Going Beyond the World of Atoms: Heinrich Hertz and the Electromagnetic Worldview

Jan Potters [1][0000-0002-5292-5369]

[1] Heinrich Hertz Fellow 2021-2022, Lichtenberg Group for History and Philosophy of Physics, Universität Bonn, Germany
Postdoctoral Fellow funded by Research Fund – Flanders (FWO), University of Antwerp, Belgium
jan.potters@uantwerpen.be

Abstract. In this paper, I argue that, contrary to what many historians have claimed, the hidden-masses mechanics introduced by Heinrich Hertz in his *Principles* did inspire reflections within fundamental physics concerning the role played by the ether. This occurred in particular in the context of the electromagnetic worldview, a program that emerged a few years after *Principles* was published. This worldview was, however, short-lived, as it was soon more or less replaced by the theory of relativity. In the second part of the paper, I will discuss how Albert Einstein and Max Planck formulated the relativistic response to the electromagnetic worldview. This will then lead me to suggest that, in a sense, this relativistic response led to the introduction of a certain hiddenness that is very similar to Hertz's conceptualization of hiddenness.

Keywords: Heinrich Hertz, Electromagnetic Worldview, Theory of Special Relativity, Max Abraham, Max Planck.

1 Introduction

Heinrich Hertz's *Die Prinzipien der Mechanik* [1] (published posthumously in 1894) famously consists of two parts. One, the introduction, provides philosophical reflections on the epistemology of physical theories, in particular the theory of mechanics. The rest of the book offers Hertz's own reconceptualization of that theory, and in particular of its central notion 'force', in terms of moving hidden masses rigidly connected to observable material masses.

Many historians (e.g. Salvo D'Agostino [2], Jesper Lützen [3] and Joshua Eisenthal [4, 5]) have argued that these two parts should be seen as closely connected. At the same time, they have also pointed out that the two parts have not enjoyed an equal reception. The first part has inspired many philosophical reflections, both by philosophers and scientists, concerning the notion of representation (see [4, p. 45, 6, 7, 8] and chapter 27 of [3] for overviews). At the same time, they all claim that the second part has not really been picked up by the physics community. D'Agostino, for example, has stated that "though widely read and commented upon, Hertz's proposal for a new form for the foundational axioms of mechanics did not find favor with his fellow physicists at the end of the century" [2, p. 96]. Similarly, Eisenthal has claimed that "even as it was admired for its elegance and scope Hertz's contemporaries could not find in it the kind of advances that they had hoped for. Indeed, there was a general sense of confusion regarding what *Principles* was supposed to have achieved" [4, p. 45]. And Joseph F. Mulligan has argued that "when Hertz's *Prinzipien der Mechanik* was published in 1894, it was severely criticized by eminent theoretical physicists like Mach, Boltzmann, Lorentz, and Fitzgerald for its impracticality and its attempt to replace forces by the motions of even more obscure entities" [9, p. 157].

That Hertz's *Principles* was seen as introducing obscurities is ironic, since as Hertz [1, p. 10] himself pointed out, his aim was precisely to remove a particular obscurity in Newtonian mechanics: while the first two laws conceptualize forces as acting on a body in a particular direction and as the source of motion, the third law pictures them as bidirectionally connecting two bodies and as the consequence of motion. According to Eisenthal (as well as e.g. Mulligan [9] and Lützen [3, p. 278]), Hertz's *Principles* was seen as only introducing more obscurities because it was approached with the wrong expectations:

> [M]ost readers of *Principles* – both historical and contemporary – have regarded it as an attempt to lay the groundwork for some future ether mechanism, the details of which could be filled in later. But the inclination towards interpreting *Principles* this way has contributed to the dissatisfaction amongst Hertz's readers, for it ties the value of this project to the prospects of filling in these details. [4, p. 47]

In this paper, I will argue that, contrary to what D'Agostino, Eisenthal, Mulligan and others have claimed, Hertz's *Principles* did inspire concrete physical developments as well. These did not primarily concern the theory of mechanics, however, but rather a newly emerging approach to fundamental physics, namely the electromagnetic worldview (mainly elaborated in the first decade of the twentieth century). Two of its foremost contributors in particular, namely Wilhelm Wien and Max Abraham, explicitly presented their electromagnetic theories as accounting for, and going beyond, Hertz's hidden masses. And these theories were, moreover, primarily concerned with the issue of how to model the interaction between electron and ether. As such, contrary to the historians' claims above, Hertz's *Principles* did inspire physical reflection on the ether, just not in mechanical terms.

In the second part, I will then argue that the theory that at the time was considered the foremost opponent of the electromagnetic worldview, namely the theory of relativity, equally well presented what one could describe as a Hertzian response (although they themselves did not explicitly label it as such). They challenged these electromagnetic theories, more specifically, by arguing that they in fact maintained the traditional framework of mechanics, and in this way also the conceptual issues that had motivated Hertz to reformulate mechanics, namely the ambiguity underlying the notion of force at play in the three Newtonian laws. In the case of the electromagnetic worldview, these ambiguities took on the form of issues concerning how to conceptualize the action-reaction principle for interactions between ether and (charged) matter. By means of their criticism, the adherents of the relativistic approach could then argue that the problem was that the electromagnetic approach assumed their theories to offer insight into the essence of the electron, while they in fact merely offered a possible representation, and that this essence had to be considered rather as hidden (in line with how Hertz had introduced a certain hiddenness). In this way, I will then conclude, we can distinguish at least two different ways in which Hertz's *Principles* inspired reflections within fundamental physics.

2 Hertz's *Principles of Mechanics*

Hertz started his *Principles* by claiming that a scientific representation is successful when it displays what he called an essential correspondence ('eine wesentliche Übereinstimmung'), which means that it provides correct predictions: as he put it, "the necessary consequences of the images in thought are always again the images of the necessary consequences of the pictured objects" [1, p. 1] (the terms 'denknotwendig' and 'naturnotwendig' are quite difficult to translate adequately. They more or less mean that the consequences of the images are impossible to be, or to be thought, otherwise. For the translation I have followed [4, p. 51]).[1] Hertz immediately emphasized that regarding the nature of the entity represented, nothing further could be inferred from such success: on the contrary, "different images of the same object are possible and these images can differ in different directions" [1, p. 2].[2]

Hertz [1, pp. 2-3] distinguished three criteria to evaluate different images of the same object. The first, correctness ('Richtigkeit'), concerns whether the images separately do indeed fulfil the essential correspondence-claim, i.e. whether their predictions are indeed successful. The second, permissibility ('Zuläßigkeit'), comes down to the claim that our images have to be logically consistent. The third, appropriateness ('Zweckmäßigkeit), in turn concerns two subcriteria, distinctness ('Deutlichkeit') and simplicity ('Einfachheit'). Hertz conceptualized these last two subcriteria in terms of a distinction he had introduced earlier in his *Untersuchungen über die Ausbreitung der elektrischen Kraft* [10], between indispensable ('unentbehrlich') and superfluous ('entbehrlich') content. He considered theoretical elements superfluous when "they cannot have any influence on [the derivation of] any possible phenomenon" [10, p. 22],[3] and essential theoretical elements are therefore those that are necessary for the derivation of a phenomenon. An image is then more distinct than another if it allows for the derivation of more actual phenomena, i.e. for more essential correspondences, and it is simpler when it has fewer superfluous elements than the other.

In his *Electric Waves*, Hertz had then used these criteria to argue that representations of Maxwellian electrodynamics that conceptualized electricity in terms of action-at-a-distance ('Fernkräfte') were conceptually ambiguous. This had led him to elaborate an alternative representation, one that relied only on contact forces, in order to eliminate

---

[1] "die denknotwendigen Folgen der Bilder stets wieder die Bilder seien von den naturnotwendigen Folgen der abgebildeten Gegenstände." [1, p. 1]

[2] "Verschiedene Bilder derselben Gegenstände sind möglich und diese Bilder können sich nach verschiedenen Richtungen unterscheiden." [1, p. 2]

[3] "sie [konnten] auf keine möglichen Erscheinungen einen Einfluss üben" [10, p. 22].

any superfluous and ambiguous elements. In *Principles*, Hertz's motivation was, according to Eisenthal [5], very similar (at least partially), namely to formulate a conceptually clear notion of force that did not give rise to the obscurities underlying its Newtonian formulation.[4] On this Newtonian formulation, as Hertz put it, "force is introduced as the cause of movement existing before, and independently of, the motion" [1, p. 5].[5] It is one of the theory's primitive notions, besides time, space and mass, and its relations to these other notions are described by means of the three laws of motions and d'Alembert's Principle. These elements together, Hertz then claimed, gave rise to a particular ambiguity, namely that whereas according to the first two laws, forces act on a body in a particular direction and bring about motion, according to the third law forces result out of motion and connect bodies. As Hertz put it:

> The force spoken of in the definition and in the first two laws acts upon a body in one definite direction. The sense of the third law is that forces always connect two bodies, and are directed from the first to the second as well as from the second to the first. It seems to me that the conception of force assumed and created in us by the third law on the one hand, and the first two laws on the other hand, are slightly different. This slight difference may be enough to produce the logical obscurity of which the consequences are manifest in the above example. [1, pp. 7-8][6]

Hertz then illustrated the issue by means of the example of a stone being swung around by means of a rope. Following the second law of mechanics, our hand exercises a force on the rock that puts it in motion. According to the third law, the stone at the same time also exercises a force on our hand, equal in value and opposite in direction. According to Hertz, however, this way of conceptualizing only introduced superfluous content by distinguishing the second force introduced – often called the centrifugal force – from the stone's inertial mass (Eisenthal [5] extensively discusses this example):

> Is that what we now call swing force or centrifugal force something different from the stone's inertia? Can we take the effect of inertia into account twice, once as mass, once as force, without disturbing the clarity of our representations? In our laws of motion force was the cause of motion

[4] I say partially here, because this was not the only motivation: Hertz equally well had issues with the notion of energy. Lützen [3] offers an extensive discussion of the context in which Hertz was working. I do not doubt or dispute these other motivations. I rather choose to focus primarily on Hertz's issues with the Newtonian notion of force, in line with how Eisenthal [5] discusses *Principles*, because, in this way a few interesting parallels with later discussions will emerge.

[5] "Die Kraft is dabei eingeführt als die vor der Bewegung und unabhängig von der Bewegung bestehende Ursache der Bewegung." [1, p. 5]

[6] "Die Kraft, von welcher die Definition und die ersten beiden Gesetze reden, wirkt auf einen Körper in einseitig bestimmter Richtung. Der Sinn des dritten Gesetzes ist, daß die Kräfte stets zwei Körper verbinden und ebenso gut vom ersten zum zweiten, wie vom zweiten zum ersten gerichtet sind. Die Vorstellung der Kraft, welche dieses Gesetz und die Vorstellung, welche jene Gesetze voraussetzen und in uns erwecken, scheinen um ein Geringes verschieden, dieser geringe Unterschied aber reicht vielleicht aus, um die logische Trübung zu erzeugen, deren Folgen in unserem Beispiele zum Ausbruch kamen." [1, pp. 7-8]

> preceding the motion. Can we now, without confusing our concepts, speak all of a sudden of forces, which emerge out of motion, which are a consequence of motion? […] All these questions are to be answered in the negative. [1, p. 7][7]

The example indicated, for Hertz, that there were conceptual issues underlying how the three laws of mechanics together conceptualized the relation between force, inertial mass, and motion. This was a purely conceptual issue: as Hertz [1, p. 10] pointed out, it solely concerned the appropriateness of Newtonian mechanics, not its correctness or permissibility. To overcome it, Hertz therefore set out to elaborate an alternative image of mechanics, which relied, in contrast to the other representations he discussed,[8] on only three primitive notions: space, time, and mass. The notion of force, on the other hand, became a derivative concept. He elaborated this concept by distinguishing two kinds of masses: the observable material masses we are all familiar with on the one hand, and hidden masses on the other. This idea of hiddenness Hertz summarized as follows:

> We can admit that a hiddenness is at play and still deny that this hiddenness is of a special kind of form. We are free to accept that the hidden is nothing other as, again, movement and mass, and precisely such movement and mass, that is not different from the observable, except in its relation to us and our normal means of observation. [1, p. 30][9]

Central to Hertz's conception of this hiddenness was that it in no way provided any kind of insight into any essences behind the phenomena: it was rather introduced solely to arrive at a conceptually clear formulation of the theory of mechanics. According to this formulation, with which the main body of *Principles* was concerned, the mechanics of moving masses could be conceptualized in terms of one fundamental law, according to which "each natural movement of an independent material system consists in the

[7] "Ist das was wir jetzt Schwungkraft oder Centrifugalkraft nennen, etwas anderes als die Trägheit des Steines. Dürfen wir, ohne die Klarheit unserer Vorstellungen zu zerstören, die Wirkung der Trägheit doppelt in Rechnung stellen, nämlich einmal als Masse, zweitens als Kraft? In unseren Bewegungsgesetzen war die Kraft die vor der Bewegung vorhandene Ursache der Bewegung. Dürfen wir, ohne unsere Begriffe zu verwirren, jetzt auf einmal von Kräften reden, welche erst durch die Bewegung entstehen, welche eine Folge der Bewegung sind? […] Alle diese Frage sind offenbar zu verneinen" [1, p. 7].

[8] Besides the Newtonian one, which had as its primitive concepts space, time, mass and force, connected by the three laws of mechanics and d'Alembert's Principle, Hertz also discussed the energeticist representation, which had as its primitive concepts space, time, mass and energy, connected by means of Hamilton's principle.

[9] "Wir können zugeben, das sein verborgenes Etwas mitwirke und doch leugnen, dass dieses Etwas einer besonderen Kategorie angehöre. Es steht uns frei anzunehmen, dass auch das Verborgene nichts anderes sei als wiederum Bewegung und Masse, und zwar solche Bewegung und Masse, welche sich von der sichtbaren nicht an sich underscheidet, sondern nur in Beziehung auf uns und auf unsere gewöhnlichen Mittel der Wahrnehmung." [1, p. 30]

system following one of its straightest paths with constant speed" [1, p. 33].[10] The idea behind it was that an unhindered single observable mass would travel in a straight path, and that the influence of forces on such a mass would then be conceptualized in terms of hidden masses, rigidly connected to it, that would constrain the observable system in such a way that its path was no longer straight but rather the straightest possible (see Lützen [3] and Eisenthal [4, 5] for extensive discussions of this idea).[11] Formulated in this way, Hertz [1, p. 33] then claimed, his fundamental law in combination with the hidden masses-hypothesis covered both the law of inertia and Gauss's Principle of Least Constraint, and hence it allowed for what he described as a purely deductive derivation of the content of mechanics. And it allowed for the formulation of a derivative notion of force that was not plagued by the conceptual obscurities underlying Newtonian mechanics. As Hertz put it:

> Force now no longer appear as something independent from us and alien to us, but rather as a mathematical auxiliary construction, whose properties we have completely under our control, und which in that way does not have anything mysterious to us. According to the fundamental law, wherever two bodies belong to the same system, the movement of the one must be co-determined by the movement of the other. The notion of force now arises from the fact that we find it appropriate ['zweckmässig'] to divide this determination of one movement by the other in two stages and to say: the movement of the first body first determines a force, which then only determines the movement of the second body. In this way, every force is always the cause of a movement, but at the same time, with the same right, it is also always the consequence of a movement; it becomes, to be precise, the merely imagined intermediary between two movements. [1, pp. 33-34][12]

As the quote indicates, Hertz's reformulation of Newtonian mechanics was to be evaluated not primarily on the basis of its correctness or permissibility – on these criteria, it scored equally well as other formulations – but rather with regards to its

---

[10] "jede natürliche Bewegung eines selbständigen materiellen Systems bestehe darin, dass das System mit gleichbleibender Geschwindigkeit eine seiner geradesten Bahnen verfolge." [1, p. 33]

[11] By means of these rigid connections, Hertz [1, p. 215] pointed out, he thus eliminated any form of action-at-a-distance from his mechanics, just as he had done in his reformulation of Maxwellian electrodynamics.

[12] "Aber die Kraft tritt nun nicht auf als etwas von uns unabhängiges und uns fremdes, sondern als eine mathematische Hilfskonstruktion, deren Eigenschaften wir völlig in unserer Gewalt haben, und welche also auch für uns nichts Rätselhaftes an sich haben kann. Nach dem Grundgesetze muss nämlich überall da, wo zwei Körper demselben System angehören, die Bewegung des einen durch die Bewegung des anderen mitbestimmt sein. Der Begriff der Kraft entsteht nun dadurch, dass wir es aus angebbaren Gründen zweckmässig finden, diese Bestimmung der einen Bewegung durch die andere in zwei Stadien zu zerlegen und uns zu sagen: die Bewegung des ersten Körpers bestimme zunächst eine Kraft, diese Kraft erst bestimme die Bewegung des zweiten Körpers. Auf diese Weise wird jede Kraft zwar stets Ursache einer Bewegung, mit gleichem Rechte aber zugleich auch stets Folge einer Bewegung; sie wird, genau gesprochen, das nur gedachte Mittelglied zwischen zwei Bewegungen." [1, pp. 33-34]

appropriateness: his reformulation of the notion of force, he claimed, allowed for the derivation of the same phenomena with less superfluous elements that were, moreover, conceptually clear. In line with his epistemology of scientific theories and his notion of hiddenness, he concluded by pointing out that this success should not be interpreted as providing any direct insight into the essence of mechanical concepts such as force. In fact, Hertz seems to have doubted whether such insight could ever be gained, since once his reformulation is achieved, "the question as to essences will not have been answered; but our minds, no longer vexed, will cease to ask unjustified questions" [1, p. 9].[13] This shows how closely connected Hertz's physical and philosophical reflections were, for, as Eisenthal [4, p. 54] puts it, according to Hertz's own epistemology of physical theories "the hypothesis of hidden masses *rules out* knowledge of the "fundamental" constituents of a system": it is a successful representation, but that only means that it provides us with successful predictions, and we should not infer from this that it in any way provides us with insight into the fundamental nature of reality. Hertz's only goal, as he put it, had been to clarify the logical-conceptual structure of the theory of mechanics, and whether forces were actually and essentially constituted by rigid connections between observable and hidden masses was a question that his theory did not purport to answer:

> In seeking the actual rigid connections we shall perhaps have to descend to the world of atoms. But such considerations are out of place here; they do not affect the question whether it is logically permissible to treat of fixed connections as independent of forces and precedent to them. [1, p. 41][14]

3 Going Beyond the World of Atoms

3.1 Lorentz and Wien on the Electron's Electromagnetic Energy

This last quote from Hertz, we will see later, became an explicit point of inspiration for Wilhelm Wien's and Max Abraham's formulation of the electromagnetic worldview. This view, which McCormmach describes as "a programmatic intent [… to focus] on problems whose solution promised to secure a universal physics based solely on electromagnetic laws and concepts" [11, p. 459] can be traced back at least to work by Wilhelm Weber from 1846 [11, p. 472].[15] The first to elaborate it in Hertzian terms was

[13] "Sind diese schmerzenden Widersprüche entfernt, so its zwar nicht die Frage nach dem Wesen beantwortet, aber der nicht mehr gequälte Geist hört auf, die für ihn unberechtigte Frage zu stellen." [1, p. 9]

[14] "[B]ei der Suche nach den wirklichen starren Verbindungen wird sie vielleicht zur Welt der Atome hinabzusteigen haben, aber diese Erörterungen sind hier nicht am Platze, sie berühren nicht mehr die Frage, ob es logisch zulässig sei, feste Verbindungen unabhängig von und vor den Kräften zu behandeln." [1, p. 41]

[15] Besides Weber, McCormmach [11] also lists Ottaviano Fabrizio Mossotti, Karl Friedrich Zöllner, Rudolf Clausius, Bernhard Riemann, Carl Neumann, and Emil Wiechert as

Hendrik Antoon Lorentz who, in a 1899 lecture, conceptualized the program as the construction of images that display something resembling what Hertz called an essential correspondence (see page 3): "as Hertz puts it, we construct inner mock images of external states of affairs in such a way that, what results out of these images according to the laws of our thinking, corresponds to what happens outside of us according to the laws of nature" [12, p. 500].[16] Lorentz distinguished three such images: a mechanical, an energeticist and an electromagnetic one. The last one was for him the most promising, because it suggested a way to account for "the fact, with which we start our physics at school, namely that all bodies in in vacuum fall equally fast" [12, p. 518].[17]

The idea was to conceptualize gravitation in terms of the assumption that all matter is completely made up of charged particles, which Lorentz called ions. While he did not elaborate this suggestion further in his lecture, he did do so in print a year later [13]. He there ascribed the ions such charge values that the attractive force between opposite charges was a little bit bigger than the repulsive force between equally charged ions [13, p. 566]. Gravitation was then to be understood in terms of the difference between these two forces as follows: "the [charges] are acted on by the same force […] in the same direction, which means that the force cannot be due to an electric field since that would move the charges oppositely. The force, Lorentz concluded, must then be gravitational" [11, p. 477]. While Lorentz found this result conceptually promising, he also pointed out that it did not agree with observations regarding Mercury's perihelion [13, p. 573].[18]

Wien [14] then further elaborated Lorentz's suggestion into a proposal for an electromagnetic worldview. He started not with gravitation, however, but rather with "[t]he inertia of matter, which besides gravitation provides the second independent definition of mass, [and which] can be derived without any additional hypothesis from the already often used concept of electromagnetic inertia" [14, p. 507].[19] The idea behind such inertia was that a charged body, when set in motion, "must […] pass through its own electromagnetic field, with a consequent decrease in velocity – just as if it had gained mass" [15, p. 220].[20] Wien conceptualized it as follows (where he called Lorentz's ions 'electrical quanta', m denoted their mass and E their electromagnetic energy):

---

predecessors. For historical discussions of the electromagnetic besides McCormmach's paper [11], see [40, pp. 227-245, 41, pp. 105-119, 42, 43, 44, 15, pp. 260-293, 45, pp. 364-373].

[16] "[W]ie Hertz es ausdruckt, wir machen uns 'innere Scheinbilder' der äusseren Gegenstande, und zwar so, dass das, was sich aus diesen Bildern nach den Gesetzen unseres Denkvermögens ergiebt, dem entspricht, was ausserhalb von uns nach den Naturgesetzen geschieht." [12, p. 500]

[17] "Die Thatsache, mit der wir auf den Schülbanken unsere Physik begannen, nämlich dass alle Körper im luftleeren Raum gleich schnell fallen[.]" [12, p. 518]

[18] For a discussion of this attempt by Lorentz, see [11, pp. 476-477].

[19] "Die Trägheit der Materie, welche neben der Gravitation die zweite unabhängige Definition der Masse giebt, lässt sich ohne weitere Hypothesen aus dem bereits vielfach benutzten Begriff der elektromagnetischen Trägheit folgern." [14, p. 507]

[20] Wien was not the first to introduce this idea. It can already be found in the work of Riemann [46], J.J. Thomson [47, 48], Oliver Heaviside [49], G. Searle [50], Wiechert [51], Theodor des Coudres [52] and Lorentz [53].

> Accordingly, the mass defined by inertia would be constant only at low velocities and would increase with increasing velocity. Since inertia is proportional to the number of quanta out of which a body is composed, as well as to the gravitation emanating from this body, it follows that the mass defined by inertia must be proportional to the inertia determined by gravity. If we have a body with mass $m = 4/3\ E\ c^2$ attract a body with mass M up to a distance r, then the electromagnetic energy supply of gravity is diminished with the value $\varepsilon\ 4/3\ E\ c^2\ M/r$, where ε denotes the gravitational constant. [14, p. 508][21]

The gravitational expression obtained in this way was quite close to Weber's, which, Wien [14, p. 509] pointed out, provided quite good predictions regarding Mercury's perihelion. Moreover, it also opened up a new way to investigate gravitation experimentally, since the velocities required to make the change in mass observable could be attained by means of cathode rays [14, pp. 509-510]. Finally, as Wien then showed, it also indicated how to obtain electromagnetic analogues for the laws of mechanics. The first law could be reformulated in terms of the principle of conservation of electromagnetic energy, and the second law came down to the claim that the work exercised by a force on a body during a certain period was equal to the corresponding change in electromagnetic energy. The validity of the third law, the action-reaction principle, was, however, restricted, but Wien did not seem bothered by this (more on this below) [14, p. 512].[22] Wien summarized the obtained results as follows:

> One can consider the reasoning sketch presented here as diametrically opposed to Hertz's. The rigid connections, which on Hertz's view are assumed in advance, show themselves here as the effect of entangled single forces. Likewise, the law of inertia is a relatively late consequence of the electromagnetic assumptions. While the Hertzian mechanics aims at providing the electromagnetic equations as consequences, here the relation is exactly the other way around. [14, p. 512][23]

---

[21] "Hiernach wäre die durch Trägheit definirte Masse nur bei kleinen Geschwindigkeiten constant und würde mit grösser werdender Geschwindigkeit zunehmen. Da die Trägheit der Anzahl der Quanten, aus denen sich ein Körper zusammensetzt, proportional ist, ebenso die von diesem Körper ausgehende Gravitation, so folgt, dass die durch die Trägheit definirte Masse der durch die Gravitation bestimmten proportional sein muss. Lassen wir einen Körper, dessen Masse $m = 4/3\ E\ c^2$ ist, bis in die Entfernung r von einem Körper von der Masse M anziehen, so ist die elektromagnetische Energievorrat der Gravitation um den Betrag $\varepsilon\ 4/3\ E\ c^2\ M/r$ vermindert, wo ε die Gravitationsconstante bezeichnet." [14, p. 508]

[22] This was not the first discussion of the action-reaction principle by Wien. In a 1898 lecture [54], he had already discussed how questions concerning its validity could arise out of different experimental results concerning the question whether matter could set the ether in motion.

[23] "Man kann die hier skizzirte Begründung der Mechanik als der Hertz'schen diametral entgegengesetzt bezeichnen. Die festen Verbindungen, welche bei Hertz zu den Voraussetzungen gehören, zeigen sich hier als Wirkung verwickelter Einzelkräfte. Ebenso ist das Gesetz der Trägheit eine verhältnismässig späte Consequenz aus den elektromagnetischen Voraussetzungen. Während die Hertz'sche Mechanik offenbar darauf abzielt, die elektromagnetischen Gleichungen als Folgerungen zu liefern, ist hier das Verhältnis gerade umgekehrt." [14, p. 512]

Wien thus explicitly conceptualized the development of the electromagnetic worldview in terms of accounting for, and going beyond, Hertz's mechanics: whereas Hertz postulated his rigid connections between observable and hidden masses as (logical) primitives (see the quote on page 7), Wien claimed that by means of the electron's velocity-dependent electromagnetic inertia, he could account for how mechanical mass and force, in the form of Hertz's rigid connections, came about.

As pointed out above, Wien's electromagnetic worldview entailed a restricted validity for Newton's third law, the action-reaction principle. The reason for this was the same as for Wien's source of inspiration, namely Lorentz's electromagnetic theory. Both theories took the ether as a substance that was absolutely at rest, to account for different experimental results concerning the possible influence of the earth's motion through the ether on optical and electromagnetic phenomena occurring in that same ether.[24] This absolute rest requirement entailed that "the ether acted on matter, but not the reverse; consequently, the violation of Newton's third law was built into Lorentz's electromagnetic theory" [16, p. 42]. The issue concerned, more specifically, the calculation of the net force exerted on charged particles in a volume, which is expressed in terms of the Maxwell stress tensor – denoting the ponderomotive action of the electromagnetic fields on the charged bodies – and Poynting's energy flux vector – which denotes a flow of energy through the ether. When the charges are at rest, the net force inside the volume is zero. When the charges are in motion, however, this is not the case: rather, "Poynting's vector changes in time, the stresses do not vanish and must therefore move the ether as a whole" [11, p. 469]. The presence of these stresses indicated that the ether acted on the material bodies present. Given, however, that the ether was also taken as an unmovable substance, matter could not act back on it, which entailed that the action-reaction principle was not valid [16, p. 43]. Neither Lorentz nor Wien saw this, however, as a really pressing issue, since abandoning it could bring closer a completely electromagnetic worldview [17, p. 22].

3.2 Poincaré, Abraham and Lorentz on the Ether's Electromagnetic Momentum

Henri Poincaré [18], however, considered jettisoning the principle as unacceptable, because that also endangered e.g. the principle of relativity and different conservation principles [16, p. 43, 19, p. 942]. Poincaré only saw one solution, namely "to abandon the stricture of an absolute stationary ether, and to confer upon it some inertia and velocity" [17, p. 18]. This was, at the same time, a problem, since on his view, the action-reaction principle should apply only to material bodies, and not to the ether, which "if it existed, had to be a very elusive thing" [17, p. 19].[25]

[24] See [55, 11, 56, 16, pp. 14-40, 17, 57, pp. 29-31] for some discussions of how these different experiments influenced the development of Lorentz's ether-model.

[25] The reason for this was that he believed that "Hertz's [electromagnetic] theory completely eliminated ether" [17, p. 20].

To address this issue, Poincaré proceeded as follows (here I follow [16, pp. 41-44, 17, pp. 23-24]). From the total electromagnetic force expression that entailed the violation of the action-reaction principle, Poincaré derived a conservation-expression for the system as a whole, i.e. consisting of both matter and ether (conceptualized as a fluid). This expression combined the material entity's velocity and mass density with Poynting's energy flux vector $c(\boldsymbol{E} \times \boldsymbol{H})$, divided by c^2 (where $\boldsymbol{E}$ and $\boldsymbol{H}$ denote the electric and magnetic forces exercised by the ether on unit charges). Poincaré then showed that if one ascribed the ether-fluid a particular mass density, the conservation equation entailed "that the total momentum of matter and fluid is conserved" and that "the fluid was only fictitious, since it was created or destroyed at [a] rate […] predicted by Poynting's theorem, and thus did not comply with the conservation of mass" [17, p. 24]. The action-reaction principle could then be retained, at least in the form of the conservation of momentum. Moreover, in this way, one did not ascribe momentum or materiality to the ether in any physically substantial sense. According to Darrigol, however, Lorentz was not convinced:

> In [Lorentz's] opinion, the principle of reaction 'should not be regarded as a fundamental principle of physics'. Regarding the conservation of momentum, he congratulated Poincaré for his 'beautiful formula' […] and suggested that the quantity $(\boldsymbol{E} \times \boldsymbol{H})/c$ be regarded as 'equivalent to momentum'. He even speculated that the equivalence could some day become an identity, 'if we manage to consider ponderable matter as a modification of the ether'. [17, pp. 29-30][26]

As Darrigol [17, p. 30] points out, it was Max Abraham who then turned this suggestion into one of the most elaborate formulations of the electromagnetic worldview. He did this in two articles from 1902, in which he elaborated a theory according to which the electromagnetic field completely determined the electron's equations of motion. Given that he took the electron to be the fundamental constituent of all matter, these equations could then be taken to provide an electromagnetic foundation for mechanics [20, p. 20].

Abraham started by pointing out that, up until then, most electromagnetic electron-models conceptualized their mass in terms of electromagnetic energy (as, for example, Wien did, see the quote on page 9). With regards to the electron's electromagnetic inertia, however, this meant that one could only obtain the electron's longitudinal mass, i.e. "the inertia opposing acceleration in the direction of its motion";[27] not, however, their transverse mass, i.e. the "inertia at play in accelerations perpendicular to the direction of its path" [20, p. 21],[28] since in the transverse case, there is no change in electromagnetic energy. This was a problem, Abraham pointed out, since the experiments on the velocity-dependency of the electron's mass carried out by Walter Kaufmann at that time indicated that it was primarily the transverse mass that was at play. To

[26] The quotes are from a letter by Lorentz to Poincaré from January 1901 (published in [64, pp. 70-71]) [17, p. 30].

[27] "diejenige Trägheit welche sich einer Beschleunigung in der Bewegungsrichtung widersetzt." [20, p. 21]

[28] "die bei Beschleunigung senkrecht zur Bahnrichtung in Betracht kommt." [20, p. 21]

overcome this, Abraham [20, p. 25] therefore proposed to reconceptualize the electron's mass by means of the notion of electromagnetic momentum.[29]

To conceptualize the electromagnetic field's influence on the electron's motion, it was essential to consider, according to Abraham [20, p. 23], that the electron's motion modifies this field, since a moving charge gives rise to a magnetic field. Hence, one could not represent the electron as a point particle: one had to take into account the charge distribution over its volume, to accommodate the fact that the electron's motion could give rise to an electromagnetic field that is, spatially speaking, distributed unequally. This vastly increased the complexity of the issue, however. To make it more graspable, Abraham followed an approach he borrowed from the study of conduction currents.[30] Such currents are called stationary when their strength is constant, in which case the magnetic field induced is determined purely by current strength. When the strength is not constant, this is strictly speaking no longer the case, but in low frequency cases, there are certain mathematical approximations to proceed as if the current is stationary, a state that is described as quasi-stationary. Abraham now proposed to proceed in a similar way: "the field will be assumed to be quasi-stationary, i.e. it will be determined completely by the instantaneous velocity of the electron" [20, p. 23].[31]

Abraham [20, pp. 25-26] then conceptualized the field's electromagnetic momentum in terms of the Poynting energy flux vector, as Poincaré had done. In contrast to Poincaré, however, Abraham [20, p. 27] took it to be a real physical property of the ether, which acted as the source for the electron's electromagnetic mass, both in its longitudinal and transverse form. In fact, Abraham concluded his first paper, this was all there was to the electron's mass: if one applied his mass-expressions to Kaufmann's first results (published in [21]), the ether's electromagnetic momentum had to be responsible for the electron's entire mass, since any kind of added mechanical mass would lead to a velocity-dependency expression that deviated further from Kaufmann's. Hence, Abraham concluded, "[t]he electron's inertia is caused exclusively by its electromagnetic field" [20, p. 40].[32]

In a second article from 1902 [22], Abraham further elaborated his theory's foundations. These consisted of three elements: (a) a kinematical equation, which constrains the electron's freedom of motion; (b) field equations that govern the electromagnetic

---

[29] According to Abraham [22, p. 110], it was Poincaré who had first introduced this notion. As Darrigol [17, pp. 30-31] has pointed out, however, ascribing to Poincaré the specific idea of an electromagnetic momentum is not completely correct: "Poincaré is often regarded as the inventor of the notion of electromagnetic momentum. This is a double historical mistake: the notion already existed in Maxwell's theory, and it was only a fiction in Poincaré's analysis of Lorentz's theory."

[30] Abraham was quite familiar with this subject, since it was the topic of his PhD dissertation (supervised by Max Planck), which "was an examination of electrical oscillations in conductors and was the first in a series of studies in which Abraham applied his specialty, Maxwellian theory, to problems of wireless telegraphy and antenna theory" [58, p. 8].

[31] Das Feld soll weiterhin als quasistationär angenommen warden, d.h. es soll bestimmt sein durch Angabe der momentanen Geschwindigkeit des Electrons [20, p. 23].

[32] Die Trägheit des Electrons ist ausschließlich durch sein electromagnetisches Feld verursacht. [20, p. 40]

field produced by the electron; and (c) dynamical equations governing the electron's motion in an external electromagnetic field [22, p. 108]. While (b) and (c) were mostly taken over from earlier work – the field equations were borrowed from Lorentz's work, and the electron dynamics was mainly a further elaboration of Abraham's earlier [20] paper –, the first part was new. Its central claim was that the electron's kinematics had to be considered as identical to the kinematics of the rigid body: "electricity adheres to the volume elements of the rigid body, in the same way as matter adheres to the volume elements of the rigid body" [22, p. 108].[33] The reason for this was that "[a] deformable electron would require the presence of non-electromagnetic forces in order to maintain its stability because of the mutual Coulomb repulsion between its constituent parts" [23, p. 216]. The resulting theory, according to Abraham, would look as follows:

> A thought similar to the one just presented may have guided Heinrich Hertz, when he allowed in his *Principles of Mechanics* only the existence of such kinematical connections that would give rise to neither the generation nor the destruction of kinetic energy. That was necessary because he wanted to trace back all energy to kinetic energy of moving masses, all forces to kinematic connections. To the objection that we find rigid connections in reality only approximately realized, Hertz [1, p. 41] replied as follows: 'In seeking the actual rigid connections our mechanics will possibly have to descend to the world of atoms'. Now, the electromagnetic mechanics will descend even further; in the atoms of negative electricity, these spheres whose radius is only the billionth part of a millimetre, it takes on the form of a rigid, unchanging arrangement of the electric charge. That it is permissible to speak of rigid connections before one speaks of forces, Hertz has shown convincingly. Our dynamics of the electron completely omits any speak of forces, that would aim to deform the electron. It speaks only of 'external forces', which are capable of endowing it with velocity or rotational velocity, and of 'internal forces' which, emerging from the electron's field, keep it in equilibrium. And these forces and rotational forces are only auxiliary concepts, defined through the kinematical and electromagnetic basic concepts. The same holds for words such as 'work', 'energy', 'momentum', which have only been chosen to let the analogy of the electromagnetic mechanics to the ordinary mechanics of material bodies clearly stand out. [22, p. 109][34]

---

[33] [W]ie die Materie an den Volumenelementen des starren Körpers, so haftet die Elektrizität an den Volumenelementen des starren Elektrons. [22, p. 108]

[34] Eine der soeben angedeuteten verwandte Überlegung mag Heinrich Hertz geleitet haben, als er in seinen 'Prinzipien der Mechanik' nur solche kinematische Zusammenhänge zuließ, deren Bestehen weder Erzeugung noch Zerstörung kinetischer Energie bedingt. Das war notwendig, weil er alle Energie auf kinetische Energie bewegter Massen, alle Kräfte auf kinematische Verbindungen zurückführen wollte. Dem Einwande, daß wir starre Verbindungen in Wirklichkeit nur angenähert realisiert finden, begegnet Hertz [1, p. 41] mit den Worten: 'auf der Suche nach den wirklichen starren Verbindungen wird unsere Mechanik vielleicht zur Welt der Atome herabzusteigen haben'. Nun, die elektromagnetische Mechanik steigt noch weiter herab; in den Atomen der negativen Elektrizität, diesen Kugeln, deren Radius nur den billionten Teil eines Millimeters beträgt, nimmt sie eine starre, unveränderliche Anordnung der elektrischen Ladung an. Daß es zulässig ist, von starren Verbindungen zu reden, bevor man von Kräften gesprochen hat, das hat Hertz überzeugend dargetan. Unsere Dynamik des Elektrons unterläßt es überhaupt, von Kräften zu reden, die das Elektron zu deformieren bestrebt

This quote shows how Hertz's *Principles* acted as a source of inspiration for Abraham's electromagnetic worldview in multiple ways. First of all, there is the claim, equally well to be found in the quote by Wien on page 9, that an electromagnetic electron-dynamics could account for the emergence of Hertz's hidden masses-mechanics (Abraham even explicitly quotes Hertz's claim that the rigid connections are possibly to be found in the world of atoms, quoted on page 7).

Second, Abraham's formulation of the electromagnetic worldview explicitly takes over an aspect of Hertz's mechanics, namely its rigid body-kinematics. The electron is to be seen as an extended material element that is constituted as a rigid body through the interaction between its charge and the surrounding electromagnetic field. No appeal to any forces is then really necessary, since this kinematic-electromagnetic equilibrium cannot in any way be disturbed or destroyed. Hence, Abraham could claim, in line with Hertz's approach, that in his theory as well, the notion of force was not a primitive but rather only a derivative notion, which could be obtained because the notion of electromagnetic momentum allowed one "to reduce the inner force to the 'impulse' and 'angular momentum' of the electron's electromagnetic field, and in this way it permits a simplified calculation of the electromagnetic mass and the electromagnetic moment of inertia" [22, p. 110].[35]

Finally, Hertz's theory formed the explicit target for the construction of electromagnetic analogues for the laws of mechanics. As we have seen on page 9, Wien's claim that the notion of electromagnetic inertia could be used to account for the constitution of Hertz's rigid connections then led him to the formulation of electromagnetic replacements for the three laws of mechanics (with the third law in a restricted form). Abraham equally well elaborated such explicit analogues to the three laws, but he did so by means of the notion of electromagnetic momentum, which he conceptualized, as he had done earlier following Poincaré and Lorentz (see pages 11 and 12), in terms of the Poynting energy flux vector [22, p. 125]. This immediately provided Newton's third law, the action-reaction principle. It also allowed him to express equations of motion for the rigid electron, which led him to the following analogue for Newton's first law: "if the motion of the electron was, from the beginning, a uniform, purely translatory one with a velocity smaller than the speed of light, then to keep it uniform no external force or

sind. Sie spricht nur von 'äußeren Kräften', die ihm eine Geschwindigkeit oder Drehgeschwindigkeit zu erteilen vermögen, und von 'inneren Kräften', die vom Felde des Elektrons herrührend, jenen das Gleichgewicht halten. Und auch diese 'Kräfte' und 'Drehkräfte' sind nur Hülfsbegriffe, die definiert werden durch die kinematischen und die elektromagnetischen Grundbegriffe. Das Gleiche gilt von Worten wie 'Arbeit', 'Energie', 'Bewegungsgröße', bei deren Wahl allerdings das Bestreben maßgebend war, die Analogie der elektromagnetischen Mechanik zur gewöhnlichen Mechanik materieller Körper deutlich hervortreten zu lassen. [22, p. 109]

[35] "die Zurückführung der inneren Kräfte auf einen vom elektromagnetischen Felde des Elektrons abhängigen 'Impuls' und 'Drehimpuls', und gestattet so eine vereinfachte Berechnung der elektromagnetischen Masse und des elektromagnetischen Trägheitsmomentes." [22, p. 110]

rotational force is required" [22, pp. 142-143].[36] He then introduced a second conceptualization of electromagnetic momentum, in the form of a Lagrangian defined in terms of the difference between the electric and magnetic energy of the electron's electromagnetic field [22, p. 143]. After showing how to derive expressions for quasi-stationary acceleration and longitudinal and transverse mass from it [22, pp. 149-151], he then used them to formulate an electromagnetic analogue of Newton's second law, which differed from the original as follows:

> The functional relation between force and acceleration is represented in the dynamics of the electron by a linear vector function of a more general kind than in ordinary mechanics. The electromagnetic mass, the coefficient system of the linear velocity function, is a tensor of rotational symmetry,[37] whose axis of symmetry is determined by the direction of motion of the electron. [22, p. 153][38]

In this way, Abraham saw his electron-theory as accounting for, and going beyond, the theory of mechanics (in the Hertzian formulation). His motivation for this commitment to the electromagnetic worldview again derived from Kaufmann's experiments, which showed, he claimed, that "the electron's mass is completely electromagnetic in nature" [22, p. 107].[39] This claim was, in turn, strengthened by Kaufmann, who argued that new experimental results obtained in 1902 [24] and 1903 [25] were completely in line with Abraham's electron-theory, which brought Kaufmann to repeat Abraham's conclusion: "their mass is completely electromagnetic in nature" [25, p. 103].[40]

Abraham was not the only one to conceptualize the electron's mass in this way. Lorentz claimed that his 1904 elaboration of his electron-theory equally well entailed "that there is no other, no 'true' or 'material' mass" [26, p. 821]. His electrons were not rigid, however: they rather "have their dimensions changed by the effect of a translation" [26, p. 818]. This was needed to account for the failures to detect any effect of the earth's motion through the ether on optical and electromagnetic phenomena (the most famous such null-result being the 1887 Michelson-Morley experiment). After introducing the equations that governed how the electron's dimensions and its field equations would transform with such changes in motion, Lorentz [26, p. 820] then pointed out that the transformation equations governing electromagnetic momentum would become

---

[36] "War die Bewegung des Elektrons von Anbeginn an eine gleichförmige, rein translatorische, und war die Geschwindigkeit kleiner als die Lichtgeschwindigkeit, so ist, um die Bewegung gleichförmig zu erhalten, keine äußere Kraft oder Drehkraft erforderlich." [22, pp. 142-143]

[37] To clarify to his audience what he meant by such a tensor, Abraham then referred to a recent article by him in the *Enzyklopädie der mathematischen Wissenschaften*, which provided "one of the first organized representations of vector analysis in Germany" [58, p. 8].

[38] "Die funktionelle Beziehung zwischen Kraft und Beschleunigung wird in der Dynamik des Elektrons durch eine lineare Vektorfunktion allgemeiner Art, als in der gewöhnlichen Mechanik, dargestellt. Die elektromagnetische Masse, das Koeffizientensystem der linearen Vektorfunktion, ist ein Tensor von rotatorischer Symmetrie, dessen Symmetrieachse durch die Bewegungsrichtung des Elektrons bestimmt ist." [22, p. 153]

[39] "Die Masse des Elektrons ist rein elektromagnetischer Natur." [22, p. 107]

[40] "[d]eren [i.e. Elektronen] Masse rein elektromagnetischer Natur ist"." [25, p. 103]

very complex, unless one restricted the electron's motion, as Abraham had done, to quasi-stationary motion. One could then easily show that, with increasing velocity, a longitudinal and a transverse mass could be distinguished [26, p. 821]. These differed, however, from Abraham's:

> The values […] which I have found for the longitudinal and transverse masses of an electron, expressed in terms of its velocity, are not the same as those that have been formerly obtained by Abraham. The ground for this difference is solely to be sought in the circumstance that, in his theory, the electrons are treated as spheres of invariable dimensions. [26, p. 826]

Lorentz [26, p. 829] concluded by pointing out that, in spite of this difference, his account was equally well in line with Kaufmann's 1903 results. This surprised Abraham, since he had argued that only a rigid electron could be fully electromagnetic in nature (see page 13). He therefore carried out a detailed comparison of the two theories, listing both the shared hypotheses as well as those on which they differed. This showed him that Lorentz's electron could not be completely electromagnetic in nature, since his deformation-hypothesis required the introduction of non-electromagnetic forces:[41]

> If one accelerates such an electron, its flattening is increased; thus work must be done against the electric forces. While for the undeformable electron the increase of the energy is equal to the work done by the external electric forces, this is not the case here anymore; the increase of energy accompanying an increase in velocity is greater than the work performed by the external forces. The consequence of this assumption is therefore that one includes, besides the inner electromagnetic forces, other, non-electromagnetic inner forces, which together determine the form of the electron. These would then do the required extra work during the contraction, which together with the work of the external forces would be equivalent to the increase in the electron's electromagnetic energy. As long as one does not specify by which law these forces are governed, the hypothesis system is incomplete. [27, p. 578][42]

[41] Here, Abraham merely stipulated this claim. He elaborated it in more detail in his 1905 textbook on electromagnetism [59, pp. 201-208], by means of the two different expressions for electromagnetic momentum he had introduced in 1902 (in terms of the Poynting energy flux vector and in Lagrangian terms, see page 14).

[42] "Beschleunigt man ein solches Elektron, so wird seine Abplattung vermehrt; es mus also gegen die elektrische Kräfte Arbeit geleistet werden. Während für das undeformierbare Elektron die Zunahme der Energie gleich der von den äusseren elektrischen Kräften geleisteten Arbeit ist, findet das hier nicht mehr statt; die Energiezunahme bei einer Geschwindigkeitsvermehrung ist grösser, als die Arbeit der äusseren Kräfte. Die konsequente Verfolgung der Hypothese zwingt also dazu, neben den inneren elektromagnetischen Kräften noch andere, nicht elektromagnetische, innere Kräfte anzunehmen, welche im Verein mit jenen die Form des Elektrons bestimmen. Diese würden dann bei der Kontraktion die erforderliche Arbeit leisten, die zusammen mit der Arbeit der äusseren Kräfte der Steigerung der elektromagnetischen Energie des Elektrons äquivalent ist. Solange man nicht angiebt, nach welchem Gesetz diese Kräfte wirken sollen, ist das Hypothesensystem unvollständig." [22, p. 578]

It was Poincaré [28] who addressed this issue. He did this by first showing that Lorentz's transformation equations form a group, and that this fact could be used to establish the invariance of different physical properties over such transformations [28, p. 146]. After investigating how the electron's electromagnetic Lagrangian (see page 14) transformed under these equations, Poincaré turned to the question what would be required, given this transformation expression, to ensure the viability of a deformable electron. This led him to postulate an internal stress acting upon the electron, which made the difference in momentum found by Abraham (see footnote 41) vanish. One advantage of this non-electromagnetic addition was that in this way the principle of relativity could be upheld [28, pp. 164-165].

The principle of relativity was valid, on Lorentz's electron-theory, because the deformability of his electron allowed him to account for those experiments that had failed to detect any influence of the earth's motion through the ether (see page 15). Abraham's electron-model, on the other hand, could not account for these null-results: according to his theory, it should be possible to detect certain influences. He did not see this as a problem, however, since "[t]he question, if and why an influence of the earth's motion on electrical and optical phenomena on the earth's surface cannot be detected, can at present not yet be answered decisively" [27, p. 579].[43] He believed that more was to be gained from new electron-experiments, especially involving Becquerel-rays, since these could attain velocities almost up to that of light, and hence could provide more insight into the precise velocity-dependency of the electron's mass. Kaufmann was carrying out such experiments in 1905-1906. These came to be seen as potentially offering a decision between the electromagnetic worldview and the relativistic approach, because, by that time, it was generally accepted that Lorentz's electron was not completely electromagnetic, and because Albert Einstein, in his first relativity paper [29], had obtained a velocity-dependency expression that was considered equal to Lorentz's. Kaufmann's conclusion was quite direct:

> These results clearly decide against the validity of the Lorentzian and hence also of the Einsteinian theory; insofar as one takes this as a refutation of these theories, one should also take the attempt, to base the whole of physics including electrodynamics and optics on the principle of relativity, as failed. […] For the moment, we will rather maintain the assumption that the physical phenomena depend on motion relative to a completely determined reference system, which we denote as the absolutely resting ether. Even if up until now such an influence of the movement through the ether by electrodynamic or optical experiments has not yet been shown, this should not be taken to mean that it is impossible to detect it. [30, pp. 534-535][44]

[43] "Die Frage, ob und wieso ein Einfluss der Erdbewegungen auf die elektrischen und optischen Erscheinungen an der Erdoberfläche sich nicht entdecken lässt, ist zur Zeit noch keineswegs spruchreif." [27, p. 579]

[44] "Die vorstehenden Ergebnisse sprechen entschieden gegen die Richtigkeit der Lorentzschen und somit auch der Einsteinschen Theorie; betrachtet man diese aber als widerlegt, so wäre damit auch der Versuch, die ganze Physik einschließlich der Elektrodynamik und der Optik auf das Prinzip der Relativbewegung zu gründen, einstweilen als mißglückt zu bezeichnen. […] Wir werden vielmehr einstweilen bei der Annahme verbleiben müssen, daß die physikalischen Erscheinungen von der Bewegung relative zu einem ganz bestimmten

4 The Construction of a Relativistic Response

Many saw this conclusion as a significant challenge for the validity of the relativity principle (see e.g. the reactions by Abraham [31, p. 196], Arnold Sommerfeld [32, p. 251], Lorentz [33, p. 203], and Poincaré [28, p. 132]).[45] And Einstein as well, in one of his few explicit discussions of the experiments,[46] admitted that, if there was no error underlying them, the results could mean "that the foundations of the theory of relativity do not correspond to the facts" [34, p. 439].[47] This was not an endorsement of the electromagnetic worldview, however, since, as a worldview, Einstein considered it too restricted: "their basic assumptions concerning the dimensions of the moving electrons are not suggested by theoretical systems that encompass larger complexes of phenomena" [34, p. 439].[48]

This restrictedness-claim was based on how he had obtained expressions for the electron's longitudinal and transverse mass in his first 1905 relativity paper [29]. There he first elaborated a kinematics that was concerned with the behavior of rigid rods and clocks constrained by the principle of relativity and the principle of the constancy of the velocity of light. Applying this kinematics to what he called the Maxwell-Hertz equations for empty space then provided him with transformation equations for the electromagnetic field and the electromotive forces they could exercise. From these, he then obtained expressions for the electron's longitudinal and transverse mass by reflecting on how the electron's equations of motions transformed between frames in relative motion [29, p. 919]. Einstein immediately emphasized two aspects of this derivation:

> Of course, with a different definition of force and acceleration we would obtain different numerical values for the masses; this shows that we must proceed with great caution when comparing different theories of the motion of the electron. It should be noted that these results concerning

Koordinatensystem abhängen, das wir als den absolut ruhenden Äther bezeichnen. Wenn es bis jetzt nicht gelungen ist, durch elektrodynamische oder optische Versuche einen derartigen Einfluß der Bewegung durch den Äther nachzuweisen, so darf daraus noch nicht auf die Unmöglichkeit eines solchen Nachweises geschlossen werden." [30, pp. 534-535]

[45] This aspect of Kaufmann's experiments has been discussed quite extensively, see e.g. [65, 16, 63, 15, 57, 66, 67].

[46] The only other place where he discussed the experiments in some way was in a small article from 1906 [60], in which he proposed an alternative way to measure the velocity-dependency of the electron's mass. Insofar as is known, these experiments were not carried out at the time [16, p. 343].

[47] "daß die Grundlagen der Relativitätstheorie nicht den Tatsachen entsprechen." [34, p. 439]

[48] "weil ihre die Maße des bewegten Elektrons betreffenden Grundannahmen nicht nahe gelegt werden durch theoretische Systeme, welche größere Komplexe von Erscheinungen umfassen." [34, p. 439]

> mass are also valid for ponderable material points, since a ponderable material point can be made into an electron (in our sense) by adding to it an *arbitrarily small* electric charge. [29, p. 919][49]

First, given that the electric charge could be arbitrarily small, the derivation concerned not merely the electron, but all material bodies in general. And second, depending on the specific mechanics (in the form of a definition of force and acceleration) used, one could obtain different expressions. The definition used by Einstein was the standard Newtonian one: "[n]umerical value of mass × numerical value of acceleration = numerical value of force" [29, p. 919].[50] However, as Kaufmann [30, pp. 530-531] was the first to point out, this provided expressions that differed slightly from Lorentz's (see [16, p. 329] for a short discussion of this). One rather had to use a relativistic mechanics, which was first developed in 1906 by Max Planck [35].

According to Planck, the relativity principle required that one replaced the ordinary Newtonian equations of motion for a free point particle ('eines freien Massenpunktes'), formulated in terms of mass and acceleration, with equations of motion formulated in terms of the time derivative of momentum, where the notion of time was to be understood in relativistic terms [35, p. 139]. In this way, Planck pointed out, one obtained a mechanics according to which the shape of entities such as the electron would change with velocity. While he acknowledged earlier discussions about how to conceptualize the work required to carry out such deformations (a reference, without mentioning any names, to Abraham, see page 16), Planck did not see these as significant, since this work could be accounted for completely in terms of the electron's kinetic energy. Proceeding in this way had two advantages, according to Planck: first, there was no need to specify whether the electron's mass was completely electromagnetic or not; and second, "it is not necessary to ascribe to the electron a spherical shape, nor in fact any specific shape at all, in order to obtain a determinate dependency of inertia on velocity" [35, p. 137].[51] In this way, Planck's work indicated that, from a relativistic perspective, two of the main issues on which the electromagnetic worldview had focused – the electron's shape and the source of its inertial mass – were not really significant at all. In two articles from 1908, Planck then further elaborated these claims.

In the first [36], he argued that work by his former doctoral student Kurt von Mosengeil on a relativistic account of thermal radiation,[52] showed that there were many different ways in which mass could be velocity-dependent, depending on how one

---

[49] "Natürlich würde man bei anderer Definition der Kraft und der Beschleunigung andere Zahlen für die Massen erhalten; man ersieht daraus, daß man bei der Vergleichung verschiedener Theorien der Bewegung des Elektrons sehr vorsichtig verfahren muß. Wir bemerken, daß diese Resultate über die Masse auch für die ponderablen materiellen Punkte gilt; denn ein ponderabler materieller Punkt kann durch Zufügen einer *beliebig kleinen* elektrischen Ladung zu einem Elektron (in unserem Sinne) gemacht werden." [29, p. 919]

[50] "Massenzahl × Beschleunigungszahl = Kraftzahl." [29, p. 919]

[51] "daß man dem Elektron weder Kugelgestalt noch überhaupt irgend eine bestimmte Form zuzuschreiben braucht, um zu einer bestimmten Abhängigkeit der Trägheit von der Geschwindigkeit zu gelangen." [35, p. 137]

[52] See [68, pp. 133-137, 16, p. 360] for discussions of von Mosengeil's work. After his death in 1906, Planck had his work published as [69].

conceptualized a body's inertial mass (in this way broadening Einstein's claim that it depended on the mechanics used, see page 18).[53] This led Planck [36, p. 27] to question the significance of searching for the specific source of that velocity-dependency, since it became almost entirely an issue of definition ('Definitionssache'). In fact, the question no longer made much sense, Planck continued, since the notion of inertial mass, "which in [classical] mechanics plays such a fundamental role, is reduced in [relativistic] dynamics to a secondary notion" [36, p. 27].[54] From a relativistic perspective, Planck [36, p. 30] claimed, it made more sense to characterize physical bodies directly in terms of energy (and in a footnote, he then pointed out that this was, in essence, a generalization of the mass-energy equivalence obtained earlier by Einstein [37]).

In the second article [38], Planck then reflected on what this all meant for one principle in particular, namely the action-reaction principle. He started by pointing out that the principle's universality had recently been threatened by Lorentz's electromagnetic theory. It was Abraham who saved the principle's generality, Planck [38, pp. 828-829] then argued, by introducing a second form of momentum, namely an electromagnetic one, besides the already existing mechanical one. Abraham argued for this, Planck [38, p. 829] claimed, by means of an analogy: just as the work-energy principle could only be upheld by introducing an electromagnetic form of energy, the action-reaction principle could only be maintained by introducing an electromagnetic form of momentum. According to Planck, however, there was an issue with this move:

> However, this comparison, which is in itself certainly indisputable, still leaves one essential difference untouched. For we already know a whole series of different kinds of energy: kinetic energy, gravity, elastic deformation energy, heat, chemical energy, and the addition of electromagnetic energy to these forms does not constitute a principled innovation. Regarding momentum, however, up until now we only know one kind: the mechanical one. While energy was already a universal physical concept beforehand, momentum was up until now a specifically mechanical concept, the reaction principle a specifically mechanical law; and because of that, the required extension has to be seen as also a fundamental change, which has turned the up until now relatively simple and uniform concept of momentum into quite a complicated one. [38, p. 829][55]

---

[53] Planck [36, p. 28] distinguished, more specifically, between transverse mass, longitudinal isothermal-isochoric mass, longitudinal adiabatic-isochoric mass and longitudinal adiabatic-isobaric mass. See [16, pp. 361-362, 66, p. 79] for discussions of this work by Planck.

[54] "Diese Größe, welche in der reinen Mechanik eine so fundamentale Rolle spielt, sinkt in der allgemeinen Dynamik zu einem sekundären Begriff herab." [36, p. 27]

[55] "Indessen läßt dieser an sich gewiß unanfechtbare Vergleich doch noch einen wesentlichen Unterschied unberührt. Denn bei der Energie kennen wir ohnehin schon eine ganze Reihe verschiedener Arten: die kinetische Energie, die Gravitation, die elastische Deformationsenergie, die Wärme, die chemische Energie, und es bedeutet daher keine prinzipielle Neuerung, wenn man diesen verschiedenen Formen als eine weitere Form noch die elektromagnetische Energie angliedert. Dagegen bei der Bewegungsgröße kannte man bisher nur eine einzige: eben die mechanische. Während die Energie von vornherein schon einen universellen physikalischen Begriff darstellt, war die Bewegungsgröße bisher speziell ein mechanischer Begriff, das Reaktionsprinzip ein speziell mechanischer Satz, und daher mußte die als notwendig erkannte Erweiterung immerhin auch als eine Umwälzung prinzipieller Art

The theory of relativity offered a way to overcome this issue, Planck claimed, by conceptualizing momentum in terms of energy, more specifically as "that vector, which denotes the flow of energy, not only Poynting's electromagnetic energy flux, but rather energy flow in general" [38, p. 829].[56] This flow, Planck continued, could be brought about in many different ways, but by dividing the vector expressing it by c^2 one obtained a very general expression for momentum. Planck [38, p. 830] called this result the law of inertia of energy, and, as Miller put it, it in fact offered "a generalization of Einstein's mass-energy equivalence which included the flow of any sort of energy, and not just the total mechanical energy [and which] asserted that the effect of forces acting on a body was transmitted by a momentum density whose source was a flow of energy" [16, p. 366]. In the last part of the paper, Planck then argued that in the case of electromagnetic momentum, this energy flow would take on the form of the Maxwell stress tensor. The significance of this result was that it addressed the issue that had plagued Lorentz's theory, namely how to conceptualize the action-reaction principle given an unmovable ether:

> It is remarkable how, through this law [of inertia of energy], the Maxwell stresses obtain a physical significance for the theory of the unmovable ether as well. For, as pressure force these stresses have no real meaning in this theory, since one cannot really make any sense of a force, which acts on something absolutely unmovable. [Here, Planck refers in footnote to Lorentz's work] That the Maxwell stresses have nevertheless maintained themselves in the theory of the unmovable ether by the fact that they often proved to be a convenient mathematical aid for certain calculations, even though they were, so to speak, officially abolished, could suggest that they do play some significant physical role, which they also played in the theory of the unmovable ether. [38, p. 830][57]

In this way, we come to see how it was primarily Planck who further elaborated Einstein's criticism of the electromagnetic approach, that as a worldview it was too restricted (see page 18) into the claim that, from a relativistic perspective, the issues that were primordial for the electromagnetic worldview were either of no real significance

---

empfunden werden, durch welche der bisher verhältnismäßig einfache und einheitliche Begriff der Bewegungsgröße einen erheblichen komplizierten Charakter erhält." [38, p. 829]

[56] "denjenigen Vektor [...], welcher die Energieströmung ausdrückt, aber nicht allein die Poyntingsche elektromagnetische Energieströmung, sondern die Energieströmung ganz im allgemeinen." [38, p. 829]

[57] "Es ist bemerkenswert, wie durch diesen Satz die Maxwellschen Spannungen auch für die Theorie des ruhenden Äthers eine physikalische Bedeutung gewinnen. Denn als Druckkraft haben diese Spannungen in dieser Theorie keinen rechten Sinn, da man doch einer Kraft, die auf etwas absolut Unbewegliches wirkt, nicht wohl eine Bedeutung beimessen kann. Daß die Maxwellschen Spannungen sich dennoch, trotzdem sie sozusagen offiziel abgeschaffen waren, in der Theorie des ruhenden Äthers behauptet haben, indem sie sich eben für gewisse Rechnungen häufig als bequemes mathematisches Hilfsmittel erwiesen, konnte schon den Gedanken nahelegen, daß ihnen doch irgendeine besondere physikalische Rolle zufällt, durch die sie auch für den ruhenden Äther legitimiert werden." [38, p. 830]

(the electron's shape and the source of its inertial mass) or could be incorporated as secondary notions (inertial masses and the Maxwell stresses). This response, I will argue now, can be characterized as Hertzian in a specific kind of way.

5 A Hertzian Response

We have seen (page 18) that Einstein obtained mass-expressions by conceptualizing the electron as a point particle endowed with an arbitrarily small electric charge. And Planck equally well claimed that specifying a body's shape was not required to obtain a definite velocity-dependency. Their approach hence differed from that of e.g. Abraham (as well as Wien and Lorentz), who claimed that one had to conceptualize the electron as extended in order to obtain any results (see page 12). Moreover, one equally well had to introduce a quasi-stationary approximation, since otherwise the question would become too complex to resolve (see pages 12 and page 15).

The importance of this approximation, however, lay not solely in the reduction of complexity. As Miller has shown, it also allowed them to elaborate electromagnetic analogues for the laws of mechanics: on Abraham's account, "unambiguous identification of the electron's mass as the coefficient of its acceleration required restricting the particle to 'quasi-stationary acceleration'" [16, p. 59]. This shows how the electromagnetic worldview, even though it proclaimed to replace the mechanical worldview, was still working within the traditional Newtonian framework in which forces are to be understood in terms of mass times acceleration. While Einstein also still employed this conceptualization, Planck soon pointed out that the theory of relativity in fact required a thorough reconceptualization of this framework, which led him to reduce the notion of inertial mass to only secondary importance, and to the claim that questions concerning the electron's shape or the velocity-dependency of its mass were of no real significance.

In this way, Planck thus not solely elaborated a relativistic mechanics, but equally well questioned the electromagnetic worldview's methodological approach. As we have seen, both Wien (page 9) and Abraham (page 14) searched for laws that were almost exact electromagnetic analogues of the laws of mechanics. In this way, however, they were not so much elaborating a simple and unified electromagnetic worldview underlying these laws, according to Planck, but rather turning them into a complicated whole, as he argued with regards to the notion of electromagnetic momentum (see page 20). In a sense, one could say that, on Planck's view, the electromagnetic worldview did not go far enough, methodologically speaking, because it retained the Newtonian framework underlying the theory of mechanics, and just endowed it with an electromagnetic coating, rather than thoroughly reconceptualizing it. At the same time, one could also say that, according to Planck, the electromagnetic worldview went too far, ontologically speaking, since there was no reason to assume that the mass of electrons and of all material bodies had to derive from one particular source, i.e.

electromagnetism. This claim, that the electromagnetic worldview overstepped its boundaries, ontologically speaking, was formulated very poignantly by Max von Laue:[58]

> [We often see] the mistaken belief, that the common behaviour of all forces under the Lorentz transformations points to a common origin, that all of them can be traced back to electrodynamic forces. The question, whether that is at all possible, is completely outside of our concerns. That commonality says nothing else than that the principle of relativity is valid in all areas of physics; and this we must assume, if this principle is to be more than a sometimes useful calculation rule. To conclude more from it would be as hasty as if one wanted to conclude from the general validity of the energy principle that all natural processes are in the end mechanical. [39, p. 186][59]

This quote shows how the relativistic response can be seen as a Hertzian one, in the sense that, from the relativistic point of view, the specific microphysical constituents underlying particular relativistic phenomena become hidden. Relativity is not concerned with the essence of matter or force: their constitution cannot be grasped from the models that the theory puts forwards. That this relativistic idea of hiddenness is very similar to how Hertz characterized his notion of hiddenness as nothing special (nothing more than masses in motion, see the quote on page 5) becomes very clear if we look at Planck's treatment of Maxwell stresses in terms of his law of the inertia of energy. The law, we have seen equally well covered electromagnetic, mechanical and other kinds of energy, without discriminating. Conceptualized in these terms, the Maxwell stresses were not, as the electromagnetic worldview assumed, an instance of a special kind of hidden entity – the ether -, but rather nothing more than hidden energy in motion, just like so many other phenomena. In line with how Hertz saw his reconceptualization of the notion of force (see page 7), the relativistic approach hence did not answer the question regarding the essence behind the electron's mass, but it did rule out the posing of unjustified questions concerning, for example, the ether's constitution.

This response can moreover also be called Hertzian in the sense that it was concerned with a conceptual problem very similar to the one that had motivated Hertz. Hertz's issue, we have seen on page 4, was that whereas according to the first two laws, forces act on a body in a particular direction, according to the third law a force connects two

---

[58] Max von Laue made this claim in 1911 in what, according to Staley [15, p. 334], was "the first and authorative textbook on relativity". In the textbook, von Laue (who worked in Berlin until 1909 as doctoral student and assistant of Planck) elaborated the relativistic dynamics developed by Einstein and Planck in Minkowskian terms [68, p. 127]. For extensive discussions of von Laue's work, see the work of Michel Janssen [57] with Matthew Mecklenburg [63].

[59] "[…] der Irrtum, daß das gemeinsame Verhalten aller Kräfte gegen die Lorentz-Transformation auf einen gemeinsamen Ursprung von ihnen hinwiese, daß sich etwa alle auf elektrodynamische Kräfte zurückführen ließen. Die Frage, ob so etwas möglich ist, steht gänzlich außerhalb unserer Betrachtungen. Jene Gemeinsamkeit sagt nichts anderes aus, als daß das Relativitätsprinzip in allen Gebieten der Physik gilt; und dies müssen wir annehmen, wenn dies Prinzip mehr sein soll, als eine manchmal nützliche Rechnungsregel. Mehr daraus zu folgern, wäre so voreilig, als wenn man etwa aus der Allgemeingültigkeit des Energieprinzips schließen wollte, daß alle Naturvorgänge in letzter Linie mechanische sind." [39, p. 186]

bodies. Similarly, Lorentz's and Wien's issues with the action-reaction principle emerged out of a tension between the principle's demand for a bidirectional interaction between ether and matter on the one hand, and their ether-model on the other, according to which matter could not act on the absolutely unmovable ether (see page 10). And similar to how Hertz overcame the issue by reconceptualizing the notion of force in terms of hidden masses in motion, Planck was able to overcome the issue of his concern by reconceptualizing the notion of electromagnetic momentum in terms of hidden energy in motion.

As such, even though Planck did not refer explicitly to Hertz's *Principles*, we can describe his response as Hertzian: just as Hertz introduced hidden masses in motion to overcome an ambiguity between two different conceptualizations of force, Planck introduced energy in motion, whose source remained hidden from the point of view of relativity, to overcome an ambiguity between mechanical and electromagnetic conceptions of action-reaction.

6 Conclusion

In this paper, I have argued that, contrary to what many historians have argued, Heinrich Hertz's *Principles of Mechanics* did inspire reflections within fundamental physics, concerning, among other things, the role played by the ether. I have argued, more specifically, that we can distinguish two different forms of inspiration. The first concerned the development of the electromagnetic worldview by Wilhelm Wien and Max Abraham. They proclaimed that their electromagnetic electron-models, which were concerned with how the interaction between charge and ether gave rise to the electron's mass, could account for Hertz's hidden masses-mechanics. This allowed them, they then claimed, to formulate electromagnetic analogues for the laws of mechanics.

The second could be found in the relativistic response to the electromagnetic worldview, elaborated in particular by Max Planck. This response can be considered as a Hertzian response, I have argued, in the sense that Planck carried out a reconceptualization of the concepts underlying the theory of mechanics that led him to claim that, from a relativistic point of view, the electromagnetic assumption made by Wien and Abraham was unjustified: models of the electron could not show that the electron's mass was completely electromagnetic in nature. Insofar as one could speak of the electron's nature, it was rather in terms of a Hertzian kind of hiddenness, embodied in Planck's law of the inertia of energy.

This suggests, I conclude, that Hertz's *Principles*, while they did not explicitly inspire the construction of mechanical ether-models at the time, still profoundly shaped the discussions that were going on in the foundations of physics.

Acknowledgments

This research has been funded by the Research Fund – Flanders (FWO) and by the Lichtenberg Group for History and Philosophy of Physics (Universität Bonn). I would like to thank Joshua Eisenthal, Dennis Lehmkuhl and the members of Lichtenberg Group for their encouragement and comments and questions.

References

1. Hertz H (1894) Gesammelte Werke von Heinrich Hertz. Band III. Die Prinzipien der Mechanik. Johann Ambrosius Barth, Leipzig.
2. D'Agostino S (1998) Hertz's Views on the Method of Physics: Experiment and Theory Reconciled?. In: Baird D, Hughes R, Nordmann A (eds). Heinrich Hertz: Classical Physicist, Modern Philosopher. Springer, Dordrecht, pp. 89-102.
3. Lützen J (2005) Mechanistic Images in Geometric Form: Heinrich Hertz's Principles of Mechanics. Oxford University Press, Oxford.
4. Eisenthal J (2018) Mechanics without mechanism. Studies in History and Philosophy of Modern Physics 62: 45-55.
5. Eisenthal J (2021) Hertz's Mechanics and a unitary notion of force. Studies in History and Philosophy of Science 90: 226-234.
6. Mulligan JF (1998) The Reception of Heinrich Hertz's Principles of Mechanics by his Contemporaries. In: Baird D, Hughes R, Nordmann A (eds). Heinrich Hertz: Classical Physicist, Modern Philosopher. Springer, Dordrecht, pp. 173-182.
7. Majer U (1998) Heinrich Hertz's Picture-Conception of Theories: Its Elaboration by Hilbert, Weyl, and Ramsey. In: Baird D, Hughes R, Nordmann A (eds). Heinrich Hertz: Classical Physicist, Modern Philosopher. Springer, Dordrecht, pp. 225-242.
8. Grasshoff G (1998) Hertz's Philosophy of Nature in Wittgenstein's Tractatus. In: Baird D, Hughes R, Nordmann A (eds) Heinrich Hertz: Classical Physicist, Modem Philosopher. Springer, Dordrecht, pp. 243-268.
9. Mulligan JF (2001) The Aether and Heinrich Hertz's The Principles of Mechanics Presented in a New Form. Physics in Perspective 3: 136-164.
10. Hertz H (1894) Gesammelte Werke von Heinrich Hertz: Band II Untersuchungen über die Ausbreitung der elektrischen Kraft. Johann Ambrosius Barth, Leipzig.
11. McCormmach R (1970) H.A. Lorentz and the electromagnetic view of nature. Isis 61: 459-497.
12. Lorentz HA (1899) Elektromagnetische Theorien physikalischer Erscheinungen. Physikalische Zeitschrift 1: 498-501, 514-519.
13. Lorentz HA (1900) Considerations on Gravitation. KNAW, Proceedings 2: 559-574.
14. Wien W (1901) Ueber die Möglichkeit einer elektromagnetischen Begründung der Mechanik. Annalen der Physik 310 (7): 501-513.
15. Staley R (2008) Einstein's Generation: The Origins of the Relativity Revolution. The University of Chicago Press, Chicago.

16. Miller AI (198) Albert Einstein's Special Theory of Relativity: Emergence (1905) and Early Interpretation (1905-1911). Addison-Wesley Publishing Company, Reading.
17. Darrigol O (1995) Henri Poincaré's Criticism of Fin De Siècle Electrodynamics. Studies in the History and Philosophy of Modern Physics 26(1): 1-44.
18. Poincaré H (1900) La théorie de Lorentz et le principe de réaction. In: Bosscha J (ed). Recueil des travaux offerts par les auteurs à H.A. Lorentz à l'occassion du 25me anniversaire de son doctorat, Martinus Nijhoff, Den Haag, pp. 252-278.
19. Goldberg S (1967) Henri Poincaré and Einstein's Theory of Relativity. American Journal of Physics 35: 934-944.
20. Abraham M (1902) Dynamik des Elektrons. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse: 20-41.
21. Kaufmann W (1901) Die magnetische und electrische Ablenkbarkeit der Becquerelstrahlen und die scheinbare Masse der Elektronen. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse: 143-155.
22. Abraham M (1902) Prinzipien der Dynamik des Elektrons. Annalen der Physik 315(1): 105-179.
23. Miller AI (1973) A Study of Henri Poincaré's 'Sur La Dynamique de l'Électron'. Archive for History of Exact Sciences 10: 207-328.
24. Kaufmann W (1902) Ueber die elektromagnetische Masse des Elektrons. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse: 291-296.
25. Kaufmann W (1903) Ueber die "Elektromagnetische Masse" der Elektronen. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse: 90-103.
26. Lorentz HA (1904) Electromagnetic Phenomena in a System Moving with Any Velocity Smaller than that of Light. KNAW, Proceedings: 809-831.
27. Abraham M (1904) Die Grundhypothesen der Elektronentheorie. Physikalische Zeitschrift 5(18): 576-579.
28. Poincaré H (1906) Sur la dynamique de l'électron. Rendiconti del Circolo Matematico di Palermo 21: 129-175.
29. Einstein A (1905) Zur Elektrodynamik bewegter Körper. Annalen der Physik 17: 891-921.
30 Kaufmann W (1906) Über die Konstitution des Elektrons. Annalen der Physik 19: 487-553.
31. Abraham M (1908) Theorie der Elektrizität. Zweiter Band. Elektromagnetische Theorie der Strahlung Zweite Auflage. B.G. Teubner, Leipzig.
32. Sommerfeld A, Eckert M, Märker K (2000) Wissenschaftlicher Briefwechsel. Verlag für Geschichte der Naturwissenschaften und der Technik, Berlin.
33. Lorentz HA, Kox A (2008) The Scientific Correspondence of H.A. Lorentz, Volume 1. Springer, New York.

34. Einstein A (1907) Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen. Jahrbuch der Radioaktivität und Elektronik 4: 411-462.
35. Planck M (1906) Das Prinzip der Relativität und die Grundgleichungen der Mechanik. Deutsche Physikalischen Gesellschaft. Verhandlungen 8: 136-141.
36. Planck M (1908) Zur Dynamik bewegter Systeme. Annalen der Physik 26(4): 1-34.
37. Einstein A (1905) Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?. Annalen der Physik 18: 639-641.
38. Planck M (1908) Bemerkungen zum Prinzip der Aktion und Reaktion in der allgemeinen Dynamik. Physikalische Zeitschrift 9: 828-830.
39. von Laue M (1911) Das Relativitätsprinzip. Fried. Vieweg & Sohn, Braunschweig.
40. Jungnickel C, McCormmach R (1986) Intellectual Mastery of Nature: Theoretical Physics from Ohm to Einstein. Volume 2. The Now Mighty Theoretical Physics. University of Chicago Press, Chicago.
41. Kragh H (2002) Quantum Generations: A History of Physics in the Twentieth Century. Princeton University Press, Princeton.
42. Seth S (2004) Quantum theory and the electromagnetic world view. Historical Studies in the Physical and Biological Sciences 35(1): 67-93.
43. Katzir S (2005) On "the electromagnetic world-view": A comment on an article by Suman Seth. Historical Studies in the Physical and Biological Sciences 36(1): 189-192.
44. Seth S (2005) Response to Shaul Katzir. "On the electromagnetic world-view". Historical Studies in the Physical and Biological Sciences 36(1): 193-196.
45. Jungnickel C, McCormmach R (2017) The Second Physicist: On the History of Theoretical Physics in Germany. Springer, Cham.
46. Riemann B (1875) Schwere, Elektricität und Magnetismus. K. Hattendorff, Hannover.
47. Thomson JJ (1881) On the Electric and Magnetic Effects Produced by the Motion of Electrified Bodies. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 5(11): 229-249.
48. Thomson JJ (1893) Notes on Recent Researches in Electricity and Magnetism. Clarendon Press, Oxford.
49. Heaviside O (1889) On the Electromagnetic Effects due to the Motion of Electrification through a Dielectric. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 27: 324-339.
50. Searle G (1897) On the Steady Motion of an Electrified Ellipsoid. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 44: 329-341.
51. Wiechert E (1894) Bedeutung des Weltäthers. Physikalisch-ökonomische Gesellschaft zu Königsberg 35: 4-11.
52. Coudres Td (1899) Handliche Vorrichtung zur Erzeugung Lenard'scher Strahlen und einige Versuche mit solchen Strahlen. Annalen der Physik 298: 134-144.

53. Lorentz HA (1899) Simplified Theory of Electrical and Optical Phenomena in Moving Systems. Proceedings of the Royal Netherlands Academy of Arts and Sciences 1: 427-442.

54. Wien W (1898) Ueber die Fragen, welche die translatorische Bewegung des Lichtäthers betreffen. Annalen der Physik 301: I-XVIII.

55. Hirosige T (1969) Origins of Lorentz' Theory of Electrons and the Concept of the Electromagnetic Field. Historical Studies in the Physical Sciences 1: 151-209.

56. Hirosige T (1976) The Ether Problem, the Mechanistic Worldview, and the Origins of the Theory of Relativity. Historical Studies in the Physical Sciences 7: 3-82.

57. Janssen M (2009) Drawing the Line between Kinematics and Dynamics in Special Relativity. Studies in History and Philosophy of Modern Physics 40: 26-52.

58. Goldberg S (1970) The Abraham Theory of the Electron: The Symbiosis of Experiment and Theory. Archive for History of Exact Sciences 7(1): 7-25.

59. Abraham M (1905) Theorie der Elektrizität. B.G. Teubner, Leipzig.

60. Einstein A (1906) Über eine Methode zur Bestimmung des Verhältnisses der transversalen und longitudinalen Masse des Elektrons. Annalen der Physik 21: 583-586.

61. P. Ehrenfest, "Die Translation deformierbarer Elektronen und der Flächensatz," *Annalen der Physik,* vol. 23, pp. 204-205, 1907.

62. A. Einstein, "Bemerkungen zu der Notiz von Hrn. Paul Ehrenfest: "Die Translation deformierbarer Elektronen und der Flächensatz"," *Annalen der Physik,* vol. 23, pp. 206-208, 1907.

63. Janssen M, Mecklenburg M (2006) From Classical to Relativistic Mechanics: Electromagnetic Models of the Electron. In: Hendricks VF, Jørgensen KF, Lützen J, Pedersen SA (eds). Interactions: Mathematics, Physics and Philosophy 1860-1930. Springer, Dordrecht, pp. 65-134.

64. Miller A (1980) On Some Other Approaches to Electrodynamics in 1905. In: Woolf H (ed). Some Strangeness in the Proportion: A Centenial Symposium to Celebrate the Achievements of Albert Einstein. Addison-Wesley, Reading, pp. 66-91.

65. Cushing JT (1981) Electromagnetic Mass, Relativity, and the Kaufmann Experiments. American Journal of Physics 49: 1133-1149.

66. Potters J (2019) Heuristics Versus Norms: On the Relativistic Responses to the Kaufmann Experiments. Studies in History and Philosophy of Modern Physics 66: 69-89.

67. Giovanelli M (2020) Like Thermodynamics before Boltzmann: On the Emergence of Einstein's Distinction between Constructive and Principle Theories. Studies in History and Philosophy of Modern Physics 71: 118-157.

68. Goldberg S (1976) Max Planck's Philosophy of Nature and his Elaboration of the Special Theory of Relativity. Historical Studies in the Physical Sciences 7: 125-160.

69. von Mosengeil K (1907) Theorie der stationären Strahlung in einem gleichförmig bewegten Hohlraum. Annalen der Physik 22: 867-904.

Key-words for analytical index example Please write many names and subjects for Index

Names Index

Subject Index