Everettian chance in no uncertain terms

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Abstract

The current landscape of views on chance in the Everett interpretation is rocky. Everettians (Wallace 2012, Sebens and Carroll 2018, McQueen and Vaidman 2019) agree that chance should be derived using principles governing uncertain or partial belief, but they cannot agree on how. Critics (Baker 2007, Dawid and Thébault 2015, Mandolesi 2019) maintain that any such approach is circular. We smooth the landscape by shifting focus from what Everettians take to be uncertain to what they should think is certain: namely, the conditions under which branches are isolated. Our approach to isolation resolves the main tensions among the different Everettian chance derivations while clarifying how they avoid circularity.

Contents

1 Introduction

The current landscape of views on chance in Everettian quantum mechanics is a curious one. On the one side, longstanding critics of many-worlds theories maintain that probability is needed to make sense of the machinery that Everettians use to derive chance values, resulting in circularity (Baker 2007, Dawid and Thébault 2015, Mandolesi 2019). On the other side, Everettians seem to agree that chances should be derived in terms of agents' uncertain or partial beliefs—but they cannot agree on how.

Perhaps the most famous of these uncertainty-based approaches is the decision-theoretic program explored by Deutsch (1999) and Wallace (2012): they purport to prove that a rational Everettian agent must order their preferences over acts in a way that recovers the Born rule. Sebens and Carroll (2018) take issue with a principle of rationality in Wallace's approach, and instead aim to derive Everettian probabilities from principles governing self-locating uncertainty. McQueen and Vaidman (2019) offer yet another self-locating uncertainty approach, taking issue with Sebens and Carroll's metaphysical view of branching. Notably, all three approaches claim that symmetries of quantum states are central to their arguments, but none attempt to characterize the symmetries at play.

We suggest a shift in focus from what these Everettians take to be uncertain to what they take to be certain. In fact, we diagnose all these Everettians as holding a tacit commitment to a functional link between chance and certainty—namely, that chances supervene on the totality of physical information relevant to (actual) relative frequencies. In the context of Everettian derivations of chance, this link is satisfied by stipulating an appropriate notion of when a subsystem is 'isolated'. We sketch a thin functional analysis of isolation that allows Everettians to identify branches as isolated subsystems without using probabilistic concepts. This analysis, in turn, affords a characterization of the symmetries of branches relevant to chance.

Then, we argue that chance's functional link with certainty does most of the explanatory work in deriving *specific* chance values: to wit, it does so by characterizing the precise sense in which symmetric, isolated states should yield equivalent chance assignments. This characterization yields a thin core approach to deriving chance in Everettian quantum mechanics that smooths the current rocky landscape. Each of the decision-theoretic and self-locating uncertainty programs discussed above can adopt this core approach while (a) changing very little of the structure of their arguments and (b) addressing the main criticisms they have levied at each other. Moreover, since certainty of a branch's 'isolation' is explicitly non-probabilistic, the core approach clarifies the sense in which these derivations avoid the usual charges of circularity.

Our plan for the article is as follows. In §2, we consider the various functional roles for objective chance which have been identified in the literature hitherto, and add to this mix the certainty link, which will play a central role in this article going forward. In §3, we show how the certainty link and a notion of system's being isolated from its environment can be leveraged in order to afford a resolution to the reference class problem. In §4, we offer a thin functional analysis of isolation: namely, that (a) 'isolated subsystems' should instantiate a

recursive structure, such as Wallace's (2022a) 'theory sectors', and (b) one should be able to create 'isolated subsystems' in a laboratory. Then, in §5 we use these functional criteria to argue that Everettians should adopt two separate notions of isolation: one for microstates and one for macrostates. We also clarify how our notion of isolation for macrostates does not depend on solving the preferred basis problem, and so in turn does not depend on a prior assignment of chances. Having done this, we demonstrate in §6 how taking chance to be an intrinsic property of isolated macrostates specifies a core approach to deriving Everettian chance. We then outline how each of the above-mentioned Born rule derivations deviates from the core, as well as how the core resolves the primary tensions among them. In §7 we wrap up.

2 Most attitudes towards 'chance' tacitly invoke certainty

Much cross-talk in philosophical literature stems from the difference between two key philosophical methodologies for confronting troubling terms: namely, *conceptual analysis* and *conceptual engineering* (Cappelen 2018). The former aims to give a precise definition of the term under consideration that yields maximal fidelity with all its uses in everyday talk. The latter aims to create a novel definition for the term that better suits a set of desired uses. Whereas the former is primarily descriptive, the latter is primarily normative.[1]

Conceptual analysis and conceptual engineering, so construed, are meant to identify endpoints on a spectrum of methodologies. For example, Quine's version of conceptual analysis (which we favor) falls somewhere in between. Quine, drawing on Carnap (1947, §2), rejects the sort of analysis that aims to 'expose hidden meanings'; instead, he thinks analysis should 'fix on the particular functions of the unclear expression that make it worth troubling about' (2013, p. 238). One might summarize the pursuit of a Quineian analysis of 'chance' as an attempt to answer the following question: what would be a happy thing for 'chance' to mean, given how we roughly use it today?

For our attempt at answering this question, we follow Quine in fixing on the *functions* that 'chance' plays or might play in daily discourse. This approach already puts us in the good company of philosophers of probability such as Hájek (2019), and Everettians such as Wallace (2012) and Saunders (2010), all of whom aim to give 'chance' a functional definition. Given our present focus on chance in Everettian contexts, we will begin our search for a functional definition with the functional links that Everettians have deemed most important. All the chance functionalists we survey assent that the following links are *descriptive* of our use of the term 'chance'. It varies whether any given chance functionalist treats any given link *normatively*, as a principle of desired use.

To begin, then: Saunders (2010), following Papineau (1996), helpfully identifies the following two functional roles which one might demand be satisfied by a notion of objective chance:

(C1) **The inferential link**: The chance of an event is measured (roughly) by (actual) relative frequencies of that event; and

[1]For a discussion of this distinction in a very different context in the philosophy of physics, see Krátký and Read (2024).

(C2) **The credential link**: All else being equal, one's subjective degree of belief or credence in an event ought to equal the chance of that event.

Brown (2011, p. 7), Wallace (2012, §4.4), and Read (2018, p. 137) all endorse that these links (or slight variations of them) are at least descriptive of chance talk.

In addition to the above two functional roles for chance, Saunders (2010, p. 181) goes on to identify another:

(C3) **The uncertainty link**: Chance events, prior to their occurrence, are uncertain.

Clearly this link is descriptively adequate, but should we add it to a list of normative requirements? Given that many worlds theories specify that all possible measurement outcomes are actual, one might be wary of the normative upshot of uncertainty. Greaves (2007), for example, aims to build an account of Everettian chance with the first two links alone. Still, Saunders and Wallace have taken pains to show how the Everettian *can* recover this link in a normative sense, if they so desire.

One immediate question to ask of the uncertainty link—whether one treats it as descriptive or normative—is this: should it have anything in particular to do with the *measure* of chance? To be sure, we might like to recover the lived experience of observers—but it seems a stretch to think that uncertainty about outcomes *per se* would be enough to inform us about the right choice of chance function.

Now, one way in which one might leverage the uncertainty link in order to get a bead on a specific chance measure is via the 'principle of indifference', which states that in the absence of any relevant evidence, agents should distribute their degrees of belief equally among all the possible outcomes under consideration—see Eva (2019) and references therein. However, even availing oneself of the principle of indifference is unlikely to fix a unique such chance measure, absent further constraints. This might be illustrated by familiar problems with applying the principle of indifference to an infinite state space. Consider the classic example of the hypothetical box factory due to van Fraassen (1989, p. 303), in which the principle of indifference is unable to discriminate between (e.g.) length- and area-preserving probability measures.[2] To paraphrase, imagine that you are told that the factory produces cubic boxes with sides of any length from 4 to 5 centimetres and given no further information. What are your best estimates for the face area and the side length of a given box? If you apply the principle of indifference to side length, you get the uniform measure over the interval $[4, 5]$ (in units of cm); if you apply the principle to face area, you get the uniform measure over $[16, 25]$ (in units of cm^2). The problem is that these choices yield estimates of 4.5cm for the box's side length and 20.5cm^2 for the area of its faces, and *no* cubic box can have both of these measurements. To get consistent answers, only one of the two probability measures can be uniform—and the principle of indifference seems to offer little guidance as to which one it should be.

Ideally, one would seek more information about the box factory to resolve the paradox. Suppose, for example, that one learns that the factory starts with an imprecise rod-cutting

[2]Van Fraassen credits Bertrand (1889) with the insight driving his example. Keynes (1978, pp. 48–49) gives a very similar version of the thought experiment (using volume and density rather than length and area), which he credits to von Kries (1886). Note that Adlam (2025) has also recently invoked Bertrand's paradox to call into question subjective uncertainty based approaches to fixing an objective chance measure.

machine, then nearly perfectly creates eleven copies of that initial rod to piece together a cubic frame. This additional information suggests that the length of that initial rod fixes all relevant parameters of the finished box—and so, at this stage one ought to pick uniform distribution over length to obtain estimates for face area. Along this line of reasoning, if one indeed strives for the recovery of a *specific* probability measure, then the following additional functional role might serve one better in making good on that:

(C4) **The certainty link:** Chance values supervene on the totality of physical information that is relevant to the (actual) relative frequencies of the event.

Following the certainty link in our reasoning about the box factory would lead us to recognize that in this case the length of the box's side is the physical parameter that is relevant to the chances.

In at least this case, the certainty link appears to be *necessary* for deriving a specific measure of chance, since self-locating uncertainty does not suffice on its own. But in fact there is nothing to stop us having both (C3) and (C4)! We can recover lived experience (via (C3), as articulated above) *and* obtain a unique chance measure (via (C4)); it's just that the explanation of the former has little to do with the explanation of the latter.

The box factory example provides some initial evidence that the certainty link latches onto something important in our usual 'chance' talk. In the next section, we adduce more evidence for this by arguing that both objectivists and subjectivists about chance appeal to the certainty link in order to solve versions of the reference class problem.

3 Certainty of a system's isolation solves the reference class problem

We claim that the link with certainty—that chance values supervene on the totality of information relevant to the actual frequencies—adequately describes most of our usual 'chance' talk. To defend this claim, we now show how it appears in several prominent objective and subjective analyses of chance as a way to solve versions of the reference class problem.

First, for objective analyses, consider various versions of frequentism. One can view frequentism as an approach to chance that begins by 'filling in' the inferential link with an analytic definition, one that identifies a chance value with with a (possibly limiting) relative frequency of a given event in an actual or hypothetical *reference class* of events of a certain type. For example, a finite frequentist might fill in the inferential link (C1) with the following definition:

FF: The chance of an attribute X in a finite reference class R is the relative frequency of actual occurrences of X within R:

$$ch(X \mid R) = \frac{n(X)}{n(R)}, \tag{1}$$

where $n(\cdot)$ gives the number of occurrences.[3]

This definition cannot recover chances that align exactly with standard quantum probabilities; we include the details here to illustrate one version of the reference class problem.[4] To wit: selecting a reference class becomes a problem if it is unclear what types of events are most relevant—and so it is unclear how to specify R. Suppose, for example, one wants to assess the chance that their grandmother Sarah, a serial smoker, will contract lung cancer. One can do so by comparing the frequency of lung cancer cases in the total population to the frequency of cases among only folks like Sarah. However, it is not immediately clear which factors making someone 'like Sarah' ought to be included in one's assessment.[5]

This issue recurs in hypothetical frequentism, where the reference class R is infinite. It is still often ambiguous why one infinite sequence contains the relevant events rather than another. A striking physical example comes from Diaconis et al.'s (2007) analysis of fair coin tosses. They argue that the usual way of tossing a coin is ever-so-slightly biased, such that there is a roughly .51 chance that the coin will land on the same side it started on before the flip. So, if you instruct your bookie to always place the coin heads up on their thumb before the flip, the resulting frequencies of heads outcomes will be ever-so-slightly altered. The hypothetical frequentist might have thought they could ignore this detail and use any infinite sequence of coin tosses R yielding a limiting frequency of one-half, but not so. Subtle details about the measurement process that produces a reference class can end up being relevant to the actual relative frequencies of an event.

Statistical physical theories offer a solution of sorts by providing one way to fill in the link with certainty: they formally specify the totality of relevant physical information. In so doing, they specify the right reference class. We might characterize this solution by saying that these theories fill in the certainty link (C4) with the following rule, which we call ontic separability, or **OS**:

OS: Suppose that a statistical physical theory T specifies that a system U contains within it an isolated subsystem S. The chances that theory assigns to outcomes X in S should be independent of the environment $U \setminus S$:

$$ch\,(X \mid U) = ch\,(X \mid S)\,. \tag{2}$$

Here, the state of the isolated system S has taken over the role of the reference class R in **FF**. S's isolation from other systems—at least as far as the relevant physical parameters are concerned—provides a justification for its playing this role.

Moreover, the reference class problem is not exclusively a problem for frequentist analyses of probability. Hájek (2007) cogently argues that it is a problem for subjectivists, too (among

[3]This particular definition of finite frequentism is due to Hàjek (1997). Throughout this paper, for ease of exposition, we allow the arguments for the chance and credence functions to be attributes, propositions, or events. However, nothing in our argument should hinge on this permissive approach. For example, the reader should feel free to rewrite a definition using attributes in terms of propositions if they prefer.

[4]This is not to say that finite frequentism cannot recover quantum probabilities approximately. Saunders (2021) presents a discrete Everettian approach that does just this. In this paper, however, we restrict our focus to approaches that aim to recover real-valued Born rule probabilities exactly.

[5]Venn (1876) presents an early version of this example.

others). To roughly paraphrase Hájek's point, subjectivist accounts of chance will generally include a judgment about what background information is admissible in assigning beliefs about chance—and it is often ambiguous what should count as 'admissible'.

Here is one way in which to illustrate the problem. In parallel with our gloss of frequentism, we view subjectivism as starting an analysis of chance by filling in the credential link (C2) with a normative principle, one reifying chances as the arbiters of certain credence assignments. Lewis (1980), for example, takes chance functions to theoretically specify chance values, and then requires that credence defer to chance following his 'principal principle', **PP**. Pettigrew (2012) characterizes this principle as follows:

PP: Let C_{ch} state that the correct chance function is ch. An agent ought to have a credence function c such that, for all possible initial chance functions ch and all propositions A and E, if E is admissible for A,

$$c(A \mid C_{ch} \wedge E) = ch(A), \tag{3}$$

providing $c(C_{ch} \wedge E) > 0$.

Lewis himself provides no formal criteria for what it means for E to be 'admissible' for A. However, it is clear that information about the future is disallowed, while nearly any information about the past is allowed (1980, pp. 272–6).

Lewis's approach to admissibility has the interesting consequence that past events can only have a chance value of zero or one, even if one is ignorant of the outcome. Although Lewis accepts this consequence (1980, p. 273), Bacciagaluppi (2020) calls it into question. To summarize Bacciagaluppi's point, suppose that a bookie tells you that she will flip a fair coin in the back room. She goes through the door, flips the coin, returns, and asks you how confident you are in the claim A that it landed heads rather than tails. Then she repeats this process, except that before she goes into the back room, she asks you how confident you are that A' the coin will land heads when she flips it. Intuitively, if you think that there is one correct credence value to report for A' (e.g., one-half), then you should think that the same value is the correct one to report for A. In other words, whether one asks about an unknown outcome of a given chance setup before or after it occurs does not matter to the integrity of that chance setup; the chance value is well defined in either case.[6]

One can accommodate the intuition that past events have well-defined, nontrivial chance values with a revision of Lewis's principle that Pettigrew (2012) attributes to Isaac Levi:

LPP: Let C_{ch} state that the correct chance function is ch. An agent ought to have a credence function c such that, for all possible chance functions ch and all propositions A and E, if E and A are stochastically independent according to ch (that is, $ch(A \mid E) = ch(A)$),

$$c(A \mid C_{ch} \wedge E) = ch(A), \tag{4}$$

providing $c(C_{ch} \wedge E) > 0$.

[6]Bacciagaluppi uses the term 'epistemic objective probabilities' to refer to well-defined, non-trival chance values for past events; see (2020, §3.5) for more details.

Following Levi's approach, it does not matter that the outcome of the toss A is information about the past, as it is clearly not stochastically independent of A. Levi thereby allows the subjectivist to report nontrivial chances for past outcomes, contradicting Lewis.

In short, insofar as subjectivists can disagree on what information E is admissible, they encounter a version of the frequentist's reference class problem. To resolve their version of the problem, they can use the link with certainty to clarify what information is 'admissible' or relevant. Once again, statistical physical theories sharpen the solution: such theories stipulate, by fiat, the system states and measurement procedures that fully characterize the admissible information. We might formalize this solution by filling out the certainty link (C4) with an *epistemic* separability principle, **ES**:

ES: Suppose that a statistical physical theory T specifies that a system U contains within it an isolated subsystem S. The credences that an agent assigns to outcomes X in S should be independent of the environment $U \setminus S$:

$$c\left(X \mid U\right) = c\left(X \mid S\right). \tag{5}$$

Note that **ES** and **LPP** together imply **OS**: in order to avoid conflict between the recommendations of **ES** (via (5)) and **LPP** (via (4)), it must be the case that any 'possible' chance function mentioned by the latter principle satisfies **OS** (via (2)). One can similarly use **OS** and **LPP** to derive **ES**. The isolation of subsystems, and thereby the certainty link, can be given primarily objective or subjective import depending on one's philosophical preference—and one can easily translate between the two.

Indeed, either **OS** or **ES** captures our solution to the box factory paradox. In that case, our physical theory T is simple. We let the total system U be given by a specification of all properties of the box—its side length, face area, and volume—at all times during its construction, if defined. Then, we let S be a specification of these properties before the faces are constructed and after the box has been fully assembled. Finally, we note that S is an isolated subsystem of U. The details of what happens in the time interval covered by $U \setminus S$ do not matter, since the final dimensions of the box are fixed once the rods are cut. Since no additional information is available, we appeal to the principle of indifference to motivate using the uniform distribution over lengths rather than a different distribution.

In this way, a wide variety of approaches to chance use isolated subsystems to fill in the certainty link (C4) and thereby resolve the reference class problem. However, this move raises the question of how we determine when a subsystem is isolated in general. In the box factory example, we simply stipulate that the right sort of isolation is achieved. In the next section, we will aim to do better.

4 But 'isolation' is ambiguous

In the previous section, we argued that both subjectivists and objectivists fill in the certainty link (C4) with an appeal to isolated subsystems in order to solve their versions of the reference class problem. In particular, we considered filling in the certainly link with appeals to the isolation of systems in a statistical physical theory. The notion of an 'isolated system' in physics, however, is a tricky one to pin down. Our strategy in this section is do for 'isolated

system' what we did for 'chance' in Section 2: namely, to give it a thin, functional analysis based on long-standing desiderata among physicists.

We start with Einstein's (1948) classic concern about how the non-locality of quantum mechanics might challenge the possibility of empirical science:

> The following idea characterises the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B; this is known as the 'principle of contiguity', which is used consistently only in the field theory. If this axiom were to be completely abolished, the idea of the existence of (quasi-)enclosed systems, and thereby the postulation of laws which can be checked empirically in the accepted sense, would become impossible. (Translation by Max Born, Einstein et al. 1971, p. 171)

Here, Einstein argues that something like a principle of local action is at least extremely useful for making sense of the idea that some systems can be effectively closed off from external influence. However, as Howard (1985) notes, Einstein is less strongly committed to local action than to separability: it is 'the existence of (quasi)-enclosed systems', however justified, that achieves the possibility of empirical physics. Prospects seem dim for non-relativistic quantum mechanics to justify this separability principle on the back of local action; it is not a field theory, for one. Still, past physicists seem to have had a coherent notion of empirically testing non-local theories like Newtonian gravity and Coulombic electrostatics; it would be strange if one could not construct an analysis of '(quasi)-enclosed systems' that did not depend on locality. Indeed, for Einstein, it only seems crucial that the physical laws—local or otherwise—can be studied in some controllable setting.

Wallace's (2022a, 2022b) notion of *subsystem recursivity* provides a useful tool for sharpening how isolation relates to testability. Wallace does not aim to analyze 'isolation' *per se*; on the contrary, he is forward about treating it as a term of art in his analysis. Nonetheless, his discussion of a theory's recursive 'sectors' is quite helpful for filling out the structural features that a theory needs for it to be 'checked empirically in the accepted sense'. Very roughly, a *sector* of theory identifies a system whose kinematics and dynamics can be fully specified without reference to any other system. As long as one is justified in claiming that they have captured a sector of theory with a system in their laboratory, one can make inferences about that theory's applicability writ large. The alignment of theory sectors and isolated subsystems, then, would seem to get very close to the thin notion of empirical testability that Einstein is after.

Wallace calls a theory *subsystem recursive* just in case it possesses this alignment. He fleshes out the idea of subsystem recursivity by illustrating how it arises in Coloumbic electrostatics:

> [Suppose] we now consider a dynamical model in an N-particle sector [of electrostatics], in which there is some sub-region R and some subset of $M < N$ particles which remain (over some time period) well inside R, while the other $N - M$ particles remain outside R; and suppose that as we approach the [boundary] of R, [the electric potential] tends to some spatially-constant value [...]. This could happen, for instance, because the other $N \leq M$ particles are very far from R, or because positive and negative charges outside R approximately balance so as

> to exert little net Couloumb force within R. Then to a good approximation, we can study the dynamics of the subsystem of those M particles autonomously from the other $N \leq M$: they will behave as if those particles were not present and as if the M particles were interacting among themselves according to the appropriate M-particle sector dynamics. If we want to study the dynamics of that subsystem of particles, in other words, under these assumptions and if we are sufficiently tolerant of small errors, we can replace the full N-particle sector with the M-particle sector. In this sense, the latter can be interpreted as modelling an isolated subsystem of the former.
>
> [...] In fact, Coulombic electrostatics has these two features:
>
> (i) Any sector of the theory can be interpreted as an idealized description of an isolated subsystem of a sector of the theory;
>
> (ii) An isolated subsystem of any sector of the theory can be described, in idealization, as a sector of the theory.
>
> I will call a theory with these two features *subsystem-recursive*: these are theories where any model can be interpreted in the first instance as modelling a dynamically isolated subsystem under certain idealizations about its environment and where, if we want to remove those idealizations, we can embed the model in a model of a larger system within the same theory—and where that larger system in turn is interpretable in the first instance as a subsystem of a still-larger system, with no assumption that we need to understand the whole setup in terms of an *ur*-system that describes the whole universe. (2022a, p. 242)

While Wallace treats 'isolation' as a term of art, he gives a formal definition of 'sector' by defining structure-preserving restriction maps that take a larger theory sector to a smaller one. For our purposes, we extract from this account three qualitative criteria that a restriction to a subsystem must satisfy for that subsystem to count as a sector: namely, the restriction must preserve the theory's rules for *kinematics*, *dynamics*, and *symmetries*. To flesh out the kinematic part for electrostatics, we might describe the trajectories of N particles at a given time slice with vectors in the N-wise Cartesian product of $\mathbb{R}^3$; in this case, we are always free to project down onto M of these trajectories to describe those particles and those alone. We ensure that these trajectories obey the Coulomb force law by stipulating that the potential does not vary across the boundary of the spatial region containing these M particles, as Wallace describes above. Time reversal, spatial translations, and rotations applied to the state space of just the M particles preserve solutions to the Coulomb dynamics just as well as they do when applied to the larger system. All such solutions will adequately describe the M particles on their own, yielding a precise sense in which they behave as though the others 'were not present'. Thus, the M-particle subsystem satisfies the three qualitative criteria and is therefore a sector of electrostatics. Moreover, in this case, we have good grounds to identify such sectors with *isolated* subsystems. The boundary's constant electric potential means, in particular, that the electric field vanishes there, and we can measure when and where there is a non-zero electric field. So, Coulombic electrostatics thereby satisfies features (i) and (ii), coming out as a subsystem recursive theory.

In contrast with Coulombic electrostatics, Wallace cites Newtonian gravity as an example of a theory where the alignment between sectors and isolated subsystems breaks down. In this theory, the dynamics of point-source masses indexed by J is given by

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}x^i_J = \nabla_i \Phi(x^i_J, t), \tag{6}$$

where Φ is a scalar potential field that satisfies the Poisson equation,

$$\nabla^2 \Phi(x, t) = -4\pi G \sum_J \delta(x - x_J) m_J. \tag{7}$$

We restrict our attention to Φ that are linear at the system boundary. Idealizing the boundary to be at $|x| = \infty$, we can write this condition as

$$\lim_{|x|\to\infty} \Phi(x, t) = \Phi_0(t) + a_i(t)x^i(t), \tag{8}$$

where the acceleration term $a_i(t)$ refers to the components of a time-dependent vector. In the case where $a_i(t) = 0$ for all t, the boundary condition is constant, and this theory recovers the usual inverse-squared force law of Newtonian gravity. We can then define sectors of the theory in much the same way that we did for Coulombic electrostatics. Just as the electric field disappears at the boundary of a Coulombic sector, the gravitational field vanishes at the boundary of a Newtonian sector. However, we now have a good reason to think that the resulting sectors are too strict to characterize a desirable notion of isolation: no such sector can describe a laboratory on Earth's surface. We need a non-zero $a_i(t)$ at the laboratory boundary to account for the Earth's gravitational field.

To fix this issue, we can simply change the definition of 'Newtonian sector' to contain dynamics with the full range of boundary conditions in (8). However, Wallace notes that one might also change the definition of 'isolation' to identify isolated subsystems with Newtonian sectors. So long as we know the specific point mass distribution generating a potential with non-zero $a_i(t)$ at a given subsystem's boundary, we can always rewrite that subsystem's dynamics within an appropriate sector. Considering only laboratories on the surface of the Earth, this issue does not appear to pose an immediate threat. However, Einstein's elevator thought experiment challenges its general tenability. When one feels a greater push towards the elevator's floor, there is no means of discriminating from within the elevator whether the cause of this effect is due to the Earth acquiring more mass or the elevator's upward acceleration. This point challenges Newtonian sectors' 'upwards' recursion, towards larger and larger systems. Progressing up this ladder, whenever we reach a cosmological system with a nonvanishing gravitational field at the boundary, Newton's theory does not license us to stop at that rung—even though the particular completion of that system within a Newtonian sector might end up being arbitrary.

Contrast this situation with what happens when we revise electrostatics to include potentials with linear boundary conditions of the form (8). This revised theory of electrostatics has sectors with empirical differences: namely, some sectors will have vanishing electric fields at their boundary, and some will not. However, this move does not challenge the isolation of these sectors from other subsystems. The dynamics can still be studied in isolation. In

other words, generalizing the form of the dynamics allows us to specify a sense in which the empirical differences between the sectors do not matter for the physics.

From these case studies, we can draw a general moral. When sectors impose conditions on the dynamics that we can check empirically but which are hard to control from within the system, we have cause to generalize those dynamics—which, in turn, disrupts that theory's notion of isolation. The converse of this moral is that tenable notions of isolation can allow for empirical differences between isolated subsystems by specifying a precise sense in which those empirical differences do not affect the dynamics.

We capture this moral as the interplay of two functional links the term 'isolated subsystem' ought to satisfy: a link to a recursive structure on the one hand and a link to empirical control on the other.

(I1) **The recursion link.** Isolated subsystems instantiate a recursive structure (like that of a theory sector).

(I2) **The control link.** We can create an isolated subsystem in a laboratory.

The recursion link (I1) sharply defines various senses in which a given subsystem might be isolated, and the control link (I2) pushes one to pick a sense of isolation that is useful for empirical practice.

One might want to be agnostic about the strength of the control link in the same way that one might be agnostic about the strength of the inferential link about chance. Subjectivists about chance often stipulate that any sequence of outcomes is possible for an event with any chance value. Thus, the actual outcomes of any finite sequence of chance events might end up being wildly misleading about the true value of the chance. Usually, these subjectivists invoke the law of large numbers to argue that such cases are rare, thus recovering the inferential link (C1) approximately. Similarly, one might want the control link (I2) to apply only to theories that describe phenomena below a certain length scale, allowing for cosmological theories with isolated systems that are impossible to create in an Earth-bound laboratory. Given that non-relativistic quantum mechanics is typically used for small length scales, we will assume that (I2) holds for our case.

Fruitfully, the symmetries encoded in subsystem structures provide a way of strengthening the separability principles that we used to fill in the certainty link (C4) in the previous section. Roughly following Wallace (2022a, p. 245), we say that a property is *intrinsic* if it is the same in any two subsystem states related by a symmetry. What would it mean for a chance assignment to be an intrinsic property of a system? While there is flexibility in answering this question, we take one happy answer to be that when a symmetry maps one system state to another, these states must assign the same chance values to any subsystems related by that map.

One can view this intrinsicity condition as a consequence of either one of the separability principles. We use **OS** to illustrate. Suppose that a theory sector S contains two subsystems X_1 and X_2 corresponding to two outcomes. By subsystem recursivity, S is isolated from the rest of the universe U. Therefore, **OS** implies:

$$ch\,(X_i \mid U) = ch\,(X_i \mid S)\,. \tag{9}$$

Now, if a symmetry maps $S = \{X_1, X_2\}$ to $S' = \{X_1', X_2'\}$, then that map should make no difference to properties that depend only on S; i.e., the intrinsic ones. Thus, (9) implies:

$$ch\,(X_i \mid S) = ch\,(X_i' \mid S')\,. \tag{10}$$

Equation (10) is powerful enough to drive the derivation of specific chance values.

Our strategy for deriving Everettian chance, then, is to first pin down appropriate theory sectors for Everettian quantum mechanics, and then apply the intrinsicity condition (10). We turn now to this first task, for which we use Newtonian and Coulombic case studies as templates.

5 Probing Everettian 'isolation' with theory sectors

In the previous section, we identified two core functional roles played by talk of 'isolated subsystems': (I1) isolated subsystems should instantiate a recursive structure, like that of Wallace's (2022a) theory sectors, and (I2) agents should be able to create them in laboratories. 'Isolated subsystems' fulfill these roles in theories with nonlocal force laws, like Coulombic electrostatics and Newtonian gravity. That bodes well for the prospects of applying this functional analysis to 'isolated subsystems' in nonrelativistic quantum mechanics.

However, there is a fly in the ointment: a theory like Newtonian gravity employs a single dynamical rule, and orthodox non-relativistic quantum mechanics infamously has two. According to orthodoxy, unitary dynamics describes self-evolution, projective dynamics describes measurement, and never the twain shall meet. In contrast, a traditional selling point of Everettian approaches is that they treat all dynamics as unitary. Thus, at first blush, one might justifiably think that orthodoxy will frustrate attempts to define a theory sector, while Everettian approaches will accommodate sectors similar to those in Newton's theory.

In fact, we find that both of these impressions are false. Orthodoxy naturally accommodates two sorts of theory sector, one for each of its dynamical rules—and prospects for Everettians getting away with just one sort of sector seem dim. On the orthodox approach, one can helpfully use von Neumann's measurement scheme to relegate projective dynamics to macroscopic objects. Similarly, Everettians rely on projections to specify the sense in which their macrostates—branches—are dynamically isolated. Thus, whether or not one is an Everettian, we recommend adopting two separate sectors and, accordingly, two separate notions of 'isolation' for non-relativistic quantum mechanics: one for microstates and one for macrostates.

The notion of isolation for Everettian macrostates amounts to what Franklin (2024) calls a 'screening-off criterion': namely, the emergence of a particular macrostate from a microstate amounts to the approximate dynamical isolation of one branch from any other branch. Macrostates include indicators of the outcomes of measuring quantum systems, such as a sharp position state of a measuring device's pointer—a 'pointer state', in Zurek's (1981, 1982) popular terminology. In short, pointer states correspond to measurement outcomes, the chance events that we care about. Thus, we use branching dynamics to fill in the recursion link (I1) and specify the notion of isolation relevant to Everettian chance.

The control link (I2) guides the precise subsystem structure that branching dynamics should instantiate. In particular, these dynamics pose two challenges to our empirical

control of macroscopic subsystems. On the one hand, we have a repeatability problem: system eigenstates may fail to strongly couple to pointer states, resulting in non-repeatable measurements. On the other hand, we have a preferred basis problem: a non-probabilistic rendering of system-environment interactions may fail to select well-localized pointer states, resulting in macroscopic states of affairs that are highly nonclassical. The repeatability problem is analogous to the case of the Newtonian who learns of Einstein's elevator scenarios; it gives us reason to generalize our macrostate sectors.

What about the preferred basis problem? Several authors use it to charge Everettians who seek to derive chance values with circularity or incoherence (Baker 2007, Dawid and Thébault 2015, Mandolesi 2019). For example, responding specifically to Wallace's (2012) decision-theoretic derivation, Mandolesi writes the following:

> Without the Born rule, Wallace's solution of the preferred basis problem does not work as expected. Branches might be nothing like our world, lacking complex structures or behaving erratically. This compromises the whole decision-theoretic approach, which depends on narratives where agents exist, their actions have the expected consequences, and rationality is possible. (2019, p. 49)

In response to such challenges, Franklin (2024, p. 299) aims to confirm the emergence of classical worlds in Everettian quantum mechanics on the basis of phenomena that one can derive from decoherence without reference to probabilities. Our strategy is slightly different. We argue that the Everettians do not need probabilities to establish a robust sense in which their worlds are isolated, even if those worlds end up being non-classical.

Note that the Everettian who allows for the theoretical possibility of deviant branches is strongly analogous to the electrostatics theorist who adopts dynamics with linear boundary conditions. The latter cannot use their weakened notion of 'isolation' to specify whether the electric field vanishes at an isolated subsystem's boundary, even though they could easily check it empirically. Likewise, the Everettian cannot use our preferred notion of 'isolation' to guarantee that a branch is quasi-classical, even though they could easily observe it to be. Quite plausibly, there is a difference in modality between these two setups. Constant electric fields at system boundaries should be common, and non-classical macrostates should be either impossible or rare enough to ignore. However, this difference does not change the fact that both theorists fulfill our two thin functional criteria for isolated subsystems.

To build up to this point, we will start by illustrating how microstate and macrostate sectors arise in a conservative approach to quantum mechanics; namely, the Schrödinger picture augmented with von Neumann's account of measurement. Then, we describe how the repeatability problem motivates a generalization of both sectors that arises in quantum measurement theory. Next, we review how Everettians revise macrostate sectors with the branching dynamics of decoherence. Finally, we flesh out the argument sketched above, which suggests that branching dynamics afford Everettians a compelling notion of isolation despite the preferred basis problem.

5.1 Orthodoxy's sectors for microstates and macrostates

We start with a conservative approach—namely, the Schrödinger picture as presented in many textbooks on non-relativistic quantum mechanics, augmented with von Neumann's

account of measurement. Infamously, this picture posits two different dynamical rules: one for a system's self-evolution, and one for measurements of that system. This immediately poses a challenge for applying Wallace's notion of subsystem structure, which tacitly assumes that a theory has just one fundamental dynamical rule. Our strategy is to propose a distinct notion of sector for each rule, associating the former with microstates and the latter with macrostates.

The kinematics of both sector types are the same, so we start there. On the Schrödinger picture, one identifies the state of a system at a given time with a vector in a complex Hilbert space. That state assigns values to observable properties via its projections $\hat{P}_i$ onto certain orthogonal subspaces of the Hilbert space—subspaces which can be equivalently described as the spectra of Hermitian operators, in line with standard practice. However, there is some arbitrariness regarding which properties we assign to which subspaces. Intuitively, if we act on the Hilbert space's vectors in a way that changes none of their relative lengths or angles, then a difference in how they project onto subspaces should amount to a difference in our choice of labels for those subspaces. Handily, unitary maps encode such transformations, making them prime candidates for dynamical symmetries. In what follows, we take a *transformation* by a unitary $\hat{U}$ to map vectors Ψ to $\Psi' = \hat{U}\Psi$ and operators $\hat{A}$ to $\hat{A}' = \hat{U}\hat{A}\hat{U}^{-1}$.

The Schrödinger equation specifies the dynamics of microstates. Anticipating our desire to define a subsystem structure, let us define these dynamics on a system S, its environment E, and a system SE containing both. We represent the state of SE at time t with a vector $\Psi(t) \in \mathcal{H}_{SE}$, where $\mathcal{H}_{SE} = \mathcal{H}_S \otimes \mathcal{H}_E$, the tensor product of Hilbert spaces describing the degrees of freedom of the system and the environment, respectively. $\Psi(t)$ must satisfy

$$i\hbar\partial_t\Psi(t) = \hat{H}\Psi(t), \tag{11}$$

for some self-adjoint operator $\hat{H}$, the Hamiltonian. The Hamiltonian defines a family of unitary operators $\hat{U}_{\hat{H}}(t, t_0) = e^{-i\hat{H}(t-t_0)/\hbar}$, and

$$\Psi(t) = \hat{U}_{\hat{H}}(t, t_0)\Psi(t_0) \tag{12}$$

solves (11). Note, too, that a unitary transformation $\hat{U}$ of the underlying Hilbert space preserves the dynamical rule (11), in the following sense. Letting $\Psi'(t_0) = \hat{U}\Psi(t_0)$ and $\hat{H}' = \hat{U}\hat{H}\hat{U}^{-1}$,

$$\Psi'(t) = \hat{U}_{\hat{H}'}(t, t_0)\Psi'(t_0) \tag{13}$$

is a solution of

$$i\hbar\partial_t\Psi'(t) = \hat{H}'\Psi'(t) \tag{14}$$

corresponding to the old dynamics in our newly-labeled space. Hence, unitary operators encode dynamical symmetries of the total system SE.[7]

Microstate dynamics and their symmetries behave well under a natural notion of restriction. Whenever we can express the kinematics of a total system as a simple tensor product, we can select any component in that product to recover a well-defined state that obeys the

[7]This is not to say that unitaries encode *all* dynamical symmetries! The time symmetry of Schrödinger dynamics, for example, is best represented by an antiunitary operator (Sakurai and Napolitano, 2010, p. 291). For the Born-rule derivations we consider, it suffices that every unitary map encodes a symmetry, but not vice versa.

same dynamical rule with the same symmetries. To illustrate, let us say that S and E are kinematically separable at time t when the state of SE is separable in the usual sense; that is,

$$\Psi(t) = \Psi_S(t) \otimes \Psi_E(t). \tag{15}$$

Similarly, let us say the dynamics are separable when the Hamiltonian is separable in the usual sense, i.e.,

$$\hat{H} = \hat{H}_S \otimes \hat{H}_E. \tag{16}$$

The separability of the Hamiltonian implies that the unitaries it generates are separable, too. Thus, we can summarize the kinematic and dynamic separability of S and E with the following equation:

$$\hat{U}_{\hat{H}}(t, t_0)\Psi(t_0) = \hat{U}_{\hat{H}_S}(t, t_0)\Psi_S(t_0) \otimes \hat{U}_{\hat{H}_E}(t, t_0)\Psi_E(t_0). \tag{17}$$

The first component of this tensor product is a solution of equation (11) defined on $\mathcal{H}_S$, the Hilbert space of the subsystem S alone. Let us introduce the notation $(\cdot)|_S$ to denote restriction to the S component of a simple tensor product state—so, e.g., $\Psi|_S(t_0) = \Psi_S(t_0)$ and $\hat{H}|_S = \hat{H}_S$. By the same reasoning as before, unitary maps $\hat{U}_S$ on $\mathcal{H}_S$ identify symmetries of the restricted dynamics. We say that the restriction to S thus yields a new microstate sector of the theory. Likewise, with light abuse of notation, we can specify extensions $(\cdot)|^{SE}$ of a microstate sector as a family of maps, each undoing the action of a given restriction from SE to S or E.

What about measurement? Although many textbook expositions of the Schrödinger picture skip it, von Neumann's account of measurement fruitfully allows us to relegate projection dynamics to macroscopic objects. To illustrate the idea driving this account, consider a spin-$\frac{1}{2}$ system S shot through a (macroscopic) Stern–Gerlach measurement apparatus A measuring spin in the z-direction. We idealize our model of A such that $\mathcal{H}_A$ is two-dimensional, just like $\mathcal{H}_S$. We suppose that SA is initially separable in the tensor-product sense described above. Then, we describe the measurement with the action of a unitary map $\hat{U}(\alpha, t_0)$ that satisfies

$$(a|\uparrow\rangle + b|\downarrow\rangle)\,|A\rangle \xrightarrow{\hat{U}(\alpha,t_0)} a|\uparrow\rangle|A_\uparrow\rangle + b|\downarrow\rangle|A_\downarrow\rangle, \tag{18}$$

where $|A_\uparrow\rangle$ and $|A_\downarrow\rangle$ are orthogonal (macroscopic) pointer states representing localized spots near the top and bottom of our observation screen, respectively. This particular Stern–Gerlach measurement is destructive, but we can imagine a non-destructive version of the experiment which prepares states $a|\uparrow\rangle|A_\uparrow\rangle$ by removing the top half of our screen. In this case, we suppose S and A return to their separable microstate dynamics after time α. Von Neumann's own account simply generalizes process (18) to apply to systems with $\mathcal{H}_S$ of arbitrary finite dimension. Note well that (18) falls entirely within the purview of a microstate sector; its dynamics can, in principle, be recovered by some Hamiltonian and equation (11). The radical break occurs when we impose the projection postulate, which stipulates that once the measurement is complete, precisely one element of the sum on the right-hand side of (18) describes the new state of SA.

Let us introduce a bit of notation to more easily get a subsystem structure out of the projection postulate. We use the time-indexed projection $\hat{P}_i(\alpha) = \hat{I}_S \otimes |\psi_i\rangle\langle\psi_i|$ to denote the

projection of SA onto the ith pointer eigenstate of A at time α. The (unnormalized) state after measurement is then given by

$$\Psi(\alpha) = \hat{P}_i(\alpha)\hat{U}(\alpha, t_0)\Psi(t_0). \tag{19}$$

These measurement dynamics straightforwardly preserve the unitary symmetries of microstates. For an arbitrary unitary transformation by $\hat{U}$, we have

$$\begin{aligned} \Psi'(\alpha) := \hat{U}\Psi(\alpha) &= \hat{U}\hat{P}_i(\alpha)\hat{U}(\alpha, t_0)\Psi(t_0) \\ &= \hat{U}\hat{P}_i(\alpha)(\hat{U}^{-1}\hat{U})\hat{U}(\alpha, t_0)(\hat{U}^{-1}\hat{U})\Psi(t_0) \\ &= \hat{P}'_i(\alpha)\hat{U}'(\alpha, t_0)\Psi'(t_0), \end{aligned} \tag{20}$$

as expected.

We can also iterate non-destructive measurements in a chain. For example, say we measure a new pointer observable at time β, perhaps with a different apparatus B, and observe the outcome j. Recycling our notation for extensions and restrictions of microstates, let $\Psi|^{SAB}(t_0) = \Psi(t_0) \otimes |B\rangle$, and let $\hat{P}_i|^{SAB} = \hat{P}_i \otimes \hat{I}_B$. The evolution of the system is then given by

$$\Psi(\beta) = \hat{P}_j(\beta)\hat{U}(\beta, \alpha)\hat{P}_i|^{SAB}(\alpha)\hat{U}|^{SAB}(\alpha, t_0)\Psi|^{SAB}(t_0). \tag{21}$$

This iterative process points to the notion of a macrostate sector for the conservative approach.

Roughly, we take a macrostate sector to be a specification of a sequence of von Neumann measurements finishing at times $(\alpha_1, \ldots, \alpha_n)$, yielding state evolution of the form (21), and equipped with a notion of extension via concatenation and restriction to substrings—that is, a contiguous sequence of characters from the original sequence without rearranging or changing them. (Note that substrings are less general than subsequences, which can omit intermediate elements of the substring.) So, for instance, for a sequence $(\alpha_2, \alpha_3, \alpha_4)$, concatenations include $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and $(\alpha_2, \alpha_3, \alpha_4, \alpha_5)$, and restrictions include (α_2, α_3). Let us suppose that extension via concatenation is precisely the process of extending the initial sequence (19) to the final sequence (21), albeit generalized to initial and final sequences of arbitrary finite length. Likewise, let us take a restriction to a substring to be the dual operation undoing one such extension. Note that the projection postulate requires us to consider only substrings and not more general subsequences, as the 'unused' superposition terms in equation (18) need to be dismissed to guarantee that the future dynamics work as expected. Crucially, restrictions allow us to dismiss systems and apparatuses that are unused in the sector under consideration. For example, suppose the non-destructive Stern–Gerlach measurement of (18) indeed yields the state $\Psi(\alpha) = a|\uparrow\rangle|A_\uparrow\rangle$ on SA. Then, letting $\Psi|_S(\alpha) = a|\uparrow\rangle$ and $\Psi|_S|^{SB}(\alpha) = a|\uparrow\rangle|B\rangle$, we have

$$\Psi|_{SB}(\beta) = \hat{P}_j|_{SB}(\beta)\hat{U}|_{SB}(\beta, \alpha)\Psi|_S|^{SB}(\alpha), \tag{22}$$

defining a macrostate sector for just apparatus B's measurement of S. Thus, there is a sense in which macrostate sectors robustly isolate systems in time: we are licensed to fully ignore past measurements of an isolated system when we wish to measure it anew. Indeed, it is straightforward to check that the extension (21) and the restriction (22) each retain unitary maps as dynamical symmetries of the sort demonstrated by equation (20).

It is instructive to look at a concrete example of a symmetry in a macrostate sector. Let $\hat{S} \in U(2)$ be a unitary map that flips the labels of 'up' and 'down' spin-$\frac{1}{2}$ states,

$$a|\uparrow\rangle + b|\downarrow\rangle \xrightarrow{\hat{S}} a|\downarrow\rangle + b|\uparrow\rangle. \tag{23}$$

Now, apply the unitary transformation $\hat{S} \otimes \hat{I}_A$ to the dynamics described by equation (18):

$$(a|\downarrow\rangle + b|\uparrow\rangle)|A\rangle \xrightarrow{\hat{U}'(\alpha,t_0)} a|\downarrow\rangle|A_\uparrow\rangle + b|\uparrow\rangle|A_\downarrow\rangle. \tag{24}$$

In particular, the symmetry $\hat{S} \otimes \hat{I}_A$ maps the final state $a|\uparrow\rangle|A_\uparrow\rangle$ to $a|\downarrow\rangle|A_\uparrow\rangle$, and it similarly maps $b|\downarrow\rangle|A_\downarrow\rangle$ to $b|\uparrow\rangle|A_\downarrow\rangle$. The underlying symmetry of the microstate carries over to the macrostate. In other words, the arbitrariness in the labeling of 'up' and 'down' states of S becomes a redundancy in descriptions of possible outcomes for SA. Symmetries in the microstate sector of the apparatus A itself similarly induce symmetries in SA. For example, $\hat{I}_S \otimes \hat{S}$ applies the frame shift defined by (23) to the pointer states of the apparatus. So, for example, the symmetry $\hat{I}_S \otimes \hat{S}$ maps the final state $a|\uparrow\rangle|A_\uparrow\rangle$ to $a|\downarrow\rangle|A_\uparrow\rangle$. Now, note that something interesting happens when the superposition weights a and b are equal. The symmetry $\hat{S} \otimes \hat{S}$ maps the state $a|\uparrow\rangle|A_\uparrow\rangle$ to $b|\downarrow\rangle|A_\downarrow\rangle$—and so the two possible final states are symmetric to *each other*.

Note well that, as of yet, we have not introduced any chance or probability rule into the conservative approach. The Born rule figures nowhere in our definitions of microstate and macrostate sectors. Moreover, the reader in the know will be keenly aware that the symmetries outlined in the preceding paragraph suffice to derive that rule rather than stipulate it.

In sum, the conservative approach to non-relativistic quantum mechanics has two separate but compatible subsystem structures, one for microstates and the other for macrostates. Macrostate sectors are specified by a sequence of microstate sectors, each of which might contain further microstate sectors as subsystems. In turn, the conservative approach yields candidates for two notions of isolation. We say that a subsystem is an isolated microstate if it is a microstate sector, and likewise for isolated macrostates.

How defensible are these conservative notions of isolation? In the introduction to this section, we flagged two problems: one about repeatability and one about preferred bases. Putting the latter on hold, we turn our attention to the former.

The repeatability problem refers to how our sectors require repeated measurements of the same sort to leave the system's microstate unchanged. For example, if one records the pointer state $|A_\uparrow\rangle$ in (18), the system S is left in the state $|\uparrow\rangle$; thus, another spin-z measurement performed immediately after the first should not change the state of the system in any way. Achieving perfect repeatability in practice is challenging, if not impossible, due to unavoidable environmental noise (Busch et al. 1995, §II.2.3). The difficulty is not so very different from the challenge of eliminating external gravitational fields to study Newtonian gravity. Fortunately, quantum measurement theory offers a revision of our dynamics that relaxes the repeatability requirement, which we cover next.

5.2 Revising the sectors to accommodate non-repeatability

In the previous subsection, we flagged that the challenge of achieving repeatable measurements renders conservative sectors too restrictive to serve as a happy characterization of isolated

subsystems. Now, we address this problem by introducing a natural relaxation of one of the assumptions in a von Neumann measurement process: namely, the notion that pointer states need not *fully* separate a system's eigenstates.

Recall our simple example of a von Neumann measurement of a spin-$\frac{1}{2}$ system, (18). Let us now suppose that the unitary dynamics couples the definite spin states to macroscopic apparatus states A_+ and A_- that are not quite orthogonal:

$$(a|\uparrow\rangle + b|\downarrow\rangle)\,|A\rangle \xrightarrow{\hat{U}(\alpha,t_0)} a|\uparrow\rangle|A_+\rangle + b|\downarrow\rangle|A_-\rangle, \tag{25}$$

where, for example, $\langle A_\pm|A_\uparrow|A_\pm\rangle \neq 0, 1$. Following Busch et al. (1995, §I.1.2), one nice way to motivate this generalization is to imagine that $\mathcal{H}_A$ encodes the spatial degrees of freedom of the spinning electron, letting A_+ and A_- refer to center-of-mass wavepackets that are not fully separated or localized. Then, a projection onto $A_\uparrow$ or $A_\downarrow$ represents a localization of one of these wavepackets via an interaction with the observation screen. In particular, the non-destructive Stern–Gerlach measurement described in the previous section would now pick out the final state

$$\Psi(\alpha) = (a\langle A_\uparrow|A_+\rangle|\uparrow\rangle + b\langle A_\uparrow|A_-\rangle|\downarrow\rangle)\,|A_\uparrow\rangle, \tag{26}$$

which, in general, is no longer a simple tensor product of states in S and A.

The generalization poses an immediate issue for conservative sectors: the final system and apparatus states can no longer be expressed as a simple tensor product. Recall that we relied on such separability to justify the isolation of a macrostate in time. In other words, separability gave us the license to ignore a system's past measurement interactions when considering new ones. Failure to recover this feature would certainly threaten the tenability of quantum mechanics as an empirical theory.

Fortunately, a minor modification will suffice. Supposing the *dynamics* of S and A remain separable in the conservative sense, we can describe S as evolving on its own by using the partial trace operation to suppress the apparatus' degrees of freedom:

$$\hat{\rho}_S(\alpha) = \mathrm{Tr}_A\left(|\Psi(\alpha)\rangle\langle\Psi(\alpha)|\right). \tag{27}$$

The relevant generalization of the conservative approach's Schrödinger dynamics is given by the standard Liouville–von Neumann equation,

$$i\hbar\partial_t\hat{\rho}_S = \left[\hat{H}_S, \hat{\rho}_S\right], \tag{28}$$

which recovers (11) in the special case that $\hat{\rho}_S$ is pure. Thus, so long as the total system follows the separable Hamiltonian $\hat{H} = \hat{H}_S \otimes \hat{H}_A$, we can take S to identify a microstate sector of the theory. The system S, after all, now behaves precisely as if the system A were not present. However, it is no longer the case that specifying states on S and A suffices to uniquely specify a state on SA that restricts to them. We have to keep in mind that to piece together the total state, we need more information than just the kinematics and dynamics of the subsystems on their own. One can revise macrostate sectors to accommodate these generalized microstates by rewriting the equations (19) through (22) with the density

operator formalism for states. So, for example, the macrostate dynamics of the generalized measurement in (25) are as follows:

$$(a|\uparrow\rangle + b|\downarrow\rangle)\,|A\rangle\langle A|\,(a^*\langle\uparrow| + b^*\langle\downarrow|) \xrightarrow{\hat{U}(\alpha,t_0)} |a|^2|\uparrow\rangle|A_+\rangle\langle A_+|\langle\uparrow| + |b|^2|\downarrow\rangle|A_-\rangle\langle A_-|\langle\downarrow|. \quad (29)$$

For ease of exposition, we avoid unpacking additional examples here.

Instead, we return to the second challenge to conservative theory sectors: the problem of the preferred basis. Loosely following Schlosshauer (2007), we split the problem into two parts. First, one might wonder whether von Neumann's scheme suffices to specify a unique basis along which measurements occur. Second, one might be concerned that the scheme fails to rule out non-classical bases. We address each in turn.

Regarding the first problem, note that we simply stipulate the pointer basis of the apparatus in (18). As Zurek (1981) notes, this choice of basis is not strictly required by the unitary dynamics of the measurement. This issue is most clearly visible in the symmetrical case discussed above, where $a = b = 1/\sqrt{2}$. In that case, we could equally well rewrite the measurement (18) as follows:

$$\frac{1}{\sqrt{2}}\,(|\uparrow\rangle + |\downarrow\rangle)\,|A\rangle \xrightarrow{\hat{U}(\alpha,t_0)} \frac{1}{\sqrt{2}}\,(|\uparrow_x\rangle|A_{\uparrow_x}\rangle + |\downarrow_x\rangle|A_{\downarrow_x}\rangle)\,, \quad (30)$$

where the z-spin system and apparatus states $\{\uparrow, \downarrow, A_\uparrow, A_\downarrow\}$ are related to their x-spin counterparts (denoted here with x subscripts) in the expected way. Without the stipulation that dynamical projection onto a state in the $\{A_\uparrow, A_\downarrow\}$ basis follows the unitary process (18), this latter process seems to equally well describe a z-spin measurement and an x-spin measurement.

Schlosshauer (2007, p. 54) takes this argument to show that the von Neumann scheme cannot, on its own, motivate a specific pointer basis for Everettians, who deny the existence of dynamical collapse. In our view, this conclusion is somewhat overstated. As Schlosshauer (2007, §2.15) also notes, due to Schmidt's (1907) theorem, any decomposition

$$|\Psi\rangle = \sum_i c_i|S_i\rangle|A_i\rangle \quad (31)$$

is unique provided that the $|A_i\rangle$ are orthogonal, each c_i is real, $\sum_i c_i^2 = 1$, and all c_i are different from each other. Note that only this last condition is violated in the symmetric case considered above. If the initial state of S can be any superposition of z-spin eigenstates, then the pointer basis of the von Neumann measurement (18) will be unique for a host of these initial states. That fact implies, in turn, that any choice of pointer basis for the measurement dynamics other than $\{A_\uparrow, A_\downarrow\}$ would have to vary based on the initial state—an undesirable feature if we think that the dynamics themselves are responsible for the pointer basis. All of which is to say: contra Schlosshauer, Everettians can use the von Neumann scheme to prescribe a specific basis for branching macrostates, even if they find the explanatory force of its prescription lacking.

That brings us to the second preferred basis problem: the fact that the von Neumann scheme accommodates any unitary dynamics of the form (18)—and so, in particular, it accommodates wildly non-classical apparatus states, which might involve macroscopic superpositions of pointer positions. The decoherence program offers a revision of macrostate

sectors that aims to address this problematic allowance of macroscopic interference. However, the solution to this problem and the condition under which macrostates are 'isolated' can fruitfully come apart. In the next subsection, we describe how.

5.3 Revising the sectors to motivate a preferred basis

One might hope that Everettians could provide tools to remedy the ugliness of requiring two notions of 'theory sector' in the conservative approach. After all, Everettians use decoherence to remove dynamical collapse and make do with only unitary evolution. However, they cannot escape the need to use different sectors for microstates and macrostates. The Everettian replaces the dynamical collapse of conservative macrostates with the dynamical 'isolation' of branches—where the relevant notion of 'isolation' is different from that for microstates.

As flagged in the previous subsection, in addition to replacing dynamical collapse with non-interacting branches, Everettians also appeal to decoherence to address an explanatory gap left by von Neumann's scheme: namely, its accommodation of unobserved macroscopic interference. However, we argue that this explanatory story is separable from the one that explains how branches become isolated—and that the latter suffices to recover a useful and robust notion of 'isolated subsystem' according to our functional criteria.

To build up to this point, recall that the standard story of how decoherence gives rise to branching begins with a single microstate sector: we let $\Psi(t_0)$ be the initial state of SE, a subsystem and its environment. Now, however, we stipulate that the dynamics must be modeled with a three-part Hamiltonian,

$$\hat{H} = \hat{H}_S + \hat{H}_{\text{int}} + \hat{H}_E, \tag{32}$$

where $\hat{H}_S$ and $\hat{H}_E$ are the self-Hamiltonians of the system S and the environment E (respectively), and $\hat{H}_{\text{int}}$ describes the interaction of the two. This interaction couples the eigenstates of some observable on S with a corresponding set of approximately orthogonal—and thereby, roughly, 'independent'—environment states. Thus, the environment states play an analogous role to the eigenstates of the pointer observable in the von Neumann measurement scheme. During a branching event, $\hat{H}_{\text{int}}$ dominates, very quickly extending any coherence in S over the designated eigenstates to coherence over the environment states. Then the self-Hamiltonians take over again, preserving this 'isolation' of each coupled system and environment state for all future times.

This story aims to address the von Neumann scheme's problematic permissiveness by leveraging the supposed naturalness of the system-environment interaction. As such, much hinges on motivating the naturalness of desirable interaction Hamiltonians. Schlosshauer (2007) illustrates the basic idea with a toy spin-$\frac{1}{2}$ system. Assuming that the system and environment are initially separable, we have (with slight abuse of notation)

$$(a|\uparrow\rangle + b|\downarrow\rangle)\,|E\rangle \xrightarrow{\hat{H}_{\text{int}}} a|\uparrow\rangle|E_\uparrow\rangle + b|\downarrow\rangle|E_\downarrow\rangle, \tag{33}$$

where $\langle E_\uparrow|E_\downarrow\rangle \approx 0$. We model the environment as a collection of systems not dissimilar to our system of interest—for example, in this case, a collection of N additional spin systems.

We take $\hat{H}_{\text{int}}$ to couple the system and the environmental spins in the following way:

$$\hat{H}_{\text{int}} = \frac{1}{2}\hat{\sigma}_z \otimes \sum_{i=1}^{N} g_i \hat{\sigma}_z^{(i)}, \tag{34}$$

where $\hat{\sigma}_z$ and $\hat{\sigma}_z^{(i)}$ are the Pauli z-spin operators of the system and the ith environmental spin, respectively, and g_i is the strength of their coupling. Now, we no longer simply stipulate that the measurement-like evolution of (33) holds between a system and an apparatus. Rather, we derive this evolution from the interactions of many systems of the same kind. Moreover, if (34) describes typical interactions of spin degrees of freedom, including those present in measurement devices, then it will be difficult to couple spin superpositions to pointer states of those devices. To summarize this idea, we can decompose S into a system S' and a measuring apparatus A, and say that we expect the following dynamics:

$$(a|\uparrow\rangle + b|\downarrow\rangle)\,|A\rangle|E\rangle \xrightarrow{\hat{H}_{\text{int}}} a|\uparrow\rangle|A_\uparrow\rangle|E_\uparrow\rangle + b|\downarrow\rangle|A_\downarrow\rangle|E_\downarrow\rangle. \tag{35}$$

In other words, we expect that apparatus states encoding definite pointer readings will couple to system states preferred by decoherence. As a bonus, we recover quantum measurement theory's generalization when the system's spin states are not the ones favored by decoherence. Recall that in the previous subsection, we took the system's spin to couple with its center of mass, and we took $A_\uparrow$ and $A_\downarrow$ to be localized center-of-mass states. Now, we might consider an environment E of air particles; when these air particles scatter off heavy objects, they strongly couple to their localized center-of-mass wavefunctions. So, if the center of mass is not well localized before reaching the screen, we expect evolution roughly like

$$(a|\uparrow\rangle + b|\downarrow\rangle)\,|A\rangle|E\rangle \xrightarrow{\hat{H}_{\text{int}}} (c|\uparrow\rangle + d|\downarrow\rangle)|A_\uparrow\rangle|E_\uparrow\rangle + (e|\uparrow\rangle + f|\downarrow\rangle)|A_\downarrow\rangle|E_\downarrow\rangle. \tag{36}$$

Here, the coupling of localized states of the system's center of mass and the apparatus' pointer states with approximately orthogonal environment states explains why we observe localized measurement records. However, in this case, the system's observables of interest—the spin observables—fail to couple strongly to the localized pointer states.

With this picture in hand, we can sketch how decoherence gives rise to 'branching' macrostates. Environmental interactions dynamically prefer a given pointer basis, supressing interference among the terms of that basis. Thus, an agent who observes a given pointer state is justified as 'forgetting' many details about the microstate: they can ignore all branches other than the one containing their pointer state, as well as any minuscule interactions between their branch and others. The key difference with conservatism is that the Everettian forgoes requiring dynamical projection to justify the agent's behavior. Instead, they require that the 'unused' branches hang around, but in such a way that they barely interfere with each other's dynamics.

To spell that out, let $|E_i^\alpha\rangle \in \mathcal{H}_{SE}$ denote the environmental state coupled to the ith pointer observable at time α, and let $\hat{P}_i(\alpha) = \hat{I}_S \otimes \hat{P}[|E_i^\alpha\rangle]$. Then,

$$\Psi_i^\alpha = \hat{P}_i(\alpha)\hat{U}(\alpha, t_0)\Psi(t_0) \tag{37}$$

defines the state of the ith branch of SE at time α. Now, suppose that another branching event occurs at a later time $\beta > \alpha$. Let $\Psi_{ij}^{\alpha\beta}$ denote the branch state recovering pointer states i at α and j at β, respectively; that is,

$$\Psi_{ij}^{\alpha\beta} = \hat{P}_j(\beta)\hat{U}(\beta,\alpha)\hat{P}_i(\alpha)\hat{U}(\alpha,t_0)\Psi(t_0). \tag{38}$$

The 'isolation' of branch states refers to the requirement that each branch state nearly determines its own future trajectory. If we idealize, we can take this determination to be complete. This move yields the *branching criterion*, which states that only one prior branch can contribute to the state of any future branch; i.e.,

$$\Psi_{ij}^{\alpha\beta}, \Psi_{i'j}^{\alpha\beta} \neq \bar{0} \iff i = i', \tag{39}$$

where $\bar{0}$ stands for the zero vector in $\mathcal{H}_{SE}$. Strictly satisfying (39) requires idealizing the environmental record states to be *exactly* orthogonal—bringing them much closer to playing the role of pointer states in a von Neumann measurement.

As such, Everettians recover something very close to conservative macrostates as an emergent feature of microstate dynamics. Note, however, that it is at best unclear how to build Everettian branches into microstate sectors: states satisfying the branching criterion will not, in general, be simple tensor products of states corresponding to the different branches. They do not quite mesh with conservative macrostate sectors, either. As such, we use the terminology 'branch sector' to denote the particular subsystem structure of Everettian branches.

A branch sector is a specification of a sequence of branching events that end at times $(\alpha_1, \ldots, \alpha_n)$, resulting in state evolution of the form (38), and equipped with notions of extension and restriction between sequences and their subsequences. Note that the dynamics of branch sectors, (38), have a similar form as the dynamics of conservative macrostate sectors, (21)—both are iterated sequences of unitary and projection operators indexed by the sequence of times α_i. The key structural difference afforded by the branch sectors is the more liberal notion of extension and restriction—to subsequences, rather than substrings. Basically, since the branching criterion ensures future measurements are (to a very good approximation) unaffected by past ones, the observer is allowed to forget any given measurement in a sequence.

In sum, Everettians do not change microstates; they can stick with the conservative ones, or they can use the generalization from quantum measurement theory.[8] Their innovation lies in replacing conservative macrostates with branches. We use branch sectors to characterize the subsystem structure imposed by the branching criterion, which differs only subtly from that of a conservative macrostate sector.

This much suffices to define a robust notion of 'isolation' of branches: to wit, we can say that a branch is isolated precisely when it can be described as a branch sector. Note, however, that the branching criterion imposes no particular form on the total system's Hamiltonian, leaving the problem of the preferred basis apparently only half solved. We claim that the thin notion of isolation sketched in this subsection is independent of the rest of the solution. In the next subsection, we argue for that claim.

[8] If the Everettian does choose to generalize their microstates, they will also have to rewrite branch sectors using the density matrix formalism analogously to equation (29). See Chua and Chen (2025) for more illustrations of this type of re-expression.

5.4 Isolation does not require a preferred basis

As flagged in the previous subsection, the decoherence program's remedy to the von Neumann scheme's permisiveness hinges on naturalistically motivating interaction dynamics that select appropriate pointer states, like spin eigenstates or well-localized wavepackets. In particular, Dowker and Kent (1996, §3.2) illustrate that the branching criterion on its own allows for desirable pointer states in one branch and superpositions of those states in another. Following Gell-Mann and Hartle (1997, 1993, 2007), Everettians like Wallace (2012) opt to treat the branching criterion as a necessary, but not sufficient, condition for (perhaps idealized) descriptions of quasi-classical histories. Gell-Mann and Hartle (1995) go further, characterizing quasi-classicality with a generalized measure of entropy for coarse-grained histories, which involves a probabilistic weighting of branches. In contrast, Wallace stops at the branching criterion, suggesting that quasi-classicality is a fuzzy notion that does not need a formal characterization (2012, p. 99).

At first blush, it would seem that an Everettian like Wallace who seeks a derivation of a quantum chance rule cannot help themselves to Gell-Mann and Hartle's probabilistic explanation of quasi-classicality. If they did, the thought goes, they would be caught in the sort of circularity that troubles Mandolesi (2019). Note, however, that once one has our thin characterization of 'isolated subsystems' on the menu, nothing stops an Everettian from having their cake and eating it. The branching criterion, on its own, fulfills the functional criteria (I1) and (I2): the branches it defines instantiate a recursive subsystem structure, and we can create at least some of them in laboratories via normal measurement processes. That is enough to make good sense of branches as 'isolated subsystems', which in turn is enough to fill in the certainty link (C4), which in turn is enough to derive chance values. Then, of course, the Everettian is free to use those chance values to recover Gell-Mann and Hartle's explanation of why we never observe the non-classical macrostates allowed by the branching criterion *per se*.

In sum, Everettians can use the machinery of decoherence to recover a notion of isolation that fruitfully decouples from their explanation of the suppression of macroscopic interference. Specifically, the branching criterion makes good sense of isolated macrosates that might be quasiclassical or not, just as a generalized Coulombic electrostatics makes good sense of isolated subsystems that might have vanishing electric fields at their boundaries or not. As flagged in the introduction to this section, there might be a modal difference between these scenarios: intuitively, non-zero electric fields are possible but non-classical branches are impossible, or nearly so. However, this difference does not interfere with (I1) or (I2). To fulfill the control link (I2), all that matters is that *some* isolated subsystem can be studied, so that the sense in which it is similar to any other isolated subsystem can be projected outward to them. Moreover, the Everettian averse to brute chances does not need to sit with the possibility of non-classical branches for long: once chances are derived, they can justify the typicality of classical branches in the usual way.

However, the Everettian is not fully off the hook. While the possibility that branches might be non-classical does not trouble their isolation, another possibility might: namely, that decoherence spreads in space as well as time. One might reasonably question whether branching is adequately modeled as occurring everywhere in space simultaneously. This modeling assumption is tacit in the dynamics (34), since the coupling constants g_i are not

functions of space or time; coupling occurs immediately at a given strength, regardless of how distant the ith spin system is from the system of interest. This modeling assumption persists in most applications of decoherence, including a de-idealization of (34) that Cucchietti et al. (2005) propose applying to nuclear magnetic resonance. In contrast, Wallace (2012, §8.5–8.6) sketches an explicitly local conception of branching, where decoherence spreads in space and time. As a first pass at unpacking this conception, we might de-idealize the decoherence interaction in (35) to occur in two steps, as follows:

$$\begin{aligned}(a|\uparrow\rangle + b|\downarrow\rangle)\,|A\rangle|E\rangle &\xrightarrow{\hat{H}_1} a|\uparrow\rangle|A\rangle|E_\uparrow\rangle + b|\downarrow\rangle|A\rangle|E_\downarrow\rangle \\ &\xrightarrow{\hat{H}_2} a|\uparrow\rangle|A_\uparrow\rangle|E_\uparrow\rangle + b|\downarrow\rangle|A_\downarrow\rangle|E_\downarrow\rangle.\end{aligned} \tag{40}$$

Here, $\hat{H}_1$ describes the decoherence of the spin system S with its environment E, after which $\hat{H}_2$ decoheres the composite system SE with the pointer states of the apparatus A. This establishes how decoherence might spread over time; if we have reason to believe that S, E, and A are each well-localized, it might also describe a sense in which decoherence spreads throughout space. Blackshaw et al. (2024) create a toy model that runs with this idea, de-idealizing the dynamics of (34) to demonstrate how decoherence spreading might work for a collection of spin systems.

Whether such de-idealizations will make an empirical difference to future physics is an interesting open question. One can imagine a robust model of spreading decoherence underpinning a covariant formulation of measurement histories. Such an approach would avoid adopting a preferred frame's time coordinate, as we tacitly do in our descriptions of both conservative and Everettian macrostates. However, it seems likely that such developments would take us beyond the purview of *non-relativistic* quantum mechanics. So, as far as the Everettian dealing with the non-relativistic domain is concerned, branch sectors provide a thin, robust, and stable characterization of the sense in which macrostates are isolated subsystems. This isolation suffices to fill in quantum chances' link with certainty, and so we take it to suffice to derive the values of those chances.

At this juncture, it is useful to consider a concern of a more metaphysical stripe. The staunch metaphysician might well demand that we derive *the* reference class defining *the* chance of an event from first principles. That metaphysician might well try to do so by embracing the truth of Everettian quantum theory and seeking a derivation of the Born rule. We do not take the subsystem recursivity of branches, on its own, to motivate the metaphysician's project. We do, however, take it to motivate the certainty link, if only provisionally, by exemplifying the Everettian theory's good behavior when it comes to testability. The theory acts as we ideally want testable theories to act—and since it is a statistical theory, it thereby offers a paragon of a well-specified reference class. Thus, the Everettian metaphysician using a separability principle is at least no worse off than any other scientific realist who defeasibly takes a well-tested physical theory as their starting point.

With that said, we turn to how our functional analysis of isolation with theory sectors begets a thin, core approach to deriving Everettian chances from symmetry. We show how this core approach helps ease disagreements among Everettians and defuse salient criticisms from non-Everettians.

6 Sectors unite and strengthen Born rule derivations

The three uncertainty-based approaches to quantum probability we consider here—the Deutsch–Wallace theorem (Deutsch 1999, Wallace 2010, 2012), Sebens and Carroll's (2018) derivation, and McQueen and Vaidman's (2019) modification of the latter—all claim that their derivations have something to do with symmetry. Wallace writes that his proof is driven by the fact that 'in Everettian quantum mechanics not just the laws, but the actual microstate of the system are invariant under a symmetry transformation, as could not be the case if only one outcome was to occur' (2012, p. 151). Sebens and Carroll do not explicitly distinguish between one- and many-outcome symmetries, but still require that transformations of subsystems are 'symmetries of the dynamical laws', i.e., symmetries of unitary evolution maps (2018, p. 48). Similarly, McQueen and Vaidman (2019) base their derivation explicitly on systems with rotational symmetry.

None of these authors attend to the difference between microstate and macrostate symmetries illustrated in the previous section. This elision might appear benign at first glance, as unitary transformation maps instantiate both sorts of symmetries. However, we take the elision to create avoidable conflict among Everettians and to obscure the core of the chance derivation. As such, our strategy in the present section is to illustrate a core approach to deriving Everettian chances with macrostate symmetries, and then illustrate how this approach resolves these tensions in the literature.

The core approach is to suppose that an assignment of chance values is an intrinsic property of a macrostate with respect to its unitary symmetries, in the sense specified by macrostates' subsystem structure (I1). As noted in Section 4, either of the separability conditions we use to fill in chance's certainty link (C4) then implies the following intrinsicity condition:

$$ch\,(X_i \mid S) = ch\,(X_i' \mid S')\,, \tag{41}$$

where a unitary transformation maps the macrostate $S = \{X_1, X_2\}$ to $S' = \{X_1', X_2'\}$. Equation (41) then drives the derivation of specific chance values. Note that this approach applies equally well to both conservative and Everettian approaches to quantum mechanics; in the latter case, we simply specify that macrostates are given by branch sectors.

Let us illustrate how the core approach works for an equal-weight, two-outcome case, returning to the symmetry we illustrated for conservative macrostate sectors in the previous section. There, we applied a symmetry map $\hat{S} \in U(2)$ that flips the labels of 'up' and 'down' spin-$\frac{1}{2}$ states, given by equation (23). This time, let's include an evironment E interacting with the spin system S before the latter interacts with our Stern–Gerlach device A:

$$(a|\uparrow\rangle + b|\downarrow\rangle)\,|A\rangle|E\rangle \xrightarrow{\hat{U}(\alpha,t_0)} a|\uparrow\rangle|A_\uparrow\rangle|E_\uparrow\rangle + b|\downarrow\rangle|A_\downarrow\rangle|E_\downarrow\rangle. \tag{42}$$

For any two states $|E_\uparrow\rangle$ and $|E_\downarrow\rangle$, there exists a unitary map $\hat{S}_E$ that permutes them. We apply $\hat{S} \otimes \hat{S} \otimes \hat{S}_E$ to obtain:

$$(a|\downarrow\rangle + b|\uparrow\rangle)|A\rangle \xrightarrow{\hat{U}'(\alpha,t_0)} a|\downarrow\rangle|A_\downarrow\rangle|E_\downarrow\rangle + b|\uparrow\rangle|A_\uparrow\rangle|E_\uparrow\rangle. \tag{43}$$

Let equations (42) and (43) specify macrostate sectors $S = \{X_1, X_2\}$ and $S' = \{X_1', X_2'\}$. Note that each contains two sub-macrostates, one corresponding to each summand of the

right-hand-side superposition. In the case that $a = b = 1/\sqrt{2}$, the map $\hat{S} \otimes \hat{S} \otimes \hat{S}_E$ leaves the state of S unchanged, so $S = S'$. Moreover, it identifies two pairs of sub-macrostates: $X_1 = X_2'$ and $X_2 = X_1'$. By the intrinsicity condition (41), that means S should assign its two sub-macrostates equal chance values:

$$ch\,(X_1 \mid S) = ch\,(X_1' \mid S') = ch\,(X_2 \mid S') = ch\,(X_2 \mid S)\,. \tag{44}$$

The rules of probability then require that each of these chance values is one-half. To make this story Everett-specific, one stipulates that equations (42) and (43) are descriptions of branching events, rather than von Nuemann measurements soon to be followed by a dynamical projection.

Each of the three symmetry-based approaches described above deviates only slightly from this core approach. In the following subsections, we illustrate how each approach deviates in turn, and we argue that returning to the core unites them while addressing approach-specific critiques from Everettians and non-Everettians alike. For each approach, we explicitly discuss only a finite-outcome, equal-weight case like the one above; such cases suffice to exhibit the key conceptual differences, and the remainder of the proofs are mostly mechanical.

6.1 Wallace's two-branch, equal-weight case

In short, Wallace's approach to deriving Everettian chances deviates from the core approach by requiring that sub-macrostates be *identical*, rather than merely symmetric. Among other things, this approach helps Wallace differentiate chance in a many-worlds theory from chance in a single-world theory: the latter could never satisfy the identity requirement, the thought goes, as different outcomes seem to correspond to different states by fiat. However, we argue that this move ends up requiring dynamics that threaten the subsystem recursivity of Everettian macrostates.

To show how, we consider the two-branch, equal-weight case in Wallace's (2012) decision-theoretic Everettian derivation of the Born rule. The full derivation utilizes a total of ten axioms: four 'richness' axioms that, roughly, enrich the state of a branch sector to closely mirror a Bayesian decision problem; and six 'rationality' axioms that, roughly, operationalize the role played by symmetries in the core approach. With this picture, Wallace refers to unitary dynamics as 'acts' (denoted U) available to agents in certain states, and he uses the term 'reward' to refer to sets of macrostates that yield the same payout for an agent.

Two of the richness axioms ('reward availability' and 'branching availability') roughly ensure that branches occur with all possible weights for all possible outcomes; we will omit these from our exposition, as we assume the possibility of the equal-weight, two-branch case by fiat. An additional two axioms ('problem continuity' and 'solution continuity') ensure the state space is rich enough to allow well-defined limiting cases. These axioms are relevant to the recovery of arbitrary real-valued chances, but they do not feature in the equal-weight case, so we will ignore them as well. That leaves us with the following six axioms (Wallace 2012, §5.3, 5.4):

> **Erasure.** Given a pair of states $\psi \in E$ and $\phi \in F$, where E and F are macrostates in the same reward, there is an act U available at E and an act V available at F such that $U\psi = V\phi$.

Ordering. The preference relation $\succeq_\psi$ is a total ordering.

Diachronic consistency. If U is available at ψ, and (for each i) if in the ith branch after U is performed there are acts V_i, V_i' available, and (again for each i) if the agent's future self in the ith branch will prefer V_i to V_i', then the agent prefers performing U followed by the V_is to performing U followed by the V_i's.

Microstate indifference. An agent doesn't care what the microstate is provided it's within a particular macrostate.

Branching indifference. An agent doesn't care about branching per se: if a certain operation leaves his future selves in N different macrostates but doesn't change any of their rewards, he is indifferent as to whether or not the operation is performed.

State supervenience. An agent's preferences between acts depend only on what physical state they actually leave his branch in: that is, if $U\psi = U'\psi'$ and $V\psi = V'\psi'$, then an agent who prefers U to V given that the initial state is ψ should also prefer U' to V' given that the initial state is ψ'.

Wallace appeals to each of these axioms in his derivation of the two-branch, equal-weight case:

> For a simple case, suppose we have two acts (A and B, say): in each, a system is prepared in a linear superposition $\alpha|+\rangle + \beta|-\rangle$ and then measured in the $\{|+\rangle, |-\rangle\}$ basis. On act A, a reward is then given if the result is '+'; on B, the same reward is given on instead '−'. The resultant states are
>
> $$\begin{aligned} &\text{A:} \quad \alpha|+\rangle \otimes |\text{reward}\rangle + \beta|-\rangle \otimes |\text{no reward}\rangle; \\ &\text{B:} \quad \alpha|+\rangle \otimes |\text{no reward}\rangle + \beta|-\rangle \otimes |\text{reward}\rangle. \end{aligned} \tag{45}$$
>
> By **erasure**, there will exist acts available to the agent's future self in the reward branch (for both A and B) which erase the result of what was measured, leaving only the reward. Performing these transformations, and the equivalent erasures in the no-reward branch, leaves
>
> $$\begin{aligned} &\text{A-plus-erasure:} \quad \alpha|0\rangle \otimes |\text{reward}\rangle + \beta|0'\rangle \otimes |\text{no reward}\rangle; \\ &\text{B-plus-erasure:} \quad \beta|0\rangle \otimes |\text{reward}\rangle + \alpha|0'\rangle \otimes |\text{no reward}\rangle. \end{aligned} \tag{46}$$
>
> Now, by **branching indifference,** the agent's future selves are indifferent to whether this erasure is or is not performed. (Branching indifference is needed because we have no guarantee that erasures are nonbranching; if we did, **microstate indifference** would suffice.) So by **diachronic consistency,** the original agent is indifferent between A and A-plus-erasure, and between B and B-plus-erasure.
>
> But now: if $\alpha = \beta$, then A-plus-erasure and B-plus-erasure leave the system in the same quantum state. So by **state supervenience,** the agent is indifferent between them. Since we know from **ordering** that preferences are transitive, the agent must also be indifferent between A and B. (2012, pp. 172–173, emphasis added)

To start, note that branching indifference appears in this derivation with something of an apology. It is needed to deal with erasures, which are in turn needed to ensure that the final states in the two cases are identical (and not merely symmetric) as required by state supervenience. If we weaken state supervenience to track symmetry rather than identity, then erasure, diachronic consistency, and branching indifference become unnecessary: microstate indifference would suffice. That makes good sense, as microstate indifference does the work of picking out branch sectors, Everettian macrostates, as the isolated subsystems that matter for chances. Thus, removing the identity requirement from Wallace's argument essentially returns us to the core approach, albeit with an explicitly epistemic way of deriving the rules of probability built in.

Moreover, on closer inspection, the availability of erasure acts seems hard to justify. Erasures are meant to reflect the fact that a system's microstate makes no difference to an agent as long as the agent gets the same reward. However, erasures do not enter the model as features of an agent's ignorance, but rather as dynamical acts that the agent can perform. Wallace flags this tension in his motivation of erasure:

> Erasure is slightly more complicated. It effectively guarantees that an agent can just forget any facts about his situation that don't concern things he cares about (i.e. by definition: that don't concern where in the reward space he is). In thinking about it, it helps to assume that any reward space has an 'erasure subspace' available (whose states correspond to the agent throwing the preparation system away after receiving the payoff but without recording the actual result of the measurement, say). An 'erasure act' is then an act which takes the quantum state of the agent's branch into the erasure subspace; the agent is (by construction) indifferent to performing any erasure act, and since he lacks the fine control to know which act he is performing, all erasures should be counted as available if any are. (2012, pp. 167)

The scare quotes around 'erasure subspace' seem to convey the difficulty of physically motivating the reification of an agent's ignorance of the microstate into a feature of the space of dynamical possibilities. Likewise, Wallace's description of agents 'throwing the preparation system away' seems only tenuously related to the acts defined in the erasure axiom, which requires two agents to map their full microstates—including the preparation system—to one and the same state.

Even worse, as Mandolesi (2019, p. 42) argues, erasures stand in strong tension with the linearity of unitary dynamics. His argument goes as follows: let $\psi \in M$ and $\phi \in N$, where M and N are orthogonal macrostates in the same reward. By the erasure axiom, there are acts U and V available at M and N such that $U\psi = V\phi$. Now, note that M and N are sub-macrostates of $M \vee N$. In the superposition microstate $\psi + \phi$, it seems like the two versions of an agent on branches M and N should be able to perform U and V at the same time, yielding an act $W = U \vee V$ available at $M \vee N$. However, that cannot be the case. If it were, then we would have $W\psi = W\phi$, and so W would fail to be unitary.

Our functional analysis of isolation enables us to sharpen Mandelosi's point: some erasures in macrostates lack natural extensions to macrostates that contain them, threatening subsystem recursivity. Recursivity is not broken outright, as U and V might each have

different extensions to $M \vee N$ if we require the two versions of the branching agent to perform their acts at different times. However, this requirement seems hard to motivate.

We wager that retaining a natural subsystem structure for branches is more important than winning a victory for many worlds over one. Thus, Wallace seems better off dropping the identity requirement, forgoing erasures, and adopting (an appropriately epistemicized version of) the core approach.

As a bonus, the core approach eschews the need for the diachronic consistency axiom. Motivated by duplication thought experiments, Sebens and Carroll (2018) reject that axiom. We will not rehearse these here; instead, we simply note that moving to the core would allow Wallace to accommodate Sebens and Carroll's criticism.

6.2 Sebens and Carroll's two-branch, equal-weight case

Sebens and Carroll's (2018) approach proposes an epistemic separability principle specific to multiverse theories, which they call **ESP**. This general principle differs from our **ES** chiefly by building in a notion of self-locating uncertainty within a multiverse:

ESP: Suppose that the universe U contains within it a set of subsystems S such that every agent in an internally qualitatively identical state to agent A is located in some subsystem that is an element of S. The probability that A ought to assign to being located in a particular subsystem $X \in S$ given that they are in U is identical in any possible universe which also contains subsystems S in the same exact states (and does not contain any copies of the agent in an internally qualitatively identical state that are not located in S):

$$c\left(X \mid U\right) = c\left(X \mid S\right). \tag{47}$$

Here, $c\left(X \mid U\right)$ means the self-locating probability of being in subsystem X given that one inhabits the universe U. Note that the terminology of S being a 'set of subsystems' might mislead one in identifying the role separability plays; to avoid confusion, one can think of S as a state description common to a set of agents in a multiverse, one that marks all such agents as 'internally qualitatively identical'. It is physical information in U above and beyond S that is meant to be redundant, as far as probability is concerned. As such, S in the above definition roughly recovers the sense in which we say S is 'isolated subsystem' in **OS** and **ES**. As Sebens and Carroll state, the 'gist' of **ESP** is this: 'The credence one should assign to being any one of several observers having identical experiences is independent of the state of the environment' (p. 40).

Helpfully for our purposes, Sebens and Carroll continue to sharpen the sense in which their subsystems S are isolated beyond the statement of **ESP** itself. After introducing the principle, they offer the following clarification:

> The essential idea is that a subsystem is a part of the larger system that can be considered as a physical system in its own right. Slightly more formally, we imagine that the overall state of a system can be decomposed into the states of various subsystems, so that two constraints are satisfied: (i) the state of each subsystem, perhaps with some additional information about how the subsystems are connected, can be used to uniquely reconstruct the original state; and (ii)

> the information contained within each subsystem's state is enough to specify its immediate dynamical evolution, as long as the other subsystems are not influencing it. (Sebens and Carroll, 2018, p. 41)

Note that (i) and (ii) cover very similar ground to subsystem recursivity. Thus, S in Sebens and Carroll's **ESP** fulfills our functional criterion (I1) for isolation.

However, Sebens and Carroll adopt the subsystem structure of microstates to define the sense in which S is isolated, rather than that of macrostates. In particular, they use the density matrix generalization of microstate sectors that we reviewed in Section 5.2, a stance they clarify in an appendix:

> In general, the reduced density matrix for the composite system AB, $\hat{\rho}_{AB}$, cannot be constructed uniquely from the separated reduced density matrices for A and B. The matrix also encodes facts about the entanglement between A and B. But as these are just facts about how A and B are connected, condition (i) is satisfied. To see that condition (ii) is met, suppose that, at least for a time, subsystem A is isolated from everything else, E. Let $\hat{U}_t$ be the unitary operator that gives the time evolution of the total state. Since A and E are non-interacting, $\hat{U}_t = \hat{U}_A \otimes \hat{U}_E$. The time evolution of $\hat{\rho}_A$ is then given by $\hat{U}_A \hat{\rho}_A \hat{U}_A^\dagger$ ($\hat{U}_E$ is irrelevant). (Sebens and Carroll, 2018, p. 67)

As such, Sebens and Carroll take the following principle, **ESP-QM**, to be the realisation of **ESP** in the quantum-mechanical context:

ESP-QM: Suppose that an experiment has just measured observable $\hat{O}$ of system S and registered some eigenvalue O_i on each branch of the wavefunction. The probability that agent A ought to assign to the detector D having registered O_i in their branch when the universal wavefunction is Ψ, $c\,(O_i|\Psi)$, only depends on the reduced density matrix of A and D, $\hat{\rho}_{AD}$:

$$c\,(O_i \,|\, \Psi) = c\,(O_i \,|\, \hat{\rho}_{AD})\,. \tag{48}$$

In their criticism of the **ESP** derivation, Dawid and Friederich (2020, p. 716) note saliently that the reduced density matrix formalism might fail to track whether branching has occurred. In our setup, this amounts to the fact that they focus on microstates rather than macrostates. Moreover, they do not derive an intrinsicity condition from their separability principle; thus, they end up tacitly adopting Wallace's identity criterion. Only identical reduced density matrices can yield the same chances, and not merely symmetric ones.

The issue of focusing on identical microstates gives rise to two curious moves in Sebens and Carroll's chance derivation. First, they adopt a global view of branching. This view amounts to a particular way of interpreting the two steps of decoherence in the de-idealized dynamics (40), where the system S first decoheres with the environment E and then SE decoheres with the measuring device A. The global view says there are two measuring devices in two universes once the first decoherence step occurs—that is, even when the device is still in its ready state. Sebens and Carroll further illustrate the view as follows:

> [...] branching happens throughout the whole wave function whenever it happens anywhere. When the universal wave function splits into multiple distinct and

effectively non-interacting parts, the entire world splits—along with every object and agent in it. (2018, p. 34)

As such, even when decoherence has not yet reached Alice in their equal-weight, two-outcome case, they can still infer that there are two copies of Alice uncertain about their location in the multiverse, because branching has occurred elsewhere.

Second, they derive chance values for just those instances in which decoherence has occurred for a system but not yet reached Alice. They present their equal-weight, two-branch case as follows:

> Alice measures the z-spin of a single particle in the x-spin up state. One display (D1) will show the result of the experiment. If the spin is up, a second display (D2) will show $\heartsuit$. If it is down, a $\diamondsuit$ will appear on the second display. Alice is not immediately affected by the result; in particular, she is for a time unaware of the experiment's outcome. The wave function of Alice, the detectors, the particle, and the environment (the rest of the universe) evolve from
>
> $$|\Psi_0\rangle = |R_0\rangle_A |R\rangle_{D1} |R\rangle_{D2} |\uparrow_x\rangle |E_R\rangle \tag{49}$$
>
> to
>
> $$|\Psi_1\rangle = \frac{1}{\sqrt{2}} |R\rangle_A |\uparrow\rangle_{D1} |\heartsuit\rangle_{D2} |\uparrow_z\rangle \, |E_{\uparrow\heartsuit}\rangle + \frac{1}{\sqrt{2}} |R\rangle_A |\downarrow\rangle_{D1} |\diamondsuit\rangle_{D2} |\downarrow_z\rangle \, |E_{\downarrow\diamondsuit}\rangle \, . \tag{50}$$
>
> To use **ESP–QM** to demonstrate that $P(\uparrow | \, \Psi_1) = P(\downarrow | \, \Psi_1) = \frac{1}{2}$, we will need to also consider an alternate scenario where the computer (part of the environment) is programmed differently so that $\heartsuit$ displays if down is measured and $\diamondsuit$ displays if up. Then the post-measurement, pre-observation wave function would be:
>
> $$|\Psi_2\rangle = \frac{1}{\sqrt{2}} |R\rangle_A |\uparrow\rangle_{D1} |\diamondsuit\rangle_{D2} |\uparrow_z\rangle \, |E_{\uparrow\diamondsuit}\rangle + \frac{1}{\sqrt{2}} |R\rangle_A |\downarrow\rangle_{D1} |\heartsuit\rangle_{D2} |\downarrow_z\rangle \, |E_{\downarrow\heartsuit}\rangle \, . \tag{51}$$
>
> Step 1: Focus first on Alice and D1. The [(Alice + Detector 1)] reduced density matrices for Ψ_1 and Ψ_2 are the same,
>
> $$\hat{\rho}_{AD1}(\Psi_1) = \hat{\rho}_{AD1}(\Psi_2) = \frac{1}{2} |R\rangle_A |\uparrow\rangle_{D1} \langle\uparrow|_{D1} \langle R|_A + \frac{1}{2} |R\rangle_A |\downarrow\rangle_{D1} \langle\downarrow|_{D1} \langle R|_A. \tag{52}$$
>
> **ESP–QM** requires that the probabilities Alice assigns to the possible spin results be the same in these two universes as they have the same [(Observer + Detector 1)] reduced density matrix,
>
> $$P(\downarrow | \, \Psi_1) = P(\downarrow | \, \Psi_2). \tag{53}$$
>
> Step 2: If we ask what probability Alice should assign to the display being $\heartsuit$, we need to consider the reduced density matrix generated by tracing over $D1$, the spin of the particle, and the environment. Ψ_1 and Ψ_2 agree on $\hat{\rho}_{AD2}$. By **ESP–QM**, the probabilities assigned to $\heartsuit$ must be equal,
>
> $$P(\heartsuit \mid \Psi_1) = P(\heartsuit \mid \Psi_2). \tag{54}$$

> Step 3: Next, note that the ♡-branches 'just are' the ↑-branches in Ψ_1 and the ♡-branches just are the ↓-branches in Ψ_2. Thus Alice is in the ♡-branch of $\hat{\rho}_{AD2}(\Psi_1)$ if and only if she is in the ↑-branch of $\hat{\rho}_{AD1}(\Psi_1)$. Similarly, she is in the ♡-branch of $\hat{\rho}_{AD2}(\Psi_2)$ if and only if she is in the ↓-branch of $\hat{\rho}_{AD1}(\Psi_2)$. Therefore, Alice must assign
>
> $$\begin{aligned} P(\uparrow \mid \Psi_1) &= P(\heartsuit \mid \Psi_1) \\ P(\downarrow \mid \Psi_2) &= P(\heartsuit \mid \Psi_2). \end{aligned} \tag{55}$$
>
> Step 4: Putting together the results in Equations (53)–(55), we see that the probability of being on a ↑ /♡-branch must be the same as that for being on a ↓ /♢-branch: $P(\uparrow \mid \Psi_1) = P(\downarrow \mid \Psi_1)$. So, the unique rational degrees of belief in the first scenario consider each branch to be equiprobable. (Sebens and Carroll, 2018, pp. 43–45)

McQueen and Vaidman (2019, p. 22) argue convincingly that this derivation applies *only* to cases where decoherence has not yet reached Alice—and so it seems that Alice's credences can only be legitimate in the split-second before decoherence reaches her. Explicitly, they note that the application of **ESP-QM** in Step 2 becomes unsound once we fast-forward to the end of the decoherence process, after which Alice has split into orthogonal states. So, labeling Alice's post-branching states $R_\uparrow$ and $R_\downarrow$, we may write the fully decohered Ψ_1 and Ψ_2 as

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|R_\uparrow\rangle_A|\uparrow\rangle_{D1}|\heartsuit\rangle_{D2}|\uparrow_z\rangle\,|E_{\uparrow\heartsuit}\rangle + \frac{1}{\sqrt{2}}|R_\downarrow\rangle_A|\downarrow\rangle_{D1}|\diamondsuit\rangle_{D2}|\downarrow_z\rangle\,|E_{\downarrow\diamondsuit}\rangle \tag{56}$$

and

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}|R_\uparrow\rangle_A|\uparrow\rangle_{D1}|\diamondsuit\rangle_{D2}|\uparrow_z\rangle\,|E_{\uparrow\diamondsuit}\rangle + \frac{1}{\sqrt{2}}|R_\downarrow\rangle_A|\downarrow\rangle_{D1}|\heartsuit\rangle_{D2}|\downarrow_z\rangle\,|E_{\downarrow\heartsuit}\rangle\,, \tag{57}$$

respectively. But now,

$$\hat{\rho}_{AD2}(\Psi_1) = \frac{1}{2}|R_\uparrow\rangle_A|\heartsuit\rangle_{D2}\langle\heartsuit|_{D2}\langle R_\uparrow|_A + \frac{1}{2}|R_\downarrow\rangle_A|\diamondsuit\rangle_{D2}\langle\diamondsuit|_{D2}\langle R_\downarrow|_A \tag{58}$$

and

$$\hat{\rho}_{AD2}(\Psi_2) = \frac{1}{2}|R_\downarrow\rangle_A|\heartsuit\rangle_{D2}\langle\heartsuit|_{D2}\langle R_\downarrow|_A + \frac{1}{2}|R_\uparrow\rangle_A|\diamondsuit\rangle_{D2}\langle\diamondsuit|_{D2}\langle R_\uparrow|_A, \tag{59}$$

and so $\hat{\rho}_{AD2}(\Psi_1) \neq \hat{\rho}_{AD2}(\Psi_2)$.

Of course, $\hat{\rho}_{AD2}(\Psi_1)$ and $\hat{\rho}_{AD2}(\Psi_2)$ specify isolated and symmetric Everettian macrostates, in our sense of these terms; note that (58) and (59) take the form of the right-hand side of (29). Thus, if Sebens and Carroll were to modify **ESP-QM** to track symmetric macrostates instead of identical microstates, they would essentially recover our core approach. This move would accommodate McQueen and Vaidman's desire to treat fully decohered branches while altering very little of the structure of their proof. We believe that they should accommodate this desire, as their cases of interest are macroscopic states of affairs—i.e., primarily, events corresponding to measurement outcomes.

McQueen and Vaidman also reject the global conception of branching on metaphysical grounds. They present an alternative metaphysics, which they illustrate as follows:[9]

[9]See also Vaidman (2020).

> [...] when you see [a notebook in which has been recorded the result of a Stern–Gerlach experiment] on your desk and ask, 'what is the probability that I am in the world with the spin-↑ result?', you ask a meaningless question. For prior to having branched, there is only one 'you' that is in *both* the world with the spin-↑ result and the world with the spin-↓ result. (McQueen and Vaidman, 2019, p. 16)

On one sensible reading of this passage, McQueen and Vaidman seek to apply a Lewisian 'overlap' view to a local de-idealization of decoherence of the sort sketched in Section 5.4, such that the number of objects or selves or whatever else in a given spatial region depends on whether decoherence has reached it.[10]

The global and local branching pictures are certainly incompatible, but we think that each seems plausible. More to the point: neither view makes a difference to the subsystem structure of Everettian macrostates for non-relativistic quantum mechanics, which isolate branches only after the interaction Hamiltonian stops dominating the dynamics of decoherence. In other words, branching could occur everywhere at once or spread in space and time, and both options yield the same isolated macrostates once decoherence has finished. Thus, if Sebens and Carroll were to adopt our core approach, they could (a) generalize their derivation to apply to agents for longer than artificially short time scales *and* (b) retain their global view of branching.

6.3 McQueen and Vaidman's three-branch, equal-weight case

McQueen and Vaidman claim to derive chance values with just two principles: *symmetry*, which says that 'if the physical situation has a particular symmetry, whatever will happen should respect this symmetry'; and *local supervenience*, which says that 'whatever happens in region A depends only on the quantum description of this region and its immediate vicinity' (2019, p. 17). They do not offer formal characterizations of these principles, but it is clear that their local conception of branching underpins both.

As such, McQueen and Vaidman's finite-branch, equal-weight case focuses on a case with spatially distant, sharply-localized wavepackets:

> Consider a particle in a superposition of three very far apart well localised wave packets centered at positions labeled A, B, and C. These three positions lie on the circumference of a perfect circle such that the particle has three-fold rotational symmetry. That is, the system appears the same when it is rotated by one third of a full turn about the circle's center. Three identical detectors, distributed over those same positions, which also exhibit three-fold rotational symmetry, simultaneously measure the presence of the particle.
>
> The quantum mechanical description is as follows. The initial state of the particle is:

$$(1/\sqrt{3})\left(|a\rangle + |b\rangle + |c\rangle\right) = \frac{1}{\sqrt{3}}\left(|1\rangle_{\mathbf{a}}|0\rangle_{\mathbf{b}}|0\rangle_{\mathbf{c}} + |0\rangle_{\mathbf{a}}|1\rangle_{\mathbf{b}}|0\rangle_{\mathbf{c}} + |0\rangle_{\mathbf{a}}|0\rangle_{\mathbf{b}}|1\rangle_{\mathbf{c}}\right), \quad (60)$$

[10] As they say in a footnote: 'We do not consider the self as a "history" in a sense which leads to a "divergent" instead of an overlapping picture' (McQueen and Vaidman, 2019, fn. 8).

where the right-hand-side is given in Fock representation, to help clarify the role that locality will play in our argument. The interaction process is described by

$$(1/\sqrt{3})\left(|1\rangle_{\mathbf{a}}|0\rangle_{\mathbf{b}}|0\rangle_{\mathbf{c}} + |0\rangle_{\mathbf{a}}|1\rangle_{\mathbf{b}}|0\rangle_{\mathbf{c}} + |0\rangle_{\mathbf{a}}|0\rangle_{\mathbf{b}}|1\rangle_{\mathbf{c}}\right)|R\rangle_{\mathbf{A}}|R\rangle_{\mathbf{B}}|R\rangle_{\mathbf{C}} \tag{61}$$

$$\begin{aligned}\rightarrow (1/\sqrt{3})(&|1\rangle_{\mathbf{a}}|0\rangle_{\mathbf{b}}|0\rangle_{\mathbf{c}}|\checkmark\rangle_{\mathbf{A}}|R\rangle_{\mathbf{B}}|R\rangle_{\mathbf{C}} + \\ &|0\rangle_{\mathbf{a}}|1\rangle_{\mathbf{b}}|0\rangle_{\mathbf{c}}|R\rangle_{\mathbf{A}}|\checkmark\rangle_{\mathbf{B}}|R\rangle_{\mathbf{C}} + \\ &|0\rangle_{\mathbf{a}}|0\rangle_{\mathbf{b}}|1\rangle_{\mathbf{c}}|R\rangle_{\mathbf{A}}|R\rangle_{\mathbf{B}}|\checkmark\rangle_{\mathbf{C}})\end{aligned} \tag{62}$$

$$\rightarrow \begin{cases} |1\rangle_{\mathbf{a}}|0\rangle_{\mathbf{b}}|0\rangle_{\mathbf{c}}|\checkmark\rangle_{\mathbf{A}}|R\rangle_{\mathbf{B}}|R\rangle_{\mathbf{C}} & prob_A \\ |0\rangle_{\mathbf{a}}|1\rangle_{\mathbf{b}}|0\rangle_{\mathbf{c}}|R\rangle_{\mathbf{A}}|\checkmark\rangle_{\mathbf{B}}|R\rangle_{\mathbf{C}} & prob_B \\ |0\rangle_{\mathbf{a}}|0\rangle_{\mathbf{b}}|1\rangle_{\mathbf{c}}|R\rangle_{\mathbf{A}}|R\rangle_{\mathbf{B}}|\checkmark\rangle_{\mathbf{C}} & prob_C \end{cases} \tag{63}$$

where signify the ready and clicked states of the detector in A, etc. The symmetry between A, B and C is explicit before measurement interaction (61). It is still explicit during the measurement unitary evolution (62). The symmetry principle then also requires that it will be manifested in the mixture (63) after the measurement. Therefore, $prob_A = prob_B = prob_C = 1/3$. (McQueen and Vaidman, 2019, p. 17)

Curiously, the fact that the symmetry is achieved by rotation of spatially distant states does not appear to play any special role in the derivation. In the end, the symmetries linking the three states are special cases of unitary transformations of branch states, as evidenced by (62). In particular, as we covered in our exposition of the core approach, there will be pairs of environmental and system unitary symmetries that translate between any pair of the three branch states in equation (63) regardless of whether these states represent spatially distant systems.

Moreover, focusing on spatially distant, well-localized states seems to hamstring the scope of the derivation. Indeed, in order to achieve the generality of the core approach, they are forced to tie arbitrary spatially-localized measurements to this rather specialized class of states:

> The proof above considers a quantum state of a particle placed in different locations, and it is not applicable directly to a quantum state of a local system like a spin. But our proof is a consistency check: Born rule violation is inconsistent with relativity theory. Born rule violation for a local measurement of a spin will also lead to inconsistency. Quantum theory considers various legitimate methods of measurements. In particular, we can arrange a unitary swap mechanism between the spin state and the spatial state of the particle:
>
> $$(\alpha|\uparrow\rangle + \beta|\uparrow\rangle)|ready\rangle \rightarrow |\uparrow\rangle(\alpha|a\rangle + \beta|b\rangle). \tag{64}$$
>
> After this procedure we make a measurement of where the particle is. We have proved that this final measurement should comply with the Born rule, and the consistency of quantum theory tells us that direct local measurements of the spin should follow the Born rule too. (McQueen and Vaidman, 2019, p. 18)

Note that it might well be the case that a system's spin degrees of freedom are isolated from its spatial degrees of freedom (in the microstate sense). This possibility licenses us to, for example, study the spin-chain decoherence model (34) without reference to any other system properties. Here, however, McQueen and Vaidman seem to render the symmetries of spin-only systems reliant on hidden degrees of freedom, threatening these systems' sectorhood.

Charitably, McQueen and Vaidman aim to give a first pass at how a chance derivation should go in a relativistic generalization of the Everettian's non-relativistic theory. They note briefly that their proof is meant to apply to 'the relativistic generalisation of the Schrödinger equation' (p. 19), but they do not specify which generalisation they have in mind (among, e.g., the Dirac equation, the Klein–Gordon equation, and so on). We think this project is a worthy one, although we would press McQueen and Vaidman to (a) provide an explicit, localized de-idealization of decoherence (of the sort sketched in Section 5.4) and (b) precisely characterize the relevant dynamical symmetries in the resulting picture. Moreover, it would be quite strange if such a project were to *invalidate* the core approach; one would naturally expect the latter to emerge from the low-velocity limit of the former. In such a case, McQueen and Vaidman should have no issue with decision-theorists or ESPers who adopt the core.

7 Discussion

We have argued that the apparent tensions among Everettian approaches to chance using uncertain or partial beliefs dissolve once we shift our attention to what these Everettians think is certain. They need to be certain of a given subsystem's isolation to address the reference class problem that hounds any approach to chance, and we offer a thin functional analysis of isolation in terms of (I1) instantiating a recursive subsystem structure and (I2) enabling empirical control. Getting clear on the subsystem structures that Everettians need enables us to unite decision-theoretic and self-locating uncertainty approaches under a thin common core—and this core clarifies why the preferred basis problem does not pose any threat of circularity.

It is not all wine and roses for Everettians, however! We ended up needing to specify two different subsystem structures corresponding to two different notions of isolation: one for microstates and one for macrostates. In one sense, this move is anathema to the Everettian ethos, which aims to unify all of quantum theory under a single dynamics. In another sense, the move amounts to a natural incorporation of lessons from statistical mechanics into an Everettian worldview. The extent to which this latter perspective might take precedence over the former, however, is a matter for future work.

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