Why Macroscopic Particles Obey Newtonian Dynamics

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It was recently shown that the Born rule for transition probabilities of a microscopic particle can be derived from the linear Schrödinger equation with a Hamiltonian represented by random matrices from the Gaussian Unitary Ensemble, taken to be independent at different time points. Under such an evolution, the particle's state undergoes an isotropic random walk on the projective state space, and the relative frequencies of reaching different regions obey the Born rule. In this work, we extend these results to demonstrate how the same framework accounts for the emergence of Newtonian dynamics for macroscopic bodies on a submanifold of state space, thereby providing a unified geometric account of the quantum-classical transition.

I. OVERVIEW

At the heart of the divide between quantum and classical physics lies the superposition principle. Microscopic systems may exist in a superposition of states, such as being in two places at once or being both decayed and undecayed, until measured. In classical physics, by contrast, a particle cannot simultaneously occupy two locations, and a cat cannot be both alive and dead. Bridging this conceptual gap, reconciling the linear framework of quantum mechanics with the inherently nonlinear character of classical physics, has been a challenge for over a century, and the debate continues to this day.

Measurement lies on the other side of this tension. In the quantum domain, measurements are intrinsically probabilistic, with outcomes determined by the Born rule, a result that does not follow from Schrödinger dynamics. In the macroscopic world, measurements, such as those of position, are usually treated as deterministic, yielding a single, definite result. Yet this determinism is only an approximation, valid within certain limits. Experimentalists measuring a macroscopic particle are well aware of unavoidable errors and the need to estimate and account for them to ensure reliable results.

Despite the probabilistic nature of measurement errors in a macroscopic system, the distribution of outcomes can, in principle, be predicted from Newtonian equations of motion. For example, interactions between the particle, the measuring apparatus, and the surrounding environment can cause the particle's position to undergo a random walk during the measurement interval, typically yielding a normal distribution of outcomes. Determining whether an analogous dynamical mechanism underlies measurement in the microworld, and clarifying the relationship between the Born rule and the classical normal distribution of measurement outcomes, is essential to resolving the question of how the quantum world gives rise to the classical one.

There exist two submanifolds of the projective state space of a microscopic particle, equipped with the Fubini-Study metric, that are directly related to the interplay between dynamics and measurement in the microscopic and macroscopic realms [1–3]. One can be identified with the six-dimensional classical phase space and the other with the three-dimensional classical configuration space of the particle, both endowed with the induced Euclidean metric. Specifically, the Schrödinger evolution of a state constrained to the six-dimensional submanifold is identical to the Newtonian evolution of the particle in classical phase space. Likewise, the Born rule for a state constrained to the three-dimensional submanifold is equivalent to the normal probability distribution for the particle's position in classical space. Conversely, under suitable conditions, the Schrödinger evolution and the Born rule are the unique extensions of Newtonian motion and the normal probability distribution, respectively, to the particles full state space. Analogous results hold for systems containing any number of particles.

The existence of these submanifolds allows the transition between the microscopic and macroscopic realms to be reformulated as two distinct tasks: (1) identifying the source of the constraint on the states of macroscopic bodies, and (2) explaining, via this constraint, the dynamical origin of the relationship between the Born rule in state space and the normal distribution of position in classical space.

The second task was addressed in [1]. It was shown that, under suitable conditions, the random walk of position used to model macroscopic measurements extends uniquely to a unitary process on the full state space. This extended walk is governed by the Schrödinger equation with a Hamiltonian represented, at each time point, by an independent random matrix from the Gaussian Unitary Ensemble. Under such evolution, the state performs a random walk with the same statistical properties as in Einstein's theory of Brownian motion [4], but now on the state space.

When constrained to the classical-space submanifold, the walk satisfies the diffusion equation in the appropriate limit; unconstrained, it yields the Born rule for transition probabilities. The transition from the normal probability distribution on the submanifold to the Born rule on state space arises from the highly nontrivial embedding of the Euclidean classical-space submanifold, as a complete set, into the state space. The linearity of Schrödinger evolution with a random Hamiltonian does not preclude Born-rule statistics, since in this setting outcome probabilities depend only on the distance between the initial and final states, not on the form of the initial state.

The appearance of random matrices in the description of measurement on a microsystem within this framework can be regarded as an instance of the Bohigas-Giannoni-Schmit (BGS) conjecture [5] in a new time-dependent setting. This conjecture originates from Wigner's work on the spectra of heavy nuclei [6] and, in general terms, asserts universal applicability of random matrix theory to fluctuations in quantum systems.

Beyond providing a unified framework for measurement in microscopic and macroscopic systems, and a clear, geometry-based link between Schrödinger and Newtonian dynamics, the approach also offers a natural explanation of how a particle can exhibit both corpuscular and wave-like behavior. When the state lies close to the classical-space or classical-phase-space submanifolds, it follows Newtonian dynamics and behaves like a classical particle, with well-defined position and momentum. When the state moves away from these submanifolds, the particle has no definite position in the classical sense, while still having a well-defined position in the state space, and exhibits wave-like properties.

Here, we complete the partial answers to the first task given in [1], clarifying why, and in what precise sense, the state of a macroscopic particle under these conditions must be constrained to the classical-space submanifold. We explain how locality in the state space manifests as nonlocality on the classical-space submanifold, further connect measurements in microscopic and macroscopic systems, and illustrate these results with the example of the double-slit experiment.

II. REVIEW OF THE FORMALISM

Consider the action functional

$$S[\varphi] = \int \overline{\varphi}(\mathbf{x},t)\left[i\hbar\frac{\partial}{\partial t} - \widehat{h}\right]\varphi(\mathbf{x},t)\,d^3\mathbf{x}\,dt, \qquad (1)$$

where the Hamiltonian $\widehat{h}$ is given by

$$\widehat{h} = -\frac{\hbar^2}{2m}\Delta + \widehat{V}(\mathbf{x},t). \qquad (2)$$

Variation of the action functional with respect to φ, yields the Schrödinger equation for a particle.

Let $M^{\sigma}_{3,3}$ denote the submanifold of the projective state space $\mathbb{CP}^{L_2}$ consisting of states of the form

$$\varphi(\mathbf{x}) = r_{\mathbf{a},\sigma}(\mathbf{x})\,e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}, \qquad (3)$$

where

$$r_{\mathbf{a},\sigma}(\mathbf{x}) = \sigma^{-\frac{3}{2}}\,r\left(\frac{\mathbf{x}-\mathbf{a}}{\sigma}\right), \qquad (4)$$

and $r \in L_2(\mathbb{R}^3)$ is any real-valued, twice-differentiable, unit-normalized function with finite variance (taken to be 1). It is further assumed that σ is sufficiently small and that $\mathbf{p} \in \mathbb{R}^3$. As $\sigma \to 0$, the sequence $r^2_{\mathbf{a},\sigma}(\mathbf{x})$ converges to the delta function [7]. A typical example of $r_{\mathbf{a},\sigma}(\mathbf{x})$ is the Gaussian function

$$g_{\mathbf{a},\sigma}(\mathbf{x}) = \left(\frac{1}{2\pi\sigma^2}\right)^{3/4}\exp\left[-\frac{(\mathbf{x}-\mathbf{a})^2}{4\sigma^2}\right], \qquad (5)$$

centered at $\mathbf{a} \in \mathbb{R}^3$.

By constraining φ in (1) to lie on the manifold $M^{\sigma}_{3,3}$ with a sufficiently small σ, the action functional takes the classical form

$$S = \int \left[\mathbf{p}\cdot\frac{d\mathbf{a}}{dt} - h(\mathbf{p},\mathbf{a},t)\right]dt, \qquad (6)$$

where

$$h(\mathbf{p},\mathbf{a},t) = \frac{\mathbf{p}^2}{2m} + V(\mathbf{a},t) \qquad (7)$$

is the Hamiltonian function of the system. Variation of (1) under the constraint that $\varphi \in M^{\sigma}_{3,3}$ then yields the Newtonian equations of motion.

The parameter σ is taken to be set by the resolution of the position-measuring instruments relevant to the particle. Since the function r can be chosen freely, the manifold $M^{\sigma}_{3,3}$ may be defined through equivalence classes of functions with spread bounded by σ. The transition to Newtonian dynamics for such states aligns with the predictions of the Ehrenfest theorem for sufficiently narrow wave packets, provided this narrow form is preserved throughout the evolution. For simplicity, we will represent the equivalence classes by Gaussian states $g_{\mathbf{a},\sigma}$.

Writing an arbitrary wave packet in the form

$$\varphi(\mathbf{x}) = r_{\mathbf{a},\sigma}(\mathbf{x})\,e^{i\Theta(\mathbf{x})}, \qquad (8)$$

and expanding $\Theta(\mathbf{x})$ in a power series around $\mathbf{a}$, we may neglect quadratic and higher-order terms when σ is small, since $r_{\mathbf{a},\sigma}(\mathbf{x})$ vanishes rapidly away from $\mathbf{x} = \mathbf{a}$. This shows that, under these conditions, (3) gives the general form of the wave packet. The packets with spread bounded by σ and with a given expectation value of the position form equivalence classes of states corresponding to points of $M^{\sigma}_{3,3}$. A precise definition of these equivalence classes will be given in Section IV.

The Fubini-Study metric on $\mathbb{CP}^{L_2}$ induces a Riemannian metric on $M^{\sigma}_{3,3}$. With an appropriate choice of units, the map

$$\Omega : (\mathbf{a},\mathbf{p}) \longmapsto g_{\mathbf{a},\sigma}\,e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}$$

is an isometry between the Euclidean space $\mathbb{R}^3 \times \mathbb{R}^3$ and the Riemannian manifold $M^{\sigma}_{3,3}$. Moreover, a linear structure on $M^{\sigma}_{3,3}$ can be induced via Ω from the linear structure on $\mathbb{R}^3 \times \mathbb{R}^3$. The restricted map

$$\omega : \mathbf{a} \longmapsto g_{\mathbf{a},\sigma}$$

is an isometry between the Euclidean space $\mathbb{R}^3$ and the Riemannian submanifold $M_3^\sigma \subset \mathbb{CP}^{L_2}$ consisting of the states $g_{\mathbf{a},\sigma}$ [8–10]. This property remains valid if $g_{\mathbf{a},\sigma}$ is replaced by any other representative $r_{\mathbf{a},\sigma}$ of the class.

The relationship between the action functionals (1) and (6) allows us to identify classical particles, i.e., systems satisfying Newtonian dynamics, with quantum systems whose states are constrained to the manifold $M_{3,3}^\sigma$ for an appropriate value of σ. The map Ω thus provides a direct identification of the Euclidean phase space $\mathbb{R}^3 \times \mathbb{R}^3$ of positions and momenta $(\mathbf{a}, \mathbf{p})$ for a classical particle with the manifold $M_{3,3}^\sigma$ of quantum states φ in (3).

As shown in [3], the velocity of a state $\varphi \in M_{3,3}^\sigma \subset \mathbb{CP}^{L_2}$ under Schrödinger evolution with Hamiltonian (2) can be decomposed into three mutually orthogonal components. The first two coincide with the classical velocity and acceleration of the particle and remain tangent to the classical-phase-space manifold $M_{3,3}^\sigma$. The third, orthogonal to the manifold, represents the spreading velocity of the particles state function.

In the Fubini-Study metric, the squared norm of the state velocity is the sum of the squares of these components and is given by

$$\left\| \frac{d\varphi}{dt} \right\|_{FS}^2 = \frac{\mathbf{v}^2}{4\sigma^2} + \frac{m^2\mathbf{w}^2\sigma^2}{\hbar^2} + \frac{\hbar^2}{32\sigma^4 m^2}, \tag{9}$$

where $\mathbf{v}$ is the classical velocity and

$$\mathbf{w} = -\frac{\nabla V}{m}$$

is the classical acceleration of the particle. The three terms in (9) have clear physical interpretations: the first term corresponds to translational motion of the wave packets center, the second to acceleration induced by the potential, and the third to intrinsic quantum spreading of the wave packet.

Imposing the classical constraint amounts to requiring that the orthogonal (spreading) component of the state velocity

$$\frac{d\varphi}{dt} = -\frac{i}{\hbar}\widehat{h}\varphi$$

vanish. In this setting, commutators of observables reduce to Poisson brackets, thereby transforming the Schrödinger dynamics of the constrained state into the Newtonian dynamics of the particle [3]. Physically, removing the orthogonal component eliminates purely quantum effects, isolating the classical trajectory in phase space while preserving the geometric structure of the underlying state space.

The embedding of classical configuration space and classical phase space into the quantum state space establishes a direct relationship between Schrödinger and Newtonian dynamics. This correspondence allows us to identify classical particles with quantum systems whose states are constrained to the manifold $M_{3,3}^\sigma$. It also underlies the connection between the normal probability distribution, typical for position measurements of a particle in $\mathbb{R}^3$, and the Born rule governing transition probabilities between quantum states.

In particular, applying the Born rule to the state $g_{\mathbf{a},\sigma}$ yields the normal distribution for the position of a classical particle in $\mathbb{R}^3$. Conversely, assuming a normal probability distribution for the particles position in $\mathbb{R}^3$ and considering the probability of finding the particle in a region recovers the Born rule for transitions between the associated quantum states. If the transition probabilities depend only on the Fubini-Study distance between states, this equivalence extends to all transitions within the state space [1].

The derivation of this result relies on the following relationship between the distance between the states $g_{\mathbf{a},\sigma}$ and $g_{\mathbf{b},\sigma}$ in the Fubini-Study metric on $\mathbb{CP}^{L_2}$ and the Euclidean distance between the corresponding points $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$:

$$e^{-\frac{(\mathbf{a}-\mathbf{b})^2}{4\sigma^2}} = \cos^2 \rho\big(g_{\mathbf{a},\sigma}, g_{\mathbf{b},\sigma}\big). \tag{10}$$

In (10), the quantity $\rho(g_{\mathbf{a},\sigma}, g_{\mathbf{b},\sigma})$ denotes the geodesic distance between the two states in the full projective state space, whereas $|\mathbf{a}-\mathbf{b}|$ is the Euclidean distance between the same states measured along a geodesic within the submanifold M_3^σ.

The distance between the states $\varphi(\mathbf{x}) = g_{\mathbf{a}}(\mathbf{x})e^{i\mathbf{px}/\hbar}$ and $\psi(\mathbf{x}) = g_{\mathbf{b}}(\mathbf{x})e^{i\mathbf{qx}/\hbar}$, measured using the Fubini-Study metric on $\mathbb{CP}^{L_2}$, is related to the Euclidean distance between the corresponding points in the classical phase space $\mathbb{R}^3 \times \mathbb{R}^3$ by a similar formula:

$$e^{-\frac{(\mathbf{a}-\mathbf{b})^2}{4\sigma^2} - \frac{(\mathbf{p}-\mathbf{q})^2}{\hbar^2/\sigma^2}} = \cos^2 \rho(\varphi, \psi). \tag{11}$$

The correspondence established between classical and quantum systems, and between the normal probability distribution and the Born rule, was used in [3] to place measurements performed on classical and quantum systems on an equal footing. In particular, the following proposition, based on Wigner's work [6], the Bohigas-Giannoni-Schmit conjecture [5], and further developed in [3], was introduced:

> **(RM)** *The dynamics of a particle's state under position measurement can be modeled as a random walk in the space of states. In the absence of drift, the steps of this random walk satisfy the Schrödinger equation, where the Hamiltonian at each instant is represented by a random matrix from the Gaussian Unitary Ensemble (GUE). The Hamiltonians at different times are statistically independent.*

Here, the abbreviation **(RM)** stands for "random matrices." Physically, the Hamiltonian in **(RM)** may result from a highly complex interaction between the measured particle and the measuring device or its environment, modeled as a complicated sum of one-particle Hamiltonians with interaction terms. This is reminiscent of Wigner's model for the Hamiltonian of a heavy nucleus [6].

A small step in the states random walk, driven by the Hamiltonian in **(RM)**, is represented by a random vector in the tangent space to $\mathbb{CP}^{L_2}$. As shown in [3], the distribution of such steps is normal, homogeneous, and isotropic. In particular, the orthogonal components of a step at any point are independent, identically distributed normal random variables. These properties imply that the transition probability between two states connected by the walk depends only on their Fubini-Study distance.

When the steps of the walk are constrained to M_3^σ, the transition probability is given by the normal probability density function. In this case, the random walk of the state approximates Brownian motion on $\mathbb{R}^3$, making it an appropriate model for classical measurement. Since the transition probability $P(\varphi, \psi)$ between two states depends only on the distance between them, and the probability density function for φ and $\psi \in M_3^\sigma$ is normal, it follows that $P(\varphi, \psi)$ is governed by the Born rule [3].

Thus, both the normal probability distribution characteristic of classical measurements and the Born rule for transition probabilities between general quantum states emerge from Schrödinger evolution with a Hamiltonian satisfying **(RM)**. Because Brownian motion is governed by the diffusion equation, the dynamical basis of the Born rule and the normal probability distribution in this model can be stated as follows: the Schrödinger equation with a Hamiltonian satisfying **(RM)** reduces to the diffusion equation on $\mathbb{R}^3$ [3]. This reduction provides the dynamical link between classical and quantum measurements, placing them on equal footing.

Consider now a system of n particles and the tensor-product manifold $\otimes_n M_3^\sigma$, whose elements have the form $g_1 \otimes \cdots \otimes g_n$, where each $g_k \in M_3^\sigma$ represents the state of the kth particle. Similarly, define the manifold $\otimes_n M_{3,3}^\sigma$ as the set of tensor products of particle states in $M_{3,3}^\sigma$. The correspondence between Schrödinger and Newtonian dynamics established for a single particle extends naturally to systems of multiple interacting particles. In particular, a two-particle system whose state is constrained to the manifold $M_{3,3}^\sigma \otimes M_{3,3}^\sigma$ evolves according to Newtonian dynamics.

The Euclidean metric on M_3^σ extends naturally to the Euclidean metric on the configuration space $\otimes_n M_3^\sigma \cong \mathbb{R}^{3n}$ of an n-particle system. This metric is induced from the metric on the tensor product of the Hilbert spaces describing the particle states. When an additional particle, described by a state φ, is considered alongside the n-particle system in $\otimes_n M_3^\sigma$, the product state $\varphi \otimes g_1 \otimes \cdots \otimes g_n$ of the full system is close to a state $g_{\mathbf{a}} \otimes g_1 \otimes \cdots \otimes g_n \in \otimes_{n+1} M_3^\sigma$ in this metric precisely when φ is close to $g_{\mathbf{a}}$ in the Fubini-Study metric on the state space of a single particle. This observation allows us to focus on the state of the particle rather than the state of the entire system when analyzing a system composed of a microscopic particle and a measuring device. This is because the state of a macroscopic object, such as a typical measuring instrument, in this framework will be shown to lie in a submanifold of the form $\otimes_n M_3^\sigma$. An analogous statement holds for the classical-phase-space submanifold of an n-particle system.

For a classical particle, Brownian motion associated with position measurement takes place in three-dimensional space $\mathbb{R}^3$, where the particle's position can, in principle, be recorded at any point. This is possible because position-measuring devices may be distributed throughout space. The situation is different for microscopic particles whose state under measurement evolves according to **(RM)**. In this case, the state of the particle propagates through the entire state space, while a measuring device can only occupy a submanifold such as M_3^σ, or products of n copies thereof. Consequently, for the position of a microscopic particle to be defined and measurable, its state must first cross the classical-space submanifold M_3^σ within the state space.

In classical terms, the situation is analogous to measuring the position of a Brownian particle in $\mathbb{R}^3$ using detectors arranged only along a line that does not intersect the particle's initial position. The particle's position can be determined only when its trajectory crosses the line, and the probability of it reaching a given segment of the line corresponds to the information encoded by the Born rule.

Our goal in this paper is to demonstrate that, if accepted, the conjecture **(RM)** provides the origin of the constraint of a macroscopic system's state to the classical-space submanifold, offers a criterion for macroscopicity, and accounts for the transition from Schrödinger to Newtonian dynamics.

III. DERIVATION OF NEWTONIAN DYNAMICS FOR MACROSCOPIC BODIES

Newtonian motion of macroscopic bodies presupposes experimentally verifiable knowledge of their position and velocity at any given moment. Physically, this condition can be met through direct observation, for example, by illuminating the body with light at selected times and detecting the scattered radiation. Yet even in the absence of deliberate measurement, the body's position is continually recorded through its unavoidable interactions with the environment and can be inferred from the scattering of environmental particles and radiation. For Newtonian motion to hold without explicitly accounting for the environment, these interactions must remain sufficiently weak. At the same time, they cannot be absent altogether: without them, neither the position nor the momentum of the body could be determined, and the very notion of Newtonian motion would lose its physical meaning.

The body's motion under these conditions may be viewed as free Newtonian motion, punctuated by regular, typically weak encounters with particles of the environment. Collectively, and neglecting damping corrections, encounters on timescales shorter than the relaxation time induce a ballistic thermal spread around the Newtonian mean trajectory. At longer times, damping effects become significant, and the motion crosses over to ordinary diffusion with drift. The body's position is identified with the center of the resulting probability distribution, as revealed by the scattering of environmental particles and radiation. The probability density for the position spreads during the encounters but contracts again whenever they provide positional information. This cycle of spreading and contraction repeats, keeping the variance bounded. This effective "continuous" measurement allows us to meaningfully describe the body's Newtonian path over long times without its positional uncertainty growing unbounded.

We propose that the behavior of a macroscopic body interacting with its environment can be derived from Schrödinger evolution together with the conjecture **(RM)**. The conjecture applies in this context because, as discussed above, the environment continually monitors the body, effectively performing a measurement on it. Before turning to the details of this derivation, we outline its main steps, beginning with an analysis of the motion of a microscopic particle in its natural environment.

For microscopic point particles, occasional interactions with the surroundings can often be neglected, so the particle evolves according to the Schrödinger equation with Hamiltonian $\widehat{h} = -\frac{\hbar^2}{2m}\Delta + \widehat{V}(\mathbf{x}, t)$. As the particle's size and its capacity to interact with the surroundings increase, for instance, progressing from beta to alpha particles or even to the nucleus of a heavier atom, interactions with the environment become unavoidable. These interactions allow the surroundings to acquire information about the particle's position. Evidence that such information is indeed recorded by the environment can be seen, for example, in the localized scintillation produced by a collision with an atom in a scintillating material, or in the vertices of scattering events with surrounding particles or radiation. According to **(RM)**, in this regime the particle's interaction with the environment is effectively governed by a random Hamiltonian with the specified properties.

Assume the particle's initial state lies in M_3^σ, representing, according to (9), a particle at rest in the absence of an external potential gradient. We now consider the action of the Hamiltonian in **(RM)** on this state. By selecting from the walk in **(RM)** only those steps that are tangent to M_3^σ, we obtain a random walk of the state confined to this submanifold. Each step corresponds to a translation of states and, under the isomorphism ω, the resulting process approximates Brownian motion on $\mathbb{R}^3$, governed by the diffusion equation. In this identification, translation of the state manifests as the displacement of the particle in physical space $\mathbb{R}^3$, as expected. In particular, the Brownian motion induced by **(RM)** corresponds to the ordinary Brownian motion of the particle in a suitable medium.

Assuming the validity of the conjecture **(RM)**, the parameters of the Hamiltonian, specifically, the variance of the random matrix entries and the time step, can be selected so as to reproduce existing observations and anticipate possible future ones. These parameters may vary depending on the particle, the measuring device, the environment, and their mutual interactions (see Section IV). Consequently, these factors also determine the characteristics of the induced random walk of the state on $\mathbb{R}^3$. This induced random walk on $\mathbb{R}^3$ approximates Brownian motion and determines the corresponding diffusion coefficient, thereby specifying the effective properties of the medium associated with the particle's induced Brownian motion. Conversely, under the embedding ω, the isotropy and homogeneity of the step distribution ensure that the walk on $\mathbb{R}^3$ originates from a unique random walk in **(RM)**.

When interactions between the particle, the measuring device, and the environment hinder the particles propagation in the associated medium, the diffusion coefficient $\mathbb{D}$ of the induced Brownian motion decreases, approaching zero. The point at which $\mathbb{D}$ becomes negligible within the medium marks the transition from microscopic to macroscopic behavior in this framework. The isotropy of the step distribution in **(RM)** implies that, at this stage, not only is the particle at rest in $\mathbb{R}^3$, but its state is effectively stationary in the state space. In this regime, sufficiently large particles and their states may remain at rest, even under interactions limited to cosmic radiation. An account of this behavior is possible within the conjecture **(RM)**, provided the diffusion coefficient $\mathbb{D}$ is properly linked to the physical properties of the particle, the measuring device, and the environment. In any case, the value of $\mathbb{D}$ alone determines the parameters of the ensemble and can ensure the stationarity of the particle's state under **(RM)**.

Having established the conditions under which **(RM)** accounts for the state of macroscopic bodies at rest, the subsequent task is to derive their Newtonian dynamics. This requires defining the classical space and classical phase space in terms of equivalence classes of states. Recall that position-measuring devices have limited resolution and cannot distinguish between states of sufficiently small support or states localized near the same point in $\mathbb{R}^3$. Consequently, any sufficiently narrow position state function centered at $\mathbf{a} \in \mathbb{R}^3$ may be regarded as representing a particle located at $\mathbf{a}$. Thus, in a position-measurement experiment, one effectively deals with equivalence classes of sufficiently narrow states, rather than with isolated eigenstates.

As shown in Section IV, the classical-space submanifold M_3^σ can be defined in terms of equivalence classes on the set U_σ of real-valued state functions whose standard deviation δ does not exceed the resolution parameter σ of the measuring device. The class associated with a point $\mathbf{a} \in \mathbb{R}^3$ consists of all such functions with expectation value of the position is $\mathbf{a}$. Equipped with a suitable metric, the set of such classes forms the manifold $\widetilde{M}_3^\sigma$, which is isometric to M_3^σ and thus to the Euclidean space $\mathbb{R}^3$.

This approach naturally extends to the classical phase-space submanifold $M_{3,3}^\sigma$. Its points can be defined as equivalence classes in $\widetilde{M}_3^\sigma$ augmented by a phase factor. As discussed in Section II, the phase factor of any sufficiently narrow state function φ can be written as $e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}$. When σ in (4) is small, the parameter $\mathbf{p}$ is approximately equal to the expectation value of the momentum operator, which in this case coincides with the mass times the group velocity of the packet. As shown in Section IV, the manifold $\widetilde{M}_{3,3}^\sigma$, consisting of such equivalence classes and equipped with an appropriate metric, is isometric to $M_{3,3}^\sigma$. As before, as long as the particle's state remains constrained to $\widetilde{M}_{3,3}^\sigma$, Schrödinger evolution is equivalent to Newtonian motion.

Importantly, each equivalence class of states in $\widetilde{M}_{3,3}^\sigma$ contains infinitely many mutually orthogonal members, thereby absorbing most of the dimensions of the state space. For a state initially on $\widetilde{M}_{3,3}^\sigma$, the only aspect of its Schrödinger evolution that drives it away from the classical phase-space submanifold, and thereby yields quantum effects observable in position measurements, is the variation of the standard deviation parameter σ. A glimpse of this is already evident in (9). A suitable coordinate s, introduced in Section IV to parametrize changes in σ, is defined as the logarithm of the scaling parameter. In this case, the metric induced on the s-axis through any state is Euclidean [1].

With **(RM)** accepted, the most general evolution of a particle's state during measurement is governed by the total Hamiltonian $\widehat{h}_{\rm tot}$, defined as the sum of the free Hamiltonian $\widehat{h}$ and the random Hamiltonian $\widehat{h}_{\rm RM}$ of **(RM)**. The free Hamiltonian generates the deterministic drift of the state, while the random Hamiltonian induces the random walk and ensures consistency with the Born rule. As the next step in deriving the Newtonian dynamics of macroscopic bodies, we examine the evolution of a state initially located on the submanifold $M_{3,3}^\sigma$ under this total Hamiltonian.

Over a short time interval, the Hamiltonian $\widehat{h}$ generates a Newtonian displacement of φ along the manifold $M_{3,3}^\sigma$, together with a step along the s-axis. Over a step-time interval, the random Hamiltonian $\widehat{h}_{\rm RM}$ generates one step of an isotropic random walk of the state in the full state space. The component of this step that leaves s unchanged induces either a random displacement within $\widetilde{M}_{3,3}^\sigma$, a trivial transformation confined to the equivalence class of φ, or a combination of the two. Since $\widetilde{M}_{3,3}^\sigma$ is isometric to $M_{3,3}^\sigma = \mathbb{R}^6$, a random displacement within $\widetilde{M}_{3,3}^\sigma$, is equivalent to the displacement experienced by a macroscopic body in the classical phase space under environmental interactions. Thus, a state initially located in $\widetilde{M}_{3,3}^\sigma$ and evolving under the total Hamiltonian $\widehat{h}_{\rm tot}$ can depart from $\widetilde{M}_{3,3}^\sigma$ and acquire non-classical features detectable by a device with position resolution σ, solely through the combined effect of Schrödinger spreading and the random walk along the s-axis.

Owing to the isotropy of the random walk along the s-axis, a state drawn from an appropriate ensemble will, after a few steps, spend approximately half the time satisfying the condition $\delta \leq \sigma$. By selecting the time step of the walk in **(RM)** for a macroscopic body to be sufficiently shorter than the characteristic time required for Schrödinger spreading of its state to become appreciable, for example, to exceed a multiple of the resolution parameter σ, the evolution under the total Hamiltonian $\widehat{h}_{\rm tot}$ ensures that the probability of finding the state in $\widetilde{M}_{3,3}^\sigma$ at any given time is approximately $1/2$. At these instants, the particle's position can, in principle, be confirmed by direct observation and, as discussed earlier, is simultaneously registered by the environment.

The acquisition of positional information resets the random walk in **(RM)**, which then proceeds from the newly registered position of the body's state. Under these conditions, and in terms of representatives of equivalence classes, the state undergoes spreading into a small neighborhood of $M_{3,3}^\sigma$, punctuated by random returns to $M_{3,3}^\sigma$ and successive confirmations of its arrival there, either through environmental monitoring or direct observation. This is fully analogous to the classical motion of a body in a natural environment. The difference is that the random walk now takes place in a neighborhood of $M_{3,3}^\sigma$ within the full state space, rather than being confined to the classical space $M_3^\sigma = \mathbb{R}^3$ or the classical phase space $M_{3,3}^\sigma = \mathbb{R}^6$.

Note again that the properties of the body, the environment, and their interactions are assumed to determine the time-step and variance parameters of the walk. Conversely, by adjusting these parameters, one may also describe the motion of microscopic particles in various media within the same dynamical framework. In particular, the observed similarity between the motion of microscopic particles in a bubble chamber and that of macroscopic bodies in natural surroundings may be explained by assigning similar values of these parameters in the two cases. It may be possible to design experiments to test the viability of **(RM)** for different particles in various media. Potential experiments to test the conjecture are outlined in Section IV.

Note that, since a point in $\widetilde{M}_{3,3}^\sigma$ fully determines the particles position, it may be useful to define the classical space submanifold $\widetilde{M}_3^\sigma$ alternatively in terms of equivalence classes of states with a fixed position expectation value, irrespective of the momentum parameter $\mathbf{p}$. This choice is more convenient when discussing measurements of position and can also be used equivalently to define the metric, as shown in Section IV. As before, we can use the states with $\mathbf{p} = 0$, that is, the states in M_3^σ, to represent the classes.

Once the state reaches $\widetilde{M}_3^\sigma$, it is identified with a position eigenstate. In this framework, "collapse" is nothing more than the random walk of the state in the s-variable toward such an eigenstate. The act of recording that the state has reached $\widetilde{M}_3^\sigma$ is not the collapse itself; by the time it is registered, the state already lies on the submanifold. As experience confirms, an immediate subsequent observation of position does not alter the state but merely verifies it, no additional collapse occurs.

Finally, the cumulative effect of the random walk in the s-variable, together with the confirmation of position whenever the state reaches $\widetilde{M}_3^\sigma$, amounts to the projection of the state onto $\widetilde{M}_3^\sigma$, thereby linking the projection postulate of quantum mechanics to the dynamics specified by **(RM)**. By connecting the successive points on $\widetilde{M}_3^\sigma$, we obtain a trajectory whose image under the isomorphism ω coincides with the Newtonian path of the particle. This completes the derivation of Newtonian dynamics from Schrödinger dynamics supplemented by **(RM)**. The next section provides the details of this derivation.

The following experiment in Newtonian mechanics serves as a classical analogue of the process just described. Consider a plane $\mathbb{R}^2$ in space, densely populated with position-measuring devices that record the particle's location whenever it crosses the plane. Suppose the particle starts on the plane with an initial velocity consisting of a tangent component and a small orthogonal component. Assume further that environmental interactions induce Brownian motion relative to its free-particle trajectory, and that the effect of the measuring devices on the particle can be neglected. The goal is to reconstruct the trajectory of the particle, undergoing such Brownian motion with linear drift, at the times when it intersects the plane. The outcome of such an experiment is consistent with Newtonian motion on $\mathbb{R}^2$, where the particle's position at each instant is normally distributed with bounded variance. Neglecting this uncertainty and the discreteness of the sampling times, the observed trajectory coincides with the orthogonal projection of the particle's full linear trajectory in $\mathbb{R}^3$ onto the plane.

IV. DETAILS OF THE DERIVATION

A. Equivalence classes of states

For simplicity, let us restrict attention to measurements in one dimension, with the Hilbert space $L_2(\mathbb{R})$ of state functions on $\mathbb{R}$ and the corresponding projective state space. Analogous to the case of Gaussian functions on $\mathbb{R}^3$, the Gaussian functions

$$g_{a,\sigma} = \left(\frac{1}{2\pi\sigma^2}\right)^{1/4} e^{-\frac{(x-a)^2}{4\sigma^2}} \tag{12}$$

form a one-dimensional submanifold M_1^σ of the state space, equipped with the induced Euclidean metric.

The set of linear combinations of translations of a given Gaussian function is dense in $L_2(\mathbb{R})$ [7]. We will therefore restrict our attention to superpositions of Gaussian functions of the form (12) and assume finite expected value μ_z

$$\mu_z = \int z|\varphi(z)|^2 dz, \tag{13}$$

and the standard deviation δ_z

$$\delta_z^2 = \int (z-\mu_z)^2|\varphi(z)|^2 dz \tag{14}$$

of the z-coordinate.

Equivalence classes of states were introduced in [1] using the example of a small scintillation screen placed near a point on the z-axis. The state function was taken as a superposition of two narrow Gaussian functions, g_a and g_b in (12), centered at points a and b on the z-axis and assumed to be nearly orthogonal. This construction is directly relevant to the analysis of the double-slit experiment in [1] , but it can be readily generalized.

A flash of light at a point c on the screen defines the particle's position only approximately, within a cell D_c of diameter σ, at least as large as an atom in the scintillation material. The screen therefore cannot distinguish between state functions localized near the same point. The expected value μ_z and standard deviation δ_z provide a natural description of such equivalence classes.

Consider the set of all functions φ in $L_2(\mathbb{R})$ with finite values of μ_z and δ_z, subject to the condition $\delta_z \leq \sigma$. Formally, the equivalence class $\{g_c\}$, referred to as a physical eigenstate of position, consists of all functions in this set with $\mu_z = c$. The conditions

$$\mu_z = c, \qquad \delta_z \leq \sigma,$$

ensure that the probability of detecting the particle in D_c is close to one, with only small weight in the tails outside the cell. If necessary, the condition $\delta_z \leq \sigma$ can be strengthened to $\delta_z \leq k\sigma$, where $k < 1$ is an appropriate parameter.

We define the Fubini-Study distance between a state φ and an equivalence class $\{g_c\}$ as

$$\rho(\varphi, \{g_c\}) = \inf_{\psi \in \{g_c\}} \rho(\varphi, \psi), \tag{15}$$

where $\rho(\varphi, \psi)$ is the Fubini-Study distance between states. For example, if $\varphi = \alpha g_a + \beta g_b$, then under the accepted conditions the distance to $\{g_b\}$ satisfies

$$\cos \rho(\varphi, \{g_b\}) = |\beta|. \tag{16}$$

A state φ reaches the physical eigenstate $\{g_c\}$ precisely when $\rho(\varphi, \{g_c\}) = 0$. Note that the equivalence class $\{g_c\}$ is quite "large": it contains many mutually orthogonal states, that is, states separated by the maximal Fubini-Study distance in state space.

The distance between equivalence classes $\{g_c\}$ and $\{g_d\}$ is defined by

$$\rho(\{g_c\}, \{g_d\}) = \inf_{\varphi \in \{g_c\}} \rho(\varphi, \{g_d\}). \tag{17}$$

This distance coincides with the Fubini-Study distance between the states g_c and g_d in (12). This allows us to identify the set $\widetilde{M}_1^\sigma$ of all equivalence classes $\{g_c\}$, with $c \in \mathbb{R}$, with the previously defined Riemannian manifold M_1^σ, which is simply the Euclidean space $\mathbb{R}$.

The two-dimensional manifolds $M_{1,1}^\sigma$ and $\widetilde{M}_{1,1}^\sigma$ are defined analogously to the manifolds $M_{3,3}^\sigma$ and $\widetilde{M}_{3,3}^\sigma$. Distances between a state and an equivalence class of $\widetilde{M}_{1,1}^\sigma$, as well as those between two equivalence classes, are defined in the same way as in (15) and (17). The isometry between $M_{1,1}^\sigma$, $\widetilde{M}_{1,1}^\sigma$, and $\mathbb{R}^2$ is established using (11). As in the Overview section, Schrödinger evolution constrained to $\widetilde{M}_{1,1}^\sigma$ reproduces the Newtonian dynamics of the particle.

Collapse of the state under a position measurement, as described by Schrödinger evolution with the Hamiltonian in **(RM)**, is the approach of the state to the manifold $\widetilde{M}_1^\sigma$ in the metric (15). This process does not require the tails of the state-function to vanish. The notion of distance between points in the support of state functions, including those in the “tails,” is replaced by the distance between the state functions themselves. State functions with infinite tails may still lie at a small distance from the classical one-dimensional space $\widetilde{M}_1^\sigma = \mathbb{R}$, allowing them to approach it within a few steps of the random walk in **(RM)**. The tails of the state function do not imply that, under measurement, the particle makes large jumps along $\mathbb{R}$. Thus, the so-called problem of tails is resolved by recognizing collapse as a motion in state space, whereby the initial state itself approaches an equivalence class $\{g_c\}$ in $\widetilde{M}_1^\sigma$.

B. Foliation of state space

Given the infinite number of linearly independent directions for propagation in state space, it might seem that the probability of reaching a position eigenstate under the random walk in **(RM)** is zero. It is important to recognize, however, that the only parameters relevant for determining whether the state has collapsed, i.e., belongs to an equivalence class $\{g_c\}$, are the expectation value μ_z and the standard deviation δ_z of the z coordinate. This suggests that the effective random walk of interest may take place on a two-dimensional submanifold of the state space, whose points are identified by specific values of μ_z and δ_z.

As already noted, the set of finite linear combinations of translations of a single Gaussian function is dense in $L_2(\mathbb{R})$. Moreover, any function in $L_2(\mathbb{R})$ can be approximated by a finite linear combination of nearly orthogonal Gaussian states, with the degree of orthogonality controlled by the standard deviation parameter σ. In light of the finite resolution of measuring devices and the resulting equivalence classes of states, such approximations remain appropriate even for a fixed σ.

Consider, therefore, the space V of finite linear combinations of Gaussian functions $g_c \in M_1^\sigma$, where $c = z_k$ corresponds to a partition $\{z_k\}$, $k = 1, 2, \ldots, N$ of the z-axis. Assume further that the Gaussians g_c are sufficiently narrow so that approximate orthogonality holds for distinct z_k and z_m in the partition. In this way, V provides a finite-dimensional approximation to the Hilbert space, reflecting the finite resolution of physical detectors.

Given an initial state $\varphi \in V$ of the particle whose position is measured, consider the two-dimensional manifold M_φ parametrically defined by

$$\varphi_{\tau,\lambda}(z) = \sqrt{\lambda}\varphi(\lambda(z - \mu_z - \tau) + \mu_z). \qquad (18)$$

The parameters τ and λ serve as coordinates on this manifold. Along the path $\varphi_\tau = \varphi_{\tau,\lambda}|_{\lambda=\lambda_0}$ with fixed λ, the expectation value of z shifts from μ_z to $\mu_z + \tau$, while the standard deviation remains constant. Conversely, along the path $\varphi_\lambda = \varphi_{\tau,\lambda}|_{\tau=\tau_0}$ with fixed τ, the standard deviation changes from δ_z to δ_z/λ, while the expectation value remains unchanged. Introducing $s = \ln \lambda$, one finds that the coordinates (τ, s) are orthogonal in the Fubini-Study metric, thereby identifying M_φ with the Euclidean plane $\mathbb{R}^2$ [1].

For each point $(\tau, \lambda) \in M_\varphi$, consider the set of all functions in V with $\mu_z = \tau$ and $\delta_z = \lambda$. This defines a foliation of V of codimension two. Each leaf $\varphi_{\tau,\lambda}$ consists of all functions in V sharing the same values of μ_z and δ_z. Because (τ, s) are orthogonal coordinates on M_φ, the corresponding components of a step of the random walk in **(RM)** from any φ are independent random variables. Since the step distribution in **(RM)** is homogeneous, the probability laws for the τ- and s-components are the same at all points φ. By definition, μ_z and δ_z remain constant along the leaves of M_φ. Consequently, step components tangent to the leaf through φ do not change μ_z or δ_z, and hence do not contribute to collapse into a physical eigenstate of position. It follows that the random walk on M_φ, identified with the (τ, s) plane $\mathbb{R}^2$, suffices to describe collapse in this setting.

To belong to an equivalence class $\{g_c\}$, the state must satisfy the condition $\delta_z \leq \sigma$. On the s-axis, the set of points with $\delta_z \leq \sigma$ corresponds to a half-line. Because the random walk in **(RM)** along the s-axis is symmetric, the state spends equal amounts of time, in the long run, on either side of this boundary. Consequently, the probability of finding the state at a random time with $\delta_z \leq \sigma$ approaches $1/2$ as the number of steps increases. Meanwhile, the random walk in τ determines the relative probabilities of reaching different eigenstates, in accordance with the Born rule [1].

C. Parameters of the random walk in (RM)

Known models of spontaneous collapse, such as CSL and DP, introduce parameters including the collapse rate

λ, localization length r_C, mass coupling, and the spectral properties of the collapse noise, all of which must remain consistent with experiment. Laboratory tests impose strong bounds. Collapse models predict spontaneous energy injection, such as X-ray or atomic radiation, which rules out large collapse rates [11, 12]. The persistence of quantum interference in large molecules and cold-atom ensembles excludes values of λ high enough to suppress interference [13]. Further constraints arise from collapse-induced heating and decoherence in optomechanical systems and from precision noise measurements in gravitational-wave detectors [14–16]. While laboratory bounds are already stringent, cosmological ones are even stronger. Collapse noise would otherwise overheat the intergalactic medium, contrary to astrophysical observations, requiring the collapse rate to be extremely small [17–20].

Unlike standard collapse models such as GRW or CSL, where collapse is a spontaneous and ever-present process that injects energy into the system, these issues do not arise in the proposed collapse by random walk in **(RM)**. Here, "collapse" is not spontaneous but is triggered by interaction with a measuring device or the environment, as specified by the conjecture **(RM)**. Consequently, interference experiments remain unaffected. Although the energy of the measured particle may fluctuate, either increasing or decreasing, after a step in the random walk of **(RM)**, the evolution is unitary, and any excess or deficit can be balanced by the measuring device or the environment, ensuring conservation of the total energy of the combined system, in agreement with classical measurement.

In **(RM)**, the Gaussian Unitary Ensemble is specified by the matrix dimension and the variance of the distribution of its entries. Since the random walk in **(RM)** evolves in time, it also requires a time-step parameter. These parameters determine the diffusion coefficient $\mathbb{D}$ of the induced Brownian motion. Conversely, the diffusion coefficient $\mathbb{D}$ and the time step of the random walk on $\mathbb{R}^3$ fully define the walk in **(RM)**.

In the standard theory of Brownian motion, the diffusion coefficient is typically expressed in terms of the particle's radius, the viscosity of the medium, and the temperature. When modeling measurement with **(RM)**, the induced random walk on $\mathbb{R}^3$ can be associated with Brownian motion in a suitable medium, thereby relating the diffusion coefficient $\mathbb{D}$ to analogous physical properties of the particle, the environment, and their mutual interactions. At the same time, the variance and time step alone fully determine the collapse process, directly analogous to how a single variance parameter at the time of observation summarizes the uncertainties arising from measurement of a classical particle.

Given the validity of the Born rule in this model, the collapse time interval remains the key experimental parameter, which must agree with the parameters of the walk in **(RM)**. At present, only an upper bound is known, set by decoherence, while the process otherwise appears instantaneous. In any case, any value of this parameter, no matter how small, can be obtained by suitably adjusting the variance parameter that defines the Gaussian Unitary Ensemble, and the time-step.

Experiments observing the trajectories of microscopic particles in different media may provide insight into the dependence of the random walk parameters in **(RM)** on the characteristics of both the medium and the particle. The question of the time distribution for forming interference dots on the screen in the double-slit experiment (i.e., the arrival times) may also be experimentally testable. Current observations indicate that this distribution is governed by the emission rate of particles and standard propagation in physics, once again pointing to an extremely fast process of state reduction.

A related experimental direction concerns the fraction of particles that actually undergo collapse, for example, those that contribute to the interference pattern in a double-slit experiment. It is well established that only a small fraction of emitted particles ever reach the detection screen. Disregarding the many losses that occur before the screen, the **(RM)** model is potentially capable of predicting the fraction of particles whose states reach the manifold M_3^g and, with a high-quality detector, are successfully registered on the screen.

V. ONTOLOGY OF $\widetilde{M_3^g}$ AND (RM)

The conjecture **(RM)** provides a unified model of measurement applicable to both macroscopic and microscopic particles. Its central idea is the identification of the classical space $\mathbb{R}^3$ of particle positions with the three-dimensional submanifold $\widetilde{M_3^g}$ of the particle's state space. When the random walk in **(RM)** is constrained to $\widetilde{M_3^g}$, it reproduces the walk that models a classical measurement and yields the normal probability distribution for the position of a classical particle. In contrast, the unconstrained walk in **(RM)** produces the Born rule for transition probabilities between states.

A related correspondence holds for dynamics: just as measurement reduces to the classical case under constraint to $\widetilde{M_3^g}$, Newtonian dynamics of a classical particle emerges from Schrödinger dynamics when the state is constrained to the six-dimensional submanifold $\widetilde{M_{3,3}^g}$ of state space. Moreover, we saw that the constraint to $\widetilde{M_{3,3}^g}$ for a macroscopic particle can be naturally explained by the way the environment records the moments when the particle's state, undergoing the walk in **(RM)**, crosses $\widetilde{M_{3,3}^g}$. This continual recording neutralizes the spreading of the probability distribution and effectively projects the state onto the classical phase-space manifold $\widetilde{M_{3,3}^g}$. This mechanism provides the basis for the emergence of Newtonian dynamics in macroscopic systems.

The scheme readily extends to N-particle systems. In this case, the classical configuration space $\mathbb{R}^{3N}$ is identified with the $3N$-dimensional submanifold $\otimes_N \widetilde{M_3^g}$, the tensor product of N copies of the manifold $\widetilde{M_3^g}$, one for each particle. Likewise, the classical phase space $\mathbb{R}^{6N}$ corresponds to the tensor product of N copies of $\widetilde{M_{3,3}^g}$. Naturally, it is equivalent to regard the system as N point particles in a single copy of $\mathbb{R}^3$, or as a single point in the $3N$-dimensional Euclidean space $\mathbb{R}^{3N}$. In the same way, one may view a system of N particles as described by a single state in $\otimes_N \widetilde{M_3^g}$, or as a collection of N particles whose states each belong to a single copy of $\widetilde{M_3^g}$.

Under the identification of $\mathbb{R}^3$ with $\widetilde{M_3^g}$, a random walk with Gaussian steps on $\mathbb{R}^3$ admits a unique extension to the full state space as a homogeneous and isotropic process, precisely the walk defined in **(RM)**. Just as classical Brownian motion emerges from Newtonian dynamics of a particle in a thermal bath, the random walk in **(RM)** can be viewed as its quantum analogue, with the step distribution on M_3^g giving rise to the Gaussian Unitary Ensemble. This analogy suggests that **(RM)** may be supported by the same kind of underlying dynamical considerations that justify Brownian motion in the classical domain.

The identification of classical space and classical phase space with submanifolds of state space, together with the established correspondence between measurement in macroscopic and microscopic systems and between Newtonian and Schrödinger dynamics, all suggest that the walk in **(RM)** should be regarded as a genuine physical process, and that state space itself should be accepted as the physically and ontologically appropriate extension of classical space. On this view, a point particle is not a point object in classical space, but rather a point object in state space. Its position is represented not by numerical coordinates in $\mathbb{R}^3$, but by a function in a Hilbert space and the corresponding element of the projective state space.

When the particle's state function broadens so that it becomes a superposition of functions in M_3^g, the point in state space representing the particle moves away from the submanifold $\widetilde{M_3^g}$ into the larger state space. This becomes evident when one computes the distance between such a state and the submanifold using the distance formula (15). That distance provides a measure of the spread of the state function, and hence a quantitative measure of its "non-classicality" or "waveness": the farther the state lies from $\widetilde{M_3^g}$, the more wave-like the particle behaves, while in the limit where this distance tends to zero the particle exhibits classical, corpuscular behavior.

The "reality" of the state space and of the walk in **(RM)** is not required for the validity of the results obtained in this paper. Nevertheless, adopting this perspective greatly simplifies the problem of classical-quantum correspondence and transition. On this view, the universe, with its macroscopic bodies, microscopic particles, and the dynamical processes between them, should be understood in terms of the state space of these systems. Dynamical processes involving macroscopic bodies take place on classical submanifolds of state space and can be equivalently described as processes in ordinary classical space. By contrast, dynamical processes involving microscopic bodies unfold in the full state space, but their states occasionally intersect the classical submanifolds, giving rise to classical features while also producing the paradoxes that puzzle our three-dimension-trained intuition.

An example of this is the double-slit experiment with particles. At the moment of emission, the particle's state is well localized and lies on the classical-space submanifold $\widetilde{M_3^g}$. In this regime, the particle behaves classically: its wave packet has a well-defined position, and the packet's group velocity corresponds to the particle's velocity. As the particle interacts with the screen containing the slits, however, its state becomes a superposition of states localized at the individual slits. This means that the point representing the state moves away from the submanifold $\widetilde{M_3^g}$ into the larger state space.

By carefully identifying both the screen and the particle within the tensor-product space $\otimes_N \widetilde{M_3^g}$ of all their constituent parts, one concludes that the particle does not pass through either slit. Instead, the point representing its state leaves $\widetilde{M_3^g}$ and moves through the larger state space, effectively passing "over" the screen. If the particle's position is measured just beyond the screen, the random walk in **(RM)** brings the state (and thus the particle) back to $\widetilde{M_3^g}$. This process ensures that the particle is detected at one of the slits, with probabilities given by the Born rule. In particular, it can only be found in the vicinity of the slits.

A particle detected at a slit becomes classical again: it acquires a well-defined position and group velocity and may continue propagating toward the backstop screen. In this case, it arrives at the screen as a point particle, and no interference pattern is observed. By contrast, if the particle's position is not measured at the slits, it propagates as a superposition of two spreading wave packets and reaches the backstop screen in the form of overlapping superposed packets. The backstop screen then acts by generating a random walk in **(RM)** from this superposition (still a point in state space!) back to $\widetilde{M_3^g}$. The Born rule derived from **(RM)** is in this case consistent with the appearance of an interference pattern on the screen.

Note again that the physics is now expressed in terms of point objects in state space and the distances between them. It is misleading to imagine a "wave" corresponding to the particle itself in classical space. There is no wave in classical space, only a point in state space. What appears as spreading of the state function in classical space is, in fact, the motion of the point representing the state *away* from the classical-space submanifold $\widetilde{M_3^g}$.

Likewise, there are no "tails" of a collapsed state function, but rather a point that may lie slightly off the classical-space submanifold. What we call collapse is not a sudden localization of a cloud in classical space, but the motion of a point in state space, from an initial position away from $\widetilde{M}_3^g$ to a final position on $\widetilde{M}_3^g$. When this motion takes the form of a random walk in **(RM)**, the probability of a specific outcome is given by the Born rule.

This reframing dispels the paradoxical imagery of waves spreading or collapsing in space, replacing it with a precise geometric motion in state space. The conjecture **(RM)** thus offers a concrete framework for understanding the quantum-classical transition, whose validity will ultimately be decided by experiment.

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