Review of "Closing the Hole Argument", by Hans Halvorson and J.B. Manchak (*British Journal for the Philosophy of Science* 76, 2025)

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For *Mathematical Reviews*

This article attempts to crack the toughest of tough nuts—*viz.*, the hole argument of general relativity (GR).[1] Here are its two central claims:

1. Weatherall (2018) has "convincingly argued" that the existence of distinct but isomorphic (i.e. isometric) models of GR "does not support the hole argument" (p. 295).

2. There is nevertheless a worry that, were there to exist multiple distinct isometries relating two isomorphic models of GR (that restrict to the same isometry on an open subset of their domain), the hole argument could re-arise. But in fact a theorem proved by Geroch (1969) rules this out, so the hole argument is thereby "closed".

Ad (1): what the authors have in mind, I take it, is what Pooley and Read (2025) call Weatherall's 'argument from mathematical structuralism'—namely, that it is mandatory to use only maps which witness the isometry of two isometric models of GR as standards of cross-model identification of mathematical points; if one does this, then (the claim goes) GR does not 'generate' a philosophical problem (of hole indeterminism).

I beg to differ that Weatherall (2018) has "convincingly argued" this point; in particular, I think one can push back in at least the following ways:

i. Given that, on standard set-theoretic foundations, models of GR—i.e. Lorentzian manifolds[2]—just are structured sets, it's not *prima facie* obvious (or at least requires some more sustained and systematic argumentation) that using maps between isometric models of GR which are not

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[1] I won't repeat the details of the argument here, but see Norton et al. (2023) and Pooley (2021) for recent reviews.

[2] At least if we set aside the stress-energy of material fields as implicitly definable from the metric via the Einstein equation, as authors typically do in these discussions.

isometries is to be implicated in "semantic ascent" (p. 314), to transcend the "language of general relativity" (p. 314), etc.[3]

ii. When one looks to mathematical and/or physical practice, it's also not obvious that practitioners don't avail themselves of such resources, which makes such stringent restrictions on what counts as GR look specious. (See Cudek (forthcoming, §5), Gomes and Butterfield (2023, §3.2), and Landsman (2023, §1).)

iii. Even if one accepts the argument from mathematical structuralism, it's not a given that the hole argument is blocked, if one has other reasons to be believe in metaphysical differences which 'cut finer' than isometries. (See Pooley and Read (2025, §5).)[4]

I would like to see from 'formalists' such as Weatherall (2018) and the authors of the present article a more thorough engagement with the above points; at least, it seems to me that it's here that the most pressing outstanding loci of disagreement can be found. On the 'category-first' view taken in the article under review, "the structure [is] defined in terms of the morphism" (p. 300)[5]—so, when we're dealing with Lorentzian manifolds, their structure is "defined" by their being objects in the category of Lorentizan manifolds; as such, it makes no sense to compare them using maps which are not morphisms in that category, including (in general) the identity map. Fair enough, and in that case the authors of the present piece will of course be unmoved by passages from Menon and Read (2023) such as the following:[6]

> There is (at this point, at least) no prohibition on comparing any two isometric models of general relativity using diffeomorphisms which do not witnesses those models' being isometric—in the above case, assuming that ψ is non-trivial, one could for example compare those models using the identity map 1_M, which [...] does not witness their being isometric [...]. (Menon and Read 2023, p. 9)

But again, what one would really like to see (if not in this paper then elsewhere) is more sustained and sympathetic argument for the categorico-structuralist out-

[3] Cf. Cheng and Read (2025) in response to Bradley and Weatherall (2022); separately, cf. Cudek (forthcoming). I take it that both Weatherall (2018) and the authors of the present piece endorse some kind of 'categorico-structuralism' as against what one might call the above 'set-theoretic constructivism'. But then I think it's helpful to be explicit that most of the authors' points are conditional on this categorico-structuralism, and I also think that it would be helpful for the authors (at some point, if not in this article) to argue against set-theoretic constructivism on its own terms (i.e., by not simply insisting—as the authors do later their article—that to talk of set theory when doing (what looks like) GR is in fact not to work with GR but rather with some augmented theory 'GR+ZF$_m$'), and moreover I further think that it would be helpful to give more explicit articulation of and argumentation for categorico-structuralism.

[4] But—fair enough—if one squints hard enough one can probably convince oneself to agree with Weatherall (2018) that these commitments are not 'generated' by GR.

[5] Cf. footnote 3.

[6] Similar remarks are to be found in Pooley and Read (2025).

look.[7] The authors proceed as if the position is uncontroversial and obvious—but it's important to remember that the view is still surely a minority one, and so (in my view, at least), the ball is in its proponents' court when it comes to rendering it compelling.[8]

In any case, let me now also for the sake of argument adopt categorico-structuralism and proceed to (2). The thought here is that, if (again, following injunctions from Weatherall (2018)) we're only allowed to consider maps which witness isometries between the models of GR under consideration, then if (as it turns out *per impossibile*) there were multiple such maps between two hole-diffeomorphic models of general relativity (that restrict to the same isometry on a proper subset of their domain), one could use one of those maps as a standard of cross-model identification of points, and the other to generate a hole diffeomorphism—in other words, two such distinct maps could, it seems, permit the hole argument to re-arise.

Good news, then, that the authors of the present piece invoke a modification of a result from Geroch (1969) in order to demonstrate that, in the case of the hole argument, there can be no two such distinct isometries. The relevant theorem is this (p. 308):

> Theorem 1: Let (M, g) and (M', g') be relativistic spacetimes. If φ and ψ are isometries from (M, g) to (M', g') such that $\varphi|_O = \psi|_O$ for some non-empty open subset O of M, then $\varphi = \psi$.

What is the significance for the hole argument of Theorem 1? The read on it presented by Menon and Read (2023) is that it can be used to 'plug a hole' is the formalist response to the hole argument offered by Weatherall (2018). Although such a reading is not presented very explicitly in the article currently under review, Manchak (for one) seems to endorse this understanding of the significance of this result, stating that "[W]eatherall acknowledges that his argument rests on the [G]eroch (1969) rigidity result in this way", and endorsing this claim attributed to Weatherall.[9]

But is it true that arguments presented by Weatherall (2018) *vis-à-vis* the hole argument require the rigidity result of Geroch (1969)? In fact, Read and Manchak (2025) show that the situation here is quite delicate. Here, I'll quickly review their results, while referring the reader to their article for the details. Before getting there, though, I first need to follow Menon and Read (2023, p. 8) in drawing a distinction between two different kinds of isometry. Letting (M, g) and (M, g') be Lorentzian manifolds and letting d be a diffeomorphism on M, we say d is an *isometry* from (M, g) to (M, g') just in case $d^*g' = g$. Then

[7] In an ideal world, one would also like to see a defence of the view that such considerations about mathematical objects have *any* bearing on the hole argument, *pace* arguments by Teitel (2022). (My thanks to Frank Cudek for raising this point.)

[8] Very likely, defending categorical-structuralism will involve engaging with older criticisms of (positions akin to) the approach such as those from Winnie (1986, §2.5)—criticisms which, in my view, deserve more attention than they have yet received.

[9] See https://www.youtube.com/watch?v=lFbfhISreFY&t=825s.

we say that an isometry d from (M, g) to (M, g') is an $\texttt{Isometry}_1$ just in case $g' = g$; we say that an isometry d from (M, g) to (M, g') is an $\texttt{Isometry}_2$ just in case $g' = d_* g$.

With this terminology clarified, the lay of the land when it comes to point (2) above, in light of the work by Read and Manchak (2025), is the following:

1. When the spacetime under consideration has no non-trivial isometries in the sense of $\texttt{Isometry}_1$, rigidity is in fact implicitly assumed, *pace* Menon and Read (2023).

2. Indeed, since all models of GR are rigid (see Manchak and Barrett (forthcoming, Proposition 1)), there is a sense in which rigidity is always assumed, when one is dealing with GR; this, however, isn't so in a broader class of spacetime theories with models $(M, O_1, \ldots, O_n)$ with the O_i geometric object fields on M, where rigidity might fail.

3. When the spacetime under consideration does have non-trivial isometries in the sense of $\texttt{Isometry}_1$, whether one regards rigidity as being necessary to "close" the hole argument depends upon the notion of determinism in play: if one construes determinism in the sense of Stein (1977), then one needn't invoke rigidity; however, if one construes determinism in the sense of Earman (1977), then one must do so.

4. Focussing on the Earmanian sense of determinism, there is a precise sense in which rigidity is both necessary and sufficient to underwrite the formalist response to the hole argument presented by Weatherall (2018).

5. Rigidity is equivalent to the condition that non-trivial hole diffeomorphisms cannot be isomorphisms from the spacetime model under consideration to itself; this makes clearer the focus on isometries in the sense of $\texttt{Isometry}_1$ in Halvorson and Manchak (2025, Corollary 2) when this might otherwise have seemed irrelevant to the hole argument (which typically deals with isometries in the sense of $\texttt{Isometry}_2$).

As I say, all of these issues are discussed in detail by Read and Manchak (2025), so I won't go into them further here. The headline summary is this: the authors' claim that rigidity is necessary to "close" the hole argument has been cleared up in the literature subsequent to their work.

Stepping back, here is how I would summarise the article under review. It is written from a still-contentious, categorico-structuralist perspective, and doesn't direct its efforts towards defending that perspective. This is fair enough—but going forward I would like to see more explicit and sustained responses from categorico-structuralists to concerns such as (i)–(iii) presented above.

Turning to the question of whether the rigidity result of Geroch (1969) is necessary to underwrite the arguments of Weatherall (2018), these issues have (one hopes!) been clarified by Read and Manchak (2025).

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