

A Conservative Theory of Semiclassical Gravity

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Abstract

We argue that semiclassical gravity is rendered consistent by considering that quantum systems emit a gravitational field only when they interact with members of Stable Determination Chains (SDCs). These are chains of non-gravitational interactions between quantum systems modeled via decoherence and test functions that obey conditions that aim to address the measurement problem and allow for a conservative theory of gravity. It is conservative because it does not need to modify the fundamental equations of quantum theory, unlike spontaneous and gravity-induced collapse approaches to semiclassical gravity, and without invoking relationalism. Furthermore, it does not appeal to nonlocal, retrocausal, or superdeterministic hidden variables. When systems do not interact with SDCs, they do not emit a gravitational field, and the expectation value of their stress-energy tensor does not enter the semiclassical equations describing the gravitational field in a region. In the absence of SDCs in a region, spacetime can be flat. This theory holds a version of the equivalence principle, which establishes that different bodies under the same gravitational field evolve similarly in the absence of non-gravitational interactions. It can be tested by experiments investigating the gravitational field emitted by quasi-isolated systems, and the lack of gravity-mediated entanglement and certain kinds of collapse in the Bose-Marletto-Vedral (BMV) experiment. It provides multiple benefits, such as a semiclassical estimation of the value of the cosmological constant and the prediction of a time-varying dark energy that weakens with time, in agreement with some evidence. More broadly, we propose a new testable framework in which there is a conditional emission of a gravitational field by quantum systems, which may undermine the main motivations for a theory of quantum gravity.

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1 Introduction

It is often claimed that a theory of quantum gravity is required, and that semiclassical gravity must be replaced by this theory to avoid unphysical consequences. In this article, we explore new theoretical and empirical possibilities for understanding semiclassical gravity via quantum field theory and propose the beginnings of a theory of semiclassical gravity that avoids well-known limitations. This theory has empirical consequences that may be tested in the future via experiments that aim to test the quantum nature of gravity. A striking one is that a system under sufficient isolation does not emit its own gravitational field. This contradicts the commonly held view that any system can source a gravitational field. Furthermore, as we will see, this theory follows a different strategy from the commonly adopted ones, which either consider gravity as quantum or as a classical stochastic field that gives rise to outcomes. Our goal is to provide a theory and framework that we hope will lead to new and productive ways of understanding gravity, while showing that the main motivations for finding a quantum theory of gravity may be undercut by rethinking core foundational assumptions.

The theory we are proposing aims to be both minimalistic and conservative. It does not modify the fundamental equations of quantum theory and only minimally modifies those of general relativity by assuming the semiclassical equations of gravity. More concretely, we will be as conservative as possible in what we see as some of the most important features of general relativity, namely general covariance and the equivalence principle. Regarding the latter, we will seriously consider the idea that a generalization of the equivalence principle, which is applicable to the quantum regime, should be valid in a theory of gravity. We will use a recently proposed approach to QT called Environmental Determinacy-based Quantum Theory (EnDQT) [88] and propose a version that accounts for gravity. The key idea is that gravity should not be quantized, and although quantum matter field systems can be affected by gravity, they cannot act as sources of (classical) gravity unless they interact with quantum matter field systems that form specific chains of interactions. These interactions are represented via quantum field theory and modeled via smearing/test functions widely used in algebraic treatments of QFT, where the origin of what these functions represent comes from stochastic interactions between systems, which give rise to a mean field. The strategy here is as follows: because we always need test functions to solve various conceptual and mathematical problems of QFT, to avoid adopting extra mathematical baggage to solve the measurement problem, we will also use these tools to help provide a solution to this problem. Furthermore, the theory proposed here can use the measurement frameworks in QFT [37, 92, 93] because it shares common tools. Therefore, in principle, it allows for measurements and local rules for state updates that are compatible with relativistic causality [91, 38, 93] in the sense of dealing with issues of the kind identified in [3, 113], which give rise to a conflict between measurement

theory and relativity.¹

In addition to this theory, we propose a framework to consider gravity in this context, which involves what we call gravitational conditions — the conditions under which systems emit a gravitational field. We will adopt a subset of those conditions. Therefore, we also present a set of underexplored features for future theoretical and empirical investigations. In addition to arguing that the semiclassical Einstein equations can be used to provide a consistent account of gravity, and that this view can circumvent some of the common objections to the semiclassical approach, we will also defend this theory by demonstrating how it can provide multiple benefits. For instance, it allows us to derive an estimate of the value of the cosmological constant from certain principles and provides an explanation for why the vacuum does not gravitate, potentially addressing the cosmological constant problem. This value comes from fluctuations in the stress-energy tensor. Interestingly, this derivation leads to the prediction that this value changes over time and that it is getting progressively smaller, in agreement with current observations that indicate that the so-called dark energy is getting progressively weaker [1]. Other conjectures concerning black holes and inflation are presented to demonstrate the potential of this theory.

We will start by motivating this theory by explaining two scenarios that can test it and distinguish it from quantum theories of gravity and theories in which gravity leads to the collapse of quantum superpositions (Section 2). Then, we will present the basic features of EnDQT (Section 3). Subsequently, we present three postulates that constitute the basis for the theory of semiclassical gravity based on EnDQT (Section 4) and explain how this theory, in a sense, generalizes the equivalence principle. In Section 6, we show how it may be able to deal with some of the common objections of the semiclassical approach and examine some other consequences of this theory, which include a conjecture that the core of black holes does not have systems that emit a gravitational field, and thus, a singularity does not arise. Indeed, we postulate that in the absence of a gravitational field, spacetime can be Minkowski. We argue that this postulate can be used to address issues regarding singularities in GR. Instead of singularities occurring within black holes or at the origin of the universe (see Appendix I), we conjecture that we can have asymptotically flat regions of spacetime, where these regions concern the progressive decrease in the gravitational field arising from interacting quantum matter fields. Instead of a gravitational field arising, we obtain a flat spacetime. In Section 7, we show how this theory allows us to estimate the value of the cosmological constant and interpret dark energy as having a time-varying value that gets increasingly smaller. Some calculations are presented in the appendices, including how this time-varying cosmological constant value leads to some of the effects that we associate with inflation and potentially new benefits associated with not having to postulate an inflaton field (Appendix I). For simplicity, we focus on real scalar fields that obey the Klein-Gordon equation. However, the approach developed is valid in principle

¹We will see how to understand this via particle detector models [92, 86] in Appendix F. The more abstract algebraic QFT framework will be discussed in future work.

for other types of fields. Throughout this article, we adopt the metric signature $(-+++)$. We will also assume mainly natural units ($\hbar = c = 1$). The context will make it clear when we do not.²

2 Semiclassical gravity and experiments to test this theory

The Einstein equations take the form

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu} \quad (1)$$

where $G_{\mu\nu}$ is the Einstein tensor, defined as $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor encoding spacetime geometry and Λ is the cosmological constant, often considered to represent dark energy. The stress-energy tensor of the matter fields is $T_{\mu\nu}$, which can source the gravitational field.

If matter and radiation fields are quantised, it is unclear what to take for the material source of the gravitational field. Multiple approaches can be used to solve this problem. The simplest approach replaces the right-hand side with the expectation value of the stress-energy operator evaluated in a state that produces meaningful results like a renormalizable stress-energy tensor (more on this in the next sections). The dynamics are governed by a modified version of Einstein's field equations called the semiclassical equations [74, 101]:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \hat{T}_{\mu\nu} \rangle_\rho - \Lambda g_{\mu\nu} \quad (2)$$

where $\langle \hat{T}_{\mu\nu} \rangle_\rho$ is the expectation value of the renormalized quantum energy-momentum tensor in a given quantum state ρ . Quantum matter fields influence the curvature of spacetime via the expectation value of the stress-energy tensor, but the gravitational field itself is not quantized, and we ignore the backaction of quantum matter fluctuations onto the gravitational dynamics. This is a form of mean-field theory and leads to well-known problems that we will approach later [62, 123].

One alternative is to formulate a theory of quantum gravity, which quantizes geometrical degrees of freedom or makes them emerge from some more fundamental quantized ones, where eq. (2) is obtained in some limit (E.g., [48, 49, 103]). Another approach is to find a consistent way to combine quantum and classical dynamics [27, 80], without making emerge or reduce the latter to the former, which leads to a gravity-induced collapse process. This can be achieved by adding a minimum amount of noise to both classical and quantum dynamics. Adding noise to the classical equations makes gravity stochastic, which can change in such a way that it does not lead to the collapse of the quantum states,

²Quantum operators will be written with a hat, except in some sections or when the context makes it clear that is an operator.

not *revealing* where the quantum system is. However, under certain conditions determined by the stress-energy tensor of the target system, such noise is reduced, and the decoherence of the quantum degrees of freedom is increased, leading to a collapse of the quantum state of this system and an outcome. Importantly, in isolated systems in a coherent superposition, a stochastic gravitational field is always present. Penrose’s theory [84] considers that a superposition of a spatial mass-density distribution corresponds to a superposition of spacetimes, which are non-stationary and tend to collapse due to their gravitational self-energy.³ Thus, both theories would consider that an outcome eventually arises, regardless of the environment of a target system. We refer to this class of theories as gravity-induced collapse theories.

In the theory that we are proposing, the stochastic gravitational field is not always present and is not directly implicated in the collapse. In addition, it is not fundamentally described by some classical state with its Hilbert space and dynamics, such as hybrid classical-quantum theories [78]. Therefore, we do not posit a classical degree of freedom with autonomous stochastic dynamics. The gravitational degrees of freedom are rather described via the semiclassical Einstein field equations seen above and account for how quantum matter fields give rise to gravity in certain contexts. More concretely, systems emit a gravitational field only under certain local decohering interactions between matter fields (even in the presence of a background *stochastic* gravitational field). The behavior predicted by the semiclassical equations occurs only under these interactions. If these interactions do not occur, quantum systems evolve in flat spacetime (if this is the default state of spacetime, as discussed in Section 4.3) or under the gravitational field emitted by other systems, and this evolution is described by flat or curved spacetime QFT, respectively. In addition, if isolated from these interactions, systems evolve unitarily indefinitely.

Furthermore, under certain assumptions, fluctuations in the stress-energy tensor lead to dark-energy effects. Other strategies are presented that show how we can minimize the fluctuations of the stress-energy tensor by allowing systems to emit a gravitational field only in contexts in which such fluctuations are minimized. In this first paper, we will not focus on how to characterize the stochastic gravitational field more conveniently or solve the semiclassical equations, although in principle, they can be solved under the circumstances we know how to solve them. Rather, we focus on the circumstances in which the gravitational field is emitted and some distinct features of this theory.

To motivate our proposal and show how it could be tested, we will look at the gravcat experiments and the so-called Bose-Marletto-Vedral (BMV) experiments [9, 71]. Let us first consider a scenario where a system is placed in a cat state [8], which is the superposition of distinguishable coherent states,⁴

$$|\psi_{\text{cat}}\rangle = \mathcal{N} \left(|\alpha\rangle + |-\alpha\rangle \right), \quad \mathcal{N} = \frac{1}{\sqrt{2 + 2e^{-2|\alpha|^2}}}, \quad (3)$$

³As it will be clearer, in the approach proposed here, one cannot place spacetimes in a superposition. If we put masses in a superposition, they do not generate a gravitational field.

⁴See Section 3.2.2 for a characterization of these states.

where this cat state is isolated from its environment such that the components of that superposition can self-interfere under suitable conditions. Now, we place a detector of the gravitational field generated by this system in a spacetime outside the location of this system. According to this theory, an isolated system cannot emit its own gravitational field; thus, the detector cannot detect the hypothetical gravitational field sourced by this target system. The target system could be subject to the gravitational field from other systems, like all quantum systems are, in principle, subject to, according to our current evidence from the Colella-Overhauser-Werner (COW) experiment [81, 19, 124], but not in a classical way as described by semiclassical gravity, unless it interacts with other members of the so-called stable determination chains (SDCs). The latter concerns certain chains of local non-gravitational decoherence-inducing interactions between systems.⁵ In the absence of these interactions, no stochastic process occurs, which selects one of the states of systems in a coherent superposition, and they do not emit a gravitational field. Decoherence applied to open systems is considered by this theory as an inferential tool for inferring and helping to represent the behavior of these chains and when the stochastic process occurs.

The above experiment can be performed in principle (see, e.g., [16, 17]). The absence of a gravitational field emitted by the degrees of freedom in a coherent superposition of particles would constitute significant evidence favoring this theory. Furthermore, according to this theory, the rate at which we can observe a gravitational field emitted by the target system of the experiment should be exclusively determined by the decoherence rate at which it is decohered by the matter fields surrounding it, which involves decoherence-inducing non-gravitational interactions.^{6,7} This contrasts with the gravity-induced collapse theories defined above, in which they postulate mechanisms where mass/energy density, stress-energy in general, or gravitational self-energy (such as in Penrose’s theory, see above) is a determining factor.

The assumption that systems do not emit their own gravitational field may seem quite radical; however, it can be considered as coming from a more abstract version of the Weak Equivalence Principle (WEP) applicable to quantum systems, and in a sense, a more general version. A precise and classical statement of the WEP is due to Clifford Will [125, 66]:

“[I]f an uncharged test body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition.”

where by an ““uncharged test body” we mean an electrically neutral body that

⁵Or more generally in the absence of processes that make systems emit a gravitational field, see below.

⁶See, e.g., [107] for an expression of the decoherence rate of an object in a spatial superposition due to air molecules.

⁷Additionally, some versions of this theory consider that the gravitational field is only emitted by systems in certain semiclassical states, or some other contexts. However, the most classical states and accessible by these experiments, e.g., coherent states, will certainly be considered to gravitate. See Section 4.2.

has negligible self-gravitational energy (as estimated by Newtonian theory) and that is small enough in size so that its coupling to inhomogeneities in external fields can be ignored.” [125] Note that the notion of a body with a negligible self-gravitational energy is considered an idealization. In principle, a probe with sufficient resolution could detect the gravitational field emitted by this body. However, the theory we propose does not consider this as an idealization and considers that under certain circumstances, a body literally does not emit a gravitational field. Thus, by taking this feature seriously, it generalizes this principle in the sense that it generalizes to be applicable to any body, not just test particles like the bodies above, and quantum phenomena by proposing what we will call the EnD Equivalence Principle:

Without being affected by other forces, any quantum system under the same gravitational field exhibits the same behavior due to this field.

The “other forces” are forces that may involve members of SDCs, which interact non-gravitationally. So, non-interacting systems (with SDCs) do not give rise to a gravitational field and behave in the same way in free fall, even if they have very different masses.

Notice that this principle is violated by quantum gravity theories. The entanglement between gravitational and material degrees of freedom will give rise to deviations from the geodesic equation, which can violate this principle [90]. It will also be violated by gravity-induced collapse theories. The collapses, independently of non-gravitational interactions, in some regime determined by the stress-energy or the gravitational self-energy of the target systems, will give rise to violations of this principle.

Now, turning to the BMV experiment, let us consider a scenario involving two particles [9, 71] that are sufficiently isolated to preserve the coherence of both their spatial and spin degrees of freedom. Each particle possesses an internal two-state degree of freedom—its spin along a given axis—which can be placed in a superposition without affecting its center of mass. Suppose that the particles are free-falling. We now use a Stern-Gerlach device to subject each particle to a force that depends on their spin. Let us consider the states $|C\rangle$, $|L\rangle$, and $|R\rangle$, which concern the center-of-mass degrees of freedom of the particles. If their spin is $|\downarrow\rangle$ the particle gets a kick of $+\Delta p$, while if the particle is in state $|\uparrow\rangle$ it will get a momentum kick of $-\Delta p$. Thus, if the particle has spin-up, it will go to the left; if it has spin-down, it will go to the right, and if it is in a superposition of spin-up and spin-down, we get

$$|C\rangle_j \frac{1}{\sqrt{2}} (|\uparrow\rangle_j + |\downarrow\rangle_j) \rightarrow \frac{1}{\sqrt{2}} (|L, \uparrow\rangle_j + |R, \downarrow\rangle_j). \quad (4)$$

The centres of $|L\rangle$ and $|R\rangle$ are assumed to be separated by a distance Δx , where each of the states $|L\rangle$ and $|R\rangle$ are localized Gaussian wavepackets with widths that are much less than Δx . Furthermore, the centres of the superpositions are separated by a distance (Figure 1) so that even for the closest approach of the masses ($d - \Delta x$), we can neglect the short-range Casimir–Polder force.

Subsequently, the two particles may have their trajectories entangled via a hypothetical distance-dependent gravitational field, which also depends on their mass. The estimate of the phases induced by gravity can be derived by assuming that the effect that dominates can be calculated via a Newtonian interaction. This approximation also retains its validity for a linearized quantum gravity model. So, if gravity is quantum, it can in principle mediate the entanglement between the trajectories of the particles (see Figure 1 for more information). In this experiment, each particle then goes over a *refocusing* Stern-Gerlach device that moves them toward the center, and we would obtain the following state,

$$|\Psi(t = t_{\text{End}})\rangle_{12} = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 \frac{1}{\sqrt{2}} (|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}} |\downarrow\rangle_2) + |\downarrow\rangle_1 \frac{1}{\sqrt{2}} (e^{i\Delta\phi_{RL}} |\uparrow\rangle_2 + |\downarrow\rangle_2) \right\} |C\rangle_1 |C\rangle_2 \quad (5)$$

with

$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d - \Delta x)}, \quad \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d + \Delta x)}, \quad \phi \sim \frac{Gm_1m_2\tau}{\hbar d} \quad (6)$$

where $\Delta\phi_{RL} = \phi_{RL} - \phi$, $\Delta\phi_{LR} = \phi_{LR} - \phi$. Measuring the spins of particles 1 and 2 at the end of the experiment provides a way to certify this so-called gravity-mediated entanglement, $\mathcal{W} = \left| \langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \rangle - \langle \sigma_y^{(1)} \otimes \sigma_z^{(2)} \rangle \right|$, where such entanglement exists if $\mathcal{W} > 1$.

According to gravity-induced collapse theories, at a certain threshold dependent on their stress-energy or gravitational self-energy, we would have a collapse and not gravity-mediated entanglement. Thus, this class of theories would consider that, independently of their non-gravitational interactions with environmental systems, the particles would eventually collapse, and we would

not be able to unitarily reverse the final collapsed state to its initial state,⁸

$$|C\rangle_1 \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 + |\downarrow\rangle_1) \otimes |C\rangle_2 \frac{1}{\sqrt{2}}(|\uparrow\rangle_2 + |\downarrow\rangle_2) \quad (7)$$

$$\longrightarrow \frac{1}{\sqrt{2}}(|L, \uparrow\rangle_1 + |R, \downarrow\rangle_1) \otimes \frac{1}{\sqrt{2}}(|L, \uparrow\rangle_2 + |R, \downarrow\rangle_2) \quad (8)$$

$$\longrightarrow \begin{cases} |L, \uparrow\rangle_1 & \text{or} & |R, \downarrow\rangle_1, \\ |L, \uparrow\rangle_2 & \text{or} & |C, \downarrow\rangle_2, \end{cases} \quad (9)$$

$$\longrightarrow \begin{cases} |C, \uparrow\rangle_1 & \text{or} & |C, \downarrow\rangle_1, \\ |C, \uparrow\rangle_2 & \text{or} & |C, \downarrow\rangle_2. \end{cases} \quad (10)$$

We propose an alternative route to these two classes of theories. Contrary to quantum gravity theories, gravitationally mediated entanglement cannot occur. In the absence of interactions with members of SDCs, systems maintain a coherent superposition and no probabilistic process occurs. Thus, systems do not source a gravitational field, which if sourced, would be classical as described by semiclassical gravity, and thus incapable of generating entanglement. Moreover, contrary to gravity-induced collapse theories, the *trigger* for the stochastic collapse is independent of their mechanism that depends on the size of the mass/energy density or gravitational self-energy associated with the particles in a superposition, but rather whether they have interacted with members of an SDC, which only involves non-gravitational decohering interactions. Thus, for particles of any mass sufficiently isolated from their environment, we can, in principle, reverse their state to their initial state contrary to the above-mentioned theories. Therefore, according to this theory, in principle, there will be no classical or quantum gravitational interaction between the particles; they will remain unentangled as they free-fall, and we can reverse the operation as follows:

$$\frac{1}{\sqrt{2}}(|L, \uparrow\rangle_1 + |R, \downarrow\rangle_1) \otimes \frac{1}{\sqrt{2}}(|L, \uparrow\rangle_2 + |R, \downarrow\rangle_2) \quad (11)$$

$$\longrightarrow |C\rangle_1 \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 + |\downarrow\rangle_1) \otimes |C\rangle_2 \frac{1}{\sqrt{2}}(|\uparrow\rangle_2 + |\downarrow\rangle_2). \quad (12)$$

⁸This matter is more subtle as discussed in [55]: “[a]s Bose et al. (2017, p. 1) put it, (...) [gravitational-induced collapse] theories imply “the breakdown of quantum mechanics itself at scales macroscopic enough to produce prominent gravitational effects.” The question of course is what counts as “prominent.” On the one hand, according to Penrose’s estimates, in the proposed experiment, with gravcats of 10^{-14} kg separated by $100 \mu\text{m}$, the gravitational collapse time should be of the order of a second, which would be fast enough for the classicality of the field to affect any observed entanglement. Therefore, it seems to be a “prominent” effect: the quantum state collapses, and no entanglement is observed. However, on the other hand, should entanglement be observed, the theories do have a tunable parameter, which could be set to prevent collapse in the currently envisioned GIE experiments, although they would place a new bound on it. But so doing is to accept that the experiment witnesses a quantum superposition of the gravitational field, which is at least against the spirit of Penrose’s position, and quite possibly falls afoul of the very arguments by which he motivates it.”

So, given the role of decoherence in the theory we propose, the extent to which we cannot reverse the state of the masses is determined exclusively by the decoherence rates and timescales due to non-gravitational interactions with other particles/matter fields present in the BVM experiment, and not by other factors present in gravitationally induced collapse theories or gravitationally mediated entanglement/decoherence. See Table 2 in [98] for a quantification of such rates and explicit expressions for the BVM experiment, which, for completeness, we have reproduced in Appendix A. If we find that gravitationally mediated entanglement, a mechanism dependent on the stress-energy or gravitational self-energy of the target particles postulated by gravity-induced collapse theories, or some other mechanism determines the which-path information of the particles in this experiment, or the decoherence rates and timescales, and not just non-gravitational environmental-induced decohering interactions, this would be decisive evidence against the theory we propose.

As we can see, these experiments can provide evidence for this theory and help distinguish it from quantum gravity and gravity-induced collapse theories. Moreover, we can also distinguish the theory proposed here from spontaneous or gravity-induced collapse theories by testing their domain of validity through other experiments. For instance, experiments have been proposed and conducted to test the Diósi-Penrose model (e.g., see [40] and references therein). If their domain of validity becomes problematic and we cannot find a satisfactory quantum theory of gravity or evidence for it, this is, in principle, evidence supporting this theory. Furthermore, the theory proposed here violates the decoherence diffusion trade-off presented in [79], which has been adopted by hybrid classical-quantum theories. This is because, according to the theory we are proposing, we can have a system in a coherent superposition without any stochasticity in the gravitational degrees of freedom, and irrespective of its stress-energy. Thus, violations of this trade-off [79] can provide evidence to support this theory.

As mentioned, in this article, we propose a series of gravitational conditions, which involve SDCs and establish when the systems emit a gravitational field. However, note that this is only one possible set of gravitational conditions. Other theories could impose different conditions for the emission of a gravitational field. For instance, one could appeal to SDCs with other rules (see next sections), or one could have certain modifications of the dynamical equation of quantum theory, which impose a collapse rule, such as in spontaneous collapse theories, and trigger a system to emit a gravitational field. One could also have a many-worlds or many-worlds-like/relational theory that states that under decoherence and branching or particular interactions, such a classical field arises. Relatedly, one could have a theory that appeals to an emergent or primitive notion of agents that trigger the gravitational field. One could even appeal to hidden variables that account for such triggering. However, many of the above classes of theories suffer from a lack of experimental evidence or well-known issues; therefore, we present a theory that also aims to circumvent them.⁹ Nevertheless, this article

⁹Regarding the debate about what is the right approach to solve the measurement problem, one could argue that if this theory ends up being confirmed, it presents important evidence

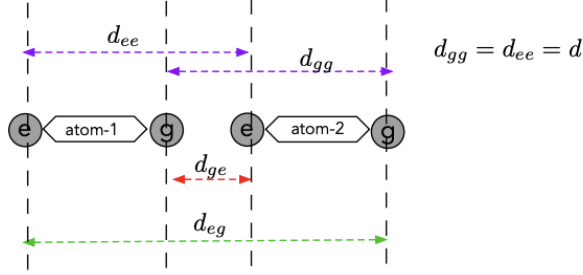


Figure 1: If there is a quantum gravitational interaction between the particles, the interaction distinguishes three paths as there are three distinct particle separations, $d_{ee} = d_{gg} = d$ and $d_{eg} > d$, $d_{ge} < d$. This will entangle the two-particle center of momentum motion in a way that depends on the mass of each particle. If we repeat the experiment with two particles of different masses, the entanglement will be different. Measurements made in a free-falling frame could thus distinguish the three paths.

could be read as opening up new, so far neglected empirical and theoretical possibilities concerning how gravity works.

3 Introduction to the framework of EnDQT

Related to gravitational conditions, there are the so-called determination conditions, which are the conditions for measurement outcomes to arise or observables to have determinate values. Different interpretations or quantum theories pose different determination conditions. Although these two kinds of conditions are related, they are distinct. As we will see in Section 4.2, we may consider that a system has a feature that we associate with a measurement outcome, having a determinate value, but it still does not emit a gravitational field. The system may only act as a test system. In this section, we present the main features of EnDQT in a non-relativistic setting and its QFT version, which adopts a set of determination conditions for the SDCs that we consider more satisfactory.

The SDCs mentioned above are like von Neumann chains [75], i.e., they involve a series of intertwined unitary evolutions and stochastic processes, but occurring in such a way that, in principle, we never lose track of the systems that belong to those chains. Local interactions modeled using test functions provide a way to track these systems and chains. In addition, we want SDCs to be compatible with relativity and with the success of decoherence in representing

against relationalist theories because they are not naturally expressed in this framework. The latter is based on non-relational determinate values arising from stochastic processes. It would also present evidence against the above-mentioned collapse theories as we have discussed, as well as hidden variable theories that do not have a satisfactory theory of gravity or no theory at all.

measurement processes, and we will see that these chains obey the key features of both. Furthermore, we want SDCs to appear in cosmological contexts and not rely on anthropocentric notions to describe them. Crucially, via the rules that will be presented, which only appeal to local QFT-based decohering interactions with a certain structure, we aim to not modify the quantum formalism significantly to provide the criteria for when an outcome arises (in a single-world and non-relational way), unlike spontaneous collapse and gravity-induced collapse theories. Also, we aim to not appeal to non-local, superdeterministic, or retrocausal hidden variables. Thus, we aim to be conservative and circumvent the issues of these approaches.¹⁰

We will now establish a set of criteria to assign definite or determinate values to observables based on SDCs. Historically, the criteria for assigning determinate values to observables in quantum theory have some underappreciated importance (see [45] for a historical overview), and come in the form of criteria such as the Eigenstate-Eigenvalue Link. This link states that a system has a determinate value q of a property or observable represented by a self-adjoint operator O if and only if it is in an eigenstate of O , which corresponds to the eigenvalue q . However, as is well known, this criterion is at odds with scientific practice because we often want to assign determinate values when systems are not in an eigenstate of some dynamical observable. In addition, for generic Hamiltonians, systems typically rapidly evolve out of those eigenstates after being measured [122]. Realistically, being in an eigenstate of a dynamical observable is better seen as something that occurs for a brief amount of time, and systems typically evolve quickly out of those states. The determination conditions below aim to provide more realistic and less problematic criteria.¹¹

For pedagogical reasons, we will initially appeal to non-relativistic quantum theory,¹² but we will see that these features become much more intuitive when we describe them using QFT. One of the main features of EnDQT comes from taking seriously the view that systems are never in eigenstates of dynamical observables, except when they are being measured and shortly after, and it is

¹⁰See, e.g., [33, 46, 44, 102, 42, 41, 52], and references therein.

¹¹A potential consequence of the Eigenstate-Eigenvalue Link is that systems have indeterminate values of certain observables outside measurement-based contexts. In previous works [88, 89], we argued that this quantum indeterminacy should be adopted when interpreting quantum theory, especially when adopting a conservative approach that does not modify this theory significantly, and when considering current no-go theorems such as Bell's theorem. Thus, we should seriously consider this potential consequence of the Eigenstate-Eigenvalue Link. In a sense, the work presented here extends this argument to the case of the hypothetical gravitational field emitted by a system outside measurement-based contexts, and by examining no-go theorems or scenarios involving gravity like we did previously for the quantum case exclusively.

¹²In the simplest pure-state-based Hilbert space formalism, a quantum system is represented by a normalized vector within a complex, complete inner product space Hilbert space. The observables of a system are described by Hermitian operators acting on these vectors, with their eigenvalues corresponding to the values of measurable quantities. The probability of obtaining a specific measurement outcome is determined by the squared magnitude of the inner product between the state vector and the observable's eigenstate or associated quantum state (see above what we mean by this). Additionally, the time evolution of the system is driven by unitary operators, which ensure that the total probability remains constant over time.

the following:

Systems have, by default, indeterminate values of any non-dynamical observable, except due to under certain interactions with systems with the determination capacity that leaves the systems involved in a certain state, and while they are in that state.

For example, consider dynamical observables such as spin in multiple directions, momentum, or energy.¹³ For EnDQT, systems have indeterminate values of all of these observables unless the above interactions occur. Moreover:

A system X can only give rise to measurement outcomes or to another system Y having a determinate value of a dynamical observable of Y when X has the determination capacity concerning Y , which we denote as DC- Y .

Furthermore, this capacity tends to spread because, under specific conditions that will be specified below, system Y can acquire this capacity and transmit it to other systems through interactions. Moreover, as we have mentioned, only under these interactions, and over the short period that systems are in an eigenstate of that observable, do systems have determinate values associated with that observable. Importantly,

it is indeterministic which determinate values of the observables O_X and O_Y systems X and Y will have under these interactions among the possible ones, where the possible values are given by the eigenstates or associated quantum states of O_X and O_Y , which were in a superposition.

We mention “associated quantum states to an observable” because, as we will see, for example, in the case of observables such as those represented via the energy-momentum operator, systems could have determinate values of energy-momentum even if they are not in an eigenstate of that observable. As we will see, coherent states are not eigenstates of the energy-momentum tensor operator, although we will consider that systems have a determinate energy-momentum when in those states under the interactions mentioned above. Furthermore, as one can see, similar to, for example, the Copenhagen interpretation, EnDQT is an indeterministic theory in the circumstances where specific interactions are involved.

One way to infer whether a system X with the determination capacity concerning a system Y acts locally as a “measurement device” for the observable

¹³And perhaps even electric charge, if not subject to a superselection rule, and hence considered as a dynamical observable. Roughly, an observable is subject to superselection rules when there are certain rules that forbid the preparation of its eigenstates in a coherent superposition. Electric charge is typically subject to this rule. However, we could allow for the more radical view that all observables are dynamical, and appeal to decoherence via SDCs (in a similar way to the so-called Environment-induced superselection) to account for why, typically, we do not see their eigenstates self-interfering.

O_Y of Y is if an eigenstate of O_X (or associated quantum state to an observable of X) contains information of an eigenstate of O_Y (or associated quantum state to an observable of Y), or via the locally induced entanglement of the degrees of freedom of X with the eigenstates or associated quantum states of the observable O_Y of Y . More precisely, it is not just entanglement but entanglement involving many degrees of freedom that give rise to a quasi-irreversible process (mathematically speaking, i.e., only in theory),¹⁴ which is often called environmental-induced decoherence [58, 127]. When decoherence occurs and X has the DC- Y , for EnDQT an indeterministic (and truly irreversible) process arises that makes *both* X and Y have a determinate value that can be represented by one of the eigenvalues of the observables O_X and O_Y whose eigenstates got entangled (or by the value associated with those states such as in the case of coherent states). So, we will regard the models of decoherence as inferential tools to infer when systems that have the determination capacity give rise to others having determinate values [88].¹⁵ Together with test functions (more on this below), they provide the main inferential tools to infer whether the conditions below are fulfilled.

More concretely, we consider that decoherence allows us to infer in open environments when SDCs act, even in the absence of knowledge about their precise locations. This is because these are considered to be the typical environments in which SDCs evolve. In addition, it allows us to infer the conditions required to shield systems from SDCs via the conditions required to shield systems from decoherence. Furthermore, if we manage to track precisely where SDCs are, it allows us to represent their behavior over spacetime. The way we use decoherence to study which outcomes associated with states $\mathcal{S} \subset \mathcal{H}_S$ arise stochastically from local interactions with members of SDCs is often via the locally established many records of the environment of S of those states, such that if the system starts in those states $\mathcal{S}$, at later times it is still well-approximated by another member of the set $\mathcal{S}$, and the environment contains records of them, having their states correlated with them,

$$|\alpha\rangle_S \otimes |0\rangle^{\otimes N} \xrightarrow{U} |\alpha_0\rangle_S \otimes |\varepsilon_1(\alpha)\rangle_1 \otimes \cdots \otimes |\varepsilon_N(\alpha)\rangle_N. \quad (13)$$

On the other hand, if it starts in a superposition of those states, it is driven over time into a mixture of $\mathcal{S}$, and we can infer that the environment has a record of $\mathcal{S}$, where this process is quasi-irreversible and the mixture of states has a probabilistic interpretation in terms of a diagonal mixture in the basis $\mathcal{S}$ over time. This can be easily observed by tracing out the states of the environment. Decoherence timescales provide an estimate of the duration required for the continuous

¹⁴Decoherence is a quasi-irreversible process in the sense that it has very high recurrence times τ_D , e.g., timescales much higher than the age of the universe or the heat-death onset of the universe (i.e., Poincaré recurrence timescales). Note that for EnDQT such recurrence never occurs. The extremely large recurrence times of decoherence for EnDQT just signal that decoherence represents a truly irreversible process.

¹⁵The determination capacity can be grounded on categorical properties, but we choose to set that characterization aside here.

stochastic process that leads to measurement outcomes to end up occurring.¹⁶ The term Stable Determination Chain (SDC) comes from the observation that to analyze whether determinacy arises in the interactions that constitute these chains, we need to analyze whether there is decoherence, which often involves a stable quasi-irreversible entanglement between the target system and its environment.¹⁷ So, examining that systems are driven quasi-irreversibly locally over time into a mixture of states $\mathcal{S}$ that have a probabilistic interpretation offers a way of inferring that they end up in one of the states $\mathcal{S}$.¹⁸ Thus, the features of the dynamics of systems play a large role in the evaluation of how (as we will sometimes say) SDCs select certain states associated with specific determinate values. Moreover, we can infer the determinate values that environmental systems will have by examining the values associated with their states that have information about the state of the target system, being correlated with the state of the target system. So, for this theory, the success of models of decoherence that we use pragmatically in physics to represent measurement-like interactions in open environments, particularly those involving only matter degrees of freedom in the case of the theory of gravity that we will present in the next sections, is justified by the fact that they are modeling the typical environments where SDCs evolve. Furthermore, models involving local interactions (i.e., strongly localized interactions modeled via test functions) represent processes that lead to measurement outcomes more realistically, although such localization is often disregarded.

System X with the DC- Y can also partially decohere system Y , which is inferred through the local quasi-irreversible partial decoherence of Y by X . This process will give rise to outcomes for both Y and X , where the possible values of X concern the partial information about the value related to the state of Y that X partially distinguishes. However, for simplicity, in this paper, we focus on the situations of complete decoherence.¹⁹

¹⁶So, note that this process is not discontinuous but rather continuous, and it does not occur at a particular instant. Nothing in the process of decoherence implies that an outcome arises discontinuously.

¹⁷The decoherence timescale typically varies inversely with the size of the bath/environment that leads to decoherence, and thus the number of members of SDCs interacting with the system influences how much time it takes for such stochastic process to occur [120].

¹⁸Another way involves having a Wigner function that is positive in some interactions involving Gaussian states.

¹⁹In [89] this situation was taken into account. It was considered that the determinacy of the values of observables of a target system Y of X will come in degrees. This is inferred through the degree of distinguishability of the states of the environment X concerning the state of the target system Y in a local quasi-irreversible partial decohering interaction. It can be assumed or not that under partial decoherence by their environment, a system Y obtains the DC concerning another system Z , where these systems fulfill the determination conditions discussed in the next section. As we have said, in this paper, we will not consider these cases for simplicity.

3.1 Conditions for the determination capacity to spread

We will now explain the conditions for the determination capacity to spread through interactions. To build some intuition, we will explain it through a non-relativistic toy model to see how this works and pretend that entanglement between two systems is sufficient for determinate values to arise in interactions (and not entanglement involving a collective of systems that give rise to decoherence). In parallel, we will explain the QFT case. For simplicity, in this article, we will focus on the case of SDCs that involve interactions between two possibly composite systems. However, SDCs in principle could have more complicated structures.

Let us consider the following Hamiltonian involving two continuous CNOT gates,

$$\hat{H}_{ABC}(t) = f_{AB}(t) \frac{\pi}{2} \left(\frac{1 - \hat{\sigma}_{zB}}{2} \right) \hat{\sigma}_{xA} + f_{BC}(t) \frac{\pi}{2} \left(\frac{1 - \hat{\sigma}_{zC}}{2} \right) \hat{\sigma}_{xB}, \quad (14)$$

which describes the interactions between systems A, B, and C. More about $f_{AB}(t)$ and $f_{BC}(t)$ below.

The initial state of these systems is

$$|\Psi(0)\rangle = |1\rangle_A \frac{1}{\sqrt{2}} (|0\rangle_B + |1\rangle_B) \frac{1}{\sqrt{2}} (|0\rangle_C + |1\rangle_C), \quad (15)$$

where the states above are eigenstates of the observable spin-z.

In the QFT case, we could have in the Hamiltonian picture a Hamiltonian density describing the interactions between scalar fields A and B , and B and C ,

$$\hat{H}_{\text{int}}(t) = \int d^3x [\lambda_{AB} f_{AB}(t, \mathbf{x}) \hat{\phi}_A(t, \mathbf{x}) \hat{\phi}_B(t, \mathbf{x}) + \lambda_{BC} f_{BC}(t, \mathbf{x}) \hat{\phi}_B(t, \mathbf{x}) \hat{\phi}_C(t, \mathbf{x})]. \quad (16)$$

λ_{AB} and λ_{BC} are coupling constants, where $x = (t, \mathbf{x})$, and $f_{AB}(t, \mathbf{x})$ and $f_{BC}(t, \mathbf{x})$ are smearing/test functions that serve to represent and infer the localization of quantum fields in a spatiotemporal region in the QFT case, which can be used to impose energy and momentum cutoffs. $f_{AB}(t)$ and $f_{BC}(t)$ in the non-relativistic case will just localize the system in time and provide energy cut-offs.

Test functions play an important role in rigorous treatments of QFT and are used to handle divergences. However, for EnDQT, they have the additional role of providing the conditions for when systems have determinate values. More specifically, test functions provide a way to specify the so-called no-disturbance condition, as we will see below. Furthermore, test functions should obey the relativistic constraints of being compatible with general covariance. Note that in this study, we are concerned with the interaction between quantum fields that are spatiotemporally localized owing to these interactions. By this, we mean that they have determinate values in bounded spacetime regions in a local manner (more on this also below).²⁰ Thus, in the simple case of only two interacting

²⁰ Additionally, to allow that local algebras are independent of the choice of the test function

fields, we consider a test function $f_{XY}(x)$, which is a function that is compactly supported within a region, or at least strongly localized around a region that smears the fields $\hat{\phi}_X(x)$ and $\hat{\phi}_Y(x)$ in that region.

To maintain general covariance, we adopt a test bump function $f_{XY}(x)$ that localizes the interaction between X and Y around a point x_{XY} inside a fixed precompact convex normal neighborhood U of x_{XY} , and we set $f_{XY} \equiv 0$ outside U . Choosing $\sigma_{XY} > 0$ sufficiently small ensures that the set $\{x \in U : |\sigma(x, x_{XY})| < \sigma_{XY}\}$ is compactly contained in U , so that the support of f_{XY} is compact.²¹ So, we have:

$$f_{XY}(x) = \begin{cases} \exp\left(-\frac{1}{1 - \left(\frac{\sigma(x, x_{XY})}{\sigma_{XY}}\right)^2}\right), & x \in U \text{ and } |\sigma(x, x_{XY})| < \sigma_{XY}, \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

Here $\sigma(x, x_{XY})$ is Synge's world function [115], which represents one-half of the squared geodesic interval between point x and center x_{XY} . In Minkowski spacetime, it simplifies to $\sigma(x, x_{XY}) = \frac{1}{2}\eta_{\alpha\beta}(x - x_{XY})^\alpha(x - x_{XY})^\beta$, i.e. $\sigma(x, x_{XY}) = \frac{1}{2}(-(t - t_{XY})^2 + |\mathbf{x} - \mathbf{x}_{XY}|^2)$. The use of Synge's world function in a test function is advantageous because it is generally covariant, it incorporates the exact spacetime geometry through geodesic intervals and thus is applicable to curved spacetimes without the need for specific coordinate systems. As can be seen, this function is smooth and goes smoothly to zero as $|\sigma(x, x_{XY})| \rightarrow \sigma_{XY}$. In Minkowski spacetime, we can write

$$f(x) = \begin{cases} \exp\left(-\frac{1}{1 - \left(\frac{-(t - t_{XY})^2 + |\mathbf{x} - \mathbf{x}_{XY}|^2}{2\sigma_{XY}}\right)^2}\right), & \begin{aligned} &|-(t - t_{XY})^2 + |\mathbf{x} - \mathbf{x}_{XY}|^2| < 2\sigma_{XY}, \\ &x \in U, \end{aligned} \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

In the non-relativistic case that our simple example is concerned with, we

and depend only locally on the interaction we could also impose that test functions to be equal to one in the support of interactions. This constraints were initially imposed by perturbative Algebraic Quantum Field Theory [97] in the context of the so-called algebraic adiabatic limit, which allows for an unproblematic and rigorous renormalization procedure (more on this in Section 5). For simplicity, we will set aside this requirement in our examples, but it can always be imposed.

²¹Examples of test functions that are not compactly supported (contrary to bump functions) are other functions that belong to space of Schwartz functions $\mathcal{S}(\mathbb{R}^n)$, such as Gaussian functions. Schwartz functions are functions that infinitely differentiable and rapidly decreasing at infinity, as well as all their derivatives.

will ignore space and relativistic considerations. Thus, we will consider that

$$f(t) = \begin{cases} \exp\left(-\frac{1}{1 - \left(\frac{t - t_{XY}}{\tau_{XY}}\right)^2}\right), & |t - t_{XY}| < \tau_{XY}, \\ 0, & \text{otherwise,} \end{cases} \quad (19)$$

where we will have $f_{AB}(t)$ and $f_{BC}(t)$. t_{XY} and τ_{XY} allow us to infer the temporal localization and duration of the interactions between quantum systems X and Y .

More broadly, $f_{XY}(x)$ allows us to make inferences about a) when systems X and Y have determinate values of their observables when interacting, and b) how this interaction-based process of having determinate values influences other processes of having determinate values if different test functions for different interactions have some of their support in common. Information a) and b) is relevant to know where interactions do not disturb other interaction because a) encodes the timing of the interaction between A and B , and B and C . b) encodes whether the interaction between A and B is disturbed by the interaction between B and C . This will be relevant to our discussion below.

Taking into account that the interactions between system X and Y in the Schrödinger picture (neglecting the self-Hamiltonian) or in the interaction picture can be given by

$$\hat{U} = \mathcal{T} \exp\left(-i \int dV \mathcal{H}_{\text{int}}(x)\right), \quad (20)$$

where $\mathcal{T}$ is the time-ordering, $dV = \sqrt{-g}d^4x$ with g being the determinant of $g_{\mu\nu}$, there are four conditions, which constitute the core of our determination conditions (and we will call them simply determination conditions), for a system B to obtain the determination capacity concerning a target system C , which we will denote as DC- C . The determination conditions are the following:

- i) if A has the determination capacity concerning B (DC- B), where A can be a composite system;
- ii) if C interacts with B , while B is interacting with A where the interaction between A and B starts first. This is translated in the centers of the test function being timelike or lightlike separated,

$$\frac{1}{2} [-(t_{BC} - t_{AB})^2 + |\mathbf{x}_{BC} - \mathbf{x}_{AB}|^2] \leq 0, \quad (21)$$

but where $t_{BC} > t_{AB}$;

- iii) if B has a determinate value due to A .

In the toy example below, this will be inferred by B being entangled with A , or in realistic cases, A locally decohering B . For instance, if A is a composite system, such as modes of a field and B is a mode of a field, this could be inferred by A decohering B ,²²

iv) if the interactions between B and C are such that C does not disturb the interaction between A and B in such a way that A probes B and both have determinate values. Considering ii), another way of expressing this condition is that A should decohere B before the interaction between B and C ends, giving rise to A and B having determinate values; and C does not disturb this process so that the unitary U that describes the interaction between A and B , and B and C is $\hat{U}\rho_X \otimes \rho_Y \otimes \rho_Z \hat{U}^\dagger \approx \hat{U}'\rho_X \otimes \rho_Y \hat{U}'^\dagger \otimes \rho_Z = \rho' \otimes \rho_Z$. U' is the unitary that describes the local decohering interaction, which typically entangles the states of X and Y , resulting in the state ρ' . ρ_Z is the state of Z , which does not get entangled with X and Y . This is the no-disturbance condition mentioned above. Considering $\hat{U}^{AB}$ as describing the interaction between A and B , and $\hat{U}^{BC}$ as describing the interaction between B and C , we may express this condition as establishing that for successive systems in an SDC, it is sufficient that the following holds in the common support of the test functions $\Omega = \text{supp}(f_{AB}) \cap \text{supp}(f_{BC})$ of $f_{AB}(x)$ and $f_{BC}(x)$,

$$\left[\hat{U}^{AB}(x), \hat{U}^{BC}(x) \right] \ll 1. \quad (22)$$

Another way to express the above comes from noticing that the commutator of the two terms in Eq.(14) is proportional to $f_{AB}(x), f_{BC}(x)$ or $f_{AB}(t), f_{BC}(t)$. Then, one could show that is sufficient that the following should hold,²³

$$\int dV f_{AB}(x) f_{BC}(x) \ll 1 \quad (23)$$

in Ω for the non-disturbance condition to be satisfied. Given the form that test functions should have, if the two test functions $f_{AB}(t), f_{BC}(t)$ have almost disjoint regions of support, the above is guaranteed to occur.²⁴

Let us then turn to the analysis of the interactions between A , B , and C in a non-relativistic toy model. Let us assume then that A has the DC- B (condition i) is fulfilled), that C interacts with B , while B is interacting with A (condition ii) is fulfilled). Moreover, we will consider that the interaction between A , B , and C is such that C does not disturb the interaction between A and B where this non-disturbing interaction is represented by the Hamiltonian in eq. (14).

²²See Section 4.2 and Appendix ?? for other ways to infer this through modes of fields that probe a target system, and the correlation functions of the latter.

²³We will see further below other ways that dispense with test functions to some degree, to fulfill this condition.

²⁴Note that the above could hold just for spatial test functions if we opted to only use them, or for both temporal and spatial test functions if they were treated separately.

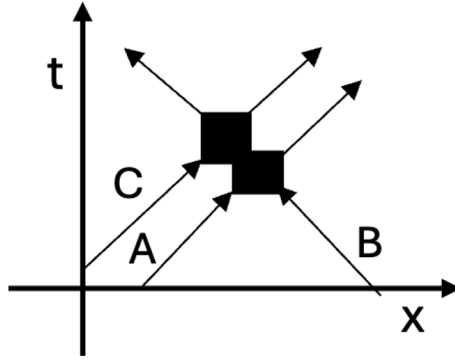


Figure 2: An SDC with systems A, B, and C in QFT interacting in overlapping regions of spacetime in agreement with i) – iv).

Thus, A and B get entangled at $t = 1$ and we can represent this interaction by

$$|\Psi(1)_{\text{approx}}\rangle \approx \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B - i|0\rangle_A|1\rangle_B) \frac{1}{\sqrt{2}}(|0\rangle_C + |1\rangle_C), \quad (24)$$

and thus condition iii) is fulfilled.²⁵

Note that according to ii), for B to have the DC- C , C needs to start interacting with B while A and B are interacting (i.e., between $t = 0$ and $t = 1$). Then, when entanglement between A and B is achieved because A has the DC- B , an indeterministic process occurs that gives rise to A and B having determinate values of their spin- z observables. Let us (for example) consider that this indeterministic process gives rise to A and B having determinate values 1 and 0, respectively. We then update the state of the system to the new state that will serve as the initial state for the next interaction,

$$|\Psi(1)\rangle \approx |1\rangle_A|0\rangle_B \frac{1}{\sqrt{2}}(|0\rangle_C + |1\rangle_C), \quad (25)$$

where condition iii) is now fulfilled. Then, since conditions i)-iv) are fulfilled, when B gets its states entangled with C at $t = 2$, i.e.,

$$|\Psi(2)\rangle = |1\rangle_A \frac{1}{\sqrt{2}}(|0\rangle_B|0\rangle_C - i|1\rangle_B|1\rangle_C), \quad (26)$$

it can give rise to C having a determinate value (1 or 0) and also to B having another determinate value (0 or 1), where one of the possible outcomes will again arise indeterministically.

As mentioned, in the realistic decoherence setting, we would not only have A but also N systems A_i that could be discretized modes of a field, each one

²⁵In Appendix B, we do a numerical study to show why the above approximation is fulfilled, given the no-disturbance condition.

with the the DC- B , or uncountably many if we treat A as a continuum of modes of a field. These modes will interact locally with B with (in the discretized case) randomly distributed coupling strengths $\lambda_{A_i B}$ (for instance, assuming uniformly distributed values from 0 to 1) that would also be multiplied by the above Hamiltonian of interaction. For large N and over time, systems A_i would decohere system B . This could, for example, be observed by off-diagonal terms of the reduced density operator of B going quasi-irreversibly to zero over time, or by the quasi-irreversible loss of purity of this operator. Furthermore, note that we would not just B but many N' systems B_j that interact with A_i . If this chain continues, they will then interact with many N'' systems C_h , each having the DC- C_h for different h , and so on. Moreover, the timescale in which interaction A_i gives rise to B having a determinate value should be of the order of the decoherence timescale. Thus, the test functions modeling this interaction should allow it to go on for at least this timescale in order for A_i to give rise to B having a determinate value. Although it might seem like an ad-hoc condition, the no-disturbance condition can be seen as a necessary condition for decoherence to occur because we do not want other systems Z to disturb a decohering process involving arbitrary systems X and Y . Note that, as we have mentioned, test functions have widespread use in rigorous approaches to QFT, and EnDQT is an approach to quantum theory that relies on them in a more diverse way than usual. We will see some examples in Sections 3.2.2 and 5.

Note also that for EnDQT, the quantum formalism (including the Hamiltonian) and quantum states primarily have a predictive and inferential role concerning the local behavior of quantum systems. Therefore, for example, there is no sense in which there is action at a distance when an agent learns about the determinate value of its entangled target system in a Bell scenario. There is only a local state update concerning the outcomes that arose indeterministically at a wing in the Bell scenario, where this update is in the future light cone of the measurement event (Appendix F). There is no collapse across hypersurfaces.²⁶ Moreover, as we have mentioned, the theory proposed here can use the measurement frameworks in QFT [37, 92, 93] because it shares common tools (i.e., test functions and interactions in compact spacetime regions mediated by probes and involving quantum fields), and this allows for rules for a state update that are independent of how we foliate spacetime in terms of spacelike hypersurfaces (see [91, 38, 92, 93] and Appendix F). The probe in these frameworks is a member of an SDC, and the state updates are formulated as not depending on states associated with specific foliations of spacetime through spacelike hypersurfaces. The latter can cause problems in terms of giving rise to conflicting expectation values of the total charge of entangled particles, conflicting with charge conservation [3]. Rather, state updates depend on the future lightcone of the measurement events involving the probe and the target system, being formulated in such a

²⁶We are adopting the view that EnDQT is a single-world theory, and that does not require some (emergent) agents. Alternative versions may deny this and consider that SDCs involve some branching process or that SDCs only tell us about interactions that ultimately require agents to make measurements to give rise to measurement outcomes. These versions are problematic and that is why we do not adopt them.

way that does not give rise to these conflicts.²⁷

So, the above are the conditions for B to act as a “measurement device” for C ; now, if C did not interact with B while B was interacting with A (i.e., if condition ii) was not fulfilled), B could not act as a measurement device for C . Therefore, A would merely act as a preparation device for B , and in this way, we would have a measurement-based preparation. Then, when B interacts with C , they would only get entangled and evolve unitarily with no indeterministic process happening. Allowing also for cases of this kind is one of the reasons why we want the DC to be transmitted via spatiotemporally overlapping interacting processes. This is because they help establish the conditions under which systems gain or lose the determination capacity, losing the capacity to constitute measurement devices for other systems, and only becoming entangled with them upon interactions without contributing to outcomes arising. More reasons for our choice of determination conditions will be provided at the end of this section.

As we can see, the determination capacity spreads through interactions, and the chain that concerns the spread of this capacity is called the stable determination chain (SDC), where these chains have a structure represented by diagrams. We can write the structure of this simple chain as $A \rightarrow B \rightarrow C$, where the arrows represent the transmission of the determination capacity or a system giving rise to another having determinate values.

One might wonder when SDCs started. One option is to invoke systems that start SDCs, called initiators. In that case, we would add a new postulate to the ones above concerning the conditions for a system B to obtain the determination capacity concerning a target system C , which we will denote as DC- C :

v) If B is an initiator, interacting with C without the need of some other system that allows it to have the $DC - C$.

As argued previously [88], to explain why initiators are, in principle, not currently observable (i.e., a measurement device or a probe seems always to need other systems that apply it or prepare it, respectively, at least according to our more direct evidence), it is because they are systems only active in the early universe such as, for example, the inflaton field. After its activity, this field sits at the bottom of its potential $V(\mathbf{x}, t)$, not being active anymore. However, there are other possibilities beyond postulating an inflaton field, which we will discuss.²⁸ Of course, we might assume that SDCs go on indefinitely, and in that case, we do not need to invoke initiators and could have some kind of cyclic universe, where the initial members of an SDC in this universe come from SDCs in a previous universe, for example. What the correct view is might end up being an empirical question. We will return to the initiators in Sections 5, 7, and Appendix I.

Notice that according to EnDQT for a system to maintain its quantum

²⁷To be precise, in the framework of [93], which is based on the other two cited frameworks, the states of the detector and the target field depends on the spacetime foliation but they are related by an equivalence relation making the slicing choice irrelevant.

²⁸An initiator may be the source of its own test function. See Section 5.

coherence, it must be isolated from SDCs. The system isolated from SDCs could be arbitrarily large, and if that isolation was achieved, the system could, in principle, be maintained for an arbitrary amount of time in a coherent superposition. This contrasts with spontaneous collapse theories, which consider that an isolated system would still collapse at some point regardless, or gravity-induced collapse theories, where a system collapses depending on its mass/energy. Also, in contrast to these theories, we do not need to modify the fundamental equations of quantum theory to represent when a system stops being in a coherent superposition and an outcome arises. This is one of the reasons why this theory is conservative.

As we will see more clearly in Section 7, the transmission of the DC between systems can be used to explain how the classical four-volume of our visible universe (inhabited by matter fields with determinate values) arises from quantum fields. This will also show what guided our choice of determination conditions. We do not tend to think of the classical four-volume of the universe as discontinuous or full of gaps, with regions with no gravitational field followed by regions with a gravitational field. However, if we only appealed to local decohering interactions (however vaguely defined) in the determination conditions, without the need for the acquisition or transmission of the DC via spatiotemporally overlapping interactions, we would not obtain this unificatory picture of the four-volume of the universe. If local decoherence sufficed for the systems involved to emit a gravitational field, it would be perfectly conceivable that we would end up having constant gaps in the emission of a gravitational field. For instance, a system could emit a gravitational field without leading other systems in adjacent spatiotemporal regions to emit such fields or without depending on other systems that previously emitted them. Thus, we think that the spatiotemporally adjacent transmission of the DC, which test functions help model, helps *unite* the classical four-volume of spacetime through regions occupied by matter fields sourcing gravity and explains why we do not obtain a fragmented gravitational universe, explaining the usual features that we attribute to the four-volume of the universe. Therefore, in our view, the above widely believed attributes of the classical four-volume supply evidence for the determination conditions postulated by EnDQT, which involve spatiotemporally overlapping test functions.

3.2 The QFT case

Let us turn to the QFT case. We focus on spacetimes where the classical dynamics governed by the Klein-Gordon equation have a well-posed initial value formulation in the sense that it admits a spacelike hypersurface where the initial data can be specified such that the entire evolution in spacetime is determined by this data. This hypersurface is a Cauchy surface, and a Lorentzian manifold is globally hyperbolic if and only if it admits a smooth Cauchy hypersurface.

So, let ϕ be a real scalar field defined in a $D = n + 1$ -dimensional globally hyperbolic Lorentzian spacetime $(\mathcal{M}, g_{\mu\nu})$, where n is the number of spatial

dimensions. The field satisfies the Klein-Gordon equation:

$$P\phi = 0, \quad P = \nabla_a \nabla^a + m^2 + \xi R, \quad (27)$$

where ξ is the curvature coupling constant, R is the Ricci scalar, and ∇_a is the Levi-Civita connection corresponding to the metric $g_{\mu\nu}$. The condition of global hyperbolicity guarantees the existence of a smooth foliation by Cauchy surfaces $\{\Sigma_t\}_{t \in \mathbb{R}}$ and a diffeomorphism $\mathcal{M} \cong \mathbb{R} \times \Sigma$. In these spacetimes, the Klein-Gordon equation admits a well-posed initial value formulation,²⁹ and we can meaningfully describe constant time slices. For instance, in Minkowski spacetime, we may identify the Cauchy surfaces $\Sigma_t \cong \mathbb{R}^n$ with spacelike hypersurfaces. Using the global inertial coordinates $(t, \mathbf{x})$, these hypersurfaces correspond to surfaces with constant t .

Although we will mostly adopt the “physicist” formalism, we also have in the background the more rigorous algebraic quantum field theoretic (AQFT) account [37] with its algebra of observables independent of a Hilbert space representation, and its smeared fields.³⁰ In the previous section, we discussed how the test function that smears fields over a spacetime region plays an important role for EnDQT in representing how SDCs propagate, and they also have an important role in AQFT.

We work on a globally hyperbolic spacetime $(\mathcal{M}, g)$ that either contains a region with a timelike Killing vector K^a (e.g., a static patch), or possesses a preferred time function t whose asymptotic or adiabatic behavior selects a “positive-frequency” notion (e.g., conformal time in the Poincaré patch of de Sitter). With n spatial dimensions the real scalar field admits the Fourier expansion,

$$\phi(x) = \int d^n \mathbf{k} \left[a_{\mathbf{k}} u_{\mathbf{k}}(x) + a_{\mathbf{k}}^\dagger u_{\mathbf{k}}^*(x) \right], \quad (28)$$

where the normalization factors were built into the mode functions.³¹ Promoting $a_{\mathbf{k}}, a_{\mathbf{k}}^\dagger$ to operators gives

$$\hat{\phi}(x) = \int d^n \mathbf{k} \left(\hat{a}_{\mathbf{k}} u_{\mathbf{k}}(x) + \hat{a}_{\mathbf{k}}^\dagger u_{\mathbf{k}}^*(x) \right), \quad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta^n(\mathbf{k} - \mathbf{k}') \mathbb{I}. \quad (29)$$

The vacuum state $|0\rangle$ is defined as the state annihilated by $\hat{a}_{\mathbf{k}}|0\rangle = 0$ for all $\mathbf{k}$. By performing quantization on a constant-time foliation $\mathbb{R} \times \Sigma_t$, where Σ_t is a spacelike Cauchy surface, we obtain the equal-time commutation relations:

$$[\hat{\phi}(t, \mathbf{x}), \hat{\pi}(t, \mathbf{x}')] = i\delta_\Sigma^n(\mathbf{x}, \mathbf{x}') \mathbb{I}, \quad (30)$$

²⁹An initial value problem consists of a differential equation together with initial data specified at a point or on an initial hypersurface, sufficient to determine a unique solution—typically the value of the unknown function and, when needed, its derivatives.

³⁰See Appendix C for some formal details regarding the quantization of the scalar field from an AQFT perspective.

³¹For proper normalization in curved space the modes $u_{\mathbf{k}}, u_{\mathbf{k}'}$ should be orthonormal under the Klein-Gordon inner product, i.e., $(u_{\mathbf{k}}, u_{\mathbf{k}'}) = \delta^n(\mathbf{k} - \mathbf{k}')$, where $(u, v) = i \int_\Sigma d\Sigma^a [u^* \nabla_a v - (\nabla_a u^*) v]$.

$$[\hat{\phi}(t, \mathbf{x}), \hat{\phi}(t, \mathbf{x}')] = [\hat{\pi}(t, \mathbf{x}), \hat{\pi}(t, \mathbf{x}')] = 0. \quad (31)$$

Here,³² the canonical momentum operator is defined in curved spacetime as $\pi(t, \mathbf{x}) = \sqrt{h} n^a \nabla_a \phi(t, \mathbf{x})$ where $h = \det(h_{ij})$ is the determinant of the induced metric h_{ij} on the Cauchy surface Σ_t , and n^a is the future-directed unit normal to Σ_t . In Minkowski spacetime, with Σ_t being a constant- t hypersurface, this reduces to the familiar definition $\pi = \partial_t \phi$.

3.2.1 Constraints on test functions and systems emitting them

To infer when and how determinate values arise under interactions, the partial trace is useful. However, it is not technically correct to assign a density matrix to the restriction of a vacuum state or any physical state of a QFT to any local subregion. Mathematically, this is because the local algebra of observables on a finite region of a relativistic QFT is a type III von Neumann algebra. This algebra does not admit any irreducible representation as an algebra of operators on a Hilbert space, and does not have any nontrivial faithful operation with the properties of a trace. Thus, operations like taking a partial trace over a subregion are unavailable, and the von Neumann entropies of the reduced density operator of a QFT on a given region are not well-defined. Therefore, we cannot use them to derive the reduced state of a QFT in a local subregion.

To circumvent this issue, we can focus on a subset of modes of real scalar fields that participate in the interactions involved in SDCs, where that selection will be inferred via the test functions. This provides one possible way to go to a type I von Neumann algebra. We often focus on this subset by quantizing a sum of discrete solutions to the Klein-Gordon equation in a bounded spacetime region,

$$\hat{\phi}(x) = \sum_{\alpha} [\hat{a}_{\alpha} u_{\alpha}(x) + \hat{a}_{\alpha}^{\dagger} u_{\alpha}^{*}(x)]. \quad (32)$$

Different positions can be taken regarding the test functions. One position is that they are fundamental and offer ways to infer how DC propagate and SDCs expand without being attached to any particular system. One could develop determination conditions that assume test functions as not being attached to systems. However, one might object to this strategy because one might consider that it renders their origin mysterious. Furthermore, there is a good case to be made that they arise from a potentials. So, the target systems of test functions are implicitly open and work can be done on them via these test functions. Thus, the above position makes it unclear which systems are responsible for the effects associated with those functions.

Another option is that test functions, among other roles, allow us to infer the localization features of systems belonging to SDCs, due to some other systems, which affects with whom they interact, and as we have been seeing, the transmission of the DC. Ultimately, as we will see, they are related to the emission of a gravitational field due to localized systems. There are multiple ways to proceed. Here, we consider that test functions arise from some systems

³² δ_{Σ}^n denotes the covariant delta distribution with respect to the volume element $d^n x \sqrt{h}$.

D in a state $\hat{\rho}_D$, which is a possibly complex-valued function, although we will focus on real-valued test functions. We restrict our attention to those systems whose mean field gives rise to a well-defined test function.³³ Such mean-field is *emitted* by a system D ,

$$f(\mathbf{x}, t) = \langle \hat{V}(x, t) \rangle = \text{Tr}(\hat{\rho}_D \hat{\phi}_D(\mathbf{x}, t)), \quad (33)$$

and to be a reliable mean-field, assuming only Gaussian states, we also need that it has low fluctuations, i.e.,

$$\langle \hat{V}(x', t') \hat{V}(x, t) \rangle \approx \langle \hat{V}(x', t') \rangle \langle \hat{V}(x, t) \rangle. \quad (34)$$

To calculate (33), we will obtain an integral over momentum $d\mathbf{k}$. To choose the bounds of the integral, we should observe that D is a set of modes of a field that have determinate values due to interactions with other members of SDCs (we are labeling the whole field $\hat{\phi}_D$ with all its modes as D , but we are actually referring to a subset of its modes that belong to an SDC). Thus, these modes should inform the bounds of this integral.³⁴ Note that we could consider only some positive or negative momentum component of the field $\hat{\phi}_D$, if we wanted f to be complex valued.³⁵ Thus, when we consider the interaction between systems A and B , we consider a system D that belongs to an SDC that we choose to ignore, and that sources the test function for A and B , where D are certain modes of a field that were left in a specific state ρ by SDCs.³⁶ Note that A could be some modes of the same field as D , and thus they can be regarded as being subsystems of a larger system. See Appendix D for an example of how coherent states can be used as the source of test functions.

Thus, the idea is that SDCs also involve systems in certain states that source the test functions. Another role of the test functions and systems that source them is, through the cutoffs they give rise to, to help determine the scales of

³³While other emission mechanisms may exist, the results we derive apply most directly to this subclass. However, the results concerning test functions presented below are general.

³⁴We may be inclined to calculate the test function via the expectation value of a continuum of modes. However, in agreement with the scale dependence of SDCs, in practice, we never work with all modes of a field. The system that emits the test function will be constituted by a series of modes, for example, up to some bound k_{max} in the UV filter case, which will have determinate values owing to the decoherence and the filtering due to some other systems (note that typically we also need a IR filter). More concretely, although to calculate a test function, we may be calculate it by integrating over a continuum of modes dk , from 0 to ∞ , in practice, we can integrate up to a $\Lambda = k_{max}$, and from $\Lambda' = k_{min}$. Thus, depending on $\Lambda = k_{max}$, we can view the test function as being emitted by many single modes or even by a single-mode $\mathbf{k} \approx 0$ if $k_{max} = \Lambda \ll 1$ for a UV cutoff. Interestingly, the inequalities (36) and (37) that we derived below for the test functions to obey the spacetime symmetries guarantee the validity of these cutoff-based bounded integrals. See at the end of Appendix D.

³⁵The bounds derived below are applicable to this case as well.

³⁶So, given information about D 's previous interaction concerning how the modes of the field were filtered, we can introduce a cutoff in the integral in eq. (33) by hand. See Appendix D. Instead of the cutoff, we could introduce in this definition a test function inside the expectation value value, inherited from D 's previous interaction with members of an SDC. However, given the bounds derived below, in principle, we do not have to. Note that we can associate the localization of this system with the test function that it emits.

systems that source gravity, i.e., what we may call the gravitational scales. It is often argued that the semiclassical approach breaks at Planck scales, but it is unclear whether any physical or gravitational phenomena occur at these scales. We hypothesize that the scales on which SDCs operate, and thus gravity, are much higher than the Planck scale. So, as we will argue, the semiclassical gravity equations may be sufficient to represent the behavior of the gravitational field.

One should notice a feature of test functions, test functions are involved in all tests of special relativity, which respect Poincaré invariance. It is expected that *ignored* external environments lead to the violation of some symmetries of target system. However, since all measurement outcomes for EnDQT involve open-systems situations, we want at least some test functions emitted by members of SDCs not to spoil the commutation relations between the generators of Poincaré transformations, which are required to preserve Poincaré invariance. Otherwise, it would be difficult for EnDQT to justify why relativity works or how reliable evidence can be found. This leads to constraints on the test functions for at least some local Hamiltonians in a (approximately) flat spacetime. Although the bounds below will be valid for any test function, for definiteness, let us suppose that we have the following temporal and spatial test Gaussian functions,

$$\Lambda(x) = \chi(t) F(\mathbf{x}) = \exp\left[-\frac{(t-t_0)^2}{2T^2}\right] \exp\left[-\frac{|\mathbf{x}-\mathbf{L}|^2}{2\sigma^2}\right]. \quad (35)$$

T and σ represent the temporal and spatial standard deviations, and characterize the region where $\Lambda(x)$ is effectively nonzero. The parameters t_0 and $\mathbf{L}$ determine the central time and position of the support of the test functions. In the case of spatial variance σ we then get the following constraint (see Appendix D),

$$\sigma \gg 1/k_{max} \quad (36)$$

where $k_{max} = |\mathbf{k}_{max}|$ is the maximum momentum of the physical processes under study, and $L_{phys} = 1/k_{max}$ is the minimal length scale of the modes of the field involved in the interactions under study. Similarly, we get the following constraint on the variance T of the temporal test function (see Appendix D),

$$T \gg 1/\omega_{max} \quad (37)$$

where ω_{max} is the maximum energy of the system in the interaction under study, and $\tau_{phys} = 1/\omega_{max}$ is the minimal temporal scale involved in this interaction. Similar inequalities need to be obeyed by the IR filter, which filters out infrared modes in flat spacetime.^{37,38} Furthermore, it can be shown that bandpass filters

³⁷Assuming for definiteness the following test function, $f(\mathbf{x}, t) = \left[\frac{\mathbf{x}^2}{\sigma^4} - \frac{d}{\sigma^2} + \frac{t^2}{T^4} - \frac{1}{T^2}\right] \exp\left(-\frac{\mathbf{x}^2}{2\sigma^2} - \frac{t^2}{2T^2}\right)$, where d is the number of spatial dimensions, whose Fourier transform is $\tilde{f}(\mathbf{k}, \omega) = -(|\mathbf{k}|^2 + \omega^2) (2\pi)^{\frac{d+1}{2}} \sigma^d T \exp\left[-\frac{\sigma^2 |\mathbf{k}|^2 + T^2 \omega^2}{2}\right]$. This filters out the long-wavelength/low-energy sector, and we can see that we can have an argument identical to the given in Appendix D; hence $k_{max}\sigma \gg 1$ and $\omega_{max}T \gg 1$.

³⁸Notice that this requirement makes test functions approximately constant, which helps implementing the requirement of being equal to 1 in the support of interactions between fields discussed in the footnote 20.

also obey these bounds.³⁹

Not only test functions concerning Poincaré symmetric spacetime can obey bounds. Test functions concerning some symmetric spacetimes can also obey certain bounds. For instance, we will also analyze the case of de Sitter spacetime with only a temporal test function,

$$f_\ell(t, \mathbf{x}) = \exp\left[-\frac{(t-t_0)^2}{2\ell_t^2}\right], \quad (38)$$

we obtain a similar inequality $\omega_{\max} \gg 1/\ell_t$ for local Hamiltonians in these spacetimes, taking into account the generators of symmetries of a de Sitter spacetime and their commutation relations.

Therefore, interactions that form some SDCs with the above symmetric features need to have couplings that obey the above constraints, which depend on the maximal momentum or energy of the modes involved in these couplings. We see that the spacetime symmetries impose that there should be emitters of the test function involving systems that live at much higher scales than the smaller systems that are subject to those smearings, which impose UV or IR cutoffs on these systems. Note that many, or perhaps most, spacetimes do not have the above bounds because they lack symmetries. Thus, relativistic symmetries are the exception rather than the rule. Moreover, they are idealizations and never hold exactly. So, small violations in the commutation relations between the generators of these symmetries are also expected when considering less idealized situations.

A feature worth noticing is that with the mean-field definition that we have adopted, not all states $\hat{\rho}$ can emit a test function; they have to be states such that $\alpha_{\mathbf{k}} := \text{Tr}(\hat{\rho}\hat{a}_{\mathbf{k}}) \neq 0$, for at least some modes $\mathbf{k}$. Thus, for instance, coherent states with $\alpha_{\mathbf{k}} \neq 0$ can in principle emit it, as well as squeezed coherent states and field amplitude eigenstates approximated by a Gaussian function. Number states, thermal states, parity symmetric, and antisymmetric cat states cannot. Note that states still need to be selected via decoherence to be able to lead a system to emit a test function.

Due to the role of the emitters of test functions in the transmission of the DC, we add a new condition to Postulate 2. As a reminder, this postulate concerns the conditions for a system B to obtain the determination capacity concerning a target system C , which we denote as DC- C (see introduction to Section 3):

vi) If B is interacting with the emitters S of a test function for its interaction with system A and C . A system S is an emitter of a test function if and only if it is in a state that can give rise to a valid test function and has the DC concerning the systems it interacts with, while having determinate values in the spacetime region where it emits such a function. So, it will emit a test function while it is in that state, where, if the system is not an initiator, it needs to be left in that

³⁹An example of that filter is $f(t, \mathbf{x}) = C e^{-\frac{t^2}{2T^2} - \frac{|\mathbf{x}|^2}{2\sigma^2}} \left(\frac{1}{T^2} - \frac{t^2}{T^4} + \frac{d}{\sigma^2} - \frac{|\mathbf{x}|^2}{\sigma^4} \right)$ where d is the number of spatial dimensions and C is some constant.

state by other members of the SDCs. Given that S 's is well-localized due to its state, giving rise to a non-zero mean-field with low fluctuations, we can use S 's emitted test function to infer its own localization. Test functions can obey conditions that satisfy relativistic symmetries or constraints.

Thus, the modes that constitute S , which are responsible for emitting the test function, are ones that were left having a determinate value due to members of SDCs. By a valid test function, we mean a function that is smooth and strongly localized, such as Schwartz functions. Note that emitters of the test functions S are necessary but not sufficient to lead to the filtering out of modes of a system S' . We also need that systems S'' interact with S' , have the DC- S' and decohere the modes that are not filtered out.⁴⁰ Test functions can obey conditions that satisfy relativistic symmetries or constraints in the sense that they may obey the bounds above, or they should meet conditions imposed via other frameworks.⁴¹ We have included these features in a postulate because it seems to be a brute fact about SDCs.⁴² In the next section, we will see that the emitters of the test function can acquire the DC concerning other systems without a third system localizing their interactions with these systems.

As we can see, SDCs have scale-dependent features. An example that supports the scale-dependence of SDCs is that decoherence in curved spacetime may occur only at certain scales, such as super-Horizon scales in the case of de Sitter spacetime, as we will see in Section 5. Another example that supports the scale-dependence perspective is based on how detector resolution determines whether a more massive system influences the decoherence of a less massive system. More concretely, it can also be shown that test functions determine whether a more massive system S' in the same spacetime region as S decoheres S or decouples from S , not decohering it. This will depend on how massive is S' compared with the temporal cutoff represented by the variance of the temporal test function. If its mass M is much larger, it will be UV filtered out (See Appendix E for more details). We will see that this case is related to the previous example because the system that emits a gravitational field in the de Sitter case is also considered to be the system that emits a test function. So, the point is that test functions emitted by systems that belong to SDCs account for a massive system decohering or not a target system, and hence it accounts for whether such a higher energetic system emits a gravitational field or not. This

⁴⁰Notice that with the determination conditions that we have assumed for EnDQT, states help defining test functions because they arise from mean fields. However, one might worry that these determination conditions are in conflict with AQFT. In the algebraic approach, the generators of algebras can involve smeared fields $\hat{\phi}(f)$, which presuppose test functions (see Appendix C). States are assumed to come after the definition of an algebra of observables and not be implicit in the definition of the generators of this algebra. We see this relation between states and observables as a self-consistent mathematical relation to represent SDCs and make inferences about them. So, we do not think that these determination conditions are in conflict with AQFT, as one might initially suspect.

⁴¹Such as the algebraic adiabatic conditions. See 20.

⁴²We could impose the obedience of the above bounds as a condition for systems to emit a test functions, but that seems too demanding.

supports the view advanced above that SDCs select systems at certain scales to emit a gravitational field. Therefore, as we have hypothesized, by adopting this theory, we can consider that it is not necessarily the case that the gravitational field is emitted at all scales, including the Planck scale.⁴³ This will depend on the structure and elements of the SDCs. We will return to this topic at the end of the next section after examining a more concrete example.

Finally, as we will discuss in Section 5, test functions and SDCs at different scales are related by renormalization group transformations. More concretely, the features of SDCs vary with energy scales, where this is described by certain laws with specific masses, couplings, etc. Systems at these energy scales have the DC concerning other systems at those scales. So, it will provide ways of understanding the scale-dependence of SDCs.

3.2.2 SDCs in flat spacetime

We now provide an example of an SDC in flat spacetime involving systems in an inertial frame by appealing to the well-known models of decoherence. This will also clarify how systems in a coherent state are selected, which we will appeal to.

Let us consider A , B , C , and D . We consider A to be a large collection of N modes of a field in a Gibbs state:

$$\hat{\rho}_A = \int \prod_i \frac{d^2\alpha_i}{\pi} \left[\prod_i (1 - e^{-\beta\omega_i}) e^{-(1 - e^{-\beta\omega_i})|\alpha_i|^2} \right] |\{\alpha_i\}\rangle\langle\{\alpha_i\}|, \quad (39)$$

and we consider that B could be a number N' of modes, each in some arbitrary state. We will focus on a single mode of B , where modes of B are in a state $|\psi\rangle_B$. Furthermore, D is a field that emits the test function, and which we will assume is in a coherent state $|\alpha\rangle_D$ due to its interactions with other members of an SDC that we chose to ignore (we will come back on how D might have ended up in that state).⁴⁴ Moreover, D is a system (composed of multiple modes) that is interacting with multiple modes that constitute A and B , while they interact, and that can also interact with B and C , if they interact, emitting the test function for these interactions. We assume that the modes of C are in an arbitrary quantum state $|\psi\rangle_C$.

We will consider that the multiple modes that constitute field A interact with a single mode of B , where the Hamiltonian of interaction is given by

$$\hat{H}_{\text{int}} = \sum_{\mathbf{k} \neq \mathbf{k}_B} C_{\mathbf{k}} \hat{X} \hat{q}_{\mathbf{k}}, \quad (40)$$

where $\hat{X}$ and $\hat{q}_{\mathbf{k}}$ are the field quadratures for the single mode of B and for the multiple modes A_i of A , respectively, and $C_{\mathbf{k}}$ are coupling constants. Assuming

⁴³Thus, we leave open the existence of systems at lower scales that cannot source a gravitational field.

⁴⁴Such coherent state has $\alpha_{\mathbf{k}} = \exp[-\frac{1}{2}(\sigma_r^2|\mathbf{k}|^2 + \sigma_t^2\omega_{\mathbf{k}}^2)] \exp[i(\omega_{\mathbf{k}}t_0 - \mathbf{k} \cdot \mathbf{x}_0)]$, where the test function associated with this state is centered at $(t_0, \mathbf{x}_0)$.

the non-disturbance condition, we choose to ignore the interaction between B_i and the rest of the systems that constitute C .

To arrive at the above H_{int} , starting from flat spacetime QFT we consider the following smeared linear interaction Hamiltonian,

$$\hat{H}_{int}(t) = \lambda \int d^3x dt f(\mathbf{x}, t) \hat{\phi}_B(\mathbf{x}, t) \hat{\phi}_A(\mathbf{x}, t), \quad (41)$$

select one of the modes of B for simplicity and insert the plane-wave decompositions in SI units,

$$\hat{\phi}_B(\mathbf{x}, t) = \frac{1}{\sqrt{V}} \sqrt{\frac{\hbar}{2\Omega}} [\hat{a}_B e^{i(\mathbf{k}_B \cdot \mathbf{x} - \Omega t)} + \hat{a}_B^\dagger e^{-i(\mathbf{k}_B \cdot \mathbf{x} - \Omega t)}], \quad (42)$$

$$\hat{\phi}_A(\mathbf{x}, t) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k} \neq \mathbf{k}_B} \sqrt{\frac{\hbar}{2\omega_{\mathbf{k}}}} [\hat{a}_{\mathbf{k}} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega_{\mathbf{k}} t)} + \hat{a}_{\mathbf{k}}^\dagger e^{-i(\mathbf{k} \cdot \mathbf{x} - \omega_{\mathbf{k}} t)}], \quad (43)$$

together with the Gaussian test function

$$f(\mathbf{x}, t) = f_r(\mathbf{x}) f_t(t), \quad f_r(\mathbf{x}) = \frac{e^{-\mathbf{x}^2/(2\sigma_r^2)}}{(2\pi)^{3/2} \sigma_r^3}, \quad f_t(t) = \frac{e^{-t^2/(2\sigma_t^2)}}{(2\pi)^{1/2} \sigma_t}. \quad (44)$$

We end up with four exponentials,

$$\begin{aligned} I_1(\mathbf{k}) &= \exp\left[-\frac{1}{2}(\sigma_r^2 |\mathbf{k}_B + \mathbf{k}|^2 + \sigma_t^2 (\Omega + \omega_{\mathbf{k}})^2)\right], \\ I_2(\mathbf{k}) &= \exp\left[-\frac{1}{2}(\sigma_r^2 |\mathbf{k}_B - \mathbf{k}|^2 + \sigma_t^2 (\Omega - \omega_{\mathbf{k}})^2)\right], \\ I_3(\mathbf{k}) &= I_2(\mathbf{k}), \quad I_4(\mathbf{k}) = I_1(\mathbf{k}). \end{aligned}$$

Given relativistic symmetries-inducing bounds derived in the previous section, $|\mathbf{k}| \sigma_r \gg 1, \omega \sigma_t \gg 1$, and thus

$$\sigma_r^2 |\mathbf{k}_B + \mathbf{k}|^2 \gg 1, \quad \sigma_t^2 (\Omega + \omega_{\mathbf{k}})^2 \gg 1, \quad (45)$$

so we have that $I_1 \simeq I_4 \ll 1$.

Furthermore, assuming that the emitter of the test function D filters out every mode except those that are quasi-resonant with the environmental probes, or assuming that it operates in narrow band, we get $|\mathbf{k} - \mathbf{k}_B| \lesssim \sigma_r^{-1}, |\omega_{\mathbf{k}} - \Omega| \lesssim \sigma_t^{-1}$. Thus,

$$\sigma_r^2 |\mathbf{k}_B - \mathbf{k}|^2 \ll 1, \quad \sigma_t^2 (\Omega - \omega_{\mathbf{k}})^2 \ll 1, \quad (46)$$

and therefore $I_2 = I_3 \approx 1$. So, $I_1(\mathbf{k}) = I_4(\mathbf{k}) \rightarrow 0, I_2(\mathbf{k}) = I_3(\mathbf{k}) \rightarrow 1$. We thus obtain,

$$\hat{H}_{int} = \frac{\lambda \hbar}{V \sqrt{2\Omega}} \sum_{\substack{\mathbf{k} \neq \mathbf{k}_B \\ |\mathbf{k}| \leq k_{\max}}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} (\hat{a}_B \hat{a}_{\mathbf{k}}^\dagger + \hat{a}_B^\dagger \hat{a}_{\mathbf{k}}), \quad (47)$$

which can be rewritten as

$$\hat{H}_{\text{int}} = \sum_{\mathbf{k} \neq \mathbf{k}_B} C_{\mathbf{k}} \hat{X} \hat{q}_{\mathbf{k}} \quad (48)$$

with $C_{\mathbf{k}} = \lambda \hbar / [V \sqrt{2\Omega} 2\omega_{\mathbf{k}}]$, $\hat{X} = \sqrt{\hbar/(2\Omega)}(\hat{a}_B + \hat{a}_B^\dagger)$, and $\hat{q}_{\mathbf{k}} = \sqrt{\hbar/(2\omega_{\mathbf{k}})}(\hat{a}_{\mathbf{k}} + \hat{a}_{\mathbf{k}}^\dagger)$.

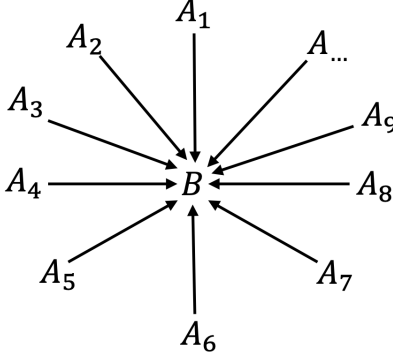


Figure 3: Multiple systems $A_1, \dots, A_N$ belonging to SDCs, and interacting with system B , giving rise to B having a determinate values and emitting a gravitational field. The inference regarding how these interactions occur is made using decoherence models. We omit systems C and D in this diagram.

It was shown that an arbitrary state $|\psi\rangle$ decohering into a statistical mixture of coherent states is a generic feature of free quantum systems that are linearly coupled to an environment in a Gibbs state. This environment can have a nonzero temperature and involve ohmic, subohmic, and supraohmic damping, and the interactions can have arbitrary coupling strengths [30]. Moreover, this Hamiltonian of interaction, depending on the specifics of the model [117], also allows for interactions that lead to systems in a Gibbs state. Multiple situations can occur. For instance, we can have a situation where modes of a system A leave multiple modes of a system B in a Gibbs state, where these modes then leave multiple modes of system C in a coherent state and possibly other modes of C in another Gibbs state, and so on. Moreover, these mixtures of coherent states stochastically give rise to single coherent states, which can then emit test functions. Thus, we have a mechanism in which systems in a coherent state arise via SDCs.

As we can see, to give rise to a system with determinate values, a source of a test function is needed, as well as some system that decoheres or whose state correlates with the state of another system. Let us call *spreading of the DC by control* the spreading of the DC between systems S due to systems S' , where S' emits the test functions that make systems S interact with each other, obey the no-disturbance condition, and transmit the DC between each other. This is

the kind of spread of the DC that we have been observing.⁴⁵

Additionally, we can consider that the emitters S' of the test function interact with all the modes of systems S up to k_{max} , which they do not filter out in case they emit a UV filter, or which they filter out in case they emit an IR filter. So, filtering is performed via the interaction of the emitter of the test function with certain modes. Then, the modes that were not filtered out can participate in a process that gives rise to determinate values. The modes that were filtered out cannot. It is in this sense that SDCs only exist at certain scales. As we have mentioned, renormalization group will also help make these inferences (Section 5).

Note that, as mentioned in the previous section, sometimes certain systems have certain features that allow them to emit test functions for their own interactions and localization. More concretely, due to the features of the state they are in, which involves giving rise to a non-zero mean-field with low fluctuations, emitters of test functions can emit the test function for their own interaction with other systems. So, we can consider that modes that constitute system D , in the example above, are emitting a test function that concerns its interaction with A and B , and with B and C . There is no need for a fifth system that localizes D .

The spread of the DC by control is not the only way to spread the DC. To see this, first notice the interesting fact that, given some Hamiltonian, there may be interactions between systems where we do not know how the way they will give rise to determinate values is going to precisely occur. This is the case where we have emitters of the test function, as mentioned earlier. Consider A interacting with B via certain modes where $\lambda_{AB} f_{AB} \hat{\phi}_A \hat{\phi}_B$ (where we disregard which modes will interact with each other). Moreover, B might develop self-interactions through certain modes via a cubic interaction $\hat{H}_{int}(t) = \int d^3x \frac{g}{3!} \left(\hat{\phi}_{B,\mathbf{k}=0}(t) + \delta\hat{\phi}_{B,\mathbf{k}\neq 0}(\mathbf{x}, t) \right)^3$ (omitting the sum over the rest of the modes $|\mathbf{k}| \neq 0$, which are inside $\delta\hat{\phi}_{B,\mathbf{k}\neq 0}$), for $g \ll 1$, which in momentum space involves terms such as one proportional to $\hat{\phi}_{B,\mathbf{k}=0}(t) \sum_{\mathbf{k}\neq 0} \hat{\phi}_{B,\mathbf{k}}(\mathbf{x}, t) \hat{\phi}_{B,-\mathbf{k}}(\mathbf{x}, t)$. $\phi_{B,\mathbf{k}=0}$ is the $\mathbf{k} = 0$ mode of B , which if it is decohered by A (let us suppose that it will), could act as an emitter of a temporal test function $f(t) = \langle \hat{\phi}_{B,\mathbf{k}=0}(t) \rangle_\rho$ for interactions involving other modes $|\mathbf{k}| > 0$ of B . However, we do not know in which state ρ , $\hat{\phi}_{B,\mathbf{k}=0}$ will end up, while A and B are interacting, where this state will determine the features of $f(t)$.

Now, notice that in this case, the $\mathbf{k} \neq 0$ modes of B that are interacting with $\mathbf{k} = 0$ of B (but are not interacting with modes of A), while the $\mathbf{k} = 0$ of B is interacting with A , fulfill the no-disturbance condition because they do not disturb the interactions between $\mathbf{k} = 0$ of B and A . So, given the

⁴⁵The features above open up the possibility of an alternative determination condition that posits that the only systems that have the DC are the emitters of the test functions, not the systems S that decohere others. However, this condition, insofar it is consistent, neglects the role of decoherence in giving rise to measurement outcomes. Intuitively, it seems that measurement instruments, modeled via decoherence, need to have some importance when measuring target systems, not only the systems that localize their interactions.

determination conditions, $\mathbf{k} = 0$ of B will obtain the DC concerning $\mathbf{k} \neq 0$ of B . This non-disturbance is trivially fulfilled because there does not need to exist a test function that makes $\mathbf{k} \neq 0$ of B interact with $\mathbf{k} = 0$ of B . Let us call *transmission of the DC by osmosis* these interactions that involve systems obtaining the DC concerning some other systems without the intermediaries that involve emitters of the test functions. Importantly, A will leave $\phi_{B,\mathbf{k}=0}$ in a state ρ , and $\phi_{B,\mathbf{k}=0}$ will emit a test function while it does not evolve away from ρ .⁴⁶

Given these determination conditions, the picture that emerges from this theory is of a tower of scale-dependent emitters of test functions and systems that are subject to such emissions, which may end up emitting the test function to other systems, and so on.

Finally, we note that systems that have the DC can be treated as probe/particle detectors, and particle detector models or measurement theory in QFT (developed in algebraic QFT) can be used to update the state of the systems. In Appendix G, we do that for particle detector models.⁴⁷

4 The theory of gravity

We now present the semiclassical gravity theory based on EnDQT. It will involve three postulates, which are added to the other features of EnDQT mentioned above and its determination conditions. Although some of these postulates may seem radical, the theory we propose is actually very conservative. String theory is not being appealed to, spacetime or gravity will not be quantized, but we also do not need to view the metric and conjugate momentum as some stochastic classical system. Therefore, it will not be a gravity-induced collapse theory, such as hybrid classical-quantum theories and the Diosi and Penrose models [26, 84]. We will also show in more detail how this theory agrees with and generalizes the equivalence principle, one of the basic principles of relativity.

4.1 Postulate 1

We have clarified above what our QFT setting is; now we need to ensure that we clarify what we can consider to be the fundamental systems studied by this theory of gravity and what affects their evolution in the absence of interactions with members of SDCs. This is the goal of the first postulate.

Postulate 1 Quantum systems involve sets of modes of quantum fields (henceforth quantum fields) that occupy localizable spacetime regions, and have quantum properties, which are properties represented by observables, such as certain field amplitude operators and energy-momentum operators, and quantum states in agreement with QFT. In the absence of interactions with SDCs and the

⁴⁶Note that there might exist other mechanisms for the transmission of the DC beyond those presented here, and consistent with our desiderata.

⁴⁷See [83] for a review of these models.

determinate values that arise from them, quantum fields in spacetime region R have indeterminate values for any of their dynamical observables in R . Quantum fields S in R that are not interacting with members of an SDC in such a way that decoherence occurs, evolve only under the dynamical equations of QFT that quantum fields obey, such as the Klein-Gordon equation, but no determinate values arise. The above equations are partially determined by the gravitational field in R , or by a flat spacetime metric in the absence of a gravitational field. However, this field is not emitted by S because S cannot emit a gravitational field.

Therefore, we are interested in studying quantum systems that occupy bounded regions of spacetime and establish local interactions with other systems. In the previous sections, we have seen how we can represent their interactions via test functions $f(\mathbf{x}, t)$. Furthermore, quantum fields in R are affected by the gravitational field in that region emitted by the sources of that field. However, they are not affected classically by the gravitational field in the sense of being test quantum fields that have determinate values or test particles obeying the geodesic equation and its deviations (we will return to this and justify it with Postulate 2). The way they are affected is described by the equations that concern the evolution of the quantum fields in that region, such as the Klein-Gordon equation or the Dirac equation for flat and curved spacetimes. However, they do not emit any gravitational field of their own. In the following sections, we will see how this postulate allows us to address some issues with the semiclassical approach.

To understand one of the consequences of Postulate 1, let us consider two scalar fields isolated from SDCs and other systems in a spacetime region R , and that these scalar fields evolve under the same gravitational field in R (e.g., the gravitational field of Earth), determined via the metric $g_{\mu\nu}$ and its derivatives. In addition, let us assume that under hypothetical interactions with SDCs, these systems would give rise to a very different determinate energy-momentum each (which could be arbitrarily different). However, their dynamics are the same, which depends on the Klein-Gordon equation for curved spacetimes that depends on the metric $g_{\mu\nu}$ and possibly its derivatives. Thus, this implies that systems with very different energy-momentum in the same region of spacetime R will evolve similarly under the same gravitational field. Therefore, according to this theory, it is possible that a feather and a very massive quantum object (such as a black hole), both in a coherent superposition of macroscopic states, evolve under the same gravitational field without affecting their spacetime, provided that these objects are not interacting with SDCs and other systems (because of their macroscopicity and decoherence, this phenomenon should be physically very unlikely). More precisely, objects in a coherent superposition behave in the same way under the influence of the same gravitational field, assuming that no other forces intervene, which includes interacting with members of SDCs.

As we have said, this principle is violated by quantum gravity theories if there is entanglement between gravitational degrees of freedom, which gives rise to violations of this principle owing to the behavior of bodies under this entanglement. It will also be violated by gravity-induced collapse theories

because systems emit a gravitational field or, in some regime independent of the environment, collapse, which will give rise to violations of this principle.

Although at first sight, the consequences of this principle seem quite radical and their features counterintuitive, we think they are not because, as we have mentioned (see Section 2 for more details), they can be regarded as a generalization of the Weak Equivalence Principle (WEP). Thus, we have the following the following EnD Equivalence Principle,

Without being affected by other forces, any quantum system under the same gravitational field exhibits the same behavior due to this field.

So, if these systems do not interact non-gravitationally with other objects, including SDCs, they will evolve similarly under the same gravitational field. Furthermore, this theory can be seen as a generalization of the so-called strong equivalence principle (SEP) of standard QFT, which does not study the back-reaction from gravity (i.e., standard curved spacetime QFT). In its simplest formulation, the SEP states that [66]:

Locally, special relativity is at least approximately valid.

This theory states the following, which we will call the Special EnD Equivalence Principle:

If a system S does not interact with SDCs, curved spacetime QFT, which considers that systems do not give rise to a gravitational field that influences their evolution, is valid to describe the behavior of S .

The idea is that curved spacetime QFT, where a system does not give rise to a gravitational field that influences its evolution (which is the standard idealization), is valid in certain circumstances (not only approximately). Given special relativity, a special case of the above principle concerns the evolution in a local region of spacetime. This special case connects this theory with the SEP and is as follows:

If systems do not interact with SDCs, locally flat spacetime QFT and, hence, special relativity are at least approximately valid to describe their behavior.

Thus, we can see that the SEP is a special case of the Special EnD Equivalence Principle. Let us now turn to the second postulate, which concerns how SDCs give rise to the gravitational field.

4.2 Postulate 2 and probing the metric through SDCs

We will now explain how a system can emit a gravitational field due to systems belonging to SDCs that probe it. We begin by showing an intuitive way of understanding how multiple systems probe a target system and give rise to that

system emitting a gravitational field in a spacetime region. Then, we present Postulate 2, which establishes the conditions under which systems can emit a gravitational field.

A model to understand how multiple systems belonging to SDCs that constitute probes (see Appendix F) give rise to a target system emitting or being subject classically to a gravitational field is based on the work of [85], which was based on [104, 60, 61]. What we mean by being subject classically is behaving like a test field, being subject to a certain metric like a field in classical physics. In this work, the authors showed how we can infer the metric that a quantum real scalar field is subject to from local measurements by particle detectors coupled to that field, where the target system is, for simplicity, in a Gaussian state fulfilling the Hadamard condition. Essentially, states that fulfill this condition allow for a finite renormalized stress-energy tensor, as we will discuss further below.⁴⁸ In Appendix F we show how the reduced state of a detector contains information about a target system via two-point correlation functions. Using this feature, the inference of the metric through particle detectors involves probes that measure two-point correlation functions, represented by the Feynman propagator and Wightman function, and extract geometric information from them. The central goal is to express the spacetime metric $g_{\mu\nu}$ in terms of these correlators.

More concretely, the starting point is the Feynman propagator, $G_F(x, x') = \langle 0|T\hat{\phi}(x)\hat{\phi}(x')|0\rangle$, and the Wightman function, where $\hat{\phi}(x)$ is the operator of the target quantum field at spacetime point x , and T represents the time-ordering operator. Assuming that the target system is in a vacuum state $|0\rangle$, we can express the metric $g_{\mu\nu}$ in D spatiotemporal dimensions as follows:

$$g_{\mu\nu} = -\frac{1}{2} \left(\Gamma \left(\frac{D}{2} - 1 \right) \frac{1}{4\pi^{D/2}} \right)^{\frac{2}{D-2}} \partial_\mu \partial_\nu \left(W_{\rho_\phi}(x, x')^{\frac{2}{2-D}} \right), \quad (49)$$

where the above equation is calculated by taking the limit $\mathbf{x}' \rightarrow \mathbf{x}$, and where Γ is a Gamma function. This equation holds for any normalized Hadamard state ρ_ϕ .

These detectors can be understood as modes of a field under certain conditions (Appendix F). We will consider that these systems give rise to a field emitting a gravitational field, or being subject classically to it, not just probing it. The detectors probe the system in separate spatiotemporal regions, forming an *array* that illustrates how the metric/gravitational field with its distances arises from these interactions. However, instead of just an array of detectors, we will consider what we fundamentally have is an array of systems belonging to SDCs in space, interacting with a target quantum field over time. Note that models mentioned above ignore any backreaction of the probes or the target quantum field on the background spacetime. However, it shows how both the field and probes have determinate values in this interactive process, and how this is associated with a metric. But, for the theory we are proposing, this metric (and the associated gravitational field) arises due to these interactions. See Appendix G for further details.

⁴⁸See Appendix C for an introduction to states fulfilling the Hadamard condition.

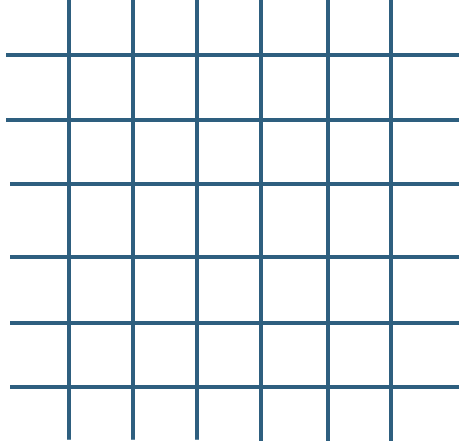


Figure 4: Two-dimensional spatial hypersurface of members of an SDC probing a scalar field at separate points of what can be illustrated as an array of detectors, giving rise to that field emitting a gravitational field in a certain region. A realistic picture would not involve points, but regions.

We will consider that settings like this one constitute environments that give rise to the target system emitting a gravitational field. Furthermore, the persistence of these interactions gives rise to systems emitting a gravitational field and are classically subjected to it in a region R . Note that this target quantum field can then probe other systems, and so on, being part of an SDC. Thus, these interactions will give rise to a set of values, correlation functions, and an associated metric. In a sense, SDCs act as rods and clocks that produce a non-flat metric, and allow gravity to spread under interactions.

Above, we established the determination conditions, and now we will establish the gravitational conditions, which, as a reminder, are the conditions for a system to emit a gravitational field. Postulate 2 establishes the gravitational conditions that we will adopt. As there are various possible determination conditions (see [89] for a discussion), there are multiple possible gravitational conditions. We will go over some of them and explain why we adopt the ones that we will adopt. The first point of division is whether systems in all states, as long as they yield a finite renormalized stress-energy tensor, such as Hadamard states or C^4 states, emit a gravitational field [72]. SDCs would leave systems in these states. Another option is that only systems in more specific states can emit a gravitational field. A possible criterion for selecting these kinds of states could be supported by Kuo and Ford criterion [64]. Some states whose second and higher moments of the energy-momentum tensor can be neglected are coherent states. These states provide trustworthy inputs to semiclassical equations for calculating the expectation value of the energy-momentum tensor. This was argued in the paper from Chung-I Kuo and L. H. Ford [64] for the case of flat

spacetime and by [6] for the more general case of globally hyperbolic spacetimes:

$$\Delta_{\mu\nu\lambda\rho}(x, x') = \left| \frac{\langle : \hat{T}_{\mu\nu}(x) \hat{T}_{\lambda\rho}(x') : \rangle - \langle : \hat{T}_{\mu\nu}(x) : \rangle \langle : \hat{T}_{\lambda\rho}(x') : \rangle}{\langle : \hat{T}_{\mu\nu}(x) \hat{T}_{\lambda\rho}(x') : \rangle} \right|. \quad (50)$$

This estimator is understood as the ratio between the covariance of the normal-ordered energy-momentum tensor and its two-point function. If this estimator is $\Delta_{\mu\nu\lambda\rho}(x, x') \ll 1$ for all x and x' , then we are inside the regime of validity of semiclassical gravity. It was found that this condition is fulfilled by coherent states.⁴⁹

Note that this estimator is useful for the specific case of Gaussian states (coherent states are Gaussian states) because, in this case, all statistical moments of quadratic observables are functions of the second and first moments, guaranteeing that the satisfaction of the Kuo-Ford criterion ensures that the system in this state gravitates semiclassically. However, there are other states, such as cat states, i.e., superpositions of distinguishable coherent states, where other moments are relevant, and thus the above criterion fails. It was shown in [6] that cat states also deliver trustworthy expectation values of the stress-energy tensor in globally hyperbolic spacetimes.⁵⁰

Therefore, according to this gravitational condition, only in certain contexts, such as those where a system ends up in minimum uncertainty states, such as coherent states and/or cat states, we would have systems in those states emitting a gravitational field and being subjected to it. We will return to this point below.

The second point of division is whether a system can have determinate values (give rise to measurement outcomes) in interactions with or without emitting a gravitational field. One option considers that we might have circumstances involving the fulfillment of the determination conditions, where systems can have determinate values but without emitting a gravitational field, where, for example, these systems are in states that are not coherent states. Another option considers that at least one of the systems involved in the interactions fulfilling the determination conditions must emit the gravitational field under the interactions, but the others do not. For instance, a system emitting a gravitational field would be in a coherent state, whereas the others would not necessarily be so.

Another option is that all systems involved in the interactions, which fulfill the determination conditions, must emit a gravitational field, and SDCs select unproblematic states, such as Hadamard states and C^4 states, that emit such a

⁴⁹Besides what is mentioned below, note that this criterion is informative in the case the expectation value of the stress-energy tensor is non-zero, which does not happen in the case of the Minkowski spacetime. This deficiency is not problematic to this theory because according to it and Postulate 3 (see the next section), the Minkowski metric can be considered the default metric and not a metric that arises from the application of the semiclassical or Einstein Field Equations.

⁵⁰More specifically it was shown that the cat state fulfills the above criterion when the coherent amplitude of the state becomes sufficiently large so that the overlap between the two superposed components becomes negligible, and for any cat state where the coefficients of the superposition are chosen such that the relative phase difference between the two coherent states equals $\pi/2$.

field. We will favor the adoption of the latter option via the Postulate 2 (see below) because we believe that it is the most conservative and open to many possible states. Furthermore, it may be the most fruitful option. As we will see, we will further hypothesize that the uncertainties in the stress-energy tensor involved in states that give rise to the gravitational field can be absorbed by a *negative* stress-energy, giving rise to a balance, which allows us to provide an account of dark energy and derive its value. However, we will be open-minded and consider another perspective via Postulate 2' below. Despite the plurality of options seen above that this theory allows for, one should see that as unproblematic because it gives us interesting new hypotheses to study, which may be testable with gravcats experiments (more on this below).⁵¹

Before stating Postulate 2, let us see one way to understand the conservation of the expectation value of the stress-energy tensor and determinate trajectories in spacetime, according to this theory.

Let us consider a system that is left in a coherent state. Via the covariant conservation equation, where we consider that

$$\nabla^\mu \langle \hat{T}_{\mu\nu} \rangle_{ren} \approx \nabla^\mu T_{\mu\nu}(x) = 0, \quad (51)$$

where $\langle T_{\mu\nu} \rangle_{ren}$ denotes the renormalized expectation value of the energy-momentum tensor operator of this system in that state; the strengthened dominant energy condition,⁵² and the support of the stress-energy tensor in an open neighborhood O , we can parametrize a smooth curve embedded in spacetime in a neighborhood O as a geodesic [43, 70]. Then, we can consider that the target system follows a determinate trajectory at least briefly for one interaction with a member of an SDC.

Note that the semiclassical covariant conservation equation and geodesic equations are only applicable once an outcome arises via the decohering interactions that constitute SDCs. Before that, the systems have an indeterminate value of their stress-energy tensor in agreement with the determination conditions (Section 3.1). Hence, the notion of a determinate trajectory given by the geodesic equations is not applicable to model the behavior of systems. Similarly, we will consider that the equations that describe deviations from the geodesic equations are only applicable when systems have determinate values due to SDCs, which also lead to certain trajectories. More generally, any behavior that follows from the Einstein Field Equations⁵³ will only be applicable when systems

⁵¹These gravitational conditions may involve assuming one of three possibilities regarding how gravity and determinate values relate, although here we just focus on the first one because it is the most conservative: a) Gravitational imperialism: all systems belonging to SDCs, when having determinate values, need also to emit a gravitational field; b) Gravitational necessitism: gravity is needed at least for one of the systems involved in interactions involved in SDCs for systems to have determinate values; and c) Gravitational dispensabilism: gravity is not needed for systems to have determinate values; we can have determinate values in a flat spacetime with systems fulfilling the determination conditions, with none of these systems emitting a gravitational field.

⁵²This condition says [70] that for all points p in the manifold M , and all unit timelike vectors ξ^a at p , $T_{ab}\xi^a\xi^b \geq 0$ and, if $T_{ab} \neq 0$, then $T^a{}_b\xi^b$ is timelike.

⁵³Except the metric and gravitational fields that follows from the default state of spacetime. More on this in the next section.

are decohered by members of SDCs.

So, Postulate 2 is the following,

Postulate 2 A system S only emits a gravitational field, has determinate values of some of its observables, and evolves classically under a gravitational field, which can involve a determinate trajectory, when it interacts with systems that belong to SDCs and while they have determinate values due to them. SDCs lead to the selection of quantum states of systems that are favorable to their emission of a gravitational field, such as Hadamard and C^4 adiabatic states. The gravitational field sourced by S and that classically affects S is given by the semiclassical Einstein field equation with the energy-momentum tensor properly renormalized:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}\langle\hat{T}_{\mu\nu}\rangle. \quad (52)$$

Thus, this equation is only valid to describe how the gravitational field affects or is affected by S when S interacts with members of an SDC.

Therefore, SDCs select certain states that are unproblematic to emit a gravitational field, but Postulate 2 leaves which states these are more open than the alternatives, as we will see. One possible alternative to Postulate 2 establishes restricted contexts $\mathcal{C}$ in which systems gravitate:

Postulate 2' A system S only emits a gravitational field, has determinate values of its observables, and evolves classically under a gravitational field, which can involve a determinate trajectory, when i) it interacts with systems that belong to SDCs, and ii) when these interactions between a target quantum matter field S and other quantum matter fields belonging to SDCs that probe the field in a region R lead S to have values that correspond to a quantum state whose second and higher moments of the energy-momentum can be neglected in the spacetime regions R where it is probed, or possibly lead to other contexts $\mathcal{C}$ that guarantee that the expectation value of the stress-energy tensor of S provides reliable results. The gravitational field sourced by S and that affects classically S is given by the semiclassical Einstein field equation with the energy-momentum tensor properly renormalized:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}\langle\hat{T}_{\mu\nu}\rangle, \quad (53)$$

and thus this equation is only valid to describe how the gravitational field affects or is affected by S when S is left in a state due to interaction with members of an SDC.

The hypothesis behind this postulate is that coherent states and/or other states whose second and higher moments can be neglected are responsible for the emission of the gravitational field, or only certain contexts where the stress-energy tensor gives trustworthy results gravitation arises. We will be neutral about which states, or more broadly contexts $\mathcal{C}$, should give rise to a gravitational field.

The advantage of this hypothesis is that it automatically considers that the only systems that emit a gravitational field have always a stress-energy tensor with low fluctuations, which guarantees that the semiclassical equation yields trustworthy results. Note that test functions, via spatial and time averaging over finite intervals of spacetime, can reduce the probability of large quantum fluctuations of the stress-energy tensor [36]. Thus, this approach allows SDCs to reduce the fluctuations of these quantities, given that we consider that SDCs give rise to systems that emit test functions. Therefore, postulate 2' could allow gravity to be emitted in these cases if they are included in the context set $\mathcal{C}$. The disadvantage of Postulate 2' is that it might restrict the domain of relativity excessively and in a problematic way. There might be states with high fluctuations, where no context can reduce them.⁵⁴ Despite the potential advantages of Postulate 2' in terms of dealing with fluctuating stress-energy tensors, we will see in Section 7 that we have other ways of dealing with these fluctuations via dark energy, which can complement Postulate 2' or be adopted by those who adopt Postulate 2.

Nevertheless, it is an empirical question as to which of the gravitational conditions and associated postulates is the right one, which could be decided by experiments involving preparing gravcat systems in specific states or contexts and then measuring their potential gravitational field to see if they emit it or not. These experiments would confirm or rule out different states or contexts.

4.3 Postulate 3: The default state of spacetime and dark energy

We will turn to the postulate that concerns the default state of spacetime in the absence of systems sourcing a gravitational field via the SDCs. It also concerns the potential source of dark energy, which we relate to the cosmological constant appearing in the Einstein field equations. Because we are dealing with very open questions and a much more speculative domain, we will consider different versions of Postulate 3. The third version of this postulate is presented in Section 7.

Postulate 3 (version 1) The effects of the cosmological constant Λ are sourced by SDCs when they emit a gravitational field. Therefore, the accelerating effects of dark energy due to this constant are the result of SDCs. In the absence of SDCs, spacetime is flat, and there is no accelerated expansion of the universe.

According to this postulate, sourceless gravitational fields do not exist because

⁵⁴There are also other possibilities such as systems having determinate values, but not emitting a gravitational field of their own. We refrain from elaborating postulates regarding this because they may be too radical. The most plausible possibility is in the case of observables, such as the spin projection. Although radical, this is still conceivable. A contextualist postulate would claim that systems that have determinate values of those observables, may evolve under a gravitational field as quantum systems, but do not emit a gravitational field.

every gravitational field is sourced by some quantum matter field. Furthermore, quantum matter fields sourced by SDCs not only give rise to a gravitational field but also to dark energy. In the absence of these sources, spacetime is flat in the sense of being described exactly by the Minkowski metric η with the derivatives of this metric being zero. For instance, when a cosmological constant $\Lambda \neq 0$ is present, the vacuum ($T_{\mu\nu} = 0$) exterior solution around a spherically symmetric mass M is the Schwarzschild–de Sitter metric:

$$ds^2 = - \left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3} \right) dt^2 + \left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3} \right)^{-1} dr^2 + r^2 d\Omega^2. \quad (54)$$

This solution includes both parameters: M (the “mass”) and Λ . They appear as separate ingredients in the metric. From this perspective, the sourcing of a gravitational field by a system with mass M would always be accompanied by dark energy. When there is no matter sourcing that field, $\Lambda = 0$. The appearance of the Λ independently of a source in the Einstein Field Equations would be an idealization. Arguably, there are always some SDCs somewhere giving rise to a gravitational field and dark energy, and the persistence of Λ would be justified in this way. Furthermore, the value of Λ would be simply a brute fact (i.e., unexplainable).⁵⁵

Note that, in this view, dark energy does not originate from vacuum fluctuations. It only comes from quantum matter fields connected to SDCs in such a way that they emit a gravitational field. An alternative view to this one, which has given rise to many problems (leading to the so-called cosmological constant problem), is that the vacuum just happens to have an inherent energy-momentum that gravitates, and thus, the vacuum energy should explain this constant. However, given the above postulate, this theory can reject this view

⁵⁵Briefly, Postulate 3 (version 1) and, as we will see, version 3, can be read as establishing that the dynamics of matter fields in the absence of a gravitational field, obey the Poincaré symmetries. In this sense, these postulates can be used to help explain the following “two miracles” of general relativity [95, 96]:

“**MR1:** All non-gravitational interactions are locally governed by Poincaré-invariant dynamical laws.

MR2: The Poincaré symmetries of the dynamical laws governing non-gravitational fields in the neighbourhood of any point $p \in M$ coincide (in the regime in which terms representing ‘tidal gravitational forces’ can be ignored) with the symmetries of the metric field in that neighbourhood.”

MR1 is the case because (given the postulates above) by default, the dynamics of matter fields in the absence of a gravitational field is given by Poincaré invariant laws, and all non-gravitational interactions locally are governed by laws that approximate this universal feature, i.e., laws that are at least approximately Poincaré invariant. This universal feature should be the case for all fields, where these fields in interactions give rise to a gravitational field, and this process is represented via the semiclassical equations. It should be unsurprising that the dynamical laws of these fields in the neighborhood of any point $p \in M$ coincide (in the regime in which terms representing ‘tidal gravitational forces’ can be ignored) with the symmetries of the metric field in that neighborhood because of this emergentist process, leading to gravity, due to quantum systems. This helps justify MR2. So, adopting this theory allows for philosophical positions concerning spacetime where these miracles do not arise.

by considering that systems in the vacuum are not interacting with SDCs. We will explain in Section 7 this idea in more detail. There is another version of Postulate 3,

Postulate 3 (version 2) Dark energy is the default gravitational field of the universe in the absence of matter.

In this view, the gravitational field determined by the cosmological constant is the default gravitational field in the universe and not flat spacetime, contrary to version 1 of Postulate 3. Thus, in the absence of matter, we would have

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -g_{\mu\nu}\Lambda, \quad (55)$$

$$\text{or equivalently in 4D, } R_{\mu\nu} = \Lambda g_{\mu\nu}, \quad R = 4\Lambda. \quad (56)$$

Note that in this view, similar to the first version of this postulate, dark energy also does not originate from vacuum field fluctuations. The gravitational field is a self-standing entity with default gravitational field values that are independent of quantum fields. Similar to the previous version, the value of Λ would also be a brute fact, contrary to the alternative postulate in Section 7.

Both postulates are, at least in principle, testable via the study of the gravitational field emitted by members of SDCs. If the gravitational field emitted by SDCs involves dark energy effects, this would be evidence for Postulate 3 (version 1). However, in our view, these postulates are not completely satisfactory because they leave the precise nature of dark energy unanswered. Postulate 3 (version 3) in Section 7 provides an answer with further consequences. Owing to its simplicity, which may help explain the origin of the cosmological constant, in this paper, we will favor Postulate 3 (version 1) and, as we will see, the related version 3.

5 SDCs in curved spacetime

To see how SDCs work in a simple curved spacetime, we will consider an example of SDCs in a flat de Sitter spacetime, which is defined by the metric,

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2 = a^2(\eta) (-d\eta^2 + d\mathbf{x}^2), \quad (57)$$

where H is constant and $a(t) = e^{Ht}$, t is the cosmological time, and η is the conformal time, which satisfies $dt = a d\eta$ with $\eta = -H^{-1}e^{-Ht} = -1/(aH)$, where $-\infty < \eta < 0$ when $-\infty < t < \infty$. So, the scale factor in conformal time is $a(\eta) = -1/(H\eta)$ and we have that late times correspond to $\eta \rightarrow 0$.

The action for scalar fields in a de Sitter spacetime is given by

$$S = - \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + \mathcal{V}_m + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\sigma, \phi) \right], \quad (58)$$

In the expression above,⁵⁶ $\mathcal{V}_m$ represents the system or systems whose energy-momentum dominates the spacetime region under study, drives its background geometry, and belongs to SDCs. We may take $\mathcal{V}_m$ to be a cosmological *constant*, but that is sourced by some system. This system could be the inflaton field; however, this does not necessarily have to be the case, and it could be some other form of radiation or matter as well. As we will see, these other fields can give rise to an inflation-like effect by sourcing a kind of time-varying dark energy that is dominant in the early universe (see Section 7 and Appendix I).

For simplicity, we assume that the same system $\mathcal{V}_m$ is both sourcing the test function and gravity, via a mode $\mathbf{k} = 0$ that is in a coherent state, which is a homogeneous and isotropic state (see Section 3.2.1 for more details). We could further interpret $\mathcal{V}_m$ as a field whose $\mathbf{k} = 0$ mode, and other modes, are interacting with SDCs through another field that we ignore for simplicity, or $\mathbf{k} = 0$ is one of the modes of the initiator mentioned in Section 3.1, which is the first system starting the SDCs with no predecessor. Multiple modes of this initiator system would exist at different scales.⁵⁷ In this perspective, the inflaton would be the first system starting the SDCs, transmitting the DC to other systems, and being active for a short amount of time, until it reaches the bottom of its potential $V(\mathbf{x}, t)$. In the typical way of understanding this scenario, a system S would be the homogeneous part $\phi(t)$ of the inflaton field, and $\phi(\mathbf{x}, t)$ could be a non-homogeneous part of the inflaton $\delta\hat{\phi}(\mathbf{x}, t)$, where the inflaton field would be split between the homogeneous and the non-homogeneous part, $\hat{\phi}(\mathbf{x}, t) = \phi(t) + \delta\hat{\phi}(\mathbf{x}, t)$. Then, ϕ has the DC concerning σ (not part of the inflaton field), and then σ could continue propagating the DC to other systems. These fluctuations have ultimately important empirical consequences, explaining the temperature anisotropies in the CMB and the seeds for structure formation (galaxies, clusters, etc.).

However, we will consider instead a more abstract case where the background gravitational field is due to a $\mathbf{k} = 0$ mode of another real scalar field ψ in a coherent state, which we can consider to be in a high occupation number so that it is a reliable semiclassical state. It is also due to some $\mathbf{k} \neq 0$ in a Bunch-Davies vacuum state approximately with a small backreaction that we choose to ignore. We assume that the timescale over which the modes of ψ change is much slower than that of the other fields under analysis, such that we can treat the field emitted by ψ as approximately constant. We assume that this field will source the de Sitter spacetime at least effectively. In de Sitter spacetime, the Hamiltonian corresponding to the real scalar fields ϕ and σ , whose interactions

⁵⁶In the expression above, we have also omitted the multiplication of the Lagrangian by a test function to obtain a generalized Lagrangian. In a more rigorous approach based on perturbative Algebraic Quantum Field Theory, we would need to consider this object. More on this below.

⁵⁷As we have said, we will consider that SDCs, and thus gravity, do not operate at the Planck scale. Thus, initiators at these scales will not exist.

will be localized in time by ψ , is given by

$$H = \int d^3x \sqrt{\gamma} \left[\frac{1}{2} \pi_\sigma^2 + \frac{1}{2} \gamma^{ij} \partial_i \sigma \partial_j \sigma + \frac{1}{2} \pi_\phi^2 + \frac{1}{2} \gamma^{ij} \partial_i \phi \partial_j \phi + \frac{1}{2} (m_{\text{env}}^2 + \xi_\phi R) \phi^2 + \frac{1}{2} (m_{\text{sys}}^2 + \xi_\sigma R) \sigma^2 + H_{\text{int}}(\sigma, \phi) \right], \quad (59)$$

where we have the spatial volume element $\sqrt{\gamma} = a^3$, ξ is the scalar curvature coupling constant, and m is the mass. The conjugate momenta are defined as $\pi_\sigma := \dot{\sigma}$ and $\pi_\phi := \dot{\phi}$, with the dot representing differentiation with respect to cosmic time.

We consider the following potential and interaction terms:

$$V = \frac{1}{2} m_{\text{env}}^2 \phi^2 + \frac{1}{2} m_{\text{sys}}^2 \sigma^2 + H_{\text{int}}(\sigma, \phi), \quad (60)$$

where the interaction Hamiltonian that we focus on is of the form

$$H_{\text{int}}(t, \mathbf{x}) = \mathcal{O}(t, \mathbf{x}) \sigma(t, \mathbf{x}), \quad (61)$$

and where $\sigma(t, \mathbf{x})$ is the operator that acts on the system's Hilbert space and $\phi(t, \mathbf{x})$ acts on the environment's Hilbert space. We consider both quadratic $\mathcal{O}_{\text{mix}} = \mu^2 f(t) \phi(t, \mathbf{x})$ and cubic interactions $\mathcal{O}_c = g f(t) \phi(t, \mathbf{x})^2$, where $f(t)$ is a Gaussian temporal test function emitted by the $\mathbf{k} = 0$ mode of the background field ψ , which is in a homogeneous and isotropic state.

Within the interaction picture, the evolution of the density operator $\rho_I(t)$ for the scalar fields is governed by the Liouville equation,

$$\partial_t \rho_I = -i [H_{\text{int}}(t), \rho_I], \quad (62)$$

where $H_{\text{int}}(t)$ denotes the interaction picture Hamilton. We are interested in the reduced density matrix $\varrho(t)$ obtained by tracing out the environmental degrees of freedom,

$$\varrho(t) := \text{Tr}_\phi[\rho_I(t)]. \quad (63)$$

The analysis of decoherence is given by the purity, $\gamma(t)$, defined as

$$\gamma(t) := \text{Tr}_\sigma[\varrho^2(t)], \quad (64)$$

where $0 \leq \gamma \leq 1$, and a state is pure if and only if $\gamma = 1$. Decoherence occurs when we end up quasi-irreversibly with a state with minimal purity under interactions (Section 3.1). To analyze the decoherence in de Sitter spacetime, we need to analyze decoherence at late times, which is when complete decoherence occurs, and systems σ and ϕ are left in a state where they emit a gravitational field.

More concretely, before exiting the horizon, the system's mode functions oscillate rapidly; thus, when we integrate them over past times when performing

perturbation theory to calculate the purity of the target system, those oscillations largely cancel, giving a small, bounded decoherence rate. After the horizon exit, these oscillations approach a nearly constant “frozen” real value. This leads to the decoherence of the super-Horizon (IR) modes of the system. Late-time calculations based on perturbation theory encounter the so-called problem of secular growth. Every extra interaction vertex in the perturbative expansion adds an integral over the past cosmic time and, once a mode has crossed the Hubble radius, its mode functions stop oscillating and start growing in such a way that invalidates the perturbative assumptions. Regardless of the strength of the coupling, waiting sufficiently long makes contributions from all perturbative orders comparable; thus, the truncated expansion loses predictability.

Open Effective Field Theory (EFT) methods address this by starting with the so-called Nakajima-Zwanzig Equation. More concretely, one starts from the Liouville equation for the system and environment and projects it onto the system. Because the Liouville equation is linear, the environmental part can be integrated, leading to a master equation for the reduced state. If the environmental correlator decays on the Hubble timescale, this equation leads to a local Lindblad equation that describes the evolution of the reduced density matrix. For Gaussian states, the Lindblad evolution leads to two simple first-order differential equations whose solutions remain accurate at arbitrarily late times, thereby providing reliable information on quantities such as purity long after the standard perturbation theory method has broken down (see Appendix H for some mathematical details of the calculations).⁵⁸

We analyze a linear $\sigma\phi$ and a cubic interaction $\sigma\phi^2$, where the system σ and the environment ϕ start in the Bunch-Davies vacuum. We again treat the target system as a collection of discrete modes, but to make inferences about the continuum of modes of the system, and we focus on a single mode of the system to make those inferences. On the other hand, the environment is treated as a large/continuous collection of modes. This environment [12] decoheres the mode of σ at the super-horizon scales, leading the target system mode $\mathbf{k}$ to be in a mixture of field amplitude states $|\sigma\rangle$,

$$\begin{aligned} \varrho_{\mathbf{k}}(t) = & \frac{1}{\pi} \int_{\mathbb{C}} d^2\sigma \left(A_{\mathbf{k}}(t) + A_{\mathbf{k}}^*(t) - B_{\mathbf{k}}(t) - B_{\mathbf{k}}^*(t) \right) \\ & \times \exp \left[- (A_{\mathbf{k}}(t) + A_{\mathbf{k}}^*(t) - B_{\mathbf{k}}(t) - B_{\mathbf{k}}^*(t)) |\sigma|^2 \right] |\sigma\rangle\langle\sigma|, \end{aligned} \quad (65)$$

where $A_{\mathbf{k}} + A_{\mathbf{k}}^* - B_{\mathbf{k}} - B_{\mathbf{k}}^*$ is a fixed point of the late-time evolution. However, given that the environment with its continuum of modes can be considered as a large reservoir that is not disturbed significantly by the single-mode system and

⁵⁸Consider a family of linear maps $\{\mathcal{E}(t_1, t_2)\}$, valid for $t_2 \geq t_1 \geq t_0$, that are trace-preserving and describe the time evolution of a system’s state $\hat{\rho}_S$ such that $\hat{\rho}_S(t_2) = \mathcal{E}(t_1, t_2)\hat{\rho}_S(t_1)$. This collection of maps is considered Markovian if it satisfies the semigroup composition rule $\mathcal{E}(t_0, t_2) = \mathcal{E}(t_1, t_2)\mathcal{E}(t_0, t_1)$ for all $t_2 \geq t_1 \geq t_0$, and if each map $\mathcal{E}(t_1, t_2)$ is completely positive, meaning it transforms positive density operators into other positive density operators for all $t_2 \geq t_1$. Although determining whether an evolution is Markovian is generally challenging, for Gaussian states, which we are examining, this task becomes tractable.

the Born approximation, we assume that the environment stays in the vacuum approximately, where, given the determination conditions, at late times, both systems and the environment have determinate values. Only then do they emit a gravitational field.⁵⁹ It can be shown (see Appendix H) that, at least in the linear and cubic Hamiltonians, this leads the target system with its different modes at late times to be in a Hadamard state $\rho(t) = \rho_{\text{IR}}(t) \otimes \rho_{\text{UV}}^{\text{BD}}(t)$ at least approximately, where $\rho_{\text{UV}}^{\text{BD}}(t)$ are the UV modes that will be in the Bunch-Davies vacuum and the different modes in the $\rho_{\text{IR}}(t)$ that are in the state (214), which should then be represented as a continuum of modes. Thus, because it is a Hadamard state, the target system has a finite renormalizable stress-energy tensor, which can then be fed into the semiclassical equations to yield a solution to those equations. The stress-energy tensor of the environment is also fed into the semiclassical equations, and its state is assumed to remain approximately in the vacuum. It is also fed into these equations the one of the system that initially emits the background gravitational field. As mentioned previously, the latter system is in a coherent state by assumption. So, all of these states are Hadamard. Furthermore, the states involved are all homogeneous and isotropic states.⁶⁰ Note that here we are treating the target system of decoherence as being an ensemble of systems (more on this below).

Therefore, the states of the systems involved being Hadamard, and the fact that we are working in a maximally symmetric spacetime, i.e., a de Sitter spacetime, allows the semiclassical equations to be solved more easily. Indeed, it was shown in [73] that for homogeneous and isotropic quasi-free fourth-order adiabatic states (which include Hadamard states) and instantaneous vacuum states, the semiclassical Einstein equation in flat cosmological spacetimes involving a massive scalar field with arbitrary coupling to the scalar curvature has unique solutions. This can involve multiple fields sourcing the gravitational field. Importantly, given the shape of test functions (which tend to have small tails) and the stochastic decohering process that affects the interacting systems, we can treat these systems, when the stochastic process occurs, as free/non-interacting fields, as well as the emitter of the test function.⁶¹ Note that we see here an important role of SDCs, which is to lead to states and conditions that one can use to solve the semiclassical equations, i.e., via Hadamard, homogeneous, and isotropic quasi-free states in the case of this scenario, which in principle allow one to solve this equation (see Postulate 2 in Section 4.2). Note again that above we have assumed each mode of σ is decohered by a continuum of modes of ψ , where there are so many decohered modes of σ that we can treat them as a continuum of decohered modes, modeled by decoherence models. Then, we can use the states obtained via the (decohered) modes to solve the semiclassical equations for a flat cosmological spacetime and find the gravitational field emitted by these members of SDCs.

⁵⁹Note that ϕ could have had some determinate values before this interaction, which allow it to have the DC concerning modes of σ .

⁶⁰See Appendix H for a proof concerning the state (214) leading to an homogeneous and isotropic for the whole system involving multiple modes.

⁶¹See Section 3.2.1 for more on this.

As we have been arguing, SDCs involve scale-dependent phenomena; here the system emitting a gravitational field, besides localizing interactions in time and filtering out some modes, allows for the scale-dependent phenomenon of decoherence at super-Horizon scales. To solve the semiclassical equation, we will need to renormalize the stress-energy tensor. Among other goals, in this view, renormalization aims to establish the scales that SDCs probe in a physically unproblematic way, dealing with UV divergences and other problematic issues. Furthermore, it allows us to infer how the couplings, masses, etc. change with the scales of SDCs (i.e., the scaling phenomenon). Indeed, in the renormalization techniques of perturbative Algebraic Quantum Field Theory and Causal Perturbation Theory (see [32], [97, 10] and references therein), test functions and operator-valued distributions play a crucial role in implementing cutoffs and in the process of renormalization. In this approach, renormalization is required to deal with ambiguities arising from the distributional features of expressions involving quantum fields. The scaling behavior that we observe in more standard renormalization group approaches such as Wilson's, where the values of couplings, masses, etc. change with scale, can also be observed here.⁶² According to the theory that we are proposing, the above ambiguities represent how SDCs change with scales. Since this theory is so far the only approach to quantum theory that makes these functions its core feature, we see the important role of these objects in dealing with both UV and IR divergences in a rigorous way as evidence for it. Note that we do not get rid of test functions in the process of renormalization according to perturbative AQFT, contrary to other approaches to renormalization.⁶³ Future work should further connect this framework with the one proposed here.⁶⁴

So, taking into account ψ , ϕ and σ , through renormalization we focus on the stress-energy tensor for certain modes/scales of ϕ and σ . Note that we consider that the cutoff in the modes of ψ , which emits the test function obeying the appropriate bounds, comes as a primitive fact if ψ is an initiator (given by some initial conditions of the universe), or comes from the previous interactions of ψ

⁶²Contrary to Wilson's, regularization, the problematic subtraction of infinity quantities, and the complicated process of coarse-graining is unnecessary.

⁶³Rather, we impose the algebraic adiabatic limit [10, 97], which assigns an unproblematic role to these functions. This is also important to deal with IR divergences. So, it shows that test functions are an integral part of theory. Moreover, note that to deal with divergences that arise from sharp cutoffs, we need smooth test functions f . For this theory, this smoothness gains an extra role of helping propagate the determination capacity.

⁶⁴Test functions are prevalent in other aspects of QFT. The use of test functions is closely related to the point-splitting technique. To the best of our knowledge, this was first observed by deWitt [25]. Furthermore, instead of adiabatic states, we can use the feature that the renormalized energy density, when smeared along a time-like curve using a point-splitting procedure, is bounded below as a function of the state. We can then find a state that minimizes this quantity [34]. Adapting this result for a test function supported on the worldline of an isotropic observer, and which minimizes each mode's contribution to the smeared energy density, the so-called low-energy states [77] were found. It was also found that they are Hadamard, and that they end up converging to the Bunch-Davies vacuum in an appropriate limit involving the support for the test function. These states make manifest the role of test functions and SDCs to establish the (Hadamard) states that we adopt, and how they are implicitly present in the semiclassical equation.

with members of an SDC (see Sections 3.2.1, 3.2.2, and Appendix E). Thus, we would have the following renormalized stress-energy tensors,

$$G_{ab} + \Lambda g_{ab} = 8\pi G(\langle T_{ab} \rangle_{\rho_\psi} + \langle T_{ab} \rangle_{\rho_{\psi'}} + \langle T_{ab} \rangle_{\rho_\sigma} + \langle T_{ab} \rangle_{\rho_\phi}). \quad (66)$$

To solve the semiclassical equation and renormalize it, we also need to solve the Klein-Gordon equations for the two-point correlation $\langle \phi(f(x))\phi(g(x')) \rangle$ of the systems involved in emitting the gravitational field while they emit that field.⁶⁵ Also, to renormalize the stress-energy tensor, we will need to perform an adiabatic subtraction, which is going to depend on the Hadamard parametrix for the Klein-Gordon equation at length scale ℓ .⁶⁶ Note that the dependence of the semiclassical equation on two spacetime points may appear to be nonlocal. However, these are effectively very close points, which can be considered to represent a single spacetime region. Even rejecting this, as in the Bell scenario case, this non-locality does not imply action at a distance between these spacetime regions.

So, in this section, we have provided an example of the beginning of an SDC in curved spacetime, where this SDC could further develop (ϕ and σ could be interacting with other fields while they interact with each other), propagating the DC. Furthermore, even if systems decohere outside the horizon, they can still reenter the horizon, interact with other systems, and also propagate the DC. Note, however, that the state (214) represents an ensemble of systems that give rise to a homogeneous and isotropic spacetime. It does not represent the state of a single system right after the stochastic process. Such a state could be represented as a Gaussian state around a field amplitude value σ_0 and with a low variance,

$$\begin{aligned} \varrho_{\mathbf{k}}^{\sigma_0, \mathbf{k}}(t) = & \frac{1}{\pi} \int_{\mathbb{C}} d^2\sigma \left(A_{\mathbf{k}}(t) + A_{\mathbf{k}}^*(t) - B_{\mathbf{k}}(t) - B_{\mathbf{k}}^*(t) \right) \\ & \times \exp \left[- \left(A_{\mathbf{k}}(t) + A_{\mathbf{k}}^*(t) - B_{\mathbf{k}}(t) - B_{\mathbf{k}}^*(t) \right) |\sigma - \sigma_{0, \mathbf{k}}|^2 \right] |\sigma\rangle_{\mathbf{k}} \langle \sigma|. \end{aligned} \quad (67)$$

This state is Hadamard (Appendix H), but is not a homogeneous and isotropic state, or leads to one.⁶⁷

In a more complicated model than the one above, if we assumed that the fields involved belong to the inflaton field, we could assume that their influence as initiators weakens owing to reaching the bottom of their potential $V(x)$ (see above). However, as we have mentioned we do not have to consider that they

⁶⁵We have added another field ψ' to source $g(x')$.

⁶⁶This is given by $H_\ell(x, x') := \frac{1}{8\pi^2} \left[\frac{\Delta^{1/2}(x, x')}{\sigma_\epsilon(x, x')} + v(x, x') \ln \left(\frac{\sigma_\epsilon(x, x')}{\ell^2} \right) \right]$. Δ is the van Vleck-Morette determinant; v is a symmetric, smooth coefficient that is determined by the Hadamard recursion relations. The subindex ϵ in σ_ϵ is an appropriate distributional regularization for the Synge world-function.

⁶⁷Future work should look at whether these states can help account for the inhomogeneities that explain the origin of cosmic structure, as well as the empirical signatures that arise from them. This could be done along the lines of what has been done with spontaneous collapse theories [87].

are initiators; SDCs may not have an initiator, and we can alternatively have SDCs that continue indefinitely. Even the field that is initially emitting the gravitational field could be interacting with other systems in that spacetime region, which we chose to ignore, and not be simply the initiator. We will come back to how we can dispense with the inflaton field further below in Section 7 and Appendix I.⁶⁸

One important aspect of the above model, and the approach we are proposing, is that the system that emits the test function, the background gravitational field, and decohering interactions drives or controls systems (that are not emitting a gravitational field) towards states that in principle can be used to solve the semiclassical equations at times t , when decoherence happens. Whether this quantum control approach, captured by Postulate 2 (Section 5) can be used in all situations of interest where gravity is manifested to help solve the semiclassical equations is a matter for future studies, but we conjecture that this will be the case.

6 Answering objections to the semiclassical approach

We will start by showing how this theory answers some of the main objections to the semiclassical theory of gravity. We will then explore some of its consequences. In the next section, we address another objection.

As we can see, according to this view, the gravitational field does not induce the collapse of the system's wavefunction. Only systems belonging to SDCs can do it. However, it has been argued that if gravity is not quantized and does not collapse the wavefunction, it can give rise to superluminal signaling, which contradicts relativity.

This argument was posed by Eppley and Hannah [31] and explained succinctly by Callender and Huggett [14]. Suppose the gravitational field is classical and adheres to relativistic principles. In this context, it is neither quantized nor subject to uncertainty principles, and does not permit superpositions of gravitational states that would introduce a quantum indeterminacy into the gravitational field.

For the sake of this discussion, we temporarily adopt the standard interpretation of quantum mechanics, where measurement interactions instantaneously collapse the wavefunction into an eigenstate of the measured observable. Next, let us investigate how this classical gravitational field interacts with a quantum system. According to Eppley and Hannah, there are only two possibilities: either gravitational interactions trigger quantum state collapse, or they do not.

According to the first horn of this dilemma, if gravitational interactions do not induce wavefunction collapse, then quantum states can transmit signals

⁶⁸Furthermore, note that given the theory that we are proposing, it is possible that system ψ is emitting a gravitational field in a subregion of the whole universe, where we assume that the rest of the universe, since it is not subject to SDCs, it is not at least yet participating in the emission of a gravitational field.

faster than the speed of light, going against the principles of relativity. Eppley and Hannah, propose multiple examples to highlight this issue. One of them involves a variant of Einstein’s thought experiment.

The key claim is that if gravitational interactions fail to collapse quantum states, then the interaction dynamics inherently depend on the wavefunction’s shape. For instance, the way a gravitational wave scatters off a quantum particle depends on its spatial distribution, akin to its interaction with a classical mass distribution. Scattering experiments with gravitational waves thus become a tool for probing the wavefunction’s properties, though they do not induce collapse. According to the authors, this assumption, along with the standard collapse postulate, leads to superluminal signaling.

To see this more concretely, suppose that we have a rectangular box containing a single quantum particle such as an electron. The particle is in a quantum state where it is equally probable to be found in either half of the box. A barrier divides the box, leading to a superposition of states where the particle is simultaneously localized in both left and right halves. The wavefunction in this case is given by

$$\psi(x) = \frac{1}{\sqrt{2}}(\psi_L(x) + \psi_R(x)), \quad (68)$$

where $\psi_L(x)$ and $\psi_R(x)$ represent the wavefunctions confined to the left and right regions, respectively.

Now, we distribute the boxes, carrying them to spatially separated locations without observing their contents and giving them to Alice and Bob. Assuming an instantaneous collapse interpretation, when Alice opens her box and finds it empty, this can immediately influence Bob’s box—even though the two boxes are spacelike separated. Assuming the collapse postulate, the wavefunction undergoes a stochastic transition upon measurement:

$$\frac{1}{\sqrt{2}}(\psi_L(x) + \psi_R(x)) \rightarrow \psi_R(x). \quad (69)$$

Now, let us consider the case where Bob employs a non-collapsing gravitational wave probe capable of interacting with the wavefunction in his box. Bob can do that by (idealizing) setting up apertures that permit gravitational waves to enter and exit the box and be detected.

Because the scattering depends on the form of the wave function in the box, any changes in the wave function will appear as changes in the scattering pattern registered by the detectors. Therefore, when Bob measures his system, a change in the gravitational wave will signal whether the particle is in the box or not, and this will instantaneously affect Alice’s box interior, enabling superluminal communication.

There are multiple issues with this experiment. Let us set aside the fact that, according to EnDQT, there would be no action at a distance in more realistic Bell scenario versions of this experiment [88]. Now, what sustains the idea that gravitational waves react to the wavefunction in the box? A way of modeling gravity classically, yet coupling it to quantum matter, is via the

weak-field (Newtonian) limit, where we derive the Poisson equation through the semiclassical equation,

$$\nabla^2 \Phi(\mathbf{r}, t) = 4\pi G \rho(\mathbf{r}, t). \quad (70)$$

$\Phi(\mathbf{r}, t)$ is the classical gravitational potential, and $\rho(\mathbf{r}, t)$ is the mass density, but now matter is described by a quantum wavefunction $\psi(\mathbf{r}, t)$ with

$$\rho(\mathbf{r}, t) = m |\psi(\mathbf{r}, t)|^2. \quad (71)$$

This means that the classical field Φ depends on the full spatial distribution of $|\psi|^2$. The potential and wavefunction in the Schrödinger–Newton equation obey the Poisson equation for Eppley and Hannah. Therefore, if there is a perturbation in the gravitational field, it should originate from the wavefunction. However, given Postulate 2, the semiclassical equation is only applicable if the target system interacts with SDCs. However, this is not the case in the scenario just described, as well as in the Bell scenario version of the experiment. We want to maintain the quantum coherence of the degrees of freedom of the systems under analysis, before interacting with the measurement devices of Alice and Bob, which involve matter degrees of freedom and SDCs; thus, we want to isolate them from SDCs.

In the case of this second horn, we suppose that gravitational interactions can collapse quantum states of matter, similar to gravitational collapse theories. More concretely, the idea is that if a gravitational wave of arbitrarily small momentum can be used to make a position measurement on a quantum particle (which “collapses” the wave function into a quantum state that concerns its position), the uncertainty principle is violated. This is because the momentum imparted to the particle by the wave would violate the uncertainty principle since it could be made arbitrarily small. We reject this horn as well because gravitational waves are not quantum matter field degrees of freedom and are not connected with SDCs (more on gravitational waves below). More concretely, to fundamentally justify the influence of gravitational waves on the particles as a probe, we would need to use the semiclassical equations, introducing the term for gravitational waves on the left-hand side of this equation. However, given Postulate 2, we can only apply these equations if the particle interacts with members of SDCs, which it does not before interacting with the measurement devices of Alice and Bob. Thus, we escape the difficulties concerning the violation of the Heisenberg uncertainty principle that arise from adopting the second horn. Therefore, the theory we propose does not require the adoption of the second horn of the dilemma.

It should be now be clear how this theory responds to Feynman and Aharonov’s thought experiment [39, 39, 4]. This thought experiment aims to show that gravity must be quantized; otherwise, the gravitational field emitted by a particle can be measured with arbitrary precision to determine the position of a particle in a double-slit experiment. Typically, the way around this scenario is to introduce some stochasticity in the coupling between the quantum degrees of freedom and the classical ones so that we do not gain information about the quantum system (and it does not collapse). However, there is another way to proceed,

which is the one adopted here. The idea is that because the quantum system that goes through the double slit does not interact with systems that belong to SDCs (because we want to maintain the system in a coherent superposition), it does not emit any gravitational field, and we cannot know where the particle is. Therefore, the response to this thought experiment is similar to that given to the above dilemma.

Other objections to the semiclassical approach were proposed by Page and Geilker [82]. The first one concerns the violation of the Bianchi identities (or more precisely the contracted second Bianchi identity) during the stochastic process (or quantum state collapse), where this identity is $\nabla_\mu G^{\mu\nu} = 0$, which implies $\nabla_\mu \langle T^{\mu\nu} \rangle = 0$. The formulation of this problem presupposes a literalistic view of the quantum state in which there is a dynamical state reduction during a measurement, which leads to this violation. However, in line with our non-literalistic (and more epistemic) view of the quantum state, a strategy to circumvent this is to hold that the semiclassical equations do not apply during the stochastic process that leads to outcomes. In other words, we are using the semiclassical equations to make inferences about the gravitational field of systems, and such inferences should not be made during this stochastic process. So, this is similar to the strategy that we are adopting, which many physicists also adopt, that considers that the Schrödinger equation is not applicable to describe the stochastic process and state update involved in measurements.⁶⁹ However, assuming this semiclassical theory, we will see in Section 7 that it is possible that $\nabla_\mu G^{\mu\nu} = 0$ and $\nabla_\mu \langle T^{\mu\nu} \rangle = 0$ hold during the stochastic process that gives rise to measurement outcomes. This will be due to fluctuations in the stress-energy tensor, which can be assumed to be maintained during this process.

The second objection is as follows: consider a mass that emits a gravitational field that exists in a superposition of two different localized states. If the gravitational field is classical but depends on the quantum wave function, the gravitational attraction generated by this system would be expected to be directed toward an intermediate, “averaged” position. Furthermore, the experimental work of Page and Geilker *has shown* that this predicted behavior does not occur. However, note that this view is based on the idea that an object in such a superposition would emit a gravitational field to an intermediate location. This is not what is expected by the theory we are proposing. Rather, what would be expected is that macroscopic systems that would form such superpositions would tend to collapse to one of the values typically associated with coherent states, and such states would serve as sources of a gravitational field, which would not be to an intermediate location.

A related objection is that semiclassical gravity and this theory are not capable of describing the Planck scale, where quantum gravity effects become strong. However, note that this assumes the quantum nature of gravity, which we deny. Also, it assumes that quantum gravity occurs at the Planck scale

⁶⁹Note that in our case, the state update would involve inserting new stress-energy tensors and states into the right hand-side of the semiclassical equations.

based on a dimensional analysis and assumptions regarding the fundamental constants, which are speculative and one can be skeptical about.⁷⁰ We have seen how this theory can address that scale dependence, which establishes in which scales gravity can occur via the scales in which SDCs evolve. Thus, this theory can clearly deny that gravity occurs at the Planck scale. It is an experimental matter whether this is the case.

Another objection is that semiclassical gravity cannot describe the interior of black holes and deal with the spacetime singularities appearing in GR, which should be the task of any theory of gravity. However, given that whatever is going on behind the event horizon is causally disconnected from the rest of spacetime, it is possible that the gravitational field emitted by systems decreases as we move towards the core of the black hole, i.e., there are no SDCs at its core to emit this field. More concretely, we would hypothetically have a rapid decrease in systems that would be decohered/distinguished by the black hole environment as we move closer to the core of a black hole, and this would be proportional to the decrease in the (determinate) mass $m(r)$ or energy density as we move closer to the core of the black hole. This would avoid a singularity, and we conjecture that it would lead to an asymptotically flat core or an asymptotically de Sitter core if the default state of spacetime is flat or de Sitter, respectively. So, to describe such black holes, we would use regular black holes (i.e., black holes devoid of singularities) with an asymptotically de Sitter [65] or Minkowski core (see, e.g., [112]) whose geometries (in our interpretation) are associated with the progressive absence of systems that emit a gravitational field. The decrease in the (determinate) mass that appears in the metrics of these black holes as we move towards their core would be interpreted in the above manner.

Models of decoherence involving black holes,⁷¹ have found that the rate of decoherence of charged systems in a coherent superposition increases the closer we are to the Killing horizon coming from outside the black hole.⁷² So, they suggest that there are regions where the activity of SDCs is at its peak towards and in the horizon of black holes, and regions where it decreases away from it. Note that just because we could have more decoherence at the event horizon, it does not mean that the gravitational field is stronger there; it also depends on the energy-momentum of the systems involved in the process of decoherence. However, for this theory, the absence of decoherence is directly related to the absence of a gravitational field. One now would have to investigate decoherence in the interior of the black holes above with appropriate matter

⁷⁰See [56] for further responses related to this objection.

⁷¹See, e.g., [22, 23]

⁷²This decoherence is due to very low frequency Hawking radiation [23]. The idea is to consider Alice's lab, where Alice conducts an experiment with a target system. The decoherence of this target system, which is a charged particle in a coherent superposition, is determined by $\langle N \rangle \sim \frac{M^3 g^2 d^2}{D^6} T$. $\langle N \rangle$ is the expected number of entangling photons, which leads to the decoherence of Alice's target system, where if $\langle N \rangle \gg 1$ this system will be completely decohered. M is the black hole mass, D is the proper distance of Alice's lab from the horizon, and T is the time in which Alice's target system is maintained in a superposition. The authors also found decoherence induced via gravitons in the perturbative quantum gravitational regimes, but according to this theory, gravitons do not exist.

fields. Furthermore, one may conjecture a lower bound in the four-volume that members of SDCs could form, similar to the following: $\Delta V \sim \frac{R_s^4}{c}$, where R_s is the Schwarzschild radius [68]. Below a four-volume like this one, inevitably, there is a gravitational collapse, and a black hole forms.⁷³⁷⁴ So, given this theory, it might be the case that the gravitational field in the interior of black holes is smooth, with much less gravitational fields being emitted as we move to its core. Therefore, it is unclear whether we need to appeal to quantum gravity theories to solve the black hole singularity problem. The semiclassical conservative theory proposed here potentially offers alternative solutions.

Related to the above objection, it is often claimed that the semiclassical approach has trouble describing black hole evaporation when the Schwarzschild radius is not large compared to the Planck scale [121]. However, it is not even clear that black holes evaporate when we examine the assumptions that go into these arguments regarding the global energy conservation of the energy-momentum tensor [18]. Thus, it is not necessarily the case that this is a real problem for the semiclassical approach. Even if black holes evaporate, given our current lack of understanding of these objects, more research is needed to see if it constitutes a problem for this semiclassical approach.⁷⁵

We will now examine some of the consequences of these postulates. One consequence is gravitational energy-momentum, as something that emits a gravitational field per se and affects systems independently of anything, does not exist because pure gravitational degrees of freedom do not source gravity according to this theory. Also, they do not affect systems unless they are interacting with members of SDCs. Thus, gravitational waves do not carry any energy-momentum. One can rather regard the pseudo-tensor or the radiative energy that appears in the equations representing gravitational waves as the maximal amount of work they can do via tidal effects [108]. However, these tidal effects are only classically felt by systems interacting with members of SDCs.

This consequence also supports the claim that, according to this theory, gravitons do not exist. Note that the above hypothesis does not imply that gravitational waves do not exist. Rather, it implies that they do not carry actual energy-momentum (or the kind of energy-momentum associated with gravity and matter fields via the Einstein Field Equations). This consequence of this theory should not be problematic because of the notorious issues involved in formulating a gravitational energy-momentum tensor, including the one for gravitational waves. The latter is rather a pseudo-tensor. See [51, 28] for a more complete

⁷³These conjectures were advanced in collaboration with Gerard Milburn, and future work will develop it.

⁷⁴Regular black holes suffer from mass inflation instabilities, but there are ways to circumvent them, e.g., [15].

⁷⁵Another alleged limitation of the semiclassical approach is that it is not able to describe the quantum fluctuations in the inflaton field [123]. However, these models are highly speculative and have their own problems. Furthermore, it is unclear whether this view cannot account for these fluctuations. Besides, we have seen in Section 5 and we will see in Appendix I that perhaps we can provide an alternative picture of inflation that does not appeal to such fluctuations, or that gets rid of the inflaton as traditionally conceived altogether.

defense of these positions.⁷⁶

Another consequence is that according to this theory there may be no default and autonomous gravitational field, and the gravitational field fully depends on matter fields. Without necessarily endorsing all the features of relationalism, this consequence can be further supported by a kind of relationalist view that would defend that matter degrees of freedom fully determine the gravitational field. Vacuum solutions to the Einstein Field Equations are regarded as idealizations; they do not exist in nature. For instance, the Schwarzschild external solution should be regarded as a solution taking into account the gravitational field sourced by a system that we idealize as a point mass.⁷⁷

7 The time-varying Λ

An issue that one might have with semiclassical gravity is that it is a mean-field theory, and therefore, it may not account for deviations in the expectation value of the stress-energy that may naturally occur. One option is to consider that these fluctuations do not gravitate. For some reason, the expectation value of the stress-energy tensor of systems is sufficient to determine the gravitational field sourced by them. Another way is to still defend the mean-field theory but additionally argue that systems only emit a gravitational field in certain states or contexts that minimize the values of the second and higher-order moments of the stress-energy tensor. However, even if one imposes this via the more contextual and restrictive Postulate 2 (version 2) one may argue that it does not completely eliminate the fluctuations of the stress-energy tensor. It turns out that a potential solution to this also potential problem is connected to dark energy and the cosmological constant problem. The idea is that those fluctuations that could contribute to gravitation are annihilated or balanced out in a sense to be specified below, and at least at some scales tend to give rise instead to a dark energy phenomenon. Another possible solution is to adopt a stochastic gravity approach [54], but such approach might not be needed as we will see.

Let us start by explaining the cosmological constant problem. We then show how this problem can be addressed in principle. The cosmological constant is

⁷⁶The interpretation of what is a gravitational wave and field gives rise to at least two distinct positions considering the ontology of this theory: a) the gravitational field emitted by SDCs affects how quantum matter fields evolve in spacetime. However, this influence of SDCs travels through spacetime via quantum matter fields because quantum matter fields are everywhere, and no determinate energy-momentum needs to be carried via gravitational waves. This view assumes that quantum systems are more fundamental than the gravitational field, assuming a kind of emergentist perspective in which matter fields give rise to spacetime and gravity. This can also support a kind of relationalism. More on this below. b) A different philosophical perspective on the ontology of this theory considers that the mathematical objects of general relativity also describe a classical gravitational field. In cases such as the propagation of gravitational waves, it amounts to changes in the values of the gravitational field throughout spacetime. Thus, this view considers that the gravitational field is as fundamental as quantum matter fields, assuming a kind of substantialist perspective.

⁷⁷See [109] for a recent nuanced relationalist account regarding the idealizations present in vacuum solutions.

used to describe the accelerated expansion of the universe. The cosmological constant problem appears when we work within a semiclassical framework, replacing the classical stress-energy tensor $T_{\mu\nu}$ with its quantum-field-theoretic vacuum expectation value $\langle T_{\mu\nu} \rangle$. Each field’s vacuum energy density then takes the form of a cosmological-constant term—a constant times the metric $g_{\mu\nu}$ —and it is claimed that it should contribute directly to the observed value of the cosmological constant. But, the standard QFT “prediction” for the combined vacuum energies overshoots the measured cosmological constant Λ by many dozens of orders of magnitude [123]. This problem can be framed as a *reductio ad absurdum* that arises when we treat General Relativity as a low-energy EFT [63].

However, according to the theory proposed here, general relativity arises from QFT under specific circumstances, but it is not a low-energy QFT. Thus, we should look elsewhere for a solution to this problem. Moreover, treating systems that are in a vacuum and in flat spacetime as gravitating according to the theory adopted here cannot be done because if such systems gravitated, it would not be in the vacuum.⁷⁸ More generally, one should not indiscriminately include systems in a given state in the stress-energy tensor of the semiclassical equation in any curved spacetime. One should only do that if we have good reasons to consider that those systems were locally decohered in some open environment (i.e., that those systems interacted with members of SDCs) and we have a realistic decoherence model that represents that process. There is a good case to be made that no realistic decoherence model favors a vacuum state. For instance, in flat spacetime, given the results from [30], we have observed that non-zero temperature environments, irrespective of the initial state, lead systems to a mixture of coherent states, which are not vacuum states. Furthermore, in realistic environments, there is not only decoherence but also diffusion, which drives the system out of the vacuum (e.g., [13, 53, 127, 105]). Even in cosmological contexts, as in the model in Section 5, the target system starts in the vacuum and then evolves into a mixed state, which, upon decohering interactions, causes the system to leave the vacuum. The environment is treated as staying in the vacuum, but this is an idealization because of its size and weak interactions. To date, there is no indication that this phenomenon of making the system leave the vacuum, represented via decoherence models, will change in future realistic models of decoherence. Considering decoherence models as good models to infer what we measure, we can hypothesize that what we realistically measure directly (when we infer the effects of the vacuum) are not quantum fields in the vacuum but rather quantum systems that were in the vacuum or that are very close to it upon measurements.

Thus, by adopting Postulate 2, we can deny that the vacuum emits a gravitational field based on models of decoherence. Therefore, the hypothesis above can be understood as showing that if there is something that the cosmological

⁷⁸Indeed, in Wald’s fourth axiom [121] (where this axiom belongs to a set of axioms that gives us a finite, well-defined, covariant, conserved, renormalizable stress-energy tensor) this tensor is set to zero in the Minkowski vacuum. Setting it to zero is equivalent to not gravitating. This is motivated by the equivalence principle.

constant problem points to, it is that we need to take into account whether systems are interacting with members of SDCs, being decohered by them, to consider whether they give rise to a gravitational field or not. Assuming the above hypothesis, we do not include the energy density of the vacuum in the semiclassical equations, and we can choose the value of the cosmological constant based on other features. Furthermore, if we consider that the cosmological constant is behind dark energy and assume the theory that we are proposing, the explanation for dark energy should only involve systems that belong to SDCs. Thus, we now pose an alternative Postulate 3 that we will justify further below:

Postulate 3 (version 3) When systems are not interacting with SDCs, they do not give rise to the relativistic four-volume, and furthermore, spacetime is flat in spacetime regions without SDCs. The relativistic four-volume is estimated by the number of events in a spacetime region involving quantum systems with determinate values of observables and emitting a gravitational field, which we will call *relativistic events*, and which constitutes what we will call *relativistic spacetime*. We estimate the volume by counting the number of these events because the four-volume of relativity arises from the interactions between systems that constitute SDCs.

The intuition behind Postulate 3 (version 3) is that, given Postulate 2 and its consequences, the full-blown notion of trajectories and four-volume from our familiar relativistic world requires systems with determinate values of observables. So, when we do not have SDCs in a spacetime region, we still have a spacetime given by a metric and a manifold. However, systems do not have determinate values or classical trajectories, and thus, the above notion of a determinate four-volume from relativity does not make sense. We crucially need matter fields with a determinate energy-momentum. A four-volume will only make sense in this emergent relativistic spacetime, where it depends on the number of relativistic events that occur.⁷⁹

To estimate the value of the cosmological constant (without invoking the vacuum energy), let us start by assuming that in a universe where SDCs had not yet formed, there were no systems with determinate energy-momentum and determinate values of any other dynamical observables. When SDCs began to form or develop in our early universe, systems with determinate energy-momentum arose, and relativistic spacetime began to expand.

The strong energy condition roughly says that gravity must be attractive.⁸⁰ Although sufficiently negative pressure violates the strong energy condition, negative energy densities of a certain magnitude over bounded spacetime regions are allowed by quantum theory, e.g., the Casimir effect (see the quantum energy inequalities in [35]). Furthermore, the cosmological constant, which we are

⁷⁹Notice that, like causal set theory, this theory assumes that the four-volume of spacetime depends on the number of events. More on this below.

⁸⁰More precisely, the strong energy condition postulates that for every timelike unit vector field v^μ , the trace of the stress-energy tensor ($T = T^\alpha_\alpha$) measured by observers is always non-negative: $(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})v^\mu v^\nu \geq 0$.

deriving, violates the strong energy condition [21]. As we have seen, according to this theory, the gravitational field is determined by quantum matter fields that belong to SDCs. Given this, let us consider that, in addition to the positive energy associated with gravitational attraction, SDCs produce negative energy and pressure, which could, for example, be measured in the comoving reference frame associated with cosmic time, i.e., the reference frame associated with hypothetical observers who are at rest relative to the expanding universe, which we will assume to be homogeneous and isotropic. Moreover, let us assume that there is a balance of energy density, pressure, and other quantities arising from the interactions that produce the gravitational field. The result of this balance is what we associate with dark energy and the expectation value of the stress-tensor.⁸¹

More concretely, if the cosmological constant has a quantum origin, it could arise as an expectation value of some observable, and it can be written as a stress-energy tensor as

$$\langle T_{\mu\nu}^{\Lambda} \rangle = -\frac{c^4}{8\pi G} \langle \Lambda \rangle g_{\mu\nu}. \quad (72)$$

Now, instead of this quantity, let us consider that on the right-hand side of this equation, we get a balance between positive and negative stress-energy tensors produced by systems and their respective uncertainties,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \left((\langle T_{\mu\nu}^{(\text{matt}')} \rangle \pm \Delta t_{\mu\nu}^{(\text{matt}')}) + (\langle T_{\mu\nu}^{\Lambda'} \rangle \pm \Delta t_{\mu\nu}^{\Lambda'}) \right) \quad (73)$$

where $\langle T_{\mu\nu}^{(\text{matt}')}\rangle$ can involve the stress-energy tensor of other, let us assume, non-interacting fields,

$$\langle T_{\mu\nu}^{(\text{matt}')}\rangle = \langle T_{\mu\nu}^{(\text{matt}1)}\rangle + \langle T_{\mu\nu}^{(\text{matt}2)}\rangle + \dots \quad (74)$$

Note that by positive/negative stress-energy tensor, we mean that all the pressures and energy densities of $\langle T_{\mu\nu}^{\text{matt}'/\Lambda'} \rangle \pm \Delta t_{\mu\nu}^{\text{matt}'/\Lambda'}$ measured in the comoving frame are positive/negative. Note also that $\Delta t_{\mu\nu}^{(\text{matt}')}$ and $\Delta t_{\mu\nu}^{\Lambda'}$ encapsulate all possible second and higher moments of these stress-energy tensors.⁸²

Now, let us hypothesize that, at least at a certain spatiotemporal scale, the balance between these quantities results in the following,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \left(\langle T_{\mu\nu}^{(\text{matt})} \rangle - t_{\mu\nu}^{\Lambda} \right) \quad (75)$$

⁸¹See [7] for another model that makes dark energy emerge from decohering interactions, and which may be related to this one.

⁸²Let us consider the simplest case of a perfect fluid in a comoving frame where $h_{\mu\nu} = g_{\mu\nu} + u_{\mu}u_{\nu}$, where u^{μ} is the unit four-velocity. A perfect fluid has no flux or anisotropic stress in this frame; therefore, $p_{\pm,1} = p_{\pm,2} = p_{\pm,3} \equiv p_{\pm}$. So, we have that $T_{\mu\nu}^{\text{tot}} = (\rho_{\text{tot}} \pm \Delta\rho_{\text{tot}}) u_{\mu}u_{\nu} + (p_{\text{tot}} \pm \Delta p_{\text{tot}}) h_{\mu\nu}$, with $\rho_{\text{tot}} = \rho_{+} + \rho_{-}$, $p_{\text{tot}} = p_{+} + p_{-}$, and $\Delta\rho_{\text{tot}} = \Delta\rho_{+} + \Delta\rho_{-}$, $\Delta p_{\text{tot}} = \Delta p_{+} + \Delta p_{-}$.

where $\langle T_{\mu\nu}^{(\text{matt})} \rangle$ is the ordinary stress-energy tensor of semiclassical gravity, and $t_{\mu\nu}^\Lambda$ is an uncertainty as we will see more clearly below.⁸³ We assume that we can write (75) as

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \left(\langle T_{\mu\nu}^{(\text{matt})} \rangle - \frac{\Delta\Lambda c^4}{8\pi G} g_{\mu\nu} \right), \quad (76)$$

Note that above we consider by convention that the expectation value associated with $t_{\mu\nu}^\Lambda$ is $\langle T_{\mu\nu}^\Lambda \rangle = 0$, where $\Delta\Lambda > 0$. The idea underlying this relation is that semiclassical gravity, as a mean field theory, holds its validity, not requiring us to consider the second and higher moments of the stress-energy tensor, via this balance that results in the emission of what we call a cosmological *constant* or dark energy. Dark energy is a manifestation of stress-energy fluctuations as we will see.⁸⁴

We hypothesize that (75) and (76) holds for all scales probed by the SDCs, which are all scales where observables have determinate values. We can also assume a weaker hypothesis that it only holds at the cosmological scales due to systems involved in local interactions. In either case, we can use the assumptions about the above scales or regimes to estimate the value Λ . If the first case is true, we can use the second case to estimate the value of Λ , which would hold at all scales (but would still evolve in time as we will see). Note that these assumptions may be unrealistic, and we need to change these hypotheses, making them more complex. For instance, $\Delta\Lambda$ could be scale-dependent or dependent on other features (e.g., features of the test functions, etc.), but we will simplify for now and assume this relation.

At this stage of research, there is some arbitrariness in what we consider to be (73), and how we go from the initial stress-energy tensors (73) to those in (76). What the initial ansätze (73) offers us is a way of justifying how the energy conditions emerge from a quantum balance that considers all possibilities of stress-energy tensors. The tuning of the sum $(\langle T_{\mu\nu}^{(\text{matt}')} \rangle \pm \Delta t_{\mu\nu}^{(\text{matt}')}) + (\langle T_{\mu\nu}^{\Lambda'} \rangle \pm \Delta t_{\mu\nu}^{\Lambda'})$ can yield quantities that satisfy or violate different energy conditions, and the changes in some postulated parameter or parameters w_i , which may be equal to a ratio involving the pressures and the energy densities, over the evolution of SDCs would influence this change. Instead of the above complex ansatz, we could also consider the simpler ansatz where $\langle T_{\mu\nu}^{(\text{matt}')} \rangle = \langle T_{\mu\nu}^{(\text{matt})} \rangle$ would obey the energy conditions for ordinary matter throughout the process of going from (73) to (76). The main point of these ansätze is that any possible fluctuations $\Delta t_{\mu\nu}^{(\text{matt})}$ in $\langle T_{\mu\nu}^{(\text{matt})} \rangle$, will be canceled by $c_{\mu\nu}$, which is a quantity coming from this balance, where the remainder will be $-t_{\mu\nu}^\Lambda$, and thus $-t_{\mu\nu}^\Lambda = c_{\mu\nu} + \Delta t_{\mu\nu}^{(\text{matt})}$. Furthermore, another assumption is that we can estimate $t_{\mu\nu}^{(\text{matt})}$ via $\frac{\Delta\Lambda c^4}{8\pi G} g_{\mu\nu}$. More on this below.

⁸³We may consider this quantity as being an uncertainty because it arises from other uncertainties. See below.

⁸⁴Our derivation of Λ could have started directly with (75) and (76) but this would be less explanatory.

As we shall see further below, these ansätze are, as far as we noticed, physically valid and open for experimental investigation because they yield predictions. Furthermore, they show how a semiclassical approach can answer the objections mentioned in the beginning of this section and in the last section. In addition, they can provide a potential solution to the cosmological constant problem without invoking other fields like other theories.

Coming back to the estimation of the value of Λ , given our interests and assumptions, the most plausible way to do it is by considering the effects of systems involved in local interactions to phenomena at the cosmological scales measured in the current epoch, and by considering a perfect fluid at the cosmological scales.

Notice that we are working with many systems that will give rise to effects visible at cosmological scales. These systems develop local interactions and belong to SDCs, giving rise to a large number of relativistic events. It is estimated that the baryon-to-photon ratio η is approximately 6×10^{-10} [2]. Given the much larger number of photons compared to baryons, a conservative assumption is to consider that the majority of the number of spacetime events throughout relativistic spacetime that arise via SDCs involves photons, or more broadly bosons, assuming that there is no other influential matter that we have not detected so far.

It is reasonable to consider that many modes of bosons that constitute bosonic fields, which give rise to effects visible at cosmological scales, are typically in a coherent state $|\alpha\rangle$. This is because models of decoherence, which represent interactions between systems that belong to SDCs, consistently consider that such states are the ones selected by interactions between bosons and a wide range of kinds of environments (see Section 3.2.2, also see e.g., [126] and [30]). Furthermore, given the large number of events that give rise to effects at the cosmological scales, it is reasonable to consider that modes of these fields are in a high mean occupation number coherent state. In addition, a Bose-Einstein condensate at high occupation numbers can be approximated as being in a coherent state. Moreover, these states minimize the fluctuations of the stress-energy tensor (see [64] and Section 4.2), showing more clearly how the semiclassical equations, approximates the classical regime that we observe at cosmological scales.

Now, estimating the value of the cosmological constant will involve relating the uncertainty of the four-volume of spacetime that SDCs give rise to in a past lightcone in the current epoch with the uncertainty $\Delta\Lambda$. Let us consider that $\hat{V}$ is an observable that represents the total relativistic four-volume that systems that belong to SDCs gave rise to, where this four-volume is in the past light cone of a spacetime point along the cosmic time. The possible determinate values of $\hat{V}$ are the different possible four-volumes that could be generated by the SDCs in the past light cone of one of its events. Then, let us consider the following uncertainty relation inspired by unimodular gravity [29, 128], and assumed in causal set theory [24, 5],

$$\frac{\Delta\Lambda}{8\pi G} \Delta V \geq \frac{\hbar}{2}. \quad (77)$$

Since we want to estimate the above uncertainty and we are working with systems in a coherent state, we can saturate it to estimate this value, and so

$$\Delta\Lambda \approx \frac{4\pi G\hbar}{\Delta V}. \quad (78)$$

Given (78), let us then estimate the uncertainty of the four-volume ΔV of the relativistic spacetime, which SDCs give rise to, in the past light cone of the current cosmic time. It might seem a bit odd to estimate the uncertainty of something that already happened (i.e., retrodict the past four-volume of the universe), but note that it becomes more plausible if we consider that the dynamics are often fundamentally indeterministic and could be otherwise, and therefore, the four-volume could be otherwise. Furthermore, we can use this to make certain predictions, as we will see. To estimate ΔV , we use Postulate 3 (version 3), which considers that the four-volume of relativistic spacetime depends on the number of relativistic events in that volume.

Relativistic spacetime and classical physics are typically concerned with particles occupying a determinate position and velocity in spacetime. However, particle number observable (i.e., the number of particles occupying a certain mode) seems to be a more appropriate observable to analyze how classicality arises from quantum field theory, and to help estimate the number of events that arise in relativistic spacetime, since position is not an observable in QFT. Thus, we assumed this. Another way to argue for this is that, fundamentally, our classical relativistic world seems to be constituted by particle-like systems; thus, particle number observables seem to be the most appropriate observables for the task of counting events. Therefore, counting particles in spacetime regions appears to play a relatively more fundamental role. A system in a coherent state has a specific quantum uncertainty in its particle number. Such a system, in interaction with other systems that probe its particle number, gives rise to particle numbers that are Poisson distributed. More precisely, the probability of the number n of particles in a single mode is

$$P(n) = |\langle n|\alpha\rangle|^2 = e^{-\langle n\rangle} \frac{\langle n\rangle^n}{n!}, \quad (79)$$

which obeys the Poisson distribution, where the average number of bosons occupying a single mode and their variance are equal to each other,

$$\langle n\rangle = \langle \hat{a}^\dagger \hat{a}\rangle = \text{Var}(n) = (\Delta n)^2. \quad (80)$$

Moreover, even for fermions, the sum of many independent modes leads to a standard deviation of the occupation number for a mode of approximately $\sqrt{\langle n\rangle}$. This further justifies the use of the Poisson distribution to estimate the number of events that arise in the relativistic four-volume.

Because our past-light-cone four-volume is the union of many subvolumes, it is reasonable to assume that event correlations between this subvolumes are negligible beyond short distances. Moreover, let us also make the plausible assumption that in the context under study, which involves phenomena visible at

cosmological scales, we have many events, and modes in a coherent state with a high occupation number, such that quantum fluctuations are small enough that we can consider the density of events (i.e., the number of events per four-volume) ρ arising from SDCs to be approximately constant. The constancy in space of ρ can also be justified because we are reconstructing a homogeneous and isotropic spacetime via SDCs.

Given the constancy of ρ , Postulate 3, our previous assumptions, and that the sum of independent Poisson processes (occurring across the different subvolumes of V) is another Poisson process we consider that $\rho = \langle N \rangle / V$ with $\langle N \rangle$ being the expectation value of the number of spacetime events in region V and the Poisson-distributed standard deviation $\Delta N = \sqrt{V\rho}$, where $\langle N \rangle = \sum_i \langle n_i \rangle$ with $\langle n_i \rangle$ being the expectation values of the particle number of modes across the different four-subvolumes of the universe in the past lightcone of an event.

In the limit of large $\langle N \rangle$, we have $N = \rho V \pm \sqrt{\langle N \rangle}$. To estimate the volume V that SDCs give rise to, we invert the above formula to consider $V = \frac{1}{\rho}(\langle N \rangle \pm \sqrt{\langle N \rangle})$. Therefore, we can estimate the uncertainty of the four-volume that SDCs give rise to as being equal to

$$\Delta V = \frac{\sqrt{\langle N \rangle}}{\rho}. \quad (81)$$

Given (78), we obtain

$$\Delta \Lambda \approx \frac{\hbar 4\pi G \sqrt{\rho}}{\sqrt{V}}. \quad (82)$$

Then, we can estimate the value of the cosmological constant by performing a dimensional analysis, assuming Planck units $\hbar = G = c = 1$, and observing that in an FLRW cosmology, V should be of the order H^{-4} where H is the Hubble parameter at the current epoch. We then obtain $\Delta \Lambda = 4\pi\sqrt{\rho}H^2$, where $H^2 = 10^{-122}$ is the magnitude of the cosmological constant in Planck units.⁸⁵

Thus, we have derived the value of the cosmological constant via a semi-classical approach, without falling into the cosmological constant problem and invoking speculative notions from quantum gravity. Note that we may use the observed value of the cosmological constant to help estimate ρ . We should clarify that the four-volume of the relativistic spacetime has nothing to do with a fundamental discretization of spacetime. That is why we distinguish between spacetime and relativistic spacetime. The fundamental spacetime is endowed with a metric and is continuous; the relativistic spacetime emerges from this spacetime via SDCs.

The picture that emerges from this theory is that, while the universe is always expanding, there are some extra quantum effects that accelerate its expansion.

⁸⁵The justification for $\Delta \Lambda$ being an uncertainty that can be estimated via a second-moment can in principle be done in multiple ways. Depending on the nature of the cancellation that leads to $t_{\mu\nu}^\Lambda$, we might assume that $t_{\mu\nu}^\Lambda$ is a second-moment quantity regardless of the state of the systems. Or we can justify it being a second-moment by associating it to the state of the system that is in a coherent state, and so only its second moments needed to be canceled and thus $t_{\mu\nu}^\Lambda$ can be understood as a second-moment. Alternatively, we might consider that this just follows from the features of how SDCs give rise to ΔV through Poisson processes.

Notice the different contributions of the energy-momentum tensor. When SDCs are expanding via interactions, leading relativistic spacetime to expand, there is an energy-momentum $\langle T_{\mu\nu} \rangle$ that gives rise to a gravitational field, influencing quantum systems in spacetime. This energy-momentum leads to an attractive force that we commonly call gravity. However, the energy-momentum also produces an extra pressure that leads to an accelerated expansion of the universe. As we can see, it is not vacuum energy that drives this accelerated expansion; it is how SDCs constituted by matter fields expand and affect the evolution of spacetime.

In summary, we estimated the value of the cosmological constant via a heuristic and simple method and provided an explanation for its origin. This method superficially resembles that of Sorkin and other workers in the causal set traditions [114] and [24] (and references therein) because they also used the above uncertainty relations and Poisson distribution methods to estimate the number of events and the associated four-volume of spacetime. So, they used the relationship between the number of events and the volume of spacetime. Indeed, with or without assuming the third version of Postulate 3, one could consider that the ordered way SDCs give rise to observables with determinate values, leading to a local effective causal-set-resembling dynamics and diagrams, but one that arises from quantum field theory. SDCs give rise not to a succession of spacetime points but to a succession of classical relativistic events based on interacting quantum matter fields. So, there is no pretense to provide a quantum theory of gravity, or a discrete theory of how spacetime arises. We would then get a causal-set-like diagram with the following structure in the simplest case,

$$(A \rightarrow B) \rightarrow (B \rightarrow C) \rightarrow (D \rightarrow E) \rightarrow \dots, \quad (83)$$

where $A \rightarrow B$ are the events in which A and B have determinate values due to their local decoherence-based interactions, which give rise to (or allow for) the events $B \rightarrow C$. The latter involves B and C having determinate values, and so on. Future work should model these structures and dynamics to gain further insight into how the four-volume arises from SDCs and dark energy changes (see also Appendix I).

So, although there is a superficial similarity between these approaches in some circumstances, they are very different. The causal set approach is mostly supported by classical dynamics and does not have a clear quantum dynamics. In addition, it is based on a yet-to-be-completed theory of quantum gravity, which is clearly not the goal of this theory. Furthermore, it remains unclear how causal sets can address the measurement problem. Also, for this theory, causal set-like structures would just provide an effective description of SDCs.

One may wonder where the uncertainty relations (77) between V and Λ in (84) come from. They could perhaps arise from the following commutation relations (in Planck units) [114, 24],

$$[\hat{\Lambda}, \hat{V}] \sim i. \quad (84)$$

However, to date, there is no proof of (84) via a canonical approach. The unimodular version of general relativity [29, 128] considers in a sense that Λ and

V are conjugate to each other in a similar way to energy and time in quantum theory. We can also see that this relation is plausible by looking at the integral of the action of GR, where we find a $-\Lambda V$ term (see also [24]). However, it is unclear whether we have to assume unimodular gravity here, some other theory, or rather just postulate this uncertainty relation and interpret it as representing something fundamental regarding how SDCs give rise to spacetime volumes and the gravitational field. If we think further about its meaning, it can be understood as a combination of the time and energy and position and momentum uncertainty relations. Furthermore, the reason why we do not have a commutation relation associated with this uncertainty relation might be related to the same reason we do not have well-defined quantum mechanically commutation relations for the time-energy uncertainty relation. Future work should investigate this from the perspective of the semiclassical approach adopted here. One may object that we are using the particle number observable to count the number of events, which is not well-defined globally without giving rise to other issues [69, 50]. However, in the macroscopic limit that we are assuming, we can consider that a global notion of particle number observable arises [20].

We will now explore some of the consequences of the prediction of Λ via our semiclassical approach. First, via $\Delta\Lambda$, this theory provides a potential way to circumvent the postulation of an inflaton field. This is because the value of $\Delta\Lambda$ will change with the evolution of the universe, as it depends on the four-volume of the universe/relativistic spacetime, and the four-volume of the universe changes with time. According to the Big Bang model, the four-volume of the relativistic spacetime was extremely small at the beginning of the universe. Thus, keeping all assumptions used to derive $\Delta\Lambda$, ΔV will also be very small, which implies that $\Delta\Lambda$ will be very large. Therefore, this means that there was a very accelerated expansion in the early universe. Given the issues surrounding the inflaton-based inflationary models, this is another benefit of this view. See Appendix I for more details on this topic.

Second, according to this theory, the introduction of Λ in the Einstein equation is interpreted as a correction to account for the consequences of SDCs. In addition, it points to the idea that the validity of semiclassical equations arises from a balance between positive and negative energy-momentum, which explains why using the expectation value of the stress-energy tensor is sufficient for predictive purposes. Perhaps a more general semiclassical equation should explicitly consider the dynamics of SDCs that lead to varying values of the cosmological constant. Note that this varying value and the associated energy density provide plausible results. For example, the smaller the four-volume of the relativistic spacetime that SDCs give rise to, the higher the energy density associated with the cosmological constant. Note also that $\frac{\Delta\Lambda}{8\pi G}$ and hence ΔV (the four-volume in a past light cone), are determined via local interactions throughout the history of SDCs. Thus, the value of the cosmological constant changes due to local processes.

Third, in the previous section, we conjectured that the emission of a gravitational field decreases as we move to the core of black holes. This hypothesis, together with the volume-number correspondence, might also provide a different

perspective on how the Bekenstein-Hawking formula for the entropy of black holes relates with its microscopic degrees of freedom. Underlying decoherence is the maximal von Neumann entropy of the reduced state of the target systems, which then gives rise locally to systems with determinate values, according to this theory. Relatedly, we hypothesize that black hole entropy is a quasi-classical concept involving counting the number of events involving systems having determinate values (due to interactions with SDCs, which involve a maximal von Neumann entropy); and that there is an area-number of events *correspondence* that allows us to count these events, similar to how the volume-number correspondence allowed us to count events. Hence, we postulate the following modification of the Bekenstein-Hawking formula: $S = \frac{A}{4G} = \frac{\rho' \langle N' \rangle}{4G}$, where ρ' is the number of these events per unit area, which is given by $\rho' = \langle N' \rangle / A$, where A is the area of event horizon and $\langle N' \rangle$ is the expectation value of the number of systems having determinate values at the event horizon due to SDCs. Note that we assume again that we have many events such that quantum fluctuations are small enough that we can consider that ρ' arising from SDCs as approximately constant at this surface.⁸⁶

This hypothesis might give insights about why the area of the horizon is so crucial to determining the area of black holes, and can be supported by noticing that the rate of decoherence of systems in a coherent superposition due to black holes is maximal at the event horizon, when taking into account the other possible rates of decoherence outside black holes [23] (see the previous section for more details on this result). We conjecture that the rate of decoherence at the event horizon is maximal in comparison with the rates in the interior of black holes, and is much bigger than the ones in the interior, in the regimes where the Bekenstein-Hawking formula is valid. Arguably, this decoherence, as a von Neumann entropy maximizing process, in principle could influence the entropy of black holes at the horizon. Given our assumptions, in the regime where the Bekenstein-Hawking formula is valid to describe the entropy of a black hole, it might well be that the entropy of black holes is determined by the maximum rate of decoherence due to SDCs, which occurs at the event horizon, and which involve discrete events that form the area of that horizon given the number-area correspondence. Of course, this is just a heuristic argument and what we want is a derivation of the above entropy formula from SDCs. Nevertheless, a semiclassical approach might help in understanding other puzzles regarding black holes, giving rise to new paths for research.⁸⁷⁸⁸

⁸⁶If we follow the logic of this section explained above, we would consider that the above relation is based on the expectation value of the particle number when the modes of fields are in a coherent state. For this to be valid, we would have to assume that systems are left in a coherent state at the event horizon. However, that might not be the case, and this is not an issue because we do not have to assume a Poisson distributed number of events here.

⁸⁷This might also help explain why the area of the horizon is proportional to the surface of the black hole because the SDCs at the surface may help accounting for the total mass of the black hole that is rendered determinate.

⁸⁸Notice that gravity arising from interactions is associated with an increase in entropy of the systems involved, but that does not mean that gravity is just an entropic force [57, 119]. We need determination and gravitational conditions, which might serve as a basis to also address

Fourth, in the previous section, we have mentioned the objection from Page and Geilker [82] to the semiclassical approach, which leads to the violation or inapplicability of the Bianchi identities during the measurement process or state update. However, this version of Postulate 3 provides a potential way of circumventing this objection and with testable predictions. Given the Bianchi identities and eq. (76),

$$\nabla^\mu G_{\mu\nu} = \frac{8\pi G}{c^4} \nabla^\mu \left(\langle T_{\mu\nu}^{(\text{matt})} \rangle - t_{\mu\nu}^\Lambda \right) = 0. \quad (85)$$

Therefore,

$$\nabla^\mu \langle T_{\mu\nu}^{(\text{matt})} \rangle = \nabla^\mu t_{\mu\nu}^\Lambda. \quad (86)$$

According to this theory, it is possible to hold that during the stochastic process, the Bianchi identities and (85) still hold because a change in the stress energy during the process of measurement, would be accompanied by a change in $t_{\mu\nu}^\Lambda$ in such a way that the Bianchi identities are obeyed during measurement. So, the local change in energy–momentum should be equal to the local changes in dark energy, and therefore, local changes in the acceleration of the expansion of the universe should be observed during a measurement process. Thus, assuming that (85) is maintained during the process in which SDCs give rise to determinate values, the local change in the acceleration of the expansion of the universe upon measurements may be another prediction by this theory, which would support the third version of Postulate 3. A better understanding of the quantity $t_{\mu\nu}^\Lambda$, its effects, and how the process maintaining (86) actually behaves under measurements is a topic for future work. Note that even if we cannot find a sensible way of maintaining (85) and (86) during the stochastic process involved in measurements, we can still maintain the third version of Postulate 3 by assuming that the semiclassical equation and the Bianchi identities are inapplicable to describe this process as we have defended in the last section.

One may ask whether other theories of gravity can derive the approximate value of the cosmological constant as we did here. Certainly, causal set theory can derive this value, but as we have explained, their approach has its limitations. It is unclear whether loop quantum gravity and string theory can make the assumption concerning discrete classical events that generate a gravitational field that we made above. This is because their gravitational degrees of freedom are quantum, and they should contribute to the value of dark energy, irrespective of classicality. On the other hand, the assumption that we have made here is that only the “semi-classical” physical states (i.e., the ones involving systems with determinate values) that arise via SDCs contribute to determining the value of the cosmological constant. Furthermore, it is unclear how quantum gravity theories and gravity-induced collapse theories can justify, in a principled way, why the vacuum does not gravitate.

We believe that the explanation of how the cosmological constant arises, providing a potential solution to the cosmological constant problem, shows the

the foundational issues associated with entropy. If in the future we derive the semiclassical gravity equation from entropic considerations, while assuming the core features of this theory, this might not invalidate it, but show in more detail how gravity emerges from certain SDCs.

potential usefulness of the theory we are proposing. In addition, recent data suggests a varying value of the cosmological constant across the history of the universe [1], as predicted by this theory. Note that arguments similar to the above argument regarding why the vacuum does not gravitate may help solve other problems in physics that are not directly related to semiclassical gravity because SDCs determine whether a system has determinate values of observables, and when we should consider those values in certain explanations.⁸⁹

The goal of this section and the previous one, to some extent, was to show that there are multiple promising strategies that we may adopt if we want to use this semiclassical approach to understand the relation between quantum theory and gravity when it comes to understanding dark energy, quantum fluctuations, and black holes.⁹⁰ Thus, we might not need to opt for more speculative theories, such as quantum gravity theories or more radical mathematically speaking semiclassical theories.

8 Conclusion and future directions

We have proposed a conservative theory that connects general relativity with quantum theory in a coherent way by appealing to semiclassical gravity, and explained how it can be empirically supported, and distinguished experimentally from quantum gravity and other semiclassical gravity theories. We have also shown multiple promising ways in which this theory can be further developed to explain different phenomena. Thus, we believe that we have proposed a theory that provides a series of interesting theoretical and empirical possibilities, which should be further explored.

We will now discuss some of the challenges or shortcomings of this approach, in addition to those mentioned in the previous sections and appendices. First, although we have focused on situations and states (e.g., Hadamard states) where the semiclassical equation can be solved in principle, solving the semiclassical equations is typically a difficult problem and is outside the scope of this article. Various methods have been developed to solve it.⁹¹ In Section 5 we conjectured that the system that emits the test function, the background gravitational field, and decohering interactions drive or control systems (that are not emitting a gravitational field) towards states that can, in principle, be used to solve the semiclassical equations at times t , when decoherence occurs. Future work will

⁸⁹For instance, the framework of EFT points towards new physics happening at the scale not far above the Higgs boson, but we have no evidence of new physics above the TeV. This, in a nutshell, is the Higgs hierarchy problem, which involves a fine-tuning problem to obtain the renormalized Higgs mass from the predictions of EFT, which predicts that the Higgs mass should receive corrections owing to its interactions. If SDCs cannot probe such scales and/or if we do not have a model of decoherence for such interactions, we should not infer the existence of such large terms that need to be canceled to account for the Higgs mass. This is because there is no mechanism that renders what these terms represent determinate. This points towards the need for an integration of the theory proposed here with the tools of EFTs and renormalization theory to make such inferences.

⁹⁰See Appendix I for the inflation case.

⁹¹See [116, 59, 47] and references therein.

explore how this approach may help solve this equation through this suggestion. Relatedly, an array of SDCs can give rise to an array of systems emitting test functions that, in principle, could lead to a lattice. Future work should investigate whether this provides a new perspective on lattice QFT and useful non-perturbative methods.

Second, future work should explore the applicability of this theory to multiple spacetimes. Essentially, one needs to explore models of decoherence in such spacetimes. Furthermore, we developed our proposal in the context of globally hyperbolic spacetimes. One may argue that these are the only realistic spacetimes, but future work could investigate this theory in the context of non-globally hyperbolic spacetimes.

Third, in this first paper, we have not provided models representing the stochasticity of the gravitational field and how it feeds into the dynamics of the matter fields. Here, using tools from hybrid classical-quantum theories, might be promising as an effective description of such stochastic gravitational fields sourced by SDCs, which in turn influences the dynamics of the systems. However, contrary to this approach, the classical stochastic gravitational field does not need to be fundamental for describing gravity. It primarily arises from the interactions between quantum fields. As mentioned, classical states, along with quantum states, where the former have stochastic behavior, do not fundamentally exist for this theory.

Fourth, future work should develop a more detailed description of the dynamics of the time-varying dark energy. Furthermore, one should explore the cosmological consequences of this theory, including for black holes (see Sections 6 and 7 for conjectures regarding these objects) and inflation (see Appendix I). As mentioned in Section 1, the strategy proposed to deal with these cosmological issues is to substitute the singularities in general relativity that arise in these contexts with the absence of a gravitational field generated by SDCs given by flat spacetime (or an asymptotically flat spacetime), or even by an asymptotically de Sitter spacetime.

Fifth, SDCs with their structural features bring about a more complex way in which quantum and classical interact than the usual simple decoherence, spontaneous, or gravity-induced collapse-based story. This may have repercussions for our understanding of how quantum features influence chemical and biological features. In these later domains, chains of interactions are more common. It is difficult to see how the above simpler quantum-to-classical transitions could be integrated with these classically described chemical or biological chains of interactions or give rise to a similar level of complexity. Therefore, we hypothesize that SDCs bring about potentially complex chains of interactions, which might be integrated with chemical and biological chains, explaining some of their features, and how quantum effects may persist or be suppressed over time. This SDC-biochemical hypothesis is an interesting possibility regarding how the quantum and the classical relate, and deserves further exploration.

Finally, we have proposed a particular set of gravitational conditions and associated determination conditions. Future work should explore other possibilities to determine whether they are also viable and propose experiments to test

which one is correct. Note that despite the conjectures proposed, as explained in Section 1, the theory we are presenting at its core is very conservative and testable in the short term.

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A Decoherence in the BVM experiment

Here we reproduce the expressions calculated in [99]. According to the theory advanced here, assuming that the environment presented is a complete description of the environments in the BMV experiment [9, 71], the extent to which we cannot reverse the state in this experiment to its initial state should only be determined by these non-gravitational decohering interactions in their appropriate domain of validity.

The master equation to describe this process of decoherence is

$$\frac{d\rho(x, x', t)}{dt} = -\frac{i}{\hbar} \langle x | [\hat{H}, \hat{\rho}(t)] | x' \rangle - \Gamma(|x - x'|) \rho(x, x', t), \quad (87)$$

in the position representation $\rho(x, x', t) = \langle x | \hat{\rho}(t) | x' \rangle$, where $\hat{H}$ is the free Hamiltonian, and we have the following typical ansatz,

$$\Gamma(\Delta x) = \Gamma_0 \left(1 - \exp \left[-\frac{\Delta x^2}{4a^2} \right] \right). \quad (88)$$

This master equation (87) leads to an exponential suppression in time of the off-diagonal terms of the density matrix in the position representation. Furthermore, given (88), decoherence depends on the localization strength Γ_0 and the localization distance a . Their explicit forms are found in Table 1.⁹² In the BVM scenario, there are only four possible position configurations, thus the problem is simplified to a discrete description. Moreover, it is assumed that decoherence acts independently on the two particles.

B No-disturbance condition approximation

To understand why the approximation in Eq. (24) is valid and the no-disturbance condition is fulfilled, let us assume that $[t_{AB}^{\text{start}}, t_{AB}^{\text{end}}]$ is the support interval for $f_{AB}(t)$, and $[t_{BC}^{\text{start}}, t_{BC}^{\text{end}}]$ is the support interval for $f_{BC}(t)$. The overlapping interval $[t_s, t_e]$ is then $t_s = \max(t_{AB}^{\text{start}}, t_{BC}^{\text{start}})$, $t_e = \min(t_{AB}^{\text{end}}, t_{BC}^{\text{end}})$.

⁹²Work in preparation will compare these different rates and timescales with the ones from alternative theories.

Table 1: Below, we find the exoressions for a and Γ_0 that enter in (88). They quantify the effects of decoherence due to collisions with air molecules (Air), as well as scattering (Sc), absorption (Ab) and emission (Em) of thermal photons on a sphere of radius R , with a dielectric constant ϵ and bulk temperature T_i [106, 100]. Here, m_{air} concerns the mass of the molecules of the residual air, T and P are the temperature and the pressure at which the experiment is performed, and $\zeta(n)$ is the Riemann zeta function. To quantify the effects, we have $\epsilon = 5.7 + i \times 10^{-4}$ for the diamond used in the BMV experiment. For simplicity, $T_i = T$ and $m_{\text{air}} \simeq 6.6 \times 10^{-27}$ kg, which corresponds to an atom of helium.

Source	a^i	Γ_0^i
Air	$\frac{\pi \hbar}{\sqrt{2\pi m_{\text{air}} k_B T}}$	$\frac{16\pi\sqrt{2\pi}}{3} \frac{P R^2}{\sqrt{m_{\text{air}} k_B T}}$
Sc	$\frac{\pi^{2/3} \hbar c}{2k_B T}$	$8! \frac{8\pi^{1/3}}{9} R^6 c \left(\frac{k_B T}{\hbar c}\right)^7 \zeta(9) \text{Re} \left[\frac{\epsilon - 1}{\epsilon + 2}\right]^2$
Ab	$\frac{\pi^{2/3} \hbar c}{2k_B T}$	$\frac{16\pi^{19/3}}{189} R^3 c \left(\frac{k_B T}{\hbar c}\right)^4 \text{Im} \left(\frac{\epsilon - 1}{\epsilon + 2}\right)$
Em	$\frac{\pi^{2/3} \hbar c}{2k_B T_i}$	$\frac{16\pi^{19/3}}{189} R^3 c \left(\frac{k_B T_i}{\hbar c}\right)^4 \text{Im} \left(\frac{\epsilon - 1}{\epsilon + 2}\right)$

The quantity *overlap* quantifies the magnitude of the overlap between the interaction B-C and A-B within the overlapping region:

$$\text{Overlap} = \int_{t_s}^{t_e} f_{AB}(t) \cdot f_{BC}(t) dt. \quad (89)$$

Relatedly, the quantity *strength* quantifies the relative influence of $f_{BC}(t)$ compared to $f_{AB}(t)$ within the overlapping region. It is defined as the ratio of the integrals:

$$\text{Strength} = \frac{\int_{t_s}^{t_e} f_{BC}(t) dt}{\int_{t_s}^{t_e} f_{AB}(t) dt}. \quad (90)$$

So, let us consider the fidelity between the above approximate state and the state $|\psi(1)_{\text{Num}}\rangle$ of the system calculated numerically, $F = |\langle\psi(1)_{\text{approx}}|\psi(1)_{\text{Num}}\rangle|^2$.⁹³ The plots in Figure 5 show that the fidelity decreases with strength and the amount of overlap, i.e., it decreases with the increase in the size of the common support between f_{AB} and f_{BC} . Thus, we will consider that there is a small overlap between the test functions concerning the interactions $A - B$ and $B - C$ because this is sufficient to fulfill the no-disturbance condition.

⁹³The simulations were made using the function NDSolve in Mathematica and the method ExplicitRungeKutta.

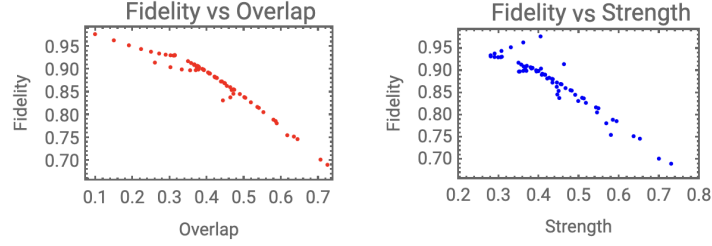


Figure 5: Strength and Overlap obtained by numerical simulations for a $t_{AB} = 0.5$ and $\sigma_{AB} = 0.13$, and for multiple values of t_{BC} and σ_{BC} within the interval $[0, 3]$ and within the common support of f_{AB} and f_{BC} . To calculate these quantities, the Schrodinger equation with the Hamiltonian in (14) was solved to yield the state $|\psi(1)_{\text{Num}}\rangle$.

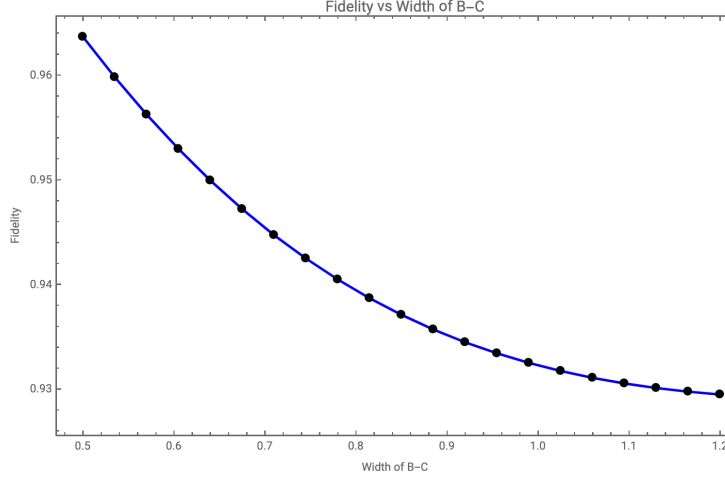


Figure 6: Fidelity as a function of σ_{BC} , assuming values between 0.5 and 1.2, and for $t_{AB} = 0.5$, $\sigma_{AB} = 0.13$, and $t_{BC} = 1.5$. We can see that the fidelity decreases as σ_{BC} increases and the size of the common support of f_{AB} and f_{BC} increases.

C Quantization of scalar fields and other definitions

For completeness, we briefly explain the quantization of the scalar field from the perspective of algebraic quantum field theory (AQFT) and explain other concepts that we will use. In addition, we will often invoke AQFT in our exposition, so it is important to be clear about that.

Let $f \in C_0^\infty(\mathcal{M})$ denote a smooth test function with compact support on

the spacetime manifold $\mathcal{M}$. The advanced and retarded Green's functions, $E^\pm = E^\pm(x, y)$, correspond to the propagators associated with the Klein-Gordon operator $\hat{P}$, where $\hat{P}\phi = 0$, $\hat{P} = \nabla_a \nabla^a + m^2 + \xi R$. Using these, we define the smeared advanced and retarded propagators, also called Green operators $E^\pm : C_0^\infty(\mathcal{M}) \rightarrow C^\infty(\mathcal{M})$, as follows:

$$E^\pm f \equiv (E^\pm f)(x) := \int dV' E^\pm(x, x') f(x'), \quad (91)$$

where the measure $dV' = d^D x' \sqrt{-g'}$ represents the invariant volume element, with $g' \equiv \det g_{\mu\nu}(x') < 0$. These propagators solve the inhomogeneous wave equation $\hat{P}(E^\pm f) = f$. The causal propagator is then defined as the difference between the advanced and retarded propagators: $E = E^- - E^+$, where we have the smeared causal propagator defined as $E(f, g) := \int dV f(x) (Eg)(x)$.

In AQFT, the quantization of the real scalar field ϕ on $\mathcal{M}$ involves a complex linear map from the space of smooth, compactly supported test functions to a unital $*$ -algebra⁹⁴ $\mathcal{A}(\mathcal{M})$ given by $\hat{\phi} : C_0^\infty(\mathcal{M}) \rightarrow \mathcal{A}(\mathcal{M})$, $f \mapsto \hat{\phi}(f)$, that fulfills the conditions of i) Hermiticity: $\hat{\phi}(f)^\dagger = \hat{\phi}(\bar{f})$ with $f \in C_0^\infty(\mathcal{O})$ and $\bar{f}$ is the complex conjugate of f (if f is real valued $\hat{\phi}(f)^\dagger = \hat{\phi}(f)$); ii) the equation for the field: $\hat{\phi}(Pf) = 0$ for all $f \in C_0^\infty(\mathcal{M})$; iii) the Canonical Commutation Relations (CCR): defining the commutator $[a, b] = ab - ba$ for $a, b \in \mathcal{A}(\mathcal{M})$, we have that $[\hat{\phi}(f), \hat{\phi}(g)] = iE(f, g)\mathbb{I}$, $\forall f, g \in C_0^\infty(\mathcal{O})$. The $*$ -algebra $\mathcal{A}(\mathcal{M})$ is referred to as the algebra of observables for the field on $\mathcal{M}$. The smeared field operator $\hat{\phi}(f)$ can be expressed as

$$\hat{\phi}(f) = \int dV \hat{\phi}(x) f(x). \quad (92)$$

Let us associate to each $\mathcal{O}$ of a globally hyperbolic spacetime a subalgebra $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{M})$, generated by $\hat{\phi}(f)$, then it can be shown that for $\mathcal{O} \subset \mathcal{O}'$ we have $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}')$ (isotony); algebras associated with spacelike separated regions commute (Einstein causality); and the algebra of a neighborhood of a Cauchy surface of a given region coincides with the algebra of the full region (time slice axiom). This feature corresponds to the well-posedness of the initial value problem. The algebras assumed in the measurement theory for QFT, [37] and in AQFT, [97] often share the above three features as axioms coming from the Haag-Kastler axioms for a net of C^* -algebras $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ associated with spacetime regions $\mathcal{O}$.

The dynamics of the field are encoded in the symplectic structure. The space of solutions $\text{Sol}_\mathbb{R}(\mathcal{M})$ to the Klein-Gordon equation (27) comes with a symplectic form $\Omega : \text{Sol}_\mathbb{R}(\mathcal{M}) \times \text{Sol}_\mathbb{R}(\mathcal{M}) \rightarrow \mathbb{R}$, defined as

$$\Omega(\phi_1, \phi_2) := \int_{\Sigma_t} d\Sigma^a (\phi_1 \nabla_a \phi_2 - \phi_2 \nabla_a \phi_1), \quad (93)$$

⁹⁴I.e., a complex algebra equipped with involution or also known as Hermitian adjoint, and that is unital because it has the identity.

where $d\Sigma^a = -t^a d\Sigma$, $-t^a$ is the inward-directed unit normal to the Cauchy surface Σ_t , and $d\Sigma = \sqrt{h} d^{D-1}x$ is the induced volume form on Σ_t . This definition is independent of the choice of Cauchy surface used in Eq. (93). The field operator $\hat{\phi}(f)$ can then be expressed as a symplectically smeared field operator $\hat{\phi}(f) = \Omega(Ef, \hat{\phi})$. The CCR algebra is reformulated as $[\Omega(Ef, \hat{\phi}), \Omega(Eg, \hat{\phi})] = i\Omega(Ef, Eg)\mathbb{I}$, where $\Omega(Ef, Eg) = E(f, g)$, as discussed above.

The Klein-Gordon inner product is given by⁹⁵

$$(\phi_1, \phi_2)_{\text{KG}} := i \int_{\Sigma_t} d\Sigma^a (\phi_1^* \nabla_a \phi_2 - \phi_2 \nabla_a \phi_1^*). \quad (94)$$

where the element $d\Sigma^a$ is given by $-t^a d\Sigma$, where $-t^a$ represents the inward-pointing unit normal vector to the Cauchy surface Σ_t . Moreover, $d\Sigma = \sqrt{h} d^n x$ denotes the volume form induced on the hypersurface Σ_t . We require that the modes are normalized according to the Klein-Gordon inner product:

$$(u_{\mathbf{k}}, u_{\mathbf{k}'})_{\text{KG}} = \delta^n(\mathbf{k} - \mathbf{k}'), \quad (u_{\mathbf{k}}, u_{\mathbf{k}'}^*)_{\text{KG}} = 0, \quad (u_{\mathbf{k}}^*, u_{\mathbf{k}'}^*)_{\text{KG}} = -\delta^n(\mathbf{k} - \mathbf{k}'). \quad (95)$$

Note that the equal-time CCRs are not manifestly covariant because they inherently single out a preferred time direction.⁹⁶ The way to do this more covariantly and arguably more satisfactorily is by using the algebraic approach.

Turning now to the Hadamard states, which we will rely on, for any such state, one can define a finite, locally covariant, renormalized expectation value of the stress-energy tensor. The two-point function for a Hadamard state of a Klein-Gordon field has to take the following form:

$$W_\omega(x, y) = \lim_{\epsilon \rightarrow 0^+} \frac{U(x, y)}{\sigma_\epsilon(x, y)} + V(x, y) \ln(\sigma_\epsilon(x, y)) + H_\omega(x, y), \quad (96)$$

which is written as a function of spacetime points x and y . $\sigma_\epsilon(x, y)$ concerns the squared geodesic distance between x and y (the Synge's world function), together with an appropriate regularization, U and V are C^∞ functions that are determined by the spacetime metric and the Klein-Gordon equation. $H_\omega(x, x')$ is a C^∞ function that concerns the state-dependent contributions.

D Coherent states examples and bounds on test functions

Let us consider a simple example of how a system in a coherent state can source a test function. Coherent states in the context of QFT are analogous to those of

⁹⁵Note that this is defined in terms of the complex form with $(\cdot, \cdot)_{\text{KG}} : \text{Sol}_{\mathbb{C}}(\mathcal{M}) \times \text{Sol}_{\mathbb{C}}(\mathcal{M}) \rightarrow \mathbb{C}$ where $(\phi_1, \phi_2)_{\text{KG}} := i\Omega(\phi_1^*, \phi_2)$, but where the symplectic form Ω is expanded to the space of solutions $\text{Sol}_{\mathbb{C}}(\mathcal{M})$ of the Klein-Gordon equation, which are complexified.

⁹⁶A related drawback of canonical quantization is that it does not inherently show the presence of multiple unitarily inequivalent representations of the CCR algebra, which is a well-known feature of quantum field theory. As previously mentioned, a more manifestly covariant approach involves first considering the entire complexified solution space of the Klein-Gordon equation [121]. However, for simplicity, we will not pursue that approach here.

the harmonic oscillator and are defined as states $|\alpha(\mathbf{k})\rangle$ that satisfy the equation

$$\hat{a}_{\mathbf{k}}|\alpha(\mathbf{k})\rangle = \alpha(\mathbf{k})|\alpha(\mathbf{k})\rangle, \quad (97)$$

where $\alpha(\mathbf{k})$ is a complex-valued function, which characterizes the state $|\alpha(\mathbf{k})\rangle$. Furthermore, for a coherent state, the uncertainty relations are minimized for the canonical quadrature pairs of a single mode. The vacuum state $|0\rangle$ is a coherent state with a zero amplitude. Nevertheless, they typically have a nonzero mean field, which makes them ideal sources of test functions. As we have seen (Section 3.2.2), coherent states tend to be selected by SDCs, being the “most-classical” states.

We can write a multimode coherent state, which depends on the complex-valued function $\alpha(\mathbf{k})$, as a displaced vacuum:

$$|\alpha\rangle = \hat{D}[\alpha] |0\rangle = \exp\left(\int d^n \mathbf{k} [\alpha(\mathbf{k}) \hat{a}_{\mathbf{k}}^\dagger - \alpha^*(\mathbf{k}) \hat{a}_{\mathbf{k}}]\right) |0\rangle, \quad (98)$$

where $\hat{D}[\alpha]$ is the unitary displacement operator for the field.

Let us examine an example. We consider that upon decoherence in flat spacetime, a stochastic process that transitions the system to one of the terms of its reduced state (together with the environment that monitors the system), and given the shape of the test function with its tails, the interaction quickly weakens, and the system (and its environment) evolve freely approximately, where its evolution is given by the free Klein-Gordon equation. From regarding ϕ_D as approximately evolving under the free Klein-Gordon equation $(\square + m^2)\hat{\phi}_D(x) = 0$, it follows that for a test function f , the following also holds $(\square + m^2)f = \text{Tr}(\hat{\rho}_D(\square + m^2)\hat{\phi}_D) = 0$, where $f(x, t) = \text{Tr}(\hat{\rho}_D \hat{\phi}_D(x, t))$. Thus, if we consider test functions as arising from mean fields of free scalar fields, it is plausible that they should be solutions to the Klein-Gordon equation.

An ideal test function is a bump function because it is compactly supported. However, the Fourier transform of this function does not have a closed analytical form. Non-compact functions, such as the Gaussian (eq. 35), provide a closed form. But this function is not a perfect solution to the free Klein-Gordon equation. A non-compact function that is a solution to the free Klein-Gordon equation is the following,

$$\begin{aligned} \Phi(t, r) = \frac{\mathcal{A}}{4r} a^{-\frac{5}{4}} \Gamma\left(\frac{5}{4}\right) & \left[(r + t') {}_1F_1\left(\frac{5}{4}; \frac{3}{2}; -\frac{(r+t')^2}{4a}\right) \right. \\ & \left. + (r - t') {}_1F_1\left(\frac{5}{4}; \frac{3}{2}; -\frac{(r-t')^2}{4a}\right) \right], \end{aligned} \quad (99)$$

with

$$\mathcal{A} = \frac{4\sqrt{2}\pi N}{(2\pi)^{3/2}}, \quad a = \frac{\sigma^2}{4}, \quad (100)$$

where N is an optional normalization constant, and the spatial and temporal variances are proportional to σ^2 . It is $\mathcal{C}^\infty$, rapidly decaying in all directions, even in $t' = t - t_0$, and spherically symmetric about $\mathbf{x}_0$.

We want to find the coherent state that gives rise to this test function. Let us obtain the result for the case of a massless scalar field that involves a continuum of modes, where the mean-field arises from

$$\langle \alpha | \hat{\phi}(t, \mathbf{x}) | \alpha \rangle = \int \frac{d^3 k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k}} [\alpha(\mathbf{k}) e^{-ikt + i\mathbf{k} \cdot \mathbf{x}} + \alpha^*(\mathbf{k}) e^{ikt - i\mathbf{k} \cdot \mathbf{x}}]. \quad (101)$$

If we consider

$$\alpha(\mathbf{k}) = N e^{-ak^2} e^{i(k t_0 - \mathbf{k} \cdot \mathbf{x}_0)}, \quad (102)$$

with $a = \frac{\sigma^2}{4} > 0$, the expectation value

$$\Phi(t, \mathbf{x}) = \langle \alpha | \hat{\phi}(t, \mathbf{x}) | \alpha \rangle \quad (103)$$

equals the test function (99).⁹⁷ In the limit where $|r|, |t'| \ll \Delta x, \Delta x = \frac{\sigma}{2}$,⁹⁸ the above test function reduces to a Gaussian,

$$\Phi_a(t, r) \simeq N \exp\left[-\frac{r^2}{2\lambda^2} - \frac{(t - t_0)^2}{2\lambda^2/3}\right], \quad \lambda^2 = \frac{12a}{5} = \frac{3\sigma^2}{5}. \quad (104)$$

Now, turning to the bounds on the test functions, consider the following single-time Poincaré algebra:

$$[H, P^i] = 0, \quad [K^i, P^j] = i\delta^{ij}H, \quad [K^i, H] = iP^i, \quad [K^i, K^j] = -i\varepsilon^{ijk}J^k. \quad (105)$$

Given, for example,

$$H_{\text{int}} = \int d^3x f(\mathbf{x}, t) \hat{\mathcal{O}}_1(\mathbf{x}, t) \hat{\mathcal{O}}_2(\mathbf{x}, t), \quad f(\mathbf{x}, t) = \exp\left[-\frac{\mathbf{x}^2}{2\sigma^2} - \frac{t^2}{2T^2}\right]. \quad (106)$$

Then, we get

$$[H_{\text{int}}, P^i] = i \int d^3x (\partial_i f) \hat{\mathcal{O}}_1 \hat{\mathcal{O}}_2, \quad (107)$$

$$[K^i, H_{\text{int}}] = i \int d^3x (t \partial_i f - x^i \partial_t f) \hat{\mathcal{O}}_1 \hat{\mathcal{O}}_2, \quad (108)$$

$$[K^i, P^j] = i \int d^3x \delta^{ij} \partial_t f \hat{\mathcal{O}}_1 \hat{\mathcal{O}}_2, \quad (109)$$

$$[K^i, K^j] = i \int d^3x (x^i t - x^j t) \partial_t f \hat{\mathcal{O}}_1 \hat{\mathcal{O}}_2. \quad (110)$$

⁹⁷As one can see, whether the system ends up emitting a temporal, a spatial, or a spatiotemporal test function depends on the state it ends up in due to decoherence by members of SDCs.

⁹⁸As we will see, considering $|r|, |t'| \ll \Delta x, \Delta x = \frac{\sigma}{2}$, if we consider $|r| \approx 1/k$ and $|t| \approx 1/\omega$, will coincide with conditions for systems to emit a test function discussed in Section 3.2.1. k and ω concern the maximum momentum and energy, respectively, of the systems subject to that test function.

The terms that spoil the Poincaré algebra commutation relations are those in which a derivative acts on the test function. Because

$$\partial_i f = -\frac{x_i}{\sigma^2} f, \quad \partial_t f = -\frac{t}{T^2} f, \quad (111)$$

every anomalous contribution carries either a factor t/T or x/σ . Fourier transforming gives

$$\tilde{f}(\mathbf{k}, \omega) = \exp\left[-\frac{1}{2}(\sigma^2 \mathbf{k}^2 + T^2 \omega^2)\right] < \varepsilon, \quad (112)$$

where we will consider $\varepsilon \ll 1$. Thus, given a physical process of spatial width L_{phys} and temporal width τ_{phys} (so $k_{\text{max}} \sim 1/L_{\text{phys}}$, $\omega_{\text{max}} \sim 1/\tau_{\text{phys}}$), one finds

$$k_{\text{max}} \gg \Lambda_k = \frac{1}{\sigma}, \quad \omega_{\text{max}} \gg \Lambda_\omega = \frac{1}{T}. \quad (113)$$

One can see that these conditions apply to any physically reasonable test function and Hamiltonian.

Now, let us show via a simple case how the inequalities (36) and (37) that we have derived for the test functions to obey the spacetime symmetries guarantee the validity of the cutoff-based bounded integrals. To show this, for simplicity, let us assume that D is a massless scalar field, where we have

$$\langle \hat{\phi} \rangle = \int_0^\infty dk \rho(k) e^{-\frac{1}{2}\Sigma^2 k^2} 2 \cos(\mathbf{k} \cdot \mathbf{x} - kt), \quad (114)$$

$\rho_0 \equiv \frac{4\pi}{(2\pi)^{3/2}}$, $\Sigma^2 \equiv \sigma_r^2 + \sigma_t^2$, $\rho(k) \equiv \rho_0 k^2$. Then we obtain the difference between the full and truncated integrals

$$\begin{aligned} \Delta(\mathbf{x}, t) &\equiv \langle \hat{\phi} \rangle - \langle \hat{\phi} \rangle_\Lambda \\ &= \int_0^\infty dk \rho(k) e^{-\frac{1}{2}\Sigma^2 k^2} 2 \cos(\dots) - \int_0^\Lambda dk \rho(k) e^{-\frac{1}{2}\Sigma^2 k^2} 2 \cos(\dots) \\ &= \int_\Lambda^\infty dk \rho(k) e^{-\frac{1}{2}\Sigma^2 k^2} 2 \cos(\dots). \end{aligned} \quad (115)$$

Note also that

$$|\Delta(\mathbf{x}, t)| \leq 2 \int_\Lambda^\infty dk \rho(k) e^{-\frac{1}{2}\Sigma^2 k^2} = \frac{8\pi}{(2\pi)^{3/2}} \int_\Lambda^\infty dk k^2 e^{-\frac{1}{2}\Sigma^2 k^2}, \quad (116)$$

where

$$\int_\Lambda^\infty dk k^2 e^{-ak^2} = \frac{\sqrt{\pi}}{4a^{3/2}} \operatorname{erfc}(\sqrt{a}\Lambda) + \frac{\Lambda}{2a} e^{-a\Lambda^2}. \quad (117)$$

For large arguments ($z \gg 1$), $\operatorname{erfc}(z) \simeq \frac{e^{-z^2}}{\sqrt{\pi}z}$, so that, keeping only the leading term we get

$$\int_\Lambda^\infty dk k^2 e^{-\frac{1}{2}\Sigma^2 k^2} \lesssim \frac{\Lambda e^{-\frac{1}{2}\Sigma^2 \Lambda^2}}{\Sigma^2}. \quad (118)$$

We then obtain

$$|\Delta(\mathbf{x}, t)| \lesssim \frac{8\pi}{(2\pi)^{3/2}} \frac{\Lambda e^{-a\Lambda^2}}{2a} = \frac{4\pi}{(2\pi)^{3/2}} \frac{\Lambda e^{-\frac{1}{2}\Sigma^2\Lambda^2}}{(\frac{1}{2}\Sigma^2)}. \quad (119)$$

Because the exponential dominates any power of $k_{max} = \Lambda$, we have

$$|\Delta(\mathbf{x}, t)| \lesssim \exp[-\frac{1}{2}\Sigma^2\Lambda^2] = \exp[-\frac{1}{2}(\sigma_r^2 + \sigma_t^2)\Lambda^2]. \quad (120)$$

Thus, whenever this cutoff satisfies (36) and (37), i.e. $k_{max} \gg 1/\sigma_r, 1/\sigma_t$, the error made by truncating the k -integral is exponentially small. This is in agreement with the bounds derived above; therefore, instead of integrating from 0 to ∞ , we only need to integrate from 0 to k_{max} .

E Filtering out systems at lower scales

Consider the projection operator onto the low-energy modes P_Λ , defined for each energy eigenstate $\hat{H}|E\rangle = E|E\rangle$ as follows:

$$\begin{cases} P_\Lambda|E\rangle = 0 & \text{if } E > \Lambda \\ P_\Lambda|E\rangle = |E\rangle & \text{if } E < \Lambda, \end{cases} \quad (121)$$

which satisfies $P_\Lambda^2 = P_\Lambda$, where Λ is a UV cutoff for the theory.

Burgess et al. [11] have shown via the “decoupling theorem” that integrating out heavy systems always leads to an evolution that cannot change a pure state into a mixed state. More concretely, acting with P_Λ on the heavy states $|E\rangle$ does not lead to a non-unitary evolution afterward.

However, recent models in flat and curved spacetimes seem to imply that the purity of states depends on the mass M of heavy states of order $\mathcal{O}(1/M)$. Consider the following Lagrangian valid to both flat and curved spacetimes,

$$\mathcal{L} = - \left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\sigma^2 \right] + \mathcal{L}_{\text{int}}, \quad (122)$$

although we will focus on flat spacetime in this example for simplicity. Now, consider the quantity purity, which measures the degree of decoherence with $\gamma = 1$ corresponding to maximal purity and no decoherence,

$$\gamma(t) := \text{Tr}_\sigma [\varrho^2(t)]. \quad (123)$$

In the case of

$$\mathcal{L}_{\text{int}} = -g^2 \phi\sigma, \quad (124)$$

in the large mass limit, i.e., the limiting case where $M \gg k, m, g$ it can be shown that when the Hamiltonian of interaction is turned on instantaneously at a certain time, for a mode $\mathbf{k}$ of a field σ decohered by ϕ we have

$$\gamma_{\mathbf{k}}(t) \simeq 1 - \frac{2\mu^4}{\omega_\sigma M^3} \sin^2 \left[\frac{1}{2} M(t - t_0) \right], \quad (125)$$

or when the interaction is turned on adiabatically in the remote past,

$$\bar{\gamma}_{\mathbf{k}}^a(t) \simeq 1 - \frac{g^4}{2\omega_\sigma M^3}. \quad (126)$$

In the case of

$$\mathcal{L}_{\text{int}} = -g \phi^2 \sigma, \quad (127)$$

it can be shown that under the above-mentioned adiabatic interaction,

$$\gamma_{\mathbf{k}} \simeq 1 - \frac{g^2}{16\pi^2 \omega_\sigma M} \int_1^\infty \frac{du}{u^3 \sqrt{u^2 - 1}} = 1 - \frac{g^2}{64\pi \omega_\sigma M}. \quad (128)$$

We see above that the purity depends on the heavy mass M of the environment, and we need to clarify how we discard heavy degrees of freedom. The correct energy selection required when discarding such degrees of freedom is automatically enforced by the so-called $i\epsilon$ prescription applied to the Wightman function. We begin by specifying precisely which $i\epsilon$ prescription is intended, which is closely related to the prescriptions that appear in particle physics, cosmology, condensed-matter physics, and quantum optics. The prescription relevant here demands that the Wightman function, $W(\mathbf{x}, t; \mathbf{x}', t')$, be evaluated with time differences $t - t_0$ possessing a small negative imaginary component. This imaginary shift ensures convergence of the sum over intermediate states in

$$\langle 0 | \phi(x) \phi(x') | 0 \rangle = \int d^3 p \langle 0 | \phi(0) | \mathbf{p} \rangle \langle \mathbf{p} | \phi(0) | 0 \rangle e^{i p(x-x')} \quad (129)$$

in purity calculations, where $p(x - x') = p_\mu (x - x')^\mu = -\omega(\mathbf{p})(t - t') + \mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')$ and $\omega(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M^2}$ is the dispersion relation of the field. The inclusion of a small negative imaginary part in $t - t'$ guarantees convergence for large $|\mathbf{p}|$. Therefore, note that $t - t' \rightarrow (t - t') - i\epsilon, \epsilon > 0$ and then

$$e^{i p(x-x')} = \exp \left[-i \omega_p ((t-t') - i\epsilon) + i \mathbf{p} \cdot (\mathbf{x} - \mathbf{x}') \right] = e^{-\epsilon \omega_p} e^{-i \omega_p (t-t')} e^{i \mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')}. \quad (130)$$

This role of $i\epsilon$ is analogous to that in quantum optics, where it regulates the finite response time of a detector, with the limit $\epsilon \rightarrow 0$ corresponding to the removal of any unresolved short-distance physics. In this ultraviolet (UV) interpretation, ϵ effectively acts as a temporal cutoff Λ^{-1} or energy cutoff Λ .

Therefore, an important function of $i\epsilon$ is to serve as a UV regulator in the Wightman function, since it suppresses contributions from energy eigenstates according to their eigenvalues—precisely what is needed when projecting out heavy modes in a decoupled basis. Furthermore, as explained in [11], the limits $\epsilon \rightarrow 0$ and $M \rightarrow \infty$ do not commute, and this non-commutativity is essential for making decoupling manifest. In the calculation of purity, if one first expands in powers of $1/M$ (for large M) and only afterward sends $\epsilon \rightarrow 0$, the system's state remains nearly pure up to exponentially suppressed corrections. This agrees with the expectation when using the exact (decoupled) energy eigenbasis, expressed via the “decoupling theorem” above. On the other hand, if one first takes $\epsilon \rightarrow 0$

and then performs a $1/M$ expansion, the resulting state appears mixed by an amount of order $\mathcal{O}(1/M)$. This matches the calculations of the purity that we have briefly seen through the above examples.

The above makes sense physically because (in the standard interpretation) $\epsilon \sim 1/\Lambda$ sets the shortest temporal resolution that the Wightman function can resolve. Heavy physics with $M > \Lambda$ produces effects that are too rapid for a low-energy detector to *observe*. Therefore, when estimating the impact of these modes, it is incorrect to take $\epsilon \rightarrow 0$ before expanding in $1/M$. In the decoupling limit, characterized by $M \gg \Lambda$, a nonzero ϵ automatically ensures that projecting out the heavy sector is equivalent to discriminating against high-energy eigenstates. On the other hand, if $M < \Lambda$, then the effects of order $1/M$ can, in principle, be discerned by low-energy experiments. In that scenario, one can safely set $\epsilon \rightarrow 0$ first and only later expand in powers of $1/M$. In doing so, significant contributions to purity arise, in agreement with the decohered-basis computations, which lead to the calculation of purity, as discussed above. Both approaches, working in the decoupled basis and working in the decohered basis, are valid within their own domains, and they yield different answers because they address different physical questions. Which approach applies depends on the relative size of M and Λ .

We now show that the shift $(t - t') \rightarrow (t - t') - i\varepsilon$ can be replaced by smearing each field with a temporal test function whose temporal profile encodes the same $i\varepsilon$ information. One then recovers exactly the factor $e^{-\varepsilon\omega_p}$ (or its square), which tames the ultraviolet behavior in the momentum integral. Thus, we can consider the existence of the emission of a test function by another system that controls whether a system is decohered by a more massive system.

To begin, we introduce two smooth, rapidly-decreasing temporal test functions $f_\varepsilon(t)$ and $g_{\varepsilon'}(t')$, together with spatial test functions $F(\mathbf{x})$ and $G(\mathbf{x}')$. Then, we define the smeared field operators by

$$\Phi_f = \int dt d^3x \sqrt{-g} f_\varepsilon(t) F(\mathbf{x}) \phi(t, \mathbf{x}), \quad \Phi_g = \int dt' d^3x' \sqrt{-g} g_{\varepsilon'}(t') G(\mathbf{x}') \phi(t', \mathbf{x}'). \quad (131)$$

Their vacuum expectation value is

$$W_{f,g} = \langle 0 | \Phi_f \Phi_g | 0 \rangle = \int d^4x \sqrt{-g} \int d^4x' \sqrt{-g'} f_\varepsilon(t) F(\mathbf{x}) g_{\varepsilon'}(t') G(\mathbf{x}') W^+(x, x'), \quad (132)$$

where $W^+(x, x')$ is the unsmeared Wightman function of a free, massive scalar field. We choose both temporal smearings to be Lorentzian with widths ε and ε' , respectively:

$$f_\varepsilon(t) = \frac{\varepsilon}{\pi (t^2 + \varepsilon^2)}, \quad g_{\varepsilon'}(t') = \frac{\varepsilon'}{\pi (t'^2 + \varepsilon'^2)}, \quad (133)$$

with Fourier transforms

$$\tilde{f}_\varepsilon(\omega) = \int_{-\infty}^{\infty} dt f_\varepsilon(t) e^{i\omega t} = e^{-\varepsilon|\omega|}, \quad \tilde{g}_{\varepsilon'}(\omega) = e^{-\varepsilon'|\omega|}. \quad (134)$$

Let us consider the Wightman function,

$$W^+(x, x') = \int \frac{d^3 p}{(2\pi)^3 2\omega_p} e^{-i\omega_p(t-t')} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}, \quad \omega_p = \sqrt{\mathbf{p}^2 + M^2}, \quad (135)$$

and let us smear spatiotemporally this correlator, and perform the spatial integrals first, which yield

$$\int d^3 x \sqrt{-g} F(\mathbf{x}) e^{i\mathbf{p}\cdot\mathbf{x}} = \tilde{F}(\mathbf{p}), \quad \int d^3 x' \sqrt{-g'} G(\mathbf{x}') e^{-i\mathbf{p}\cdot\mathbf{x}'} = \tilde{G}^*(\mathbf{p}). \quad (136)$$

The remaining time integrals yield

$$\int_{-\infty}^{\infty} dt f_{\varepsilon}(t) e^{-i\omega_p t} = \tilde{f}_{\varepsilon}(-\omega_p) = e^{-\varepsilon\omega_p}, \quad \int_{-\infty}^{\infty} dt' g_{\varepsilon'}(t') e^{+i\omega_p t'} = \tilde{g}_{\varepsilon'}(\omega_p) = e^{-\varepsilon'\omega_p}. \quad (137)$$

Consequently, the smeared two-point function becomes

$$W_{f,g} = \int \frac{d^3 p}{(2\pi)^3 2\omega_p} \tilde{F}(\mathbf{p}) \tilde{G}^*(\mathbf{p}) e^{-(\varepsilon+\varepsilon')\omega_p}. \quad (138)$$

If the same spatiotemporal smearing is used on both F and G , then $W_{f,f}$ reduces to

$$W_{f,f} = \int \frac{d^3 p}{(2\pi)^3 2\omega_p} |\tilde{F}(\mathbf{p})|^2 e^{-2\varepsilon\omega_p}. \quad (139)$$

In the case of no spatial smearing ($F(\mathbf{x}) = G(\mathbf{x}) = \delta^{(3)}(\mathbf{x})$), one recovers exactly the factor $e^{-2\varepsilon\omega_p}$ that arises from imposing the $i\varepsilon$ prescription directly on the time difference. Thus, the Lorentzian temporal smearing allows us to rederive the exponential damping without the above trick of shifting times into the complex plane.

Now, test functions are emitted by a mean-field of some other field. Thus, we need to find a state ρ of a real scalar field ϕ such that we obtain the temporal smearing

$$f_{\varepsilon}(t) = \langle \phi(t) \rangle_{\rho} = \text{Tr}[\rho \phi(t)] = \frac{\epsilon}{\pi(t^2 + \epsilon^2)}. \quad (140)$$

For simplicity, we work in 1+1 D, set the spatial point $x = 0$, and take ϕ massless ($\omega_k = |k|$). The coherent state $|\alpha\rangle = \exp\left[\int_0^{\infty} dk (\alpha_k a_k^{\dagger} - \alpha_k^* a_k)\right] |0\rangle$ has the classical expectation value

$$\langle \phi(t) \rangle_{\alpha} = \int_0^{\infty} \frac{dk}{\sqrt{4\pi k}} [\alpha_k e^{-ik t} + \alpha_k^* e^{ik t}]. \quad (141)$$

To match the $i\epsilon$ prescription we need

$$\alpha_k = \sqrt{\frac{k}{\pi}} \tilde{f}_{\varepsilon}(k) = \sqrt{\frac{k}{\pi}} e^{-\epsilon k}. \quad (142)$$

Plugging this into the expression for coherent states gives

$$\begin{aligned}\langle \phi(t) \rangle_\alpha &= \int_0^\infty dk \left[e^{-\epsilon k} e^{-i k t} + e^{-\epsilon k} e^{i k t} \right] \\ &= \frac{\epsilon}{\pi (t^2 + \epsilon^2)},\end{aligned}\quad (143)$$

which yields the Lorentzian. Hence, the system in the state

$$\rho = |\alpha\rangle\langle\alpha| \quad \text{with} \quad \alpha_k = \sqrt{\frac{k}{\pi}} e^{-\epsilon k} \quad (144)$$

generates a test function that yields the $i\epsilon$ prescription.

Following the same logic applied in 3 + 1 D, it can be shown that a single coherent state of a massless real scalar can be tuned such that its time-dependent expectation value at the spatial origin reproduces the Lorentzian test function that encodes the $i\epsilon$ -prescription.

We expand the free, massless field at $\mathbf{x} = 0$ in the usual basis:

$$\phi(t) \equiv \phi(t, \mathbf{0}) = \int \frac{d^3 k}{(2\pi)^{3/2} \sqrt{2k}} \left[a_{\mathbf{k}} e^{-i k t} + a_{\mathbf{k}}^\dagger e^{i k t} \right], \quad k \equiv |\mathbf{k}|. \quad (145)$$

Now, we choose a real coherent profile $\alpha_{\mathbf{k}} = \alpha(k) = \alpha^*(k)$ and define

$$|\alpha\rangle = \exp \left[\int_0^\infty dk k^2 \alpha(k) \int \frac{d\Omega_{\mathbf{k}}}{(2\pi)^{3/2}} (a_{\mathbf{k}}^\dagger - a_{\mathbf{k}}) \right] |0\rangle. \quad (146)$$

Because $a_{\mathbf{k}}|\alpha\rangle = \alpha(k)|\alpha\rangle$, the one-point function at the spatial origin is

$$\langle \phi(t) \rangle_\alpha = \int \frac{d^3 k}{(2\pi)^{3/2} \sqrt{2k}} 2 \alpha(k) \cos(kt). \quad (147)$$

Using $d^3 k = k^2 dk d\Omega$ and $\int d\Omega = 4\pi$, this becomes

$$\langle \phi(t) \rangle_\alpha = \frac{8\pi}{(2\pi)^{3/2} \sqrt{2}} \int_0^\infty dk k^{3/2} \alpha(k) \cos(kt) = \frac{2}{\sqrt{\pi}} \int_0^\infty dk k^{3/2} \alpha(k) \cos(kt). \quad (148)$$

We wish to obtain the Lorentzian temporal profile

$$\langle \phi(t) \rangle_\alpha = \frac{\varepsilon}{\pi (t^2 + \varepsilon^2)}, \quad \varepsilon > 0. \quad (149)$$

This is achieved by choosing

$$\alpha(k) = \frac{1}{2\sqrt{\pi}} \frac{e^{-\varepsilon k}}{k^{3/2}}. \quad (150)$$

Indeed,

$$\langle \phi(t) \rangle_\alpha = \frac{2}{\sqrt{\pi}} \frac{1}{2\sqrt{\pi}} \int_0^\infty dk e^{-\varepsilon k} \cos(kt) \quad (151)$$

$$= \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + t^2}, \quad (152)$$

because

$$\int_0^\infty dk e^{-\varepsilon k} \cos(kt) = \frac{\varepsilon}{\varepsilon^2 + t^2}. \quad (153)$$

Gaussian states can also be used to source test functions. Let ρ be a Gaussian density operator—for example, the thermal state

$$\rho = Z^{-1} \exp\left[-\beta \int d^3k \omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}}\right]. \quad (154)$$

Applying the displacement operator $D[\alpha] = \exp\left[\int d^3k (\alpha_{\mathbf{k}} a_{\mathbf{k}}^\dagger - \alpha_{\mathbf{k}}^* a_{\mathbf{k}})\right]$ with the same coherent profile

$$\alpha_{\mathbf{k}} = \frac{1}{2\sqrt{\pi}} \frac{e^{-\varepsilon k}}{k^{3/2}}, \quad k = |\mathbf{k}|, \quad (155)$$

then, we define the mixed state $\rho = D[\alpha] \rho_G D^\dagger[\alpha]$. Because $D^\dagger a_{\mathbf{k}} D = a_{\mathbf{k}} + \alpha_{\mathbf{k}}$, the mean field is shifted, and we obtain

$$\langle \phi(t, \mathbf{0}) \rangle_\rho = \langle \phi(t, \mathbf{0}) \rangle_{D[\alpha]|0\rangle} = \frac{\varepsilon}{\pi(t^2 + \varepsilon^2)}. \quad (156)$$

The above Lorentzian function does not have a rapid decay; thus, it is not what we often associate with a test function, such as a Schwartz function. However, we can achieve this by replacing these Lorentzians with order- n super-Lorentzian functions

$$f_{n,\gamma}(t) = \frac{C_n}{[t^2 + (\gamma/2)^2]^n}, \quad g_{n',\gamma'}(t) = \frac{C_{n'}}{[t^2 + (\gamma'/2)^2]^{n'}}, \quad (157)$$

where

$$C_n = \frac{\Gamma(n)}{\sqrt{\pi} \Gamma(n - \frac{1}{2})} \left(\frac{\gamma}{2}\right)^{2n-1}, \quad C_{n'} = \frac{\Gamma(n')}{\sqrt{\pi} \Gamma(n' - \frac{1}{2})} \left(\frac{\gamma'}{2}\right)^{2n'-1}. \quad (158)$$

Their Fourier transforms (for $\omega \neq 0$) are given by

$$\tilde{f}_{n,\gamma}(\omega) = e^{-\frac{\gamma}{2}|\omega|} \mathcal{P}_{n-1}\left(\frac{\gamma}{2}|\omega|\right), \quad \tilde{g}_{n',\gamma'}(\omega) = e^{-\frac{\gamma'}{2}|\omega|} \mathcal{P}_{n'-1}\left(\frac{\gamma'}{2}|\omega|\right), \quad (159)$$

where $\mathcal{P}_m(z)$ is a finite polynomial of degree m with $\mathcal{P}_m(0) = 1$.

Proceeding exactly as in the original calculation, one finds that the smeared two-point function becomes

$$W_{f,g} = \int \frac{d^3p}{(2\pi)^3 2\omega_p} \tilde{F}(\mathbf{p}) \tilde{G}^*(\mathbf{p}) e^{-\frac{\gamma+\gamma'}{2}\omega_p} \mathcal{P}_{n-1}\left(\frac{\gamma}{2}\omega_p\right) \mathcal{P}_{n'-1}\left(\frac{\gamma'}{2}\omega_p\right), \quad (160)$$

so the exponential damping that codifies the $i\varepsilon$ can be seen here. With identical smearings ($n = n'$, $\gamma = \gamma'$, $F = G$), this reduces to

$$W_{f,f} = \int \frac{d^3p}{(2\pi)^3 2\omega_p} |\tilde{F}(\mathbf{p})|^2 e^{-\gamma\omega_p} \left[\mathcal{P}_{n-1}\left(\frac{\gamma}{2}\omega_p\right)\right]^2. \quad (161)$$

For $n = 1$ (so that $\mathcal{P}_0 \equiv 1$, with $\gamma = 2\varepsilon$), one recovers the factor $e^{-2\varepsilon\omega_p}$. For $n > 1$, the same exponential is multiplied by a finite polynomial of degree $2n - 2$ in ω_p yielding a stronger ultraviolet cutoff.

In the limit $n \rightarrow \infty$ with

$$\gamma_n = \frac{2\sigma}{\sqrt{n}}, \quad (162)$$

we obtain the following temporal gaussian,

$$f_{n,\gamma_n}(t) = \frac{C_n}{[t^2 + (\gamma_n/2)^2]^n} \rightarrow \frac{1}{\sqrt{4\pi}\sigma^2} e^{-t^2/4\sigma^2} \quad (163)$$

where σ is a temporal variance associated with γ and thus with the cutoff Λ mentioned above, while the corresponding momentum-space function tends to $\exp(-\sigma|\omega|^2)$. Thus, a super-Lorentzian not only reproduces the $i\varepsilon$ prescription but also allows us to recover in the limit of large n the Schwartz test functions.

F Measurement theory in QFT given SDCs

We will briefly show in this section how this theory fits with measurement theory in QFT, in particular, particle detector models.⁹⁹ We will focus on two real scalar fields A and B , where A is decomposed into modes. We will assume that the modes of A belong to an SDC and already have the $DC - B$ in agreement with the determination conditions explained in Section 3.

We will assume that the target system B and its modes are initially in a zero-mean Gaussian state, as well as A . Gaussian states are completely characterized by their first and second moments (i.e., mean values and covariance matrices). Examples of these states include thermal, coherent, and squeezed states. Furthermore, we assume that the vacuum of the states under study fulfills the Hadamard condition. As is well known, in QFT, there are many unitary inequivalent Hilbert space representations. However, the consensus is to select a subclass of states known as Hadamard states that fulfill the idea that all states should look similar locally and be as close to flat space QFT as possible.

In the covariant picture, the interaction between system A and system B is described by the following Lagrangian density,

$$\mathcal{L} = \frac{1}{2}(\nabla_\mu \phi_A)(\nabla^\mu \phi_A) + \frac{1}{2}(\nabla_\mu \phi_B)(\nabla^\mu \phi_B) - \lambda_{AB} f \phi_A \phi_B \quad (164)$$

where λ_{AB} is the coupling constant with dimensions of energy squared and f is a dimensionless smooth, real-valued test function with support in some compact coupling spacetime region R .¹⁰⁰ The above Lagrangian omits the systems that give rise to the background gravitational field.

We adopt the canonical picture in 3+1 globally hyperbolic spacetime, where we regard a (3+1)-dimensional spacetime $\mathcal{M}$ as foliated by a family of spacelike

⁹⁹We will follow closely the calculations and results obtained in [86] with some appropriate adaptations.

¹⁰⁰We thus express the test function for each mode A and B in terms of this function.

3-dimensional hypersurfaces Σ_t , labeling the hypersurfaces by a time parameter t , and assume the following split of the metric,

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (165)$$

where h_{ij} is the spatial metric, N is the lapse function that describes the amount of proper time that elapses between two hypersurfaces along the direction normal to the spatial slice, and N^i is the shift vector that describes how the spatial coordinates change when moving from one hypersurface to another.

So, we have the following evolution,

$$\hat{U} = \mathcal{T} \exp \left(-i \int dt \hat{H}_{\text{int}}(t) \right), \quad (166)$$

where $\mathcal{T} \exp$ denotes a time-ordered exponential concerning any time parameter and

$$H_{\text{int}}(t) = \lambda_{AB} \int_{\Sigma_t} d^3x \sqrt{h} \chi(t) F(\mathbf{x}) \phi_A(t, \mathbf{x}) \phi_B(t, \mathbf{x}). \quad (167)$$

with λ_{AB} being a coupling constant, and $\chi(t)$ and $F(\mathbf{x})$ being the temporal and spatial test functions, respectively, over the spacelike hypersurfaces Σ_t . Furthermore, they fulfill the no-disturbance conditions jointly with other temporal and spatial test functions concerning other interactions in this SDC.

Assuming that λ_{AB} is sufficiently small we can have the following Dyson expansion,

$$\hat{U} = 1 + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda^3), \quad (168)$$

where

$$\hat{U}^{(1)} = -i \int dt \hat{H}_{\text{int}}(t), \quad (169)$$

and

$$\hat{U}^{(2)} = - \int dt dt' \hat{H}_{\text{int}}(t) \hat{H}_{\text{int}}(t') \theta(t - t'), \quad (170)$$

where $\theta(t)$ is the Heaviside theta function.

Now, let us consider the initial state of the systems, where we focus on the interaction of one of the modes of A , which was previously decomposed into finite modes,

$$\hat{\rho}_0 = |0_A\rangle\langle 0_A| \otimes \hat{\rho}_B. \quad (171)$$

The interaction between other N modes of A with the DC- B are omitted. We could also consider A as a series of modes, which we idealize as a simple system, and which will decohere B . On the other hand, B could be a single mode or a whole continuum of modes that we choose not to decompose for simplicity. In Sections 5 and Appendix H, we saw a model in de Sitter spacetime where A decoheres a single mode of B in a more complex situation.

Taking into account that

$$\hat{\rho}_f = \hat{U} \hat{\rho}_0 \hat{U}^\dagger, \quad (172)$$

we get that the final states of the fields are represented by

$$\hat{\rho}_f = \hat{\rho}_0 + \hat{\rho}^{(1)} + \hat{\rho}^{(2)} + \mathcal{O}(\lambda^3), \quad (173)$$

where

$$\begin{aligned} \hat{\rho}^{(1)} &= \hat{U}^{(1)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(1)\dagger}, \\ \text{and } \hat{\rho}^{(2)} &= \hat{U}^{(2)} \hat{\rho}_0 + \hat{U}^{(1)} \hat{\rho}_0 \hat{U}^{(1)\dagger} + \hat{\rho}_0 \hat{U}^{(2)\dagger}. \end{aligned} \quad (174)$$

More concretely,

$$\begin{aligned} \hat{\rho}^{(2)} &= \lambda_{AB}^2 \int dV dV' \left[\hat{M}(t, \mathbf{x}) \hat{\phi}_B(t, \mathbf{x}) \hat{\rho}_0 \hat{\phi}_B(t', \mathbf{x}') \hat{M}(t', \mathbf{x}') \right. \\ &\quad - \hat{M}(t, \mathbf{x}) \hat{M}(t', \mathbf{x}') \hat{\phi}_B(t, \mathbf{x}) \hat{\phi}_B(t', \mathbf{x}') \hat{\rho}_0 \theta(t - t') \\ &\quad \left. - \hat{\rho}_0 \hat{M}(t, \mathbf{x}) \hat{\phi}_B(t', \mathbf{x}') \hat{\phi}_B(t, \mathbf{x}) \hat{M}(t', \mathbf{x}') \theta(t' - t) \right]. \end{aligned} \quad (175)$$

with $\hat{M}(t, \mathbf{x}) = \chi(t) F(\mathbf{x}) \phi_A(t, \mathbf{x})$.

We will now partial trace the final state over the degrees of freedom of B , focusing only on A , i.e., $\hat{\rho}_A = \text{Tr}_B \hat{\rho}_f$, to see how system A probes the field B .

Since B starts as a zero-mean Gaussian, we have that $\text{tr}_B (\hat{\phi}_B(t, \mathbf{x}) \hat{\rho}_B) = \langle \hat{\phi}(t, \mathbf{x}) \rangle_{\rho_B} = 0$. Moreover, given that $W(x, x') = \langle \hat{\phi}(x) \hat{\phi}(x') \rangle_{\rho_B}$ we have that

$$\begin{aligned} \text{tr}_B (\hat{\rho}^{(2)}) &= \lambda_{AB}^2 \int dt dt' W(x, x') \left[\hat{M}(t', x') |0_A\rangle \langle 0_A| \hat{M}(t, x) \right. \\ &\quad - \hat{M}(t, x) \hat{M}(t', x') |0_A\rangle \langle 0_A| \theta(t - t') \\ &\quad \left. - |0_A\rangle \langle 0_A| \hat{M}(t, x) \hat{M}(t', x') \theta(t' - t) \right]. \end{aligned}$$

Thus, we have

$$\begin{aligned} \hat{\rho}_A &= |0_A\rangle \langle 0_A| + \lambda_{AB}^2 \int dV dV' W(x, x') \left[\hat{M}(t', \mathbf{x}') |0_A\rangle \langle 0_A| \hat{M}(t, \mathbf{x}) \right. \\ &\quad - \hat{M}(t, \mathbf{x}) \hat{M}(t', \mathbf{x}') |0_A\rangle \langle 0_A| \theta(t - t') \\ &\quad \left. - |0_A\rangle \langle 0_A| \hat{M}(t, \mathbf{x}) \hat{M}(t', \mathbf{x}') \theta(t' - t) \right] + \mathcal{O}(\lambda^4). \end{aligned} \quad (176)$$

As we can see, A 's final state contains information of the values of B through the correlation function of B . Assuming that the modes of A decohere B (note that this model does not analyze this process), we infer that it gives rise to B having determinate values, which the state $\hat{\rho}_B$ represents, over a spacetime region represented via the test functions.

A closer comparison with particle detector models becomes possible if space-time is static and the metric is such that there is a separation between space and time. This also allows us to clarify how the systems probe each other. So, in

this case, the solutions $u_{\mathbf{k}}(x)$ decompose as $u_{\mathbf{k}}(x) = e^{-i\omega_{\mathbf{k}}t}\Phi_{\mathbf{k}}(\mathbf{x})$. Then, writing $\zeta(x) = \chi(t)F(\mathbf{x})\Phi_{\mathbf{k}}(\mathbf{x})$, the interaction Hamiltonian becomes

$$\hat{H}_{\text{static}}(x) = \lambda_{AB} \left(\zeta(x)e^{-i\omega_{\mathbf{k}}t}\hat{a}_{\mathbf{k}} + \zeta^*(x)e^{i\omega_{\mathbf{k}}t}\hat{a}_{\mathbf{k}}^\dagger \right) \hat{\phi}_B(x). \quad (177)$$

To obtain the expression of a particle detector evolving in a given “trajectory,” let us consider x_0 as the spatial coordinate that concerns the center of $\Phi_{\mathbf{k}}(\mathbf{x})$. More concretely, let us consider a particle detector whose center of mass has a trajectory given by the Fermi normal coordinates $z(\tau) = (\gamma\tau, x_0)$, where τ is the proper time and γ is the redshift factor relative to t . Then, the proper energy gap is defined as $\Omega = \gamma\omega_k$, so the effective interaction Hamiltonian becomes

$$\hat{H}_{\text{eff}}(x) = \lambda_{AB} \left(\zeta(x)e^{-i\Omega\tau}\hat{a}_{\mathbf{k}} + \zeta^*(x)e^{i\Omega\tau}\hat{a}_{\mathbf{k}}^\dagger \right) \hat{\phi}_B(x). \quad (178)$$

This corresponds to the interaction Hamiltonian of a harmonic oscillator detector with an energy gap Ω that is interacting with a scalar field $\hat{\phi}(x)$. By appropriately balancing the units of $F(\mathbf{x})$, the switching function, and the coupling strength, one can match this model to the harmonic-oscillator Unruh-DeWitt detector (UdW) detector model. Note that UdWs are idealized quantum two-level systems that couple locally to the quantum field, evolving with respect to their proper time. For “one-particle” excitations in mode $\mathbf{k}$, the Hamiltonian can be restricted to a two-level system spanned by $\{|0_{\mathbf{k}}\rangle, |1_{\mathbf{k}}\rangle\}$, reducing to the leading-order interaction of a two-level detector A with a scalar field.

It can be shown [92] that quantum field theories, which assume the principle of microcausality (where observables commute at spacelike-separated points) will generate time-evolution operators that remain independent of the specific time parameter used for time ordering. However, in the above approximation using smeared operators, this is not what happens. To see this, let us first observe that the algebra of creation and annihilation operators restricted to act on a two-dimensional space is isomorphic to the ladder operators $\hat{\sigma}_+$ and $\hat{\sigma}_-$ also acting on such a space. Given that the monopole moment operator is $\hat{\mu}(\tau) = e^{i\Omega\tau}\hat{\sigma}_+ + e^{-i\Omega\tau}\hat{\sigma}_-$, it can be shown that $[\hat{H}_{eff}(x), \hat{H}_{eff}(x')] = \lambda^2\Lambda(x)\Lambda(x')[\hat{\mu}(\tau(x)), \hat{\mu}(\tau(x'))]\hat{\phi}(x)\hat{\phi}(x')$ for spacelike separated regions x and x' due to $[\hat{\mu}(\tau), \hat{\mu}(\tau')] = 2i\sin(\Omega(\tau - \tau'))\hat{\sigma}_z$ just vanishes for certain times. However, in cases where covariance violation occurs at the leading order, this is due to the spatial smearing of the detector. To see this more intuitively, note that $\hat{H}_{eff}$ couples non-locally a single quantum degree of freedom of the detector to multiple spacelike-separated points. Consequently, the effect is suppressed as the smearing decays with time. Furthermore, when we have point-like detectors (which arise as a limit of very sharply localized test functions), we also obtain full covariance in the above sense.

Given this, [92] smeared particle detector models lead to a quantifiable breaking of covariance because, in a covariant formalism, the time evolution operator concerning the same Hamiltonian should yield the same results regardless of the reference frame used, i.e., $\hat{\mathcal{U}}_\tau = \hat{\mathcal{U}}_t$. Particle detector models provide a measurement theory for QFT with a series of update rules for different measurements,

which can be used in this framework. Nevertheless, because the theory presented here starts fundamentally from quantum fields and particle detectors arise from it, we consider that this breaking of covariance is merely an emergent feature, not present in the fundamental theory, and is under control.

Given the above formalism, we can formulate an update rule that takes into account the state update for the individual observers (or observers with probes that belong to SDCs) as well as their jointly correlated state. To remain both fully predictive and causal, these update rules treat in different ways the observables that can be measured locally and those whose statistics are only jointly accessible. These update rules were proposed in [91], and are covariant by construction in the sense of being defined only in terms of the causal structure of spacetime and in the sense of being frame independent. This was called the polyperspective formalism, involving polystates.¹⁰¹

In this framework, we introduce an extended Hilbert space

$$\tilde{\mathcal{H}}_{AB} := \mathcal{H}_A \oplus \mathcal{H}_B \oplus (\mathcal{H}_A \otimes \mathcal{H}_B), \quad (179)$$

whose sub-spaces accommodate, respectively, *Alice-only*, *Bob-only*, and *joint* observables, where the space of physical operators is given by

$$\mathcal{L}(\tilde{\mathcal{H}}_{AB})_{\text{phys}} := \mathcal{L}(\mathcal{H}_A) \oplus \mathcal{L}(\mathcal{H}_B) \oplus \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B). \quad (180)$$

The complete quantum state is a *polystate*

$$\tilde{\rho}_{AB} := \hat{\rho}_A \oplus \hat{\rho}_B \oplus \hat{\rho}_{AB} \in \mathcal{L}(\tilde{\mathcal{H}}_{AB})_{\text{phys}}, \quad (181)$$

where $\hat{\rho}_A$ and $\hat{\rho}_B$ reproduce all expectation values of *individual* observables while $\hat{\rho}_{AB}$ reproduces the joint ones. Furthermore, in this framework, in general $\hat{\rho}_A \neq \text{Tr}_B \hat{\rho}_{AB}$ and $\hat{\rho}_B \neq \text{Tr}_A \hat{\rho}_{AB}$. To see this, note that in this framework because Alice and Bob carry their own proper clocks, the state may depend on two time parameters:

$$\tilde{\rho}(\tau_A, \tau_B) = \hat{\rho}_A(\tau_A) \oplus \hat{\rho}_B(\tau_B) \oplus \hat{\rho}_{AB}(\tau_A, \tau_B). \quad (182)$$

Then, the local sectors evolve only due to the physics inside their individual causal pasts,

$$\hat{\rho}_A(\tau_A) \propto \text{Tr}_B \Psi_{J^-(x_A(\tau_A))}(\hat{\rho}_{AB}), \quad (183)$$

$$\hat{\rho}_B(\tau_B) \propto \text{Tr}_A \Psi_{J^-(x_B(\tau_B))}(\hat{\rho}_{AB}), \quad (184)$$

where $J^-(x)$ is the causal past of $x \in \mathcal{M}$ and $\Psi_S : \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B) \rightarrow \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B)$ is a completely positive map that replays every unitary evolution and every actually realized measurement in the region $S \subset \mathcal{M}$. The joint sector must then take into account the past of both parties,

$$\hat{\rho}_{AB}(\tau_A, \tau_B) \propto \Psi_{J^-(x_A(\tau_A)) \cup J^-(x_B(\tau_B))}(\hat{\rho}_{AB}). \quad (185)$$

¹⁰¹These rules aim to deal with the issues presented in [3] concerning the absence of a relativistic covariant postulate on a Cauchy surface in relativistic quantum mechanics, which was mentioned in the main text. See the citations above to see how this framework and the algebraic one deal with it.

The proportionality factors in Eqs. (183)–(185) are fixed by normalization.

Now, let us consider an example involving a Bell scenario. Suppose the qubits start in the Bell state $|\Phi^+\rangle = (|\uparrow_A\uparrow_B\rangle + |\downarrow_A\downarrow_B\rangle)/\sqrt{2}$ and Alice measures $\hat{\sigma}_{z,A}$ at proper time τ_A^* , obtaining the outcome $+1$. The map then reduces to

$$\Psi_S(\hat{\rho}) = \begin{cases} \hat{\rho}, & x_A(\tau_A^*) \notin S, \\ |\uparrow_A\rangle\langle\uparrow_A| \hat{\rho} |\uparrow_A\rangle\langle\uparrow_A|, & x_A(\tau_A^*) \in S, \end{cases} \quad (186)$$

where $x_A(\tau_A^*) \in S$ means that we have a region S containing Alice's measurement event. Then, the polystate becomes

$$\tilde{\rho}(\tau_A, \tau_B) = \begin{cases} \frac{1}{2} \mathbb{I}_A \oplus \frac{1}{2} \mathbb{I}_B \oplus |\Phi^+\rangle\langle\Phi^+|, & \tau_A < \tau_A^*, \quad \tau_B < \tau_B^*, \\ |\uparrow_A\rangle\langle\uparrow_A| \oplus \frac{1}{2} \mathbb{I}_B \oplus |\uparrow_A\uparrow_B\rangle\langle\uparrow_A\uparrow_B|, & \tau_A \geq \tau_A^*, \quad \tau_B < \tau_B^*, \\ \frac{1}{2} \mathbb{I}_A \oplus |\uparrow_B\rangle\langle\uparrow_B| \oplus |\uparrow_A\uparrow_B\rangle\langle\uparrow_A\uparrow_B|, & \tau_A < \tau_A^*, \quad \tau_B \geq \tau_B^*, \\ |\uparrow_A\rangle\langle\uparrow_A| \oplus |\uparrow_B\rangle\langle\uparrow_B| \oplus |\uparrow_A\uparrow_B\rangle\langle\uparrow_A\uparrow_B|, & \tau_A \geq \tau_A^*, \quad \tau_B \geq \tau_B^*. \end{cases} \quad (187)$$

This construction is valid for whichever outcome and eigenstate is realised. For instance, if Alice obtained the outcome -1 , we would swap $\uparrow$ by $\downarrow$. If she made a measurement of $\hat{\sigma}_x$ we would replace $\uparrow, \downarrow$ with $+, -$. Furthermore, this framework works for multipartite correlations, and as we can see, it can accommodate the freedom of choice of the observers. Note that the above quantum states assigned to systems to predict the outcomes are built exclusively from physical operations in the causal pasts of observers. Because different observers have different causal pasts, they may legitimately assign different density operators to the same underlying physical system. However, the outcomes are absolute and not perspectival or relational. The observers will agree on the outcomes that arise. Notice that the assigned states are not foliation-dependent but rather depend on the past light cone of the local observers. Other frameworks (e.g., [38, 93]) offer other local update rules. In principle, the theory we are proposing also accommodates these frameworks because they share common tools.

G How SDCs allow us to infer a gravitational field and give rise to it

In this section, we provide an intuition about how a classical metric may arise via how probes interact and decohere a quantum field (49). First, it is assumed that particle detectors interact with the quantum field at specific spacetime points, giving rise to detection events. By analyzing the probabilities associated with the detection events, we can extract both the real and imaginary parts of the Wightman function. Second, as we have mentioned, the scenario below ignores any backreaction of the probes or the target quantum field on the background

spacetime. However, it shows how both the field and probes have determinate values in this interactive process, and how we can associate this with a classical metric that systems emit or are subjected classically to, which for the theory that we are advocating, can only arise due to these interactions.

Assuming an interaction between the detector and the field that quickly turns on and off, represented by a delta coupling, which occurs at two distinct times $\tau_i = t_1$ and $\tau_i = t_2$, and for timelike separated events, the two-point correlation function between detectors at points x_1 and x_2 is expressed through measurable quantities as:

$$\cos(\Omega\Delta t)\Re\langle\hat{\phi}(x_1)\hat{\phi}(x_2)\rangle+\sin(\Omega\Delta t)\Im\langle\hat{\phi}(x_1)\hat{\phi}(x_2)\rangle=L_{ii}-P_i(x_1)-P_i(x_2), \quad (188)$$

where Ω is the energy gap of the detector, Δt is the time difference between events, and $P_i(x_1)$ and $P_i(x_2)$ are the individual detection probabilities at points x_1 and x_2 , respectively, L_{ii} is the probability that the detectors at two different spacetime points x_1 and x_2 fire together due to their interaction with the quantum field. Assuming point-like detectors again, the correlation function between detectors i and k that are spacelike separated can be given by the expression

$$C(i, k) = 4\lambda^2 \sin(\Omega(t_i + \tau_0)) \sin(\Omega(t_k + \tau_0)) \langle \hat{\phi}(x_i) \hat{\phi}(x_k) \rangle, \quad (189)$$

where $\tau_0 = \frac{\tau_i^0 + \tau_k^0}{2}$, and the proper time at which each detector interacts is labeled $\tau_i = \tau(x_i)$, leading to $z(\tau_i) = x_i$. So, we have a way of inferring the correlators from the probabilities of the detectors having determinate values. Note that pointlike interactions are unphysical and this is only an approximation to the more realistic smeared interaction in spacetime.¹⁰²¹⁰³

Now, to obtain the above first- and second-order derivatives of the correlator, and hence the metric, the key is to *place* an array of probes throughout space. This grid allows the measurement of field correlations across multiple points over time, providing a detailed map of the behavior of the quantum field. More concretely, in this setup, each detector interacts with the quantum field at specific spacetime points, which are labeled by multi-indices corresponding to the coordinates in spacetime. Let us break it down further with an emphasis on the labeling of detectors and their corresponding spacetime positions. The set of

¹⁰²We should regard the delta coupling as a mathematical tool that represents very rapid interactions. This coupling leads to divergences in the models as prior work has investigated [94, 110, 111]. However, these divergences are restricted to the local terms related to each individual interaction and have no impact on the correlations between detectors, which are the primary focus of reconstructing the metric. For instance, if one replaces Dirac deltas with sharply peaked Gaussians, the results for the detector correlations would remain largely unchanged. This approach avoids divergences in the system but increases the complexity of the calculations, which is beyond the scope of this work.

¹⁰³While a perfect delta-coupling interaction is unrealistic (see previous footnote), it serves as an approximation for small systems that can interact with the quantum field over times comparable to the light-crossing time. To model this fast interaction, the delta-coupling assumption leads to the following spacetime test function: $\Lambda_i(x) = \frac{\delta(x-z_i(t_1))}{\sqrt{-g}} + \frac{\delta(x-z_i(t_2))}{\sqrt{-g}}$. Note that the authors also consider temporal smearing.

detectors is parameterized by $j := (j_0, j_1, \dots, j_n)$, where each index corresponds to the detector's position in spacetime. The index j_0 represents the time coordinate, whereas the remaining $j_1, j_2, \dots, j_n$ represent the spatial coordinates. In total, there are N^n detectors, where N is the number of detectors along each spatial direction, and n is the spatial dimension.

The spacetime location of a detector is denoted by its coordinates $x_j^\mu = (x_{j_0}^0, x_{j_1}^1, \dots, x_{j_n}^n)$. Each detector interacts with the quantum field at specific times $x_{j_0}^0$ and spatial positions $(x_{j_1}^1, \dots, x_{j_n}^n)$.

Once interactions between the quantum field and detectors occur, the Wightman function $W(x, x')$, which encodes the two-point correlation between spacetime points, can be computed from the detector readings. The derivative of this function at the positions corresponding to the two detectors labeled j and l is discretized as

$$\begin{aligned} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x'^\nu} W^{\frac{2}{2-D}}(x, x') \Big|_{x=x_j, x'=x_l} &\approx \frac{W^{\frac{2}{2-D}}(x_{j+1\nu}, x_{l+1\mu}) - W^{\frac{2}{2-D}}(x_j, x_{l+1\mu})}{(x_j^{\mu+1\mu} - x_j^\mu)(x_l^{\nu+1\nu} - x_l^\nu)} \\ &\quad - \frac{W^{\frac{2}{2-D}}(x_{j+1\nu}, x_l) - W^{\frac{2}{2-D}}(x_j, x_l)}{(x_j^{\mu+1\mu} - x_j^\mu)(x_l^{\nu+1\nu} - x_l^\nu)}. \end{aligned} \quad (190)$$

Here, the spacetime positions $x_{i+1\mu}^\mu$ and $x_{l+1\nu}^\nu$ refer to the locations of the detectors separated by a small coordinate distance L in the μ - and ν -directions, respectively. Parameter L represents the coordinate separation between the detectors in each direction, including time. This means that the detectors are spaced at regular intervals in both the spatial and temporal directions, allowing for a systematic sampling of the quantum field at different spacetime points. The coordinates of the nearby detectors can then be written as $x_{i+1\mu}^\mu = x_i^\mu + L1_\mu$.

This arrangement of detectors allows us to approximate the derivative of the Wightman function, which is required to recover the spacetime metric. Thus, by measuring the Wightman function at the positions of the detectors, we can obtain the metric tensor in eq. (49). More concretely, by refining the detector grid and taking the limit where $L \rightarrow 0$, we can infer the metric via Eq. (49). The *precision* of the metric recovery depends on the detector spacing and resolution of the measurements.

For instance, in the hyperbolic static Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime example, the metric is expressed in terms of comoving coordinates. Detectors are placed at spacetime intervals L in the η -direction (conformal time) and spatial directions such as χ . The Wightman function for this spacetime was calculated explicitly as follows:

$$W(x, x') = \frac{i\mu(\chi - \chi')H_1^{(2)}(\mu[(\eta - \eta')^2 - (\chi - \chi')^2])}{8\pi a^2 \sinh(\chi - \chi')[(\eta - \eta')^2 - (\chi - \chi')^2]}, \quad (191)$$

and its derivatives were used to recover the metric components by employing a discrete approximation of the Wightman function through detector readings.

The *precision* of the recovery of this metric depends on the detector spacing and resolution of the measurements, where a lower spacing results in a more accurate recovery.

Finally, it was proposed [85] the following setup to recover the spacetime metric using local measurements of a quantum field at different spacetime points: couple local detectors to the target quantum field, measure the correlations between detectors that are located at different spacetime points, and use these correlations to calculate the two-point function of the quantum field concerning the different events and determine the spacetime metric by applying the coincidence limit described in Eq. (49).

Note that it was assumed that the probes were fixed in space and evolved over time, but this is an idealization. Rather, what we consider that occurs are interactions with SDCs give rise to values in an extended region of spacetime. However, systems belonging to SDCs can be approximated as evolving particle detectors in a fixed spatial region (see Section F).

H Decoherence in a de Sitter spacetime; symmetric and Hadamard states

We will adopt the following Gaussian test function emitted by $\mathcal{V}_m$,

$$f(t) = N \exp\left[-\frac{(t - t_c)^2}{2\sigma_t^2}\right], \quad (192)$$

obeying the bound in Section 3.2.1, i.e., $\omega_{max}\sigma_t \gg 1$, where ω_{max} is the maximum energy of the mode involved in these interactions between fields. This test function arises from a field in a $\mathbf{k} = 0$ coherent state, which is a homogeneous and isotropic state. Being a homogeneous and isotropic state is relevant because the system can emit a gravitational field in agreement with the symmetries of the de Sitter spacetime.

The interaction Hamiltonian that we will analyze will be of the form

$$H_{\text{int}}(t, \mathbf{x}) = \mathcal{O}(t, \mathbf{x})\sigma(t, \mathbf{x}), \quad (193)$$

where $\sigma(t, \mathbf{x})$ is the operator that acts on the system's Hilbert space and $\phi(t, \mathbf{x})$ acts on the environment Hilbert space. We consider both quadratic $\mathcal{O}_{\text{mix}} = \mu^2 f(t)\phi(t, \mathbf{x})$ and cubic interactions $\mathcal{O}_c = gf(t)\phi(t, \mathbf{x})^2$. Using Open EFT techniques summarized in Section 5, under the Born approximation, to the second-order in perturbation theory, this yields the following non-Markovian

equation that we want to use to infer the behavior of the system at late times,

$$\begin{aligned} \partial_t \varrho(t) = & -i \int d^3x a^3(t) [\sigma(t, \mathbf{x}), \rho(t)] \langle \mathcal{O}(t, \mathbf{x}) \rangle \\ & - (i)^2 \int d^3x a^3(t) \int d^3y a^3(s) \int_{t_0}^t ds \left\{ [\sigma(t, \mathbf{x}), \sigma(s, \mathbf{y}) \rho(s)] W(t, \mathbf{x}; s, \mathbf{y}) \right. \\ & \left. - [\sigma(t, \mathbf{x}) \rho(s), \sigma(s, \mathbf{y})] W^*(t, \mathbf{x}; s, \mathbf{y}) \right\} + \mathcal{O}(V_{\text{int}}^3), \end{aligned} \quad (194)$$

where

$$W(t, \mathbf{x}; s, \mathbf{y}) = \langle \delta \mathcal{O}(t, \mathbf{x}) \delta \mathcal{O}(s, \mathbf{y}) \rangle, \quad \delta \mathcal{O} = \mathcal{O} - \langle \mathcal{O} \rangle, \quad (195)$$

and $\langle X \rangle = \text{Tr}_{\text{env}}[X \rho_{\text{env}}]$ is the vacuum expectation value. The first-order term merely generates unitary evolution under the Hamiltonian of interaction $V_{\text{eff}} = \langle V_{\text{int}} \rangle$ and therefore cannot produce decoherence. Thus, our attention is directed to the second-order contribution, which yields the dominant decoherence effect.

This expression can be simplified into a Lindblad equation depending on how sharply peaked in time the environmental correlator, which depending on the Hamiltonian of interaction can be expressed as

$$\langle \delta \mathcal{O}(t, x) \delta \mathcal{O}(s, y) \rangle = \mathcal{W}(t, x; s, y) = \begin{cases} \mu^4 W(t, x; s, y) \\ 2g^2 [W(t, x; s, y)]^2 \end{cases} \quad (196)$$

with $W(t, \mathbf{x}; s, \mathbf{y}) = \langle \delta \mathcal{O}(t, \mathbf{x}) \delta \mathcal{O}(s, \mathbf{y}) \rangle$.

An alternative to the expression above requires more than simply expanding in V_{int} using perturbation theory. What is additionally needed is a clear separation of scales, or the so-called hierarchy of scales, which allows us to consider that the bath changes much faster than the system changes (where this change is related with the decoherence timescale τ), and which allows us to implement the Markovian approximation. That separation is provided by the ratio of the Hubble scale (which determines the size of environmental correlations) to the decoherence timescale τ (which depends on $\mu \ll H$ or $g \ll H$). If the correlator $\langle \delta \mathcal{O}(t, x) \delta \mathcal{O}(s, y) \rangle$ decays rapidly for $H|t - s| \gg 1$, the evolution for time intervals exceeding H^{-1} allows the remainder of the integrand of eq. (194) to be expanded as a Taylor series around $s = t$. Successive terms are suppressed given that $(H\partial_t)^n \ll 1$ when acting on what remains of the integrand. This leads to an overall contribution diminished by the powers of $(H\tau)^{-1}$, corresponding to $H^{-1} \ll \tau$. Thus, if the correlators in (196) are sharply peaked, we can expand them, drop the subdominating terms, neglect the memory effects and treat the evolution as Markovian [12],

$$\begin{aligned} \partial_t \varrho(t) \approx & -i [V_{\text{eff}}(t), \varrho(t)] - \int d^3x d^3y a^6(t) \kappa(t, \mathbf{x}, \mathbf{y}) \left[\left\{ \sigma(t, \mathbf{x}) \sigma(t, \mathbf{y}), \varrho(t) \right\} \right. \\ & \left. - 2 \sigma(t, \mathbf{y}) \varrho(t) \sigma(t, \mathbf{x}) \right], \end{aligned} \quad (197)$$

with

$$V_{\text{eff}}(t) = \int d^3x a^3(t) \sigma(t, \mathbf{x}) \langle\langle \mathcal{O}(t, \mathbf{x}) \rangle\rangle + \int d^3x d^3y a^6(t) h(t, \mathbf{x}, \mathbf{y}) \sigma(t, \mathbf{x}) \sigma(t, \mathbf{y}), \quad (198)$$

where we have the expression for the Lamb-shift and dissipator kernels,

$$\kappa(t, \mathbf{x}, \mathbf{y}) = \frac{1}{2} [C(t, \mathbf{x}, \mathbf{y}) + C^*(t, \mathbf{y}, \mathbf{x})] \quad \text{and} \quad h(t, \mathbf{x}, \mathbf{y}) = -\frac{i}{2} [C(t, \mathbf{x}, \mathbf{y}) - C^*(t, \mathbf{y}, \mathbf{x})] \quad (199)$$

and

$$C(t, \mathbf{x}, \mathbf{y}) := \int_{t_0}^t ds \langle\langle \delta \mathcal{O}(t, \mathbf{x}) \delta \mathcal{O}(s, \mathbf{y}) \rangle\rangle, \quad (200)$$

We consider $H^{-1}, \tau \ll \sigma_t$. Given this and the expansion around $s = t$, $f(s) \approx f(t)$, and we also have an approximately constant temporal envelope that we can treat as a constant.

We will not focus on that, but we can have another simplification if the correlation function as a function of position also decreases sufficiently quickly as a function of position. If the falloff is sufficiently steep, the spatial integrals are well-approximated by expanding any fields evaluated at position y in powers of $|y - x|$ and the leading-order evolution equation becomes local in space. In a sense, besides Markovianity (or a notion of “temporal locality”), “spatial locality” can also arise upon decoherence in this picture.

We now express the equation (197) in the k-space.¹⁰⁴ First, note that translation invariance leads to the following expression for the field operator

$$\phi(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} [v_k(t) c_{\mathbf{k}} + v_k^*(t) c_{-\mathbf{k}}^*] e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (201)$$

and similarly for $\sigma(t, \mathbf{x})$ in terms of $a_{\mathbf{k}}, a_{-\mathbf{k}}^*$ and the mode functions $u_{\mathbf{k}}(t)$. As usual, the ladder operators satisfy

$$[a_{\mathbf{p}}, a_{\mathbf{q}}^\dagger] = \delta^3(\mathbf{p} - \mathbf{q}) \quad \text{and} \quad [c_{\mathbf{p}}, c_{\mathbf{q}}^\dagger] = \delta^3(\mathbf{p} - \mathbf{q}). \quad (202)$$

¹⁰⁴We should distinguish between the comoving wavelength and momentum, where the comoving momentum is $k = |\mathbf{k}|$ and the corresponding comoving wavelength is $\lambda_{\text{com}} = \frac{2\pi}{k}$, from the physical wavelength and momentum, which are time-dependent: $\lambda_{\text{phys}}(t) = \frac{2\pi}{p(t)} = \frac{2\pi a(t)}{k}$, with $p(t) = \frac{k}{a(t)}$. The comoving momentum k is most convenient for solving the field equations on an expanding background because each Fourier mode decouples and k remains constant in time. The physical momentum $p(t)$, on the other hand, redshifts with the expansion and is what we compare to physical scales such as the Hubble radius. Hubble crossing occurs when the physical wavelength equals the Hubble radius at the Hubble crossing time t_* (see also the main text below), i.e., $\lambda_{\text{phys}}(t_*) = \frac{1}{H} \iff \frac{2\pi a(t_*)}{k} = \frac{1}{H} \iff k = 2\pi a(t_*) H$. Equivalently, in terms of physical momentum, this crossing condition is: $p(t_*) = \frac{k}{a(t_*)} = 2\pi H$.

Now, we have

$$\begin{aligned} \partial_t \varrho(t) = & -i \left[V_{\text{eff}}(t), \varrho(t) \right] - a^6(t) \int d^3 \mathbf{k} \kappa_{\mathbf{k}}(t) \left[\{ \sigma_{\mathbf{k}}(t) \sigma_{-\mathbf{k}}(t), \varrho(t) \} \right. \\ & \left. - \sigma_{\mathbf{k}}(t) \varrho(t) \sigma_{-\mathbf{k}}(t) - \sigma_{-\mathbf{k}}(t) \varrho(t) \sigma_{\mathbf{k}}(t) \right] \end{aligned} \quad (203)$$

with

$$\sigma_{\mathbf{k}}(t) = \sigma_{-\mathbf{k}}^*(t) = u_{\mathbf{k}}(t) a_{\mathbf{k}} + u_{\mathbf{k}}^*(t) a_{-\mathbf{k}}^\dagger, \quad (204)$$

$$\kappa_{\mathbf{k}}(t) = \int_{t_0}^t ds \Re[W_{\mathbf{k}}(t, s)], \quad (205)$$

with

$$V_{\text{eff}}(t) = (2\pi)^{3/2} a^3(t) \langle\langle \mathcal{O}(t, \mathbf{x}) \rangle\rangle \sigma_{k=0}(t) + \int d^3 k a^6(t) h_{\mathbf{k}}(t) \sigma_{\mathbf{k}}(t) \sigma_{-\mathbf{k}}(t), \quad (206)$$

and

$$h_{\mathbf{k}}(t) = \int_{t_0}^t ds \Im[W_{\mathbf{k}}(t, s)]. \quad (207)$$

Having the right-hand sides of (206) and (198) be quadratic in $\sigma_{\mathbf{k}}$ ensures that there is no mode mixing, so that the state for each momentum mode $\mathbf{k}$ remains uncorrelated as time evolves, provided that this was true of the initial conditions. In particular, if one starts with $\varrho(t_0) = \bigotimes_{\mathbf{k}} \varrho_{\mathbf{k}}(t_0)$, then the factorized form is preserved, $\varrho(t) = \bigotimes_{\mathbf{k}} \varrho_{\mathbf{k}}(t)$, and thus (197) can be written as a separate evolution equation for each mode's density matrix:

$$\begin{aligned} \partial_t \varrho_{\mathbf{k}}(t) = & -i \left[V_{\text{eff}}(t), \varrho_{\mathbf{k}}(t) \right] - a^6(t) \kappa_{\mathbf{k}}(t) \left[\{ \sigma_{\mathbf{k}}(t) \sigma_{-\mathbf{k}}(t), \varrho_{\mathbf{k}}(t) \} \right. \\ & \left. - \sigma_{\mathbf{k}}(t) \varrho_{\mathbf{k}}(t) \sigma_{-\mathbf{k}}(t) - \sigma_{-\mathbf{k}}(t) \varrho_{\mathbf{k}}(t) \sigma_{\mathbf{k}}(t) \right]. \end{aligned} \quad (208)$$

A consequence of (203) and (204), which is quadratic in $\sigma_{\mathbf{k}}$ is that if the system starts as a Gaussian, it remains a Gaussian. Thus, we can solve the evolution of (208) through the following Gaussian ansatz written in the field amplitude basis $\{|\sigma\rangle, |\tilde{\sigma}\rangle\}$,

$$\langle \sigma | \varrho_{\mathbf{k}}(t) | \tilde{\sigma} \rangle = Z_{\mathbf{k}}(t) \exp \left[-A_{\mathbf{k}}(t) \sigma^* \sigma - A_{\mathbf{k}}^*(t) \tilde{\sigma}^* \tilde{\sigma} + B_{\mathbf{k}}(t) \sigma \tilde{\sigma} + B_{\mathbf{k}}^*(t) \sigma^* \tilde{\sigma}^* \right], \quad (209)$$

with the following evolution that is equivalent to the evolution in eq. (208),

$$\begin{aligned} \partial_t A_{\mathbf{k}} = & -\frac{i}{a^3} (A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2) + a^3 \left[i \left(m^2 + \frac{k^2}{a^2} \right) + a^3 h_{\mathbf{k}} \right] + a^6 \kappa_{\mathbf{k}}(t), \\ \partial_t B_{\mathbf{k}} = & -\frac{i}{a^3} (A_{\mathbf{k}} - A_{\mathbf{k}}^*) B_{\mathbf{k}}^* + a^6 \kappa_{\mathbf{k}}(t), \end{aligned} \quad (210)$$

where $A_{\mathbf{k}}(t_0) = A_{\mathbf{k}0}$ and $B_{\mathbf{k}}(t_0) = B_{\mathbf{k}0}$ with $A_{\mathbf{k}0} + A_{\mathbf{k}0}^* = \frac{1}{|u_{\mathbf{k}}|^2}$ and $B_{\mathbf{k}0} = 0$, whose exact solution yields the expression for purity at late times,

$$\gamma_{\mathbf{k}}(t) := \text{Tr}[\rho_{\mathbf{k}}^2(t)] = \int d\sigma d\sigma^* \langle \sigma | \rho_{\mathbf{k}}^2(t) | \sigma \rangle = \frac{1 - \mathcal{R}_{\mathbf{k}}}{1 + \mathcal{R}_{\mathbf{k}}}, \quad \mathcal{R}_{\mathbf{k}} := \frac{\mathcal{B}_{\mathbf{k}} + \mathcal{B}_{\mathbf{k}}^*}{\mathcal{A}_{\mathbf{k}} + \mathcal{A}_{\mathbf{k}}^*}. \quad (211)$$

The evolution of purity at late times is given by

$$\begin{aligned} \partial_t \gamma_{\mathbf{k}} &= 2 \frac{(B_{\mathbf{k}} + B_{\mathbf{k}}^*) \partial_t (A_{\mathbf{k}} + A_{\mathbf{k}}^*) - (A_{\mathbf{k}} + A_{\mathbf{k}}^*) \partial_t (B_{\mathbf{k}} + B_{\mathbf{k}}^*)}{(A_{\mathbf{k}} + A_{\mathbf{k}}^* + B_{\mathbf{k}} + B_{\mathbf{k}}^*)^2} \\ &= - \frac{4 a^6 \kappa_{\mathbf{k}} \gamma_{\mathbf{k}}}{A_{\mathbf{k}} + A_{\mathbf{k}}^* + B_{\mathbf{k}} + B_{\mathbf{k}}^*}. \end{aligned} \quad (212)$$

Through the above equations, it can be shown [12] that at late times ($-k\eta \ll 1$ or in cosmic time, $t \gg t_* + \frac{1}{H}$ with t_* being the Hubble crossing time with $t_* = \frac{1}{H} \ln(\frac{k}{2\pi H})$), given the system initially in a Bunch-Davies vacuum, the purity becomes minimal, and the system decoheres. Furthermore, upon decoherence, we get that $\varrho_{\mathbf{k}}$ becomes a mixture of field amplitude states whose diagonal terms are given by,

$$\langle \sigma | \varrho_{\mathbf{k}} | \sigma \rangle = \mathcal{Z}_{\mathbf{k}} \exp[-(A_{\mathbf{k}}(t) + A_{\mathbf{k}}^*(t) - B_{\mathbf{k}}(t) - B_{\mathbf{k}}^*(t)) |\sigma|^2], \quad (213)$$

with $|\sigma\rangle$ being the field amplitude basis, and where properly normalized, we have

$$\begin{aligned} \varrho_{\mathbf{k}} &= \frac{1}{\pi} \int_{\mathbb{C}} d^2\sigma \left(A_{\mathbf{k}}(t) + A_{\mathbf{k}}^*(t) - B_{\mathbf{k}}(t) - B_{\mathbf{k}}^*(t) \right) \\ &\quad \times \exp \left[- (A_{\mathbf{k}}(t) + A_{\mathbf{k}}^*(t) - B_{\mathbf{k}}(t) - B_{\mathbf{k}}^*(t)) |\sigma|^2 \right] |\sigma\rangle \langle \sigma|. \end{aligned} \quad (214)$$

Note that $A_{\mathbf{k}} + A_{\mathbf{k}}^* - B_{\mathbf{k}} - B_{\mathbf{k}}^*$ is a fixed point of the late-time evolution, which is obtained by solving equations (210), and yields a finite value. This leads to a stochastic process that probabilistically selects one of the terms of this mixture.¹⁰⁵

We can use the above state to calculate the purity at late times for diverse fields. For example, we end up with the following expression for the purity of the target field for the case of a massless environment,

$$\gamma_k(\eta) \simeq \left[1 + \frac{g^2}{32\pi^2 H^2 \nu_{\text{sys}}} |2^{\nu_{\text{sys}}} \Gamma(\nu_{\text{sys}})|^2 (-k\eta)^{-2\nu_{\text{sys}}} \right]^{-1}. \quad (215)$$

We will now show that the state $\varrho(t)$ for the entire system under analysis is homogeneous and isotropic. First, the state of the modes below a certain cutoff, as we have seen above, is the initial state of the entire system under analysis, and is the Bunch-Davies state, and it is a homogeneous and isotropic state. We will not consider the states above the UV cutoff. Now, let us analyze

¹⁰⁵As we have discussed, notice that when we have decoherence, we have approximately Markovian dynamics.

the homogeneity and isotropy of states that involve the state (214) where for this state, $\alpha_k(t) = A_{\mathbf{k}}(t) + A_{\mathbf{k}}^*(t) - B_{\mathbf{k}}(t) - B_{\mathbf{k}}^*(t)$ is a positive real function that depends only on the magnitude $k = |\mathbf{k}|$. The field-amplitude operator has as an eigenvector the field amplitude state $|\sigma\rangle_{\mathbf{k}}$,

$$\sigma_{\mathbf{k}}(t) |\sigma\rangle_{\mathbf{k}} = \sigma |\sigma\rangle_{\mathbf{k}}, \quad (216)$$

which obeys the resolution of the identity

$$\int_{\mathbb{C}} \frac{d^2\sigma}{\pi} |\sigma\rangle_{\mathbf{k}} \langle\sigma| = \mathbb{I}_{\mathbf{k}}. \quad (217)$$

To check translation invariance, let $T(\mathbf{a}) = e^{-i\mathbf{a}\cdot\mathbf{P}}$. It acts on the ladder operators as

$$T(\mathbf{a}) a_{\mathbf{k}} T^\dagger(\mathbf{a}) = e^{-i\mathbf{k}\cdot\mathbf{a}} a_{\mathbf{k}}, \quad T(\mathbf{a}) a_{\mathbf{k}}^\dagger T^\dagger(\mathbf{a}) = e^{+i\mathbf{k}\cdot\mathbf{a}} a_{\mathbf{k}}^\dagger, \quad (218)$$

Similarly, in the case of $-\mathbf{k}$, and hence on the field amplitude operator

$$T(\mathbf{a}) \sigma_{\mathbf{k}}(t) T^\dagger(\mathbf{a}) = e^{-i\mathbf{k}\cdot\mathbf{a}} u_k a_{\mathbf{k}} + e^{+i\mathbf{k}\cdot\mathbf{a}} u_k^* a_{-\mathbf{k}}^\dagger. \quad (219)$$

Acting with $T(\mathbf{a})$ on $\varrho_{\mathbf{k}}(t)$ amounts to replacing each projector $|\sigma\rangle\langle\sigma|$ by $|e^{-i\mathbf{k}\cdot\mathbf{a}}\sigma\rangle\langle e^{-i\mathbf{k}\cdot\mathbf{a}}\sigma|$. Changing variables $\sigma' = e^{-i\mathbf{k}\cdot\mathbf{a}}\sigma$, with $|\sigma'| = |\sigma|$ and unit Jacobian, shows that

$$T(\mathbf{a}) \varrho_{\mathbf{k}}(t) T^\dagger(\mathbf{a}) = \varrho_{\mathbf{k}}(t), \quad (220)$$

for every $\mathbf{a}$. Thus the full state satisfies

$$T(\mathbf{a}) \varrho T^\dagger(\mathbf{a}) = \varrho, \quad (221)$$

which shows that this state is homogeneous.

To check rotation invariance, let $R(\Lambda)$ implement $\mathbf{k} \mapsto \Lambda\mathbf{k}$. Then

$$R(\Lambda) \sigma_{\mathbf{k}}(t) R^\dagger(\Lambda) = \sigma_{\Lambda\mathbf{k}}(t), \quad (222)$$

and acting on each mode's density operator gives

$$R(\Lambda) \varrho_{\mathbf{k}}(t) R^\dagger(\Lambda) = \frac{\alpha_k}{\pi} \int d^2\sigma e^{-\alpha_k |\sigma|^2} |\sigma\rangle_{\Lambda\mathbf{k}} \langle\sigma|_{\Lambda\mathbf{k}}. \quad (223)$$

Relabeling $\mathbf{k}' = \Lambda\mathbf{k}$, doing something similar for the case of $-\mathbf{k}$, and noticing that this Gaussian state depends only on $k = |\mathbf{k}|$ we have that

$$R(\Lambda) \varrho R^\dagger(\Lambda) = \varrho \quad \forall \Lambda \in SO(3), \quad (224)$$

Hence, the state $\varrho(t)$ is isotropic, and the state $\varrho(t)$ is both homogeneous and isotropic.

We want the interaction between systems, some of which belong to SDCs, to give rise to them emitting a gravitational field. This interaction is modeled via decoherence. A key step is to see if the state,

$$\langle\sigma_{\mathbf{k}}(\eta)|\hat{\rho}_{\mathbf{k}}(\eta)|\tilde{\sigma}_{\mathbf{k}}\rangle = Z_{\mathbf{k}}(\eta) \exp \left[-A_{\mathbf{k}}(\eta)|\sigma_{\mathbf{k}}|^2 - A_{\mathbf{k}}^*(\eta)|\tilde{\sigma}_{\mathbf{k}}|^2 + B_{\mathbf{k}}(\eta)\sigma_{\mathbf{k}}\tilde{\sigma}_{\mathbf{k}} + B_{\mathbf{k}}^*(\eta)\sigma_{\mathbf{k}}^*\tilde{\sigma}_{\mathbf{k}}^* \right] \quad (225)$$

upon decoherence at late times is Hadamard, where $Z_{\mathbf{k}}(\eta) = \frac{C_{\mathbf{k}}(\eta)}{\pi}$, $C_{\mathbf{k}}(\eta) = A_{\mathbf{k}} + A_{\mathbf{k}}^* - B_{\mathbf{k}} - B_{\mathbf{k}}^*$, together with its complement valid at higher k . This is because, to have a renormalizable expectation value of the stress energy tensor of a system in the state ρ (which can emit a gravitational field), the state of the system should be Hadamard or at least differ from a Hadamard state by a C^4 function at increasingly lower distances. The above state ρ was derived under the assumption of an approximate Markovian evolution at late times.

To make this analysis, let us consider the adjoint master equation ([12], Appendix D) for a system operator $\mathcal{O}$ at super-Horizon scales, which is related to Eq. (203),

$$\partial_t \langle \mathcal{O} \rangle(t) = i \langle [H_0(t) + V_{\text{eff}}(t), \mathcal{O}] \rangle(t) - a^6(t) \int d^3\mathbf{k} \kappa_{\mathbf{k}}(t) \langle [\sigma_{\mathbf{k}}, [\sigma_{-\mathbf{k}}, \mathcal{O}]] \rangle(t), \quad (226)$$

where H_0 is the free quadratic Hamiltonian of the system, V_{eff} is a renormalized quadratic potential generated by the environment, $\sigma_{\mathbf{k}}$ is the system field operator in Fourier space, and $\kappa_{\mathbf{k}}$ is the noise kernel.

Given the covariance matrix of a single Fourier mode $\mathbf{k}$,

$$\Sigma_{\mathbf{k}}(t, s) := \frac{1}{2} \langle z_{\mathbf{k}}(t) z_{-\mathbf{k}}^T(s) + z_{-\mathbf{k}}(s) z_{\mathbf{k}}^T(t) \rangle - \langle z_{\mathbf{k}}(t) \rangle \langle z_{-\mathbf{k}}(s) \rangle^T, \quad (227)$$

and the phase-space vectors,

$$z_{\mathbf{k}}(t) := \begin{pmatrix} \phi_{\mathbf{k}}(t) \\ \pi_{\mathbf{k}}(t) \end{pmatrix}, \quad \pi_{\mathbf{k}}(t) = a^3(t) \dot{\phi}_{\mathbf{k}}(t), \quad (228)$$

from (226), the transport equation for $\Sigma_{\mathbf{k}}$ can be derived,

$$\partial_t \Sigma_{\mathbf{k}}(t, s) = \Omega H_{\mathbf{k}}(t) \Sigma_{\mathbf{k}}(t, s) - \Sigma_{\mathbf{k}}(t, s) H_{\mathbf{k}}^T(s) \Omega + D_{\mathbf{k}}(t) \delta(t - s), \quad (229)$$

where $H_{\mathbf{k}}(t)$ is the 2×2 Hamiltonian matrix of the homogeneous system evolution, $D_{\mathbf{k}}$ is the diffusion matrix, and with the symplectic matrix

$$\Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (230)$$

Equation (229) is solved in terms of the retarded Green matrix $G_{\mathbf{k}}$ of the homogeneous evolution,

$$\Sigma_{\mathbf{k}}(t, t) = G_{\mathbf{k}}(t, t_0) \Sigma_{\mathbf{k}}(t_0, t_0) G_{\mathbf{k}}^T(t, t_0) + \int_{t_0}^t dt' G_{\mathbf{k}}(t, t') D_{\mathbf{k}}(t') G_{\mathbf{k}}^T(t, t'), \quad (231)$$

where

$$D_{\mathbf{k}}(t) = \begin{pmatrix} 0 & 0 \\ 0 & a^6(t) \kappa_{\mathbf{k}}(t) \end{pmatrix}, \quad \kappa_{\mathbf{k}}(t) = \int_{t_0}^t ds \text{Re } W_{\mathbf{k}}^{\text{env}}(t, s), \quad (232)$$

where we add a superscript to the correlator to emphasize that it concerns one with the states of the environment. Equivalently, in conformal time η we have

$$\kappa_{\mathbf{k}}(\eta) = \int_{\eta_0}^{\eta} d\eta' a(\eta') \text{Re } W_{\mathbf{k}}^{\text{env}}(\eta, \eta'). \quad (233)$$

The Green's matrix is expressed in terms of normalized mode functions $u_{\mathbf{k}}$ and $\pi_{\mathbf{k}}(t)$ as

$$G_{\mathbf{k}}(t, t') = 2 \begin{pmatrix} \text{Im}[u_{\mathbf{k}}(t) \pi_{\mathbf{k}}^*(t')] & -\text{Im}[u_{\mathbf{k}}(t) u_{\mathbf{k}}^*(t')] \\ \text{Im}[\pi_{\mathbf{k}}(t) \pi_{\mathbf{k}}^*(t')] & -\text{Im}[\pi_{\mathbf{k}}(t) u_{\mathbf{k}}^*(t')] \end{pmatrix}, \quad (234)$$

and in particular

$$G_{12}(t, t') = -2 \text{Im}[u_{\mathbf{k}}(t) u_{\mathbf{k}}^*(t')] = i(u_{\mathbf{k}}(t) u_{\mathbf{k}}^*(t') - u_{\mathbf{k}}^*(t) u_{\mathbf{k}}(t')). \quad (235)$$

The field two-point function at equal times is the $(1, 1)$ entry of the covariant matrix,

$$W_{\mathbf{k}}(t) := \langle \phi_{\mathbf{k}}(t) \phi_{-\mathbf{k}}(t) \rangle = [\Sigma_{\mathbf{k}}(t)]_{11}. \quad (236)$$

Using (231)–(234), one obtains the equal-time decomposition

$$W_{\mathbf{k}}(t) = |u_{\mathbf{k}}(t)|^2 + \Delta W_{\mathbf{k}}(t), \quad \Delta W_{\mathbf{k}}(t) = \int_{t_0}^t dt' a^6(t') \kappa_{\mathbf{k}}(t') [G_{12}(t, t')]^2. \quad (237)$$

A commonly used test to determine whether a state is Hadamard involves calculating the unequal time correlation function and comparing the two-point correlation function of the state under analysis with the two-point correlation function of another Hadamard state (such as the Bunch–Davies vacuum), and determining whether their difference (i.e., $W_{\psi}(t, x; t, x') - W_{BD}(t, x; t, x')$) yields a smooth function when $x \rightarrow x'$ and $t \rightarrow t'$. However, a Hadamard test that is more convenient to implement in our case rather compares $W_{\psi}(t, x; t, x') - W_{BD}(t, x; t, x')$ at a single t . But, to implement this test we should also compare $\partial_t(W_{\psi}(t, x; t', x') - W_{BD}(t, x; t', x'))_{t=t'}$ and $\partial_t \partial_{t'}(W_{\psi}(t, x; t', x') - W_{BD}(t, x; t', x'))_{t=t'}$ at some given time slice, and evaluate whether they are smooth. If this is the case, a quasi-free state $|\psi\rangle$ is Hadamard since $W_{\psi}(t, x; t, x') - W_{BD}(t, x; t, x')$ satisfies the equation of motion in both (t, x) and (t', x') . This feature is because smooth initial data for the equation of motion imply a smooth solution.¹⁰⁶

$|u_{\mathbf{k}}(t)|^2$ already concerns the two-point function of the Bunch–Davies vacuum, which we aimed to subtract to evaluate whether the state is Hadamard. Thus, we need to show that $\Delta W_{\mathbf{k}}(\eta)$ in the position space is a C^∞ at low distances, and adopting a strategy explained above for the linear and cubic coupling, which corresponds to the different ways the two-point function can evolve considered in this paper. Since ρ concerns super-Horizon scales, we will focus on the case where $|k\eta| \ll 1$ first.

To evaluate whether $\Delta W_{\mathbf{k}}(\eta)$ is smooth at lower distances, let us then find the expressions for G_{12} and $\kappa_{\mathbf{k}}$ at super-Horizon scales. Given these expressions, we will evaluate whether $\Delta W_{\mathbf{k}}(\eta)$ and its derivatives in order of r are bounded

¹⁰⁶A slight subtlety here is that in a general spacetime, the singular behavior of equal-time correlation functions of Hadamard states does depend on the geometry in a neighborhood of such an equal-time surface (for instance via time derivatives of the metric), and for this reason, checking that a state is Hadamard from its equal-time correlation functions is not always convenient. However, if we already know the correlation functions of a reference Hadamard state (as we do), this subtlety is already taken care of.

by smooth functions to verify the smoothness of $\Delta W_{\mathbf{k}}(\eta)$ at lower distances r . We will also want to do the same to the following temporal derivatives of $\Delta W_{\mathbf{k}}(\eta)$,

$$\partial_{\eta} \Delta W_{\mathbf{k}}(\eta) = 2 \int_{\eta_0}^{\eta} d\eta'' a^7(\eta'') \kappa_{\mathbf{k}}(\eta'') G_{12}(\eta, \eta'') \partial_{\eta} G_{12}(\eta, \eta''). \quad (238)$$

$$\partial_{\eta'} \partial_{\eta} \Delta W_{\mathbf{k}}(\eta, \eta') \Big|_{\eta'=\eta} = \int_{\eta_0}^{\eta} d\eta'' a^7(\eta'') \kappa_{\mathbf{k}}(\eta'') [\partial_{\eta} G_{12}(\eta, \eta'')] [\partial_{\eta'} G_{12}(\eta', \eta'')] \Big|_{\eta'=\eta}. \quad (239)$$

The mode function for a free scalar in de Sitter spacetime is $u_{\mathbf{k}}(\eta) = \frac{\sqrt{\pi}}{2} H e^{i(\nu+\frac{1}{2})\pi/2} (-\eta)^{3/2} H_{\nu}^{(1)}(-k\eta)$ where $\nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2} - 12(\xi - \frac{1}{6})}$, H is the Hubble constant, m is the mass, and ξ is the curvature coupling. The parameter ν may correspond to either the system or the environment. On the super-horizon scales, given $z = -k\eta$ with $|z| \ll 1$, the expansion of the Hankel function for the system s yields

$$H_{\nu_s}^{(1)}(z) = A_s z^{-\nu_s} (1 + a_1 z^2 + O(z^4)) + B_s z^{\nu_s} (1 + b_1 z^2 + O(z^4)), \quad (240)$$

with coefficients

$$\begin{aligned} A_s &= -\frac{i}{\pi} 2^{\nu_s} \Gamma(\nu_s), \\ B_s &= \frac{1 + i \cot(\pi \nu_s)}{\Gamma(\nu_s + 1)} 2^{-\nu_s}. \end{aligned} \quad (241)$$

and with $a_1 = -\frac{1}{4(1-\nu_s)}$ and $b_1 = -\frac{1}{4(\nu_s+1)}$ and we keep the first and second-order terms of the expansion above.

Hence the system's mode functions can be approximated as

$$u_{\mathbf{k}}(\eta) = C_s |\eta|^{3/2} [A_s z^{-\nu_s} + B_s z^{\nu_s}] + C_s |\eta|^{3/2} O(z^{-\nu_s+2}, z^{\nu_s+2}), \quad C_s = \frac{\sqrt{\pi}}{2} H e^{i(\nu_s+\frac{1}{2})\pi/2}. \quad (242)$$

Moreover, given (235),

$$\begin{aligned} G_{12}(\eta, \eta') &= -2 \Im[u_{\mathbf{k}}(\eta) u_{\mathbf{k}}^*(\eta')], \\ \partial_{\eta} G_{12}(\eta, \eta') &= -2 \Im[u'_{\mathbf{k}}(\eta) u_{\mathbf{k}}^*(\eta')], \\ \partial_{\eta} \partial_{\eta'} G_{12}(\eta, \eta') &= -2 \Im[u'_{\mathbf{k}}(\eta) u'_{\mathbf{k}}^*(\eta')]. \end{aligned} \quad (243)$$

Differentiating (242) and using $z = -k\eta$ with $dz/d\eta = -k$ gives

$$\begin{aligned} u'_{\mathbf{k}}(\eta) &= C_s \frac{3}{2} |\eta|^{1/2} [A_s z^{-\nu_s} + B_s z^{\nu_s}] + C_s |\eta|^{3/2} (-k) [A_s (-\nu_s) z^{-\nu_s-1} + B_s (\nu_s) z^{\nu_s-1}] \\ &\quad + C_s |\eta|^{3/2} O(z^{-\nu_s+1}, z^{\nu_s+1}) (-k), \end{aligned} \quad (244)$$

Thus, by differentiating the mode functions, and using (243) it can be seen that

$$\begin{aligned} G_{12}(\eta, \eta') &= \mathcal{C}_{\text{sys}}(\eta, \eta') + O((k\eta)^2, (k\eta')^2), \\ \partial_\eta G_{12}(\eta, \eta') &= \mathcal{C}_{\text{sys}}^{(1)}(\eta, \eta') + O((k\eta)^2, (k\eta')^2), \\ \partial_\eta \partial_{\eta'} G_{12}(\eta, \eta') &= \mathcal{C}_{\text{sys}}^{(2)}(\eta, \eta') + O((k\eta)^2, (k\eta')^2). \end{aligned} \quad (245)$$

for some smooth functions $\mathcal{C}_{\text{sys}}(\eta, \eta')$, $\mathcal{C}_{\text{sys}}^{(1)}(\eta, \eta')$, and $\mathcal{C}_{\text{sys}}^{(2)}(\eta, \eta')$. Thus,

$$|G_{12}(\eta, \eta')| \leq \mathcal{C}_{\text{sys}}(\eta, \eta'), \quad (246)$$

is bounded by a k -independent function, and similarly for its derivatives.

Let us turn to $\kappa_{\mathbf{k}}$. For the linear coupling $O(\chi) = \mu^2 \chi$ [12],

$$W_{\mathbf{k}}^{\text{env}}(\eta, \eta') = u_{\mathbf{k}}^{\text{env}}(\eta) u_{\mathbf{k}}^{\text{env}*}(\eta') = \frac{\pi H^2}{4} (\eta \eta')^{3/2} H_{\nu_{\text{env}}}^{(1)}(-k\eta) [H_{\nu_{\text{env}}}^{(1)}(-k\eta')]^*. \quad (247)$$

Writing $\alpha := \Re \nu_{\text{env}}$, for $\alpha > 0$, we obtain

$$\text{Re } W_{\mathbf{k}}^{\text{env}}(\eta, \eta') = C(\nu_{\text{env}}) H^2 k^{-2\alpha} |\eta|^{\frac{3}{2}-\alpha} |\eta'|^{\frac{3}{2}-\alpha} + O(k^{-2\alpha+2}) \quad (248)$$

with $a(\eta') = -1/(H\eta')$ and where $C(\nu_{\text{env}})$ is a function that depends on ν_{env} , which leads to

$$\begin{aligned} \kappa_{\mathbf{k}}^{\text{lin}}(\eta) &= \int_{\eta_0}^{\eta} d\eta' a(\eta') \text{Re } W_{\mathbf{k}}^{\text{env}}(\eta, \eta') \\ &= C(\nu_{\text{env}}) H k^{-2\alpha} |\eta|^{\frac{3}{2}-\alpha} \int_{\eta_0}^{\eta} d\eta' |\eta'|^{\frac{1}{2}-\alpha} + O(k^{-2\alpha+2}), \end{aligned} \quad (249)$$

and where the η' -integral is

$$\int_{\eta_0}^{\eta} d\eta' |\eta'|^{\frac{1}{2}-\alpha} = \begin{cases} \frac{|\eta|^{\frac{3}{2}-\alpha} - |\eta_0|^{\frac{3}{2}-\alpha}}{\frac{3}{2}-\alpha}, & \alpha \neq \frac{3}{2}, \\ \log(|\eta|/|\eta_0|), & \alpha = \frac{3}{2}. \end{cases} \quad (250)$$

Hence, for $|k\eta| \ll 1$ and $\alpha > 0$,

$$|\kappa_{\mathbf{k}}^{\text{lin}}(\eta)| \leq C^{\text{lin}}(\eta) k^{-2\alpha}. \quad (251)$$

For $\alpha = 0$, we have $\nu_{\text{env}} = i\mu$, and we obtain

$$\kappa_{\mathbf{k}}^{\text{lin}}(\eta) = \int_{\eta_0}^{\eta} a(\eta') \Re W_{\mathbf{k}}^{\text{env}}(\eta, \eta') d\eta' = H |\eta|^{3/2} \int_{\eta_0}^{\eta} |\eta'|^{1/2} \mathcal{B}(\ln(k\eta), \ln(k\eta')) d\eta' + O(k^2). \quad (252)$$

Here $\mathcal{B}(\cdot, \cdot)$ denotes a bounded oscillatory function. So, for $\nu_{\text{env}} = i\mu$

$$|\kappa_{\mathbf{k}}^{\text{lin}}(\eta)| \leq C^{\text{lin}}(\eta). \quad (253)$$

For the cubic coupling $O(\chi) = g\chi^2$,¹⁰⁷

$$W_{\mathbf{k}}^{\text{env}}(\eta, \eta') = 2g^2 \int \frac{d^3 p}{(2\pi)^3} W_{|\mathbf{k}-\mathbf{p}|}^{\text{env}}(\eta, \eta') W_p^{\text{env}}(\eta, \eta'). \quad (254)$$

Given the asymptotic form of the Hankel function for $|k\eta| \ll 1$,¹⁰⁸ considering the dominant term, and Rescaling $\mathbf{p} = k\mathbf{q}$ isolates the k scaling inside $W_{|\mathbf{k}-\mathbf{p}|}^{\text{env}}$ for $\alpha > 0$:

$$\int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^{2\alpha} |\mathbf{k} - \mathbf{p}|^{2\alpha}} = k^{3-4\alpha} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{q^{2\alpha} |\hat{\mathbf{1}} - \mathbf{q}|^{2\alpha}}, \quad (255)$$

which is finite for $\frac{3}{4} < \alpha < \frac{3}{2}$, IR divergent at $\alpha = \frac{3}{2}$, which can be regularized, and UV-divergent for $\alpha = \frac{3}{4}$, which can be renormalized. Given also the equations above,¹⁰⁹ we obtain the following dependence

$$|\Delta W_k(\eta)| \lesssim \begin{cases} C_*^{\text{lin}}(\eta) \times \begin{cases} k^{-2\alpha}, & \mu = 0, \alpha > 0, \\ 1, & \alpha = 0, \mu > 0, \end{cases} \\ C_*^{\text{cub}}(\eta) \times \begin{cases} k^{3-4\alpha}, & \mu = 0, 0 < \alpha < \frac{3}{2}, \alpha \neq \frac{3}{4}, \\ |\ln(k^2/\bar{\mu}^2)|, & \mu = 0, \alpha = \frac{3}{4}, \\ k^{-3}[1 + |\ln(\Lambda/k)|], & \mu = 0, \alpha = \frac{3}{2}, \\ k^3, & \alpha = 0, \mu > 0, \end{cases} \end{cases} \quad (256)$$

where Λ is a cutoff and $\bar{\mu}$ is the renormalization scale.

We will now prove that $\Delta W_{\mathbf{k}}$ in the position space and its first and second η -derivatives are smooth as $r \rightarrow 0$. Let $r := x - x'$, at fixed η , we have the following Fourier transform of the IR part of the correlation function,

$$\Delta W(\eta; x, x') = \int_{|\mathbf{k}| \leq K_*} \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{r}} \Delta W_{\mathbf{k}}(\eta), \quad (|k\eta| \ll 1). \quad (257)$$

As we will see below, the complementary UV piece of this integral is the two-point function concerning the Bunch-Davies vacuum, which does not pose problems. To prove that $\Delta W(\eta; x, x')$ and its first and second η -derivatives are C^∞ in r , it suffices to justify that for every index $\beta \in \mathbb{N}$, the integral

$$\partial_r^\beta \Delta W(\eta; x, x') = \int_{|\mathbf{k}| \leq K_*} \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{r}} (i\mathbf{k})^\beta \Delta W_{\mathbf{k}}(\eta), \quad (258)$$

¹⁰⁷See eq. (A.17) in [12].

¹⁰⁸The asymptotic forms of the Hankel function are given for small z by $H_\nu^{(1)}(z) \simeq (\frac{z}{2})^{-\nu} \left[-\frac{i\Gamma(\nu)}{\pi} + \mathcal{O}(z^2) \right] + (\frac{z}{2})^\nu \left[\frac{1+i\cot(\pi\nu)}{\Gamma(\nu+1)} + \mathcal{O}(z^2) \right]$, where $z = k\eta$ (eq. C.12 in [12]).

¹⁰⁹And the mathematical identity:

$$\int \frac{d^d p}{(2\pi)^d} (p^2)^{-\alpha} [(\mathbf{k} - \mathbf{p})^2]^{-\beta} = \frac{(k^2)^{\frac{d}{2}-\alpha-\beta} \Gamma(\frac{d}{2}-\alpha) \Gamma(\frac{d}{2}-\beta) \Gamma(\alpha+\beta-\frac{d}{2})}{(4\pi)^{d/2} \Gamma(\alpha) \Gamma(\beta) \Gamma(d-\alpha-\beta)}.$$

is absolutely convergent. Furthermore, we want to evaluate the case where $r \rightarrow 0$, and thus

$$\lim_{r \rightarrow 0} \int_{|\mathbf{k}| \leq K_*} \frac{d^3 k}{(2\pi)^3} e^{i \mathbf{k} \cdot \mathbf{r}} (i \mathbf{k})^\beta \Delta W_{\mathbf{k}}(\eta) = \int_{|\mathbf{k}| \leq K_*} \frac{d^3 k}{(2\pi)^3} (i \mathbf{k})^\beta \Delta W_{\mathbf{k}}(\eta). \quad (259)$$

Thus a sufficient condition for (258) as $r \rightarrow 0$ is

$$\int_{|\mathbf{k}| \leq K_*} d^3 k |\mathbf{k}|^{|\beta|} |\Delta W_{\mathbf{k}}(\eta)| < \infty \quad \text{for all } \beta \in \mathbb{N}. \quad (260)$$

Given the spherical coordinates and (256), (260) reduces to

$$4\pi \int_0^{K_*} k^{2+m} B(k) dk < \infty, \quad m := |\beta|, \quad (261)$$

where $B(k)$ denotes the scalings that bound $|\Delta W_k(\eta)|$ derived above. We now evaluate (261).

In the case of the linear coupling, from (256) we have that the integrands are $k^{2+m-2\alpha}$ or k^{2+m} . In the $\mu = 0, \alpha > 0$, the integral $\int_0^{K_*} k^{2+m-2\alpha} dk$ converges iff

$$2 + m - 2\alpha > -1 \iff \alpha < \frac{3+m}{2}. \quad (262)$$

For $\alpha = 0, \mu > 0$ the integral is obviously finite: $4\pi C_*^{\text{lin}}(\eta) K_*^{3+m}/(3+m)$.

In the case of the cubic coupling, from (256) we have that multiplying by k^{2+m} gives:

- (i) $k^{5+m-4\alpha}$ is integrable iff $5 + m - 4\alpha > -1$, i.e. $\alpha < \frac{6+m}{4}$;
- (ii) $k^{2+m} |\ln(k^2/\mu^2)|$ is integrable for every $m \geq 0$;
- (iii) $k^{m-1} [1 + |\ln(\Lambda/k)|]$ diverges for $m = 0$ and is finite for $m \geq 1$;
- (iv) k^{5+m} is integrable for all $m \geq 0$.

Therefore (261) holds for all m in every cubic case except $\alpha = \frac{3}{2}$, where it already fails at $m = 0$. However, this case already has IR pathologies that could be solved via regularization.

Thus, under the above conditions $\Delta W(\eta; x, x')$ is C^∞ in r as $r \rightarrow 0$. The same argument applies to $\partial_\eta \Delta W_k(\eta)$ and $\partial_\eta \partial_{\eta'} \Delta W_k(\eta, \eta')|_{\eta'=\eta}$ because their super-horizon k -scalings coincide with that of ΔW_k . Therefore $\partial_\eta \Delta W(\eta; x, x')$ and $\partial_\eta \partial_{\eta'} \Delta W(\eta, \eta'; x, x')|_{\eta'=\eta}$ are also C^∞ in r .

Given the super-Horizon scales, the Gaussian state above concerns k_{phys} up until $k_{phys} \leq K_*/a(\eta_*)$ where η_* is some late time. Regarding the case where $k_{phys} > K_*/a(\eta_*)$ (i.e., the UV tail that goes up until a cutoff that we choose to omit), this concerns the sub-Horizon scales with at least approximately

$\rho_{UV}^{BD} = |\Omega_{BD}\rangle\langle\Omega_{BD}|$. Thus, at least approximately we will consider that the overall state,

$$\rho(t) \approx \rho_{IR}(t) \otimes \rho_{UV}^{BD}(t) \quad (263)$$

is one whose UV tail coincides with the Bunch-Davies vacuum (which has the singularity structure that characterizes a Hadamard state), with $\text{Tr}_{UV} \rho(t) = \rho_{IR}(t)$, $\text{Tr}_{IR} \rho(t) = \rho_{UV}^{BD}(t)$, and the different modes in the $\rho_{IR}(t)$ are equal to the state in (214), which should then be represented as continuum of modes. Therefore, the Gaussian state (225) and its complement valid at the UV scale, form a state $\rho(t)$ that is a Hadamard state.

I Accounting for the universe's accelerated expansion and inflation simultaneously via a time-varying dark energy

We will now present briefly a toy model to explain how this theory may account for the accelerated expansion of the universe and inflation simultaneously via a time-varying dark energy. The goal is to provide further arguments in favor of this theory, and our derivation of the cosmological constant with its time-varying features and dependence on the four-volume of the universe in a past light cone.¹¹⁰

Consider the following FLRW metric,

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (264)$$

where t is the cosmic time, $a(t)$ is the scale factor, k is the spatial curvature constant, where $k = 0$ (flat), $k = +1$ (closed), $k = -1$ (open). (r, θ, ϕ) are the comoving spatial coordinates.

According to this theory, we can assume that at $t = 0$, no quantum systems that belong to SDCs interacted, and because the gravitational field arises from these interactions, the FLRW metric is not applicable. Since there are no interactions and we are modeling the whole universe, no metric except the flat metric is applicable,

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2, \quad (265)$$

or more realistically a small perturbation around it as we will see.

For simplicity, we can assume that at $t = 0$, we have only a target real scalar field ϕ_1 and a set of probes that have the DC concerning ϕ_1 (DC- ϕ_1). Additionally, since interactions need to be localized by some other field, we can assume that at $t=0$ we have a system that is in a state that emits a test function to

¹¹⁰We will be assuming that the dark energy-like effects of Λ still hold in the early universe given by the equation (76), and that most systems with determinate values in the early universe are in a coherent state so that we can implement the assumptions in Section 7 to estimate $\Delta\Lambda$. More on this at the end of this section.

the interactions between ϕ_1 and the other probes, and emits a gravitational that is extremely weak while in that state. Such a gravitational field generates only a small perturbation around the Minkowski spacetime. At this point, we have at least two options. One of them assumes the cosmological constant as a brute fact still arising from SDCs. Once the systems start interacting, the cosmological constant kicks in as well (sourced by these systems), and the description of this scenario would be given via the Einstein Field Equations effectively or the semiclassical equations in agreement with the standard cosmological models. One of the issues with this option is that we would also need to assume the inflaton field or some other field that explains the accelerated early expansion of the universe, and the usual inflationary story has issues.¹¹¹

A second option does not require postulating an additional field to explain the expansion of the universe. It rather postulates a time-varying cosmological constant, as explained in Section 7. We will focus on this option, which is as follows: once we have the first interaction between the ϕ_1 and the probes, a small four-volume will arise. Given that, in Planck units and given $\Lambda := \Delta\Lambda$,

$$\Lambda \sim \frac{1}{\Delta V}, \quad (266)$$

we will obtain a high value of the cosmological constant, and therefore, a rapid expansion of the universe. We suppose that when ϕ_1 , or its mode, in a homogeneous and isotropic state has a determinate energy-momentum tensor, it gives rise to a perfect fluid that leads to the FLRW metric (see Sections 5 and Appendix H), possibly together with the probes. Then, we can run the story presented in Sections 5 and 7 by assuming some initial systems that have the DC concerning some other systems.

Note that posing such special initial conditions at the beginning of the universe that we have postulated above is common in cosmology. However, we think that we may end up, under a more realistic and detailed model, having an advantage compared with these other theories because we do not have to postulate dark energy as a primitive or the inflaton field. So, the prospects of this proposal are positive in terms of in the future providing similar benefits to inflation without its issues. To understand why we think this is the case, let us consider the two main problems that inflation claims to solve: the flatness and horizon problems.

The flatness problem arises from the observation that the current universe appears very close to being spatially flat (i.e., having zero curvature). Consider the Friedmann equation that governs the expansion of the universe, which can be derived from the FLRW metric and the Einstein Field Equations with a perfect fluid as a source:

$$H^2 = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}(\rho_M + \rho_R) - \frac{\kappa c^2}{a(t)^2} \quad (267)$$

¹¹¹Another option is that de Sitter spacetime is the default geometry of spacetime and is not sourced by any systems. See Section 4.3.

where H is the Hubble parameter, which measures the expansion rate of the universe. The terms ρ_M and ρ_R represent the energy densities of the matter and radiation, respectively. The parameter κ represents the curvature of the universe, with $\kappa = 0$ for a flat universe, $\kappa > 0$ for a closed universe, and $\kappa < 0$ for an open universe. The scale factor, a , roughly describes the size of the universe at a given time.

The curvature term $-\kappa c^2/a(t)^2$ falls off as a^{-2} , while the energy densities of matter and radiation decay more rapidly with the scale factor. Specifically, $\rho_M \propto a^{-3}$ for matter and $\rho_R \propto a^{-4}$ for radiation. This seems to imply that as the universe expands and the scale factor a increases, the relative contribution of the curvature term becomes increasingly dominant over the energy densities of matter and radiation. Thus, the fact that we observe the universe to be so close to flat today suggests that the universe must have been very finely tuned to be near flat in the early universe. This is because any small deviation from flatness would have grown over time, making the universe today either highly curved or very open.

The horizon problem is roughly the following: if we observe two widely separated parts of the Cosmic Microwave Background (CMB), we will see that we have distinct patches of the CMB that were causally disconnected at recombination (i.e., the period when protons and electrons combined to become atoms of hydrogen). However, we observe with high precision that they have a similar temperature. The problem is to explain how they have the same temperature if they were never in causal contact.

Now, let us turn to the Friedmann equation with the cosmological constant,

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho_{\text{matter/radiation}} - \frac{\kappa c^2}{a(t)^2} + \frac{\Lambda c^2}{3}, \quad (268)$$

Let us consider that in the beginning of the universe, $\Lambda \gg 1$ because of the small four-volume, and that we can treat Λ approximately as a constant in this short period, and so this model is effective. In the early universe, due to its small volume and the (determinate) energy density of matter/radiation being low (because not many systems with determinate values are arising), it is thus plausible that

$$\frac{\Lambda c^2}{3} \gg \frac{8\pi G}{3}\rho, \kappa c^2. \quad (269)$$

Then, we obtain that

$$a(t) \approx Ae^{\sqrt{\frac{\Lambda c^2}{3}}t}. \quad (270)$$

where A is a constant of integration.¹¹²

This exponential expansion is similar to the exponential expansion predicted by inflation. This expansion, in principle, will allow this theory to address the horizon problem. The explanation is as follows: before the onset of inflation, the universe was much smaller and denser. During this phase, the entire region

¹¹²Note that the scale factor can be very small in the early universe, but the cosmological constant can be arbitrarily very large in such a way that it compensates for that.

that would later become the observable universe was contained within a single causally connected patch. This implies that any two points within this region could influence each other and reach thermal equilibrium. Exponential expansion stretched these regions beyond the current particle horizon. The particle horizon is the maximum distance from which particles can travel to an observer in the age of the universe. This means that regions that were once close enough to interact and equilibrate went far apart, beyond each other's ability to *communicate*. In the case of this theory, this exponential expansion is due to the SDCs.

We now turn to a sketch of the potential resolution of the flatness problem. To see how this theory might be able to deal with this problem, let us rewrite the Friedmann equation in the way below,

$$\Omega_{total} - 1 = \frac{kc^2}{a(t)^2 H^2} \quad (271)$$

where $\Omega(t) = \frac{\rho(t)}{\rho_{crit}(t)}$ with $\rho_{crit}(t)$ being the critical density defined as $3\tilde{m}_P^2 H^2(t)$, and we consider ρ to include the dark energy density. When the actual and critical densities are equal, the geometry of the universe is flat. Thus, we consider that $\Omega = \Omega_{radiation} + \Omega_{matter} + \Omega_{\Lambda}$ (note that following the standard approach, we are including dark energy as part of the energy density of the universe). As we can see, in order for the universe to be flat ($k = 0$), $\Omega_{total} = 1$. Since $a(t) \approx e^{\sqrt{\frac{\Lambda c^2}{3}}t}$, with enough e-folds, the early Friedmann universe, in principle, can become flat regardless of the initial densities of matter/energy.

Another problem that we will not go into deeply here, which inflation addresses, is the following: inflation is typically considered to have been driven by a scalar field ϕ which is the inflaton. It is hypothesized that the zero-point fluctuations of the quantized inflaton scalar field in some regions (i.e., fluctuations of the field in the vacuum state) and the associated energy-momentum fluctuations and gravitational field, amplified by the rapid expansion of inflation, attracted more matter than in other regions. Then, it is hypothesized that this phenomenon gave rise to the unevenly distributed cosmic structure in our universe (e.g., galaxies, galaxy clusters, etc.) [67]. This explanation can, in principle, also be given via the above picture if we take into account that SDCs involve quantum fields that are subject to quantum fluctuations, which, upon stochastic processes, give rise to inhomogeneous states, as we have seen in Section 5 with eq. (67).

Furthermore, note that the inflaton field is often treated classically, and the effects of these fluctuations are observed via slight temperature anisotropies in the Cosmic Microwave Background. There is also the problem of explaining how these quantum fluctuations became classical during the early stages of the evolution of the universe. Adopting this theory helps address this problem, given that SDCs involve indeterministic processes that give rise to classicality. Furthermore, although this theory proposes a time-varying cosmological constant, current evidence points towards a time-varying dark energy, as mentioned previously.

Whether this approach to the early universe cosmology may end up being better than competing approaches will need to be settled via a more physically realistic and detailed model; however, we believe that it is a promising one.

Several models impose a varying cosmological constant, such as quintessence models [118], and attempt to unify inflation and dark energy, such as inflationary quintessence models. However, to our knowledge, none have predicted the precise value of the cosmological constant based on quantum theory and a conservative approach. For example, quintessence models add a new quantum field and hence a new particle (so far unobserved). This theory just starts from the basic principles of quantum theory. Moreover, it addresses the measurement problem, including the measurement problem that occurs right at the beginning of the universe. More concretely, note that in models based on the inflaton or some other field, one must explain why (loosely speaking) there was a collapse of the quantum state at the beginning of the universe to account for the inhomogeneities of matter distribution that gave rise to cosmic structures. Otherwise, all inflation gives us is a superposition of quantum states that does not lead to a single cosmic structure. Decoherence per se, which many appeal to in order to solve this problem, does not solve the problem because it is a vaguely defined physical process. This theory, in principle, does not fall into this problem because it establishes clear criteria for when determinate values arise. Furthermore, if we adopt this approach, we do not need to fall into the issues of eternal inflation and the multiverse problem that plagues inflation.

Future work should develop a more accurate cosmological model that can address the cosmological singularity problem. Our toy model above already indicates how this might be done. We would assume an asymptotically flat spacetime in the early universe, where the activity of SDCs would slow down towards the beginning of the universe in terms of giving rise to a gravitational field. Indeed, some alternative inflationary cosmological models exist in which the universe starts expanding from Minkowski spacetime (see [76] and references therein). Furthermore, future work should develop empirical signatures of this theory in the Cosmic Microwave Background. Finally, note that our dark energy cancellation hypothesis (Section 5) allows for significant fluctuations in the stress-energy tensor in the early universe because the early universe has a very small past four-volume and a large $\Delta\Lambda$, balancing and canceling these fluctuations. Future work should examine the size of the fluctuations in the stress-energy tensor that could be canceled.

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