Relational Unsharp Properties

A POVM-Based Ontology for Relational Quantum Mechanics

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Abstract

Since its inception in 1996, Rovelli's Relational Quantum Mechanics (RQM) has left the status of non-interacting systems notably underexplored. When discussed at all, one prevailing view has maintained that attributing properties to isolated systems within RQM is simply *meaningless* (Laudisa and Rovelli, 2008). However, it has become increasingly evident that without a satisfying account of non-interacting systems, RQM risks sliding toward antirealist interpretations—at least insofar as such systems are concerned (Dorato, 2016; Calosi and Mariani, 2020). In response to this crucial challenge, two main approaches have been advanced. One is a form of *dispositionalism* (Dorato, 2016), which suggests that isolated physical systems possess only dispositional properties. The other appeals to a determinable-based account of *metaphysical indeterminacy* (Calosi and Mariani, 2020), suggesting that a non-interacting quantum system may bear a determinable property without possessing any unique determinate of that determinable at a given time. In this paper, we offer a new response to the question of what can be meaningfully said about quantum systems prior to measurement in the context of RQM. In particular, drawing on the formalism of Positive Operator-Valued Measures (POVMs), we emphasize that (1) *preparation*—like measurement—constitutes a physical interaction capable of assigning relational properties to a system, and that (2) POVMs are indispensable for the proper description of observables with continuous spectra. On this basis, we propose that RQM can accommodate an entity-realist interpretation of non-interacting systems by attributing to them what we term *relational unsharp properties*. As we will show, this theoretical move preserves the relational nucleus of RQM while enabling a physically and philosophically coherent account of non-interacting quantum systems.

Keywords: *Quantum Measurement*, *Relational Quantum Mechanics*, *Unsharp Properties*, *POVM*

Contents

1 Introduction

Since Rovelli (1996)'s proposal, Relational Quantum Mechanics (RQM) has sparked a sustained and stimulating philosophical debate (see Calosi and Riedel, 2024). One of the core tenets of this interpretation is the abandonment of the notion of an absolute or apparatus-independent quantum system. In RQM, quantum systems and their associated properties are always defined relative to other physical systems, reflecting an ontological commitment to relationality rather than intrinsic individuation.[1] This shift has had profound implications for the interpretation of quantum phenomena: it redirects attention from abstract wave-function dynamics to concrete, observer-relative interactions and measurement outcomes. Despite the considerable literature elaborating the relational metaphysics of quantum events

[1]This interpretive shift has a clear historical precedent in Bohr (1935)'s reply to the EPR argument, where he argued that quantum properties can only be defined relative to the entire measurement context, including the apparatus. See Covoni, Fano, and Sanchioni (manuscript) for a first historical reconstruction.

and the measurement process (Candiotto, 2017; Oldofredi, 2021; Dorato and Morganti, 2022; Adlam and Rovelli, 2023; Adlam, 2024; Rovelli, 2025), there still is an important foundational question that has received comparatively little attention. Namely, what can be said about quantum systems in the absence of interaction.

Put differently, RQM appears to treat measurement as the fundamental moment of property realization. But if all properties are relational and require interaction to be meaningful, then what is the ontological status of a system before it interacts—before it is measured? Is the system simply undefined, or does RQM allow for some account of pre-interaction existence? This question, which we take as central in this article, concerns the treatment of non-interacting systems[2] within RQM. More precisely, we are interested in whether, and how, RQM can accommodate meaningful talk about quantum systems prior to measurement. This issue is not merely semantic. It bears directly on the interpretation of a range of foundational and applied issues—such as the Aharonov-Bohm effect (Aharonov and Bohm, 1959)—[3] in which the pre-measurement behavior or properties of a system appear to play a crucial role.

Following the schematization proposed by Calosi and Mariani (2020), we distinguish two closely related but analytically separable questions:

(NIS) Non-Interacting Systems. In the context of RQM, where the properties of physical systems are defined only in relation to other systems, is it possible to make any meaningful statements about a quantum system prior to its interaction with another system?

(RP) Relevant Properties. Does RQM imply that only those properties which acquire definite values through measurement (i.e. through interaction) are physically or metaphysically relevant?

These questions point to a potential tension in the relational framework: if all properties are realized through interaction, and no intrinsic properties are assumed, does this mean that systems lack ontological content before interaction?

Several philosophical responses have been offered to the question of what, if anything, can be meaningfully said about non-interacting quantum systems in the relational framework. These can be grouped into three main categories:[4]

[2] Throughout this paper, we will use the term *non-interacting systems* to align with the few existing papers that address this issue in the literature on RQM (Dorato, 2016; Calosi and Mariani, 2020). However, we would like to emphasize that this terminology is somewhat ambiguous. In our account, we show that both state preparation and measurement constitute *interactions*, which makes the term non-interacting systems problematic in nature. We will elaborate further on this point later in the paper.

[3] We are currently preparing a follow-up study that develops a relational interpretation of the Aharonov-Bohm effect, building on the framework introduced in this article.

[4] We introduce these three positions and their associated critiques as part of an argumentative trajectory leading to our own proposal. Our aim here is not to offer a full-dress critique of each view. A more systematic comparison between these approaches and the relational unsharp property framework is the subject of a follow-up paper.

(i) *The "Meaninglessness" View.*[5] According to this view—arguably implicit in Rovelli's early formulations—it is incoherent to ascribe any properties to a system prior to interaction. The very idea of a quantum system possessing a state or property in isolation is held to be ill-posed. Reality, in this picture, is constituted exclusively by interactional events; outside of such events, there is literally nothing to describe. While this view is maximally faithful to the relational ontology of RQM, it faces serious limitations. In particular, it precludes any ontological account of the system between interactions, or of its contribution to counterfactual scenarios, preparation stages, or dynamical evolution. Some critics (Dorato (2016), Calosi and Mariani (2020)) argue that this stance risks narrowing the distance between RQM and instrumentalist stances, whereby the formalism describes measurement outcomes but not an underlying physical reality. This issue becomes even more pressing when one considers that RQM is already presented as a perspective that bears notable similarities to the Copenhagen interpretation.[6] Therefore, to clearly establish RQM as a realist approach to quantum mechanics, it is essential to prioritize a satisfying treatment of systems prior to measurements.

(ii) *The Dispositionalist View.* This approach—defended, for instance, by Dorato (2016)—argues that systems do possess properties even in isolation, but these are not categorical: they are dispositional. A system carries the potential to yield a determinate outcome if it were to interact with a suitable measuring apparatus. Dispositions are thus understood as non-actualized but real properties, grounded in the system's internal structure or potentialities. This view retains a realist stance without requiring sharp value definiteness, and allows talk about systems between interactions. However, it pays a high ontological cost: any given system must be treated as bearing an extensive—possibly infinite—set of counterfactual properties across all possible observables. In this respect, the ontology becomes heavily inflationary, especially when applied to continuous observables.

(iii) *The Metaphysical Indeterminacy View.* A third position, recently advanced by Calosi and Mariani (2020), holds that non-interacting systems are rightly accounted for by a determinable-based account of metaphysical indeterminacy. For instance, a system may possess the determinable property having a spin along the z-axis, without instantiating any specific determinate such as spin-up or spin-down. The indeterminacy is thus not epistemic or semantic, but ontological: the world genuinely fails to determine a specific value. This account preserves realism while rejecting the classical assumption of sharp value-definiteness. However, it rests on a controversial metaphysical framework—namely, that determinables can exist independently of any determinate.

[5]It is noteworthy that a quite similar perspective on non-interacting systems can already be found in Reichenbach's treatment of what he termed *interphenomena* (Reichenbach, 1944, 21). In his account, *phenomena* are verifiable, unambiguous occurrences "closely" connected to macroscopic apparatuses, as there is a "short causal chain" (1944, 21) linking these microscopic events to the relevant macroscopic instruments. In contrast, *interphenomena* refer to all the occurrences between these phenomena and derive from a more complex inferential process, grounded in the theoretical framework of quantum mechanics.

[6]However, since "Copenhagen" can refer to a variety of philosophical positions, it may be more accurate to say that this view adds nothing substantially new beyond Bohr's original formulation.

While such models have been proposed (notably by Wilson (2013)), their application to quantum mechanics (Calosi and Wilson (2019)) raises several challenges. In particular, one may question whether primitive value-indeterminacy is the best way to capture the structure of quantum probabilities, and whether this framework can be coherently integrated within the relational ontology of RQM without reintroducing hidden absolutes.

In this article, we propose a fourth response to the **(NIS)** and **(RP)** questions. Our contribution unfolds along two complementary lines. First, we argue that a quantum system can possess relational properties even in the absence of ongoing interaction, insofar as it stands in a preparatory relation to another system. In RQM, preparation—understood as a past physical interaction—can ground the ascription of a property in the same relational sense as measurement. This provides a principled response to **(NIS)**: systems do have properties before being measured, namely those relationally instantiated through preparation.

Second, we investigate the nature of the properties thus ascribed. While in some cases (e.g., discrete-spectrum observables) preparation yields sharp properties corresponding to projective measurements, many physically relevant observables—such as position or momentum—have a continuous spectrum, making sharp ascriptions idealized and empirically unattainable. In such contexts, the standard treatment via projection-valued measures (PVMs) becomes ontologically inadequate. We argue that POVMs provide the appropriate formal framework for capturing the structure of real preparation-induced ascriptions. This motivates the introduction of relational unsharp properties, which we formalize through the subclass of regular effects within the POVM formalism. As we will discuss throughout the paper, these unsharp properties are neither hidden variables, nor dispositional potentials, nor indeterminate gaps. They are genuinely instantiated relational properties, probabilistically structured but objectively grounded in physical interactions. As such, they offer a novel ontological category that is both realist and relational, and that remains closely tied to the empirical and operational architecture of quantum theory in the relational interpretation.

The paper is structured as follows. §2 addresses **(NIS)** by arguing that preparation procedures—understood as past physical interactions—can ground the relational ascription of properties. §3 introduces the formalism of Positive Operator-Valued Measures (POVMs) and motivates the need for unsharp representations in the case of continuous-spectrum observables. In §4, we develop the ontology of relational unsharp properties answering to **(RP)**, focusing on the subclass of regular effects as candidates for genuine property ascriptions. In §5, we illustrate the concrete applicability of our framework by revisiting the double-slit experiment from a relational perspective. Section §6 concludes.

2 Relational Sharp Properties and the Ontological Role of Preparation

To introduce our framework, we begin with the case of discrete-spectrum observables, focusing on spin. This allows us to present the notation of relational property ascription in its simplest form, using PVMs and avoiding the technical complications of continuous spectra. Spin provides an ideal case: it is well-understood, geometrically intuitive via the Bloch sphere, and governed by the spin-polarization principle, which ensures that each pure spin state determines a definite direction of polarization. Starting from this context allows us to clarify how both measurement and preparation can be understood as interactions that ground the attribution of relational sharp properties—a structure we will later generalize to the unsharp case.

In RQM, properties are not intrinsic to systems but are always ascribed relative to other systems. A property is ascribed when a concrete interaction occurs—typically a measurement—between a system S and an apparatus f, resulting in a definite outcome. Note that, here, the notion of "measurement" is not meant to denote a special category of interaction, but simply refers to those interactions that allow epistemic access to relational properties. All interactions instantiate properties; measurement interactions are merely those through which such properties become accessible to us. Following the relational notation developed by Fano and Sanchioni (2025), the ascription of a property is expressed using the symbolic form:

$$(A = a, S, O = o)_f, \tag{1}$$

which means that the value a of observable A is ascribed to system S relative to the value o of observable O ascribed to system f, where f plays the role of the measurement reference. Importantly, when f is a macroscopic apparatus, $O = o$ does not refer to a sharp quantum property of f, but to the registration of a classical outcome—such as a pointer position or detector click. This aligns with the standard treatment of measurement devices as effectively classical systems, where the relevant "observables" are classical records of quantum interactions (cf. Busch et al. 1997).

In the case of sharp observables, the property ascribed to S is represented by a projection operator P_a on the Hilbert space $\mathcal{H}_S$. The probability of this ascription is given by the Born rule:

$$p(a) = \langle\psi|P_a|\psi\rangle, \tag{2}$$

where $|\psi\rangle$ is the state of S relative to f. The key point is that the property does not pre-exist the interaction; it is instantiated by the relation established through measurement.

To illustrate, consider a spin-½ particle and the measurement of the spin component $S_{\vec{n}}$ along some unit vector $\vec{n}$. The outcome $+\hbar/2$ corresponds to the projection $P_{\vec{n}} = |\uparrow_{\vec{n}}\rangle\langle\uparrow_{\vec{n}}|$, and the ascription

$$(S_{\vec{n}} = +\hbar/2, S, O = o)_f \tag{3}$$

represents the relational property of being spin-up along $\vec{n}$, relative to a classical outcome o registered by apparatus f. Here, $O = o$ can be understood as a specific macroscopic event—such as a detector click in a Stern–Gerlach apparatus oriented along the direction $\vec{n}$. This framework makes explicit that sharp properties in RQM are not absolute features of a system, but relational facts grounded in physical interactions and encoded formally through projection operators.

This relational framework naturally raises the following question: can a quantum system be ascribed a spin property even prior to measurement? After all, measurement is not the only form of interaction. In particular, a general feature of spin-½ systems—the so-called *spin-polarization principle*—suggests a positive answer. The principle states that any pure spin state is an eigenstate of some spin component $S_{\vec{n}}$. That is, for every $|\psi\rangle \in \mathbb{C}^2$, there exists a unique direction $\vec{n}$ such that

$$S_{\vec{n}}\,|\psi\rangle = +\hbar/2\,|\psi\rangle. \tag{4}$$

This implies that, even before any measurement takes place, the system possesses a definite spin value along a well-defined axis. While this direction $\vec{n}$ may be epistemically inaccessible, it is ontologically determinate in virtue of the system's state. Crucially, this definite state does not arise spontaneously: it is the result of a prior physical interaction, typically a preparation procedure. Thus, a natural proposal within the relational framework is to interpret the act of preparation itself as the moment in which relational sharp properties are ascribed to the system. In practical terms, this means that if a spin measurement is subsequently performed along the same direction $\vec{n}$, the outcome $+\hbar/2$ will be observed with near certainty.

Hence, in the relational framework, we take it that both measurement and preparation are physical interactions through which properties are ascribed to quantum systems. Although these processes serve distinct epistemic purposes—preparation aims at producing systems with controlled properties, while measurement retrieves information from them—their ontological function is symmetric: both establish relational facts through interaction.

To make this concrete, consider again a spin-½ particle. A paradigmatic preparation consists in sending the particle through a Stern–Gerlach apparatus aligned along a direction $\vec{n}$, and post-selecting those particles deflected into the spin-up channel. The resulting state is $|\uparrow_{\vec{n}}\rangle$, and the system can be relationally ascribed the property:

$$(S_{\vec{n}} = +\hbar/2,\ S,\ O = o)_f, \tag{5}$$

which reads: system S has spin component $+\hbar/2$ along direction $\vec{n}$, relative to the preparation apparatus f, which registers the classical outcome o (e.g. deflection toward the upper detector). This ascription has exactly the same logical form as the one that occurs in measurement. In both cases, the observable $S_{\vec{n}}$ acquires a definite value through a physical interaction with a reference system. The difference lies in the epistemic context: in preparation, we select systems with a desired outcome; in measurement, we read which outcome occurred. But ontologically, both processes establish a relational property grounded in a specific interaction.

This symmetry provides a principled response to **(NIS)**: a quantum system can possess a definite property even prior to any measurement, provided that a previous interaction—namely, a preparation procedure—has relationally defined such a property. In this view, preparation is not merely an operational stage, but a fully physical and relational event that grounds ontological content.

3 From Sharp to Unsharp: POVMs and the Ontology of Continuous Observables

In the standard formalism of quantum mechanics, physical observables are represented by self-adjoint operators on a Hilbert space. By the spectral theorem, each such operator is associated with a projection-valued measure (PVM), which maps measurable subsets of possible outcomes to projection operators on the Hilbert space (Moretti, 2013). This framework reflects the classical idealization that physical properties are sharply instantiated: for any given state, a system either possesses the property associated with a projection, or it does not.

This sharp characterization, however, is increasingly at odds with both the theoretical structure of quantum mechanics and the practical realities of experimental physics. Two principal limitations challenge the adequacy of PVM-based observables. First, quantum theory imposes fundamental constraints on the simultaneous definability of certain observables. Canonical examples include position and momentum, whose mutual incompatibility—expressed by the Heisenberg uncertainty principle—precludes assigning sharp values to both simultaneously. Second, real-world experimental setups invariably involve finite precision. Preparation devices cannot produce perfectly sharp quantum states: even highly collimated beams or well-defined energy sources yield states with finite spreads in position and momentum. Similarly, measurement devices possess finite resolution: no detector can resolve exact positions or momenta but only intervals within a finite tolerance.

A particularly vivid illustration of these issues arises in the case of observables with continuous spectrum, such as position or momentum. In such cases, the idealized notion of a system possessing a sharp property—like being exactly located at a point x—corresponds to a projection onto a generalized eigenstate $|x\rangle$. However, these ideal states are not physical states in the usual sense: they are not square-integrable and are never strictly realized in practice. More importantly, quantum theory itself predicts that the probability of finding a particle at an exact point is zero, making such sharp property ascriptions empirically and ontologically problematic. This motivates the need for a more realistic formal framework—one that accounts for finite-resolution measurements and captures how position-like properties are actually instantiated in the world.

These considerations suggest that the use of PVMs reflects a mathematical idealization that neither corresponds to the operational procedures used in laboratories nor to the ontological status of physical systems under realistic conditions. A more adequate formal framework should account not only for probabilistic outcomes but also for the intrinsic unsharpness of quantum interactions. In this section, we address these limitations by introducing the framework of POVMs. Section §3.1 defines POVMs and shows how they arise naturally from the need to model finite-resolution procedures. Section §3.2 responds to the objection that POVMs are conceptually redundant due to Naimark's Theorem, and argues instead that unsharp observables represent a genuinely distinct and ontologically significant class of properties.

3.1 POVMs: the Mathematical Formalism and Logical Implications

In the standard formulation of quantum mechanics, measurement outcomes are typically represented by projection-valued measures (PVMs), which associate each possible outcome of a measurement with a projection operator on a Hilbert space. These projection operators correspond to sharp properties, meaning that the system either definitively has or does not have a given property. However, in realistic experimental contexts, perfect sharpness is unattainable. This is especially evident for observables with continuous spectra (like position and momentum), where the associated "eigenstates" (e.g., $|x\rangle$ or $|p\rangle$) are not normalizable vectors in the Hilbert space but instead generalized functions (distributions) like Dirac deltas. As such, the projection operators onto these states are idealizations that cannot correspond to physically realizable states or measurements.

To describe these limitations more faithfully, quantum theory employs positive operator-valued measures (POVMs). A POVM is a more general kind of observable, defined as a mapping:

$$E : \mathcal{B}(\mathbb{R}) \to \mathcal{E}(\mathcal{H}), \tag{6}$$

where $\mathcal{B}(\mathbb{R})$ is the Borel algebra of measurable subsets of the real line, and $\mathcal{E}(\mathcal{H})$ is the set of effects on the Hilbert space $\mathcal{H}$. Each effect $E(B)$ is a bounded, positive operator such that $0 \leq E(B) \leq \mathbb{I}$.[7] The measure is normalized:[8]

$$\sum_i E(B_i) = \mathbb{I}, \quad \text{for disjoint } B_i \text{ such that } \bigcup_i B_i = \mathbb{R}, \tag{7}$$

where the sum denotes the operator addition of effects.[9]

Unlike projections (which are idempotent: $P^2 = P$), effects are not necessarily idempotent. This reflects the fact that they correspond to unsharp properties—properties whose occurrence is not deterministic even in principle, but fundamentally probabilistic due to either experimental imprecision or quantum limitations (e.g., preparation spread, detector resolution). A POVM allows one to compute the probability of obtaining an outcome in a set B when measuring a system in state $|\psi\rangle$, via:

$$\mathbb{P}(B) = \langle\psi|E(B)|\psi\rangle. \tag{8}$$

Importantly, POVMs also support the modeling of fuzzified observables: starting from a sharp observable O represented by a PVM, one can define an unsharp version using a Markov kernel $\mu(B, r)$, which describes how the ideal value r is smeared over other possible outcomes

[7]Here, the ordering $A \leq B$ is defined in terms of the operator ordering: $B - A$ is a positive operator. This partially ordered structure is crucial for defining the logic of quantum properties.

[8]Although we focus here on real-valued observables, the formalism is readily generalizable to POVMs over $\mathbb{R}^n$ or more general measurable spaces.

[9]The addition of effects $E_1 + E_2$ is simply the usual operator addition, but must remain within the set of effects, i.e., $0 \leq E_1 + E_2 \leq \mathbb{I}$. The countable additivity of POVMs thus presupposes that these sums remain within the operator interval $[0, \mathbb{I}]$.

due to finite resolution. The unsharp version is given by:

$$E(B) = \int_{\mathbb{R}} \mu(B, r)\, dO(r). \tag{9}$$

This process reflects the intrinsic unsharpness of quantum reality: the unsharp observable E captures the way in which a system exhibits probabilistically distributed properties, even in ideal preparation or measurement scenarios. Rather than being a mere artifact of imprecise instruments, unsharpness is a structural and unavoidable feature of quantum theory—at least in the continuous-spectrum case.

The mathematical shift from PVMs to POVMs reflects a deeper conceptual shift: from a logic of sharply defined yes/no properties to a logic of intrinsically unsharp, probabilistically instantiated properties. In standard quantum mechanics, the set of projection operators $\mathcal{P}(\mathcal{H})$—used to represent sharp properties—forms an orthocomplemented lattice. This structure has well-defined logical operations: conjunction ($\wedge$), disjunction ($\vee$), and orthocomplementation ($^{\perp}$). Each projection P has a unique orthocomplement $P^{\perp} = \mathbb{I} - P$, and these satisfy classical-looking logical laws:

$$P \wedge P^{\perp} = 0, \quad P \vee P^{\perp} = \mathbb{I}. \tag{10}$$

These relations reflect the principles of non-contradiction and excluded middle: a system either has a property or it has its complement, and never both. However, despite appearances, this structure is not Boolean. The lattice of projections is non-distributive, meaning that classical logical identities like

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R) \tag{11}$$

may fail. This non-Boolean behavior is at the heart of the so-called quantum logic, first systematically studied by Birkhoff and Von Neumann (1975). In finite-dimensional spaces, the lattice is modular, but in infinite-dimensional Hilbert spaces—typical in quantum field theory or continuous-variable systems—even modularity fails. This mathematical structure encodes the logical strangeness of quantum properties.

Now, when we move from projections to effects—the operators forming the building blocks of POVMs—we leave even this non-classical logical structure behind. Effects are positive operators E satisfying $0 \leq E \leq \mathbb{I}$. Unlike projection operators, they are generally not idempotent (i.e., $E^2 \neq E$), and they do not form a lattice. This has several striking consequences: While one can formally define a "complement" of an effect as $E^{\perp} := \mathbb{I} - E$, this does not play the same role as logical negation. For instance, it is generally false that:

$$E \wedge E^{\perp} = 0. \tag{12}$$

In fact, the meet $\wedge$ and join $\vee$ may not even be well-defined for arbitrary pairs of effects. It is perfectly possible for both $\langle\psi|E|\psi\rangle > 0$ and $\langle\psi|\mathbb{I} - E|\psi\rangle > 0$ to hold in the same quantum state $|\psi\rangle$. This means that the system instantiates both a property and its "complement" simultaneously—something that is logically inadmissible in any classical, Boolean, or orthomodular setting. This reflects a deep feature of unsharpness: quantum properties represented by effects admit degrees, rather than strict yes/no truth values. The logic of unsharp observables is not a classical propositional logic, nor even quantum logic in the traditional lattice-theoretic sense—it is a probabilistic structure defined over an infinite, ordered family of effects.

Another crucial difference is coexistence. In the PVM framework, commutativity is necessary for joint measurability: *two* projections P and Q can be jointly measured only if they commute, i.e., $[P, Q] = 0$. In contrast, with POVMs, coexistence of effects does not require commutativity. Two effects E_1 and E_2 may fail to commute, yet still be part of the same POVM or arise from compatible measurement setups. This mathematical fact reflects a powerful physical insight: unsharp observables can encode compatibility where sharp ones cannot. For example, while exact position and momentum cannot be sharply measured together due to the Heisenberg uncertainty principle, one can define joint unsharp position and momentum observables whose effects are coexistent and hence jointly measurable.

3.2 Naimark's Theorem and the Ontological Status of Unsharp Observables

A possible objection to the use of POVMs in the ontology of quantum mechanics is based on a classical result known as Naimark's Theorem (Naimark, 1940). This theorem shows that for any POVM A defined on a Hilbert space $\mathcal{H}$, there exists a larger Hilbert space $\mathcal{H}^+$, an isometric embedding of $\mathcal{H}$ into $\mathcal{H}^+$, and a PVM A^+ on $\mathcal{H}^+$ such that the probability distributions generated by A on $\mathcal{H}$ can be exactly reproduced by restricting the outcomes of the A^+ on $\mathcal{H}^+$ back to the original system. This result is mathematically powerful and elegant. However, it is sometimes interpreted as undermining the conceptual significance of POVMs. According to this reading, unsharp observables would merely be coarse-grained shadows of "truly" sharp properties residing in an enlarged Hilbert space—useful for modeling, perhaps, but lacking any ontological depth. On this view, POVMs would be dispensable: anything expressible via a POVM A can, in principle, be recovered from a PVM A^+. Yet such a conclusion is both philosophically naive and empirically misleading.

First, it is essential to distinguish between formal representability and ontological commitment. The fact that the statistical predictions of a POVM can be mathematically simulated via a PVM in an extended space does not entail that unsharp observables are ontologically reducible to sharp ones. The Hilbert space $\mathcal{H}^+$ used in Naimark's construction is an auxiliary artifact: it has no physical instantiation in real experiments unless an explicit ancillary system is introduced and dynamically coupled to the target system in a way that mirrors the mathematical embedding. But this is precisely what is absent in most empirical contexts. For instance, in quantum optics, weak measurements, and finite-resolution detection, what is manipulated, prepared, and recorded is unambiguously modeled by POVMs—and no $\mathcal{H}^+$ appears anywhere in the laboratory. The gap between mathematical convenience and physical practice is critical. A theoretical reconstruction using $\mathcal{H}^+$ offers no new empirical predictions, no enhanced explanatory power, and no operational role. As such, it fails to meet the usual standards by which theoretical posits earn ontological status in the philosophy of science. The mapping from POVMs to PVMs in a higher-dimensional space is structural and syntactic, not semantic. It tells us that a POVM's statistical content can be captured in another formalism—not that the physical properties it models are reducible to those of another system.

Second, interpreting unsharp observables as mere epistemic shadows of unknown sharp values mischaracterizes their internal logic. As discussed in Section §3.1, the algebra of effects $\mathcal{E}(\mathcal{H})$ differs fundamentally from the structure of projections. Effects are not idempotent, they do not support strict orthocomplementation, and they generally violate classical logical principles such as:

$$E \wedge E^{\perp} = 0 \quad \text{and} \quad E \vee E^{\perp} = \mathbb{I}. \tag{13}$$

Indeed, the meet and join of two effects may not exist at all. This reflects a genuinely non-classical logic, where a system may instantiate both an effect and its complement: for a given state $|\psi\rangle$, it may be the case that both

$$\langle\psi|E|\psi\rangle > 0 \quad \text{and} \quad \langle\psi|\mathbb{I} - E|\psi\rangle > 0. \tag{14}$$

Such a condition is unthinkable in classical logic, or even within the orthocomplemented lattice of PVMs, but is natural and expected in the POVM framework. It demonstrates that unsharpness is not just a matter of incomplete knowledge or finite experimental resolution. Rather, it reflects the fact that unsharp effects correspond to an infinite, ordered family of quantum properties, each associated with a distinct degree of precision or resolution, and not reducible to classical binary alternatives.

Moreover, the Naimark dilation often fails to track anything physically meaningful. If the enlarged space $\mathcal{H}^{+}$ and the associated ancillary systems have no empirical role—no state preparation, no interaction dynamics, no observables—then their only function is to reproduce statistics already available from the original POVM. In such cases, the Naimark dilation becomes an invisible scaffold: it supports the mathematics, but says nothing about the ontology. This becomes especially clear in quantum optics experiments, where detectors have well-characterized resolution profiles, and the observables involved—such as position, polarization, or quadrature amplitudes—are modeled by POVMs from the outset. There is no experimental procedure that introduces or manipulates the ancillary system required by the dilation; the extended Hilbert space H^{+} remains a purely mathematical construction with no laboratory counterpart. If one were to insist on a realist reading of $\mathcal{H}^{+}$, one would thereby be positing hidden degrees of freedom with no experimental access, introducing precisely the kind of metaphysical inflation that relational and operationalist interpretations of quantum mechanics seek to avoid. From a relational realist perspective, then, it is more natural and more economical to accept POVMs as genuine carriers of physical content—as expressions of relational, unsharp properties instantiated through actual system–apparatus interactions. They are not approximations to sharp values. They are not epistemic compromises. They are the correct ontological category for describing how quantum systems relate to one another under finite, concrete, non-ideal conditions.

Finally, it is worth emphasizing a further limitation of the "dilation" strategy. As argued by Beneduci (2012), there is in general no single Hilbert space $\mathcal{H}^{+}$ that can accommodate all POVMs as restrictions of PVMs. Each unsharp observable may require a distinct dilation space, depending on its specific structure. This means that the Naimark construction is not only empirically idle, but also fragmented and context-sensitive. There exists no universal sharp ontology ”behind” all unsharp observables. This reinforces the point that POVMs cannot be dismissed as mere projections of a hidden PVM-based reality. Rather, they must be taken seriously as representing irreducible aspects of the physical structure of quantum theory.

Hence, Naimark's Theorem shows that every POVM can be embedded within a PVM, but it does not show that unsharp observables are reducible to sharp ones. The mathematical possibility of dilation does not entail a physical or metaphysical equivalence. If anything, the distinct logical, algebraic, and empirical roles played by POVMs reinforce the idea that unsharpness is a fundamental and irreducible feature of quantum reality—and not merely a convenient abstraction. Hence, the ontology of quantum mechanics must accommodate these unsharp properties as "first-class citizens", not as "second-tier derivatives" of idealized PVMs.

4 Relational Unsharp Properties

The relational notation introduced in §2 accounts for sharp properties ascribed through interaction—typically in measurement. However, this framework rests on idealizations that rarely hold in actual experimental practice. Extending the relational framework in the direction of POVMs requires reinterpreting the procedure of property ascription. In the sharp case, a system is assigned a determinate value of an observable, encoded by a projection. In the unsharp case, by contrast, the system instantiates a property that is formally represented by an effect $E \in \mathcal{E}(\mathcal{H}_S)$, where $\mathcal{E}(\mathcal{H}_S)$ is the set of positive operators between 0 and $\mathbb{I}$ on the system's Hilbert space. These effects represent unsharp properties: probabilistically structured, relational features of the system that emerge from interaction with a finite-resolution device.

Not all effects, however, can correspond to physically meaningful unsharp properties. To select those that do, we follow Busch et al. (1996) and adopt the criterion of regularity. An effect E is regular if there exist two states $|\psi\rangle$ and $|\phi\rangle$ such that:

$$\langle\psi|E|\psi\rangle > \tfrac{1}{2}, \quad \langle\phi|E^{\perp}|\phi\rangle > \tfrac{1}{2}, \tag{15}$$

where $E^{\perp} = \mathbb{I} - E$ is the orthocomplement of E.[10] This ensures that E and $E^{\perp}$ are neither trivially true nor false across all states, and that the property represented by E plays a genuine probabilistic role. In fact, regularity captures the intuition that the property is empirically salient, responsive to different preparation contexts.

To illustrate, consider again the position observable. In the idealized sharp case, exact localization is represented by a projection-valued measure $P_R = \int_R |x\rangle\langle x| dx$, where the formal entities $|x\rangle$ correspond to position eigenstates. However, such eigenstates are not square-integrable functions, and thus do not belong to the Hilbert space $L^2(\mathbb{R}^3)$. While

[10] While $\mathbb{I} - E$ is commonly referred to as the *orthocomplement* of the effect E, this usage should not be confused with the strict notion of orthocomplementation found in the lattice of projection operators. The set of effects $\mathcal{E}(\mathcal{H})$ does not form an orthocomplemented lattice, and $E^{\perp}$ lacks many of the formal properties expected of a true orthocomplement. It is best interpreted operationally, as the effect corresponding to the counter-event of E, rather than as its logical negation in a classical or quantum propositional sense.

these projectors are mathematically well-defined within the theory's spectral framework, they correspond to idealized configurations that cannot be physically prepared or realized with finite energy and resolution. As such, they should be understood as idealizations—useful for the formal structure of quantum mechanics, but not strictly representative of any physically attainable configuration. Instead, in realistic preparation procedures—say, producing a Gaussian wave packet centered at x_0 with spread Δ—the system is better described by a regular effect:

$$E = \int_{\mathbb{R}^3} g_{x_0}(x)\, |x\rangle\langle x|\, dx, \tag{16}$$

where $g_{x_0}(x)$ is a normalized Gaussian peaked at x_0 with width Δ. This effect encodes the unsharp relational property "the particle is approximately at x_0", as ascribed through the physical interaction constituting the preparation. Its complement $E^{\perp}$ encodes the negation—"the particle is elsewhere," again understood relationally. Choosing states narrowly localized around x_0 or far from it easily confirms the regularity condition.

This shift from sharp to unsharp properties is not merely technical. It reflects a deep change in the logical and ontological structure. In the sharp case, the relational ascription:

$$(A = a, S, O = o)_f \tag{17}$$

assigns to system S a definite property (value a of observable A) relative to the apparatus f, which registers classical outcome o. In the unsharp case, the generalized form is:

$$(E, S, O = o)_f, \tag{18}$$

where E is a regular effect, representing a property whose truth value is given by the probability $\langle\psi|E|\psi\rangle \in (0, 1)$.[11] These unsharp properties violate classical logic: both a property and its complement may be instantiated. Yet they are still objective relational features of the quantum system, not mere epistemic uncertainties. This leads naturally to the next question: how are such properties grounded?

As argued above (§2), unsharp properties do not arise solely in measurement; they are already present in preparation. Within a relational framework, both processes—preparation and measurement—are forms of physical interaction through which a quantum system is assigned a property relative to another system. These processes differ in epistemic intent. Measurement extracts information; preparation imposes constraints. Yet ontologically, they share a common structure: both interactions define relational facts about the system—facts that are encoded, in the unsharp case, by regular effects. The physical system produced by preparation is not a disembodied mathematical object, but the outcome of a concrete interaction that establishes a probabilistic property profile for the system.

For instance, a source emitting Gaussian packets centered at x_0 with spread Δ prepares the

[11] An unsharp property, formally represented by an effect $E(B)$, corresponds to a well-defined statement—e.g., "the position lies within the Borel set $B \subset \mathbb{R}$." What is probabilistic is not the definition of the property, but the likelihood that it is instantiated in a given interaction, quantified by $\langle\psi|E(B)|\psi\rangle$. In this sense, any preparation confers a well-defined property (e.g., position in a given region B), and unsharpness reflects the structure of B and the finite resolution of the physical process, not a vagueness in property attribution.

system in such a way that we may ascribe the relational unsharp property "being approximately at x_0"—represented by a regular effect E. This property is not a placeholder for a hidden sharp value, but an actual feature of the system's property relative to the preparation device. Likewise, when a detector with resolution Δ yields an outcome o, it does not indicate that the system had a sharp value of the measured observable, but rather that the interaction has instantiated an unsharp property represented by a regular effect E_o, whose form depends on the resolution profile of the apparatus.
In both cases, the formal structure of the ascription is:

$$(E, S, O = o)_f, \tag{19}$$

where f is the apparatus—whether preparing or measuring—and $O = o$ denotes the classical outcome anchoring the relational fact. This structural continuity highlights an essential point: unsharpness is not a defect or a limitation of our measurements. It is a genuine, irreducible ontological feature of quantum systems, captured relationally through interactions and formalized via the algebra of effects. Thus, preparation and measurement are not merely symmetric in logical form; they are united in their metaphysical role: both establish unsharp relational properties. By recognizing this, we restore physical significance to unsharp observables, and we allow the relational framework to describe not only measurement outcomes but the ontological structure of quantum systems under all physically realistic conditions.

5 Addressing (NIS) and (RP): Relational Unsharp Properties as an Ontological Proposal

As outlined in §1, the two central challenges for the ontology of RQM can be framed as follows:

(NIS) **Non-Interacting Systems.** In the context of RQM, where the properties of physical systems are defined only in relation to other systems, is it possible to make any meaningful statements about a quantum system prior to its interaction with another system?

(RP) **Relevant Properties.** Does RQM imply that only those properties which acquire definite values through measurement (i.e. through interaction) are physically or metaphysically relevant?

In response, we propose a realist interpretation that preserves strict relationality while avoiding both the ontological inflation of dispositionalism and the metaphysical vagueness associated with indeterminacy. Our view—based on relational unsharp properties—offers a unified framework for addressing both questions.
Within our framework, the answer to **(NIS)** is affirmative: a quantum system can possess properties even in the absence of a current interaction, since it has undergone a preparation process, that is an interaction. In RQM, what matters ontologically is not whether a system is being measured, but whether it has interacted—past or present—with another system. We emphasize that preparation and measurement are ontologically symmetric: both are

physical interactions that give rise to property ascriptions. When a system is prepared in a specific state—for example, through collimation, filtering, or polarization—it acquires relational properties relative to the preparation device. These properties are not hypothetical dispositions but real, instantiated features. In the case of discrete observables such as spin, the system acquires a relational sharp property (e.g., $S_{\vec{n}} = +\hbar/2$) relative to the preparation apparatus. In the case of continuous observables such as position, the system acquires a relational unsharp property, formally captured by a regular effect in the POVM framework. Thus, **(NIS)** is resolved: a quantum system without current interactions can still bear relational properties, because past interactions—specifically, preparation—have already defined them. These properties are not potentialities, but relationally realized features.

The **(RP)** question concerns whether only sharp, projection-valued properties should be considered metaphysically robust. We argue that unsharp properties, as modeled by regular effects, are no less real. The empirical and theoretical structure of quantum mechanics strongly supports this view. In realistic scenarios, sharp measurements are the exception, not the norm. Continuous-spectrum observables—like position and momentum—are typically not sharply realized in practice, not only because of mathematical limitations, but because experimental procedures invariably involve finite-resolution wave packets rather than idealized eigenstates. Indeed, detectors always have finite resolution, and preparation devices yield wave packets rather than point states. The standard formalism of PVMs fails to capture these features. In contrast, the POVM framework accounts for both the finite resolution of experimental setups and the probabilistic structure of quantum systems.[12] What we propose is that relational unsharp properties are fully determinate, even if not sharp. A Gaussian wave packet centered at x_0 instantiates the property "being approximately at x_0," represented by a regular effect peaked at that location. This is not a vague or indeterminate property; it is a precisely defined unsharp property, relative to the apparatus that prepared or measured it. Our position thus rejects both the idea that only sharp properties are real and the view that unsharp properties imply a dererminable-based account of metaphysical indeterminacy.[13] We do not say the particle is in an indeterminate state "between" positions; we say it has a determinate, unsharp property of being approximately localized.

To summarize, our proposal answers both challenges with a consistent ontological strategy:

> To **(NIS)**, we respond that preparation grounds properties. A quantum system acquires relational properties through interaction—even if measurement has not yet occurred.

[12]We are aware of the important distinction between two types of uncertainty: (i) epistemic uncertainty due to limited experimental resolution, and (ii) structural uncertainty as encoded in the formalism of quantum theory—most notably in Heisenberg's relations. While the former depends on the imperfections of measurement apparatuses, the latter expresses a principled limit on simultaneously well-defined values of certain observables. No experiment can ever violate this structural bound: in this sense, experimental uncertainty is always bounded by and subordinate to Heisenberg-type indeterminacy. Our notion of unsharpness refers to this second, intrinsic feature of quantum theory, not directly to contingent instrumental imprecision.

[13]Some might be inclined to argue that, even if we reject a determinable-based account of metaphysical indeterminacy, the notion of relational unsharp properties still falls within the domain of metaphysical indeterminacy. At least insofar as it involves a commitment to unsharp properties. In this respect, it is important to emphasize that a degreed conception of property does not, in itself, entail metaphysical indeterminacy. This crucial point has been convincingly defended by Nørgaard (2025). For now, we will not delve further into this issue and will leave it for future work.

To **(RP)**, we respond that both sharp and unsharp relational properties are ontologically robust. Sharp properties arise in discrete-spectrum cases (like spin), while unsharp properties govern continuous-spectrum cases (like position and momentum).

Crucially, the properties we describe are not to be understood as mere dispositions or potentialities. They are actual features of quantum systems—relationally instantiated through concrete physical interactions, whether during measurement or preparation. In this respect, our view departs sharply from dispositionalist accounts, which treat quantum properties as conditional capacities to produce certain outcomes in hypothetical future measurements. For us, no such hypothetical scaffolding is required: a prepared or measured system already possesses the relational property in question, albeit in a form that may be unsharp.

At the same time, these properties do not involve any form of metaphysical indeterminacy. In the case of sharp observables—such as spin along a given axis—the system possesses a definite property relative to the reference system, and this property is determinate at all times after preparation. In the case of unsharp observables—such as position or momentum—what the system possesses is likewise determinate, though not sharply defined in the classical sense. Rather, these unsharp properties are graded and probabilistically structured, reflecting the finite resolution of physical interactions. Yet their indeterminacy is not metaphysical: the system does not lack a fact of the matter, nor is reality gappy. One might say that unsharp properties are "sharply unsharp": precisely defined within the formalism, operationally instantiated, and grounded in actual physical relations.

Taken together, these claims support a coherent and parsimonious relational ontology. Properties are never absolute; they are always defined with respect to a particular physical system—typically a measuring or preparing apparatus. But once such a relation is established, the property it defines—whether sharp or unsharp—has full ontological status. It is not merely a bookkeeping device, nor a convenient summary of possible outcomes, but a real feature of the world as described by quantum theory. This approach preserves the central insight of RQM—that reality is fundamentally relational—while showing that such relationality is entirely compatible with a robust, non-instrumental realism about quantum properties.

6 Application: a Relational Reading of the Double Slit Experiment

In this section, we apply the framework of relational unsharp properties developed in the previous sections to the paradigmatic case of the double slit experiment. We begin in §6.1 by presenting the standard theoretical description of the experiment using Gaussian wave packets and show how the use of POVMs becomes conceptually necessary to account for realistic experimental conditions. We then discuss (§6.2) the Merli–Missiroli–Pozzi experiment as a key empirical realization, highlighting how both preparation and detection processes involve ascriptions of unsharp properties. Finally (§6.3), we provide a relational reading of the double slit experiment.

6.1 Theoretical Description of the Double Slit Experiment

In order to study the behavior of approximately localized quantum systems—such as those involved in the double slit experiment—Gaussian wave packets provide a natural and physically grounded model of single-particle dynamics. They allow us to describe both the coherent splitting at the slits and the resulting interference on the detection screen. We thus consider in what follows Gaussian wave packets, which are minimal uncertainty states that saturate the Heisenberg bound and offer a controlled balance between position and momentum localization. In one spatial dimension, the position-space representation of a free-particle Gaussian wave packet centered at position x_0 with mean momentum p_0 is given at time $t = 0$ by:

$$\psi(x,0) = \left(\frac{1}{2\pi\sigma^2}\right)^{1/4} \exp\left[-\frac{(x-x_0)^2}{4\sigma^2} + \frac{ip_0x}{\hbar}\right], \tag{20}$$

where σ controls the spatial width of the packet and consequently its momentum spread $\Delta p \sim \hbar/(2\sigma)$. This wavefunction describes a particle that is simultaneously localized around x_0 and has a well-defined average momentum p_0, subject to the uncertainty constraint $\Delta x\,\Delta p \geq \hbar/2$. Under free evolution, the packet spreads over time due to dispersion. The time-dependent solution to the Schrödinger equation for a free Gaussian packet is:

$$\psi(x,t) = \left(\frac{1}{2\pi\sigma_t^2}\right)^{1/4} \exp\left[-\frac{(x-x_t)^2}{4\sigma_t^2} + \frac{i}{\hbar}p_0(x - x_t/2)\right], \tag{21}$$

where $x_t = x_0 + \frac{p_0}{m}t$ is the classical trajectory and $\sigma_t = \sigma\sqrt{1 + \left(\frac{\hbar t}{2m\sigma^2}\right)^2}$ describes the time-dependent spatial spread due to quantum dispersion.

Gaussian wave packets are relevant in the context of the double slit experiment because they offer a precise description of a particle that is both partly spatially localized and partly dynamically evolving toward the barrier. Their finite spatial extent ensures that the particle interacts with the slits at a specific time and in a controlled region, while their momentum distribution governs how sharply or diffusely they propagate. The balance between spatial localization and momentum spread—encoded in the uncertainty relation—determines whether the packet coherently illuminates both slits, thereby enabling interference. Too narrow a packet in space will have a wide momentum spread, potentially degrading spatial coherence. Conversely, a packet that is too narrow in momentum will be overly extended in space, risking interaction with regions of the barrier distant from the slits. In this sense, Gaussian packets are not just mathematically convenient—they embody the physical constraints required for coherent splitting, interference, and ultimately meaningful measurement in real interferometric setups.

Having justified the use of Gaussian wave packets, we now turn to the emergence of the interference pattern, i.e. how the model predicts the characteristic fringe structure observed in the double slit experiment. To model the emergence of an interference pattern, we thus consider the time evolution of a Gaussian wave packet incident on a barrier with two narrow slits. When the packet reaches the barrier, its spatial support must cover both slits coherently. In this case, the interaction with the slitted barrier leads to a coherent splitting of the initial packet into two transmitted components, $\psi_1(x,t)$ and $\psi_2(x,t)$, each corresponding to the contribution from one slit. Neglecting decoherence and assuming no which-path information is acquired,[14] the resulting state is the superposition:

$$\psi(x,t) = \frac{1}{\sqrt{2}} \left[\psi_1(x,t) + \psi_2(x,t) \right]. \tag{22}$$

Each $\psi_i(x,t)$ evolves as a free Gaussian packet originating from the corresponding slit. Due to dispersion, the two wave packets broaden as they propagate and begin to overlap in the far field (i.e. at the detection screen). The probability density for detecting the particle at position x is given by:

$$|\psi(x,t)|^2 = \frac{1}{2} \left[|\psi_1(x,t)|^2 + |\psi_2(x,t)|^2 + 2\mathrm{Re}(\psi_1^*(x,t)\psi_2(x,t)) \right]. \tag{23}$$

The last term represents the interference between the two wave packets. Its presence requires not only spatial overlap but also a stable phase relationship between ψ_1 and ψ_2. The resulting detection probability exhibits oscillations in space—the familiar interference fringes—whose spacing and visibility depend on the relative phase, the central momentum p_0, the slit separation, and the distance to the screen. These features can be analyzed semi-classically by evaluating the phase difference between the two paths and applying the stationary phase approximation (Feynman and Mechanics, 1979; Schulman, 2012), or more rigorously using Fresnel or Fraunhofer diffraction integrals under suitable assumptions (Born and Wolf, 2013). In either case, the interference pattern is a direct manifestation of the coherent superposition of spatially separated components of the wavefunction, which have been shaped by the geometry of the experimental setup.

The double slit experiment, when treated realistically, involves not only the free evolution of a quantum wave packet, as just outlined, but also its physical preparation and detection, both of which depart from the idealizations of textbook quantum mechanics. The initial state of the particle is not simply postulated but results from a series of interactions—emission, collimation, spatial filtering—that shape the wave packet reaching the slitted barrier. These preparation processes do not correspond to projective measurements but rather to a sequence of unsharp operations, whose net effect can be captured using the formalism of POVMs. In this context, the wave packet is best understood not as an intrinsic state of the system but as the conditional outcome of a preparation procedure, described by a POVM that filters certain position-momentum components while discarding others. Likewise, the final detection at the screen, implemented by a spatially extended and imperfect apparatus, does not yield a sharp value of position. Instead, it registers a coarse-grained, probabilistic outcome, again modeled by a POVM whose elements reflect the finite spatial resolution of the detectors.

Thus, the use of POVMs in the double-slit experiment is not an optional refinement but a conceptual necessity: it enables a consistent account of the entire experimental context—from the probabilistic preparation of localized wave packets to their unsharp detection.

[14] The presence of which-path information—whether due to environmental interaction, measurement devices, or internal degrees of freedom—effectively destroys the coherence between the two components ψ_1 and ψ_2, suppressing the interference term. This can be formally described using decoherence theory, where tracing out the environment leads to a mixed state with vanishing off-diagonal terms in the position basis.

6.2 Experimental Description: The Merli–Missiroli–Pozzi Experiment

A seminal realization of the quantum double slit experiment with electrons was conducted by Merli, Missiroli, and Pozzi, Merli et al. (1976). Their experiment demonstrated that individual electrons, emitted one at a time, could form an interference pattern over repeated trials — a direct and striking confirmation of quantum coherence at the single-particle level.[15].
In the experiment reported in Merli et al. (1976), electrons were emitted individually from a thermionic source and accelerated to energies around 30 KeV. The beam was collimated and shaped using electrostatic lenses, ensuring a high degree of transverse spatial coherence—sufficient for each electron's wavefunction to extend over both paths of the interferometer. The core of the setup was a modified electron biprism, originally developed by Möllenstedt and Düker (1955),[16] consisting of a thin $(1-2\mu m)$ gold-coated quartz filament held at a small positive voltage and suspended between two grounded electrodes. This filament produces a localized electrostatic field that deflects the wavefront of the incident electron beam, effectively splitting it into two coherent partial waves. These diverging components are then redirected to overlap in the far field, where their superposition gives rise to an interference pattern. By adjusting the biprism voltage, the angular separation between the paths—and thus the spatial frequency of the resulting fringes—could be precisely tuned.
Importantly, no material double slit was used; the wavefront splitting was entirely electrostatic, which allowed for a clean, tunable, and minimally perturbative implementation of the two-path setup. Detection was carried out either via long-exposure photographic plates, as in the original 1974 setup Merli et al. (1976), or using a TV image intensifier system that allowed the real-time observation of individual electrons contributing to the interference pattern, as documented in later accounts Rosa (2012). In both configurations, electrons were sent through the apparatus one at a time, with time intervals long enough to exclude any two-particle effects. The resulting interference pattern emerged statistically, as a spatial distribution of localized detection events. This pattern revealed not only the wave-like behavior of individual electrons, but also the coherence properties of the prepared wave packet, shaped through a combination of emission, collimation, and interaction with the biprism field.
The Merli–Missiroli–Pozzi setup also suggests a natural framework for modeling the electron's quantum state and its interaction with the apparatus. Since the prepared electron is neither localized in position nor sharply defined in momentum, it is best represented by a Gaussian wave packet: a minimal uncertainty that captures both spatial coherence and finite momentum spread. Such wave packets are well-suited to describe the coherent propagation and deflection that occur in the biprism field. Furthermore, the measurement process—whether via photographic emulsion or a TV image intensifier—does not correspond to an idealized projection onto a position eigenstate. Rather, it involves spatially extended detectors with finite resolution and temporal integration. This makes POVMs the appropriate formalism: they allow one to model the actual detection as an unsharp spatial localization,

[15] A detailed historical and philosophical reconstruction of this experiment, including its significance in the interpretation of quantum mechanics, is provided in Rosa (2012)

[16] For a historical reconstruction of the invention and early development of the electron biprism, including its conceptual origins and experimental implementation, see Möllenstedt (1991).

reflecting both the physical extent of the wave packet and the coarse-grained nature of the apparatus. In this light, both the preparation and detection stages of the experiment can be understood as generalized quantum operations that define a sequence of relational, context-sensitive properties of the electron—not as pre-existing attributes, but as emergent from specific interactions.

6.3 A Relational Reading of the Double-Slit Experiment

From the perspective of RQM, the double slit experiment exemplifies how quantum properties are not absolute attributes of isolated systems but relational features that emerge through specific interactions. Crucially, both the preparation and measurement stages of the experiment ascribe unsharp relational properties to the quantum system, formalized through regular effects.

During preparation, the electron interacts with devices that shape its spatial and momentum distributions, such as collimators and electrostatic lenses. This does not result in a sharp localization but in the ascription of an unsharp property—formally represented by a regular effect E_{prep}—relative to the preparation apparatus A. This can be expressed as:

$$(E_{\text{prep}}, S, O = o)_A, \tag{24}$$

where the system S (the electron) possesses the relational unsharp property E_{prep}, and the apparatus records a classical configuration o corresponding to this preparation. The Gaussian wave packet used to model the initial state is precisely a manifestation of this unsharp relational property, characterizing the electron as "approximately localized" with a finite spatial spread Δ and momentum uncertainty. Crucially, during the preparation phase, the electron can be ascribed *both* a relational unsharp position and a relational unsharp momentum, despite the fact that these observables are formally incompatible in standard quantum mechanics. This apparent paradox dissolves once we recognize that the ascriptions involve unsharp relational properties rather than sharp, definite values.

Indeed, incompatibility strictly applies to sharp properties associated with projective measurements, where the assignment of a definite value to one observable precludes the simultaneous assignment of a definite value to an incompatible observable. However, when properties are unsharp and described by regular effects, this strict exclusivity no longer holds. Instead, both position and momentum can be relationally attributed with a probabilistic profile characteristic of regular effects, reflecting the finite precision allowed by the uncertainty principle. This is precisely what happens in the preparation of a Gaussian wave packet. The electron is assigned an unsharp position property—"being approximately at position x_0"—represented by a regular effect E_x, and an unsharp momentum property—"having approximately momentum p_0"—represented by another regular effect E_p. These two effects do not correspond to mutually exclusive states but coexist within the relational framework, constrained by the uncertainty relation:

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}. \tag{25}$$

The preparation apparatus effectively "balances" these relational unsharp properties, shaping the electron's state to have finite spreads in both position and momentum, compatible with experimental requirements. This dual ascription reflects the experimental reality: the apparatus prepares a particle that is sufficiently localized to interact meaningfully with both slits, yet also has a well-defined momentum to reach the detection screen and produce the interference pattern. In relational terms, the electron bears both unsharp position and momentum properties relative to the preparation apparatus—neither sharply defined, yet both jointly attributable within the precision constraints imposed by the structure of quantum theory.

The detection phase similarly involves the ascription of an unsharp relational property through interaction with the detector D. Since the second screen performs a position measurement, the effect $E_{\text{meas}} \equiv E_x(B)$ corresponds to the unsharp position property associated with a Borel set $B \subset \mathbb{R}^2$, reflecting the finite resolution of the detection apparatus. This interaction is formally described by:

$$(E_{\text{meas}}, S, O = o')_D, \tag{26}$$

where the measurement apparatus D registers a definite classical outcome o' (e.g., a localized detection event on the screen), but the property ascribed to the electron remains unsharp, corresponding to a finite spatial resolution rather than a sharp projection.

In both stages, relational properties are not determined by hidden variables or pre-existing intrinsic features but emerge dynamically through interaction. Preparation and measurement ascribe regular effects that express the relational nature of quantum properties. The resulting interference pattern on the detection screen is not evidence of an underlying absolute trajectory but rather a manifestation of the coherent interplay of relational unsharp properties acquired during preparation and measurement. This reading of the double slit experiment illustrates the core insight of RQM extended to unsharp properties: quantum phenomena do not reveal hidden certainties but relational, probabilistic structures that depend on the specific physical contexts in which systems interact.

7 Conclusions and Outlooks

This paper has proposed and defended a novel ontological framework for Relational Quantum Mechanics centered on the notion of relational unsharp properties. Our goal has been to respond constructively to two longstanding challenges within the RQM literature: **(NIS)** problem, which questions whether non-interacting systems can be said to possess any properties at all, and the **(RP)** problem, which casts doubt on the metaphysical status of unsharp observables.

We have argued that both challenges can be met without appealing to dispositions, metaphysical indeterminacy, or instrumentalist reinterpretations. The core idea is that preparation and measurement are ontologically symmetric processes: both instantiate properties through interaction. When a system is prepared (e.g., as a Gaussian wave packet), it acquires determinate unsharp properties relative to the preparation context. These are not potentialities waiting to be actualized, but objective, probabilistically structured features of

the system—formally represented by regular effects in the POVM formalism.
In the case of discrete-spectrum observables, sharp relational properties can be assigned in a determinate and stable manner. In the case of continuous observables, such as position or momentum, relational unsharp properties are the appropriate ontological primitives. These properties are not epistemically vague nor metaphysically indeterminate, but sharply unsharp: they possess well-defined probabilistic profiles grounded in physical interactions, and their unsharpness reflects the structure of quantum phenomena rather than a deficiency in the theory.
Moreover, we argued for the practical and conceptual relevance of this framework through a detailed analysis of the double slit experiment, including both theoretical modeling via Gaussian wave packets and POVMs, and an empirical case study of the Merli–Missiroli–Pozzi experiment. In both preparation and detection, the assignment of unsharp properties proved not only natural but necessary, providing a coherent relational account of interference without recourse to hidden trajectories or pre-existing sharp values. In sum, relational unsharp properties provide a coherent, parsimonious, and ontologically robust extension of RQM. They allow us to: preserve realism without invoking dispositions; preserve determinacy without invoking metaphysical indeterminacy; extend the relational paradigm beyond sharp measurements to encompass the full range of realistic experimental interactions. By grounding quantum properties in the concrete interactions that prepare and measure them—whether sharply or unsharply—this framework enriches the metaphysical landscape of RQM and opens a path toward a more complete understanding of quantum ontology, rooted in the actual structure of experimental practice.
While we have framed this contribution as a coherent and innovative response to both **NIS** and **RP**, we wish to register that it provides, at least in part, a clarifying response to a similar problem. In their recent work, Calosi and Riedel (2024) outlined future challenges that philosophical works on RQM must address. Among these, one relevant point they highlighted concerns the relationship between Rovelli's interpretation and weak measurements:

> The Problem of Generic Measurements: So far, the principles of RQM have always been illustrated using the example of ideal, non-disturbing measurements. How do property acquisition and state update generalise to more generic types of interactions, including so-called 'interaction-free', weak, and continuous measurements? (Calosi and Riedel, 2024, 20)

By clarifying for the first time the interplay between RQM and weak measurements (or POVM), our work provides a first step in the right direction. In this regard, we also wish to emphasize that our foundational work opens up several new research directions. A forthcoming companion paper will engage directly with the two dominant alternatives we have set aside here: dispositionalism and metaphysical indeterminacy. There, we will offer a systematic comparison, evaluating their respective costs and theoretical commitments in light of the framework of relational unsharp properties. Our aim is to show that the present view offers a more economical and empirically grounded ontology, without sacrificing the explanatory depth that these approaches attempt to provide. Beyond this philosophical comparison, the formalism we have developed lends itself naturally to reinterpret core quantum phenomena within a relational, unsharp framework. One particularly promising direction is a reanalysis of the Aharonov–Bohm (AB) effect. Traditionally seen as challenging for local and relational interpretations, the AB effect can instead be read in terms of unsharp relational properties instantiated between the charged particle and the global configuration of the electromagnetic field apparatus. Another avenue concerns the nature of quantum information in RQM. A follow-up project will articulate how entanglement underlies relational information structures, providing an objective, non-epistemic account of information grounded in physical interaction. In this light, information is not what an agent knows, but what a system is, relative to another, as encoded in the entanglement structure and its associated POVMs. Lastly, our reliance on POVMs invites deeper investigation into the logical foundations of quantum theory. The transition from projection-valued to effect-valued representations is not merely technical: it suggests a logical shift in which properties are compatible not because they are jointly sharp, but because they are jointly regular. Exploring the philosophical implications of this shift could offer new insights into the nature of quantum logic.

References

Adlam, E. (2024). What does ?(non)-absoluteness of observed events? mean? *Foundations of Physics 54*(1), 1–43.

Adlam, E. and C. Rovelli (2023). Information is physical: Cross-perspective links in relational quantum mechanics. *Philosophy of Physics 1*(1).

Aharonov, Y. and D. Bohm (1959). Significance of electromagnetic potentials in the quantum theory. *Physical review 115*(3), 485.

Beneduci, R. (2012). Semispectral measures and feller markov kernels. *arXiv preprint arXiv:1207.0086*.

Birkhoff, G. and J. Von Neumann (1975). The logic of quantum mechanics. In *The Logico-Algebraic Approach to Quantum Mechanics: Volume I: Historical Evolution*, pp. 1–26. Springer.

Bohr, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical review 48*(8), 696.

Born, M. and E. Wolf (2013). *Principles of optics: electromagnetic theory of propagation, interference and diffraction of light*. Elsevier.

Busch, P., M. Grabowski, and P. J. Lahti (1997). *Operational quantum physics*, Volume 31. Springer Science & Business Media.

Busch, P., P. J. Lahti, and P. Mittelstaedt (1996). *The quantum theory of measurement*. Springer.

Calosi, C. and C. Mariani (2020). Quantum relational indeterminacy. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics 71*, 158–169.

Calosi, C. and T. Riedel (2024). Relational quantum mechanics at the crossroads. *Foundations of Physics 54*(6), 1–24.

Calosi, C. and J. Wilson (2019). Quantum metaphysical indeterminacy. *Philosophical Studies 176*(10), 2599–2627.

Candiotto, L. (2017). The reality of relations. *Giornale di Metafisica 2*, 537–551.

Covoni, N., V. Fano, and M. Sanchioni. Relational quantum mechanics and (soft) perspectivism. Manuscript, https://philsci-archive.pitt.edu/25250/.

Dorato, M. (2016). Rovelli's relational quantum mechanics, anti-monism, and quantum becoming. In A. Marmodoro and D. Yates (Eds.), *The Metaphysics of Relations*, Mind Association Occasional Series. Oxford: Oxford University Press. Online edition published 17 Dec. 2015, Oxford Academic.

Dorato, M. and M. Morganti (2022). What ontology for relational quantum mechanics? *Foundations of Physics 52*(3), 1–19.

Fano, V. and M. Sanchioni (2025). Relational properties and relational quantum mechanics. *Foundations of Physics 55*.

Feynman, R. and A. H. Q. Mechanics (1979). Path integrals. *Lecture Notes Phys 106*, 64.

Laudisa, F. and C. Rovelli (2008). Relational quantum mechanics. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Fall 2008 ed.). Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/fall2008/entries/qm-relational/.

Merli, P. G., G. F. Missiroli, and G. Pozzi (1976). On the statistical aspect of electron interference phenomena. *Am. J. Phys 44*(3), 306–307.

Möllenstedt, G. (1991). The invention of the electron fresnel interference biprism. In *Advances in optical and electron microscopy*, Volume 12, pp. 1–23. Elsevier.

Möllenstedt, G. and H. Düker (1955). Fresnel scher interferenzversuch mit einem biprisma für elektronenwellen. *Naturwissenschaften 42*, 41–41.

Moretti, V. (2013). *Spectral theory and quantum mechanics*, Volume 64. Springer.

Naimark, M. (1940). Spectral functions of a symmetric operator. *Izvestiya Akademii Nauk SSSR. Seriya Matematicheskaya 4*, 277–318.

Nørgaard, M. (2025). Quantum indeterminacy: A matter of degree? *European Journal for Philosophy of Science 15*(1), 1–24.

Oldofredi, A. (2021). The bundle theory approach to relational quantum mechanics. *Foundations of Physics 51*(1), 1–22.

Reichenbach, H. (1944). *Philosophic Foundations of Quantum Mechanics.* Mineola, N.Y.: Dover Publications.

Rosa, R. (2012). The merli–missiroli–pozzi two-slit electron-interference experiment. *Physics in perspective 14*, 178–195.

Rovelli, C. (1996). Relational quantum mechanics. *International journal of theoretical physics 35*, 1637–1678.

Rovelli, C. (2025). Relational Quantum Mechanics. In E. N. Zalta and U. Nodelman (Eds.), *The Stanford Encyclopedia of Philosophy* (Spring 2025 ed.). Metaphysics Research Lab, Stanford University.

Schulman, L. S. (2012). *Techniques and applications of path integration*. Courier Corporation.

Wilson, J. M. (2013). A determinable-based account of metaphysical indeterminacy. *Inquiry 56*(4), 359–385.