z}:(-1)\\ S_{1z}:(-1)\wedge S_{2z}:(1)\end{gathered} \tag{7}$$

(2nd) Each actual case-property of the total system bundle is equivalent to multiple type-relations depending on the number of admissible tensor product structures that leave the state of the total system non-separable. The properties of the total system bundle are open to determination in a double sense: first, according to which are the subsystems into which it is partitioned by a certain tensor product structure (as stated in this clarification); and second, according to the different combinations of possible values that these subsystems can take (as stated in the 1st clarification).

(3rd) in postulate (3') it is established that a physical system in pure state is in the ontological domain a bundle of instances of intrinsic type-properties. If the physical system that we are referring to is the whole universe, then the definition (3') can be reformulated simply deleting the words 'instances of'. In fact, if the physical systems in pure states are many, then they can only possess particular instances of universal properties. But if the system in pure state is single (the universe), then universals exhaust in a single instance. The properties of the universe, if a strong form of QH is adopted (as we aim to do), are both universal and particular.

(4th) presumably only certain elements of $\{TPS\}$ leave the state of the universe non-separable. It may be possible to choose a TPS for which the state of the universe is separable. Nevertheless, in a universe partitioned by such a TPS, quantum correlations would be completely deleted. Each part of the universe would be in a pure state and so each of them would constitute somehow a closed universe by itself. Particularly, a TPS as such does not seem to be the one that stands between us as observers and the rest of the world.

(5th) what the above postulates establish for bipartite structures can be generalized for multipartite structures, obtaining so not only dyadic relations but polyadic ones.

(6th and last) OPR avoid well-known issues about the failure of property composition and decomposition that plague most modal interpretations (see Vermaas 1998, 110). Such failure occurs when the state of the composite system is non-separable. In that case, it is not possible to assign to the composite system a property that can be decomposed into intrinsic properties of the subsystems. We avoid that failure since, in OPR, intrinsic properties corresponding to state-dependent observables are not assigned to subsystems in such circumstance, but a set of possible relations among them.

5. OPR as an instance of Priority Monism

Unlike the QM ontology proposals inspired in RQM in which it makes no sense to conceive of the quantum state of the universe (since it could not be relative to another system), OPR takes the possible relations between parts of the universe as equivalent to intrinsic properties of the whole. This fact raises the possibility of drawing links between OPR and PM. Schaffer (2010b) defines priority monism as the metaphysical thesis that maintains that there is only one basic concrete object and that is the universe. The opposite theses are priority pluralism (PP), according to which there is a plurality of basic concrete objects, and priority nihilism (PN), which holds that there are no basic concrete objects. Formally

$$
\begin{aligned}
&\text{PM: } \exists x\left(Bx \wedge \forall y\left(By \rightarrow x = y\right)\right) \\
&\text{PP: } \exists x \exists y\left(Bx \wedge By \wedge x \neq y\right) \\
&\text{PN: } \neg \exists x Bx
\end{aligned}
\tag{8}
$$

Where B is the property of being a basic concrete object (see 'Monism' in SEP authored by Schaffer, 2018). These are theses that alternatively conceive the way in which the relation of ontological dependence between different concrete objects occur. Schaffer (2010b) argues that PM can be defended based on the internal relations that link all objects (note that internal relations in Schaffer terms are the aforementioned inherent relations). He states that: 'the substantial unity of the whole universe is inferred from the interdependence of all of its parts' (p. 342). Of all the possible definitions of internal relations that have been put forward in the literature, Schaffer (2010b) highlights the following: an internal relation is such if it is 'modally constraining'. That is, an internal relation is such that prevents the free recombination of its relata. Formally, a relation is internal if:

$$
\left(\forall x_1\right)\ldots\left(\forall x_n\right) R x_1 \ldots x_n \rightarrow \neg M^n x_1 \ldots x_n
$$

Where M^n is the *n*-adic relation that $x_1 \ldots x_n$ have when they can recombine their properties freely. Therefore, if $x_1 \ldots x_n$ have the internal relation R then they cannot freely recombine their properties. The definition of internal relation that Schaffer proposes adjusts to the type of relations that have been proposed in OPR as constituents of the subsystems bundles. Indeed, it has been said that each type-relation $R_{|\Psi_{rr}\rangle\langle\Psi_{rr}|}\left(S^1, S^2\right)$ is equivalent to a set of possible case-relations $R_i\left(S^1, S^2\right)$. That is, the type-relation $R_{|\Psi_{rr}\rangle\langle\Psi_{rr}|}\left(S^1, S^2\right)$ is an instance of the internal relation defined by Schaffer (2010b) since it precisely restricts the modal freedom of its relata. That is, it enables only certain *possible* combinations of *possible* values of the systems S^1 and S^2 (those encoded in the *possible* case-relations $R_i\left(S^1, S^2\right)$) and excludes any others.

Starting from the assumption that all objects are internally related (what is consistent with OPR by assumption (6), see Section 4) and some other metaphysical definitions and logical and mereological principles, Schaffer (2010b) offers two proofs of PM. In the first proof it is assumed that (1) there are no two modally free objects (an assumption that follows from the assumption that all objects are internally related), (2) there is at least some basic object, and (3) any basic object is modally free with respect to any other object with which it does not overlap. It is further assumed (to reduce to absurdity) that the object *a* that is part of the universe is basic. Then we define the object *b* that does not overlap with *a*. By (3) *a* and *b* are modally free. However, by (1) there are no two modally free objects. It is proved by the absurd that no part of the universe is basic. Since by (2) there is at least some basic object, that basic object cannot be other than the universe itself. From the perspective of this proof, what is unique about the universe is that it is the only object that overlaps with any other. This is consistent with OPR in which bundles of relations are always parts of the universal bundle. Furthermore, from this proof it follows that the universe is the only modally free object, a fact consistent with OPR in which the system of the universe is the only one with a pure state and therefore corresponds to the only bundle of intrinsic properties.

The second proof (somewhat more intricate) runs as follows. It is assumed that (1) there are no two modally free objects and that (4) non-overlapping and modally constrained objects are interdependent. It is further assumed (to reduce to absurdity) that the universe is not basic and that it depends on object *a*. Then, since the dependence relation is asymmetric, *a* cannot be dependent on the universe. Then, since the dependence relation is irreflexive, *a* cannot be the universe. Since any object that is not the universe is a proper part of the universe, then *a* is a proper part of the universe. Then the complement *â* of *a* is defined. The objects *â* and *a* cannot be modally free by (1). Then, since neither object overlaps with its complement and (4), *â* and *a* are interdependent. Then, since two interdependent objects either overlap each other or form a common whole, and since *â* and *a* do not overlap each other, then *â* and *a* form a common whole. Then, since *â* and *a* depend on a common whole, *â* and *a* must depend on the universe since the universe is the only whole that *â* and *a* have in common. Therefore, *a* depends on the universe, against the initial assumption. Therefore, the universe is basic. Now suppose (and then reduce to absurdity) that some other object *c* (which would have to be a proper part of the universe) is basic. Define the complement *ĉ* of *c*. The objects *ĉ* and *c* are interdependent for the same reason that *â* and *a* are interdependent. If *c* is interdependent then it is dependent and not basic, against the assumption that *c* is basic. Therefore, no other object but the universe is basic. From the perspective of this second proof, what makes the universe special is that it is the only object that has no complement. This fact is also consistent with OPR, in which the bundles of relations are always defined with respect to their complement, while the properties of the universal bundle are intrinsic and therefore it has no complement.

It should be noted that PM does not require that all properties of objects be internal relations. This is consistent with OPR, in which intrinsic properties corresponding to state-independent observables are allowed. From this it follows that PM is not in itself a form of OSR, but it could be compatible with MSR. This makes possible to contend that OPR is both an instance of MSR and PM.

6. OPR as an instance of MSR

We mentioned in Section 2 that Candiotto's proposal is an OSR instance and Oldofredi's is an MSR instance. Now we ask ourselves how OPR is positioned with respect to OSR and MSR. The natural response indicates that OPR is a MSR instance. It seems that in OPR relata and relations are in ontological parity and have equal explanatory power. They simultaneously arise from the universal bundle once a TPS is given. To change the parts amounts to changing the relations, and to change the relations is equivalent to redefining the parts together with their intrinsic properties. So we cannot explain the relations in terms of the parts and their intrinsic properties nor the parts in terms of their relations. The parts, relations and intrinsic properties corresponding to each possible TPS are all encoded in the Hilbert space and its algebra of operators representing the universe and its physical magnitudes. Neither the related bundles nor their intrinsic properties corresponding to state-independent observables are there before the relations (as if they were thin particulars or bundles of intrinsic properties), but they are constituted by the inherent or internal relations between the parts of the universal bundle. The related bundles are indeed parts of the whole, but they are not basic objects and do not have ontological priority with respect to the whole and its internal structure, since OPR is a PM instance. Nevertheless, it could be argued that, since OPR is framed in a bundle theory, which is a constituent ontology, the relations should have ontological priority over the bundles they constitute. It seems that, after all, any ontological proposal based on bundle theories (OPR and MBT included) could not become an MSR instance. We think that the objection holds if the bundles constituents have to be taken as basic items, as it seems to be the case in MBT. That is certainly not the case in OPR, in which the single basic item is the universal bundle. Taking OPR as an instance of PM makes it possible to avoid this objection.

Although Oldofredi's (2021) proposal is also understood as an MSR instance, there are some noticeable differences. First, the relations in Oldofredi's proposal and the OPR relations have a different physical nature. While in RQM they are interaction relations, in OPR they are entanglement relations. RQM postulates prevent the existence of massive entanglement relations as the ones we assume. A second difference is that Oldofredi takes mereological composition as the bundle relation at issue, while OPR dispenses with a bundling relation of any type (more on this on Section 7). A third difference is that MBT adopted by Oldofredi takes external spatio-temporal relations to ensure the identity of the bundles, while OPR takes the bundled relations themselves to create conditions of relational identity for the bundles they constitute (more on this on Section 9). A point in which Oldofredi's proposal and OPR agree is that in both cases bundles are allowed to possess intrinsic properties as well as relations, corresponding to non-contextual, state-independent observables. The only requirement, in order to allow intrinsic properties in a manner consistent with MSR, is that they do not underlie relations nor create conditions of individual identity. It is convenient to admit (from the point of view of the adequacy between an ontology proposal and physical practice) that quantum systems have some intrinsic properties, since in physical practice the state-independent magnitudes are used to characterize types of particles. In that sense, Candiotto's (2017) metaphysics of relations, in which intrinsic properties are eliminated (along with any QM ontology that adopts the general metaphysical framework provided by OSR), face an additional challenge. To face that challenge, it would be possible to argue that every proper quantum physical quantity is an observable capable of taking on a definite value only in relation to another physical system with respect to which its state is entangled (see e.g. Mermin, 1998), thus not considering state-independent magnitudes as proper quantum. There are even attempts to reduce intrinsic magnitudes to relational ones (see Muller 2011, Esfeld and Lam 2010). In any case, it exceeds the limits of this work to offer a detailed account of this matter.

It is worth mentioning in this section that Esfeld (2017) proposes a minimalist ontology that is also an instance of MSR. According to him, all that there are in the world are 'distance relations that individuate simple objects, namely matter points'. Esfeld considers that the following two ontological statements are equivalent: '(a) there is one whole (i.e. the universe) exhibiting an internal differentiation in terms of relations that individuate a plurality of simple objects within the whole. (b) There are relations that individuate simple objects so that the relations and the objects make up a configuration that is the universe'. It seems that OPR is an instance of statement (a) (or at least approximates to (a) since in OPR objects are not necessarily simple and relations do not generally individuate) and Esfeld's minimalist ontology is an instance of (b). While (a) seems to be a form of holism compatible with PM, (b) seems to be a form of atomism compatible with PP. It must be recognized that Esfeld himself considers that (b) is also a form of (relational) holism, which is reasonable since simple objects do not have intrinsic properties but only inherent or internal relations. Despite their alleged equivalence, Esfeld endorses an ontology which is an instance of (b) because he considers that instances of (b) are more parsimonious than instances of (a). We consider that the topic of parsimony is still open, since it not clear that an ontology in which a universe is dependent on a plurality of simple objects is more parsimonious than an ontology in which a plurality of objects is dependent on a simple universe. At the end of the day, we have in both cases a complex structure that internally differentiates (a); or make up a whole (b). Note that Esfeld's minimalist ontology adds to (b) that the relations are *distance* relations and that the objects are matter points. From a certain point of view, it looks like Esfeld is being in his ontology proposal a bit more specific that we are in OPR. He commits himself with a specific type of relations that ought to be actualized: they are always distance relations. In OPR it is not specified which context is actualized (that issue is not settled, so OPR has to be complemented by the property ascription rule or actualization rule provided by the concrete modal interpretation one is willing to endorse, more on this on Section 8). Esfeld's commitment (which resembles the commitment that Bohmian interpretation makes with the position operator) may create a conflict with quantum contextuality, unless, of course, Bohmian mechanics is adopted instead of QM. Another important difference between OPR and Esfeld's minimalist ontology is that in the latter there is an ontological commitment with one particular TPS of the total system: the one that yields the simplest objects possible (in this sense it seems to have an atomistic aspect). OPR, contrarily, is not committed with any particular TPS (it only demands that the TPS leaves the state of the universe non-separable). We can choose fine-grained TPSs or coarse-grained TPSs. All of them are dependent on the whole.

7. OPR and bundling relation

Every bundle theory faces the issue of specifying some bundling relation that holds between the bundle constituents. The nature of the bundling relation is much disputed among advocates and critics of bundle theories (see e. g. Grupp 2004, Shiver 2014). Schematically, the bundle theories in which the bundle constituents are particular instances of universals make use of the coinstantiation relation and the theories in which the constituents are tropes (particular and concrete basic items) make use of the compresence relation. It must be considered that bundle theories were originally developed to address the metaphysics of concrete particulars, which are naturally assumed to be individuals, in an allegedly more parsimonious way than traditional substratum theories. The bundling relation at issue ensures that an aggregate of properties or tropes become modally connected and thus become a proper bundle, with determinate identity conditions. The bundling relation has to be taken as primitive. This maneuver has led many advocates of substratum theories to rise objections about the allegedly more parsimonious status of bundle theories. In fact, the bundling relation turns out to be an additional *ad hoc* primitive item in ontology, analogous to the substratum (see Loux 2006).

As mentioned in Section 2, in Oldofredi's (2021) proposal, the bundling relation adopted is the mereological relation that mediates between the whole and the parts (Paul 2017). Namely, the properties according to this approach are the parts of the bundle that corresponds to a quantum system. Contrarily, in OPR the mereological relations do not hold between the bundles and their properties, but between the universal bundle and the subsystems bundles. So in OPR mereological relations could not work as the bundling relation. Remember that OPR is both a PM and MSR instance, such that the only bundle that has an absolute identity (entailed by intrinsic properties) is the universal bundle. Then, a first point is to specify a bundling relation that holds between the intrinsic properties of the universe. It can be argued that there is no need to specify such a relation since the constitution of the universe can be taken as a primitive metaphysical fact (this is in line with the PM assumption that the only concrete object that is basic is the universe). Indeed, the issue of specifying a bundling relation that yields proper bundles arises when there are multiple ways of aggregating properties, imposing the need to distinguish between aggregates that constitute proper bundles and other that do not. Since in OPR there is a single aggregate of properties that constitutes an object with absolute identity, namely the universe, the constitution of this bundle is given primitively without the need to justify this fact by subsuming it under some principle.

As a corollary, in OPR there is no need to distinguish between the universal and its particular instances. Namely, the intrinsic properties of the universe are by the same token universal and particular. Their concreteness and particular character depend on the concreteness and particular character of the object they constitute (and not the other way around as in trope theories or as in MBT adopted by Oldofredi). At the same time, their universality depends on the fact that the object they constitute is the only object (with absolute identity) and that is the universe. From the perspective of monistic holism, the traditional problem of the nature of universals as 'one-in-many' turns out to be dissolved. Universals from this perspective are exhausted in a single instance to the extent that the only particular in which the universals instantiate is the universe. This OPR feature can make it appear more parsimonious with respect to other QM ontology proposals.

It remains to be specified what is the bundling relation in the case of the subsystems bundles. A possible answer to this question is radical and consists in denying that the bundles of relations have identity conditions at all. In such a case there would be no need to specify a particular bundling relation, since any aggregate of relations could count as a proper bundle. This solution keeps continuity with the OPP proposal regarding the denial of the status of objects of quantum systems. Recall that according to OPP authors quantum bundles do not have identity conditions at all (see Lombardi 2023, more on this on Section 1). However, a less extreme way of addressing this issue is to propose a relativized notion of object. Indeed, the bundles of relations could possess certain identity conditions not deriving from their intrinsic properties but precisely from the relations in which they stand, which are, in turn, the same relations that constitute them. The identity conditions of each bundle may be relativized to the bundle (or bundles) with which it is related. In this sense, the proposal is close to that of Muller and Saunders (2008) in which they treat quantum systems as 'relationals' instead of individuals. For them, individuals have absolute identity, i. e. they are discerned by intrinsic properties, and relationals have relative identity, i. e. they are discerned by means of relations. In case two objects have the same intrinsic and relational properties, their numerical difference can still be accounted for by means of the so-called *weak discernibility*, if the symmetric relations in which they stand are irreflexive, as it is argued for identical quantum systems (more on this on Section 9, when we come to the issue of indistinguishability).

Consider that the constitutive items of the OPR bundles cannot be assimilated to intrinsic properties that can be defined independently of their bundling into a certain array or another, as it is the case in traditional bundle theories. On the contrary, in OPR bundles constituents are defined at the same time as the pairs (or n-tuples if multipartite TPSs are chosen) of reciprocal bundles that are constituted by them. No ontological or explanatory priority between OPR bundles and their constituents can be established. Consequently, unlike the bundle theories in which it is necessary to postulate, in addition to the basic constituents, a primitive relation of a purely metaphysical nature (the coinstantiation or compresence relation), it seems that in OPR, the same relations that are bundled have the required bundling effect. In fact, subsystems bundles in OPR are not constituted one by one in virtue of the aggregation of a set of constituents, but are bundled all at once in virtue of the internal differentiation between parts that certain set of relations yields within the universal bundle (each set corresponds to a TPS). The OPR relations are bundled in such a manner that they give rise to complementary parts of the whole. But the partition of the whole is in turn defined by the OPR relations. So it seems that the OPR relations are capable of bundle themselves, as long as they differentiate the internal parts of the whole. These relations correspond exactly to the correlations that QM formalism describes when the state of the total system is non-separable. They do not have a primitive and purely metaphysical nature, thus avoiding well-known issues that arise in bundle theories about the nature of the bundling relation. In short, along with the dissolution of the problem of universals that enables us to take the constitution of the universal bundle as a primitive fact, this possibility of reducing the bundling relation of subsystems bundles to clearly defined physical relations, without any appealing to relations of a purely metaphysical nature, may constitute an additional advantage of OPR in terms of the methodological principle of parsimony.

The related question about the identity over time of the universal bundle and the bundles of relations in OPR has not been addressed so far. However, it can be suggested that this problem would vanish if together with OPR we took a 'timeless approach to time' (see Marletto and Vedral 2017, Page and Wootters 1983), which reduces external time to internal correlations between subsystems of the universe as those postulated in OPR. To achieve that, it would suffice to add to the assumption (6) that the state of the universe is stationary, that is, a Hamiltonian eigenstate, and that subsystems do not interact with the chosen clock. This would also shed light on the issue of the dynamical change of properties. Assuming the 'timeless approach to time', it would be obtained that the dynamics of properties can be spelled out as a series of possible case-relations. Let us consider a simplified example:

$$\left|\Psi\right\rangle = \sum_i c_i \left|t_i^1\right\rangle \otimes \left|a_i^2\right\rangle \tag{9}$$

Where $\left|a_i^2\right\rangle$ are the eigenstates of some observable A of a subsystem of the universe; $\left|t_i^1\right\rangle$ are the eigenstates of the time observable of the universe subsystem which is taken as a clock; and $\left|\Psi\right\rangle$ is a Hamiltonian eigenstate of the universe. Also, the Hamiltonian of the universe should be decomposed

$$H^1 \otimes I^2 + I^1 \otimes H^2 \tag{10}$$

which ensures that there is no interaction between the subsystems. This scheme meets the three conditions required by the timeless approach (see Marletto and Vedral 2017, p. 2): (i) *timelessness*, because the state of the universe is stationary; (ii) *noninteracting*, because there are no interaction terms in the Hamiltonian of the universe decomposition (see the right-numbered expression 10); and (iii) *entanglement*, as it is clear in the state in right-numbered expression (9). Interestingly, it is possible to derive the Schrödinger equation from this scheme (see Marletto and Vedral 2017, p. 3). Within the OPR framework, this scheme allows to define the possible case-relations $R_i\left(S^1, S^2\right)$, each corresponding with a correlated pair of definite values t_i^1 and a_i^2 (see postulate 9). This way, the dynamical change of properties $a_1, a_2 \ldots a_i$ with respect to time $t_1, t_2 \ldots t_i$ is spelled out as a series of possible case-relations R_i. This means that the change of properties with respect to time can be accounted for by a set of possible case-relations that all subsystems bundles in a stationary universe have with respect to a specific bundle that has been chosen as a clock.

8. OPR and contextuality

Regarding quantum contextuality, OPR exactly mimics the approach of OPP. As in OPP, a possibilist metaphysics of modality is fully embraced (see Menzel 2023, about the debate actualism-possibilism). That is, it is accepted as a principle that not only actual but also merely possible properties (*possibilia*) can exist in concrete particulars. Both in OPP and OPR, the bundles (constituted either by solely intrinsic properties or by fundamental relations along with intrinsic properties) are defined directly in the domain of the possible. They are bundles of non-determined type-properties (see OPP postulate 3 in Section 1) or type-relations (see OPR postulate 4' in Section 4). To each type-property or type-relation corresponds respectively a set of possible case-properties or possible case-relations. In this way, OPP and OPR conform to the restrictions imposed by quantum contextuality, reinforced by Kochen and Specker (1967) theorem. Of course, this feature of OPR makes it especially suitable for modal interpretations, intendedly for most of them. OPR is by itself silent about which particular context is to be taken as actualized. That specific point is reserved to be made explicit by means of the property ascription or actualization rule included within the interpretive postulates of modal interpretations. For instance, if we embrace MHI, then we have that the context defined by the Hamiltonian operator is the one chosen (Lombardi y Castagnino 2008). If we are Bohmians the preferred context is the one defined by position operator (Bohm 1952). If we adopt BDMI, SDMI or PMI the context that ought to actualize is the one defined by the density operator that represents the state of the system (see Dieks 2021). Different motivations (in addition to empirical adequacy) may guide the choice of one property ascription rule or another.

This is the right place to provide some examples of applications of OPR to concrete physical situations, complemented with the property ascription or actualization rule of some modal interpretations. First, let us address the problem of quantum measurement, the touchstone against which the interpretations of QM are put under test. Assuming the von Neumann model of quantum measurement (1932), we have that after the interaction between a quantum system S^1 being measured and a measurement device S^2, the state of the composite system is

$$|\Psi\rangle = \sum_i c_i |a_i^1\rangle \otimes |p_i^2\rangle \tag{11}$$

Where $\left|a_i^1\right\rangle$ are the eigenstates of some observable A being measured and $\left|p_i^2\right\rangle$ are the eigenstates of the device pointer. For simplicity, we here are assuming that the universe is a bipartite bundle where only S^1 and S^2 exist. In this case, the quantum bundles are constituted by (at least) possible case-relations $R_i\left(S^1,S^2\right)$, each corresponding with a correlated pair of definite values a_i^1 and p_i^2 (see postulate 9). Let us now adopt the property ascription rule belonging to one of the modal interpretations, say BDMI. According to this particular modal interpretation, the preferred contexts that acquire definite values correspond to the bases that are obtained by means of the biorthogonal or Schmidt decomposition (Kochen 1985, Dieks 1988, 1989). The state in the right-numbered expression (11) is precisely a Schmidt decomposition that results of the entanglement between the system being measured and the measurement device. So BDMI picks out precisely $\left|a_i^1\right\rangle$ as the preferred basis for system being measured and $\left|p_i^2\right\rangle$ as the preferred basis for the device. So, applying the BDMI property ascription rule within the OPR framework, it is obtained that a certain possible case-relation $R_k\left(S^1,S^2\right)$ (one of the many $R_i\left(S^1,S^2\right)$), corresponding to outcome p_k^2, is picked out as an actual case-relation, with probability $\left|c_k\right|^2$.

This account of measurement is clean and straightforward, but fails in case of non-ideal measurements. MHI (Lombardi and Castagnino 2008) propose an actualization rule that is able to properly address not only ideal but also non-ideal measurements, at the price of obtaining a model of measurement a bit more intricate. As mentioned a few paragraphs above, MHI fixes the context of actualization in a specific observable, the Hamiltonian. As a result, in this interpretation, the Hamiltonian and those observables that commute with it acquire definite values. To apply the actualization rule we need to restate the right-numbered expression (11) on the Hamiltonian basis

$$\left|\Psi\right\rangle = \sum_j c_j \left|h_j^1\right\rangle \otimes \left|h_j^2\right\rangle \qquad (12)$$

Where $\left|h_j^1\right\rangle$ are the eigenstates of the Hamiltonian of system being measured and $\left|h_j^2\right\rangle$ are the eigenstates of the Hamiltonian of the measurement device. In this case, the possible case-relations $R_j\left(S^1,S^2\right)$ corresponding with each correlated pair of definite values h_j^1 and h_j^2, suffice to define the bundles corresponding to systems S^1 and S^2. Applying the MHI actualization rule within the OPR framework, it is obtained that a certain possible case-relation $R_k\left(S^1,S^2\right)$ (one of the many $R_j\left(S^1,S^2\right)$), corresponding to definite value h_k^2, is picked out as an actual case-relation, with probability $\left|c_k\right|^2$. By the very way that measurement devices are constructed, it must be assumed that the pointer commutes with the measurement device Hamiltonian, otherwise the lecture of the pointer could not be possible. This means that the actualization of the case-relation $R_k\left(S^1,S^2\right)$ entails that the pointer also has a definite value, that is the outcome p_k^2. MHI authors have proposed models for many usual physical scenarios: the free particle with spin, the harmonic oscillator, the free hydrogen atom, the Zeeman effect, fine structure, the Born-Oppenheimer approximation (see Lombardi and Castagnino 2008). It has also been proposed a model of consecutive measurements, which has always been a difficult task for any interpretation that avoids collapses (see Ardenghi, Lombardi y Narvaja 2013).

Additionally, let us briefly address an EPR-Bohm type experiment. We go back to our previous example, in Section 3. There are two half-integer spin systems S^1 and S^2 in the singlet state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle \otimes |\downarrow_2\rangle - |\downarrow_1\rangle \otimes |\uparrow_2\rangle\right) \qquad (13)$$

In this case, the quantum bundles are constituted by two possible case-relations $R_i\left(S^1, S^2\right)$, one corresponding with the correlated pair of outcomes $\uparrow_1$ and $\downarrow_2$ and the other with the correlated pair $\downarrow_1$ and $\uparrow_2$. Adopting the property ascription rule belonging to BDMI, since the two terms sum in the right-numbered expression (13) is a Schmidt decomposition, it is obtained that one of the two possible case-relations, say $R_k\left(S^1, S^2\right)$ corresponding to definite values $\uparrow_1$ and $\downarrow_2$, is picked out as an actual case-relation, with probability $\frac{1}{2}$. Adopting in this case the MHI actualization rule would be less straightforward. We would have to expand the state on the Hamiltonian basis, then pick out an actual case-relation, and finally, assuming that systems S^1 and S^2 are being spin-measured by two devices, find the corresponding pointers eigenstates correlated with the actual case-relation selected.

Since most standard physical situations presupposes interactions that give rise to entanglement between different quantum systems, the solutions sketched here are probably applicable to most of them. Of course, a full analysis of the applicability of different property ascription or actualization rules within the OPR framework, to already studied physical situations in which those rules work properly, ought to remain as pending work. It must be emphasized that the complement of OPR with a property ascription or actualization rule belonging to a particular modal interpretation, allows OPR to account not only for the qualitative profile of entangled quantum systems, but also for those quantum systems that, in virtue of a measurement interaction or any analogous physical process, acquire definite values. While, for instance, orthodox interpretation associates definite values to collapsed, disentangled states, thus corresponding to intrinsic properties in the ontology, OPR assumes quantum systems to remain entangled even in the face of the acquisition of definite values. According to OPR, all possible case-relations remain there, without collapses. Definite values are accounted for by the property ascription or actualization rule at issue, which picks out an actual case-relation among all possible case-relations.

A last remark is in order. A point in which OPR differs from OPP is regarding the actual properties of the only bundle of intrinsic properties that OPR accepts, that is, the universal bundle. Indeed, according to postulate (3’), that bundle is defined not only as a set of possible case-properties corresponding to instances of type-properties. In addition, certain actual case-properties are specified. What are to be the actual properties of the universal bundle is determined by the state of the universe. This movement is necessary since OPR selects precisely those actual case-properties of the universal bundle that are equivalent to the type-relations that the subsystems have when the state of the total system is non-separable with respect to those subsystems. If we here drop EEL and apply to the universal bundle an alternative actualization rule, that would give actual case-properties corresponding to eigenstates of the total system that may be separable with respect to the subsystems. Consequently, the intrinsic properties of the total system would not amount to relations between the subsystems. They would be properties of the whole that can be decomposed into intrinsic properties of the parts. This would attempt against the holistic and relational image that OPR intends to offer. Certainly, that minimal actualization rule built into OPR results in a partial concession to EEL. Remember the EEL formulation: a system has a certain property, i. e. a certain observable possesses a certain definite value, iff its state is the eigenstate corresponding to that eigenvalue. OPR takes the actual properties of the universe to correspond to the eigenstates of observables that coincide with the universe pure state. Then, if certain modal interpretations are embraced jointly with OPR, their property ascription or actualization rule will have to be fixed in such a manner that, when they are applied to the universal bundle, they go against the EEL 'only if' conditional, but not against the first one. That would be the case of the Atomic Modal Interpretation (AMI, Bacciagaluppi y Dickson 1999) and the MHI, the only two modal interpretations that reject EEL both conditionals.

9. OPR and indistinguishability

PII was conceived to account for the identity conditions of bundles of properties without appealing to the allegedly obscure metaphysical notion of substratum. Of course, adopting a bundle theory to account for quantum systems that are indistinguishable but numerically different, imposes a challenge. As stressed above, OPP addresses this challenge by appealing precisely to the modal character of the properties that constitute the bundles (see Section 1). Two instances of the same type-property are indistinguishable if they have the same possible case-properties. If the instances of type-properties constituting two bundles were all determined, that is, if one (and only one) actual case-property could be mapped to each of the instances of type-properties (as is to be assumed occurs in the classical bundles) and if those instances of type-properties, in addition to having the same possible case-properties, possess the same actual case-property, then PII would be applied and the numerical distinction between the two bundles would vanish. But, due to quantum contextuality, quantum bundles always have some instances of non-determinate type-properties, i.e. for which possible case-properties can be specified, but not which of them is the actual case-property. This is precisely the point that allows OPP to overcome the challenge that quantum indistinguishability imposes to PII. Indeed, two bundles of instances of type-properties can be indistinguishable with respect to their possible case-properties and yet be numerically different since they may still differ according to some actual case-property, even though that never actually happens. In that sense it is acceptable that PII does not apply to bundles of merely possible properties.

Let us move on to the same issue in OPR. First of all, it must be recognized the legitimacy of the strategy adopted by OPP. If the universal bundle could be compared with a universe indistinguishable in terms of its possible properties, it is reasonable to suppose that *solo numero* difference would subsist between them by virtue of the principles adopted in the OPP strategy. However, in the case of subsystems bundles, the strategy could be another. Instead of blocking the application of PII, one can resort to a form of PII in which it is admitted that the symmetrical but irreflexive relations are capable of establishing numerical difference. This is the weak discernibility approach due to Muller and Saunders (2008). In fact, due to the metaphysical nature of such relations, the numerical difference between at least two relata is required, since nothing can stand in an irreflexive relation to itself (Esfeld and Lam, 2010, 148). There is a natural correspondence between the relations from which it would be possible to weakly discern numerically and the quantum correlations between indistinguishable systems. Indeed, QM asks to symmetrize or anti-symmetrize the state of a system composed of indistinguishable subsystems. Such a symmetrization or anti-symmetrization operation necessarily yields non-separable states for the subsystems, which is the condition that assumption (6) generally establishes in OPR for all subsystems in the universe. It is not a contingency that the relations defined in postulate (4') of OPR are exactly the sort of symmetric and irreflexive relations in which Muller and Saunders focus (2008, 535). As an additional advantage of embracing weak discernibility in the context of OPR it is obtained that, in virtue of the OPR assumption (6), not only fermions are discerned categorically but also bosons. In fact, product states in which bosons are only probabilistically discerned (but not categorically) are forbidden by the assumption (6). The possibility of this natural blend between weak discernibility and OPR represent a comparative advantage of OPR with respect to, e. g. Oldofredi's proposal, in which numerical difference between identical systems has to be accepted as an 'ungrounded difference'.

Final remarks

In the present work, a modal bundle-theorist relational proposal for quantum mechanics (OPR) has been advanced, which is an instance of both moderate structural realism (MSR) and priority monism (PM) and could be associated with a quantum modal interpretation. The theoretical core of the present proposal is the already established modal ontology of bundles of possible intrinsic properties (OPP). With respect to OPP, OPR has the advantage of accommodating in a single proposal not only contextuality and indistinguishability, but also those relational aspects of quantum mechanics based on the phenomenon of quantum entanglement.

OPR can be compared with other quantum ontologies framed in a metaphysics in which relations are taken as fundamental items. They are Oldofredi's and Candiotto's proposals for relational quantum mechanics (RQM). It has been argued that OPR addresses in a more straightforward manner the issues of contextuality and indistinguishability than these RQM ontologies. For instance, it avoids, in Oldofredi's proposal, the *ad hoc* addition of a metaphysical principle to allow numerical difference among indistinguishable bundles (‘ungrounded difference’) and the *ad hoc* violation of the ‘requisite determination’ to account for quantum contextuality. OPR, as an instance of moderate structural realism, also avoids the objection that can be raised against Candiotto's proposal about the dependence of relata on relations.

We point out some additional metaphysical advantages of the advanced proposal: (i) OPR may be considered more parsimonious than its rivals because it dispenses with both substrata and *ad hoc* bundling relations; (ii) the traditional metaphysical problem about the relation between universals and their instances may turn out to be dissolved; (iii) by means of the embracement of the weak discernibility program, OPR saves the principle of identity of indiscernibles in the quantum domain; (iv) by means of the embracement of the timeless approach to time, a deflated metaphysics of time can be adopted; (iv) hopefully, OPR may constitute a metaphysical framework appropriate to fully spell out the metaphysics of quantum entanglement. We end up by highlighting that point (iv) may be considered a promising future development of the advanced proposal.

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References

Ardenghi, J. S., Lombardi, O. y Narvaja, M. (2011). “Modal interpretations and consecutive measurements”. En Karakostas, V., y Dieks, D. (eds.), EPSA 2011: *Perspectives and Foundational Problems in Philosophy of Science*. Dordrecht: Springer, pp. 207-217.

Bacciagaluppi, G. y Dickson, M. (1999). “Dynamics for modal interpretations”. *Foundations of Physics*, 29: 1165-1201.

Bene, G. and Dieks, D. (2002). A perspectival version of the modal interpretation of quantum mechanics and the origin of macroscopic behavior. *Foundations of Physics,* 32: 645-671.

Bohm, D. (1952). “A Suggested Interpretation of Quantum Theory in Terms of Hidden Variables”. Physical Review, 85: 166-193

Calosi, C. (2022). Quantum modal indeterminacy. *Studies in History and Philosophy of Science* 95, 177-184.

Candiotto, L. (2017). The reality of relations. *Giornale di Metafisica*, vol 2/2017, pp. 537-551.

da Costa, N. and Lombardi, O. (2014), Quantum mechanics: Ontology without individuals. *Foundations of Physics*, 44: 1246-1257.

da Costa, N., Lombardi, O. and Lastiri, M. (2013). A modal ontology of properties for quantum mechanics, *Synthese*, 190: 3671–3693.

Dieks, D. (1988). The formalism of quantum theory: an objective description of reality? *Annalen der Physik*, 7: 174–190.

Dieks, D. (1989). Resolution of the measurement problem through decoherence of the quantum state. *Physics Letters A*, 142: 439-446.

Dieks, D. (2021). Modal interpretations of quantum mechanics. In O. Freire (ed.), *Oxford Handbook of the History of Interpretations of Quantum Mechanics*, Oxford: Oxford University Press, Cap. 47.

Earman, J. (2015). Some Puzzles and Unresolved Issues About Quantum Entanglement. *Erkennis* 80, 303–337.

Einstein, A., Podolsky, B., and Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47: 777–780.

Esfeld, M. (2001). The Meaning of Quantum Holism. In: *Holism in Philosophy of Mind and Philosophy of Physics*. Synthese Library, vol 298. Springer, Dordrecht. https://doi.org/10.1007/978-94-017-1787-8_8

Esfeld, M. (2004). Quantum Entanglement and a Metaphysics of Relations, *Studies in History and Philosophy of Modern Physics*, 35: 601–17.

Esfeld, M. (2017). A proposal for a minimalist ontology. *Synthese* 197: 1889–1905.

Esfeld, M. and Lam, V. (2010). Ontic Structural Realism as a Metaphysics of Objects. In: Bokulich, A., Bokulich, P. (eds) *Scientific Structuralism*. Boston Studies in the Philosophy and History of Science, vol. 281. Springer, Dordrecht, pp. 143-159.

Everett, H. (1957). ‘Relative state’ formulation of quantum mechanics. *Reviews of Modern Physics*, 29, 454–462. Reimpreso en DeWitt, B. S. and Graham, N. (Eds.) (1973). *The many-worlds interpretation of quantum mechanics*. Princeton: Princeton University Press, pp. 141–149.

French, S. and Krause, D. (2006), *Identity in Physics: A Formal, Historical and Philosophical Approach*, Oxford: Oxford University Press.

Grupp, J. (2004). Compresence Is A Bundle: A Problem For The Bundle Theory Of Objects. *Metaphysica* 5 (2):63-72.

Howard, D. (1989). Holism, Separability, and the Metaphysical Implications of the Bell Experiments. In: Cushing, J. T. and McMullin, E. (eds.). *Philosophical Consequences of Quantum Theory*. University of Notre Dame Press, pp. 224-253.

Kochen, S. (1985), A new interpretation of quantum mechanics. In: Lahti, P. J. and Mittelsteadt, P. (Eds.). *Symposium on the Foundations of Modern Physics*. Singapore: World Scientific, pp. 151-169.

Kochen, S. and Specker, E. (1967), The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics* 17: 59-87.

Ladyman, J. (2020), Structural Realism, *The Stanford Encyclopedia of Philosophy* (Winter 2020 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2020/entries/structural-realism/>.

Ladyman, J., Ross, D., Spurrett, D. and Collier, J. (2007). *Every Thing Must Go: Metaphysics Naturalized*. Oxford University Press, Oxford.

Laudisa, F. and Rovelli, C. (2021). Relational Quantum Mechanics, *The Stanford Encyclopedia of Philosophy* (Winter 2021 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2021/entries/qm-relational/>.

Lombardi, O. (2023). “Not individuals, nor even objects: On the ontological nature of quantum systems.” En: Arenhart, J. y Arroyo, R. (eds.), *Non-Reflexive Logics, Non-Individuals, and the Philosophy of Quantum Mechanics. Essays in Honour of the Philosophy of Décio Krause*, Dordrecht: Springer-Synthese Library.

Lombardi, O. and Castagnino, M. (2008), A modal-Hamiltonian interpretation of quantum mechanics. *Studies in History and Philosophy of Modern Physics* 39: 380-443.

Loux, M.J. (2006) *Metaphysics: A Contemporary Introduction*. 3rd ed., Routledge, New York and London.

Marletto, C. y Vedral, V. (2017). Evolution without evolution and without ambiguities. *Physical Review D* 95: 043510.

Menzel, Christopher, "The Possibilism-Actualism Debate", *The Stanford Encyclopedia of Philosophy* (Fall 2023 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = <https://plato.stanford.edu/archives/fall2023/entries/possibilism-actualism/>.

Mermin, N. D. (1998). What is quantum mechanics trying to tell us? *American Journal of Physics* 66, 753–767.

Muller, F. A. (2011) Withering away, weakly. *Synthese* 180, 223–233. https://doi.org/10.1007/s11229-009-9609-6

Muller, F. A. and Saunders, S. (2008). Discerning Fermions. *The British Journal for the Philosophy of Science* 59 (3): 499-548.

Oldofredi, A. (2021). The Bundle Theory Approach to Relational Quantum Mechanics. *Foundations of Physics* 51 (1):1-22.

Page D. N. and Wootters W. K. (1983). Evolution without evolution: Dynamics described by stationary observables. *Physical Review D* 27: 2885.

Paul, L (2017). A One Category Ontology. In: Keller, J. (ed.) *Being, Freedom and Method. Themes from the Philosophy of Peter van Inwagen*. Oxford University Press, Oxford.

Rovelli, C. (1996). Relational quantum mechanics. *Int. J. Theor. Phys.* 35(8): 1637–1678

Schaffer, J. (2010a). Monism: The Priority of the Whole. *Philosophical Review* 119: 31–76.

Schaffer, J. (2010b). The Internal Relatedness of All Things. *Mind*, Volume 119, Issue 474, April 2010, Pages 341–376, https://doi.org/10.1093/mind/fzq033.

Schaffer, J. (2018). Monism, *The Stanford Encyclopedia of Philosophy* (Winter 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2018/entries/monism/>.

Shiver, A. (2014). Mereological bundle theory and the identity of indiscernibles. *Synthese* 191, 901–913. https://doi.org/10.1007/s11229-013-0298-9

Teller, P. (1986). Relational holism and quantum mechanics. *British Journal for the Philosophy of Science* 37, 71–81.

Vermaas, P. (1998). The pros and cons of the Kochen–Dieks and the atomic modal interpretation. In: D. Dieks & P. Veermas (Eds.). *The modal interpretation of quantum mechanics*. Dordrecht: Kluwer: pp. 103–148.

Vermaas, P. and Dieks, D. (1995). The modal interpretation of quantum mechanics and its generalization to density operators. *Foundations of Physics* 25: 145-158.

von Neumann, J. (1932). *Mathematische Grundlagen der Quantenmechanik*. Berlin: Springer.