Quantum Probabilities as Emergent from Interacting Wavefunctions

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Abstract

We present a conceptual framework in which quantum probabilities arise from discrete events generated by real-valued alignments of inner products between two dynamically evolving wavefunctions. In this perspective, discreteness and probabilistic behavior emerge from the temporal structure of such events rather than being imposed axiomatically. Illustrative calculations show that the Born rule can appear as the limiting frequency of these events, without invoking wavefunction collapse, many-worlds branching, or decision-theoretic postulates. A two-state example demonstrates consistency with standard quantum predictions and suggests how outcome frequencies track Born weights. Extensions to interference scenarios, quantization heuristics, and multidimensional systems indicate that this proposal provides a fresh conceptual angle on the origin of quantum probabilities. This work is exploratory and aims to highlight the underlying idea rather than provide a completed alternative theory; questions concerning dynamical equations, general proofs, and experimental signatures remain open for future research.

1 Introduction

Quantum mechanics has achieved extraordinary empirical success through its Hilbert-space formalism, where physical states are represented by vectors and observables by self-adjoint operators. The probabilistic character of measurement outcomes is encoded in the Born rule [1], which prescribes that the probability of obtaining an eigenvalue is given by the squared modulus of the corresponding amplitude. While this rule underpins all quantum predictions, its conceptual origin remains unresolved: why should probabilities arise from $|\psi|^2$, and why should the eigenvalues of operators exhaust the set of possible outcomes?

Several interpretations have sought to address these questions. Bohmian mechanics [2] supplements the wavefunction with deterministic trajectories, introducing an equilibrium distribution to recover Born weights. Collapse models [3] modify the Schrödinger equation by stochastic terms to enforce outcome definiteness. Everett's relative-state formulation [4] eliminates collapse, interpreting all outcomes as realized in parallel branches of a multiverse. Decoherence theory [8] accounts for classicality by suppressing interference but does not by itself explain probability assignments.

Beyond interpretation, structural reformulations have also been pursued. Noncommutative geometry [5] and generalized probabilistic theories [6] seek alternatives to Hilbert space while preserving operational predictions. Operational and resource-theoretic approaches have highlighted constraints on measurement compatibility and disturbance [11], leading to renewed attention on the physical underpinnings of probability. Recent experimental and theoretical works have probed measurement dynamics, measurement-induced entanglement and phase transitions, and constraints from joint measurability, deepening links between dynamics and foundational principles [11, 12, 13, 14].

Attempts to derive the Born rule often appeal to either structural or rationality arguments. Gleason-type theorems [7] associate probability measures with the geometry of Hilbert space, while axiomatic and operational reconstructions, such as Hardy's framework [6], seek to recover quantum theory from simple postulates. Everettian and decision-theoretic programs [4, 10] interpret probability as a rational constraint on branching agents. Frequency-based interpretations, dating back to von Mises [9], identify probabilities with limiting frequencies of repeated events and have inspired recent operational approaches [11, 12, 13, 14].

In this work, we propose an alternative conceptual framework in which probabilities emerge dynamically from the interaction of two evolving wavefunctions. Specifically, we consider a pair of wavefunctions $f(t)$ and $g(t)$ whose inner product

$$I(t) = \langle f(t), g(t) \rangle$$

is generally complex but becomes real at discrete times $\{t_j\}$. These *real-valued events* play the role of measurement-like occurrences. At such events, the observed outcome corresponds to the eigenstate whose contribution dominates the real part of $I(t_j)$. Probabilities then arise as limiting frequencies of these events rather than being postulated.

This paper does not claim to present a fully developed theory or a rigorous derivation of the Born rule in full generality. Instead, it introduces an exploratory mechanism and examines its implications in simplified settings. Illustrative examples, including a two-state system and an interference scenario, demonstrate consistency with standard quantum predictions and suggest a new way to conceptualize quantization and measurement.

The remainder of the paper is structured as follows. Section 2 introduces the mathematical structure of the event-based formalism. Section 3 presents a two-state example providing an illustrative link to Born probabilities. Section 4 explores interference effects, a selection rule for quantization, and extensions to multidimensional systems. Section 5 presents the conclusions of this work, including its current limitations and possible directions for future research.

1.1 Physical Motivation

A central puzzle in the foundations of quantum mechanics is why the Born rule privileges the inner product of a wavefunction with its own conjugate, $|\psi|^2 = \langle\psi|\psi\rangle$, as the measure of physical probability. From a mathematical point of view, complex numbers are not just magnitudes but carry a phase structure that directly influences interference and superposition. It is therefore natural to ask: why restrict to the self-conjugate inner product when the Hilbert space structure allows the more general overlap $\langle\psi|\varphi\rangle$ between two possibly distinct wavefunctions?

This question motivates the present framework. We suggest that measurement-like events may be understood in terms of the dynamical interaction of two wavefunctions $f(t)$ and $g(t)$, rather than a single wavefunction with its conjugate. In this view, the *real-valued occurrences* of their inner product provide the basis for discrete outcomes. Probabilistic behavior can then be interpreted as arising from the limiting frequencies of these real events, with Born-type weights appearing as a possible special case. From this perspective, the emergence of such weights is not imposed axiomatically, but may be traced to structural features of complex numbers and their role in governing the interplay of two dynamical wavefunctions.

2 Mathematical Formalism

We begin with two time-dependent wavefunctions $f(t)$ and $g(t)$ defined on a Hilbert space. Their inner product is

$$I(t) = \langle f(t), g(t) \rangle = \int_{\mathbb{R}} f^*(x,t)\, g(x,t)\, dx. \tag{1}$$

In general, $I(t)$ is complex-valued. The central **postulate** of this formalism is that

$$\exists \{t_j\} \text{ such that } I(t_j) \in \mathbb{R}. \tag{2}$$

That is, there exists a discrete sequence of event times $\{t_j\}$ at which the inner product becomes purely real. These instants represent the "measurement-like" events in the dynamics.

2.1 Discrete Event Structure

To represent the occurrence of measurement-like events along a continuous time axis, we define the outcome process as

$$O(t) = \sum_j x_{i(j)}\, \delta(t - t_j), \tag{3}$$

where $x_{i(j)}$ denotes the eigenvalue associated with the j-th event time t_j, and $\delta(t - t_j)$ is the Dirac delta distribution. This ensures that contributions occur only at the event instants t_j, while $O(t)$ vanishes elsewhere.[1]

Physical Interpretation of the Contextual State

In this framework, the second wavefunction g does not correspond to an additional physical system in the usual sense, but acts as a *contextual state* that provides the relational background against which discrete events are defined. This state can be interpreted as representing an apparatus or reference frame, aligning with the spirit of relational formulations of quantum theory [15], where measurement outcomes arise from correlations rather than absolute properties. The evolution of g determines the instants when the inner product $\langle f, g \rangle$ becomes purely real, signaling an event in the model.

[1] If the time parameter were restricted to a discrete grid, the Dirac delta would be replaced by the Kronecker delta δ_{t,t_j}.

2.2 Outcome Selection from Eigenstate Contributions

To see how the outcome $x_{i(j)}$ is determined, expand the wavefunctions in an orthonormal eigenbasis $\{\varphi_i\}$:

$$f(t) = \sum_i f_i(t)\varphi_i, \qquad g(t) = \sum_i g_i(t)\varphi_i. \tag{4}$$

The inner product is then

$$I(t) = \sum_i f_i^*(t) g_i(t). \tag{5}$$

At an event time t_j, the real-valued condition requires

$$\Im[I(t_j)] = 0. \tag{6}$$

The **dominance rule** is defined as follows: at an event t_j, select the eigenvalue x_i corresponding to the term $\Re[f_i^*(t_j)g_i(t_j)]$ that contributes most strongly to $I(t_j)$, while other terms are either suppressed or purely imaginary. Thus the outcome is

$$x_{i(j)} = x_i \quad \text{if } \Re[f_i^*(t_j)g_i(t_j)] \text{ dominates in } I(t_j). \tag{7}$$

For clarity, this rule can be formalized as

$$i(j) = \arg\max_i \left|\Re[f_i^*(t_j)g_i(t_j)]\right|,$$

with tie-breaking by minimal index. This ensures a well-defined selection for every event.[2]

2.3 Emergent Probabilities from Frequencies

Over many cycles, different outcomes x_i recur with different frequencies. One may define the probability of outcome x_i as the limiting frequency of its occurrence across events:

$$P(x_i) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \delta_{i,i(j)}. \tag{8}$$

Here $\delta_{i,i(j)} = 1$ if the j-th event produced outcome x_i, and 0 otherwise. In simple superposition settings, this frequency definition suggests a correspondence with the standard quantum

[2]In principle, two eigenstates could contribute equally to the real part of the inner product at an event, creating a tie. Such cases are extremely rare under generic conditions and do not affect the main argument. If they occur, one may resolve the tie by an arbitrary rule, for example by selecting the eigenstate with the smaller index.

weights,

$$P(x_i) \approx |c_i|^2, \tag{9}$$

where c_i are the coefficients in the conventional quantum expansion. The weighting thus appears not as a postulated rule, but as a possible outcome of the relative frequencies with which the real-valued condition selects different eigenstate contributions. In this picture, the Dirac delta enforces discreteness at the level of single events, while the inner product dynamics of f and g influence how often each eigenvalue x_i is realized. Born-type behavior can therefore be interpreted as emerging from the structure of repeated real-valued inner product events, rather than assumed from the outset.

For clarity, we summarize below the central postulate and the supporting assumptions that underpin the event-based formalism developed in this section. The postulate (A0) captures the conceptual novelty of the approach, while (A1)–(A4) specify technical conditions adopted in this work for concreteness.

Assumptions and Postulate of the Event-Based Formalism:

(A0) **Event Postulate:** There exists a discrete set of times $\{t_j\}$ such that

$$I(t_j) = \langle f(t_j), g(t_j) \rangle \in \mathbb{R}.$$

These instants correspond to "measurement-like" events.

(A1) The time axis is continuous; events occur at discrete instants $\{t_j\}$.

(A2) Event localization on the time axis is represented by Dirac deltas $\delta(t - t_j)$.

(A3) Outcome selection at an event t_j follows the dominance rule:

$$i(j) = \arg\max_i \left| \Re[f_i^*(t_j) g_i(t_j)] \right|,$$

with tie-breaking by minimal index.

(A4) Probabilities arise as limiting frequencies over events:

$$P(x_i) = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} \delta_{i,i(j)}.$$

The limit is assumed to exist under ergodicity or uniform phase sampling.

3 Illustrative Two-State Example

To demonstrate the consistency and implications of the formalism, consider a two-level system (e.g., a spin-$\frac{1}{2}$ system) with eigenstates φ_1 and φ_2, associated with discrete outcomes x_1 and x_2, respectively. Let us define

$$f(t) = (e^{i\theta} + e^{-i\theta})\varphi_1 + \varepsilon(e^{i2\theta} + e^{-i2\theta})\varphi_2, \tag{10}$$

$$g(t) = (i\,e^{-i\theta} - i\,e^{+i\theta})\varphi_1 + i\varepsilon[(e^{i\theta} + e^{-i\theta}) + (e^{i\theta} - e^{-i\theta})]\varphi_2, \tag{11}$$

where $\theta = \omega t$ and $0 < \varepsilon \ll 1$.

Then, the inner product is

$$I(t) = 2\sin(2\theta) + 2i\varepsilon^2(\cos\theta + \cos 3\theta) - 2\varepsilon^2(\sin 3\theta - \sin\theta). \tag{12}$$

Events occur when $I(t)$ is real, i.e., when $\Im I(t) = 0$. This condition reads

$$\cos\theta + \cos 3\theta = 0 \iff \cos\theta(4\cos^2\theta - 2) = 0. \tag{13}$$

Define the real parts of contributions for convenience:

$$\Re I_1 := \text{real contribution from } \varphi_1,$$
$$\Re I_2 := \text{real contribution from } \varphi_2.$$

Family A (Outcome x_2). If $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$, then $\cos\theta = 0$, so $\Im I = 0$. At these points $\sin(2\theta) = 0$, making the φ_1 channel vanish. The φ_2 channel contributes a small but nonzero real part, $\Re I_2 = \mp 4\varepsilon^2$, which therefore determines the outcome. Hence these events yield x_2. They repeat with period π, giving two x_2 events per full cycle $0 \to 2\pi$.

Family B (Outcome x_1). If $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, then $\cos\theta = \pm 1/\sqrt{2}$, so again $\Im I = 0$. At these points $|\sin(2\theta)| = 1$, making $\Re I_1 = \pm 2$, while the φ_2 contribution is negligible. Thus the event is dominated by φ_1, and the outcome is x_1. These occur with period $\pi/2$, giving four x_1 events per full cycle.

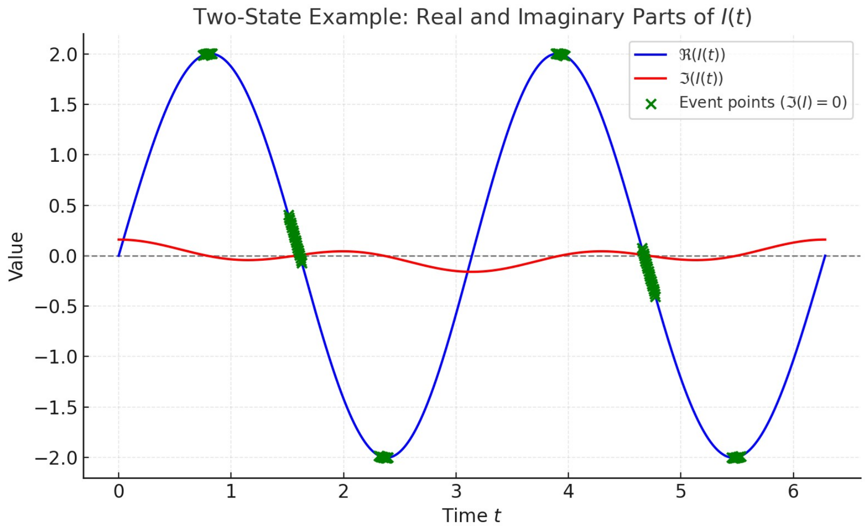

Figure 1: Real and imaginary parts of the inner product $I(t)$ for the two-state system. Green markers indicate event points where the imaginary part vanishes ($\Im(I) = 0$), corresponding to measurement-like occurrences. This illustrates the discrete event structure predicted by the formalism.

θ	$\cos\theta + \cos 3\theta$	$\Im I$	$\sin(2\theta)$	$\Re I_1$	$\sin 3\theta - \sin\theta$	$\Re I_2$	Dominant	Outcome
$\frac{\pi}{2}$	0	0	0	0	-2	$+4\varepsilon^2$	φ_2	x_2
$\frac{3\pi}{2}$	0	0	0	0	$+2$	$-4\varepsilon^2$	φ_2	x_2
$\frac{\pi}{4}$	0	0	1	2	0	0	φ_1	x_1
$\frac{3\pi}{4}$	0	0	-1	-2	0	0	φ_1	x_1
$\frac{5\pi}{4}$	0	0	1	2	0	0	φ_1	x_1
$\frac{7\pi}{4}$	0	0	-1	-2	0	0	φ_1	x_1

Table 1: Real and imaginary parts of $I(t)$ at the six event angles. Two events select x_2, four select x_1, giving the 2:1 split.

The probability of outcome x_i is then

$$P(x_k) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \delta_{i(j),i},$$

which for this construction gives $P(x_1) = 4/6 = 2/3$ and $P(x_2) = 2/6 = 1/3$. These values are consistent with what would be expected from the standard Born-rule weights for a state proportional to $\sqrt{2/3}, \phi_1 + \sqrt{1/3}, \phi_2$. [3]

[3]If the phases oscillate faster, the real-valued events happen more often. This makes the time gaps with no events so small that they become practically unnoticeable, even though outside those instants there are no events and therefore no outcomes. The outcome ratios, however, remain the same because they depend on the relative contributions, not on the speed.

4 Discussion

The proposed event-based formalism views Born-type weighting as a possible emergent feature of temporally localized events where the inner product of two evolving wavefunctions becomes real. Rather than postulating collapse to enforce discreteness, this approach suggests that measurement-like events could arise from intrinsic relational dynamics between two wavefunctions. The framework is explicitly conceptual and intended to illustrate plausibility rather than to provide a final formulation.

A key strength of this approach lies in its frequency-based foundation: probabilities are not treated as primitive axioms but as limiting frequencies of real-valued events, in line with von Mises' interpretation of probability [9]. At the same time, the construction appears consistent with standard quantum predictions, as illustrated in the two-state example. This perspective suggests that measurement outcomes are not pre-assigned eigenvalues but dynamically permitted results conditioned by a phase-dependent reality criterion. Whether an outcome can occur depends on whether its associated interaction term achieves real alignment during the evolution. Thus, phase relations between wavefunctions acquire a physical role in determining the possibility of discrete outcomes.

The following subsections present illustrative extensions and conceptual implications of the formalism. First, we examine a simplified interference scenario showing how alternating constructive and destructive events emerge from the real-valued condition. Next, we introduce an interaction-based selection rule that offers a physical interpretation of quantization, reframing eigenvalues as emergent rather than assumed. Finally, we outline a schematic extension to multidimensional and entangled systems.

4.1 Illustrative interference scenario

To further illustrate the mechanism, we consider a simplified interference model where alternating constructive and destructive events emerge from the real-valued condition. While this example is heuristic, it demonstrates how phase relations can determine event timing and outcome frequencies in a manner that mirrors Born weights. Let's consider a simple two-component ansatz:

$$f(r,t) = a(r) + b(r)e^{-i2\Omega t}, \qquad g(r,t) = a(r) + b(r)e^{+i2\Omega t},$$

where $a(r), b(r) \in \mathbb{C}$ and Ω is a characteristic angular frequency. Taking the Hermitian conjugate of f gives $f^*(r,t) = a^*(r) + b^*(r)e^{+i2\Omega t}$, so the inner product becomes

$$f^*(r,t)\, g(r,t) = |a|^2 + (a^*b + ab^*)e^{+i2\Omega t} + |b|^2 e^{+i4\Omega t}. \tag{14}$$

Define

$$R := a^*b + ab^* = 2\Re(a^*b) \in \mathbb{R}.$$

The imaginary and real parts separate as

$$\Im\big(f^*g\big) = R\sin(2\Omega t) + |b|^2 \sin(4\Omega t), \tag{15}$$

$$\Re\big(f^*g\big) = |a|^2 + R\cos(2\Omega t) + |b|^2 \cos(4\Omega t). \tag{16}$$

Writing $a^*b = |a||b|e^{i\varphi}$ gives $R = 2|a||b|\cos\varphi$. The event condition $\Im(f^*g) = 0$ thus becomes

$$|b|^2 \sin(4\Omega t) + 2|a||b|\cos\varphi\, \sin(2\Omega t) = 0. \tag{17}$$

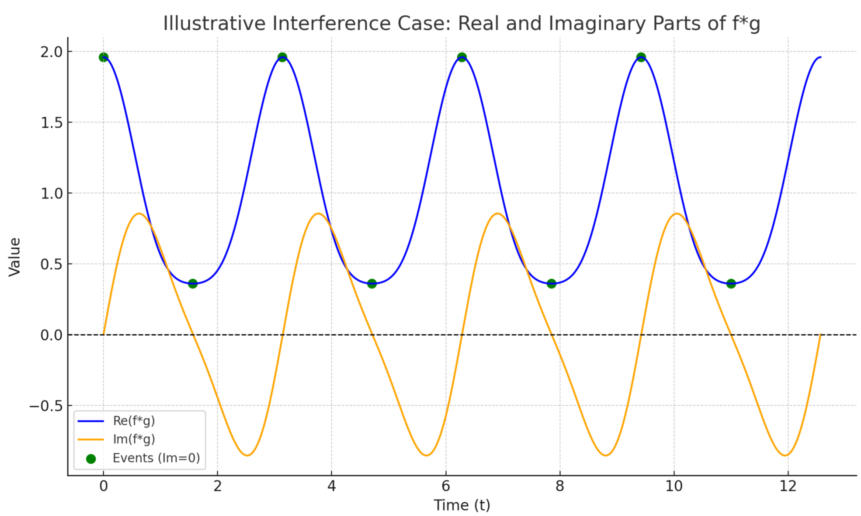

Figure 2: Illustration of the interference scenario. The plot shows the real and imaginary parts of the inner product $f^*(r,t)g(r,t)$ for the two-component model. Event points (green markers) occur when $\Im(f^*g) = 0$, corresponding to discrete measurement-like occurrences. Successive events alternate between constructive and destructive branches, with outcome frequencies governed by the relative magnitudes of $|a|$ and $|b|$, reproducing Born-type weighting.

Approximation: Alternating-event picture. If the 4Ω-harmonic term is negligible (e.g., $|b|$ small or under a rotating-wave approximation) and $\varphi \in \{0, \pi\}$, then (17) simplifies to

$$\sin(2\Omega t) = 0 \implies 2\Omega t = n\pi, \quad n \in \mathbb{Z}.$$

At these instants $\cos(4\Omega t) = 1$, and the real part evaluates to

$$\Re(f^* g) = |a|^2 + |b|^2 \pm 2|a||b| = (|a| \pm |b|)^2.$$

Thus, successive events alternate between constructive and destructive branches. The dominance rule then selects outcomes with relative frequencies governed by $|a|$ and $|b|$, reproducing Born-type weighting in this illustrative scenario. In the general case, event times are determined by the full equation (17).

4.2 Interaction-based selection and quantization

The formalism invites an interpretation of quantization as a phase-alignment effect: eigenvalues become observable only when the corresponding interaction term satisfies the real-alignment condition. This interpretation is suggestive rather than definitive, serving as a conceptual heuristic for how discrete spectra could emerge dynamically. In the standard formulation, measurable quantities are introduced as eigenvalues of self-adjoint operators. Here, by contrast, an eigenvalue x_i becomes physically realizable only if the associated interaction term

$$I_i(f, g) = f_i^* g_i$$

satisfies the reality condition

$$\Im(I_i) = 0,$$

where f_i, g_i are the expansion coefficients of f and g in the eigenbasis. If this condition is met at one or more instants, x_i can occur as an outcome. If not, it remains inaccessible despite belonging to the formal spectrum.

This principle reframes quantization as a dynamic selection rule based on phase alignment between two evolving wavefunctions. Observable values are not intrinsic properties of operators but relational features determined by temporal conditions on interaction terms. For example, the two outcomes of a spin-$\frac{1}{2}$ measurement correspond to subspaces in which the interaction term can achieve real alignment. Similarly, discrete energy levels may be viewed as configurations allowing such alignment under unitary evolution.

This interpretation elevates phase to a determining role: whether an eigenvalue is observable depends on an evolving relational property between two wavefunctions, rather than a static operator structure. Consequently, the framework offers an objective criterion for outcome selection and provides a fresh angle on the measurement problem.

4.3 Toward multidimensional and entangled systems

The Dirac-delta event structure extends naturally to systems with multiple degrees of freedom. Let $\{t_j\}$ denote event times and define the discrete process

$$O(t) = \sum_j x_i(j)\,\delta(t - t_j),$$

where $x_i(j)$ is an n-dimensional eigenvalue vector. The limiting frequency of outcomes defines the joint distribution

$$P(x_i) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \delta_{i,i(j)}, \tag{18}$$

where $\delta_{i,i(j)}$ is a multi-index Kronecker delta comparing entire outcome vectors. The multi-dimensional dominance rule selects the joint eigenvector whose contribution $\Re[f_i^*(t_j)g_i(t_j)]$ dominates among all joint terms. This construction may recover the standard joint probability structure in composite systems and accommodates entanglement when f and g are multi-particle wavefunctions. While the present discussion is only conceptual, it outlines how the Kronecker-delta event structure can naturally generalize to composite systems. A rigorous treatment of entanglement, correlations, and open-system dynamics remains an important direction for future work.

5 Conclusion and Outlook

This work proposes a novel perspective on the measurement problem, suggesting that quantum probabilities may emerge from the interaction of two evolving wavefunctions rather than from primitive postulates. The event-based mechanism sketched here hints at how Born-type weighting could arise from real-valued inner product occurrences, linking discrete outcomes to phase-dependent features of the dynamics.

The framework remains exploratory: no explicit dynamical laws have been formulated, the Born rule is only illustrated in simple cases, and the physical interpretation of the dual wavefunctions is unsettled. Extensions to relativistic or many-body settings and possible experimental implications are also open questions. Future work may aim to develop the underlying dynamics, clarify interpretation, and explore whether the approach yields testable distinctions from standard quantum mechanics.

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