The Enhanced Enhanced Indispensability Argument

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1. Introduction

W.V.O Quine and Hilary Putnam constructed a well-known and popular argument for believing in the existence of mathematical objects. This argument is known as the "Indispensability Argument" (IA) for mathematical realism, and depends upon showing that mathematics is indispensably used and, from a logical standpoint, existentially quantified over in our best scientific theories. Being the types of naturalists that they were, Quine and Putnam took this to indicate that we should believe in the existence of mathematical objects. However, there has recently been some dissatisfaction with this approach to justifying mathematical realism, and has led to theorists proposing a new, 'enhanced' version of the IA (the EIA). This enhanced version argues that since mathematics is indispensably quantified over in our best *explanations*, we should believe in the existence of mathematical objects. In this paper, we will explore how the EIA emerged from the IA, and put forward some problems with it. We will argue that unclarity in both the meaning of 'explanation', the explanatory standards for when to classify an explanation as 'best', and the ontological relevance of those standards has plagued discussion of the EIA from the start and has created a confusing and muddled literature. As a remedy, we propose a new, enhanced version of the EIA, or the EEIA. Through this version, we hope to maintain clarity about what exactly should be shown in order for us to justify ontological inferences from explanatory considerations, *regardless* of what one takes 'explanation' to mean, the explanatory standards one uses to determine the best explanation, and the ontological import of those standards. We do this by invoking a higher order inference to the best explanation, where the best explanation of why mathematics is indispensably used in our best scientific theories is because it establishes dependency backing relations. One upshot from this discussion is that theorists should spend more time focusing on how mathematics could establish dependency backing relations, which may require developing in greater detail one's favored ontology of mathematics and the world. Another upshot that will be touched on briefly at the end

is that this will have ramifications for how one argues for scientific realism from inference to the best explanation.

2. The Indispensability Argument

Since W.V.O Quine, it has been common to run indispensability arguments to establish mathematical realism. The argument is roughly as follows:

1. Our best scientific theories indispensably quantify over mathematical objects (indispensability thesis).
2. Our best scientific theories are true.
3. The evidence for the truth of our best scientific theories being true is evidence for every part of the theory being true (confirmation holism).
4. If some object is indispensably quantified over in a true statement, then that object exists (Quine's Criterion of Ontological Commitment, QC).
5. Therefore, mathematical objects exist.[1]

Recently, this indispensability argument has started to be rejected by both realists and anti-realists in favor of a new indispensability argument. This is due to numerous reasons, only two of which will be canvassed here. First, the above argument relies on the doctrine of confirmation holism, which is now widely rejected. Confirmation holism states that whatever empirical evidence that is used to support our best theories supports *all of* that theory, not just some parts of that theory. This is required for the argument to go through, for otherwise nominalists could argue that just because a scientific theory is true (or we believe it to be true) does not mean that we have to believe that all of its statements are true, and hence don't have to believe that statements quantifying over mathematical objects are true. If we don't have to believe that such statements are true, then we won't have to be committed to mathematical objects via QC.[2]

One reason for rejecting Confirmational Holism is the role of idealizations. Idealizations are used heavily in the sciences, and these are not taken to actually represent ways the world really

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[1] We could rephrase this argument to not establish the existence of mathematical objects, but instead that we should believe that mathematical objects exist, if we want to hedge our bets and not say that our best scientific theories are true, but that we think they are true, and that the evidence for the truth of the theory is evidence for believing the truth of each of its parts.

[2]The realist could respond with "well when we say 'the earth is 93 million miles from the sun', we are making a true statement that quantifies over numbers. So even if we don't need to accept every part of a true scientific theory, we still make statements which quantify over numbers that we accept as true." There are a few problems with this response. First, in order for the truth of this statement to be ontologically relevant, we must adopt a correspondence theory of truth. Second, this requires viewing the existential quantifier as always being ontologically committing, which, as we will point out below, is contentious. Finally, the Quinean naturalist would want to ask how do we know that the statement 'the earth is 93 million miles from the sun' is true? For them, we know its truth because it is part of a larger theory that has received confirmation. We cannot test parts of a theory in isolation, for every empirical consequence that we derive from a part of a theory depends upon numerous background assumptions that compose different parts of that theory. Thus any test could confirm/disconfirm any one of those parts of the theory, and we cannot know which one. Thus the famous Duhem-Quine thesis that our theories are only ever confirmed as a whole, and that confirmation accrues to each of its parts. Hence if the realist wishes to use this response, then they must be rejecting at least part of Quine's naturalism.

is, but instead are just instrumental. For instance, the ideal gas law makes explicitly false assumptions about the behavior of actual gasses (e.g. that they are dimensionless, that they have perfectly elastic collisions, and that they have no mutual attraction), and yet is still widely adopted in scientific practice due to its usefulness in approximating the behavior of many gasses. Alternatively, we could look at ceteris paribus laws which only hold given that nothing else is at play. For example, the law of gravity, $F=Gm_1m_2/r^2$, is not universally true, for there are other forces at play which interfere with this law's application. If there were no other forces at play, then this law would be true, but there are, and so, strictly speaking, it is false. Thus not all components of our true scientific theories are to likewise be taken as true.[3,4]

The second reason why many philosophers have rejected this argument is a more critical appreciation of QC. The existential quantifier was originally thought by Quine to be ontologically committing because it provides a logical regimentation of ordinary language phrases like 'there is/are' and 'there exist/s', and such language seems to be ontologically committing. It seems to be baked into the meaning of these phrases that we are committing ourselves to the existence of something or other. Therefore, the existential quantifier was argued to be ontologically committing. However, Jody Azzouni, Eli Hirsch, and others have argued that this is not necessarily so. For example, the quantifier variantist, like Hirsch, would hold that there is not a single set meaning to the existential quantifier, and may hold that as a result the existential quantifier is not always implying ontological commitments (sometimes it has an ontologically committing meaning, and sometimes it does not).[5] The deflationary nominalist, like Azzouni, may argue that our ordinary language phrases like "there is/are" and "there exist/s" are many times used in an ontologically non-committing way, unless we want to say that ordinary speakers are committed to the mind/language-independent existence of things like Mickey Mouse, goddesses in Greek mythology, and imaginary people.[6] Thus, many have found that Quine's criterion of being ontologically committed to whatever is a bound variable of the existential quantifier to be unduly demanding and unwarranted.[7]

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[3] The example from Newton's law of gravity may not be fitting here since we don't accept Newton's theory as being true anymore, but the point can be applied to other ceteris paribus laws that are parts of theories which we do accept as being true. For more examples of false statements used in our best scientific theories, see Catherine Elgin, "True Enough", *Philosophical Issues* 14, 2004: 113-31.

[4] For more reasons why we should reject confirmational holism, see Elliot Sober, "Quine I: Quine's Two Dogmas", *Proceedings of the Aristotelian Society Supplementary Volume* 74 (2000): 237-80.

[5] See Eli Hirsch, *Quantifier Variance and Realism: Essays in Metaontology*, (New York: OUP, 2010). See also Jody Azzouni, "Applied Mathematics, Existential Commitment, and the Quine-Putnam Indispensability Thesis", *Philosophia Mathematics* 5 (1997): 193-209.

[6] See Jody Azzouni, *Deflating Existential Consequence*, (Oxford: OUP, 2004): ch. 4 and Jody Azzouni, "Ontology and the Word 'Exist': Uneasy Relations", *Philosophia Mathematica* 18 (2010): 74-101 for more on this. He argues that we in fact do not take such figures to exist in a mind/language independent way, and since our linguistic community's criterion for what exists is mind/language-independence, we should conclude that these idioms are not ontologically committing.

[7] We do not take these to be knock-down arguments to the Quinean ontological enterprise, and believe that much more needs to be said in evaluating the Quinean position. However, these are some of the major reasons that have driven philosophers away from taking the original IA to be a convincing argument.

3. The Explanatory Indispensability Argument

In light of the objections to the IA, recent realists have proposed a new and 'enhanced' indispensability argument, one that does not depend upon QC or confirmation holism. This has become known (creatively) as the 'enhanced indispensability argument', or the EIA. It runs roughly as follows:

1. Our best scientific explanations indispensably existentially quantify over mathematical objects.
2. Some explanation's being best is evidence of its truth (commitment to IBE)
3. If something is indispensably existentially quantified over in true explanations, then we should believe in the existence of that thing (the explanatory criterion of ontological commitment, EC).
4. Therefore, we should believe in the existence of mathematical objects.[8]

The EIA depends both upon taking IBE to be truth conducive and the intuitive sounding claim that those things indispensably used in our true explanations must exist.[9] It is targeted towards philosophers who have already accepted scientific realism on the basis of IBE, i.e. philosophers who already believe in the existence of unobservable posits like quarks, electrons, the Big Bang, because of their indispensable usage in our best explanations. By arguing that mathematical objects are just another case of unobservable objects indispensably used in our best explanations, the EIA becomes another instance of the general argument schema already accepted by scientific realists, and so should be just as convincing as other already accepted instances. It gets its dialectical power through parity of reasoning. The sorts of explanations that are being appealed to in the EIA are what are called 'extra-mathematical explanations,' namely**,** explanations of physical facts that depend upon mathematics in order to do the putative explanatory work. This is contrasted with 'intra-mathematical explanations,' namely**,** explanations of mathematical facts that depend upon mathematics. The purpose of the distinction is to prevent us from begging the question against the mathematical anti-realist from the start.[10]

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[8] See Alan Baker, "Are There Genuine Mathematical Explanations of Physical Phenomena?", *Mind* 114 (2005): 223–38, Alan Baker, "Mathematical Explanations in Science", *The British Journal for the Philosophy of Science* 60 (2009): 611-33, and Mark Colyvan, *The Indispensability of Mathematics*, (Oxford: OUP, 2001) for expositions of this argument.

[9] One issue with this formulation is that it moves from the truth of some explanation to ontological commitments. That is, it adopts a link between semantic realism (that our theories/explanations are true) and metaphysical realism (that the entities, processes, and events posited in our true theories/explanations exist). If one is a correspondence theory about truth, then this link should follow unproblematically. However, metaphysical realism is not entailed by semantic realism if one rejects the correspondence theory of truth. Thus, premise 3 would be unwarranted without further argument given a rejection of the correspondence theory of truth. For the sake of this paper, we will be presupposing a correspondence theory of truth, as that is the most commonly accepted theory of truth. However, the EIA could be run without this assumption by modifying P2 to say "some explanation's being best is evidence of its capturing the metaphysical structure/ontology of our world", and modifying P3 accordingly.

[10] Baker, "Mathematical Explanations": 613. However, Sam Baron has recently proposed an explanatory indispensability argument from intra-mathematical explanations (see Sam Baron, "Platonism and Intra-Mathematical Explanations", The Philosophical Quarterly 75 (2025): 812-33). His argument is quite simple: "(1) There are intra-mathematical explanations. (2) All explanations are backed by dependence relations between parts of

Over the past 20 years of debate, a few problems with the EIA have emerged. The main problem is that the discussion is unclear and confusing through different theorists using different standards of explanation, and different standards of what makes an explanation best. The double confusion creates challenges for assessing whether P1 and P2 are true. Through using different standards of explanation, theorists disagree over whether alleged cases of extra-mathematical explanations (EMEs) are actually explanatory, which then sidetracks the debate into issues on the nature of explanation.[11] To make matters worse, much of the time people are not clear about what standard of explanation they are using, and so whether we should agree with them that certain cases of EMEs are or aren't explanatory (i.e. whether these cases of explanation satisfy certain accounts of what makes something explanatory, e.g. unification, increasing the probability of the explanandum, being a covering law, etc.). For example, Alan Baker uses the increase in unification and explanatory depth that mathematics provides as evidence that they are best explanations, but fails to state what sort of explanation mathematics is providing/when something counts as an explanation (his appeal to the unifying power of mathematics suggests, but does not imply, explanation via unification).[12] This makes it difficult to assess whether his alleged cases of EMEs should in fact be counted as explanations (and whether such explanations are ontologically relevant, more on that below). Further, some theorists are operating off of one or another various unspecified forms of naturalism, whereby we should take something to be explanatory given that scientists say that they are explanatory, whereas others are more willing to

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the world. (3) If there are intra-mathematical explanations and all explanations are backed by dependence relations between parts of the world, then mathematical entities exist. Therefore, (4) Mathematical entities exist." (Ibid, 815). He does consider the objection above that this might be question begging, as explanation requires at least weak facticity, that at least the explanandum is true. He says that his argument gets around this by not requiring the link between truth and ontology and not relying on the facticity of explanation. He is linking ontology through dependence and not truth (Ibid, 817). While more deserves to be said about his argument, we think that it does not satisfactorily evade the issue at hand. The issue is not how we make the link to ontology, but whether we believe that there are intra-mathematical explanations in the first place. The fictionalist will say that there are not *because* there are no mathematical facts and explanation requires weak facticity, and so Baron's appeal to dependency relations is irrelevant. There will be no relata for the dependency relation to hold between. The whole argument hinges upon (1), for just about everyone accepts the weak facticity of explanation, and so if (1) is true, then there are mathematical truths. Thus the rest of the premises become superfluous. Baron's arguments in favor of (1) rehash the same points that others have made, which is that mathematicians take themselves to be providing explanations or to be talking about truths, and we should trust mathematical practice more than philosophical critiques of it (see John Burgess and Gideon Rosen, *A Subject With No Object: Strategies for Nominalistic Interpretation of Mathematics*, (Oxford: OUP, 1997) for this same sort of critique). Because part of what we mean by 'explanation' is that the explanandum is true, one cannot believe that there are intra-mathematical explanations without thereby believing that there are mathematical truths. Hence the only relevant premise is (1).

[11] For example, a recent paper by Mary Leng spends some time arguing about the nature of explanation and how we should allow increasing cognitive salience to be a kind of explanation. See Leng, "Models, Structures, and the Explanatory Role of Mathematics in Empirical Science", *Synthese* 199 (2021): 10415-40. Marc Lange in a recent paper also puts much emphasis on counterfactual dependence being part of what 'explanation' means, and uses this to indict platonism and nominalism as failing to provide an account of EMEs (see Marc Lange, "What Could Mathematics be for it to Function in Distinctively Mathematical Scientific Explanations?", *Studies in History and Philosophy of Science* 87 (2021): 44-53). See also Sorin Bangu, "Mathematical Explanations of Physical Phenomena", *Australasian Journal of Philosophy* 99 (2021): 669-82, and Davide Rizza, "Magicicada, Mathematical Realism, and Mathematical Explanation", *Erkenntnis* 74 (2011): 101-14.

[12] Baker, "Mathematical Explanations": 620-21. The same goes for Colyvan (Colyvan, *Indispensability*: ch. 4).

let their philosophy guide what they think scientists *should* say about what is explanatory and what isn't. Most of the time, these commitments are not explicitly stated, but are implicitly guiding their discussions, and so in turn affecting agreement over whether there are cases of EMEs. These reasons are why many theorists have disagreed over whether mathematics is indispensable to *explanations* or to something else.

Further, the discussion gets further muddied by considerations of whether a certain kind of explanation is ontologically relevant (and so is apt to be considered 'approximately true' by being our best explanation). Even if we can get clear on what criteria something must meet to count as an explanation, there is disagreement over whether certain kinds of explanations are ontologically relevant. While there does seem to be a growing consensus about what sort of explanations are ontologically relevant, at least at the start of the debate many philosophers were silent on this matter. Their main goal was to show that mathematics was involved in our best explanations, regardless of the kind of explanation. This is because they focused on showing how the EIA mirrored arguments for scientific realism, which they understand as arguing that we should believe in unobservable scientific posits because they are indispensable to our best explanations, regardless of the kind of explanation. Others disagreed, arguing that the arguments for scientific realism depended upon specific forms of explanation, and so the EIA had not yet been proved as it failed to show that mathematics explained in these ontologically relevant ways.[13] Further, there are issues involving the usage of idealizations and fictional models in our best explanations. These explanations are false, and so they show that the explanans doesn't need to be true in order to be explanatory. Sometimes they are even counted as our 'best' explanations for certain phenomena. This then leads to issues about when the 'bestness' of an explanation licenses us to infer that it is approximating truth.[14]

Nowadays, there is a growing consensus that certain kinds of explanations (e.g. modal explanations, explanations by increasing cognitive salience, etc.) simply are not ontologically relevant (and hence don't need to be true or approximately true in order to be our best explanations), whereas other kinds (e.g. ontic explanations like counterfactual explanations) are. However, even here the debate is unclear, as some theorists are explanatory monists, believing that there is only one type of 'explanation', whereas others are explanatory pluralists, where there are many types of explanation that are distinct from and non-reducible to one another. The monist believes that by allowing only one type of explanation, e.g. counterfactual explanations, they can more easily account for what makes something 'explanatory' in all cases of explanation, e.g. providing counterfactual information. The pluralist has to adopt different accounts of what

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[13] See, for example, Juha Saatsi, "On the 'Indispensable Explanatory Role' of Mathematics", *Mind* 125 (2016): 1045-1070, and Jacob Busch and Joe Morrison, "Should Scientific Realists be Platonists?", *Synthese* 193 (2016): 435-49. Saatsi goes even further and argues that even if it can be shown that alleged EMEs are explanatory in an ontologically relevant way, it has not yet been shown that it is the mathematics that is doing the explanatory work as opposed to some representational or instrumental work.

[14] See Mancosu, Paolo, Francesca Poggiolesi, and Christopher Pincock, "Mathematical Explanation", The Stanford Encyclopedia of Philosophy (Fall 2023 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = <https://plato.stanford.edu/archives/fall2023/entries/mathematics-explanation/> for more on this.

makes something 'explanatory' for the different types of explanation, but believes that this more satisfactorily captures the actual explanatory practices in science. If one is a monist, then those other types of 'explanation' simply aren't explanations to begin with, and so one gets drawn back into a discussion of the nature of explanation. Here we see yet again different types of naturalism at work.

There is also no general agreement upon what standards we should use for counting some explanation as 'best'. Most EIA proponents, like Baker and Colyvan, have argued that mathematical explanations are our best explanations because they increase explanatory virtues like depth and unification.[15] Others, like Christopher Pincock, have argued that the history of IBE in science shows that those virtues are insufficient to classify the sort of explanations mathematics is indispensable to as being the 'best' explanation for scientific phenomena.[16] For example, the hypothesis that God created the universe and life on earth is a more unifying explanation and one that is deeper than the alternatives, however it was seen by scientists as a worse explanation than the competing Darwinian explanation of life. Further, as Robert Knowles and Juha Saatsi have argued, some of these explanatory criteria appealed to to suggest that EMEs are the 'best' explanations are valuable because they increase our cognitive salience, which is not obviously connected to truth/ontology.[17] They argue that through increasing unification, mathematics is only helping us to more easily grasp counterfactual information about physical dependencies, and that no ontological conclusions about mathematics can be drawn from that. Because of this, it is unclear whether the data being used to support the EIA actually does support ontological commitments/ (either by not actually being the best explanation or by being the best explanation in pragmatic, non-truth tracking ways).

Finally, even explanations that are treated as ontologically relevant (i.e. explanations whose explanatory power is related to their truth or truth-likeness or to the world being some way) need not always be true. After all, even paradigmatic ontic accounts of explanation such as Woodward's counterfactual analysis allow for explanations to sometimes be merely instrumental. False explanations can still be counted as explanatory, as they can derive their explanatory power through accurately mapping the true ontic structure of the world (e.g. Newton's theory of gravity, though incorrectly describing the ontology and relations of the world, can nonetheless be explanatory as it can accurately map the correct ontic structure of the world in many cases).[18] Given that explanations don't need to be true in order to be explanatory, even in this ontologically relevant way, it is possible that these explanations could be our best explanations and still not be thought of as true (this is further supported by the fact that we already treat model and idealized explanations as sometimes being our best explanations even

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[15] Baker, "Mathematical Explanations": 620-21. Mark Colyvan, *The Indispensability of Mathematics*, (Oxford: OUP, 2001): 78-80, and Colyvan, "Road Work Ahead: Heavy Machinery on the Easy Road", *Mind* 121 (2012): 1041-2.

[16] Christopher Pincock, *Mathematics and Explanation*, (Cambridge: CUP, 2023): 69-73.

[17] Robert Knowles, Juha Saatsi, "Mathematics and Explanatory Generality: Nothing But Cognitive Salience", *Ekenntnis* 86 (2021): 1119-37.

[18] See Saatsi, "On the 'Indispensable Explanatory Role'": 1061-3.

though we know they are false, and that under some understanding of 'best', Newtonian gravitational explanations are treated as 'best').[19] From there, it is possible that our best explanations indispensably quantifying over mathematics, even if they were ontic, need not be true. Thus, even appealing to ontic explanations is not enough to secure their truth through IBE, even if one is a scientific realist (since many realists restrict the applicability of IBE, as they don't take our best model explanations to be true). This is because something can be playing a representational role in an ontic explanation. Hence we need to go even further and argue that mathematics does not play such a representational role in order to move from their being used in best ontic explanations to these explanations being true.

Given these issues with the EIA, we think it best to try out a revamped version of the EIA. This new argument will draw inspiration from the explanatory characteristics of the EIA and incorporate philosophical developments that have emerged from the past two decades of discussion about it. Like the EIA it will seek to use explanatory considerations to argue in support of mathematical realism, but unlike the EIA will be explicit about what its explanatory standards are. It will also seek to make it clear that it is not enough to provide an account of how mathematics can meet such explanatory standards; one must go beyond and argue that it in fact does. Thus it focuses the discussion around mathematical indispensability and its consequences for realism on developing accounts of mathematical explanation and arguing that this account is the proper account for understanding how mathematics explains, and so will help illuminate why such discussions have naturally moved towards that area.

4. The Enhanced EIA

What we are proposing could be called an enhanced enhanced indispensability argument, or an EEIA. The argument is as follows:

1. Mathematics is indispensable to our best scientific theories.
2. The best explanation for why mathematics is indispensable to (at least some of) our best scientific theories is that in those theories they are establishing dependency backing relations between them and the physical explanandum.
3. We should believe in the existence of those things that indispensably establish dependency backing relations in our best scientific theories (dependency criterion of ontological commitment, DC).
4. Therefore, we should believe in the existence of mathematical objects.

Like its predecessor, the EIA, the EEIA supports realism through explanatory considerations. Thus it mirrors discussions of scientific realism. Both the EIA and the EEIA use a criterion of ontological commitment that is based around explanation, though the EEIA's criterion is more restricted than the EIA's. The EEIA's criterion, DC, refers to 'dependency backing relations'.

[19] For example, scientists many times prefer to use the Newtonian equations for gravitational phenomenon instead of the equations of GTR due to their simplicity and ease to work with. On these pragmatic dimensions, these explanations can be counted as 'best'.

What we have in mind here is something like this: explanations hold between propositions. Some, but maybe not all explanations function by describing ways in which the explanandum (or what the explanandum is about) is dependent upon the explanans (or what the explanans is about).[20] One of the most common forms of explanatory dependencies is counterfactual dependency (that if the explanans were different, then the explanandum would be different as well). Further, these dependencies are 'backed' or hold via some dependency backing relation (e.g. causation, grounding, constraint, law-like regularity, etc.). These relations are ontic; they are real relations that hold in the world and hold between things that exist in the world. Hence, intuitively, if one can show that some x stands in a dependency backing relation to y, then one has shown that x exists.[21] Some features of dependency backing relations are that they are objective (i.e. they hold between things/events in the world and their holding is not dependent upon our beliefs about them), non-monotonic, asymmetric, and irreflexive. Some relations are or at least can be thought of as 'productive.'

This is akin to arguments for scientific realism based upon the indispensable role of unobservable posits to our best explanations. Arguably, the EEIA is more akin to those arguments than the EIA is, for as Jacob Busch and Joe Morrison have argued, arguments for scientific realism typically depend upon showing that some unobservable posit is playing an indispensable explanatory role in our best *causal* explanations (and so are providing a dependency backing relation).[22] If this is correct, then the EEIA seems to be a step up over the EIA in terms of the strength of the analogy between them and the arguments for scientific realism, and so should be of more appeal to the scientific realist.

While both the EIA and the EEIA rely upon indispensability of mathematics and an explanation based criterion of ontological commitment, the EEIA does not start off by assuming that what mathematics is indispensable for are *explanations*. Instead, it is more akin to the original IA, where the naturalized philosophical datum is the indispensability of mathematics to our best scientific theories –or models. The purpose of this is to get around questions about what is and is not the nature of explanation, and to focus more specifically on the important aspects of explanation for ontological commitment: that of establishing dependency backing relations. By doing this, we can sidestep questions about what does and does not count as explanatory/explanations and instead start from a place that most people can agree upon: that mathematics is indispensable to our best scientific theories. Explanatory considerations are not

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[20] Hence we are not committing ourselves to monism or 'realism' about explanation, that all explanations are of the same kind, e.g. providing information about what metaphysically determines the explanandum.

[21] For more on the dependency-backing model of explanation, see Elanor Taylor, "Against Explanatory Realism", *Philosophical Studies* 175 (2018): 197-219 and Eleanor Taylor, "Backing Without Realism", *Erkenntnis* 87 (2022): 1295-1315. There she talks about explanations being backed directly by dependency relations, whereas we are talking about explanations describing dependencies and those being backed by dependency backing relations. The reason we have decided to use our terminology is to help connect our position with others in the philosophy of scientific explanation. There you find talk about explanation, for example, being counterfactual in nature, describing counterfactual dependencies, which are backed by relations like causation.

[22] Busch and Morrison, "Should Scientific Realists be Platonists?": 442.

initially present in the data that we are arguing from.[23] Instead, explanation only appears as part of a sub-conclusion from that data, and then only a form of explanation that is generally agreed upon to be ontologically relevant, and not just 'explanation' in general. We take it that the failures of the IA show that we cannot jump straight from the indispensability of mathematics to scientific theories to the conclusion of mathematical realism. We also take it that the appeal of the EIA is that explanatory considerations *are* ontologically relevant, but that the tangled discussions about whether there are extra-mathematical explanations and whether that even supports mathematical realism show that only *some* explanatory considerations are generally accepted as ontologically relevant, and not 'explanation' in general. Thus we believe that what is dialectically most helpful for the realist is to take as their data the generally accepted fact that mathematics is indispensable to our best scientific theories (P1), as well as the generally accepted belief that DC is a good and sufficient criterion for ontological commitment (P3). This helps the realist to escape debates over what does and does not count as explanation, and to instead focus on one specific mode of explanation that is generally agreed upon to be ontologically relevant. This will illuminate the relevance of providing accounts of extra-mathematical explanation, and encourage the realist to go the extra step and to argue that accounts which have mathematics establishing dependency backing relations are the best accounts.

By having its criterion of ontological commitment be based around establishing dependency relations, the EEIA is able to avoid the problems plaguing the IA and the EIA. Unlike the IA, the EEIA does not depend upon confirmation holism or QC. Thus it is not vulnerable to the attacks lodged against them (while also maintaining the value the IA had by working from the indispensability of mathematics to science in general). Likewise, it does not depend upon a broad understanding of 'explanation' and account of when IBE is truth-tracking, but instead narrows down when we should take IBE to be truth-tracking. This will be in cases when the relevant explanation is ontic, and when we have reason to believe that the terms in the explanation are establishing dependency backing relations. The EEIA is able to retain the intuitive appeal of the relevance of explanation to ontology, while avoiding the messy discussions of the nature of explanation and when IBE is truth-tracking, as well as making it clear what the realist should look for in an account of extra-mathematical explanation, i.e. an account of how mathematics establishes dependency backing relations.

Because of this, the EEIA can remain quite neutral on questions about whether explanation is monistic, i.e. that all explanations work via appealing to the same dependencies (e.g. counterfactual dependencies) or dependency backing relations (e.g. causation). It can also

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[23] For examples of practices of construction of kind-quantities and similarity measures, for instance, see Jordi Cat, "The Performative Construction of Natural Kinds: Mathematical Application as Practice", in C. Kendig, ed., *Natural Kinds and Classification in Scientific Practice*, New York: Routledge, 2016, 87-105, and Jordi Cat, "Synthesis and Similarity in Science: Analogy in the Application of Mathematics and Application of Mathematics to Analogy", in S. Wuppuluri and A.C. Grayling, eds., *Metaphors and Analogies in Sciences and Humanities*, Cham: Springer, Synthese Library, 2022, 115-145.

remain neutral on the question of whether explanatory 'realism', which is the view that *all* explanations give information about metaphysical determination, or generative/productive dependency backing relations, is true.[24] This sort of view holds that all explanations are 'realistic' in the sense that they are supported by worldly non-reductive, productive views of dependency backing relations like causation or grounding.[25] Non-realist views hold that some explanations involve dependency backing relations, but that these are not determination relations (i.e. they are non-productive, such as conceptual dependency). Anti-realist views hold that no explanations involve determination relations (and so would hold that, for example, causation is not productive/generative). As we have characterized dependency backing relations, there is nothing that requires that they be determination relations, and so nothing that will require one to be an explanatory realist or not, just that one believe that there are at least *some* dependency backing relations in the world and that these back at least *some* explanatory dependencies.

One might worry that by being a higher order IBE, the EEIA will run into the same problems as the EIA did with regards to explanatory standards. After all, the objection against the EIA was that some explanatory standards for when to consider an explanation 'best' did not seem to be relevant to ontology, but more relevant to our abilities to grasp and understand the explanation. Thus explanations that are considered 'best' via these standards may have nothing to do with truth and hence our ontological commitments. If the EEIA is going to rely upon IBE as well, then would it not run into the same problem with ontologically irrelevant explanatory standards?[26]

It is not clear that the EEIA would in fact suffer from the same problem of explanatory standards as the EIA. As we see it, there are two routes that one could go down to resolve this issue. First, suppose that IBE is truth-tracking. That does not automatically mean that our best explanations are telling us about metaphysical reality. After all, there is a difference between semantic realism and metaphysical realism, so some explanation could be true while it nonetheless does not describe an actual state of affairs.[27] Similarly, given that IBE is truth-tracking, some explanation's being 'best' only tells us that it is capturing something true about the world, or mapping onto the ontic structure of the world, but not necessarily that all of the components in the explanation are accurate depictions of the world. This is analogous to how, on

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[24] See Taylor, "Against Explanatory Realism" and "Backing Without Realism" for characterizations of this view. Her title "Backing Without Realism" is a bit confusing here, as she is not talking about *ontological* realism as we are, but *explanatory* realism.

[25] See Martin Glazier, "Explanation", in *The Routledge Handbook of Metaphysical Grounding*, ed. Michael Raven, (New York: Routledge, 2020): 124-5. On the complications for causal and other realists adopting, for instance, negative descriptors of entities, states and causation, see Jordi Cat, "Failure in Practice and Dialectic in Theory: More Value, Context, Relativity and Plurality in Science", forthcoming in *Scientific Mistakes and Mistake Science: Error, Failure, Malfunction*, eds. A. Tuboly and A. Karakas (London: Routledge, 2026).

[26] More needs to be said about the relationship between the explanatory virtues and truth/ontology in IBE. Pincock has provided some argument that the most important explanatory virtues for truth are explanatory conservativeness and modesty (see his *Mathematics and Explanation*: 69-73). In his view, all of the explanatory virtues are relevant to truth, but some are more relevant than others. We do not have the time to pursue this issue here, and instead wish to consider it more in detail in a later work.

[27] See Colyvan, *Indispensability*: 2-3.

Woodward's account of explanation, we can have good explanations that are tracking truth (by mapping onto the ontic structure of the world) but are nonetheless literally false or merely instrumental, like Newton's theory of gravity. The explanatory standards like unity, simplicity, and depth seem to be truth-tracking but (so people like Pincock, and Knowles and Saatsi have argued) they don't necessarily give us reason to believe that the components of the explanation, or the things that the explanation quantifies over, exist.[28] There needs to be something about the explanation itself that suggests that these components exist. Thus one could very well accept that IBE is truth-tracking while also being skeptical that we can always read off our ontological commitments from our best explanations.

So then, how does this help the EEIA proponent? The above provides reasons to not move too quickly from our best explanations to our ontological commitments. Thus it would make us skeptical about the EIA, which does exactly that. However, things are different once we move to the EEIA. This is because the best explanation for why mathematics is used in our best theories/explanations is the fact that they are establishing ontological dependency relations. Whatever virtues are used to support that this is the best explanation would lead us to conclude that it is true (or tracking truth) to say that mathematics is establishing dependency relations. From there we seem justified in concluding that mathematical objects exist, as mathematics could not establish dependency relations without existing. IBE as used in the EEIA is not directly supporting ontological claims; we are not reading our ontology straight off of our best explanation in the EEIA. What we are doing is instead supporting the truth of some claim that itself supports, in a rather plausible way through DC, some ontological claims. Hence we are not reading our ontology off of our best explanations; we are reading truth (or closeness to truth) off of our best explanation, and the truth of this then, in conjunction with DC, supports ontological claims.[29]

The second route is to argue, along with Marc Lange, that the disputed explanatory virtues may be mainly pragmatic, and hence are not necessarily truth-tracking. However, they can still increase the plausibility of some explanation's being true, and so could help support ontological claims.[30] What the EIA fails to do is show that the pragmatic virtues of depth and unification are in fact increasing the plausibility of the alleged EME's being true, as opposed to

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[28] See, for instance, the aforementioned Cat, "Performantive Construction", "Synthesis and Similarity" and "Failure in Practice".

[29] Essentially, the difference comes down to the fact that the EEIA makes use of DC, whereas the EIA makes use of EC in tandem with IBE, which as we have seen is not apt for making ontological inferences, whereas DC is. Now one might ask "if that's the case, then why didn't you just say that? Why invoke this rather convoluted structure of having a higher order IBE?" The reason for this is because one cannot necessarily conclude, simply from looking at an explanation, what sort of explanatory work its parts are doing. Going back to Woodward, one cannot conclude from the fact that gravitational forces are being used in some of our best explanations that the gravitational forces are establishing dependency backing relations, as opposed to being cognitively useful stand-ins for whatever is in fact establishing these relations. Thus even in ontically relevant explanations, one needs further arguments for claiming that specific components of the explanation are establishing dependency backing relations. Hence, without the higher order IBE, one could not appropriately apply DC, and since the higher order IBE does not invoke EC, there are no problems from explanatory standards.

[30] See Lange, "What Could Mathematics be": 51.

being purely pragmatic, in this case. This is what the EEIA seeks to do. In order to do this, we are claiming that the realist should show that the best explanation for why mathematics is used is that mathematics is establishing dependency backing relations between it and the physical world, and how it does this. This would make it plausible that the mathematics in this case is not merely pragmatically useful, but is capturing something about the world and our ontology. Without such an argument (which will require providing an account of how mathematics establishes these relations), it is open to the nominalist to conclude that the mathematics in EMEs is only being used pragmatically, and hence that ontological commitments are not being supported. Such a route is not open in light of the EEIA.

The EEIA also makes explicit that the real work which needs to be done by the realist to justify their realism from the indispensability of mathematics in science is to provide an account of extra-mathematical explanation that involves mathematics establishing dependency relations, and shows how. For without such an account, it is always open to the anti-realist to argue that, despite the indispensability of mathematics to science, it is unclear how it could be doing anything ontologically relevant (e.g., establishing dependency relations), and because of this we have strong prima facie reasons to reject indispensability arguments (Mark Balaguer is one of the main proponents of this, and something like this seems to inspire the claim that mathematics is playing a purely representational/indexing role in science).[31] Providing such an account is essential for the EEIA to work, for without it, the realist cannot complete their argument. Without an account of extra-mathematical explanation that involves mathematics establishing dependency relations, the realist cannot infer that the best explanation for the indispensability of mathematics in science is because mathematics establishes dependency relations. This is so because of the alternative explanations of mathematics' indispensability, that mathematics is playing a merely representational role. To decide which of these two provide the best explanation of indispensability, we need to flesh out what each of these explanations would look like, i.e. provide an account of what mathematics is doing in science based upon these proposals. This in turn makes it explicit that the anti-realist should provide accounts of what mathematics is doing in scientific theories, accounts that do not require mathematics to establish dependency relations, to undermine the realist's IBE (it is irrelevant whether one counts these as accounts of extra-mathematical explanations or not).

In light of this, it will also become important for the realist and anti-realist to develop their respective ontological theories about mathematics. Recently, some authors have argued that certain ways of understanding EMEs either fits best with certain ontologies or completely excludes certain ontologies. For example, Marc Lange has argued that if we believe in EMEs and that EMEs establish dependency backing relations through 'constraint', then one cannot be a platonist. He goes on to argue that Aristotelian realism is better suited to making sense of mathematics explaining by constraint.[32] Sam Baron has recently argued that a 'Pythagorean'

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[31] See Mark Balaguer, *Platonism and Anti-Platonism in Mathematics*, Oxford: OUP, 1999:136.

[32] Marc Lange, "What Could Mathematics be for it to Function in Distinctively Mathematical Scientific Explanations?", *Studies in History and Philosophy of Science* 87 (2021): 44-53.

proposal, that physical systems possess mathematical properties, is the best way to understand how mathematics explains, and that this proposal requires Platonism.[33] Similarly, Mary Leng has presented a theory of EMEs which *can be* nominalist friendly, given that one pairs it with a form of modal structuralism.[34] Hence, by requiring authors to focus on developing theories about how mathematics explains, the EEIA will become intertwined with deeper issues in the metaphysics of mathematics, and so certain ways of arguing for/against mathematical realism via the EEIA may rise or fall with the sort of metaphysics that fits best with its account of EMEs. For example, if aristotelian realism fails as a metaphysics of mathematics, then one could not support the EEIA through appealing to explanation by constraint, and if Baron's pythagorean proposal fails to provide a satisfactory account of EMEs, then it would seem like one could not support platonism through the EEIA.

With all these benefits of the EEIA over the EIA and IA, what might it look like? Given that it could (in theory) use any data of mathematical indispensability in science, does that make that EEIA much easier to run successfully than the EIA? We think not. For, though it can use any instances of the indispensability of mathematics in science, we believe that the most compelling data for the EEIA are the currently debated instances of mathematical explanation in science. For it is in those cases where we are most likely to have mathematics establishing dependency backing relations as the best explanation for why mathematics is indispensable to science. Other non-explanatory or non-explanatory-like cases do not seem puzzling enough to motivate concluding that the best explanation for why mathematics is indispensable in those cases is because of its establishing dependency backing relations (nor would many of those cases have anything to do with dependencies, such as cases where we are using mathematics to express some measured quantity). The main advantage that the EEIA has over the EIA is that it does not require us to agree from the start that these cases are all cases of explanation; rather, that can be the conclusion of one's investigations in support of premise 2 for whatever instance of indispensability one uses.

5. Conclusion

It has been argued that the indispensability argument and the enhanced indispensability argument run into issues, both philosophically and dialectically. The indispensability argument requires adopting two controversial premises: confirmation holism, and Quine's criterion of ontological commitment. And while we believe that the enhanced indispensability argument is on the right track for how to establish ontological commitments, it suffers from similar problems with regards to its criterion of ontological commitment, requiring a sort of 'explanatory' confirmation holism, and dialectical issues stemming from being unclear about what theorists

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[33] Sam Baron, "Mathematical Explanation: A Pythagorean Proposal", *British Journal for the Philosophy of Science* 75 (2024): 663-685.

[34] Mary Leng, "Models, Structures, and the Explanatory Role of Mathematics in Empirical Science", *Synthese* 199 (2021): 10415-40

mean by ‘explanation’. While they suffer from these problems, there is something appealing about each of these arguments. The indispensability argument does not require making controversial claims about what is and what is not an explanation (and especially that all explanations are ontologically relevant), and instead relies upon less controversial claims about what is and is not our best scientific theory and what is indispensable to those theories. The enhanced indispensability argument ties ontological commitments to explanatory demands, an intuitive connection, as seen in the widespread appeal of explanationist arguments for scientific realism.

In response to these points, we have developed a new indispensability argument that incorporates each of the valuable components of the previous indispensability arguments, while avoiding their problems. We call this the enhanced enhanced indispensability argument. This argument is a sort of higher order inference to the best explanation, where we take as our data the indispensability of mathematics to our best scientific theories, and infer that the best explanation for why this is the case in some instances is because they establish dependency relations. As a result of this, we should believe in the existence of mathematical objects. This argument ties our criterion of ontological commitment to a specific form of explanation (dependency relations) instead of to indispensability in general, and does not require making controversial claims about what is and isn’t an explanation. It helps to clarify why it is so important for the realist to provide an account of extra-mathematical explanation, as without one, they cannot argue that mathematical objects establishing dependency relations is the best explanation of indispensability. This argument thus does require us to get mired in discussions about the nature of explanation more broadly, but rather the nature of extra-mathematical explanation, which is what is relevant for the realist’s argument. We believe that this argument provides a clear exposition of how the realist should argue for mathematical objects from explanatory considerations, and avoids the pitfalls of each of the previous indispensability arguments. Further discussion is required to determine whether or not this argument succeeds in establishing realism.

Additionally, the arguments in this paper have implications for how IBE is used in arguing for scientific realism. For if the arguments here are correct, local arguments for realism about some posit because it is used in our best explanations may not work because that does not guarantee that the posit is playing an ontologically relevant role in that explanation (or is a part of an ontologically relevant explanation). This thus would help to reshape the way one argues for scientific realism through IBE. Further, it may also have implications for the global argument for scientific realism from IBE, that the best explanation for our theory’s succeeding is that the posits they quantify over exist and have the properties ascribed to them. The sense of ‘explanation’ here may impact whether this argument is capable of supporting realism, because the best explanation may not be an ontologically relevant one. However, more work is needed to tease out these implications, and this will be left for a separate paper.