Black Holes are about *Quantum* Information

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Abstract

We review and analyse, with a philosophical readership in mind, recent quantum information-theoretic approaches to black hole evaporation that purport to bring us closer to a resolution of the black hole information paradox. Our focus is on a precise version of the paradox, called the 'Page time paradox,' and on how it is addressed in the models first proposed by Page and later developed by Hayden and Preskill. In these models, the black hole is taken to be a finite-dimensional quantum system whose dynamics is represented by a quantum circuit, and its communication abilities are studied. We identify the assumptions underlying this modelling and highlight the two key features, fast scrambling dynamics and entanglement as a resource, that underpin the expectation that the Page time paradox can be resolved.

This quantum information-theoretic reasoning exemplifies a broader change in how black holes are conceived within fundamental physics, captured by the slogan '*Black holes are about quantum information.*' This slogan reflects a conceptual and methodological perspective where quantum information theory is employed to answer precise questions about black holes, particularly concerning their role in transmitting quantum information. We argue that this application of information theory avoids earlier philosophical objections and marks a significant advancement in the discussion of the black hole information paradox. As such, this line of reasoning has become central to the debate and warrants greater attention from the philosophy of physics community.

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1 Introduction

Recent years have witnessed a wave of research on various aspects of black holes, giving rise to what may be called an emerging 'philosophy of black holes'. Among the questions that have been addressed, the black hole information paradox, along with related issues such as black hole thermodynamics, are of particular significance. Some work has examined the sense in which the black hole information paradox is or is not paradoxical (see e.g. Maudlin (2017); Wallace (2020)), while other work has focussed on highlighting explicit and implicit assumptions in the latest incarnation of this paradox (Cinti and Sanchioni, 2025). Additional work has explored the thermal properties of black holes, investigating whether the relation between black hole thermodynamics and thermodynamics is more than an analogy: see for example Curiel (2014); Dougherty and Callender (2017); Wallace (2018, 2019). This line of research has also explored whether the Bekenstein-Hawking entropy of a black hole can be interpreted as an information-theoretic quantity, especially in the framework of Shannon's information theory, as Bekenstein (1973) initially proposed. The criticism here is that information is an epistemic concept, while thermodynamical entropy pertains to the physical process of heat transfer at a given temperature (see Dougherty and Callender (2017); Wüthrich (2017); Prunkl and Timpson (2019)).

To the best of our knowledge, the philosophy of physics literature has so far not engaged with recent and more detailed *quantum* information-theoretic studies of black holes in physics, particularly those that apply detailed ideas from quantum computation (such as quantum error-correction) and quantum communication (such as entanglement-assisted communication) to black holes.[1] Instead, prior philosophical work by Dougherty and Callender (2017) and Wüthrich (2017)) has been limited to critiques of the application of *classical* information theory to classical black holes.

In this paper, we aim to fill this gap by first introducing, and then analysing, the ground-breaking work of Hayden and Preskill (2007). There are three related reasons why we consider it worthwhile to undertake this task.

First, quantum information theory, the theory which addresses quantitative questions about the acquisition, transmission and processing of information in quantum systems, offers useful conceptual and technical tools for studying black holes. We will argue that tools from quantum information theory can help refine our understanding of the version of the information paradox at issue in this paper, namely, the Page time paradox (Page, 1993c). The debate around this version of the paradox goes well be-

[1] The interpretation of the AdS/CFT correspondence as a quantum error-correcting code, proposed by Almheiri et al. (2015); Pastawski et al. (2015), has been addressed by Bain (2020).

yond simply asking whether the evolution is unitary, and this is precisely where the analysis by Hayden and Preskill offers new insight. To some degree, this may not be very surprising, since similar techniques have proven useful in other branches of physics and philosophy for addressing such questions as locality and causality.

Second, our engaging with quantum information theory will allow us to argue that we can model a dynamical black hole as an object that effectively performs a specific computation. Namely, a black hole unitarily processes the information that is thrown in, without destroying it. This model assumes the 'central dogma' about black hole information (Almheiri et al., 2021): in short, seen from the outside, a black hole is a quantum system that evolves unitarily and is made up of a finite but large number of degrees of freedom. Establishing this central dogma requires providing proof of unitarity, which is not the claim made by Hayden and Preskill (2007). This assumption makes a daunting problem more tractable, and, as such, it is valuable to study its conceptual implications for the release of information. Nonetheless, we will complement our analysis of the Hayden-Preskill model by also discussing both recent evidence and counter-arguments to the central dogma.

Third, while Hayden and Preskill were not the first to apply quantum information-theoretic ideas to black hole physics, their work significantly advanced this perspective and has been taken as a major clue for later investigations. Notably, papers such as Almheiri et al. (2019); Penington (2020), widely seen as crucial to resolving the Page time paradox, built on the Hayden–Preskill framework by implementing it in holographic settings and giving spacetime interpretations to some of its key insights. Their paper also played a pivotal role in formulating another version of the paradox: the firewall (or AMPS) paradox (Almheiri et al., 2013), which arises from the possibility that an infalling observer could extract information from Hawking radiation before crossing the horizon. Their approach has further motivated analogue gravity experiments, such as those reported in Landsman et al. (2019) and Jafferis et al. (2022). A careful analysis of their original work is thus essential for assessing both the strengths and limitations of the research it inspired.

In more detail, Hayden and Preskill (2007) view a black hole as playing a role in a quantum communication system and model the transmission of information within this system by an assembly of discrete sets of components known as a quantum circuit. They consider a small amount of quantum information (namely, the information in Alice's diary), thrown into an existing (old) black hole, and ask two well-defined questions about its release into the radiation:

(i) Physical mechanism of information release: *How does a small amount of quantum* *information escape from the black hole? What is the physical mechanism for information release?*

(ii) Amount of time: *How long will Bob, who is outside the black hole, need to wait to be able to recover Alice's diary?*

Because of its relative simplicity and precision, this kind of model is well suited to addressing conceptual questions. The goal is to gain qualitative insight into a plausible (schematised) mechanism through which information can be released from a black hole, idealising away from the complex interactions between the particles that instantiate this information in the real world. In particular, we will identify two distinct roles that entanglement plays in addressing questions (i) and (ii): first, as means of protecting information against local errors via scrambling; and second, as a consumable resource that facilitates the transport of information.

Hayden and Preskill's model being a *model* means that it contains a number of simplifying assumptions and idealizations that are good approximations to some of the behaviours of the black hole. Thus a major part of our analysis will be to state eight *modelling assumptions* that go into answering questions (i) and (ii) above. In this way, we aim to clarify what this model requires for it to give a successful description of a black hole.

Another important point we make is that Hayden and Preskill's approach aligns with the view articulated by Christopher Timpson regarding the role of quantum information theory. His idea is that quantum information theory can serve as an alternative framework of enquiry, supplementing our fundamental physical theories, for exploring certain interesting features of material objects; notably, their capacity to store, process, and transmit information (Timpson, 2013, p. 237). We argue that Hayden and Preskill's approach was to apply this information-theoretic lens to black holes. As such, it does not rely on epistemic reasoning and helps to counter certain forms of scepticism about the use of the concept of 'information' in black hole physics.

The paper is structured as follows: Section 2 lays the conceptual and theoretical groundwork for applying quantum information theory to the study of black holes, introducing, motivating, and challenging the main modelling assumption; namely, the central dogma. Section 3 begins by introducing the basic features of a quantum circuit, and then presents Page's pioneering use of this framework to model black hole dynamics. Section 4 then analyses Hayden and Preskill's treatment of a black hole as part of a communication protocol, identifying its underlying modelling assumptions and explaining the new insights their use of quantum information theory offers regarding information release. Section 5 concludes.

2 Black Hole Entropy and the Central Dogma

In this section, before delving into the specifics of the approach taken by Hayden and Preskill (2007), we provide the necessary groundwork to justify the application of quantum information theory to the study of black holes.

Section 2.1 gives an overview of the issues surrounding the use of the concept of 'information' in relation to black holes. Section 2.2 introduces the central dogma, the key assumption in Hayden and Preskill's work, that enables information to play a different and, as we will argue, unproblematic role. Section 2.3 presents the main pieces of theoretical evidence supporting the central dogma, which convincingly establish black holes as thermodynamic objects, with their thermodynamics rooted in the unitary dynamics of many microscopic degrees of freedom. Section 2.4 discusses the 'Page time Paradox' as a challenge to the central dogma (Sect. 2.4.1) and how recent developments in black hole physics possibly counter this rebuttal, albeit without providing direct proof of the central dogma (Sect. 2.4.2).

2.1 The Information Controversy in Black Hole Physics

The Bekenstein-Hawking relation (Bekenstein, 1973; Hawking, 1976a) between the entropy, S_{BH}, and horizon area, A_{bh}, of a black hole[2]

$$S_{BH} = A_{bh}/4G \tag{1}$$

is often presented as joining the quantum realm and Einstein's gravity in a profound and mysterious way. The exact interpretation of this relationship still eludes physicists. However, assuming that Hawking radiation exists (Hawking, 1975), one can infer from this expression that the entropy has to do with the thermodynamics of black holes. If this were the case, it would be natural to assume that the thermodynamic behaviour of a black hole is underpinned by (quantum) statistical mechanics. There are essentially two types of arguments supporting this hypothesis in the literature. Assuming the black hole is made up of a large number of microscopic degrees of freedom, concepts from information theory may be applied to relate the entropy to the horizon area of the black hole. These different 'routes' are described below.

[2]From now on, we adopt units where $c = \hbar = 1$ unless explicitly stated otherwise.

Information route: Bekenstein's epistemic understanding of entropy. The so-called "information route" was followed by Jacob Bekenstein in his famous 1973 article, where he interprets S_{BH} as a measure of the uncertainty an external observer has about the black hole's microstate, given its observed macrostate:[3]

> The connection between [statistical] entropy and information is well known. The entropy of a system measures one's uncertainty or lack of information about the actual internal configuration of the system (Bekenstein, 1973, p. 2335).

This quotation reflects an *epistemic* understanding of the Bekenstein-Hawking entropy, and more broadly, the view that statistical mechanics concerns our knowledge of the world, rather than the world itself.[4] As such, the information-theoretic interpretation of relation (1) has been the subject of a number of criticisms. Among them are Dougherty and Callender (2017), who establish their critique in the light of a long-standing debate in the foundations of statistical mechanics. The debate concerns the reduction of thermodynamic entropy, which is undeniably "mind-independent," to (a conception of) Shannon entropy, i.e. the classical information-theoretic conception of entropy, which changes based on an agent's knowledge about the world, even when the world itself remains unchanged.

> The thermodynamic entropy is directly connected to efficiency and the amount of work a system can do. These are perfectly objective facts about boxes of gas, steam engines, and the like. A steam engine's efficiency doesn't care about whether anyone is looking or not, our uncertainty, or our beliefs. Since Shannon entropy does, it therefore cannot be identified with the thermodynamic entropy (Dougherty and Callender, 2017, p. 21).

Applying this discussion to a black hole, the argument is essentially that the facts about its microconstituents should not concern our beliefs or the information we have about them. In other words, only mind-independent entropies are identifiable with the Bekenstein-Hawking entropy understood as in thermodynamics.

Physical route: black hole entropy counts microstates. The alternative route sees the Bekenstein-Hawking entropy as counting the number of microstates available to a black hole with specified macroscopic properties, wherein the entropy increases as a result of

[3]As noted above, a precondition for this interpretation is that the observer assumes the system possesses microstates; otherwise, knowledge of its macrostates would entail full certainty.

[4]For an analysis of Bekenstein's original argument that black holes have entropy, and that this entropy is given by the Bekenstein-Hawking relation, see Wüthrich (2017) and Prunkl and Timpson (2019).

the detailed dynamics of the microphysics. This is an *ontological* statement about real degrees of freedom realising these microstates and their behaviour. This interpretation finds support from computation of statistical entropies in various approaches to quantum gravity (effective field theory, AdS/CFT duality, and string theory) which we will revisit in Section 2.3. Based on these developments, Wallace (2019) convincingly argues that, independent of any notion of information, the evidence strongly suggests that the thermodynamics of black holes is underpinned by statistical mechanics.

More critiques of information theory. Further scepticism about the concept of 'information' concerns its role in communication, in the sense of Shannon (1948)'s 'Mathematical Theory of Communication'. Wüthrich (2017) points out that, before talking about information, a communication system must be in place. This communication system has an inherent minimal complexity, consisting of essentially two elements: an 'information source' producing the information and a 'communication channel,' through which the information, once suitably encoded, is transmitted. And so, Wüthrich retorts:

> the complexity of the set-up should not be required of a physical system described by a fundamental physical theory, which is, in general, too impoverished to incarnate all these roles at once at their level (Wüthrich, 2017, p. 218).

The notion of information being intimately linked to its transmission, Wüthrich asserts that black holes, as described by general relativity, cannot realise the components of the communication system. The conditions for being able to talk about information are therefore not met.

Another complaint, not specifically tailored to black holes, was enunciated by Christopher Timpson (2013), who argues against the thesis of 'informational immaterialism,' that material elements such as particles and fields at the fundamental physical level are replaced by an immaterial basis of information. This view is, for instance, reflected in John Wheeler's famous slogan 'It from bit' (Wheeler, 1989, pp. 310-311). Timpson contends that the most basic components of the world are not immaterial in nature, and that abstractions such as information must be instantiated by something concrete.[5] In other words, we should not regard quantum information theory as introducing a new fundamental material component to the world; some thing called

[5] Timpson's view is based on his understanding of Shannon (1948)'s famous 'source coding theorem,' and its extension to the quantum realm by Schumacher (1995). The source coding theorems provide a notion of how much (classical or quantum) information a given (classical or quantum) source produces, as measured by the extent to which its output can be compressed. (Timpson, 2013, p. 4) argues that these theorems not only provide measures of these quantities, but also introduce the concept of what it is that is transmitted; of what it is that is measured.

'information.'

A question arises: What role other than an ontological fundamental one can a concept like information play in fundamental physics? Where does quantum information theory stand in relation to fundamental physics if it does not describe the behaviour of nature's most basic elements? In addressing this, Timpson argues that the success of information theory is in the introduction of an alternative general framework of enquiry in which new questions can be asked and in which devices are developed to answer them (Timpson, 2013, p. 237). The idea is to look at the system studied through the prism of quantum information theory, and to investigate its information-carrying, storing, and processing abilities.

In the context of black hole physics, this suggests that while information may not be essential to ground black hole thermodynamics, this alone is not a sufficient reason to dismiss the notion entirely. The question of viewing black holes through this prism is not new. In the past few decades, the framework of enquiry provided by quantum information theory has actually been vastly used in the study of black holes: old puzzles they posed were reformulated, new questions were asked and answers were sought using the techniques provided by theory. This paper aims to show that Hayden and Preskill (2007) were among the first to implement Timpson's proposal with interesting results.

The Hayden-Preskill model and the way forward. Hayden and Preskill (2007) make an information-theoretic analysis of an evaporating black hole by modelling it as a quantum channel through which a small amount of quantum information is transmitted. This allows them to address quantitative questions about retrieving a given amount of quantum information from an evaporating black hole. If a black hole does not destroy this information, how and when does it come out? What physical processes are involved in the information release? The first step in tackling these questions involves treating black holes as ordinary quantum systems, i.e. as objects described from the outside by a finite but large number of degrees of freedom obeying the laws of quantum mechanics. This idea was first proposed by 't Hooft (1985), further developed by Page (1993b), and has since become so central to the field that it is now referred to as the 'central dogma' of black hole physics (Almheiri et al., 2021, p. 13).

The second step is to take advantage of specific tools from quantum information theory, including the theory of quantum error-correction and the concept of entanglement-assisted communication. In the context of this model, these concepts explain how black holes process new information and how this information is released. As we will argue below, the physical processes by which information escapes in the Hayden and Preskill model involve entanglement in two ways: to help (via entanglement-assisted communication) and to protect (via error-correction) the transmission of quantum information.

Thus our slogan 'Black holes are about *quantum* information' should not be understood in the sense of information immaterialism, but rather as a *conceptual and methodological* slogan that reflects the use of quantum information in the Hayden-Preskill (and cognate) models. Quantum information theory offers an alternative perspective, alongside analytical tools, that are relevant to fundamental physics. The idea that we will illustrate in the next section is that there are precise questions about black holes that can be answered by analysing their information-theoretic properties. This framework shows that black holes have something to do with quantum information: they enable its transmission!

2.2 The Central Dogma of Black Hole Physics

In this section, we discuss the sense in which black holes can be modelled as ordinary quantum systems. This important idea in the development of black hole physics is called the 'central dogma' (Almheiri et al., 2021, p. 13).

> ***Assumption 0: Central Dogma.***
> *As seen from the outside, a black hole can be described in terms of a quantum system with $A/4G$ degrees of freedom, which evolves unitarily under time evolution.*

The central dogma states that the whole system composed of the black hole and its atmosphere, up until a certain cutoff surface of area A beyond which spacetime is treated as fixed, forms an ordinary quantum system that carries a thermodynamic entropy given by $S = A/4G = A/4\ell_{Pl}^2$, where $\ell_{Pl} = \sqrt{\hbar G/c^3}$ denotes the Planck length. In our units where $\hbar = c = 1$, this implies $G = \ell_{Pl}^2$. Roughly, there is one of degree of freedom per Planck unit of area.[6] Imagine enclosing the black hole and the spacetime region surrounding it up to that surface inside a "box" from which we could send signals to and receive signals from. If this hypothesis is true, we would be unable to determine whether the quantum system inside the box is a black hole.

An example of a clear realisation of this picture is in the context of AdS/CFT, where a cutoff surface is placed near the boundary of an AdS spacetime containing a black hole. The gravitational system within this region is then dual to a quantum mechanical system defined on the boundary by imposing a high-energy cutoff on the

[6] This statement pertains to the logarithm of the dimension of the Hilbert space, without differentiating between qubits, fermions, or other types of degrees of freedom.

CFT. Beyond AdS/CFT, it is enough to say that the two systems are equivalent (Cinti and Sanchioni, 2024, pp. 26-27).

This idea can also be understood as a restatement of postulate 1 of black hole complementarity (Susskind et al., 1993), which asserts that a distant observer can describe the formation and evaporation of a black hole entirely within the framework of standard quantum theory, with unitary evolution. Unlike in black hole complementarity, however, this assumption does not rely on the membrane paradigm or the existence of a stretched horizon. As discussed by Cinti and Sanchioni (2024), the central dogma makes no commitment about the spatial location of the black hole degrees of freedom. This point will become important later, when we examine in Section 4.1 Hayden and Preskill's assumption that these degrees of freedom are uniformly distributed on a stretched horizon.

Up to this point, nothing has been said about the interior structure of the black hole: the central dogma gives a picture of the black hole as seen from the outside. In this external picture, the black hole-plus-atmosphere system is portrayed as an ordinary quantum system, with the following properties:

(a) It is made up of a finite but large number of degrees of freedom, not required to be spatially located at the black hole horizon.

(b) These degrees of freedom obey the laws of quantum statistical mechanics, which in turn imply the laws of thermodynamics. Notably, the dynamics is unitary, and the system's evolution can be generated by a Hamiltonian.

Given that quantum circuits serve as models for the time evolution of quantum systems (cf. Sect. 3), the central dogma is key in justifying the very use of quantum circuits to represent the way in which black holes process information. To what extent is this core hypothesis established? It is supported in two ways: first, a number of theoretical developments provide explicit backing for the claim that the Bekenstein-Hawking entropy is a thermodynamic entropy; second, the rejection of certain rebuttals prevents concluding that this claim is simply false. We argue that the theoretical evidence supporting the central dogma is robust. Further details are provided in the following two sections.

2.3 Evidence from Black Hole Thermodynamics

This section explains why the degrees of freedom in the central dogma are responsible for the thermodynamic behaviour of black holes. This argument proceeds in two steps, both discussed by Wallace (2018, 2019). The first step is limited to phenomenological thermodynamics. The point is to demonstrate that, without going into microscopic considerations, these black holes behave like thermodynamic objects. Once the thermodynamic behaviour has been established, the second step is to derive it from statistical mechanics (with or without relation to information).[7]

We will be brief on the step concerning phenomenological thermodynamics, as we share the view of Wallace (2018), who regards Hawking radiation to be the central result that transforms the apparent analogy into an identity.

Hawking radiation allows a black hole to exchange energy and heat with its environment, and thus places the black hole in thermal contact with its surroundings.[8] Several constructions of Carnot-like cycles suggest that this interaction occurs in the same way as it would for a thermodynamic system with the corresponding entropy (Kaburaki and Okamoto, 1991; Curiel, 2014; Bravetti et al., 2016; Prunkl and Timpson, 2019; Wallace, 2018).

Dougherty and Callender (2017), on the other hand, took a different approach to the question. They challenged the thermodynamic understanding of the black hole entropy, arguing that the supposed thermodynamic behaviour of black holes relies on a caricature of thermodynamics. As Wallace stresses, however, unlike the other authors, their work does not correctly reflect the role of Hawking radiation in establishing the black hole's thermal contact with its environment (Wallace, 2018, appendix).

We now turn to the second step, which is to seek, as we do for any other thermodynamic system, in statistical mechanics the fundamental justification for this phenomenological thermodynamic behaviour. As mentioned earlier, Bekenstein certainly followed the "information route." Nevertheless, Wallace (2019) argues that this is no longer necessary: modern developments provide a strong foundation for black hole thermodynamics in statistical mechanics, requiring no further mention of 'information.' They instantiate what we have called the 'physical route' and stand on their own with no need for Bekenstein's informational motivation. For this reason, we set aside the controversial link between information and thermodynamics, and briefly outline the recent

[7] Curiel (2024) argues that a good thermodynamics conception of entropy "alone" is essential for these computations to serve as evidence of black hole statistical mechanics. Without this, there is no justification that these computations are counting the relevant microstates—those whose dynamics give rise to thermodynamical behaviour.

[8] As noted by Curiel (2021), an important conceptual issue with Hawking radiation is that it is not standard blackbody radiation, since it is not produced by the random movements of degrees of freedom of the horizon. Rather, Hawking radiation consists of excited modes of an *external* quantum field that are not coupled to the spacetime geometry, as the derivations do not account for back-reaction on spacetime.

progress Wallace has in mind.[9] These recent developments are important because, by highlighting the existence of real degrees of freedom, they pave the way for an abstract characterisation of black holes through information-theoretic reasoning—an alternative role that information can play, as this article aims to demonstrate.

These developments include different microstate countings in fairly different theories of quantum gravity. Although it needs not be so, they all give the same result: the existence of a space of dimension $\exp(A_{bh}/4)$ for the black hole microstates, in agreement with the Bekenstein-Hawking entropy. For Wallace (2019, p. 30), it is not remotely plausible that all of this is a massive coincidence.

Among the calculations able to reproduce the Bekenstein-Hawking entropy, both Gibbons and Hawking (1977); Gibbons et al. (1978) analyse general relativity as a low-energy field theory. That is, they take this theory as giving a coherent description of physics only at scales below a certain cut-off scale. They can thus write the path integral for the gravitational field and use it to compute the partition function, using the saddle-point approximation.[10] The use of standard statistical methods then gives an answer for the entropy but does not specify what microstates are being summed over. Wallace (2019, p. 3) argues that the ability of partially developed theories of quantum gravity to reproduce with precision the black hole entropy formula is difficult, if not impossible, to make sense of unless these theories really were partial descriptions of a consistent quantum gravity theory grounding black hole thermodynamics.

The Bekenstein-Hawking entropy has also been calculated using string theory, which is among the most popular of the more speculative approaches to cutoff-free quantum gravity. On the one hand, Strominger and Vafa (1996) use an arsenal of mathematical tools from string theory, combining dualities, variation of relevant parameters and symmetry-protected properties to compute the entropy of some special black holes (extremal and near-extremal) using strings and D-branes (higher-dimensional hyperplanes where strings can end).[11] According to Wallace (2019, p. 22), it is quite challenging to account for the derivation (of the leading-order terms, higher-derivative corrections, and quantum corrections) without acknowledging that the entropy of extremal and near-extremal black holes has a statistical-mechanical origin and that string theory offers an ultraviolet completion of low-energy quantum general relativity, at least in the regimes appropriate to those black holes. If this is true, one might say that the central dogma

[9] Readers interested in the details can find them in his article (Wallace, 2019).

[10] The idea is to approximate the path integral by the value the integrand takes when the action is stationary. Since this is a gravitational path integral, the saddle points are geometries.

[11] The companion papers by De Haro et al. (2020); van Dongen et al. (2020) provide a thorough analysis of the argument, clarifying key assumptions and its context in mid-1990s string theory. See also (De Haro and Butterfield, 2025, Sect. 10.2) for a brief philosophical summary.

has been formally derived in the context of string theory.

On the other hand, in the specific context of the AdS/CFT correspondence (Maldacena, 1999), one can use the so-called Cardy formula in two-dimensional CFTs (Cardy, 1986) to compute the entropy form the CFT side of the duality, where the black hole is a fluid of strongly interacting particles. Not only can the computation be carried out, but the situation is arguably more conceptually tractable in the absence of gravity. Independently of this, Wallace's point is that such calculations provide evidence for the statistical mechanics of black holes well beyond those specifically addressed by string theory (Wallace, 2019, p. 22). This, however, comes at the cost of defining the cutoff-free quantum gravity theory in an asymptotically Anti-de Sitter spacetime.

2.4 Possible Rebuttal of the Central Dogma from a Paradox

The previous section discussed that, by combining the evidence coming from phenomenological thermodynamics, with the full recognition of the role of Hawking radiation, and the results of different microstate countings, we have good reason to believe that the Bekenstein-Hawking entropy is a *thermodynamic entropy*, underpinned by quantum statistical mechanics. For an outside observer, the black hole can be considered a quantum statistical mechanical system whose number of degrees of freedom is fixed by the thermodynamic entropy.

If the central dogma is true, the black hole dynamics, as it appears from the outside, must be *unitary*. But anyone aware of Hawking (1976b)'s 'information loss paradox' will burn to object that the complete evaporation of a black hole is a process that violates unitarity. While it is correct that there is a tension with the laws of quantum mechanics, it is wrong that this tension appears only when the black hole has completely vanished. For, first, jumping to the conclusion that such a process violates unitarity may be premature since a full understanding of quantum gravity is required to describe the final moments of evaporation. Second, we will explain how inconsistencies between Hawking's perspective of evaporation and the external picture given by the central dogma appear well before the end of the evaporation.

The realisation that troubles arise earlier than Hawking anticipated stems from a second version of the paradox, known as the 'Page time paradox,' introduced by Page (1993b).[12] Wallace (2020) regards this version as genuinely paradoxical, because the

[12]It is somewhat anachronistic to say that Page conceived his work as a challenge to the central dogma, as this terminology entered the vocabulary of high-energy physicists much later. However, it is indeed the central dogma that he implicitly commits to when he considers the black hole from the perspective of quantum mechanics.

quantum statistical mechanical description of black holes, as articulated in the central dogma, conflicts with the predictions of QFT in curved spacetime derived by Hawking (1975), at a length scale where both these results are expected to be reliable. Which one should be abandoned?

Recent work based on the path integral approach to gravity suggests that Hawking overlooked certain small non-perturbative corrections, which become important after the black hole has radiated more than half of its initial entropy and thus keep open the possibility that the central dogma holds.[13]

In Section 2.4.1, we explain how the results of Hawking's calculation seem to be in conflict with the central dogma, even when the black hole might still be very large. Then, in Section 2.4.2, we discuss how new developments offer a way out of this paradoxical situation, consistent with the central dogma.

2.4.1 Page time Paradox

Following Wallace (2020), Page (1993c)'s idea was to incorporate evaporation into the simple picture of black hole given by the central dogma. As a general rule, one considers the evaporating black hole to be in a pure state before evaporation. By unitarity, at any time after the evaporation started, the composite system of the black hole and the Hawking radiation should still be in a pure state.[14]

Page demonstrated that, if the central dogma is true and the black hole can be described from the perspectives of both statistical mechanics and quantum mechanics, respecting unitarity results in a bound, imposed by the Bekenstein-Hawking entropy, on the number of Hawking modes entangled with the black hole degrees of freedom.

[13] Almheiri et al. (2021) provides a review of these findings tailored for readers with minimal prior knowledge of the subject and includes the relevant references. Cinti and Sanchioni (2024) give more details on the calculations, focusing on the crucial differences with previous approaches.

[14] What is specific to Page's approach is that, at any time after evaporation begins, he assumes the black hole and its radiation to be in a *randomly* chosen pure state. The meaning and significance of this assumption is discussed in Section 3.2.

The so-called 'Page bound' reads:[15]

$$S_{vN}(\rho_R(t)) \leq S_{BH}(t)\,. \tag{2}$$

The expression on the left-hand side, i.e. $S_{vN}(\rho_R(t)) = -\operatorname{tr}[\rho_B(t)\log\rho_B(t)]$, is the von Neumann entropy of the radiation. It is a quantum information-theoretic entanglement entropy, and it is the expression to which Hawking's calculation in quantum field theory on a curved background applies. The expression on the right-hand side, i.e. $S_{BH}(t) = \log\dim\mathcal{H}_B(t)$, is a quantum Boltzmann entropy, obtained as the (logarithm of the) number of black hole microstates.

The *Page time paradox* is as follows: by producing radiation that is exactly thermal, i.e. emitting quanta of radiation in highly mixed states, and evaporating, the black hole eventually violates the above bound—and thus unitarity—long before the black hole completely evaporates. This comes from two effects. First, the fact that the black hole evaporates means that $S_{BH}(t)$ decreases over time. Thus the bound on the entanglement of the radiation with the black hole degrees of freedom, Eq. (2), becomes tighter as evaporation proceeds because *the right-hand side decreases*. Second, Hawking (1975) showed that the density matrix of the radiation is perfectly thermal. This is a consequence of the entanglement of the QFT vacuum state across the event horizon and from tracing over the physically-inaccessible partner mode just inside the horizon. More technically, this means that in $\rho_R(t)$, no emitted quanta can be entangled with one another, but each is entangled with a degree of freedom of the black hole system. Therefore, since each emission of a Hawking quantum increases the entanglement with the black hole degrees of freedom, $S_{vN}(\rho_R(t))$ on *the left-hand side of Eq.* (2) *increases.*

Thus if Hawking's calculation is valid, evaporation increases the left-hand side and decreases the right-hand side of the Page bound, making it increasingly difficult to maintain unitarity. The time at which the Page bound saturates and there are no longer enough black hole modes to maintain the thermal nature of the radiation is known as the 'Page time.' The Page time is approximately half of the evaporation time for a Schwarzschild black hole, a moment at which an initially large black hole can still

[15] A quantum statistical mechanical black hole can be assigned two types of entropy: a thermodynamic entropy $S_{BH}(t)$, which, à la Boltzmann, counts the number of independent degrees of freedom, and a von Neumann entropy $S_{vN}(\rho_B(t))$, which, given that, from unitarity, the composite system is pure, counts the number of entangled degrees of freedom between the black hole ρ_B subsystem and the radiation subsystem ρ_R. In particular, $S_{vN}(\rho_B(t)) = S_{vN}(\rho_R(t))$. These two notions of entropy are connected because the number of degrees of freedom that make up a quantum system limits its entanglement with the outside. This stems from a fundamental property of entangled systems called the 'monogamy of entanglement:' a degree of freedom can be maximally entangled with only one degree of freedom at a time.

be considered reasonably large. The paradox is visualized in Figure 1, which depicts the behaviour of the entropies during black hole evaporation.

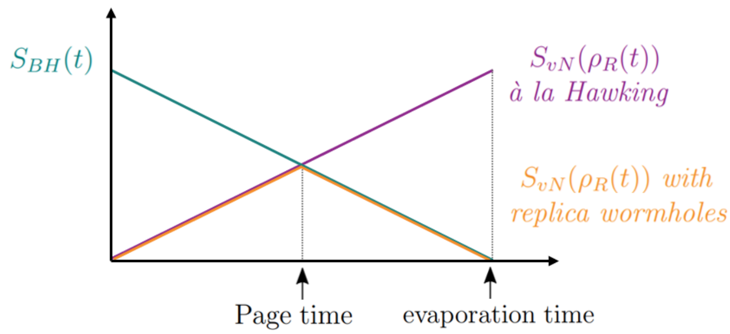

Figure 1: Page time paradox. As evaporation proceeds, the Bekenstein–Hawking entropy (blue curve) decreases, while the entanglement entropy of the radiation, as computed by Hawking (purple curve), increases. At the Page time, there are no longer enough black hole modes to purify the radiation. Computing the entanglement entropy using the gravitational entropy formula yields the orange curve, which respects the Page bound.

There are at least two possible conclusions from this violation of the Page bound (Eq. (2)), i.e. from the Page time paradox, neither of which is particularly attractive:

(a) One can abandon the central dogma, and with it the explanation for the black hole's thermodynamic behaviour. Under this assumption, black holes, like no other known thermodynamic system, are not underpinned by statistical mechanics. Furthermore, one would have to find an alternative, non-miraculous explanation for the precise derivation of the entropy by various forms of quantum gravity (effective field theory; string theory; AdS/CFT).

(b) One can call the accuracy of Hawking's calculations into doubt and argue that Hawking radiation has non-thermal components. This is the option taken by Page in his paper, where he proposes (based on information theoretic reasoning described in Section 3.2.2) that the Hawking radiation emitted after the Page time, instead of being entangled with the black hole system, is entangled with the radiation emitted earlier. In this way, the early radiation purifies the state, preserving the purity of the black hole-plus-radiation system and restoring unitarity. If this is the case, then the breakdown of QFT in a curved background, i.e. the fact that QFT predicts loss of unitarity, while Page's argument says that unitarity is maintained, would have to be explained, especially since it occurs well within the apparent domain of applicability of the theory.

2.4.2 A Potential Resolution of the Page Time Paradox

In this section, we review recent works that identify a flaw in Hawking's quantum field theory calculation and propose a resolution that renders black hole evaporation consistent with the central dogma (thus favouring option (b) above).

The core of recent advances in black hole physics is the derivation of a general gravitational formula for computing the von Neumann entropy of quantum systems coupled to gravity (Lewkowycz and Maldacena, 2013; Faulkner et al., 2013; Dong et al., 2016; Dong and Lewkowycz, 2018).[16] This formula is derived employing a method akin to the one of Gibbons and Hawking (1977) previously mentioned. The novelty is the incorporation of 'replica wormhole saddles,' namely non-perturbative abstract geometric saddles that modify the gravitational path integral and, ultimately, the entropy derived from it.

Therefore, the proposal is that Hawking (1975) used the wrong formula for computing the von Neumann entropy of Hawking radiation, namely that of quantum fields in a fixed curved spacetime. Even though the radiation lives in a region where the gravitational effects are small, the fact that we are describing a state within a theory of gravity means that we should use the gravitational formula for the entropy. The idea is that the states in a QFT without gravity differ from the states in a theory of quantum gravity even in the limit where gravity is weak.

This proposal was implemented for the case of an evaporating black hole in AdS/CFT (Almheiri et al., 2019, 2020; Penington, 2020). To build an evaporating black hole in AdS/CFT, one takes a black hole formed from collapse and places it into a bulk geometry whose boundary is not reflecting but absorbing. This change of boundary conditions allows the Hawking quanta to escape into a reservoir. From a boundary perspective, this procedure is the same as (generically) coupling the boundary system B dual to the black hole to an auxiliary system R, whose bulk dual need not be further specified. The black hole-plus-radiation gravitational system is thus holographically dual to the boundary-plus-auxiliary quantum system. Using the gravitational entropy formula to compute $S_{vN}(\rho_R(t))$, it was shown that the Page bound (2) is respected throughout the evaporation process (see Figure 1).

One might wonder about the relevance of these results for black holes in the "real world," which does not seem to be AdS spacetime. As it turns out, subsequent works studied the process of black hole evaporation in two-dimensional gravitational model

[16] Prescriptions for calculating this entropy in holographic settings had already been published in a series of works Ryu and Takayanagi (2006); Hubeny et al. (2007); Engelhardt and Wall (2015). For an evaporating black hole in AdS/CFT, this is sometimes called the 'island formula.'

starting from the gravitational path integral, but without appealing to holography, and reached the same conclusion: namely, that the Page bound is respected (Penington et al., 2019; Almheiri et al., 2020). In these calculations, the path integral prescription employed by Hawking was revised by incorporating the contribution of replica wormhole saddles.[17] In other words, the error in Hawking's calculation stemmed from an incomplete application of the saddle-point approximation, neglecting the contribution of certain saddles, that do not have a counterpart in a non-gravitational QFT. Before the Page time, the additional saddle is heavily suppressed, and the Hawking saddle dominates the gravitational path integral. After the Page time, however, the replica wormhole becomes important and renders the evolution of the entropy of Hawking radiation consistent with unitarity.

Summing up: recall that unitarity implies the Page bound, Eq. (2), which is violated by Hawking's calculation leading to the conclusion of a genuine loss of information. This is the Page time paradox. Recent calculations, both for two-dimensional gravity and for AdS spacetime, have pointed to a shortcoming of Hawking's calculation that goes into Page's result: namely, Hawking did not include non-perturbative corrections in the path integral, and specifically replica wormholes, in his calculation of the entropy. If replica wormholes are included, then the Page bound in Eq. (2) is respected and the Page time paradox is resolved.

Since Hawking's original calculation is some sense incomplete, and more recent calculations are consistent with the assumption of unitary evolution, it is reasonable to expect that the ultimate resolution of the Page time paradox will be consistent with unitarity. We say 'ultimate resolution', because the argument given so far relies on idealizations, like taking low dimensions or considering a black hole in AdS, and so the Page time paradox has not received a definitive solution for the Schwarzschild black hole. In other words, although the paradox has not been resolved definitively other than in these cases, the argument clearly suggests that Hawking's conclusion is unwarranted, and the paradox for the Schwarzschild black hole can be solved along similar lines. If that is the case, then the central dogma may well remain a robust statistical mechanical foundation for explaining black hole thermodynamics.

[17] There is still a great deal of philosophical analysis that is required to unpack these works. For a start, see Cinti and Sanchioni (2024), who give more detail on the calculation of Penington et al. (2019); Almheiri et al. (2020), including a discussion of how the degrees of freedom appearing in the central dogma also describe part of the black hole's interior. See also Cinti and Sanchioni (2025).

3 The Dawn of Quantum Circuits: Page's Model and Scrambling

Quantum circuits serve as models for the time evolution of quantum systems. If, as the central dogma suggests, black holes are ordinary quantum systems, then their unitary evolution can be modelled using a quantum circuit. Such models aim to highlight the information-theoretic properties of black holes. The quantum information of interest is constituted by the initial state of (a subset of) the qubits. Subject to the black hole's unitary dynamics, the multi-qubit state evolves unitarily over time. In this way, the black hole performs a computation in which information is processed but not destroyed.

This computational perspective prompts several questions that can be explored within the conceptual framework provided by quantum information theory. For example, one can ask how much information has been transformed, whether it can be efficiently recovered, and even whether black holes can be used for communication.

These questions are important because the debate surrounding information loss paradoxes extend beyond the issue of unitary evolution. As discussed in the previous section, Page (1993b) assumed unitary evolution yet arrived at a conclusion that truly contradicts the description provided by quantum field theory (cf. Sect. 2.4.1).

Page went beyond merely pointing out the Page time paradox: he pioneered the modelling of black holes using quantum circuits. To study the time dependence of the entanglement between the black hole and its radiation, he assumed that the black hole started in a certain state of a finite number of qubits as well as a certain unitary dynamics for its evolution, which we explain below are the essential features of a quantum circuit model. His results show that the entanglement starts out small, increases until half the entropy has been radiated, and then decreases, ensuring that the Page bound (2) is respected throughout evaporation.

Another key concept in Page's legacy is the notion of 'scrambling:' a dynamical process through which information falling into a black hole becomes highly entangled and distributed across the system's degrees of freedom, rendering it inaccessible through local observations. This suggests that black holes conceal quantum information, making it accessible only to sufficiently "patient" observers who wait for enough radiation to be emitted.

Section 3.1 provides a general introduction to quantum circuits, followed by an outline of the key features that specify any quantum circuit (QC) model of black holes. Section 3.2 delves into Page's model, where the concept of scrambling first emerges and which forms a cornerstone of both his approach and the later model proposed by

Hayden and Preskill.

3.1 Quantum Circuit Model

In Section 3.1.1, we explain what quantum circuits are made of and what they aim to represent. Then, in Section 3.1.2, we specify the key features that need to be defined when considering them as a model for how black holes process information.

3.1.1 Introduction to Quantum Circuits

Quantum circuits are both visual and algebraic representations of the set of changes occurring to the state of a quantum system, in terms of the action of a unitary operator on an input state to produce a certain output state. They consist of abstract wires and logical gates. The wires are thought to carry the quantum information (they correspond, for example, to a physical particle moving in space and/or time, or more generally to one or several qubits), while the gates are simple operations that manipulate this information. By 'simple operation,' we mean operators involving a small number of qubits. More precisely:

> A k**-local gate** is a non-trivial k-qubit unitary operator chosen from a universal gate set, where a universal gate set is a collection of gates whose finite sequences can approximate any unitary operation to arbitrary accuracy.[18]

The 'k-local' character of the gates refers to the small number of qubits on which they act, and not to any spatial location. Typically, only 2-local gates are considered, since 1-local gates cannot create entanglement (Susskind, 2016, p. 5), making 2-local gates necessary. Moreover, any arbitrary unitary operator can be expressed *exactly* as a product of unitary operators acting non-trivially on two or fewer qubits. Thus, 2-local gates are sufficient (Nielsen and Chuang, 2010, Chap. 4.5).

What is more surprising is that the number of different types of 2-local gates needed to *approximate* any unitary operation to arbitrary accuracy is relatively small.[19] An example of such a universal gate set includes the Hadamard, phase, controlled-NOT

[18] An additional assumption is made that if a gate is in the set, so is its Hermitian transpose.

[19] Approximating a unitary operator means implementing another operator that behaves sufficiently similarly to the desired one. This similarity is quantified by how close the measurement outcomes are when using the approximation versus the exact operator. When multiple gates are approximated in a sequence, their individual errors accumulate at most linearly. By keeping each approximation within a small enough error bound, the overall accuracy of the quantum circuit can be guaranteed to stay within a specified tolerance (Nielsen and Chuang, 2010, p. 194).

and $\pi/8$ gates (Boykin et al., 2000).[20]

We can now give a precise definition of quantum circuits made of these gates:

> A k**-local all-to-all quantum circuit** is a sequence of k-local gates that permits any group of k qubits to interact.

A given quantum circuit thus represents the preparation of a particular unitary operator U by acting with a sequence of gates from the universal set $\{g_1, g_2, \ldots, g_n\}$ on the identity operator $\mathbb{1}$:

$$U = g_i g_{i-1} \ldots g_1 \mathbb{1}. \tag{3}$$

Equivalently, it models how the output state $|\phi\rangle$ is obtained from the input state $|\psi\rangle$ by acting with this unitary, i.e. $|\phi\rangle = U|\psi\rangle$.

The all-to-all character of a circuit is a particular component of its 'architecture.' The 'architecture' of a quantum circuit is the spatial structure of the circuit, i.e. the way in which the quantum gates and qubits are connected and organised within the circuit. In this case, the architecture is such that interactions between the qubits are not limited by any constraints, spatial or otherwise. Another feature of a quantum circuit's architecture is that it can exhibit 'parallel processing' if all qubits interact exactly once at each step.

One useful aspect of quantum circuits is that they can be easily illustrated using a few lines and boxes (see Fig. 2). The qubits are represented as lines, while the gates are shown as thick horizontal lines or boxes that act on those qubits. This visual representation depicts the step-by-step transformation of a quantum state, or how a unitary operator acts on an input state to produce a certain output state.

[20] The Hadamard gate creates superpositions, transforming $|0\rangle$ and $|1\rangle$ into equal superpositions. The phase gate adds a phase of i to $|1\rangle$, while the $\pi/8$ gate introduces a phase of $e^{i\pi/4}$. The controlled-NOT gate, a two-qubit operation, flips the target qubit if the control qubit is $|1\rangle$.

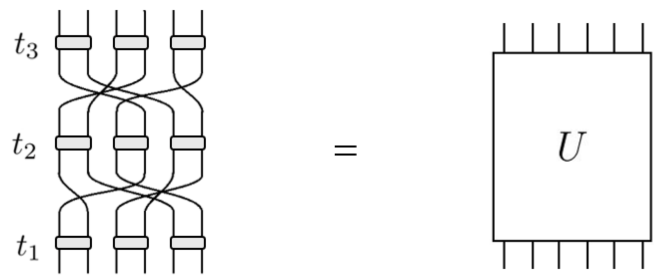

Figure 2: 2-local all-to-all circuit with parallel processing, where any qubit pairs with any other qubit at each step and interacts therewith by means of a 2-local gate, but no qubit interacts with more than one other qubit. This depicts the gradual transformation of a quantum state of six qubits (the lines) by the action of gates (the boxes). Equivalently, it represents the preparation of the unitary operator U.

One can equip quantum circuits that exhibit parallel processing with a clock. The notion of time introduced in this way is called the 'circuit time' and is denoted by t_n, where n indexes the number of steps in the circuit. In the diagram, it runs in the vertical direction. Other important definitions for circuits include their 'depth,' which refers to the number of time steps, and their 'size', which refers to the the total number of gates.

Finally, even if we said that quantum circuits can generate any unitary operation, it is not always possible to do this efficiently. That is, given a unitary transformation U on K qubits, there does not always exist a circuit of size polynomial in K approximating U. As an example that will be important later, generating a random unitary is highly inefficient: it takes a circuit of size exponential in K.[21]

3.1.2 Quantum Circuit Model of Black holes

Let us now discuss the specificities of quantum circuits as a model of black holes.

Assuming the central dogma is true, black holes are ordinary quantum systems and their dynamics can be modelled by a quantum circuit. To say this in another way, a quantum circuit features, at the very least, a finite number of qubits and a sequence of gates acting on them. These components are exactly what the central dogma gives us (cf. Sect. 2.2): property (a) stipulates that a finite number of degrees of freedom make up the black hole. It says nothing about their exact nature, but these subsystems can be treated as the qubits on which the gates act. Property (b) indicates that the

[21] See Knill (1995) for a detailed investigation of the hardness of approximating arbitrary unitary operations using quantum circuits.

dynamics is unitary, meaning that it may be approximated to arbitrary accuracy by a quantum circuit involving simple operations chosen from some universal gate set.

That being said, additional modelling assumptions and idealisations remain to be made in order to determine the exact characteristics of the QC-model and make it applicable to problems in black hole physics. In particular, three features must be specified to define the quantum circuit used to model a black hole. We will call these the *features of the model.* Each of these features reflects details of the physical object it represents, namely the black hole:

(1) the *number of qubits* determines the dimension of the Hilbert space characterising the black hole,

(2) the *circuit architecture* and *size* represents the type of Hamiltonian generating the internal dynamics, and the efficiency with which the latter can be approximated,

(3) the *initialisation of the qubits* is the state in which the black hole starts.

To fully specify the circuit model, a choice has to be made for each of these features. These choices are influenced by what is known, believed and desired about black holes, but in the absence of a theory of quantum gravity confirmed by observations, they remain ultimately hypotheses.

As is to be expected from a model, the representation of a black hole by a quantum circuit is simple, or at least simplified, and idealisations are necessary in its achievement. One such idealisation is the foundational reliance on the central dogma, which provides an external perspective on the black hole. As a result, the model does not attempt to describe what happens very close to the event horizon, or behind it. Another major simplification is that the quantum circuit model operates at a highly abstract level: the processing of information is represented independently of the detailed physical interactions between the particles that instantiate that information in the real world. These underlying interactions are not modelled explicitly.

The specific assumptions and idealisations underlying the black hole model proposed by Hayden and Preskill are laid out in Section 4.1, along with the theoretical motivations or supporting arguments for each.

3.2 Page's Model in Relation to Scrambling

This section focuses on scrambling, a concept first introduced in the work of Page (1993a) and now regarded as central in black hole physics. Later, the fact that the unitary dynamics of the black hole had scrambling properties was an important feature of the QC-model of Hayden and Preskill (2007) in obtaining their results. This drew significant attention to the concept, leading, for instance, Sekino and Susskind (2008) to conjecture that black holes are the fastest scramblers of information in nature, and Shenker and Stanford (2014) to establish a connection between their evolution and that of chaotic systems.[22]

To approach this concept, however, we must be cautious as there are multiple, non-equivalent definitions of scrambling in the literature, and the understanding of this relatively "recent" phenomenon has evolved over time—and may continue to do so. While a more in-depth analysis is left for future work, the discussion presented here serves as a starting point.

Section 3.2.1 introduces the concepts of 'Haar scrambling' and the weaker notion of 'Page scrambling,' both of which originate from Page's approach.[23] This will help to clarify the intermediate definition used in the model of Hayden and Preskill (2007), which we present in Section 4.1. It also clarifies the connection between Page scrambling and thermalisation. Section 3.2.2 discusses Page's very first QC-model of a black hole, treating it as a quantum system with specific dynamics and analysing the evolution of the entanglement between the black hole and its radiation during evaporation. The relationship between Page's results, scrambling, and information release is also explained.

3.2.1 Page and Haar Scrambling: Definitions

Scrambling is typically considered a property acquired by the state of a quantum system after it evolved under a specific unitary dynamics. The most stringent definition of scrambling is based on the concept of a 'Haar-random unitary.' A Haar-random unitary is a unitary transformation that is selected uniformly at random according to the Haar measure on the corresponding unitary group, i.e. $U(2^b)$ for a system of b qubits.[24] The Haar measure properly weighs different regions of the unitary group, allowing one to sample its elements uniformly given the structure of the group.

By applying a Haar-random unitary to a fixed reference state (such as the input of a quantum circuit), one can generate quantum states uniformly at random. This leads to the following definition of a 'Haar-scrambled' system:

> A system is **Haar-scrambled** when its quantum state has been randomised with respect to the Haar measure over the entire Hilbert space.

[22]For a review of recent developments on chaos in holographic settings, see Jahnke (2019).

[23]The terminology was introduced by Sekino and Susskind (2008).

[24]For the mathematical foundations of the Haar measure, see Duistermaat and Kolk (1999).

In other words, a Haar-scrambled system is one that has evolved from a reference state under the action of a Haar-random unitary transformation. With overwhelming probability, so generated random pure states have near-maximal entanglement (Hayden et al., 2006). We will omit the prefix 'Haar-' when referring to random unitaries in what follows, with the understanding that this remains the intended meaning unless stated otherwise.

As any other unitaries, such random unitaries can be generated by a quantum circuit (cf. Sect. 3.1.1). However, this is very inefficient and requires a quantum circuit of exponential size in the number of qubits. This is because the space of unitary operators acting on a b-qubit system has a volume that is exponentially large in the dimension of the Hilbert space (Nielsen and Chuang, 2010, Sect. 4.5.4). Sampling uniformly with respect to the Haar measure thus requires a circuit whose size is exponential in b.[25]

A much weaker notion of the scrambling of a state is the following:

> A system in a pure state is **Page-scrambled** when all subsystems smaller than or equal to half of the whole system are nearly maximally mixed.

As we will explain, the notion of Page scrambling underpins our expectation that the Page-time paradox can be resolved. It is this property that Hayden and Preskill also leverage to propose a mechanism for information release using quantum information-theoretic tools (see Sect. 4.2).

Following the work of Hayden and Preskill, Page scrambling wsa initially understood as a stringent form of thermalisation that applies to the evolution of closed quantum systems (e.g. Lashkari et al. (2013) and Sekino and Susskind (2008)). Thermalisation refers to the growth of entanglement between parts of a closed quantum system undergoing chaotic evolution. As the system evolves from an initially unentangled (e.g., product) state, couplings between microscopic degrees of freedom develop, causing small subregions to become increasingly entangled with the rest. These subsystems then appear thermal: their reduced density matrices approach maximal entropy. In other words, thermalisation causes the system to relax to a state that is *locally* indistinguishable from a thermal state. The process of scrambling that would lead to a Page-scrambled state was understood as a strong form of thermalisation making all smaller-than-half subsystems very nearly maximally mixed.

However, Hosur et al. (2016) showed that the entanglement growth associated with thermalisation does not occur on a timescale fast enough to allow for information release.

[25] This fact forces Hayden and Preskill (2007) to rely on a weaker definition of scrambling in their QC-model of a black hole, as we explain in Section 4.1. Notably, this weaker form of scrambling is still sufficient for achieving Page scrambling, despite being efficiently implemented by a quantum circuit.

As a result, Page scrambling should not be equated with thermalisation.

The modern understanding of scrambling instead centres on the *delocalisation* of initially local quantum information. Information about a small part of the input becomes rapidly distributed across the entire system, rendering it inaccessible from any small part of the output.[26] Thus, a system is said to be Page scrambled when no information about the input can be recovered from any subsystem smaller than half the system's total size.

This delocalisation is closely related to the growth of operators: under chaotic dynamics, initially simple operators evolve into increasingly complex ones that act non-trivially on larger and larger subsets of the system (Shenker and Stanford, 2014; Kitaev, 2015). As a result, an operator that initially perturbs only a few degrees of freedom becomes spread across the entire system over time. Once this spreading has occurred, it becomes impossible to recover information about the operator's original action without access to the full set of degrees of freedom it now acts on. The scrambling of quantum information in this way can be diagnosed by the decay of 'out-of-time-order correlators,' which vanish at late times when the initial information has been effectively scrambled. In the context of black holes, this decay occurs rapidly enough to support the Hayden–Preskill argument. We will return to this point in Section 4.1.

3.2.2 Page's Modelling and Results

Our focus now shifts to Page's QC-model of an evaporating black hole. We will explain how Page used it to demonstrate that a system that is Haar-scrambled is also Page-scrambled. In the context of black hole evaporation, this guarantees that the Page bound is satisfied.

Page sought to 'examine a more natural model, in which the black hole and its surrounding radiation are two subsystems of a combined system which is assumed to be in a random pure state' (Page, 1993c, p. 3). In his approach, Page described the black hole by a quantum system B made of $b = A_{bh}/4G$ qubits and modelled its dynamics by a Haar-random unitary. Specifically, B is divided into two parts: one contains d qubits and the other $b - d$ qubits. A unitary transformation, chosen uniformly with respect to the Haar measure on $U(2^b)$, is applied to the whole system. After this transformation, the d-qubit subsystem is identified as the radiation and the $(b - d)$-qubit subsystem as the remaining black hole.

Page showed that the entanglement entropy of the radiation subsystem is close to

[26] In Section 4.2, we will see how this spreading can be analysed by studying four-party entanglement between the input and output using an information-theoretic measure called 'mutual information.'

maximal in the Haar-scrambled pure state, so long as it is smaller than half the size of the whole system. Quantitatively,

$$S_{vN}(\rho_D) = d - O\left(e^{2d-b}\right) \tag{4}$$

for all $d < b/2$, which means that the black hole-plus-radiation system is Page-scrambled (Page, 1993a, p. 4). This equation expresses that the entanglement between the radiation and the black hole starts small, increases until half the black hole entropy has been radiated, and then decreases (cf. orange curve in Fig. 2). In other words, Page demonstrated that combining a Haar-random dynamics with evaporation ensures that the Page bound (2) is respected throughout evaporation.

In relation to information loss (or more accurately, information release), this implies that the information swallowed during the formation of a black hole remains hidden until the black hole's original area has shrunk by more than half. After this point, the information emerges at a constant rate as evaporation proceeds, with non-thermal components of the radiation becoming increasingly apparent. This corresponds to option (b) for resolving the Page time paradox, as described in Section 2.4. In this sense, the information within the black hole-plus-radiation system is mixed up so thoroughly that it can only be recovered by accessing to at least half of the degrees of freedom.

4 From Scrambling to Communication: Hayden and Preskill's Model

The previous section presented Page's model, which suggests that an evaporating black hole, modelled as a Haar-scrambled system, begins releasing information at a constant rate once half its entropy has been radiated. However, this does not resolve all aspects of the Page time paradox. In particular, Page's model does not explain why scrambling is crucial for information to escape, nor what conditions an observer must satisfy to recover it.

To address these questions, Hayden and Preskill (2007) extended Page's QC-model. Their work focuses on determining, in principle, when and how a given amount of quantum information becomes accessible in the radiation. With the addition of certain assumptions, they explored how a black hole can participate in a communication protocol, using its quantum properties—namely entanglement—to protect information against error and release it relatively quickly.

These authors set up a thought experiment, which we briefly outline. Enter the main protagonists Alice and Bob, with Bob being the top forensic scientist of Alice's era. Alice writes her secrets in a quantum diary (e.g., the memory of a quantum computer) which she throws into a black hole. The black hole evaporates, loses mass and eventually vanishes, encoding Alice's diary in the outgoing Hawking radiation where it might be decrypted by Bob. Since the black hole evaporates extremely slowly (the evaporation time is of the order of M_{bh}^3 in Planck units), Alice believes that her secrets will be safeguarded for many generations. Bob wants to access Alice's confidences from Hawking radiation. He does not want to wait forever and would like to decode the radiation before the black hole disappears. See the representation by the quantum circuit in Figure 3.

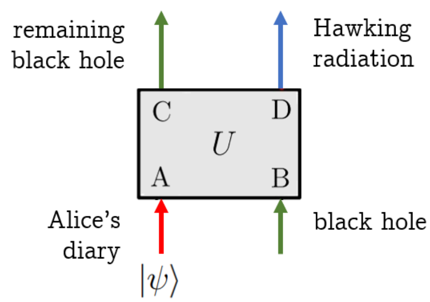

Figure 3: Hayden and Preskill Thought Experiment: Alice's diary is modelled as a quantum state $|\psi\rangle$ of a system A, consisting of a qubits (in red), which is thrown into an existing black hole B, made of b qubits (in green). Bob attempts to spy on Alice by capturing a portion of the Hawking radiation, represented by subsystem D (in blue). The operator U represents the unitary evolution of the black hole.

Within the set-up provided by this thought experiment, Hayden and Preskill are interested in two quantitative questions about the retrieval of a given amount of quantum information from the black hole. We briefly recall them from the Introduction:

(i) Physical mechanism: *What are the processes involved in the information release?*

(ii) Amount of time: *If the black hole processes, rather than destroys, the quantum information, how long does Bob need to be able to decode the state of the diary?*

We identify these two as *new questions* that can be precisely formulated and answered using the distinctive possibilities for information processing and communication that quantum information theory provides.

An important clarification at this point concerns the meaning of the phrase 'Bob can decode the information' as it appears in question (ii). Specifically, this does not imply that Bob has enough information to specify the quantum state of Alice's diary in classical bits, nor that he can directly access the content of the diary—reading out the content of a quantum state is notoriously subtle. Rather, we mean that Bob can manipulate the recovered state in any way he could have, had he had direct access to the original quantum system before Alice attempted to destroy it. This includes applying unitary operations, performing measurements, or, in the case of a multi-qubit state, obtaining the reduced state of a subsystem via a partial trace.

In this sense, decoding the information means transforming the quantum state of (part of) the radiation emitted by the black hole into Alice's original quantum state. To make this distinction clearer, we will henceforth refer to decoding as 'recovering the information.'

An important point we will revisit is that Hayden and Preskill were able to address these questions without specifying the exact recovery procedure Bob must follow. Instead, their focus is on the conditions under which such a recovery strategy is guaranteed to exist.

Their answers to the questions above are discussed in Section 4.3. Before that, Section 4.1 outlines the assumptions defining their QC-model, their physical significance, and how these differ from Page's. Section 4.2 then explains how this model represents the black hole as a quantum channel relying on entanglement to protect (cf. Sect. 4.2.1) and assist (cf. Sect. 4.2.2) the transmission of information. Finally, Section 4.4 addresses how Hayden and Preskill's approach avoids earlier criticisms about the use of information in black hole physics.

4.1 Specifying the Model: Assumptions and Idealisations

In this section, we outline the anatomy of the Hayden and Preskill quantum circuit model of a black hole in terms of three main model features (namely, the number of qubits, the architecture and size of the circuit, and the way the qubits are initialised: see Sect. 3.1.2), together with eight assumptions that define the model. We also discuss the theoretical basis of each assumption (some of which was developed after Hayden and Preskill published their work), and specify their importance in the argument that leads to the model's results.

This model does not claim to provide a definitive answer to the complex questions surrounding black hole evaporation. Rather, it offers a proposal for understanding certain features of black holes differently, especially in light of their role in addressing communication-based questions such as (i) and (ii) above (more on this in Sect. 4.3).

4.1.1 First Model Feature: Number of Qubits

The central dogma implies that, from the outside, a black hole can be modelled as a quantum system consisting of $b = A/4G$ qubits. In the formalism of quantum mechanics, this means that the overall Hilbert space is given by the tensor product:

$$\mathcal{H}_B = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_b \,, \tag{5}$$

where the $\mathcal{H}_i$'s are the single-qubit Hilbert spaces.

The spatial location of these degrees of freedom is not provided by the central dogma.[27] Hayden and Preskill assume that, for a distant outside observer, the b degrees of freedom are uniformly distributed over the black hole's 'stretched horizon,' a timelike surface or 'membrane' extending around the event horizon of the black hole at a Planck-length proper distance from it.

Modelling Assumption 1: Location of the Degrees of Freedom.
The black hole degrees of freedom are uniformly distributed on the stretched horizon, with about one degree of freedom per Planck unit of area.

This reflects Hayden and Preskill's subscription to the 'membrane paradigm' (Thorne et al., 1986a; Susskind et al., 1993), according to which only the stretched horizon could legitimately be considered the repository of the quantum information absorbed by the black hole, from the perspective of external observers.[28] The stretched horizon was introduced to treat the black hole as a thermodynamic *object* and to provide a statistical-mechanical underpinning for it.[29]

Cinti and Sanchioni (2024) have qualified this choice of spatial location as *conventional* because it does not reflect any fundamental feature of the full quantum gravity theory. They explain that 'the idea of a smooth membrane exactly located one Planck length from the horizon is exactly the sort of geometric notion one expects to not make

[27] We use the phrase 'location of the degrees of freedom' in the usual sense in quantum mechanics: namely, one takes the tensor product of each qubit Hilbert space $\mathcal{H}_i$ and a copy of $L^2(U_i^3)$ on an open subset $U_i \subset \mathbb{R}^3$, representing the region of space that the degree of freedom "lives" in. See also the discussion below for the case of quantum gravity.

[28] The quantum extensions of the membrane paradigm are discussed in Wallace (2018, 2019).

[29] On the one hand, as long as we are dealing with energy levels well below the Planck scale, the stretched horizon may be treated as a one-way barrier, just as the event horizon can be. For crossing it would require extremely high accelerations or extremely short wavelengths. On the other hand, unlike the event horizon—which, in its original definition by Hawking and Ellis (1973); Wald (1984b), is defined globally as a region *of spacetime* with implicit reference to all future times (Curiel, 2019, p. 9)—the stretched horizon can be treated as an enclosed region *of space that evolves through time* and can be ascribed time-dependent properties (Wallace, 2018, p. 18).

sense in quantum gravity in which spacetime structure is not well-defined' (Cinti and Sanchioni, 2024, p. 27).

With recent developments leading to the formulation of the central dogma, locating the degrees of freedom that describe a black hole on the stretched horizon is no longer the only possibility considered by high energy physicists. For instance, in holographic approaches, these degrees of freedom are instead thought to reside on the boundary of spacetime. Another example comes from string theory, where the counted microstates correspond to configurations of D-branes, extended objects that are not understood to be localised at the horizon. Indeed, the very notion of classical horizon is not manifest at this level of description.

Thus, what will remain of *Modelling Assumption 1* in the non-spatiotemporal context of full quantum gravity, is the specification of the number of degrees of freedom, but not the requirement that those degrees of freedom are located at the stretched horizon. This specifies the first feature of the QC-model of this black hole: there are $b \approx A_{bh}/4G = (r_S/2\ell_{Pl})^2$ qubits, where A_{bh} is the area of the event horizon at the Schwarszchild radius r_S, corresponding to the black hole system B in the quantum circuit. As it would be tedious to draw all these lines on paper, it suffices to represent a single line of weight b. This is the green line in Figure 3.

4.1.2 Second Model Feature: Circuit Architecture and Size

As noted earlier, the central dogma further stipulates that, from the viewpoint of an external observer, the evolution of a black hole's microscopic degrees of freedom is governed by a Hamiltonian, although its exact form remains unspecified. In contrast, the gravitational description is subject to a Hamiltonian constraint, which ensures that the full spacetime geometry evolves consistently with Einstein's equations.[30] This constraint is inherently global, incorporating both the interior and exterior regions of the black hole. Consequently, it is not straightforward to extract a Hamiltonian that describes only the evolution accessible to an external observer (Almheiri et al., 2021, p. 14).

Nevertheless, if the central dogma is correct, such a Hamiltonian must exist and must reproduce the expected spacetime dynamics, even if it is not manifest in the classical gravity description. As we will discuss, it is expected to be strongly interacting and to generate dynamics characteristic of quantum chaos. In the QC-model developed by Hayden and Preskill, this evolution is implemented by a Hamiltonian constructed à

[30] For a conceptually clear introduction to the Hamiltonian constraint in general relativity, see Wald (1984a, Chap. 10).

la Page: as repeated discrete random unitary operations acting in parallel. Physically, this reflects the hypothesis that black holes rapidly scramble information, converting it into a form that is very difficult to access in practice.

As a starting point, and for the sake of generality, it is assumed that interactions between all the degrees of freedom are allowed, but coupling only a few degrees of freedom at a time within each interaction—making the interactions local in this sense. The internal dynamics of the black hole is governed by a so-called "easy" Hamiltonian.

Modelling Assumption 2: Easy Dynamics.
The internal dynamics of the black hole is governed by an 'easy Hamiltonian,' i.e. a sum of simple Hermitian operators involving interactions between all degrees of freedom, but coupling only two of them at a time.

This modelling assumption is an idealisation of the dynamics of the black hole, which makes it suitable for modelling through a 2-local all-to-all quantum circuit with parallel processing. Indeed, the evolution by an easy Hamiltonian is analogous to the evolution by a quantum circuit with this architecture (Susskind, 2016, p. 6), discussed in Section 3.1.1.

In spite of the fact that the black hole dynamics is modelled as being "easy", Hayden and Preskill assume that it evolves according to a Haar-random unitary, which introduces complexity through scrambling:

Modelling Assumption 3: Scrambling Dynamics.
The black hole's dynamics conceals or encodes the information by Haar scrambling it, i.e. thoroughly mixing it among all the black hole degrees of freedom in a highly non-local form.

So far, the choices and hypotheses made by Hayden and Preskill closely resemble those made by Page (1993c). Their major refinements start with the next requirement.

For Hayden and Preskill, it is important that the information deposited by Alice is *rapidly* Haar-scrambled by the black hole's dynamics, on a time scale comparable to the time interval between the emission of successive radiation quanta. If this were not the case, the quantum information in the diary would not have sufficient time to interact with all the other qubits in the black hole, allowing radiation to escape without carrying any information about the quantum state of the diary.

To see this, suppose we have a Haar-scrambled system to which we add a single qubit in a pure state. Our goal is to recover *this* qubit of information by looking at some part of the scrambled-plus-one qubit system. At this point, the system is no longer completely Page-scrambled (and therefore not Haar-scrambled either) because the new information can be accessed from the single qubit, i.e. the added degree of freedom. The other degrees of freedom, however, do not carry this information.

Over time, interactions modelled via 2-local gates cause the added information to spread across all degrees of freedom, eventually restoring the system to a Page-scrambled state. The information is no longer accessible from a single degree of freedom but instead requires access to at least half of them. This is characterised by an important time scale:

> The **scrambling time** $t_{\rm scr}$ is the time needed for a Page-scrambled system to re-scramble when a qubit of information is added.

Thus, Hayden and Preskill propose that:

Modelling Assumption 4: Rapid Scrambling Constraint.
The scrambling time is rapid, i.e. it occurs on a time scale comparable to the time interval r_S, as measured by a distant observer, between the emission of successive radiation quanta.[31]

This bound poses a physical constraint on the scrambling time and ensures that any radiation escaping the black hole contains some information about Alice's quantum state.[32] Combined with other hypotheses (notably *Modelling Assumptions 7 and 8*), we will see that this constraint makes the speed of information release depend on a much shorter time scale.

How is the rapid scrambling constraint expressed in the QC-model? Addressing this question requires a slightly more technical discussion. We discussed that Page introduced a definition of scrambling in quantum circuits by the action of a Haar-random unitary transformation on the input state (cf. Sect. 3.2). However, such a transformation requires a quantum circuit of exponential size in the number of qubits. Consequently, if the evolution operator of the black hole is described by a Haar-random unitary, the quantum circuit that models its internal dynamics has to be of exponential size. Not only is this unrealistic for physical evolution; it is also highly inefficient and prevents information from being mixed quickly enough to satisfy the rapid scrambling constraint.

It turns out that the only way Hayden and Preskill (2007) find to accommodate *Modelling Assumption 3* with *Modelling Assumption 4* is to rely on a *weaker* notion

[31] To determine the time interval between successive radiation quanta, one can divide the black hole's evaporation time, M_{bh}^3, by the total number of emitted qubits, M_{bh}^2, and use the relation $r_S \sim M_{bh}$ to express the results in terms of the Schwarszchild radius.

[32] This is related to the property of 'entanglement-protection' that we will discuss in Section 4.2.

of Haar scrambling in their protocol: the unitary U is sampled with respect to the Haar-measure, but only for the *approximate* analogue of the task they were initially considering. This is reflected in the following hypothesis:

Modelling Assumption 5: Approximate Decoding.
The information that reaches Bob's end contains a small error, quantified by a fixed parameter δ, which determines the precision with which Bob can recover Alice's diary by observing the radiation system.

In other words, Bob only needs to recover from the radiation a quantum state that is *close*—according to some suitable measure of similarity—to Alice's original state.[33] The key idea is that if exact recovery is not required, then the process that encodes Alice's information into the black hole, i.e. the unitary U, can also be approximate. This relaxation allows the encoding process to be implemented using simpler and more efficient circuits.

Specifically, instead of using truly random quantum operations which are extremely complex and require very large circuits, the model uses a carefully chosen subset of operations that behave like random ones for the purposes of this protocol. The unitary U is taken to be an element of a subset of the full unitary group known as the 'ϵ-approximate unitary 2-design' (Dankert, 2005; Dankert et al., 2009). This subset can adequately (but not perfectly) approximate the Haar-measure for the Hayden-Preskill protocol and can be generated by gate sequences of smaller size. In particular, generating such a unitary requires a circuit of depth approximately $\log(b)\log(\epsilon^{-1})$ (neglecting the diary size compared to the number of black hole qubits), where b is the number of black hole qubits and ϵ controls the quality of the approximation, but not smaller.

Finally, since the black hole is a physical object in spacetime, one expects that the arrangement of its microscopic degrees of freedom is constrained by its geometric structure. If we only consider gates that act on spatially close qubits, there is an additional cost in the circuit depth from the time it takes for the information to propagate from one location to the other, which is determined by the linear size of the system. As the qubits are uniformly distributed over the stretched horizon, with a radius of order $r_S \sim \sqrt{b}$, the additional cost of using geometrically-local gates is, at worst, a multiplicative factor of the order of $\sqrt{b}$. Therefore, the circuit scrambling time estimate becomes

$$t_{n,\mathrm{scr}} \approx \sqrt{b}\log(\sqrt{b})\log(\epsilon^{-1}), \tag{6}$$

where we neglected a factor of 2.

[33] More details on the definition of the parameter δ are provided in Section 4.2.2.

Of course, a black hole does not evolve through a (quasi) Haar-random unitary selected—by whom?—at random from all possible unitaries in the group. Is there another physically meaningful way in which black holes scramble information? And if so, does this process occur quickly enough to satisfy the rapid scrambling constraint?

The black hole scrambling time was first estimated by Hayden and Preskill (2007, Sect. 4). This estimate is based on an essentially classical argument concerning the exponentially rapid diffusion of a charged particle dropped on the stretched horizon, originally derived by Thorne et al. (1986b). A charged particle dropped into a black hole spreads rapidly on the stretched horizon, with the proper area of a droplet of charge growing like $\exp(t_S/r_S)$.[34]

Hayden and Preskill assume that this exponential spreading also takes place when a *piece of quantum information* is deposited on the horizon. This reasoning leads to a Schwarzschild scrambling time estimate of the order $r_S \log(r_S/\ell_{Pl})$. Despite the correcteness of the expression, we are sceptical about this particular argument: Hayden and Preskill model information as if it were a locally conserved fluid deposited on the horizon—an interpretation that contrasts the view of information endorsed here.[35]

This classical picture was later extended by Sekino and Susskind (2008), who elaborated on the estimate in the context of string theory, arriving at a similar scrambling timescale.

However, the modern understanding of black hole scrambling goes beyond these early estimates. It emphasizes the role of chaotic time evolution, as diagnosed by the decay of out-of-time-order correlation functions. These four-point correlation functions measure the failure of commutation between operators at different times, and their decay signifies the chaotic nature of the system, where initially commutative operators become non-commutative due to the exponential sensitivity to initial conditions.[36]

In gravitational systems, out-of-time correlation functions can be explicitly computed using gravitational shockwaves. When a weak perturbation falls into a black hole, it experiences an exponential blueshift and creates a shockwave near the horizon that disrupts later infalling matter. This gravitational amplification was first studied for Schwarzschild black holes by Dray and 't Hooft (1985), and has since been extended

[34] This is a kinematic effect that does not depend on the details of the Schwarzschild metric, but is very general for any horizon which is locally Rindler-like (Susskind, 2011, p. 2).

[35] As argued by Timpson (2005), this failure to recognise the logical status of 'information' as an abstract noun may be misleading. We will come back to this point in Section 4.3.

[36] Maldacena et al. (2016) have conjectured a bound on the influence of chaos on such correlators: $\langle O_A(0)O_B(t)O_C(0)O_D(t)\rangle = 1 - \exp\{\lambda t\}/n$, where $O(t) = U^\dagger O U$, n is the number of degrees of freedom in the system, and the Lyapunov exponent λ, which characterizes the rate of exponential divergence of initially close quantum states, is bounded by $\lambda \leq 2\pi T$.

to other spacetimes.[37] Kitaev (2015) was the first to emphasise the connection between gravitational dynamics and quantum chaos, arguing that gravitational shockwave calculations effectively compute out-of-time-order correlation functions in disguise. The shockwave near the horizon causes the out-of-time-order correlation functions to decay from its initial value, an effect that signals the onset of scrambling and becomes significant at timescales of order $r_S \log(r_S/\ell_{Pl})$.

In conclusion, there is substantial evidence that

$$t_{\text{scr}} \approx r_S \log(r_S/\ell_{Pl}), \tag{7}$$

which satisfies the rapid scrambling constraint. This timescale arises not from Haar-random unitary dynamics, but from the chaotic evolution governed by gravitational interactions near the horizon. As mentioned, this process can be diagnosed by the decay of out-of-time-order correlators and is captured in a number of gravitational shockwave calculations.

On the other hand, it is worth noting that the logarithmic scaling of the scrambling time plays no essential role in the argument by Hayden and Preskill; what matters is simply that scrambling occurs much faster than evaporation.

Comparing this result with Eq. (6), we can deduce the relationship between the time-steps in the quantum circuit and the continuous time evolution. Since t_n represents a number of steps and is dimensionless, while the asymptotic Schwarszchild coordinate t has units of time, we must have $t_n = t/\Delta t$, where Δt time interval between successive circuit steps. Using the estimate $b \approx A_{bh}/4G \approx (r_S/\ell_{Pl})^2$, and noting that $\ell_{Pl} = t_{Pl}$ in natural units (since $c = 1$), we find that, from the perspective of observers far from the black hole, $\Delta t = t_{Pl}$.[38]

4.1.3 Third Model Feature: Initialisation of the Qubits

The third model feature is about the initial state in which the various components of the thought experiment start. To this end two maximally entangled states in the form of EPR pairs are considered. A state in which all the qubits form EPR pairs is the simplest case of a maximally entangled multi-partite state. Introducing these maximum entanglement structures amounts to assuming that the joint state in which the black

[37] These spacetimes include AdS black holes (Shenker and Stanford, 2014), Reissner–Nordström black holes (Sfetsos, 1995), and Kerr black holes (BenTov and Swearngin, 2019).

[38] An intriguing consequence of this identification is that, due to gravitational blueshift, the local dynamics near the stretched horizon proceed on timescales shorter than one Planck time per gate operation from the viewpoint of a near-horizon observer.

hole, the diary and the reference system start, is simple.

The first maximally entangled state is introduced by coupling the diary qubits to an external reference system. Instead of throwing a specific quantum state $|\psi\rangle$ into the black hole, this setup allows one to analyse how the decoding works for any possible diary state at the same time.

To do so, one assumes that Alice's quantum diary is maximally entangled with a reference system, and the qubits of A are then thrown into the black hole:

Modelling Assumption 6: Reference System.
The quantum diary A is maximally entangled with a reference system R of the same dimensionality, i.e. their initial joint state is the pure state:

$$|\mathrm{EPR}\rangle_{RA} = \frac{1}{\sqrt{2^a}} \sum_i |i\rangle_R \otimes |i\rangle_A \,, \tag{8}$$

where $\mathcal{H}_R \equiv \mathcal{H}_A$, $\{|i\rangle\}$ denotes the standard basis of $\mathcal{H}_A$.

The reference system does not necessarily represent a physical system. Its introduction is a mathematical trick that enables a quantitative discussion (in terms of information-theoretic measures) of when the information is successfully transmitted. The key insight is that successful transmission to the radiation implies, in principle, that the information is recoverable. The existence of such a decoder can be diagnosed via the amount of entanglement between the reference system and the radiation. We will return to this point in more detail when we describe the procedure for information release in Section 4.2.

To make this concrete, suppose Bob employs a decoding strategy aimed at extracting information from the Hawking radiation. In this setting, Bob would be able to isolate a subsystem of the radiation of dimension $|A|$ (where $|\cdot|$ denotes the size of the corresponding Hilbert space) that is maximally entangled with the reference system R. If the initial state of A was a pure state $|\psi\rangle$ (i.e., not entangled with any reference system), this would imply that Bob can recover $|\psi\rangle$ from the radiation. In other words, he has successfully reconstructed the quantum information originally stored in Alice's diary.

The next assumption reflects the pioneering work of Page (1993d) and, together with the one that follows, allows one to take advantage of the fact that the black hole-plus-radiation system is scrambled:

Modelling Assumption 7: Old Black Hole.
The black hole, which is initially in a pure state, has been evaporating for a time longer

than the Page time.

This modelling assumption is a choice of system (namely, an old black hole), together with an initial condition for the black hole (namely, that its initial state is pure).

After the Page time, the radiation Hilbert space exceeds that of the black hole, and each black hole degree of freedom is nearly maximally entangled with a previously emitted Hawking quantum. A distant observer with access to all the emitted radiation can then probe more than half the system and detect non-thermal features (cf. Sect. 3.2.2). For this to be possible, the observer must be far enough from the black hole for the central dogma's picture to hold, viewing it as a quantum system in a box, emitting Hawking radiation that the observer can collect. Hayden and Preskill assume the existence of such an observer, introducing a second maximally entangled state:

Modelling Assumption 8: Bob's Unlimited Control.
By collecting all the Hawking radiation emitted by the old black hole, Bob holds a quantum memory, within which a subsystem B' is maximally entangled with the state of the black hole B in the EPR state

$$|\mathrm{EPR}\rangle_{BB'} = \frac{1}{\sqrt{2^b}} \sum_i |i\rangle_B \otimes |i\rangle_{B'}\,, \tag{9}$$

where $\mathcal{H}_B \equiv \mathcal{H}_{B'}$, $\{|i\rangle\}$ denotes the standard basis of $\mathcal{H}_B$.

Like the previous modelling assumption, this is also a choice of physical system (Bob's quantum memory) and of an initial condition for Bob's subsystem (it is maximally entangled with the black hole). Again, the maximal entanglement between the remaining black hole and the previously emitted Hawking radiation is expected for a black hole that has already radiated away more than half of its initial entropy. Moreover, it is assumed that this entanglement has a simple structure consisting of familiar EPR pairs.

The key part of this assumption, and the difference with Page's setting, is that Bob has been able to collect all the previously radiated Hawking quanta in a quantum memory. The resulting quantum entanglement that Bob's quantum memory shares with the black hole before Alice throws her diary in will facilitate the transmission of its quantum state (cf. Sect. 4.2). In other words, Bob "knows" not only the dynamics of the black hole but also its initial state.

The last three assumptions about the initialisation of the qubits imply that the whole system composed of the diary A, the black hole B, the reference system R, and the quantum memory B' is initially in the state:

$$|\Psi_{in}\rangle \equiv |\text{EPR}\rangle_{RA} \otimes |\text{EPR}\rangle_{BB'} \,. \tag{10}$$

4.2 Zooming in: the Procedure behind Information Release

In this section, we explain how techniques from quantum information theory can be used to describe the release of a given amount of quantum information (namely, the quantum state of Alice's diary) from the black hole. We will delve into the details of the communication protocol proposed by Hayden and Preskill (2007) and unravel two distinct roles that entanglement plays in its functioning: first, as means of protecting information against local errors; and second, as a consumable resource that facilitates the transport of information.

Hayden and Preskill's QC-model is illustrated in Figure 4. This model is completely specified through the eight hypotheses outlined in the last section. Alice's diary (system A with $|A| = a$), maximally entangled with a reference system (system R) of the same size, falls into an "old" black hole (system B with $|B| = b$), one that has passed the Page time. Bob has been collecting its Hawking radiation in a quantum memory (system B' with $|B'| = |B|$) ever since evaporation started, and by now, the black hole's internal state is maximally entangled with Bob's quantum memory in the state $|\text{EPR}\rangle_{BB'}$.

The black hole's unitary dynamics U_{AB} can be viewed as an encoding map that encodes the a-qubit diary state into a $2(a+b)$-qubit output state:[39]

$$U_{AB} : \mathcal{H}_A \otimes \mathcal{H}_B \to \mathcal{H}_C \otimes \mathcal{H}_D, \quad |\Psi_{out}\rangle \equiv U_{AB}|\Psi_{in}\rangle, \tag{11}$$

where $|\Psi_{in}\rangle$ was defined in Eq. (10). We can define the density matrix $\rho_{RCDB'}$ associated with this state as

$$\rho_{RCDB'} \equiv |\Psi_{out}\rangle\langle\Psi_{out}| \;, \tag{12}$$

and use it to compute the entanglement entropy of arbitrary subsystems by tracing out the irrelevant degrees of freedom and applying the standard von Neumann entropy formula to the resulting reduced density matrix.

Bob's objective is to recover Alice's original quantum state from the information contained in the newly emitted radiation (system D) and his quantum memory. In other words, he seeks to apply a unitary operation that reconstructs the diary by acting solely

[39] Mathematically, this pure state can be interpreted as a representation of U_{AB} via the Choi–Jamiołkowski isomorphism (Jamiołkowski, 1972; Choi, 1975). This operator-to-state mapping allows us to study the entanglement properties of U_{AB} using standard quantum information measures of entanglement (Hosur et al., 2016, Sect. 2.1).

on subsystems under his control. The decoding unitary is denoted $V_{DB'}$.

Importantly, Hayden and Preskill do not attempt to construct this decoding operation explicitly. Instead, their focus is on a simpler question: whether such a decoding operation *exists* in principle. To address this, they use the theory of quantum error-correction in the context of entanglement-assisted communication, which are the focus of the upcoming Sections 4.2.2 and 4.2.1.

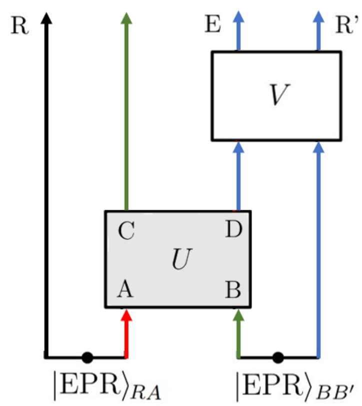

Figure 4: Hayden and Preskill's QC-model. The quantum diary A, in red, is maximally entangled with a reference system R, in black. The green lines represent the black hole: B at the Page time, and C after the diary has fallen in and additional radiation has been emitted. The blue lines represent the subsystem accessible to Bob: B' is the Hawking radiation emitted before the Page time; D is the newly emitted radiation. R' is a subsystem of DB', of the same size as A, were the diary information is reproduced; E is its complement. The unitary U represents the black hole's scrambling dynamics, and V the decoding operation.

4.2.1 Entanglement-Protected Communication

The basic idea of any error-correction scheme, quantum or classical, is to *redundantly* encode the information in a larger system so that errors and modifications along the way are not fatal to its successful transmission. The easiest way to protect a classic message is to send it several times. Repetition, however, does not work for quantum systems because of the 'No-cloning Theorem' (Wootters and Zurek, 1982) that says it is impossible to make copies of an arbitrary unknown quantum state.

Quantum information, therefore, must be protected differently. Rather than duplicating the state, the so-called '*quantum* error-correcting codes' encode the information into the entanglement structure of a larger system. This spreads the information across many degrees of freedom, allowing the original state to be recovered even if some qubits are lost or corrupted. Such schemes rely crucially on entanglement, and their existence was first demonstrated in foundational work by Shor (1995) and Steane (1996).

The main insight of Hayden and Preskill is to use this framework in the context of black hole evaporation. The type of errors relevant in this context are 'erasures,' defined as the loss of access to a known subset of the physical degrees of freedom. This is because, after encoding the quantum state of the diary across a larger number of degrees of freedom (as in Eq. (11)), the black hole eventually reveals only a subset of qubits to Bob, namely those emitted after further evaporation (subsystem D). The qubits retained in the black hole (subsystem C) are in fact *erased* as far as Bob is concerned. In other words, we see that the black hole acts as a specific type of noisy channel, known as a 'quantum erasure channel,' for the transmission of the diary.

In their QC-model, the black hole's dynamics is assumed to effectively conceal the information through scrambling, modelled by a random unitary (cf. *Modelling Assumption 3*). If the diary's state is well-encoded under this unitary, quantum error correction suggests that it can be preserved, despite the partial erasure. The central question is therefore: Is this random unitary a good encoder for the purpose of error correction in a quantum erasure channel?

To answer this question, let us take a look at the effect of applying the scrambling unitary to the input state. Initially, subsystem A (the diary) is maximally entangled with a reference system R, and subsystem B (the old black hole) is entangled with Bob's quantum memory B'. In this simple entanglement structure, the composite AB' is maximally entangled with RB, and any of their subsystems is maximally entangled with its complement. This is illustrated on Figure 5 (left).

Once the random unitary U_{AB} is applied to the input state $|\Psi_{in}\rangle$, the initial EPR pairs are transformed into a more complicated arrangement of multipartite entanglement between the input and output subsystems. Given that Haar scrambling implies Page scrambling (cf. Sect. 3.2.1), the output state $|\Psi_{out}\rangle$ of the entire system $RCDB'$ is a Page-scrambled state. By definition, this means that any subsystem of whose size is less than or equal to half the total size is nearly maximally mixed, reflecting the fact that the entanglement structure has become more complex. This is illustrated on Figure 5 (right).

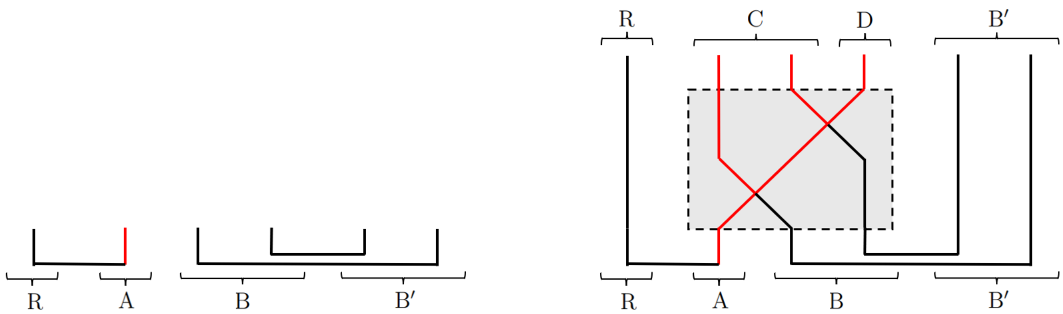

Figure 5: Entanglement structure before and after applying the scrambling unitary U_{AB}. Left: the initial state consists of simple bipartite entanglement. Right: after applying U_{AB} (dashed box), the system becomes Page-scrambled, with the entanglement redistributed into a complex multipartite structure. Subsystems up to half the total size are nearly maximally mixed.

To understand the implications of this entanglement structure for how the diary information is encoded in particular, we can use the concept of 'mutual information,' defined for two subsystems R, $\widetilde{R}$ as

$$I(R:\widetilde{R}) \equiv S_{vN}(\rho_R) + S_{vN}(\rho_{\widetilde{R}}) - S_{vN}(\rho_{R\widetilde{R}}). \tag{13}$$

This non-negative function quantifies the amount of (classical and quantum) correlations between R and $\widetilde{R}$. In our case, we are interested in the mutual information between the reference system (which purifies Alice's quantum state) and other parts of the system. As noted in the discussion of *Modelling Assumption 6*, if this mutual information is near its maximum, then Alice's quantum state is approximately recoverable from that subsystem.

Because the global state is Page-scrambled, the mutual information between R and any subsystem that is smaller than or equal to half the size of $RCDB'$ is approximately zero. (To a good approximation, each entropy term is individually maximal, according to Page's result (4).) This implies that no small subset of the system reveals any significant information about Alice's diary. However, once a subsystem exceeds half the size of $RCDB'$, the mutual information approaches its maximum, $2a$, meaning the diary can be recovered from that subsystem.

The conclusion is that the information about Alice's quantum state cannot be recovered from small, local subsystems. Instead, it is non-locally encoded across many degrees of freedom in a highly redundant way. The corollary is that, as long as it is not too large, the erasure of any particular subsystem does not lead to information loss. This is the hallmark of an effective quantum error-correcting code: it spreads information redundantly so that small erasures do not threaten recovery.

Hayden and Preskill formalise this insight using a simple decoupling argument, described below, which shows that the reference system R becomes nearly uncorrelated with the erased degrees of freedom (in C) and highly correlated with the subsystem accessible to Bob (D and B'). The second role that entanglement plays in this protocol is precisely to grant Bob such access.

Thus, the black hole's scrambling U_{AB} dynamics is able to protect the information in the quantum erasure channel by entangling it within a larger system, thereby ensuring that no single subsystem is essential for its recovery and rendering small erasures inconsequential. By being "well hidden," the information in Hayden and Preskill's QC-model is also well protected, a dual aspect the authors describe as a "delicious irony" (Hayden and Preskill, 2007, p. 15).

A potential problem, mentioned in Section 4.1, is that generating these random unitary operations with a 2-local quantum circuit is inefficient, whereas for the Hayden-Preskill model it is specifically important that the scrambling is fast (cf. *Modelling Assumption 4*). Fortunately, approximate encoding unitary (cf. *Modelling Assumption 5*), which are sufficiently random in this restricted context, are good enough to protect against errors in such a circuit and can be generated much more efficiently (Dankert, 2005; Dankert et al., 2009).

4.2.2 Entanglement-Assisted Communication

We proceed to discuss the second role played by entanglement in the Hayden-Preskill QC-model: not as a means for protecting information, but as a resource that enables its recovery. This idea is a cornerstone of quantum information theory and underlies protocols such as superdense coding (Bennett and Wiesner, 1992) and quantum teleportation (Bennett et al., 1993). In these protocols, pre-existing entanglement between the transmitting and receiving ends of the channel can be *used up* to enable more efficient or otherwise impossible forms of communication. In Hayden and Preskill's protocol, this corresponds to Bob attempting to decode the diary information by exploiting the entanglement his quantum memory shares with the black hole.

To see how this is possible, we must consider the interplay between entanglement-assisted communication and entanglement-based protection, discussed earlier. Since the final state is Page-scrambled, Bob can recover the diary only after gaining access to more than half of the total system $RCDB'$. As we discuss now, the assistance provided by entanglement allows for this situation to quickly arise.

At the start of the protocol, Bob already has access to a significant part of the system. As established in *Modelling Assumptions 7* and *8*, he has been collecting Hawking radiation since evaporation began. As a result, the old black hole is maximally entangled with his quantum memory. This means that before Alice throws in her diary, Bob already controls half of the Page-scrambled system BB'.

When the diary falls in, the previously scrambled system expands, and one must wait a scrambling time for it to reach a new scrambled state. At this point, B' is smaller than half of the total system and appears maximally mixed to Bob, so that $I(R : B') \approx 0$. In other words, the old radiation are devoid of any information about Alice's quantum state. So how can Bob exploit entanglement to recover the diary?

Hayden and Preskill used a simple decoupling criterion to show that Bob does not need access to the entire system. He only needs to wait until the black hole emits enough additional radiation so that the combined system DB' (newly emitted radiation plus quantum memory B') becomes larger than its complement CR (remaining black hole plus reference system). The idea is as follows: for the diary to be recoverable from DB', the remaining black hole C must have lost all memory of the diary information content and become decoupled from the reference system, implying that R is now maximally entangled with a subsystem of DB'. This condition is captured by the approximate factorisation:

$$\rho_{RC} \approx \rho_R \otimes \rho_C \tag{14}$$

where closeness is quantified by the decoding error δ:

$$||\rho_{RC} - \rho_R \otimes \rho_C||_1 \leq \delta. \tag{15}$$

Here, the trace norm $||A||_1 = \mathrm{Tr}\sqrt{A^\dagger A}$ measures the distinguishability of two states.[40]

Hayden and Preskill proved that for an ϵ-approximate unitary 2-design U, the average trace-norm distance between ρ_{RC} and the product state $\rho_R \otimes \rho_C$ becomes exponentially small once $d = a + k$ qubits have escaped the black hole. Here, a denotes the number of diary qubits, while k is a small additional number of qubits emitted beyond a. In other words, after only a relatively small number of extra qubits are emitted, the original diary information becomes recoverable to Bob.[41]

[40]Specifically, if the trace norm of the difference between two density matrices is less than δ, then the probabilities predicted by these operators for any measurement will differ by at most δ.

[41]Yoshida and Kitaev (2017) later formalized this result as a theorem, proving that if black hole's dynamics is Haar scrambling and $|D| \gtrsim |A|$, then the subsystems R and C are decoupled. This can be further understood through the decay of out-of-time-order correlators, which imply that the mutual information between any small subsystem of the input and any partition of the output remains small.

Consequently, when the size condition

$$|D||B'| > |C||R| \tag{16}$$

is met, the mutual information between R and DB' becomes nearly maximal. This guarantees the existence of a decoding unitary

$$V_{DB'} : \mathcal{H}_D \otimes \mathcal{H}_{B'} \to \mathcal{H}_E \otimes \mathcal{H}_{R'} \tag{17}$$

that acts only on the subsystems under Bob's control and approximately achieves

$$V_{DB'}|\Psi_{out}\rangle \approx |\mathrm{EPR}\rangle_{RR'} \otimes |\phi\rangle_{CE}, \tag{18}$$

where $R' \subset DB'$ is a subsystem of size $|A|$ (cf. Fig. 4) that is nearly maximally entangled with the reference system, E is its complement, and and $|\phi\rangle_{CE}$ is an arbitrary state of the remaining degrees of freedom. If Bob were to apply this decoding operation, and we considered the special case where the initial state of A was a pure state $|\psi\rangle$ (i.e. not entangled with any reference system), then he would recover $|\psi\rangle_{R'}$ from the radiation.

Notice that if Alice had dumped a qubits into a "young" black hole, one that is not yet highly entangled with its surroundings and with which a substantial pre-existing entanglement reserve does not exist (that is, if initially $|B'| \ll |B|$) then entanglement protection would still guarantee that decoding is eventually possible. However, Bob would have to wait significantly longer for the size of subsystem DB' to satisfy Eq. (16). Specifically, he would need to wait until the black hole that absorbed the diary had emitted more than half of its Bekenstein–Hawking entropy. The corresponding delay would be of the order of the evaporation time, which is much longer.

4.2.3 Summary: Communication Prespective

To conclude, we briefly summarise the main points of this section. By throwing her quantum diary into an old black hole, Alice is effectively transmitting an arbitrary quantum state $|\psi\rangle$ through a quantum erasure channel with the following two characteristics: (1) it is robust against the erasure of any subsystem smaller than half of the total, thanks to entanglement-protected encoding; and (2) the successful recovery of the state is enabled by the pre-existing entanglement between Bob's quantum memory and the black hole's internal degrees of freedom, allowing for entanglement-assisted decoding. Entanglement, in other words, is both the mechanism of protection and the currency of recovery. This communication system, in which the black hole functions as a quantum erasure channel, is depicted in Figure 6. It can be related to a 90° clockwise rotation of the quantum circuit shown in Figure 4, after discarding the subsystems that do not play a direct role in the transmission and recovery of quantum information (namely, the reference system and the qubits retained by the black hole), and by decomposing the black hole dynamics into a unitary encoding followed by an effective erasure.

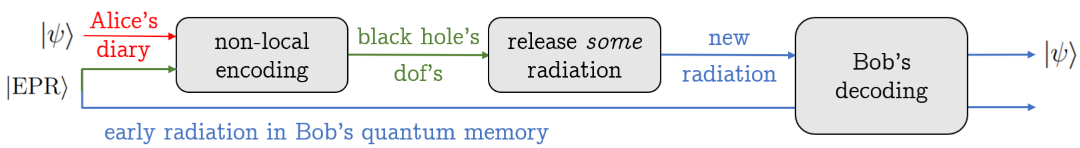

Figure 6: Transmission of the quantum state $|\psi\rangle$ through a black hole, modelled as a quantum erasure channel that releases only some radiation. The communication is assisted by entanglement, represented by the $|\mathrm{EPR}\rangle$ pair shared between the old black hole and the early radiation under Bob's control.

4.3 Zooming out: Questions Tackled and their Answers

In the last section, we explained Hayden and Preskill's proposal that information escapes in the Hawking radiation due to the entanglement inherent to the black hole's dynamics that protects and assists the transmission of quantum information. Now, we turn to the two main questions addressed by these authors using this quantum information-theoretic approach (see also the Introduction or Sect. 4). The first question is:

(i) *How does a given amount of information escape from the black hole?*

Within Hayden and Preskill's communication perspective, this amounts to answering the question: 'How does the information get from Alice to Bob in this communication protocol?' However, this question has historically led to conceptual puzzles, particularly in the context of quantum teleportation, which, according to Timpson (2013, Chap. 4), dissolve once the nature of a piece 'information' is properly understood.[42] If we encounter similar issues in trying to answer this question for the Hayden and Preskill protocol, it might be questioned whether an answer was provided in the first place.

The confusions to which the present protocol is prone are related to the process of scrambling, which underlies information protection in the Hayden–Preskill setup.

[42]This is evidenced by the answers to this question, which are sometimes surprising; for instance, through the use of backwards time travel of classical information (Penrose, 1998; Jozsa, 2003) or by noting that quantum information can be unexpectedly hidden in classical bits (Deutsch and Hayden, 2000).

Scrambling involves a strongly mixing unitary dynamics which encodes information non-locally across many degrees of freedom in a highly redundant manner (cf. Sect. 4.2.1). On a naive reading, the fact that the information can be decoded from different locations simultaneously suggests an exotic form of non-locality, one in which the same piece of information appears to exist in multiple places at once.

This puzzle can be resolved similarly to those encountered in quantum teleportation. At their source is the same mistaken assumption: that the word 'information' in the question 'How does the information get from Alice to Bob?' refers to a concrete spatiotemporal entity whose trajectory we must track. In Timpson's view, information is an abstract noun; it denotes an abstract sequence of symbols produced by an information source, which may be instantiated in various physical systems, but does not itself have a location or path through space and time.

Similarly, the scrambling of the diary's quantum information should not be understood as the movement of a physical substance along a definite spatiotemporal path. Rather, scrambling distributes the information non-locally across many degrees of freedom in a highly redundant way. That is, there are distinct particles, located in different spacetime locations, that instantiate the same piece of information. This offers a clear example of what Timpson describes as 'information flow,' which is a change in the set of locations where the quantum state can be reconstructed (Timpson, 2013, p. 34).

Recognising information in this way, Timpson concludes: "The central moral is that one should not be seeking, in an information-theoretic protocol—quantum or otherwise—for some particular 'the information', whose path one is to follow, but rather concentrating on the physical processes by which the information is transmitted, that is, by which the end result of the protocol is brought about (Timpson, 2013, p. 5)."

Following Timpson's moral, we take the question 'How does a given amount of information escape from the black hole?' not as a request to track the flow of a *physical object* out of the black hole, but rather as an inquiry into the physical processes that enable the transmission of a quantum state from Alice to Bob. The notion of information involved in Hayden and Preskill's protocol is precisely this: a quantum state, which, via the different steps and physical conditions of the protocol, is instantiated in different physical systems and eventually transmitted to Bob. Thus, answering how information escapes from the black hole requires a description of the physical processes by which the state is recovered. The question is thus better reformulated as:

(i') *What are the physical processes that allow quantum information to escape?*

The physical processes involved in the protocol (cf. Sect. 4.2) include the rapid scrambling of information by the black hole's unitary dynamics, the evaporation of the black hole via Hawking's process, and, potentially, the collection and decoding of this radiation by a powerful observer.

Alice's diary in a specific quantum state, falling toward the black hole, induces a gravitational shockwave that scrambles its information among the black hole's degrees of freedom, which are distributed across the event horizon. This scrambling occurs on a timescale much shorter than that of evaporation. Consequently, any radiation emitted after the diary has reached the horizon can carry information about its quantum state. Because the black hole's degrees of freedom are already entangled with previously emitted Hawking radiation, the diary's quantum information becomes delocalised across this entangled structure.

Once sufficient additional radiation has been emitted, the black hole effectively "forgets" the information content of the diary. At this point, the diary information becomes accessible from the radiation. In other words, the information has been released. Operationally, if a powerful observer collects this radiation, they can, in principle, apply a specific decoding operation to recover Alice's original quantum state on a subsystem of their own. The observer can then manipulate the recovered state just as if they had direct access to the original system, prior to Alice's attempt to destroy it.

Identifying these processes does not imply that we have given a definitive answer. In fact, we have identified two possible concerns. First, the combination of these physical processes explains how information can escape from a black hole, but only when it is modelled as a quantum circuit. As we mentioned earlier, we cannot be certain that this is a good model in the sense that the multiple assumptions it relies on reflect true facts about nature. Despite the different theoretical arguments that are used to argue for the model's accuracy, it remains unclear whether the black hole's dynamics is scrambling, Hawking radiation has not been observed, and providing an observer with unlimited control over the radiation (as in *Modelling Assumption 8*) seems unrealistic for astrophysical black holes. However, the rationale behind these assumptions where given in Section 4.1.

Second, and in more general terms regarding any information-theoretic protocols, since they consist of various quantum mechanical operations, they inevitably inherit the conceptual challenges of quantum mechanics itself. In particular, the way one constructs a narrative about the physical processes involved in the protocol will naturally depend upon one's interpretative stance. In this regard, Timpson (2013, p. 86) points out that it is somewhat reassuring that we stand in "known territory:" the remaining controversy concerns what interpretation of quantum mechanics one adopts.

Despite these two points, it is clear that entanglement is crucially involved in the physical processes of the Hayden and Preskill protocol. If black holes are quantum in some respect, which they certainly are at a level of description that goes beyond general relativity, we expect that the conclusion that entanglement is crucial in information release will survive. The peculiarity of the quantum world seems to go against the classical conclusion that black holes are hairless and black.

The second question Hayden and Preskill considered is the following:

(ii) *How long does Bob have to wait to be able to recover Alice's quantum state with a high degree of confidence?*

They show that the decoding task is information-theoretically possible, i.e. there exists a unitary decoder $V_{DB'}$, provided

$$d \geq a + k, \tag{19}$$

where k is a constant linked to the decoding error, and where d and a are the number of qubits in the late radiation system D and the diary A. Indeed, if there are only $d = a + k$ qubits that are released to Bob, this means that there are $c = b - k$ qubits retained by the black hole, which ensures that the condition Eq. (16) is met.

What this inequality means is that Bob can recover Alice's quantum state after waiting for just a few more than a qubits to be emitted. Once evaporation reaches the half-way point, the delay of the information release is thus equal to the time needed for the black hole to scramble the diary qubits plus the time needed to radiate $a+k$ qubits. This conclusion holds as long as scrambling occurs on timescales much shorter than the evaporation time, without requiring the scrambling time to follow the logarithmic scaling of Eq. (7).

This led Hayden and Preskill to propose the following metaphor: the QC-model of a black hole is hardly black at all; rather, it may be more accurately regarded as an 'information mirror' (Hayden and Preskill, 2007, p. 3). That is, even though Hawking radiation escapes slowly and Alice's diary can potentially contain many degrees of freedom, her secrets, after being masked for a while, are "reflected" to the exterior.

Yet, we insist that this information mirror has peculiar properties. First, it is a deforming mirror in the sense that the information decoded or reflected is only an approximation of the original, because small errors were allowed in its encoding. Errors occur for efficiency reasons (rapid scrambling requires an approximate random unitary) and from erasures (only some radiation are released after diary falls in). Second, not every observer can see the information being reflected at this fast rate; doing so requires access to a resource: pre-existing entanglement shared with the black hole.

4.4 Information without Controversy

We close this Section by discussing how these results bear on the discussion in Section 2.1. There, we outlined three distinct complaints in the debate surrounding the use of the term information in fundamental physics and to study black holes in particular. This section aims to highlight how Hayden and Preskill's use of quantum information is immune to the issues raised in that controversy.

The first critique, formulated by Dougherty and Callender (2017), targeted the epistemic reasoning employed by Bekenstein to explain black hole thermodynamics, particularly the increase of the horizon area, in terms of the knowledge of an agent about the underlying microphysics. In any case, Bekenstein's motivation has proven unnecessary since black hole thermodynamics can now be grounded in statistical mechanics using an alternative approach we called the *physical route*, described in detail by Wallace (2019).

While information may not be essential for grounding black hole thermodynamics, we believe this is not a sufficient reason to discard the concept of information. On the contrary, the consistent microstate counting across various approaches to quantum gravity indicates the existence of real degrees of freedom (cf. Sect. 2.3). These degrees of freedom can then be abstracted using the concept of information. By supporting the existence of real microscopic degrees of freedom, these developments pave the way for an alternative role that information theory can play in the study of black holes, one that is distinct from its role in grounding black hole thermodynamics. The central idea defended in this paper, as exemplified by the work of Hayden and Preskill (2007), is that this approach is fruitful because it offers a new perspective on the system under study, one that raises novel questions and suggests possible answers (cf. Sect. 4.2 and Sect. 4.3).

The second complaint emphasised that the notion of information is inherently tied to its transmission. Therefore, there must be a communication system in place for information to be present. According to Wüthrich (2017), the description of a black hole provided by general relativity is too impoverished to account for the complexity of such a communication system.

In contrast, the theoretical basis on which Hayden and Preskill based their QC-model goes beyond general relativity and incorporates quantum effects. The theoretical results supporting the central dogma are obtained when one considers a theory of gravity coupled to quantum fields. Once the theoretical framework is extended beyond general relativity, black holes reveal themselves to be considerably more complex. This added complexity provides the necessary components for a communication system: if the central dogma is true, the black hole can act as a quantum channel that transmits quantum information. As Hayden and Preskill's work shows, this quantum channel can be used in a communication protocol where the quantum state of Alice's diary is transmitted to the radiation, allowing Bob, who has been collecting the radiation from the start, to decode it.

The third aspect concerns the thesis of informational immaterialism, which claims that all material things are information-theoretic in origin and replaces particles and fields with an immaterial basis of information. Against this thesis, Timpson (2013) argues that information is an abstract concept that must be instantiated by something concrete in the material world. Hayden and Preskill seem to align with Timpson's view: they do not abandon physical reality but instead propose a model that represents a selected part of the physical world; a black hole as seen from far away.

5 Conclusion and Outlook

In this paper, we introduced and analysed the Page and Hayden-Preskill models of black hole evaporation. These are pioneering examples of toy models based on quantum circuits and quantum information-theoretic tools that have recently been widely used by theoretical physicists in their investigation of the black hole information paradox. Our goal was twofold: first, to illustrate that *this* use of information theory overcomes previously raised scepticism; and second, to assess the strengths and limitations of this approach, which is often relied upon in further investigations.

Taking the influential work of Hayden and Preskill as our primary focus, we have proposed that the particular use of quantum information theory by these authors aligns with the perspective articulated by Timpson (2013, p. 237) in the conclusion of his book. That is, quantum information theory can serve as a general framework of enquiry, functioning as an alternative to, though not a replacement for, other fundamental theories that describe the behaviour of Nature's most basic elements, for exploring certain interesting features of material objects. In this sense, Hayden and Preskill investigated the black hole's properties, such as fast scrambling dynamics and evaporation, for the purpose of enabling communication.

We have argued that the information route to black hole entropy has been rightly criticised by philosophers. Indeed, the thermodynamics of black holes is currently best understood through the physical route, wherein black holes thermodynamics is underpinned by the counting of microstates in quantum statistical mechanics. The present application of information theory, however, does not draw on epistemic reasoning to infer facts about the black hole's microconstituents. Quite the contrary: the quantum information-theoretic perspective is only possible once it is established that a system possesses underlying degrees of freedom from which one can abstract.

In the present context, the strength of the quantum information framework is that it is well-suited for formulating and answering quantitative questions about retrieving a given amount of quantum information, i.e. the quantum state of a number of qubits, from an evaporating black hole. Specifically, Hayden and Preskill focused on determining when and how such quantum information leaks out in the radiation, and what would be required of an observer to recover it in principle.

As the Page time paradox illustrates, these are important questions in the context of information loss, which extends beyond the issue of whether black hole evolution is unitary. Hayden and Preskill's answer is that an observer who has access to all the previously emitted Hawking radiation does not need to wait for the black hole to completely evaporate, but only slightly longer than the time required to emit the number of qubits in the quantum state. After this delay, the observer can, in principle, recover the original state by performing a sophisticated quantum operation on the radiation.

This relatively rapid release occurs because the black hole's dynamics and evaporation process allow for quantum error-correction combined with entanglement-assisted communication. These are well-established tools in quantum information theory that each make use of entanglement, albeit for different purposes.

On the other hand, the weakness of the Hayden–Preskill model lies in the assumptions it relies on, which, like anything concerning the quantum mechanics of black holes, are not supported by direct experimental data. We have identified nine assumptions under which the black hole can be reliably used in a communication protocol. Among these, the most important is the central dogma, which provides the necessary rationale for quantum circuit modelling. The next eight assumptions are modelling assumptions that specify the key features of the model: the number of qubits, the architecture and size of the quantum circuit, and the initial state of the qubits.

The central dogma draws an analogy between a black hole and a finite-dimensional quantum system that is well-motivated by several distinct approaches to quantum gravity. However, this perspective is challenged by the Page time paradox. Although recent developments, especially those involving the inclusion of replica wormholes in the path integral approach to gravity, suggest a resolution to the paradox consistent with unitarity, they do not provide an explicit unitary description of black hole dynamics.

However, as we hope to have illustrated, when addressing precise questions about the retrieval of information, which require a description of black holes beyond that of general relativity, quantum information theory appears to be indispensable. We expect that the general conclusion, namely that entanglement is crucial for information release, will remain valid.

We end this article by examining how Hayden and Preskill's framework has informed recent experimental realisations of quantum information recovery protocols in both non-gravitational and holographic settings. These experiments implement a completed version of the Hayden–Preskill protocol, including an explicit decoding operation for Bob to recover the information. Two decoding mechanisms were proposed and later implemented, with different experimental objectives.

The first, known as the 'Yoshida–Kitaev decoding' (Yoshida and Kitaev, 2017), enables Bob to recover Alice's state using an ancillary EPR pair. Bob applies the complex conjugate of the black hole unitary dynamics to his memory and half of the EPR pair, then performs a Bell measurement on the emitted Hawking radiation and the corresponding part of his memory. For a specific outcome, this measurement effectively teleports Alice's state to the reference qubit in Bob's EPR pair, without requiring classical communication. Because it applies to any scrambling unitary and is independent of gravitational physics, the Yoshida–Kitaev protocol captures a general feature of chaotic quantum systems, including but not limited to those believed to describe black holes. This has motivated experimental tests in controllable, non-gravitational systems. For example, Landsman et al. (2019) implemented this protocol on a seven-qubit trapped-ion quantum computer (see also Blok et al. (2021) for an implementation in a superconducting qutrit system), to study the scrambling properties of the system's unitary dynamics given its teleportation capabilities.

The second mechanism, the traversable wormhole protocol, operates in a holographic setting. It was first proposed by Gao et al. (2017) and later refined by Maldacena et al. (2017), who also emphasised its connection to the Hayden–Preskill protocol. The protocol takes place within a boundary system where the black hole and Bob's memory are entangled in a thermofield double state. Explicit quantum circuit models for the evolution of this system were proposed in Gao and Jafferis (2021); Brown et al. (2019). Its gravitational dual is the AdS-Schwarzschild geometry (Maldacena, 2003; Maldacena and Susskind, 2013), which features two asymptotically AdS boundaries connected by a wormhole. In this dual picture, information recovery corresponds to the wormhole becoming traversable. The protocol was realised experimentally by Jafferis et al. (2022) through a many-body simulation on the Google Sycamore processor of a quantum system believed to admit a gravitational dual, with the goal of demonstrating wormhole traversability via the corresponding process in the boundary theory.

Thus, these experiments pursue different objectives, reflecting their distinct connections to the original ideas proposed by Hayden and Preskill. We believe it would be valuable to explore in greater detail how the quantum information-theoretic framework they introduced in black hole physics relates to analogue gravity experiments.

Acknowledgements

We are grateful to Enrico Cinti and Manus Visser for helpful feedback and discussions, and to Erik Verlinde for important discussions. We would also like to thank Eric Curiel for his thoughtful comments.

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