Space-Time Normalisation in GRWf Theory

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Abstract: Roderich Tumulka's GRWf theory offers a simple, realist and relativistic solution to the measurement problem of quantum mechanics. It is achieved by the introduction of a stochastic dynamical collapse of the wavefunction. An issue with dynamical collapse theories is that they involve an amendment to the Schrödinger equation; amending the dynamics of such a tried and tested theory is seen by some as problematic. This paper proposes an alteration to GRWf that avoids the need to amend the Schrödinger equation via what might be seen as a primary set of solutions to the Schrödinger equation that satisfy a normalisation condition over space and time. The traditional Born-normalised solutions are shown to be conditionalisations of these primary solutions.

Keywords: space-time normalisation; flash ontology; statistical interpretation.

1. Background

Quantum mechanics is one of the most successful scientific theories we have, underlying much of today's technology, yet after a century of debate there is no consensus as to how we should interpret the formalism of the theory. The traditional view interprets the square modulus of the wavefunction as a probability distribution associated with the outcome of measurement. The problem is that it is not clear exactly what constitutes a measurement. We may see this as part of a more general problem of reconciling the indefinite or superposed nature of the quantum world with the definiteness of our experience. One of the main realist approaches to this problem, known as the measurement problem, is via the dynamical collapse of the wavefunction.

The idea of dynamical collapse was proposed by Philip Pearle (1976, 1979), it was then developed by Ghirardi, Rimini and Weber (1986) and a number of variant theories have since been proposed, some extending into the domain of quantum field theory[1]. The approach prioritises quantum dynamics and elegantly brings the classical and quantum realms together under a unified dynamical theory. The Ghirardi, Rimini and Weber theory, known as GRW, supplements the standard linear evolution of the wavefunction prescribed by the Schrödinger equation, with a stochastic non-linear localisation of the particle wavefunction in configuration space. The measurement problem is resolved by the spontaneous dynamical collapse.

[1] For example, Oreste Nicrosini and Alberto Rimini (2003), Tumulka (2006a) and Daniel Bedingham (2011).

Roderich Tumulka (2006a, 2006b) develops the GRW theory by proposing a flash ontology, an idea discussed by John Bell (1987).[2] Tumulka (2006b, p.2) says "We follow Bell in taking as the primitive ontology, or local beables, of the model the space-time points where the collapses are centred. 'A piece of matter then is a galaxy of such events.' We will call these points 'flashes' ". What is significant about Tumulka's proposal (GRWf) is that he shows the theory, for *N* non-interacting particles, can be made Lorentz invariant.

GRWf achieves Lorentz invariance by assigning nomological status to the wavefunction (see Allori et al 2008, p.11). The primitive ontology is one of flashes, the occurrence of the flashes is governed by physical law, and this law is described by the wavefunction. Non-local processes are thereby confined to a nomological realm, leaving a relativistic description of the primitive ontology provided by a relativistic Schrödinger equation[3]. Tim Maudlin (2011, p.247) points out that it is the relative-time invariance of quantum correlations of entangled particles that facilitates the relativistic description. Whilst Tumulka does not consider interacting particles, he shows that the non-locality exhibited in Bell-type experiments can be accommodated within a relativistic quantum theory.

A concern with Tumulka's GRWf theory, as with all dynamical collapse theories, is the amendment of the Schrödinger equation[4] entailed by a dynamical collapse mechanism. Changing such a well-corroborated and widely used equation, absent of inaccurate measurement predictions, limits the appeal of the proposal[5]. Tumulka (2006b, p.16) talks of GRWf paying "the price of a certain deviation from quantum mechanics". Whilst Stapp (1989, p.157) says "The collapse mechanisms so far proposed could… be viewed as ad hoc mutilations designed to force ontology to kneel to prejudice".

This paper seeks to address the concern, and specifically address the need to amend the Schrödinger equation whilst maintaining wavefunction collapse. It is argued that the amendment can be avoided by exploiting an alternative set of solutions to the Schrödinger equation that utilise a *space-time* normalisation (STN) condition. Under the proposed interpretation, referred to as STN for brevity, solutions to the Schrödinger equation provide a probability distribution, over configuration space-time, of the occurrence of flashes. *Quantum mechanics under the STN interpretation is a statistical theory describing the location of a primitive ontology of flashes*, thereby avoiding the measurement problem without the need to modify the Schrödinger equation.

2. STN Interpretation

The idea behind the proposed STN interpretation is to pick out a non-traditional set of solutions to the Schrödinger equation that are more suited to a primitive ontology of flashes,

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[2] Flashes are events, and there is a long history of event ontologies both within quantum theory and within the study of philosophy. The discontinuity of events makes them a natural place to look for an understanding of the discontinuous nature of measurement that forms part of traditional quantum mechanics. Alfred Whitehead (1925) and Bertrand Russell were early advocates of an event ontology from a philosophical perspective. Russell (1927, p.427) writes "the physical object to be inferred from perception is a group of events, rather than a single 'thing'. Percepts are always events, and common sense is rash when it refers them to 'things' with changing states. There is therefore every reason, from the standpoint of perception, to desire an interpretation of physics which dispenses with permanent substance." Leemon McHenry (2015) provides a wide-ranging discussion on event ontologies.

[3] Tumulka uses the Dirac equation for spin ½ particles in his proof.

[4] The term Schrödinger equation will be used to refer to both relativistic and non-relativistic versions.

[5] Quantum interferometry could potentially detect spontaneous collapse, see Adler (2007), and if spontaneous collapse is detected, these grounds for concern at the amendment of the Schrödinger equation disappear.

where a flash is assumed to be a moment of definite position of a particle[6]. The set of solutions result from a primary space-time normalisation (STN) condition that is consistent with, and in a sense sits behind, the traditional Born normalisation (BN) condition. The BN condition is seen as being perfectly correct, though it is a conditionalisation over measurement outcome of the more fundamental STN condition. Specifically, the proposed primary STN condition is such that the square modulus of each component of the multi-particle wavefunction provides a probability distribution over space *and time* of the occurrence of a flash, given a certain set of initial conditions. Let us elaborate the proposal by comparing it to GRWf, before considering a more formal description of the STN wavefunction.

- STN is, like GRWf, a theory about a primitive ontology of flashes that provides the likelihood of the occurrence of a flash in space-time. And a flash, as in GRWf, is at the centre of a collapse.
- Unlike GRWf, STN makes no change to the Schrödinger equation to determine the likelihood of when and where the flashes occur. GRWf adds a dynamical reduction process to the Schrödinger equation to predict the location of flashes in space and time, whereas STN interprets an alternative set of solutions to the Schrödinger equation as providing a probabilistic description of the location of flashes in space and time.
- STN proposes a normalisation condition over space and time that is more fundamental or less restrictive than the Born normalisation condition of GRWf. The STN condition results in solutions to the Schrödinger equation that provide probability distributions over space and time of the occurrence of flashes. Specifically, the square modulus of the STN wavefunction provides a Poisson rate parameter of the occurrence of flashes over space-time – the details are discussed in the next section.
- Collapse for GRWf is part of the dynamics of a *nomological* wavefunction; the wavefunction reduces to a narrow Gaussian structure that induces a flash at its centre.[7] The STN wavefunction, however, is not a nomological entity, it is a *statistical* entity for predicting the spatiotemporal location of flashes. The STN collapse does not involve a *Gaussian* reduction of the STN wavefunction, it describes the change in predictions of future flashes associated with the occurrence of a flash. Collapse is not part of the dynamics of the STN wavefunction, formally it is a reset of initial conditions of STN solutions to the Schrödinger equation after the occurrence and inclusion of a flash.
- Predictions of GRWf and STN will differ slightly as a result of the Gaussian structure introduced by GRWf theory in the collapse process. The reason for this difference is the tail of the Gaussian; if we consider a position measurement on a single particle system in GRWf theory, it is possible that the particle is found far from a flash that has just occurred. STN denies this possibility because the location of a flash is by definition the particle position.

The existence of a physically significant set of solutions to the Schrödinger equation that satisfy the space-time normalisation condition is perfectly consistent with traditional quantum mechanics. STN is a statistical interpretation of quantum mechanics that does not change the traditional formalism; it is consistent with both the traditional Born interpretation of the wavefunction and the Born normalisation condition. The STN solutions to the

[6] Particles in this context are assumed to be emergent phenomena, though one might conceive of flashes within the STN interpretation as point particles, or particles that exist at a particular point in space-time.

[7] The idea of a nomological wavefunction is not unproblematic and has been challenged by, for example, Belot (2012), Romano (2020), Oldofredi and Öttinger (2020).

Schrödinger equation are a secondary and more fundamental set of solutions, and when the STN solutions are conditionalised over measurement outcome we get the Born probabilities, as will be discussed next.

3. Formalism of STN

The Schrödinger equation is a linear equation which means that if ψ is a solution for a given set of boundary conditions, then $C\psi$ is also a solution, for any value of constant C. Because of this linearity, a normalisation condition is required to identify particular solutions that are relevant for a given interpretation or use, and the Born normalisation condition provides solutions that are used to determine probabilities of outcomes of position measurements. Which then are the STN solutions to the Schrödinger equation that can be interpreted as probabilistic descriptions of the spatiotemporal location of flashes?

(i) Infinite Square Well Potential

It may be helpful to initially consider a particular STN solution to the Schrödinger equation. The (non-relativistic) ground state solution for a particle of mass m in an infinite square-well potential between 0 and 1 is

$$\psi_{BN}(x,t) = \sqrt{2}\,\sin(\pi x)\,\exp(-i\hbar\pi^2 t/2m) \qquad (1)$$

The "$_{BN}$" here means Born-normalised, the traditional normalisation condition. The proposed STN solution to the Schrödinger equation with a constant flash frequency λ is then assumed to be

$$\psi_{STN}(x,t) = \sqrt{\lambda}\,\sqrt{2}\,\sin(\pi x)\,\exp(-i\hbar\pi^2 t/2m) \qquad (\ = \sqrt{\lambda}\,\psi_{BN}(x,t)\ ) \qquad (2)$$

The two solutions, (1) and (2), are both perfectly good solutions to the Schrödinger equation, and they have quite distinct, though related, interpretations. The square modulus of the traditional solution $\psi_{BN}(x,t)$ is interpreted as a probability density of finding a particle at x (between 0 and 1) given a measurement at time t. The square modulus of the solution $\psi_{STN}(x,t)$, on the other hand, is interpreted as a Poisson rate parameter of the occurrence of a flash at general position x (between 0 and 1) and at general time t.

Although $\psi_{STN}(x,t)$ and $\psi_{BN}(x,t)$ have different interpretations, they are consistent with each other and are indeed closely related. The proposed relationship in this case is simply

$$|\psi_{STN}(x,t)|^2 = \lambda\,|\psi_{BN}(x,t)|^2 \qquad (3)$$

where $|\psi_{BN}(x,t)|^2$ is the conditionalisation of the underlying probability function $|\psi_{STN}(x,t)|^2$, over the outcome of position measurement. This conditional relationship of $|\psi_{STN}(x,t)|^2$ and $|\psi_{BN}(x,t)|^2$ can be demonstrated as follows.

The proposed interpretation of $|\psi_{STN}(x,t)|^2$, for the single particle, is a Poisson rate parameter of a flash occurring at x (between 0 and 1) and t. Therefore, the Poisson rate parameter for a flash occurring at t, somewhere in the well, is

$$\begin{aligned} \text{PRP (flash at } t) &= \int_0^1 |\psi_{STN}(x,t)|^2\,dx \\ &= \int_0^1 \lambda\,|\psi_{BN}(x,t)|^2\,dx \quad \text{by (3)} \\ &= \lambda \qquad (4) \end{aligned}$$

Now, the conditional rate parameter of a flash occurring at x, given that it occurs at t, is obtained as follows,

$$\begin{aligned} \text{PRP (flash at } x \mid \text{flash at } t) &= \text{PRP (flash at } x \cap \text{flash at } t) / \text{PRP (flash at } t) \\ &= |\psi_{STN}(x,t)|^2 / \lambda \qquad \text{by (4)} \\ &= |\psi_{BN}(x,t)|^2 \qquad \text{by (3)} \end{aligned}$$

We thereby see that $|\psi_{BN}(x,t)|^2$ is the conditionalisation of $|\psi_{STN}(x,t)|^2$ over the outcome space of a position measurement at time t in the square-well potential.

(ii) System of N Identical Particles

We may extend the treatment of the infinite square-well potential to a system of N identical particles with a constant particle flash frequency λ in region R. The STN solution to the Schrödinger equation is defined as follows,

$$|\psi_{STN}(\underline{x}_1,\ldots,\underline{x}_N,t)|^2 = \lambda\, |\psi_{BN}(\underline{x}_1,\ldots,\underline{x}_N,t)|^2$$

It is then proposed that the space-time normalised wavefunction provides a Poisson rate parameter of the occurrence of a particle i flash at x and t, as follows,

$$PRP_i(\underline{x},t) = \int_R \delta(\underline{x}-\underline{x}_i)\, |\psi_{STN}(\underline{x}_1,\ldots,\underline{x}_N,t)|^2\, d\underline{x}_1,\ldots,d\underline{x}_N$$ where $\delta(\underline{x}-\underline{x}_i)$ is the delta function.

Defining $PRP_i(\underline{x},t)$ in this way may be seen as the STN condition. So, we may say that $\psi_{STN}(\underline{x}_1,\ldots,\underline{x}_N,t)$ satisfies the STN condition, whilst $\psi_{BN}(\underline{x}_1,\ldots,\underline{x}_N,t)$ satisfies the BN condition. Both wavefunctions are solutions to the Schrödinger equation that provide different, though consistent, information about the quantum system. In effect the STN formalism places the GRWf flash frequency λ within the wavefunction, rather than treating it as part of a supplementary dynamics.

4. Is STN-GRWf an Improvement?

GRWf proposes an amendment to the Schrödinger equation, a foundation of modern science, in order to explain the measurement problem. STN, on the other hand, avoids the foundational concern. The Schrödinger equation yields physical and non-physical solutions[8], one might perhaps interpret this distinction in terms of their epistemic value, and the standard Born normalisation condition provides a basis for determining solutions to the Schrödinger equation of epistemic value. There is, however, no reason why we should expect only one class of solutions with epistemic value (i.e. the Born normalised solutions), and it does not diminish their epistemic value if we find more. The metaphor of an Easter egg hunt comes to mind; the more we can find in the Schrödinger equation, the better. There is no foundation of modern science that requires there to be only one epistemically valuable set of solutions to the Schrödinger equation. GRWf and STN both address the measurement problem in a similar way, and since STN avoids the need to modify the Schrödinger equation, STN offers a preferable solution to the measurement problem.

[8] This is of course not uncommon; we often deem time-reversed solutions to dynamical equations as non-physical.

5. Conclusion

The objective of this paper is to remove one objection (the modification of the Schrödinger equation) to an important interpretation of quantum mechanics (GRWf). It is conjectured that interpreting the Schrödinger equation as *solely* providing probabilities associated with the outcome of measurement is overly restrictive, and that there are additional solutions to the Schrödinger equation that facilitate the interpretation of quantum mechanics. If we utilise a normalisation condition over space *and time*, the resulting solutions can be naturally interpreted as providing probabilistic descriptions of the occurrence of flashes over space and time. And there is nothing within the traditional formalism that precludes the existence of such space-time normalised solutions.

Quantum mechanics under the STN interpretation is a statistical theory of a primitive ontology of flashes, where the STN wavefunction directly provides Poisson rate parameters for their occurrence. Importantly, STN makes no amendment to the Schrödinger equation.

Quantum mechanics under STN diverges from GRWf in at least three areas. (1) GRWf is not a statistical theory, the GRWf wavefunction is interpreted in *nomological* terms. (2) The GRWf wavefunction is prescribed by a modified Schrödinger equation. (3) The evolution of the GRWf wavefunction involves stochastic collapses into a Gaussian structure that is not part of the STN interpretation. Consequently, GRWf predictions deviate from those of STN.

The STN interpretation of quantum mechanics avoids the measurement problem without "the price" of modifying the Schrödinger equation, and on the basis that the Schrödinger equation is one of the foundations of modern science, STN may be seen as an improvement on GRWf theory.[9]

[9] The proposed STN interpretation has similarities to the Sf theory of Allori et al (2008, p.377). It should be noted that the aims of the STN interpretation and Sf differ; the aim of the former, as described above, is to avoid the need to modify the Schrödinger equation within GRWf theory, whilst the aim of the latter is to elucidate possible types of theories available to primitive ontologies. Sf successfully achieves the aim of STN, via a different route, and with a different outcome. Sf avoids the modification of the Schrödinger equation by removing the GRWf wavefunction collapse associated with each flash. The result is a many worlds–GRWf hybrid in which "different non-interacting families of flashes correspond to different terms of the superposition" (Allori, 2015, p.117). In contrast, the STN maintains a single-world metaphysics.

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