Positive Possibility and Representation in Modeling

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Abstract: Apparently modally laden terms and relations figure in scientific models, especially though not only possibility. There is an old empiricist tension between measurements as returning actual values, and stronger forms of modality. How could we measure what didn't happen, or use measurement to distinguish what didn't happen but could have, from that which did not happen and could not have? I offer several pragmatist points in the context of modeling and possibility specifically, by which to see this tension as a fruitful one. Peirce offers a view of positive possibility, where it prior to and required for actuality, and which can be differentiated from merely in-principle possibility. He also offers a view of doubt where it requires one suspend use of a belief and initiate an inquiry; views of possibility like those in Humean accounts, even pragmatic Humean views, are offering what amounts to a doubt about models that do not represent these, and only represent positive possibilities. This leads to a distinction by Price between e-representation and i-representation, where some representation is constrained externally by the environment, and some constrained internally by inferential connections. Putting these together provides a better way to understand how models can represent modality such as positive possibility, and how models can be used in ways that settle direction of fit.

Keywords: pragmatism, representation, possibility, modality, doubt, inquiry

1. Introduction

It has long been presumed that empiricism requires prioritization of verification, operationalization, or some other method by which to check claims against the world to establish them as true, and thereby has special difficulty with apparently not-directly-measurable features such as possibility or necessity. This falls out of, on the one hand, emphasizing experience as the central source of knowledge, and on the other hand, is a result of thinking about evidence in empiricism as involving something like individual sense data, or an intersubjective version such as outcomes of individual measurements.

This sets up an apparent tension between measurement and modality, such as possibility, necessity, or impossibility, where measurement may seem unable to differentiate these. The critical concern can be summarized like this. How could measurements, of any kind and even in principle, be sufficient to establish that some un-measured outcome was possible though not actual, and to also differentiate it from an unmeasured outcome that was impossible, not merely unmeasured but

unmeasurable? There are different specific forms this concern takes for different modalities; I will focus here on possibility. This is an old problem (as old is Aristotle, it turns out). It came up for logical empiricism and for pragmatism in early 20th century. It is a variation of the problem to which Quine responded with "Two Dogmas of Empiricism." The criticism that pragmatism has difficulty with modality because of the emphasis on measurement involves a false dichotomy. Consider this for the case of models (rather than Quinean webs of belief). The false dichotomy is to insist that either models are empirically checkable via measurements, *or* models can make modal claims that outstrip any possible individual measurement, but not both. Pragmatism can both/and these two options: pragmatism as a kind of empiricism is accountable for empirical accuracy and relies on measurement, among other tools, for this; *and also* pragmatists can analyze the representational features of models that, both implicitly and explicitly, represent or embed modal claims. One of Quine's main take-aways is that statements don't encounter experience one by one through a hardline verificationism; they don't have meaning, taken out of context, much less have single definitive verification procedures for measuring whether they are true or false. It is already familiar that measurements confirm or disconfirm far more than just individual numerical predictions. A second take-away of Quine's is that individual statements don't have a fixed status as analytically true, or synthetic and in need of verification through measurement. If we treat some statement in a model as analytic, for example, that just means we treat it as giving definitions for the terms and as unable to be directly falsified. But it can eventually be overturned if we decide it is the most effective way to rectify an issue in the model. Similarly, being synthetic is just to be treated as synthetic, or, to be treated as more directly verifiable via measurement (for more detail, see Andersen 2018).

Contemporary pragmatist philosophy of science emphasizes models and modelling far more than older discussions that may have involved laws as a central unit for analysis, or Quinean beliefs of individuals. This makes it a good time to re-evaluate how models can convey modal information while also being tied to the world, and checkable for accuracy, using measurements that only return single values as outcomes and can by definition only return outcomes that are actual. Holism about verification and meaning have new iterations for contemporary discussions in terms of measurement and representation in models.

Part of the reason the tension between empiricism, including but not limited to pragmatism, and modality, appears, is an underlying assumption about how measurement works in conjunction with various ways of representing modal status. Both logical empiricism and pragmatism emphasized empirical knowledge and measurement as a sharp point of that where it hits the world most clearly and directly. The problem of measurement for modality is a problem about atomism for meaning, of terms or of statements. In other words, it is misunderstanding about how models represent.

There have been recent Neo-Humean or best systems accounts of laws that are labeled pragmatist (or pragmatic). These, I will argue, don't help solve how a pragmatist should think about modality, especially possibility. No matter how pragmatist these approaches attempt to be, as Neo-Humean approaches they still require a 'flat' or trivialized notion of possibility, where anything that is not incoherent or metaphysically impossible has the same kind of possibility. I will use the shortcoming

of pragmatist Humean accounts as a springboard to motivate a Peircean distinction between positive or genuine possibility, and merely in-principle possibility, using doubt as a marker.

Peirce has a broadly Aristotelian view of positive possibility, well suited for analyzing science. This pairs well with his view on doubt and inquiry. Peirce holds that doubt has a kind of action-implicating character: if we genuinely doubt something, this is a kind of intellectual itch of sorts, one that requires us to pursue the question until we have resolved it one way or another. He rejects doubt as passive or merely in principle. Put together, it means that asserting that something is possible in some in-principle way just is to doubt any model that does not admit it as a possibility. This undermines the Neo-Humean approach to modality where all these possibilities are in-principle available. Either they are not genuine possibilities - if they were, they would necessarily induce doubt-resolution; or they are genuine possibilities, differing in degree only and not kind, in which case they thereby doubt all models in science.

This leads to a discussion by Price, by which a pragmatist can avoid the false dichotomy of measurement versus modality while highlighting holist conditions for evaluating models and their representational content. Price distinguished between two senses of representation, one of which is environment-tracking and one of which is internal, about inference and meaning. These can vary independently; the external form is especially apt for measurement, and the internal form is especially apt for representing modality, including possibility.

This is an integrated history and philosophy methodology. I am not engaging in Peirce exegesis; this is not intended to be a full account of Peirce's views on these issues. This is more of a Peircean account, plus an opportunity to looks closely at several specific passages in his writings. Similarly, I will pull one particular distinction out from a complex discussion by Price, and put it to somewhat different purposes than Price did.

2. Neo-Humean possibility

Out of post-Lewisian approaches came a version of possibility where everything in possible in some world, and the laws in a given world are just the most efficient set of descriptions of the actual distributions of perfectly natural properties, as he calls it, in the Humean mosaic. Because Neo-Humean approaches have generally focused on laws as a locus of necessity, I will be coming at this discussion sideways. Laws in the sciences don't really appear are free-floating propositions that can be evaluated one by one, though. At least since Cartwright (1983) argued that laws by themselves don't tell us much about the world, and that if we took them literally, what they tell us about the world is false, the move has been away from laws as free-standing special objects of representation or modality in the sciences, and towards models and modelling practices. Models don't have to replace theories; a generic way to construe the relationship here is that theories are much broader, more encompassing, and abstract, and only make predictions or get put to use in the science by being fitted out in the form of models, which are much more specific, tailored to particular systems

or dynamics, etc. So, I will rely on discussions of laws here, while moving to models in subsequent sections.

All Neo-Humean approaches in post-Lewisian metaphysics have to start from the presupposition that there is no necessity in *this* actual world. The Humean mosaic must be made of inert bits; they cannot be magnetic tiles. On this approach, measurements return actual values, and incoherent outcomes are impossible, but otherwise, all other outcomes have to be considered possible. This does generate a problem if one wants to say that there are some outcomes that are not genuinely possible, at all, in this world, and some that will be unactualized but are genuinely possible. Since this is a common occurrence in the sciences, this tension appears dire from a Neo-Humean perspective.

This approach requires a lot of up-front detailed metaphysical commitment: perfectly natural properties, or some substitute; that everything is in principle possible; that possibility involves comparisons across worlds, no matter how worlds are construed. Centrally, it requires the assumption that statements, like those in or derivable from models, have a single stable meaning or truth conditions in or when removed from their modelling context, and that they can be checked or verified individually.

Recently, some Neo-Humeans have developed version of this approach where they draw on pragmatic considerations. The best system of laws for our world might be impossible for us to reach, or to comprehend if we had it. More immediately, though, it simply can't make sense of why our current laws seem to be so effective at describing and predicting since they are very unlikely to be part of that best system. Regarding the pragmatic turn, consider Demarest (2025)

> Lewis's best system analysis says that we are to systematize *as much of the actual Humean mosaic as possible* (so long as we can do it simply enough). But a great deal of what goes on in the universe is irrelevant to pragmatic human goals. The patterns in the movement of distant galaxies or prehistoric dinosaur life will not help me make my way about in the world.

There are several things to note here. First, Demarest is not talking about pragmatism as a philosophical approach or movement. She is talking about something like merely pragmatic, or perhaps, of immediate utility. Second, this involves a misunderstanding of what it means to make a difference to experience. Many predictions about the movement of distant galaxies have testable It attributes to best systems accounts a surface level understanding of pragmatism that is more pragmatic, or involves a strawman version of pragmatism.[1]

Furthermore, it highlights how Neo-Humean accounts struggle with models and modeling. Pragmatist Humeanism was partially meant to salvage the fact that we seem to be able to make a lot of very well confirmed, projectible, and apparently law-like generalizations in the sciences, despite these being almost overwhelmingly unlikely to be the final, or genuinely 'best' on this approach, set of laws. There is an intrinsic and irreconcilable tension here: in order to count as any kind of best

[1] For reasons of space, I am focusing on illustrative examples of pragmatic Neo-Humeanism; there are a wider variety of these views in Hicks, Jaag, and Loew (2023).

system, the laws cannot be built from the bottom up. Whatever laws we currently have can be useful, the Humean can endorse them in the meantime. But what of models? They will never be part of a best system of laws. In fact, they are usually obtained by processes like fitting out a set of laws for a more concrete setting. They limit generalizability, by design. Even if pragmatic Humeanism can salvage some laws, models are the wrong kind of beast to ever be salvaged, yet they are among the most useful scientific tools we have.

Dorst (2019) offers a different formula for obtaining the laws from a Humean base, which takes into account the utility of the laws for creatures like us. But this still encounters the same problem: what is currently useful to us, given our comparative ignorance of much of the world to be captured in any set of laws, and what would be useful to us if we have perfect knowledge of that world, have no reason to overlap. More concerns about this pragmatic approach can be found, inter alia, in Callender (2023) and Hildebrand (2023).

Thus, one dominant approach to modality involves a notion of possibility that is flat, where there is only one way in which something is possible, and involves a version of pragmatism that is merely pragmatic, unable to account for the apparently modal claims made in current sciences since they are by definition false on the Humean approach. This is unsatisfactory in a number of regards (another will be explored in the next section), but in particular, it precludes treating models as a locus of possibility. They are not laws, and even when they involves laws, it is laws as fitted out onto more specific kinds of phenomena or systems in the world, not the totality of the world as required by Humean laws.

This then points towards a major feature that is required to make sense of modality, especially possibility, in our world as that world is studied by the sciences. A major pragmatist maxim concerns making a difference to experience, construed more broadly. There has to be a sense of live possibilities, no merely in-principle ones, available for use in a scientific context. Some possibilities are represented in models as tgenuine possibilities in that model, and others are either represented as impossible or as not genuine possibilities, and others are simply not represented, thereby implicitly represented as impossible or not genuine possibilities. This difference is more than probability can provide. A great example is by Ruyant and Guay (2024), looking at possibilities in Lagrangian mathematical structure for physics. In order to adequately analyze what goes on in Langrangian mathematical models, there are some outcomes that are simply not possible, and of those that are represented as possible, there are still two different sets of possibilities. They differentiate kinematical from dynamical possibilities. Everything in the kinematical space is possible, in one sense, but for example, there will be points that are kinematically possible but which are not in the dynamical space, so, not dynamically possible. Whatever this difference is between kinematic and dynamical possibility is, it is not adequately expressed by just assigning a probability distribution over possible worlds.

It is the job of the sciences, in a strong way, to say what the actual possibilities are. Insisting on some outcome as 'really' possible in the face it its being completely absent, or represented as of 0 probability, or as impossible, in a reasonably well confirmed model, fails the pragmatist test. It

makes no conceivable difference to experience, construed broadly, to insist that 'really' these other possibilities are genuinely possible in the same *way,* just to a different *degree,* as e.g. the kinematical possibilities represented in a Lagrangian space. In many (arguably, all) theories or models, there are many outcomes that are simply not genuinely possible. Those outcomes just are not there, and so we have *no reason* to expect them, and more strongly, *reason to not* expect them.

New attention is being paid to how modality shows up in the sciences, especially in modeling (for example, see Knuuttila, Grüne-Yanoff, Koskinen, & Wirling 2025). Koskinen (2023) provides a detailed overview of these trends, including how-possibly modeling and exploratory or hypothetical modeling. The points I will make below should be taken to complement this literature, adding to it where there are lacuna. For example, in how-possibly modeling (e.g. Bokulich 2014), models involve notions of possibility, but are not thereby modeling a space of what is possible, nor modeling possibility as contrasted with necessity. How-possibly modeling can provide plausible explanations that require further investigation into how-actually models (e.g. Resnik 1991) but they generally pick out a possible mechanism or explanation, not something like a set of possible outcomes or states.

A pragmatist thus is looking for a notion of possibility that has the following features. It should differentiate between genuine possibilities, and merely trivial or in-principle ones. It should be consistent with modelling practices in the sciences, but more strongly, it should make sense of those practices and enable us to use this notion of possibility to get more out of the models already in use, by helping us identify how possibilities are represented, explicitly and implicitly, in them. It should avoid imposing philosophical distinctions that may not fit the practices well and which are generally antithetical in pragmatism as likely-to-be-false dichotomies. For example, a pragmatist may want to avoid having to make a hard distinction between possibilities that are epistemic versus those that are ontological (e.g. Wirling and Grüne-Yanoff 2021, 2023). Finally, it should situate measurements as direct ties to experience for which pragmatists generally, and models usually, are on the hook, but not in an oversimplified way; there should be room for different contexts of use to return different answers about what is being treated as necessary, or possible, or contingent, and what the direction of fit between model and world is for that use.

3. Peirce on inquiry and possibility

This section will examine some specific passages in Peirce's work that illuminate a way to think about modality, especially possibility, in the context of pragmatism and the sciences. Perhaps counterintuitively, a helpful place to start on Peirce's views of possibility is to start with his views on inquiry and doubt.

Philosophers generally conceive of doubt in a passive fashion, and as something that is always at least in principle deployable, against any potential belief. The idea is roughly that one can always say, of any belief, that it might not be true, or request without cost that one produce justification. Then the onus is on the believer to provide justification for it, not on the asker to provide reason why such justification should be produced. One can always ask, but do you *know* that, and thereby put

anyone asserting a belief in the position of having to provide justification for it. In normal conversational contexts, used in normal ways, this is likely quite reasonable. But of course, it can get mis-used in philosophical contexts to turn the request for justification into an almost undefeatable weapon of skepticism. In the face of arbitrarily strong justification, one can still in principle say, yes this is justified, but is it *true*? and, thus, wedge in that bit of doubt between justification and truth. This passive construal of doubt, where there is nothing more involved than asking the question, makes sense against, and only makes sense against, the backdrop of the JTB view of knowledge. This has already been criticized in detail as an account of knowledge, justification, and truth (inter alia, Chang 2022, Andersen 2023). Here, I will raise concerns about its feasibility as a picture of doubt.

For Peirce, doubt is not a passive stance, nor does it leave us able to continue operating in the meantime, just with doubt instead of without it. He took doubt to be something more like an itch. If we come to genuinely doubt something, then, he says, that is saying that we feel a compulsion to investigate it further, to satisfy the doubt, to find further evidence or to assess our evidence more clearly. Doubt is an active state with respect to some belief, with a distinctive phenomenological feel, and with immediate consequences for action, namely, triggering inquiry into the doubted belief.

> We generally know when we wish to ask a question and when we wish to pronounce a judgment, for there is a dissimilarity between the sensation of doubting and that of believing.
>
> But this is not all which distinguishes doubt from belief. There is a practical difference. Our beliefs guide our desires and shape our actions. .... The feeling of believing is a more or less sure indication of there being established in our nature some habit which will determine our actions. Doubt never has such an effect.
>
> Nor must we overlook a third point of difference. Doubt is an uneasy and dissatisfied state from which we struggle to free ourselves and pass into the state of belief...
>
> Thus, both doubt and belief have positive effects upon us, though very different ones. Belief does not make us act at once, but puts us into such a condition that we shall behave in some certain way, when the occasion arises. Doubt has not the least such active effect, but stimulates us to inquiry until it is destroyed. This reminds us of the irritation of a nerve and the reflex action produced thereby... (1877, section III)

Doubt involves suspending a belief. One does not use it to guide action until the doubt is resolved, and doubt is the trigger to actively inquiring into justification for the belief until the doubt is resolved one way or another. Doubt doesn't *cause* change in using a belief, or cause the start of inquiry. Doubt just *is* that change. It would be false, or a kind of category mistake, to claim to doubt something that one continues to use in the meantime. If one doubts it, one's basic behavior, one's first-personal attention, must be altered. Inquiry is thus pursued until there is a resolution of doubt.

Doubt requires, it doesn't just point towards, inquiry to resolve it. "The irritation of doubt causes a struggle to attain a state of belief. I shall term this struggle inquiry... With the doubt, therefore, the

struggle begins, and with the cessation of doubt it ends. Hence, the sole object of inquiry is the settlement of opinion." (ibid. section IV)

Inquiry, then, is not limited to but also includes inquiry for the purpose of resolving some doubt. It's helpful that Peirce's example involves a belief one thought was sure, about which one comes to have doubt. He thinks that we start out with a collection of beliefs that we take to be true, by the time we even consider these questions of doubt and belief. On this picture, we don't start with candidate beliefs at some imagined starting line where all are in doubt, and then evaluate justification for them until one 'wins' the race and is believed while the rest are rejected. Instead, we start with a collection of beliefs on which we rely in reasoning and in guiding actions so we know how best to work towards some aim. Then, we may come to doubt one of the beliefs, for many reasons. But we cannot then continue relying on it the same way as before, if we genuinely doubt it. This doubt triggers a process by which we inquire into this belief which we may not have ever acquired via inquiry in the first place.

The doubt is 'live' in the sense that one cannot just ignore, and have it count as genuine doubt. Doubt that is merely in principle and does not change anything about one's reasoning or actions, is not real doubt. Inquiry in these cases is shaped by what, precisely, one doubts, so that the inquiry is suited to resolve the doubt. An analogy might be made with driving a stick shift vehicle. A belief that is doubted is, by dint of being doubted and what doubt means, something that is no longer safe to drive on. Like an engine in the wrong gear, it is taken out of use. This is like putting a car into neutral while driving it; the whole car is not immediately stopped, nor does the engine seize up. But we have to do something as the driver to put the car back into a more suitable gear, and once we do, we re-engage it. This is more active than just having an in-principle concern. The doubted belief is disengaged, and only re-engaged for driving if the doubt is resolved. If the doubt persists, or inquiry uncovers that the belief should be replaced, then some new belief is put into gear instead. Meanwhile, one doesn't simply drive the car as normal while it is not in gear.

The take-away here is that doubt is an active state that precipitates inquiry to resolve it, and any purported doubt that does not precipitate inquiry is not genuine doubt. Doubt as a kind of experience, with a distinctive feel and characteristic changes to how we think and act, is a useful way to approach the issue of possibility. If something is *really* a possibility, then that means we have to allow it into our reasoning, and change actions in ways that reflect it as a possibility. Conversely, if some outcome is insisted on as possible, even if very improbable, the fact of that possibility must be reflected in our reasoning and actions. To say that something is possible is to positively commit to a belief about it, one which is not in a state of doubt, or put another, one which is not currently the target of inquiry.

> Some philosophers have imagined that to start an inquiry it was only necessary to utter a question whether orally or by setting it down upon paper, and have even recommended us to begin our studies with questioning everything! But the mere putting of a proposition into the interrogative form does not stimulate the mind to any struggle after belief. There must be a real and living doubt, and without this all discussion is idle. (Ibid., section IV)

To say that something is possible is to thereby offer it as something that must be taken into account. If that 'possibility' is not, for example, represented as possible in a model, then to say it is in-principle possible just is to say that you doubt the model. The pragmatist can reject this trivial notion of possibility that involves some in-principle doubt to which we are never actually responsive. To genuinely doubt a model, we would have to suspend use of it, and engage in inquiry until the doubt is satisfied. In-principle doubt, which includes the Humean insistence that some outcome or situation that conflicts with a model is 'in-principle' possible, is thus on these dilemma horns. Either it is not genuine doubt, because it induces no action; or it is genuine doubt, and thereby commits us to re-opening inquiry on some or all of the model as empirically inadequate.

Consider the Volterra Principle (Weisberg and Reisman 2008, Räz 2017): in the predator-prey system of the Lotka-Volterra equations, any general biocide that affects both predator and prey will result in the prey population springing back much faster than the predator population. This can be derived purely by rearranging the 2x2 system of differential equations that is the toy version of the model. The L-V model thus says that there is no possibility, no matter how in-principle, whereby the predator population springs back before the prey one. Whatever necessity is involved in Volterra Principle, it is a belief commitment to ruling out the possibility of the Principle not holding. This is more than just saying that one would be rejecting the model if it didn't allow for this possibility. The Humean approach requires that it be in principle possible to have those equations hold and yet still have the mathematically incoherent in-principle possibility that the Volterra Principle not hold.

This leads to the next part of Peirce, his views on Firstness, Secondness, and Thirdness. It is very helpful as an angle into that discussion to start with how he introduces it, with Aristotle's discussion of the sea battle. In the Aristotelian discussion, the sailors in the evening discuss the possibility of there being a sea battle tomorrow, weighed against the possibility of the generals reaching some negotiated peace without a sea battle. Peirce points out that it had to be genuinely possible, in a substantive way, for there is both be, or not be, a sea battle tomorrow, to count those as live possibilities. This is not merely an epistemic point. If, unbeknownst to the sailors, the generals had already negotiated a truce, then it is already true that there will be no sea battle tomorrow. It is just that they do not know about this. This contrast illuminates the original notion of possibility: if negotiations are ongoing, it is not mere epistemic limitation. It has to be a genuinely open possibility either way for there to be a sea battle, or to not be a sea battle.

> Aristotle, on the other hand, whose system, like all the greatest systems, was evolutionary, recognized besides [actuality, the being which consists in the object's crowding out a place for itself in the universe, so to speak, and reacting by brute force of fact, against all other things] an embryonic kind of being, like the being of a tree in its seed, or like the being of a future contingent event, depending on how [one] shall decide to act. (1931, Chapter 1, sections 21-22)

Peirce connects the Aristotelian notion of possibility and actuality for existence: in order for something to count as real, some instance of it must actually exist. There are no species of which there are not nor ever will be any members. But, prior to the first instance of a kind actually existing,

there must have been some sense of possibility such that it could *possibly* exist, in a way that is more substantial than mere in-principle possibility.

> My view is that there are three modes of being. I hold that we can directly observe them in elements of whatever it is at any time before the mind in any way. They are the being of positive qualitative possibility, the being of actual fact, and the being of law that will govern facts in the future (ibid., section 23)

Actuality, what he calls Secondness, presupposes or requires that prior to it, there be Firstness: positive qualitative possibility. Drawing on more Aristotle, the acorn has this Firstness of positive possibility of becoming an oak. But it does not have Firstness of possibly-a-pine. A pinecone does have that Firstness, that positive possibility, of possibly-a-pine. In order for there is actually be a pine, there must have already been that possibility into which it could actualize. Even if an acorn never grows into an oak tree, the notion of positive possibility here still picks out an oak as what it could have possibly grown into, and never a pine. That positive possibility is a form of existence.

> Firstness is the mode of being which consists in its subject's being positively such as it is regardless of aught else. That can only be a possibility. For as long as things do not act upon one another there is no sense or meaning in saying that they have any being, unless it be that they are such in themselves that they may perhaps come into relation with others. The mode of being a redness, before anything in the universe was yet red, was nevertheless a positive qualitative possibility. And redness in itself, even if it be embodied, is something positive and sui generis. That I call Firstness. We naturally attribute Firstness to outward objects, that is we suppose they have capacities in themselves which may or may not be already actualized, which may or may not ever be actualized, although we can know nothing of such possibilities [except] so far as they are actualized. (Ibid., section 25)

For Peirce, anything actual has a kind of larger, encompassing halo of possibility, and this possibility has a special status. It is not mere in-principle possibility. It is the space into which the actual comes into existence. This substantive possibility involved in Firstness is required for, and both conceptually and temporally prior to, actual existence.

This kind of possibility is what we need for talking about models, which differentiate what is genuinely possible via the bounds of the model itself. Consider again the example of a toy version of the Lotka-Volterra equations. There are certain possibilities there, prior to actualization. Any actual biocide that then conforms to the Volterra Principle does so because that is the space for the actual that was made available by the positive possibility represented via the differential equations.

There are also additional outcomes that simply don't appear. The predator population cannot decide to have fewer offspring for judicious reasons; there simply is nothing in there that represents this as any possibility. It is impossible, in a *different* way than a predator-first biocide rebound. We would lose important modal information conveyed by the model if treat these two impossibilities as equivalent. One is a mathematical relationship; the other is a deliberately built part of the equations.

Peirce's notion of positive possibility is helpful for modality in the sciences in a way that differentiates kinds of possibility. It is also, as noted above, bluntly incompatible with Neo-Humean approaches, even ones that center human oriented concerns. His view of doubt is also quite helpful for distinguishing a trivial or in-principle notion of possible, and a positive or substantive notion of possibility. Offering an in-principle possible outcome or state is not the same as offering a genuine possibility, because it does not involve any of the distinctive features of doubt.

4. Price on representation

In the introduction, I noted how the apparent tension between measurement as returning actual point-outcomes, and modality such as outcome possibilities that are unactualized versus outcomes that are impossible, reiterates the logical empiricist problem with which Quine took issue. Peirce has delivered something quite useful with his notion of positive possibility. Models represent some outcomes as possible, whether they actually occur or not; these are Peircean positive possibilities. This is, in some ways, a new iteration of Quine's point about holism. These positive possibilities have the shape that they do, many models, because of the shape the entirety takes in light of, among other factors, measurements. Even toy models that don't make numerical predictions and thus cannot be directly confirmed or disconfirmed with measurements, have been shaped by their ties to some system, from which these toys versions were then abstracted.

Given the vast heterogeneity of models and modeling practices, it is hard to make any sweeping claims about all of them that would be informative. There are some things that come up consistently, though, which are germane to this apparent tension between measurement and modality as resolved through holism about verification. Quine's further point had to do with meaning, as well. How do we even know what a particular statement means, in order to see if it is true? Abstractions and idealizations serve useful purposes inside a model, but it is important to remember that they are not straightforwardly false (see, inter alia, Rice 2021, Andersen 2023). By removing some statement from a model, we have likely thereby changed it meaning, since that was at least partially given by the inferential connections it had in that model. At the same time, pulling in the opposite direction, surely we need to leave open that some statements or elements in a model are wrong, or less accurate than it should be, or could be improved. One way to navigate positive possibility and measurement in models while accommodating this vast range of truth-aptness in models is with a distinction between two senses of representation from Price (2008).

Price begins with an analogy using the sticker matching game. There are children's sticker books where there are precisely shaped shadows in which only one of the provided stickers will exactly fit. The puzzle is to find which sticker goes with which shadow. There is a tendency to think of what makes a statement true is that it is like the sticker that fits precisely the spot onto the world where it goes. An ill-fitted sticker would be false. Our statements, then, are like the stickers, and the world is like the shadows. A problem arises, though, in that we seem to have far more stickers than we have places in the world to put them. He calls this big-R representationalism: " This proto-theory accords a key role to the idea that the function of statements is to "represent" worldly states of affairs, and

that true statements succeed in doing so." (Price 2008, p. 2). Having more stickers than shadows is like having more statements than truthmakers. Price argues, through these lectures, that we should give up this picture of how language says things about the world.

One extant strategy he considers for this is that of claiming some statements are not making true or false claims about the world. They are expressive of something else, perhaps like fiction, or moral claims, etc. The problem with any of the strategies that take this line, though, according to Price, is that "they think that for any of our claims or commitments, there's a genuine issue whether it is really factual, or descriptive. In other words, they take for granted what Rorty calls the Bifurcation Thesis – the doctrine that there is a line to be drawn in language, between descriptive and nondescriptive uses." (Ibid., p. 7)

This means that, to solve the placement problems that result from thinking of statements like little stickers that get made true by fitting onto the right bits of the world in the right ways, is that they all lead to the Bifurcation Thesis. where statements get sorted into distinct piles, descriptive versus nondescriptive, and only descriptive are capable of being true or false. This just sneaks back in the atomism of logical empiricism, that each statement has, all by itself independently of use or context, some determinate "descriptive: yes or no?" answer. Price is right to caution us against this (as is Rorty).

He then goes through two poles along which one might think of representation. One is from Blackburn, and it highlights how representation involves environment tracking. The prototype of this is perception: content has to co-vary with the parts of the environment in order for a sensory modality to count as sight, for example. Fuel gauges in vehicles are environment tracking, when they work; when they are not working, it is because they are not providing an accurate description of the fuel in the tank.

Price contrasts with the Brandomian pole, where representation is inferential. Word meaning involves a nexus of usage in language, and language has rules or norms that we choose to follow, by dint of using language. No one gets to decide on their own that they want to change the rules; that would count as just failing to use language, failing to give or ask for reasons. The Lotka-Volterra equations again illustrate here: what it means to be the predator population is given by the role that population plays in the system of differential equations. If we ask of a population of coyotes, for example, whether it is predator or prey, more would be required: what is the other population with which they will be modeled? If it is cougars, then the coyotes are prey. If it is mice, then the coyotes are predators. The meaning of 'predator' in the context of the model is given by the internal inferential connections; this is especially apt in the case of systems of differential equations where various terms in either equation are defined by terms in the other equation.

Price thus solves the problem of how statements (in my case, parts of models and the models themselves) can be truth-apt by claiming both kinds of representation. He calls these e-representation and i-representation, respectively.

In e-representation, the co-variation with the front foot. This is often what philosophers of science expect from models. The parts that are like an indicator on a model-dashboard that hopefully reliably indicate the values of some quantity 'out there' in the system being measured and modeled.

> On the one hand, we have the environment-tracking paradigm of representation, dependent on such notion s as covariation and (what Field calls) indicator-relations – think of examples like the position of the needle in the fuel gauge and the level of fuel in the tank, the barometer reading and air pressure, and so on. In these cases, the crucial idea is that some feature of the representing system either does, or is (in some sense) 'intended to', vary in parallel with some feature of the represented system… In biological cases, for example, this notion gives priority to the idea that the function of a representation is to co-vary with some (typically) external environmental condition: it puts the system-world link on the front foot. (Ibid., p. 13)

In i-representation, the inferential connections between statements are the front foot, as he puts it. The meaning of something, and what makes it truth-apt, is how it is used in that inferential position.

> i-representation: On the other hand we have a notion that gives priority to the internal functional role of the representation: something counts as a representation in virtue of its position or role n some kind of cognitive or inferential architecture. Here it is an internal role of some kind… that takes the lead." (Ibid., p. 13)

E-representation provides external constraints on models, such that they are answerable to something beyond themselves, against which they can evaluated. I-representation provides a different kind of constraints on models, such that they have to meet in-model or in-language constraints about usage and meaning. These two kinds of constraints, externally imposed and internal, can both be operative: we needn't pick just one. "My New Bifurcation Thesis claims that these are not two competing accounts of a single species of representation, but two distinct species…" (Ibid., p. 15).

Models thus require both e- and i-representation. Models have to have elements that facilitate some empirical contact with the system(s) being modeled, by which the model is checked for fit, altered by means such as changing parameter values, etc. Measurement is a central avenue by which this is accomplished. These e-representing parts provide confirmation that suffuses the whole model. The individual variables for prey and predator population numbers can be used this way: they can generate detailed predictions about next season's population numbers, for example, to be checked against that via measurement of actual population numbers, or as estimates of current population. In order to do this, i-representation plays a key role in tying the model together internally. The predictions for the prey population next year could not be generated without measuring the population this year, or measuring the birth rate, but also could not be generated without the equations that allow inferences between these quantities, and which must meet additional constraints about how to use the system of equations.

Price also offers two notions of constraint, external and internal, that go along with two notions of representation, e- and i-representation. These provide a new solution to avoiding the problem of atomism about measurement and about meaning. Price does not specifically develop these in the context of models and how models represent; owever, these distinctions also apply to models. There is no reason to think that models somehow have a sui generis form of representation such that these couldn't apply. That means they are especially helpful for addressing this question of possibility, how possibilities appear in models, and how they differentiate positive possibilities that are unactualized from impossibility.

5. Positive possibility and representation in models

I will conclude by putting together the ideas on doubt and on positive possibility from Peirce, and the distinction between e- and i-representations from Price.

I propose using Price's two notions of representation as something like two axes in a graph.[2] These two axes capture different parts of the representational work done by a model. These axes can vary independently: increasing or decreasing the e-representation content of a model can be done without doing so by changing its i-representational axis and vice versa.

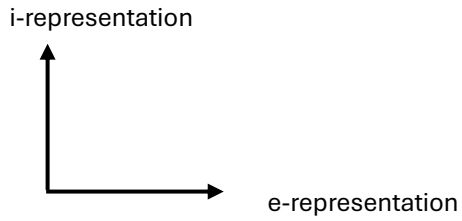

Figure 1: e- and i-representation

For a model to be further out on the e-representation axis, it would need to involve more directly measurable quantities, and have higher standards for producing empirically correct numerical predictions. Such a model could be moved closer to the origin by making a toy version of it: treating it as indicative of dynamics but taking out many of the complicating parameters that might be necessary for accurate prediction but obscure underlying relationships. Some models might have low i-representation, such as models with a large number of parameters that are set based on ongoing updated measurement values instead of being calculated from within the model. A model might have a very high degree of i-representation, such as the Lotka-Volterra models, which are no longer

[2] This goes beyond what Price says, and so this should not be taken as Price's direct view; it could be he would disagree with some or all of this. As such, it is more like a Pricean view, as the previous section laid out a Peircean view.

usually about any particular system in the world, but which are so inferentially well connected that they can be studied on their own.

I-representation directs attention inside the model: to the meaning and definitional relationships. It is how the model is inferentially tied together. It is also how we get from one measured/measurable quantity to another; Chang's (2004) notion of nomic measurement involves i-representation, for example. E-representation directs attention outward, to the parts of the world that the e-representing variables or quantities or terms pick out. For models that are not quite about any particular part of the world, such as toy versions of models, these e-representational elements still stick out, like the parts that could be attached to something, but which need not be actually attached in order to serve their purpose.

My claim is that all models can be better understood by considering where they fall along each of these two independent axes. This also provides a better way to think about what makes models true or false, better or worse in terms of fitting the systems they purportedly represent, using direction of fit. There are three distinguishable directions of fit here. One is from world to model. In that case, the model must fit onto the world, and its truth or veridicality is adjudicated by the world being 'right', as it were, and the model having to conform to it. There is another important direction of fit, though, from world to model. The earlier example of the Lotka-Volterra equations illustrates this. It is a poor fit for many ecosystems, but one could find one that is isolated in just the right way so as to illustrate the general biocide example for the Volterra Principle. In that case, the model is 'right' and the modeller goes looking for a bit of the world that fits onto the model. Finally, there are models that don't have fit: we might examine some system of equations that we take to have potential physical significance but neither try to make predictions with it, nor try to find a system that fits onto it. There may be even more (entheorization from Beauchemin and Staley 2025 might be yet a different direction of fit).

Models on a pragmatist approach involve both e-representation and i-representation. Some parts hold the model together, and some co-vary with parts of the system being modeled. These tend to involve the parts that represent various modal features, and measurement, respectively. The tension between modality and measurement is like the tension that holds up a tent. It needs to be there - it is doing structural work.

The same model can be used with different directions of fit, depending on modeling purposes. And, the same model can be tweaked to increase or decrease the e- or the i-representation. It is not an intrinsic feature of any model that it only goes one direction for fit, or only contains one set kind and specificity of representation. Models can be used in ways that have direction of fit, or specified degrees of each kind of representation. This is entirely in keeping with Quine's rejection of a hard analytic-synthetic distinction, and makes more sense of his rejection in the context of models (rather than individual beliefs).

The usage of models ties us back to the beginning, with Peirce's view of doubt and inquiry, and positive possibility. Peirce holds that doubt is something that positively affects our thoughts and actions, not merely a passive lack of belief. Doubt is itself a kind of action, where, doubts requires one to actually suspend reliance on a belief and to initiate a process of inquiry until the doubt is resolved. Similarly, modality and possibility is connected to what is, or is not, doubted. To insist that some outcome is 'really' possible, on a flattened notion of possibility where literally anything is possible (Neo-Humeanism), is to claim that a model where it is not represented, or even representable, as a positive possibility, is false or misleading. But that then implies that one ought to be doing something to *change* the model, or to investigate that possibility further. To insist on a possibility is to insist it must be taken earnestly in inquiry, and to doubt when it is not.

Substantive or positive possibilities, on Peirce's view, are those that also warrant inquiry. Non-substantive possibilities would be a distraction from inquiry, or at best an incoherent claim that one is doubting but also not doing anything about it. There are only so many inquiries in which we can engage. Models guide inquiry, and sometimes do so *by* conveying modal information about what the genuine possibilities are. Because of holist of meaning in models, idealizations cannot be taken out and examined on their own without thereby irremediably altering their meaning. Changing the inferential connections and locus of a statement *just is* to change its i-representational content. Finally, because of the holism, the e-representing parts of models thus confirm the whole model, not just the individual quantities but all the apparatus that was used to generate the prediction, which includes those inferential connections that make the model into something cohesive.

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