Can Self-Locating Uncertainty Ground the Born Rule? An Inconsistency Argument

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn

December 25, 2025

Abstract

Vaidman's self-locating uncertainty (SLU) represents an innovative and influential approach to deriving Born rule probabilities in the Many-Worlds Interpretation (MWI) of quantum mechanics, offering a promising epistemic foundation for understanding probabilities in a deterministic multiverse. This paper examines a potential inconsistency in the SLU framework: while the concept relies on observer branching to generate multiple copies and thus enable uncertainty about branch location, such branching results in pure, deterministic local states for each copy, making the mixed reduced density matrix (RDM)—a key element in deriving amplitude-squared probabilities—unavailable. Conversely, if the mixed RDM is retained, the observer remains unentangled and spans all branches, eliminating the multiplicity essential to SLU. The derivations by McQueen and Vaidman appear to draw on both SLU (which presupposes branching) and the mixed RDM (which presupposes no branching), without fully addressing this tension. Our analysis suggests that the inconsistency poses a significant challenge to grounding the Born rule purely in SLU, independent of specific branching models. The results highlight unresolved questions in the foundations of Everettian probability and point toward the need for further refinement or supplementation of the SLU approach.

1 Introduction

The Many-Worlds Interpretation (MWI) of quantum mechanics, originally proposed by Everett [1], offers a compelling resolution to the measurement problem by eliminating wave function collapse and asserting that all possible outcomes of a quantum measurement are realized in distinct, branching histories of a unitarily evolving universal wave function. While this approach preserves the determinism and linearity of the Schrödinger equation, it faces a long-standing conceptual challenge: how to recover the Born rule—the probabilistic axiom of standard quantum mechanics—within a framework in which every outcome deterministically occurs.

Early attempts to derive probabilities in MWI, such as naive branch-counting, encounter difficulties due to the continuous and approximate nature of branching induced by environmental decoherence [7]. In response, Vaidman introduced the concept of self-locating uncertainty (SLU) as an insightful epistemic foundation for probabilities in MWI

[6]. This idea posits that, following a measurement but prior to the observer's awareness of the result, the observer experiences uncertainty about their location among the newly formed branches. Such epistemic uncertainty, it is argued, provides a natural basis for assigning credences proportional to the squared amplitudes of the branches, thereby aligning with the Born rule [4]. SLU has proven influential, inspiring further work such as Sebens and Carroll's use of the Epistemic Separability Principle (ESP-QM) to derive Born-rule-compliant credences [5] (see also [2, 3]).

However, a closer examination reveals a potential tension at the heart of the SLU program. On the one hand, meaningful self-locating uncertainty requires the observer to have branched into multiple distinct copies, each associated with a definite outcome. On the other hand, existing derivations of the Born rule within the SLU framework typically rely on a mixed reduced density matrix (RDM) to represent the local state of the observer before they learn the outcome. Yet, if the observer has already branched, each copy is in a pure, determinate local state, and no single mixed RDM can represent the perspective of a unified epistemic agent. Conversely, if the observer remains unentangled and is described by a mixed RDM, there is only one observer spanning all branches, and the question of self-location becomes ill-posed. This suggests that the very conditions that make SLU epistemically meaningful may undermine the formal tools used to derive the Born rule.

In this paper, we analyze this potential inconsistency in detail. Using the familiar example of a spin-$\frac{1}{2}$ measurement, we show how branching purifies local states and eliminates the mixed RDM as a description of the observer's epistemic situation. We then examine key steps in McQueen and Vaidman's derivation of the Born rule [4], arguing that it implicitly conflates the pre-branching mixed RDM with the post-branching epistemic perspective of a branched observer. We consider and respond to several natural objections, such as appeals to idealized pre-observation windows, approximate decoherence, and spatial analogies. Ultimately, we conclude that while SLU offers an intuitively attractive account of Everettian probability, reconciling it with a rigorous derivation of the Born rule remains an open challenge—one that highlights deeper questions about branch ontology, locality, and the nature of uncertainty in a multiverse.

The paper is structured as follows. Section 2 reviews Vaidman's SLU proposal and the associated derivation of the Born rule. Section 3 presents the inconsistency argument in detail, including ontological and empirical considerations. Section 4 addresses potential objections. Section 5 concludes with implications for the MWI and future directions in the foundations of quantum probability.

2 Vaidman's Self-Locating Uncertainty Proposal

Vaidman's SLU offers an epistemic foundation for probability in the deterministic framework of MWI. The central idea is that after a quantum measurement has occurred but before an observer becomes aware of the outcome, the observer is uncertain about *which branch* of the wave function they inhabit. This uncertainty is not about what will happen—since all outcomes deterministically occur—but about the observer's self-location among the newly created branches. From this subjective uncertainty, Vaidman proposes that rational credences should be assigned in proportion to the squared amplitudes of the branches, thereby recovering the Born rule [4, 6].

2.1 The Incoherence Problem and the Role of SLU

In MWI, a measurement does not select a single outcome; instead, the universal wave function evolves unitarily into a superposition of distinct branches, each containing an observer who records a definite outcome. This raises the so-called *incoherence problem*: if all outcomes are realized, what could it mean to assign probabilities to them? Vaidman's answer is that probability emerges not from objective chance, but from the observer's *ignorance* about their own branch location after branching has occurred but before they have registered the result. This post-measurement, pre-observation uncertainty provides a meaningful basis for probabilistic statements within an otherwise deterministic multiverse.

To illustrate, consider an observer Alice who measures the z-spin of a spin-$\frac{1}{2}$ particle initially in a superposition:

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_S + |{\downarrow_z}\rangle_S)\,|\mathrm{R}\rangle_D\,|\mathrm{R}\rangle_A\,|E_0\rangle\,, \tag{1}$$

where $|\mathrm{R}\rangle_A$ is Alice's "ready" state. After the measurement, decoherence produces an entangled state:

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_S\,|{\uparrow_z}\rangle_D\,|\mathrm{R}\rangle_A\,|E_1\rangle + |{\downarrow_z}\rangle_S\,|{\downarrow_z}\rangle_D\,|\mathrm{R}\rangle_A\,|E_2\rangle), \tag{2}$$

with $|E_1\rangle$ and $|E_2\rangle$ orthogonal. At this stage, Alice has not yet registered the outcome; she remains in the ready state $|\mathrm{R}\rangle_A$ and is therefore ignorant of which branch she is in. It is during this brief pre-observation window that self-locating uncertainty arises, and Vaidman argues that Alice should assign credence $\frac{1}{2}$ to each outcome, matching the Born rule.

2.2 Types of Self-Locating Uncertainty

McQueen and Vaidman (2019) distinguish three categories of SLU, which clarify when probabilistic questions are meaningful in MWI.

Absent self-location uncertainty occurs when the observer has not branched, so there is only one copy spanning all branches. For example, if a student records a measurement result in a notebook placed parallel to a table edge, and you, the professor, later see the notebook but have not yet opened it, your state may remain unentangled. Here, there is only one "you"; the question "Which world am I in?" is ill-posed, and probability has no epistemic footing.

Tainted self-location uncertainty arises when the observer has branched, but the uncertainty stems from ignorance of correlations that could in principle be resolved. Suppose the notebook's orientation depends on the outcome (parallel for spin-up, rotated for spin-down). Your descendants see the orientation but do not know the correlation with the spin. Each descendant knows their local orientation but not the spin result. Though the uncertainty is "tainted" by incomplete knowledge, the probability of being in the spin-up world remains $\frac{1}{3}$, given by the squared amplitude.

Clean self-location uncertainty is genuine uncertainty that persists even with complete knowledge of the universal wave function, because the observer's local experiences are identical across branches. In a variant where the entire building is rotated for the spin-down outcome, your perceptions are the same in both branches. This situation is analogous to Vaidman's sleeping-pill experiment, where an observer is split into identical rooms. Clean uncertainty provides the ideal epistemic basis for deriving probabilities, as it cannot be resolved by further physical information.

2.3 Derivation of the Born Rule via Symmetry and Locality

McQueen and Vaidman (2019) provide a derivation of the Born rule within the SLU framework, relying on symmetry and locality principles. The proof proceeds in several steps.

Step 1: Symmetric superposition. Consider a particle in a symmetric superposition over three spatially separated locations A, B, C:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|a\rangle + |b\rangle + |c\rangle) = \frac{1}{\sqrt{3}}(|1\rangle_a |0\rangle_b |0\rangle_c + |0\rangle_a |1\rangle_b |0\rangle_c + |0\rangle_a |0\rangle_b |1\rangle_c). \quad (3)$$

Identical observers at each location branch into "ready" (R) or "found" (F) states upon measurement. By symmetry, each observer must assign the same self-location probability to being in the F room. Since there are three symmetric possibilities, the probability is $\frac{1}{3}$. This generalizes to N symmetric branches, yielding probability $|\alpha|^2 = 1/N$ for equal amplitudes $\alpha = 1/\sqrt{N}$.

Step 2: Non-symmetric case via locality. The symmetry argument is extended using *local supervenience*: events in a region depend only on the quantum description of that region and its immediate vicinity. Suppose the wave packets at B and C are redirected to interfere at a distant location D, breaking the global symmetry. The local description at A remains unchanged, so by locality, the self-location probability for an observer at A must still be $\frac{1}{3}$. This reasoning applies to arbitrary modifications outside the local region, establishing that probability depends only on the local amplitude.

Steps 3–4: Rational and irrational coefficients. For rational amplitude ratios M/N, the configuration can be subdivided into N equal-amplitude sub-branches, M of which correspond to the outcome of interest. Symmetry among sub-branches implies probability $M/N = |\alpha|^2$. For irrational amplitudes, continuity arguments extend the result.

Step 5: Extension to local measurements. The spatial case can be mapped to local measurements (e.g., spin) via unitary swaps that transfer the internal degree of freedom to a spatial degree of freedom. Consistency with the spatial derivation ensures that local measurements also obey the Born rule.

This derivation grounds the probability postulate—that self-location probabilities equal squared amplitudes—in physical principles (symmetry and locality) rather than decision-theoretic axioms. It thereby strengthens Vaidman's SLU framework and provides a response to the incoherence problem. Pre-measurement agents, though certain of all outcomes, can rationally align their bets with the credences their branched descendants will hold, effectively reproducing Born-rule statistics in their behavior.

2.4 Summary

Vaidman's SLU account transforms the problem of probability in MWI from one of objective chance to one of epistemic self-location. By distinguishing absent, tainted, and clean uncertainty, it clarifies when probabilistic questions are meaningful. The derivation based on symmetry and locality offers a principled route to the Born rule, making SLU a coherent and compelling candidate for grounding probability in Everettian quantum mechanics.

3 A Potential Inconsistency in Self-Locating Uncertainty

Vaidman's SLU offers a conceptually elegant route to probability in the deterministic framework of MWI, but its formal implementation—particularly in the derivation of the Born rule—may conceal a subtle tension. To make probabilities meaningful in MWI, SLU requires observer branching to create multiple copies for uncertainty about "which branch am I in?" However, such branching renders the mixed reduced density matrix (RDM)—essential for deriving Born rule probabilities—unavailable, as each branched copy has a pure, deterministic local state with credence 1 for its outcome. Conversely, obtaining a mixed RDM for Born rule probabilities requires no branching of the observer, but then there is no multiplicity and thus no SLU to ground the probabilities epistemically. This analysis suggests that SLU and Born rule probabilities cannot coexist coherently in Vaidman's framework.

To illustrate this inconsistency, consider the spin-1/2 measurement setup from Section 2, where Alice measures the z-spin of a particle in a superposition:

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_S + |{\downarrow_z}\rangle_S)\,|\mathrm{R}\rangle_D\,|\mathrm{R}\rangle_A\,|E_0\rangle\,. \tag{4}$$

For SLU to exist, branching must generate multiple Alice copies, entangling her with the outcome:

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_S\,|{\uparrow_z}\rangle_D\,|{\uparrow_z}\rangle_A\,|E_1\rangle + |{\downarrow_z}\rangle_S\,|{\downarrow_z}\rangle_D\,|{\downarrow_z}\rangle_A\,|E_2\rangle). \tag{5}$$

Each copy (e.g., Alice-up) has a pure local RDM (e.g., $|{\uparrow_z}\rangle_D\,|{\uparrow_z}\rangle_A\,\langle{\uparrow_z}|_D\,\langle{\uparrow_z}|_A$), assigning deterministic credence 1 to its branch's outcome. The ontology of branching splits Alice into distinct, non-interacting copies, with no unified observer persisting to assign credences across branches via a mixed RDM. Thus, SLU's multiplicity precludes the mixed RDM needed for Born probabilities (e.g., 1/2).

Conversely, to obtain a mixed RDM for Born probabilities, Alice must remain unentangled post-measurement (no branching):

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle_S\,|{\uparrow_z}\rangle_D\,|\mathrm{R}\rangle_A\,|E_1\rangle + |{\downarrow_z}\rangle_S\,|{\downarrow_z}\rangle_D\,|\mathrm{R}\rangle_A\,|E_2\rangle), \tag{6}$$

yielding $\rho_{AD} = \mathrm{Tr}_E(|\Psi_1\rangle\langle\Psi_1|)$:

$$\rho_{AD} = \frac{1}{2}\,|{\uparrow_z}\rangle_D\,|\mathrm{R}\rangle_A\,\langle{\uparrow_z}|_D\,\langle\mathrm{R}|_A + \frac{1}{2}\,|{\downarrow_z}\rangle_D\,|\mathrm{R}\rangle_A\,\langle{\downarrow_z}|_D\,\langle\mathrm{R}|_A\,, \tag{7}$$

with 50% credences. However, without branching, there is only one Alice spanning branches, making self-location questions ill-posed—no multiplicity for uncertainty.

This inconsistency is compounded by two further issues, which demonstrate the necessity of a branch-local perspective and the empirical inadequacy of a mixed-RDM approach [2]. First, the ontology of branching inherently splits observers into distinct copies within non-interacting branches. For Alice in the spin-1/2 setup, her detector's measurement creates Alice+ (in the $|\uparrow_z\rangle_S$ branch) and Alice- (in the $|\downarrow_z\rangle_S$ branch), each with a pure local RDM (e.g., $|\uparrow_z\rangle_D |\uparrow_z\rangle_A \langle\uparrow_z|_D \langle\uparrow_z|_A$). Assigning credences to a unified "Alice" is logically incoherent, as no such agent persists post-branching. This ontological splitting compels a branch-local RDM interpretation, as the mixed RDM requires a non-existent unified subsystem.

Second, a mixed-RDM approach leads to inconsistencies with QM and MWI predictions. After Alice's first measurement (observing, e.g., spin-up), she occupies a definite branch. QM and MWI require that a second identical measurement on the spin-1/2 particle yields spin-up with probability 1 (repeatability). The branch-local RDM for Alice's subsystem correctly assigns credence 1 to spin-up. However, a mixed RDM, incorporating the decohered spin-down branch, remains a mixed state (as in ρ_{AD} above), yielding a credence of 1/2 for spin-up, contradicting the certain outcome required by QM and MWI. This empirical failure reinforces that the mixed RDM is inappropriate post-branching, as it violates the deterministic outcome structure within branches.

The derivation of the Born rule given by McQueen and Vaidman [4] provides a useful case study of this inconsistency. In the symmetric superposition (step 1), the authors invoke SLU: "Alex branches into a descendant who is still in room R and a descendant who is placed in the F room," [4] using symmetry among branched descendants to assign probability 1/3 to the F room. Yet, they immediately appeals to the mixed local RDM at A (**??**) as reflecting this probability, treating it as the "complete local description." This mixes incompatible elements: SLU assumes branching (entanglement), purifying local states to deterministic pure states for each descendant, while the mixed RDM assumes no branching (unentanglement) of the observer. The authors use both without noticing they cannot apply simultaneously—the branched descendants have pure states, not the mixed RDM.

In step 2 (non-symmetric case), the authors redirect wave packets from B and C to D, keeping the local RDM at A unchanged. By local supervenience, probabilities remain 1/3. Again, this relies on the mixed RDM for probabilities but presupposes SLU from the symmetric case, where branching was invoked. The redirection maintains the mixed RDM only because observers at B and C are distant and not entangled with A, but in local measurements, branching entangles the observer, purifying the state and invalidating the mixed RDM. The authors overlook that SLU (branching) precludes the mixed RDM they use for probabilities.

Steps 3-4 extend this to general cases, but the inconsistency persists: the derivation derives probabilities from mixed RDMs in non-branched (spatial) setups but claims grounding in SLU, which requires branching and purifies states. Thus, Vaidman's framework cannot coherently combine SLU and Born rule derivation.

To summarize, while SLU offers an appealing epistemic route to probabilities, the inconsistency—requiring branching while precluding the mixed RDM—combined with the ontological splitting and empirical inconsistency of a mixed-RDM approach, prevents a coherent derivation of Born rule probabilities. Whether this amounts to a fatal inconsistency or a subtlety in need of further clarification is an open question, but it highlights a crucial point of conceptual friction in the SLU program. The following section will examine possible rebuttals to this line of criticism.

4 Addressing Potential Objections

We now address several potential objections to the inconsistency argument and the argument that Vaidman's derivation overlooks the inconsistency between SLU (requiring branching) and the mixed RDM (requiring no branching).

Objection 1: An idealized pre-observation window allows the observer to remain unentangled after measurement, preserving the mixed RDM while branches exist in the environment, enabling SLU.

This objection fails to resolve the inconsistency. For SLU to be meaningful, the observer must have multiple copies across branches, implying branching and entanglement with the outcome. If the observer remains unentangled, as in state $|\Psi_1\rangle$, there is only a single observer spanning all branches, rendering self-location questions ill-posed—no multiplicity exists for uncertainty about "which branch am I in?" This situation corresponds to "absent self-location uncertainty" as defined by McQueen and Vaidman (2019), where probabilistic self-location lacks an epistemic basis. Thus, the pre-observation window preserves the mixed RDM but eliminates SLU, in accordance with the inconsistency argument.

Objection 2: In realistic scenarios, local branching via decoherence propagation provides a brief window where the observer experiences SLU before full entanglement, allowing use of the approximately mixed RDM.

Local branching does not evade the issue. Decoherence propagates gradually, but the inconsistency is ontological rather than dynamical: once branching creates distinct observer copies (even approximately), each copy resides in a definite branch with a locally pure state, precluding a unified mixed RDM for assigning probabilistic credences across branches. Moreover, in the limit of complete decoherence, the states become orthogonal, solidifying the pure local RDMs and the absence of a unified observer. The empirical inconsistency remains, as repeated measurements would incorrectly predict probabilistic outcomes if relying on the mixed RDM post-branching.

Objection 3: Approximate decoherence means branching is never exact, so an effective mixed RDM is always available, resolving the inconsistency and allowing coherent SLU with Born probabilities.

Approximation does not eliminate the inconsistency. Even in approximate decoherence, the MWI ontology posits emerging branches with nearly orthogonal states, demanding that credences be assigned from the perspective of branch-local observers with nearly pure states. Unified credences across branches become ill-posed, as there is no exact unified subsystem post-decoherence. Furthermore, relying on approximation would lead to slight deviations from certainty in repeated measurements, contradicting the exact repeatability required by QM and MWI in the decoherence limit.

Objection 4: Vaidman's derivation uses local RDMs in spatial setups where observers are pre-separated, avoiding the need for local branching and thus resolving the inconsistency for general cases via extension to local measurements.

This misses the point of the inconsistency argument. The spatial setups derive Born probabilities from mixed local RDMs without observer branching (observers are distinct from the start), but SLU requires branching to create multiplicity for uncertainty. Extending to local measurements "via consistency with spatial permutations" ignores that local cases involve splitting a unified observer, purifying states and precluding the mixed RDM. Vaidman's implicit mixing of branched descendants (for SLU) and mixed RDMs (for probabilities) in the same argument highlights the overlooked incompatibility, as spatial analogies do not capture the ontological splitting in local branching.

5 Conclusions

Vaidman's concept of SLU offers an elegant epistemic route to probability in the Many-Worlds Interpretation, but its compatibility with a rigorous derivation of the Born rule faces a significant challenge. For SLU to be meaningful, the observer must branch into distinct copies, each inhabiting a definite outcome branch with a pure local state. In contrast, derivations of the Born rule typically rely on a mixed reduced density matrix, which describes a unified, unentangled observer—a scenario in which self-location questions are ill-posed. This creates a tension: branching eliminates the mixed RDM needed for Born-rule probabilities, while retaining the mixed RDM eliminates the multiplicity required for SLU.

This inconsistency persists despite appeals to pre-observation windows, approximate decoherence, or spatial analogies. The challenge is thus not merely technical but conceptual, touching on the ontology of branching and the nature of epistemic perspectives in a multiverse. While SLU remains a valuable explanatory tool, a fully coherent derivation of the Born rule from it may require either a reformulation of the probability rule or integration with complementary approaches. Resolving this tension is essential for advancing the foundations of probability in Everettian quantum mechanics.

References

[1] Everett, H. (1957). Relative State Formulation of Quantum Mechanics. *Reviews of Modern Physics*, 29(3), 454–462.

[2] Gao, S. (2025). Global Branching and Everettian Probability: A Critique of Sebens and Carroll's Proposal. *Foundations of Physics*, 55, 78.

[3] Kent, A. (2015). Does it Make Sense to Speak of Self-Locating Uncertainty in the Universal Wave Function? Remarks on Sebens and Carroll. *Foundations of Physics*, 45 (2), 211-217.

[4] McQueen, K. J., & Vaidman, L. (2019). In Defence of the Self-Location Uncertainty Account of Probability in the Many-Worlds Interpretation. *Studies in History and Philosophy of Science Part B*, 66, 14–23.

[5] Sebens, C. T., & Carroll, S. M. (2018). Self-Locating Uncertainty and the Origin of Probability in Everettian Quantum Mechanics. *British Journal for the Philosophy of Science*, 69(1), 25–74.

[6] Vaidman, L. (1998). On Schizophrenic Experiences of the Neutron or Why We Should Believe in the Many-Worlds Interpretation of Quantum Theory. *International Studies in the Philosophy of Science*, 12(3), 245–261.

[7] Wallace, D. (2012). *The Emergent Multiverse: Quantum Theory According to the Everett Interpretation*. Oxford University Press.