Determinism and Indeterminism as Model Artefacts: Toward a Model-Invariant Ontology of Physics

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Abstract

This paper argues that the traditional opposition between determinism and indeterminism in physics is representational rather than ontological. Deterministic–stochastic dualities are available in principle, and arise in a non-contrived way in many scientifically important models. When dynamical systems admit mathematically equivalent deterministic and stochastic formulations, their observable predictions depend only on the induced structure of correlations between preparations and measurement outcomes. I use this model-equivalence to motivate a model-invariance criterion for ontological commitment, according to which only structural features that remain stable across empirically equivalent representations, and whose physical effects are invariant under such reformulations, are candidates for realism. This yields a fallibilist form of structural realism grounded in modal robustness rather than in the specifics of any given mathematical representation. Features such as conservation laws, symmetries, and causal or metric structure satisfy this criterion and can be encoded in observable relations in mathematically intelligible ways. By contrast, the localisation of modal selection—whether in initial conditions, stochastic outcomes, or informational collapse mechanisms—is not invariant under empirically equivalent reformulations and is therefore best understood as a gauge choice rather than an ontological feature. The resulting framework explains how certain long-standing problems in the foundations of physics, including the measurement problem and the perceived conflict between physical determinism and free agency, arise from the reification of representational artefacts. By distinguishing model-invariant structure from modelling conventions, I offer a realist ontology for modern physics that combines empirical openness with resistance to metaphysical overreach.

1 Introduction

Since the Scientific Revolution, it has been common to treat the contrast between determinism and indeterminism as a deep metaphysical divide in our description of the natural world. With the advent of quantum mechanics, fundamental physics has largely shifted from deterministic equations of motion to ones with intrinsically stochastic features, and it is often taken for granted that the underlying structure of the world must therefore consist of an admixture of lawlike determination and intrinsic randomness. This paper challenges that assumption. I argue that both determinism and indeterminism as commonly understood are representational artefacts of our models rather than ontologically significant features of the world.

Beginning with the Bernoulli map, I illustrate how a deterministic dynamical model can be reformulated as a stochastic one in a way that preserves the preparation–measurement correlation structure, rendering the two models empirically indistinguishable for the relevant observables. Drawing on work by Werndl (2009) and Ornstein & Weiss (1991), I argue that representational duality of this kind is not confined to this specific toy case but is a recurring feature of deterministic dynamical systems exhibiting chaotic behaviour under suitable coarse-grainings.

If attention is focused on structure that is invariant under equivalent reformulations, there is a sense in which this duality also extends to dynamical regimes not characterised by chaotic divergence, but exhibiting either apparently irreducible stochasticity or long-term dynamic stability. I indicate how this extends to quantum theory: coarse-graining necessarily leads to stochastic transition rules that may be governed by either deterministic or indeterministic completions of empirically verifiable quantum transition rules. Thus there exist fully deterministic and fully stochastic formulations of quantum mechanics that display the same general pattern of model-equivalence.

These specific cases reflect a general and well-established structural duality between deterministic and stochastic models. On the one hand, any stochastic process can be represented deterministically on an appropriately defined path space, with its stochasticity carried by a probability measure over trajectories (Kolmogorov 1950). On the other hand, deterministic models with fine-grained but inaccessible microstates routinely admit stochastic coarse-grained descriptions that capture observable behaviour while suppressing microstate detail (Van Kampen 2007; Sklar 1993). As Werndl (2011) emphasises, these two representational possibilities underwrite a broad class of observational equivalences between deterministic and indeterministic models. Taken together, they show that whether a model is presented in deterministic or indeterministic form depends on representational choices.

This raises an obvious question: if deterministic and stochastic formulations can be empirically equivalent in this way then what, if anything, distinguishes the features of a model that we should regard as ontologically significant from those that merely reflect representational choices? The fact that a model is deterministic or indeterministic cannot bear ontological weight if an empirically equivalent representation reverses that classification. To address this problem, I propose a model-invariance criterion for ontological commitment: only structural features that remain stable across empirically equivalent formulations, and whose empirically accessible physical effects are preserved under such reformulations, qualify as candidates for realism. This criterion shifts attention away from the specific mathematical form of a model and towards the modal and relational structure that persists through representational variation. On this basis, I outline a fallibilist form of structural realism that treats invariant modal structure as the primary locus of ontological commitment while classifying features such as determinism and indeterminism as representational artefacts. Unlike standard ontic structural realism (e.g. Ladyman & Ross 2007), which treats structure as ontologically exhaustive, and epistemic structural realism (e.g. Worrall 1989), which is explicitly agnostic about underlying ontology, the present view treats only model-invariant structure as eligible for ontological status.

The central claim developed in this paper is that underdetermination of modal structure is not merely epistemic in the familiar sense, but reflects a genuine gauge freedom induced by finite empirical resolution.[1] Once fine-grained state spaces are quotiented by observational equivalence classes, multiple inequivalent descriptions—deterministic or stochastic—can generate the same empirically accessible transition structure. The question of where modal selection is located within such models, whether in initial conditions, in stochastic outcomes, or in collapse-like update rules, is therefore underdetermined by the theory's empirical content. What remains invariant across these reformulations is the induced modal and relational structure of the dynamics. It is this invariant structure, rather than any particular completion beyond the empirical horizon, that I will argue to be the appropriate content of ontological commitment. In what follows, I will make this claim precise by analysing concrete cases—classical and quantum—in which deterministic and indeterministic representations are related by well-defined, structure-preserving transformations. In the quantum case, the point may be sharpened by observing that otherwise opposed realist proposals—such as primitive-ontology approaches (e.g. Maudlin 2007) and Everettian approaches

[1]This gauge freedom is a redundancy in representational structure relative to a fixed empirical domain, not a redundancy in the underlying physics in the familiar field-theoretic sense. The claim is not that any underlying dynamics is unreal, but that its specific form is underdetermined by the theory's empirical support.

(e.g. Wallace 2012)—agree on the empirically accessible transition structure, leaving the localisation of modal selection underdetermined.

2 Model-equivalence: illustrative examples

2.1 The Bernoulli map

To illustrate deterministic–stochastic model-equivalence in a maximally transparent way, consider the Bernoulli map, the discrete-time transformation

$$T(x) = 2x \bmod 1, \qquad x \in [0, 1).$$

This map doubles the value of x and shifts it back into the unit interval; it is a standard example of a chaotic dynamical system. When this transformation is applied iteratively, we get a sequence of values that depend deterministically on the choice of the initial point. If we partition the interval into two halves $[0, 1/2)$ and $[1/2, 1)$, and record at each iteration which half contains the point, we obtain a binary sequence. For a random starting point, this binary sequence is mathematically equivalent to an independent sequence of fair coin tosses.

Thus the Bernoulli map, although strictly deterministic, generates the same observable behaviour—or "symbolic dynamics"—as a genuinely stochastic (in this case, Bernoulli) process when examined through this coarse partition. For the purposes of the coarse-grained measurements, the two models are empirically indistinguishable: the deterministic map behaves stochastically, and the stochastic process can be viewed as a deterministic path-space system with randomness encoded in the initial condition. To put this in more technical language, the modal profile—that is, the set of allowed histories and their distribution—is the same, and this, common to both models, is what I will eventually argue to be a candidate for realism.

Viewing a deterministic theory through a coarse-grained partition of this kind is designed to mimic the real-world use of a scientific theory given a (necessarily) finite level of empirical access. In the example above, if we have access only to a measuring apparatus that can distinguish left and right states, we have no way of adjudicating between the deterministic and stochastic models; the empirical content is exactly the same. Nor, in this case, can we appeal to the structure of the models themselves for extra clues; both are manifestly applicable to a wide range of real-world situations, and are in fact so-used. Moreover, coarse-graining treats factual limits on preparation and measurement not as accidental deficiencies of our instruments, but as constitutive of what the theory itself counts as observable. When the coarse-grained variables coincide with the empirical domain within which the theory has been tested, the limits of that domain become the limits of the theory's observables.

Now, at the level of empirical access granted by the left/right partition, the doubling law has no dynamical content, so that it could well be regarded not as the structure of a genuine dynamical updating rule but as a bookkeeping device for encoding observational outcomes in the "initial" condition (or decoding them from that condition). If the situation being modelled were, in fact, a series of coin flips, for example, we would be unlikely, in the absence of any empirical evidence, to give much credence to the Bernoulli map as a serious candidate for a "hidden" dynamical structure controlling the way the coin lands. Even if there were some hidden deterministic dynamics controlling the outcome of each flip, its structure would probably be something else entirely. In general, there will be many such possible deterministic completions—or extensions (they need not be completely deterministic)—and there is no reason to privilege any of them unless or until we have independent empirical access to their internal variables. Nevertheless, we remain cognisant that there could be some such underlying dynamics in play, and any proposal for what this might be (including the Bernoulli map) is something that we could take seriously as soon as we had a strategy for probing it empirically.

But what of the bookkeeping? This term is not dismissive; it merely expresses the fact that quantities underdetermined by the theory need to be encoded or represented in some way. According to the deterministic representation of the coarse-grained model, the observed history is determined for the entire history by the initial condition, while in the indeterministic representation, the left/right position is decided for each step only at the point of observation. The usual view is that this distinction is epistemic: there is a matter of fact about which of these is true, it just happens to be unknown to us. Yet, this assumption is unmotivated if (i) no empirical distinction is possible even in principle given the level of access the theory itself presupposes, and (ii) the purported mechanism is unconstrained by any feature of the model that has successfully captured the observed regularities. In this specific toy model, by assumption, the observational content of the two representations is identical, including their shared modal structure. So why should we see the difference between them as anything other than representational?

In this paper, I will be arguing that, to a high level of generality, multiple scientifically plausible (and mathematically specifiable) indeterministic and deterministic completions exist for coarse-grained theories—that is, for any theory of science as it can actually be applied to the real world—and that this undercuts realism about any of these completions given the de facto context of finite empirical resolution and/or limited operational accessibility. Thus the difference between them should be seen as representational rather than ontic. This may seem counterintuitive, for example, in the case of a deterministic theory that makes concrete dynamical claims beyond a currently established empirical threshold, but it should be emphasised that nothing stands in the way of trying to push back that threshold so that these predictions can be tested—indeed, this belongs to the very nature of science. What I am arguing against is any unsupported assumption that it is reasonable to be confident in advance that these tests will affirm the predictions of the theory. For it also belongs to the nature of science that the dynamical laws that it attempts to isolate to a controllable level of precision, are only ever valid in an idealised and circumscribed sense, and that the contravening factors that will, at some scale, or in some context, render those laws inapplicable are by no means controllable beyond the empirical horizon, because they are, by definition, completely unknown there.

A tendency to overgeneralise observed patterns is a psychological fact about the human being, one without which we would perhaps not be doing science at all, and one that the remarkable success of science has only intensified. It is therefore crucial to distinguish between what a theory may legitimately include as part of its representational apparatus and what we are warranted in treating as real, in the sense of a well-founded ontic commitment.

I will argue that the dynamical structure of an empirically well-tested theory, at the coarse-grained level of its empirical support, is conceptually intelligible, mathematically describable, and genuinely deserving of ontic status. By contrast, any further dynamical structure posited beyond the empirically accessible level reflects a representational choice—one that, in the case of Hamiltonian dynamics, can be understood as a gauge choice with respect to preservation of phase-space volume in the Liouville sense: any reorganisation of microstates that preserves the induced Liouville measure on empirically distinguishable cells leaves the observable transition structure unchanged. This leaves open the question of which such choices might fruitfully be developed into working hypotheses about as-yet unprobed dynamics, while preserving a clear distinction between established structure and speculative extension.

2.2 Effective Dynamics and Gauge Invariance

To begin to generalise the above example, consider the case where we can improve our observational resolution of the Bernoulli map (imagining that it is applied as a model to some real-world situation) to an arbitrary finite extent. Suppose we can increase the resolution of our apparatus to gain empirical access to a more precise, but not exact, value of the initial condition and/or the subsequent values instantiated by the system. Our epistemic situation is not now qualitatively different: at any finite resolution the deterministic model remains empirically indistinguishable from a suitable stochastic model induced by the refined partition and governed by a new symbolic dynamics. Our improved knowledge may allow us to predict certain aspects of the coarse-grained behaviour for a finite number of forward steps, but the transitions between kinematically distinguishable positions again follow a well-defined, realistic and non-trivial stochastic process. For example, if we divide the unit interval into cells of length $1/2^n$ and possess an apparatus that allows us to distinguish which of these cells contains the value of x at any given step, then we will now be able to predict the left/right position for n steps of the process (note that the relationship of length scale to predictive horizon illustrates the exponential divergence typical of chaotic systems). Alternatively, for these n steps, we retain some information about which of the 2^n refined cells the system can occupy. Beyond that horizon, the past is completely screened off. Even in the stochastic formulation, the dyadic dynamic plays a mathematically specifiable geometric role that is encoded exactly in the transition probabilities. The kinematically distinguishable states—namely, the distinguishable positions defined by the refined partition—correspond to initial sequences of length n in the binary expansion of x, and the induced dynamics is a Markov process on these configurational states.

If, perhaps more realistically, our partition does not fit neatly into the structure governed by the underlying dynamic (we could, for example, take cells of length $1/3$) the induced dynamics will not, in general, be Markovian, but by using the aforementioned states as a basis, we can approximate it by a Markov process to an arbitrary degree of precision (in the sense of ε-convergence: see Werndl (2009), Ornstein and Weiss (1991)). More generally, whenever preparational states do not encode sufficient information to fully characterise future evolution—either because the partition is not dynamically aligned or because states are represented by distributions over cells—the induced one-step dynamics will typically be non-Markovian. In what follows, therefore, the use of dynamically aligned cells should be understood as a simplifying construction introduced for illustrative clarity, rather than as a feature to be expected of a fully general representational framework.

It is important to recall that the Bernoulli map itself does make claims about the underlying dynamics beyond the empirical horizon set by any finite precision. The coarse-grained model and its stochastic counterpart, by contrast, are both indifferent about these claims, as well they might be, for nothing in the empirical record can speak for or against them. All bets are off about what happens there until we can find a way to probe it. This is simply how scientific models work.

The upshot is that we are justified in taking what is invariant across these models as ontic, while the deeper claims of the fine-grained theory are at most provisional. Yet this does not rule out a discussion of the respective virtues of different representations.

What is important about the coarse-grained theory is that its symbolic dynamics reproduces the exact evolution at all scales to which we have empirical access, while remaining agnostic about microstructural behaviour beyond that empirical horizon. Within each equivalence class of empirically indistinguishable microstates, it is possible to permute or reshuffle the microstates deterministically or indeterministically—in a number of distinct senses—while preserving the induced symbolic dynamics. This is a genuine gauge freedom arising from quotienting the fine-grained phase space by observational equivalence classes; we will explore in the next section what phase-space structure it may or may not preserve. What is invariant under this gauge freedom is the entire empirical content of the theory, together with its modal structure, and as we will see, the invariants of the dynamic structure also survive at the coarse-grained level in a mathematically precise sense.

We should, however, be clear about what the empirically relevant states actually are here. These will not, in general, be simple equivalence classes of empirically indistinguishable positions (Liouville or kinematic states), but equivalence classes of such positions together with probability measures over their possible forward histories, induced by the observable dynamics (empirical states). As we have seen, the circumstance in which transitions are automatically Markovian on coarse-grained outcomes is a somewhat artificial one. Nevertheless, at sufficiently high observational resolution, trajectory-like transitions may be observable over finite scales, revealing partitions on which the transition structure simplifies.

In practice, the coarse-grained theory tracks possible evolutions by implicitly presupposing how sub-cell structure would evolve were it accessible—effectively retaining notional positional information about fibres within partition cells—whereas the corresponding stochastic theory generically provides a family of non-Markovian finite-history transitions defined over distributions of Liouville states, yielding measurable correlations across forward histories. The circumstances under which such evolutions admit a Markovian approximation, possibly on an enlarged phase space, therefore constitute non-trivial empirical probes of the invariant dynamics. This opens up multiple, methodologically distinct routes for investigating how far established regularities persist under extension of the empirical domain.

Whether deterministic or indeterministic formulations are more appropriate may thus depend on the empirical strategy adopted: whether one seeks to probe deeper levels of the dynamics by increasing positional resolution, or by analysing multi-step correlations over longer histories. In both cases, what is being tested is whether the observed dynamical structure persists at smaller scales, or equivalently, across longer temporal horizons. The fine-grained map—which embodies the simplifying assumption that the dynamics generalises uniformly across scales—therefore retains a legitimate representational and constructive role. What must be kept in view, however, is that this assumption will almost certainly fail beyond some level, so that any claims made about unprobed regimes remain hypothetical rather than ontologically grounded. The simplicity of the present toy model makes this particularly transparent: while the fine-grained theory exhibits formal simplicity and scale continuity, these features are not themselves ontological. By contrast, the coarse-grained and stochastic formulations bring into focus what is invariant across empirically equivalent representations—which the distinction between determinism and indeterminism does not appear to be.

2.3 Reversibility

Lest one suspect that the preceding discussion depends upon the inherently irreversible character of the Bernoulli map, it is natural to turn next to its simplest invertible generalisation, the Baker's map. This reversible system also provides a first indication of how the gauge freedom introduced in the previous section manifests in the more general setting of symplectic invariance and Hamiltonian flows.

The Baker's map $B\colon [0,1) \times [0,1) \to [0,1) \times [0,1)$ is defined by[2]

$$B(x,y) = \begin{cases} (2x,\ y/2), & 0 \le x < 1/2, \\ (2x-1,\ (y+1)/2), & 1/2 \le x < 1. \end{cases}$$

This map is area-preserving and, because it is invertible, every point on a trajectory encodes the full past and future behaviour. The projection onto the x-coordinate reproduces the Bernoulli map, but the encoded trajectory information that previously appeared to be carried solely by the initial condition can now be localised at any point, or even distributed across the entire trajectory. This is, of course, a general feature of reversible dynamical systems, yet it underlines the fact that trajectory information underdetermined by the dynamics cannot be given a privileged temporal location even in the deterministic case. As before, coarse-graining the system by dividing the kinematic state space into finite cells introduces a gauge freedom: microstates (and, if we wish, microdynamics) can be reorganised in any way that preserves the modalities of the coarse-grained theory. The resulting coarse-grained dynamics is mathematically equivalent to a suitable induced stochastic process (a two-sided Markov shift or one that may be so approximated in the sense of ε-congruence), and the greatly expanded scope of how one may represent the redistributed trajectory information makes the deterministic–stochastic distinction appear even more clearly as a representational choice rather than an ontic feature at the coarse-grained level.

It is worth emphasising that the stretching–and–stacking mechanism embodied in the Baker's map is not a mere mathematical curiosity but a simplified representative of a dynamical pattern that occurs widely in physical systems. Whenever a flow exhibits local expansion in one direction together with contraction or folding in another—as in return maps for chaotic Hamiltonian systems, Poincaré sections of geodesic flows, the mode structure of optical cavities, or even the mixing behaviour of certain fluid flows—the induced discrete map shares these qualitative features, along with the possibility of empirically equivalent coarse-grained and stochastic formulations.

The connection to Hamiltonian dynamics can be made clear as follows. The area-preserving property of the Baker's map can be seen as a special case

[2]The dyadic rationals $x = k/2^n$ have non-unique binary expansions, and are sometimes excluded from the domain. As they form a zero-measure set, this does not affect anything either way at the level of the coarse-grained, gauge invariant theory, whose partitions are of finite measure. The map as defined here is bijective.

of symplectic invariance; in the language of differential forms, we can define a Hamiltonian flow preserving the non-degenerate closed 2-form $\omega = dx \wedge dy$. A convenient choice of Hamiltonian is the area: $H(x, y) = xy$; the associated Hamiltonian vector field generates uniform expansion in one coordinate and contraction in the other. Given the discrete-time nature of the map, the forward difference takes the place of the derivative, so that the Hamiltonian flow equation is replaced by

$$\Delta z_n = J\nabla H(z_n),$$

where $z_n = (x_n, y_n)$, J is the canonical symplectic matrix, and $\Delta z_n = z_{n+1} - z_n$. In this sense the Baker's map may be viewed as a piecewise-linear, discontinuous analogue of a symplectic integrator: each branch preserves the symplectic form exactly and hence preserves phase-space volume in the Liouville sense, which here is again the area. There is a slight change of perspective here, in that we are now treating x and y as canonical coordinates rather than configuration space ones, but it is a natural interpretation in this setting.

Viewed through the finite partition, reversibility in the sense of bijective transitions is lost, because the mapping between Liouville states is no longer one to one, but the stochastic transitions between these kinematic states continues to encode the invariant dynamics in a way that respects symplectic invariance. If we fully discretise the state space, taking cells of length $1/2^n$ in both the x and y directions, and using the midpoints x_i, y_j, $i, j = 1 \ldots n$ of the resulting faces as the discrete coordinates, then the Hamiltonian may be taken as $H_{i,j} = x_i y_j$, and this induces a discrete flow on cells $X_H = (x_i, -y_j)$ with exactly the same symbolic form as before. The passage from the continuous to the discrete theory may be understood abstractly as a passage from differential forms to cochains, preserving the relevant algebraic structure but replacing pointwise relations by cell-level ones.

Specifically, the Hamiltonian vector field defines, for each cell, a pattern of directed fluxes across its boundaries. Because the discrete Hamiltonian still represents a conserved area, these fluxes satisfy a discrete divergence-free condition: the total outgoing flux from any cell equals the total incoming flux. In this sense the discrete flow preserves a discrete analogue of the Liouville measure, corresponding to the pushforward of phase-space volume to the partition. In the present construction, this is just a uniform counting measure on cells.

When viewed at the level of individual microstates, the dynamics is no longer reversible: multiple fine-grained states within a cell are mapped into multiple successor cells, and distinct microhistories become observationally indistinguishable. However, when the dynamics is projected onto the finite partition, the induced evolution is a stochastic process whose transition probabilities exactly encode the discrete Hamiltonian fluxes. Each transition matrix is bistochastic: probabilities sum to unity both row-wise and column-wise, reflecting the underlying preservation of phase-space volume. This bistochasticity is not a peculiarity of the Baker's map, but the discrete signature of Liouville invariance under coarse-graining.

Crucially, the stochastic description is not an ad hoc replacement of the Hamiltonian dynamics, but a coarse-grained representation of the model-invariant structures. The non-reversibility of the effective dynamics is a representational feature, not a claim about the underlying ontology: it leaves open whether model-invariant features are governed by an underlying reversible dynamics. Symplectic invariance survives the coarse-graining not as reversibility at the level of trajectories, but as invariance of measures and conserved flux structure. What appears as stochasticity is simply one representational approach to the bookkeeping required once micro-information within cells is quotiented out.

This example illustrates a general pattern. For arbitrary Hamiltonian flows, coarse-graining by a finite partition replaces exact symplectic diffeomorphisms with stochastic dynamics on cells. Yet the defining invariant content of Hamiltonian mechanics survives: Liouville invariance is realised as conservation of probability flow, and symplectic structure remains in the form of constraints on admissible transition matrices. Deterministic and stochastic descriptions are thus not rival dynamical hypotheses, but alternative representations of the same invariant flow at the available level of resolution. There may be fundamental obstructions—which need not be epistemic in nature—to resolving the partition beyond a given level, in which case the fine-grained flow becomes simply another method of bookkeeping.

In the concrete example of the Baker's map, the underlying expanding and contracting dynamics were specified by stipulating that the same structure holds uniformly at all length scales. This assumption can be read in two ways that are not intrinsically incompatible, but which must be kept conceptually distinct: as a substantive hypothesis about the microdynamics of the system, and as a particularly economical way of organising measures over possible forward histories. The former is hypothetical and must ultimately be tested to carry any weight; the latter reflects a representational choice whose adequacy may be evaluated only relative to the resolution at which the system is probed.

More generally, many distinct assumptions about what occurs below the level of empirical access—whether deterministic, stochastic, or hybrid—may give rise to the same observable transition statistics. These alternatives may be understood as gauge-related representations of a single empirically fixed dynamical structure, corresponding to different choices of a Liouville potential whose associated probability current is (at least approximately) closed at the level of partition cells.

What remains invariant across such gauge choices is not a particular micro-dynamical law, but a finite-resolution analogue of Liouville invariance: an observed conservation of probability flux across partition cells. At this level, the empirically relevant content of the dynamics may be represented by a coarse-grained probability current, defined only relative to the chosen partition and constrained by flux balance between cells. This structure may persist even when the underlying evolution is non-Markovian, and/or when no exact Hamiltonian description is available at finer scales.

Deterministic flows that preserve symplectic structure exactly, stochastic dynamics that realise the same transition statistics, and mixed constructions that interpolate between them can thus be regarded as gauge-equivalent completions of the same coarse-grained theory. The distinction between determinism and indeterminism does not correspond to a difference in invariant dynamical content, but to alternative ways of extending that content beyond the empirically established domain. In this context, the stochastic formulation can be interpreted not as postulating a rival dynamics, but as summarising what is invariant across a family of empirically equivalent representations.

2.4 Quantum two-level system

Model invariance in quantum mechanics may be illustrated by considering a two-level system (qubit) evolving under a fixed Hamiltonian (e.g. a uniform magnetic field) and subjected to a coarse-grained measurement described by a positive operator-valued measure (POVM). The aim at this stage is not to revisit foundational questions about quantum measurement, but to show explicitly in this setting how a deterministic dynamics induces a non-trivial stochastic dynamics once one restricts attention to empirically accessible coarse-grained states.

The state of a two-level quantum system is represented by a density operator

$$\rho \in \mathcal{D}(\mathbb{C}^2), \qquad \rho \geq 0, \qquad \operatorname{Tr} \rho = 1.$$

Any such state admits a Bloch representation

$$\rho = \tfrac{1}{2}(I + \mathbf{r}\cdot\boldsymbol{\sigma}), \qquad \mathbf{r}\in\mathbb{R}^3, \qquad \|\mathbf{r}\| \le 1,$$

with pure states corresponding to points on the unit sphere $\|\mathbf{r}\| = 1$ and mixed states to interior points. The Bloch sphere thus provides a convenient geometric representation of the quantum state space, though it should be emphasised that it is a space of equivalence classes of preparation procedures, not a classical phase space of simultaneously measurable quantities.

Let the system evolve under a time-independent Hamiltonian

$$H = \tfrac{\hbar\omega}{2}\,\mathbf{n}\cdot\boldsymbol{\sigma},$$

where $\mathbf{n}$ is a fixed unit vector. The unitary evolution

$$\rho(t) = U(t)\rho(0)U^{\dagger}(t), \qquad U(t) = e^{-iHt/\hbar},$$

induces a rigid rotation or precession of the Bloch vector $\mathbf{r}$ about the axis $\mathbf{n}$ with angular velocity ω. The dynamics of the underlying model is therefore deterministic and invertible, and may be viewed as a smooth Hamiltonian flow on the state space.

In this model, the density matrices evolve in closed orbits under the unitary dynamics, and it is tempting to identify these orbits with "trajectories", especially since density matrices provide a natural representation of the correlation structure between preparations and observables that I have argued constitutes the appropriate content of ontic commitment. It is important, however, to remember that quantum theory does not provide empirical access to anything corresponding to the classical notion of a trajectory. There are no well-defined simultaneous values of non-commuting observables, and no operational access to a continuous history of intermediate states. Any attempt to probe the state at intermediate times requires a state update that interrupts the unitary evolution and thereby defines a new dynamical situation. This is not a limitation arising from experimental imprecision, but a structural feature of the quantum description itself.

To make this explicit, consider a two-outcome POVM defined by the effects

$$E_{\pm} = \tfrac{1}{2}(I \pm \eta\,\sigma_z), \qquad 0 < \eta \le 1.$$

For any state ρ, the probabilities of the two outcomes are

$$p(\pm \mid \rho) = \mathrm{Tr}(E_{\pm}\rho) = \tfrac{1}{2}(1 \pm \eta\, r_z),$$

where r_z is the z-component of the Bloch vector. The parameter η controls the sharpness of the measurement: $\eta = 1$ corresponds to a projective measurement of σ_z, while smaller values of η define increasingly coarse observational partitions of the state space. In this sense, η plays the role of a resolution parameter directly analogous to the scale of a coarse partition in the classical phase-space examples.

This POVM defines a many-to-one observational map from the space of density operators to two outcome probabilities. States that differ in their transverse components (r_x, r_y) but agree in r_z are empirically indistinguishable at this level of description.

If one now insists on a closed description at the level of these coarse-grained outcomes—i.e. on a dynamics defined purely in terms of the POVM statistics—then the underlying unitary evolution induces a stochastic map between the two outcome "cells". Concretely, choosing any reasonable measurement instrument compatible with the effects $E_\pm$, one obtains transition probabilities of the form

$$T_{ji} = \mathrm{Tr}\Big(E_j\, U(\Delta t)\, \tilde{\rho}_i\, U^\dagger(\Delta t)\Big)\,,$$

where $\tilde{\rho}_i$ denotes a representative state associated with outcome i. Examples include $(0, 0, r_z)$ in the minimal Kraus realisation, or $(\mu r_x, \mu r_y, r_z)$, with $\mu = 1 - \lambda^2$, in the minimally disturbing Lüders realisation. For Hamiltonians generating rotations that mix the z-axis with other directions, the resulting transition matrix is non-degenerate: each coarse state evolves into a probability distribution over both outcomes.

The key point is that this stochasticity is not added by hand. It arises because the coarse-graining discards information about the state that is dynamically relevant within a fine-grained representation of the unitary evolution but empirically inaccessible relative to the observational restrictions imposed by the chosen POVM. At the formal level, a POVM does not fix a unique state-update rule: many distinct quantum instruments realise the same POVM effects and hence the same observable statistics. The different state-update rules—commonly represented by means of Kraus operators—fix, purely as a matter of bookkeeping, the degrees of freedom of the post-POVM density matrices that lie outside the observational content of the POVM, such as their transverse Bloch components. While different choices of update rule would, in general, lead to different predictions for finer-grained measurements, they are empirically equivalent so long as attention is restricted to the same coarse-grained observational level. This freedom in the underlying realisation, devoid of observable consequence at that level, is the quantum analogue of the gauge freedom relating empirically indistinguishable microstates that we encountered in the classical case.

Once attention is restricted in this way, distinct state evolutions that agree on all statistics defined by the chosen POVM lead to identical empirical data, and any closed description at that observational level must therefore take a stochastic form.

This quantum example mirrors, in a structurally precise way, the classical situations analysed earlier. In both cases:

- one may represent the dynamics deterministically within a sufficiently fine-grained model;
- empirical access is mediated by a many-to-one observational map (phase-space partition or POVM);
- closure of the description at the observational level yields an induced stochastic dynamics;
- there is a non-trivial freedom in how states are represented within a coarse cell (Liouville-gauge freedom in the classical case, instrument or Kraus freedom in the quantum case), without any empirical consequences at that observational level. Note that the choice of Kraus representation is also a gauge freedom corresponding to unitary changes of basis in an auxiliary space.

The quantum case nevertheless reformulates the general lesson by making the role of observation explicit and dynamically relevant. The adoption of a POVM does not merely specify which aspects of the state are recorded; it also constrains the state update, and hence the character of the effective dynamics at that level. The resulting description consists of unitary evolution—understood as part of a fine-grained representation—interspersed with irreducibly probabilistic transitions associated with observation. At the chosen observational resolution, what is empirically accessible is therefore a stochastic process over coarse-grained outcomes, rather than a deterministic history supplemented by incomplete information.

This makes the situation more subtle than in the classical case, where the stochastic dynamics of the coarse-grained model can (optionally) be thought of as underpinned by an underspecified local deterministic dynamics. In the quantum case, the stochastic and deterministic formulations can no longer be thought of as related by a gauge symmetry in the strict sense, owing to the irreducibly stochastic element introduced by POVM interventions.

Note that one could preserve a closer parallel with the classical case by coarse-graining a suitable quantum phase-space representation and deferring an explicit consideration of measurement. In that setting, it would still be possible to speak of a gauge symmetry linking empirically indistinguishable representations of the dynamics. There would, however, still be no local symplectic geometry, since the non-commutativity of quantum observables precludes the existence of jointly definable local phase-space coordinates.

Coarse-graining induced by POVM measurement is only one of many possible coarse-graining types, but it is an instructive one for our purposes. From the perspective of model invariance, what emerges in this context is not that deterministic and stochastic descriptions are freely interchangeable, but that the presence of irreducible stochasticity is intrinsic to the theory and does not, by itself, warrant any particular ontological reading. Though this stochasticity constrains the form of admissible descriptions, it does not fix the structure of the theory at levels lying beyond the observational horizon.

Once a particular observational coarse-graining is fixed, stochastic elements enter through the state-update map and cannot be eliminated by retaining additional bookkeeping information about degrees of freedom lying outside the observational content of the model. What remains model-relative is the representation of this unobserved structure and its incorporation into the effective dynamics. As in the classical case, the distinction between deterministic and stochastic formulations of the evolution between observational interventions reflects the level at which the theory is taken to encode empirically meaningful structure. A modest stance that remains agnostic about structure beyond the empirical horizon therefore retains the possibility of a mathematically well-defined structural description of the invariant empirical facts.

3 General applications

3.1 Continuous time classical Hamiltonian dynamics

Having looked at some simple examples of how empirical model-equivalence can be realised as a gauge or equivalence-class freedom, we need to consider how generally this idea can be applied. In the classical case, the most important existing results are those of Ornstein and Weiss (1991) and Werndl (2009, 2011). In what follows, the discussion will be restricted to classical systems whose empirically accessible dynamics exhibits a Hamiltonian-type structure, in the sense of being compatible with a measure-preserving phase-space description. This restriction concerns the structural features of the empirical dynamics itself, and does not privilege any particular deterministic or stochastic formulation of that structure.

Ornstein and Weiss establish at the level of rigorous ergodic theory that the distinction between deterministic and stochastic descriptions is not invariant under measure-preserving isomorphism. In particular, they show that a wide class of deterministic measure-preserving transformations—including paradigmatic examples arising from classical dynamics—are measure-theoretically isomorphic to Bernoulli shifts, and hence to i.i.d. stochastic processes characterised entirely by their Kolmogorov–Sinai entropy. More precisely, these results concern ergodic, measure-preserving transformations and establish equivalence up to measure-theoretic isomorphism, so that all statistics associated with finite measurable partitions—and hence all finite-resolution observations—are preserved, even though finer-grained structural properties need not be. From the standpoint of all finite-resolution observations, such systems are therefore empirically indistinguishable from genuinely stochastic processes, despite being generated by strictly deterministic dynamics. These results demonstrate that, even in exact classical mechanics, determinism is not a structural property preserved under empirically equivalent reformulations, but rather a feature of a particular representational choice.

It is worth noting that the central results of Ornstein and Weiss are formulated for discrete-time measure-preserving transformations, whereas classical mechanics is most naturally presented in terms of continuous-time Hamiltonian flows. This distinction, however, does not limit their relevance. Empirical access to a continuous-time system is necessarily mediated by finite-resolution, discrete observations—whether through stroboscopic sampling, Poincaré sections, or symbolic dynamics induced by finite partitions—and it is at this level that observational equivalence is assessed. From the standpoint of the empirical horizon, continuous-time dynamics and their discrete-time representations stand or fall together: if a continuous flow admits a discrete-time description whose finite-resolution statistics are indistinguishable from those of a stochastic process, then no empirically meaningful distinction between deterministic and stochastic dynamics survives. The discrete-time setting of the Ornstein–Weiss results therefore captures precisely the structure that is relevant for the present discussion, rather than representing a limitation of scope.

Werndl's subsequent analysis sharpens and extends this conclusion by bringing it explicitly into contact with the modelling practices of the sciences. She shows that deterministic and stochastic models—both taken to be "science-like" in the sense of employing familiar dynamical structures—can be observationally equivalent at every finite observation level, relative to a given choice of coarse-graining. Formally, her notion of observational equivalence is defined relative to a fixed observation function (or finite-valued partition), and requires that the induced stochastic processes over observable histories agree at all observation levels. Her results make precise the sense in which no amount of finite-resolution empirical data suffices to distinguish between deterministic and indeterministic descriptions, even when one restricts attention to models that are independently well-motivated within classical physics. At the same time, Werndl is careful to delimit the scope of her claims, emphasising that observational equivalence alone does not warrant the unrestricted conclusion that determinism and indeterminism are always mere modelling artefacts. This qualification is developed explicitly in her later philosophical analysis of the choice between deterministic and indeterministic models, which appeals to indirect evidential support from wider theoretical frameworks rather than observational equivalence alone (Werndl 2013).

In the present context, the focus is somewhat different. The central question is not whether the competing formulations qualify as "science-like," let alone candidates for empirical refinement, but whether they preserve the same dynamical and probabilistic structure at the level of empirical accessibility. Where such structure is invariant, it supports an ontic reading of the corresponding theoretical description at that level, even while remaining agnostic about the character of any underlying microdynamics, which may admit refinements of an unspecified—or presently unimagined—kind. As we have noted, these competing descriptions may differ substantially in their modelling virtues, and such differences may well matter when attempting to extend the empirical horizon or to probe for hitherto undiscovered structure. Within the established empirical domain, however, there is a genuine sense in which they stand on an equal footing: despite their differing formal and interpretative commitments, they are—in various precisely definable ways that themselves carry a rich mathematical structure—gauge-equivalent representations of the same empirical facts.

Taken together, these results provide a rigorous foundation for the idea that, in classical mechanics, the deterministic or stochastic character of a model need not reflect any invariant feature of the empirically accessible dynamics. They also clarify the sense in which such distinctions may persist as perfectly legitimate modelling choices, while nonetheless failing to carry ontological weight within the empirical horizon. The task of the present section is to situate these insights within the more general framework developed above, and to assess how far they support a genuinely model-invariant understanding of classical dynamics once finite resolution, coarse-graining, and predictive horizons are taken into account.

Werndl is careful to emphasise that not every classical model admits a deterministic–stochastic duality of the specific kind she analyses. In particular, there exist systems for which no stochastic formulation can be shown to be observationally equivalent, at every observation level, to a deterministic one, and vice versa. This caution is well-founded: the existence of a clean duality between deterministic and stochastic descriptions is a substantive property of particular classes of systems, not a universal feature of classical dynamics. Nevertheless, the examples she identifies as falling outside her strongest equivalence results remain instructive for present purposes, as there is an important sense in which an equivalence class of possible deterministic and stochastic formulations still captures the dynamical content accessible at any observable level.

Consider first purely stochastic models that do not arise, even implicitly, from an underlying deterministic dynamics. Such models always admit a formal deterministic completion in the Kolmogorov sense, by passing to an enlarged state space encoding entire histories. From a purely mathematical standpoint, this establishes that stochasticity can always be represented as determinism at a higher descriptive level. Empirically, this move effects no obvious refinement of observable structure. From the point of view of articulating what is invariant across permissible models, this hardly matters: the stochastic behaviour of the original model remains an adequate summary of the observed transitions, and any possible empirical support for this or any other deterministic completion would only exist relative to a different empirical level. What is retained across all empirically equivalent formulations can thus reasonably be taken—in the absence of any independent empirical handle on the sub-dynamics—to be the stochastic transition structure alone.

At the opposite end of the spectrum are non-dispersive or weakly dispersive dynamical regimes, in which uncertainty in initial conditions fails to amplify significantly over timescales of interest. In such cases, coarse-grained stochastic descriptions—where they exist at all—exhibit highly concentrated transition probabilities, and empirical evolution is well approximated by a deterministic flow. To describe such regimes as deterministic is not, however, to say that they modulate between their own modal possibilities, but almost the opposite: that the choice between decisively different realised histories lies outside the regime itself. Within the regime, what we observe is the existence of a long predictive horizon: over the relevant range of resolutions and timescales, initially nearby states remain sufficiently correlated that future behaviour can be robustly inferred from present data.

Precisely because this determinism is regime-relative, the decisive modal facts—namely, which of several macroscopically distinct trajectories is realised—thus depend on features of the dynamics that lie outside the descriptive remit of the regime in which determinism holds approximately. Within the regime itself, such distinctions are neither resolvable nor dynamically consequential, and therefore play no significant role in the empirically accessible evolution. At the same time, deterministic–stochastic duality is not eliminated: at any observational level, one cannot rule out trivial stochastic variations that remain confined within the predictive envelope of the regime. What distinguishes these cases is not the absence of stochastic alternatives, but the fact that all such alternatives are dynamically equivalent over the timescales and resolutions of interest. In this sense, the regime may reasonably be described as approximately deterministic in a predictive sense, even though the ultimate modality of the dynamics that selects between individual trajectories of interest remains unspecified.

Between these limiting cases lie the examples analysed by Ornstein and Weiss and by Werndl, in which deterministic and stochastic descriptions coexist as fully equivalent representations of the same empirical structure. What distinguishes these systems is not that they alone exhibit model equivalence, but that the equivalence takes a particularly symmetric form: deterministic and stochastic formulations can be placed in direct correspondence without privileging either description at the level of observation. It should be noted that none of these representational alternatives are guaranteed to be of use beyond the empirical level with respect to which they are defined. They may nonetheless be methodologically valuable, insofar as they systematically explore the space of dynamical extensions compatible with the observed structure.

Seen in this light, Werndl's warning does not mark a boundary beyond which the present analysis ceases to apply, since the claim advanced here is not that every classical system admits a deterministic–stochastic duality, but that every empirically adequate classical description belongs to an equivalence class of models that agree on all empirically accessible structure. Across the cases considered—irreducibly stochastic models, effectively deterministic regimes, and systems admitting a meaningful duality—the common feature is that distinctions in modal character track features of the representation rather than invariant features of the empirically accessible dynamics. What varies from case to case is not whether model equivalence occurs, but how it manifests: as stochastic irreducibility, as effective determinism defined by a long predictive horizon, or, in the most revealing instances, as a genuine duality between deterministic and stochastic formulations—precisely those cases in which the most substantive choices arise about which representation is most useful for further analysis. In every case, however, the invariant empirical structure underdetermines the ultimate modality of the dynamics, leaving open whether the decisive causal transitions are deterministic or stochastic in origin. Nevertheless, the above classification into characteristically stochastic, predictively stable, or dual regimes is robust; any future model refinement or conditioning of modal possibilities would apply to new empirical regimes, not the ones so-classified under the model invariance criterion.

This convergence suggests a more general lesson. Even where deterministic–stochastic duality in Werndl's strict sense fails to obtain, the empirically supported structure of a classical model remains stable across a range of admissible redescriptions, while modal attributions shift with representational choices. It is this pattern—rather than the existence of any particular duality—that motivates the more general model-invariance perspective developed here.

At the level of the idealised, dynamically aligned dyadic partitions of the Baker's map, it is easy to see how these representational choices correspond to gauge transformations of the Liouville potential, corresponding to probability currents that are at least approximately closed over complete cells. The situation is more complicated if we allow for the possibility of more realistic states, represented as densities over possibly overlapping or fuzzy partition cells. The POVM-based partitions illustrated in the previous section provide some clues as to how we might deal with such a situation, but we should note that Liouville invariance remains a unifying principle in the classical situation at least.

What is common to all empirically equivalent classical descriptions is that empirical access constrains only a finite-resolution dynamical structure, while leaving open multiple, representationally distinct ways of realising that structure at sub-empirical scales. Gauge equivalence, in this setting, consists in transformations between such realisations that preserve the empirically fixed probability current defined over coarse-grained states.

At the most restrictive level, one may consider exact conservation of the probability current induced on a partition. Representations related in this way are related by gauge transformations of the Liouville potential by an exactly closed form, leading to distinct Hamiltonian representations that preserve the empirical current observed between coarse states.

More generally, one may relax the requirement of exact current conservation and allow for approximately current-preserving gauges, in which the empirical probability current is conserved only up to the resolution and tolerance of the observational scheme. Such situations arise naturally when empirical states are represented by densities over overlapping or fuzzy coarse observables. In this setting, the induced dynamics is generically non-Markovian, since there is a choice of instrument associated with any realisation of a coarse observation—closely paralleling the Kraus freedom noted in the quantum case—and different realisations typically preserve information about the underlying densities that is not captured by the observational outcomes alone. From the empirical point of view, memory effects therefore distribute probability flow over extended histories rather than between sharply defined states.

In such cases, the underlying microdynamics need not preserve symplectic structure pointwise, and need not admit any exact Hamiltonian representation at all, even though the empirically accessible evolution continues to satisfy a finite-resolution analogue of Liouville invariance. There is also the possibility of a Hamiltonian description on an enlarged phase-space, incorporating the effects of previously neglected degrees of freedom.

At the most permissive level lies what may be termed an empirical gauge, under which only those features fixed by observed transition statistics are held invariant. Here, distinctions between deterministic and stochastic realisations, between Hamiltonian and non-Hamiltonian generators, or between smooth and diffusive sub-dynamics are entirely representational. All such constructions are equivalent insofar as they realise the same empirical probability current, and hence the same observable correlations over histories. This is the level at which deterministic and indeterministic completions can be seen as gauge choices rather than competing claims about empirically accessible dynamics.

These levels form a nested hierarchy of gauge freedoms, reflecting progressively weaker commitments about sub-empirical structure relative to a fixed empirical resolution, while preserving the same invariant dynamical content at finite resolution. What model-invariance licenses as ontologically significant is not any particular representative within this hierarchy, but the empirical probability current itself, understood as a resolution-dependent yet structurally robust feature of the observed dynamics.

Framed in this way, the present analysis naturally aligns with a programme of "reverse physics" advocated by Carcassi et al. (2018), which reconstructs dynamical structure from empirically motivated physical constraints rather than from postulated microdynamics. Thus, rather than beginning with Hamiltonian dynamics and deriving observable behaviour by quotienting on equivalence classes of empirical states, one may ask what minimal assumptions suffice to characterise invariant dynamics directly at the level of empirical support.

One interesting question, then, is under what conditions partition-level dynamics, considered on their own terms, admit description within the general Hamiltonian framework. In the simplified case of the Baker's map, the dyadic structure of the dynamics permitted the construction of a partition for which the induced transition matrix is exactly Markovian and invertible (as a linear operator on probability distributions), and probability-preserving, making a discrete Hamiltonian description available without appeal to any underlying continuous dynamics. Werndl's ε-convergence technology allows this analysis to be extended to a much wider class of classical systems, even when such exact alignment is absent.

In closer analogy with Carcassi and Aidala's appeal to "minimal assumptions" (2023), what is required in this context is a set of empirically motivated conditions under which the discrete dynamics can be regarded as Hamiltonian in structure. These include (i) effective Markov closure up to a fixed ε-tolerance, so that the dynamics closes on an empirically meaningful state space; and (ii) a discrete analogue of Liouville invariance, understood as conservation of total probability, i.e. a divergence-free probability current at finite resolution, again within ε-tolerance. When these conditions are met, the discrete dynamics fits naturally within a Hamiltonian framework and the resulting discrete Hamiltonian provides a convenient summary of symmetries observable at the empirical level.[3] The examples considered here suffice to show that such a reconstruction is possible across a wide and physically significant class of systems.

3.2 A model-invariance criterion for ontological commitment

The analyses of the preceding sections motivate a general criterion for distinguishing between structural features of a theory that warrant ontological commitment and features that reflect representational choice. The need for such a criterion arises whenever a single body of empirical data admits multiple, mutually incompatible formulations—deterministic and stochastic, Markovian and non-Markovian, Hamiltonian and non-Hamiltonian—that are nevertheless observationally equivalent at the relevant level of resolution.

The guiding idea of the present framework is that ontological commitment should attach not to individual models, but to equivalence classes of empirically indistinguishable representations. A feature of a theory there-

[3] Note that the associated discrete Hamiltonian generator may be dynamically trivial in particular regimes—for example, in states satisfying detailed balance, where no circulating probability current is present (as for example in the ground state of the hydrogen atom in Nelson's stochastic formulation).

fore qualifies as ontologically significant only if it is invariant across all representations that reproduce the same empirically accessible structure—where empirical accessibility is understood relative to a specified resolution, timescale, and observational scheme.

More precisely, let a class of models be said to be empirically equivalent if they agree on all observable statistics over finite histories up to a specified tolerance. A structural feature F is a candidate for realism only if:

1. F is fixed across this entire equivalence class;
2. F can be formulated purely in terms of empirically accessible relations, such as correlations, probability currents, or symmetry constraints on observable transitions; and
3. F is robust under admissible representational choices, including changes of coarse-graining, instrument realisation, and dynamical gauge.

This criterion licenses ontological commitment to features such as conservation laws, symmetry relations, and invariant probability-current structures, all of which persist across deterministic and stochastic descriptions and can be characterised without reference to unobservable microstructure. By contrast, distinctions typically associated with determinism and indeterminism—such as the localisation of modal selection in initial conditions, stochastic outcomes, or collapse mechanisms—fail to satisfy this invariance requirement. Where empirically equivalent formulations differ on these points, such distinctions must be regarded as representational artefacts, or at most working hypotheses for further empirical investigation, rather than reflections of physical reality.

Importantly, the present criterion is fallibilist and regime-relative. It does not deny that additional structure may become empirically accessible at finer resolution or over longer timescales, nor does it preclude the heuristic or explanatory value of adopting particular dynamical representations when attempting to extend a theory beyond its established domain. It allows for the possibility that structure may be misidentified and subsequently falsified by further empirical results. What it denies is that features underdetermined by the totality of empirically accessible data at a given observational level can bear ontological weight at that level.

Understood in this way, the opposition between determinism and indeterminism is not a metaphysical divide in nature, but a difference in modelling practice that reflects how invariant empirical structure is encoded. The task of ontology, on this view, is not to choose between such representations, but to identify and articulate the structural features that remain stable across them.

3.3 Quantum foundations

As a roadtest of the model-invariance criterion, I would like to sketch its application to two approaches to quantum foundations that are close in spirit to the present discussions, but completely opposite in their representational choices.

The Thermal Interpretation (Neumaier 2019; 2020) proposes a fully deterministic ontology, according to which expectation values of physical quantities evolve deterministically and constitute the primary ontic structure. On this view there is no fundamental collapse and no irreducible randomness; apparent stochasticity arises only at the level of coarse-grained description, when the detailed dynamics of interactions—typically involving degrees of freedom that have become effectively inaccessible through decoherence and environmental coupling in measurement-like contexts—are no longer empirically resolvable. By contrast, Barandes' stochastic–quantum correspondence (2018) posits an ontology in which state evolution is intrinsically indeterministic: microstates undergo genuine stochastic transitions, collapse corresponds to real stochastic state updates, and the dynamics is governed by a generally non-Markovian law defined over equivalence classes of configuration-space histories. In order to reproduce quantum correlations, including entanglement, this law exhibits explicit dependence on entire past histories and departs from straightforward forward-time causal structure. By construction, both approaches reproduce the standard quantum-statistical predictions.

This raises the now familiar question of whether the determinism or indeterminism of these formulations—together with other allegedly ontic features such as expectation values, memory, collapse, or trajectory structure—should be regarded as features of the world itself, or as representational choices internal to a given formulation.

Barandes' choice to begin with configuration space is motivated by a desire to keep the central objects of study as close as possible to the familiar framework of classical mechanics. This is a representational move that is not clearly motivated by empirically observable facts, and the example of the qubit shows that it is not a natural starting point for all quantum systems. The central technical step of the construction is to represent a bistochastic transition matrix as the modulus square of a complex matrix, which can then be interpreted as a Kraus-type or unitary evolution operator on a Hilbert space. Hamiltonian dynamics, together with the Schrödinger, von Neumann, and Ehrenfest equations, then emerge given differentiability and a unitary lift of the dynamics—a lift that is mathematically guaranteed to exist within the framework, but only by virtue of representational choices built into the formalism. The construction has the virtue of not postulating Hilbert space structure at the outset, but the choice of lift itself requires further representational choices. How surprising the appearance of familiar quantum dynamical equations is considered to be given the setup is perhaps a matter of perspective. What the framework does succeed in demonstrating is that a stochastic completion of the empirical quantum transition structure is possible with a high degree of generality. It also illustrates clearly the way in which wave-functions are not fundamental, but derive from features that vary across representations.

The equivalence classes of configuration-space histories considered in this approach are very broad, to the point that they would in practice be impossible to rule out empirically. If a much more restricted class were selected, this could in principle be tested via multi-time correlations probed by minimally invasive POVM measurements. Given that entanglement statistics cannot be reproduced in this scheme by strictly forward-time causal dynamics alone, one natural alternative would be to restrict attention to retrocausal histories that remain local in both space and time. The purpose of pointing this out, however, is not to advocate any particular restriction, but to emphasise that a range of distinct choices—each with different conceptual advantages—is available. As the framework currently stands, the trajectories treated as ontic by the formalism are not empirically accessible at all, which raises the question of what explanatory role they play beyond illustrating one possible class of representational realisations of the invariant empirical structure.

Neumaier's thermal interpretation takes a markedly different route. Rather than reconstructing quantum dynamics from a more classical substrate, it treats expectation values of observables as the primary elements of physical reality and regards the quantum state as a tool for organising these expectations. Questions concerning trajectories, hidden variables, or underlying configuration spaces are accordingly set aside as either ill-posed or physically unmotivated.

This approach aligns naturally with several themes developed earlier in this paper. In particular, it respects the empirical horizon imposed by finite resolution and avoids over-interpreting unobservable microstructure. Expectation values are typically robust under coarse-graining and refinement, and they often capture precisely the information that is operationally accessible. In this sense, Neumaier's ontology can plausibly be understood as a fixed point of empirical refinement, at which further increases in descriptive detail no longer yield additional empirically resolvable structure.

However, by insisting on the universal ontological status of this level of description, the thermal interpretation risks elevating a descriptively stable but representation-dependent structure to a status that is not empirically warranted. On Neumaier's view, stochastic outcomes are still understood as arising from coarse-graining over strongly decohered pointer states, but the expectation values associated with these states are then identified with an operator algebra that is taken to be ontic. From the perspective adopted in this paper, this move conflates empirical stability with model invariance. Decoherence of apparatus and environment-entangled degrees of freedom can indeed be studied in a controlled and systematic way, and it is entirely plausible that the robust expectation values that emerge represent a stable endpoint of empirical refinement. What has been achieved, however, is to relocate the uncontrollable degrees of freedom to deep within the empirically inaccessible domain, where it is to be expected that alternative theoretical descriptions—potentially very different in character—could equally well mediate between the underlying dynamics and the observed, apparently stochastic outcomes.

By contrast, the model-invariance criterion proposed in this paper suggests that the emergence of stochasticity is an inevitable consequence of coarse-graining on empirically accessible states, independently of whatever structure may exist beyond the observable horizon. On this view, stochasticity is not a feature to be explained by appeal to a deeper ontic level, but a structural consequence of finite empirical resolution.

From this perspective, the traditional formulation of the measurement problem is revealed to depend on a prior representational commitment. The problem arises only if one assumes that a particular level of description—typically a fine-grained, unitary dynamics on Hilbert space—must remain universally valid, even in regimes where the empirical distinctions required to sustain that description are no longer accessible. The apparent tension between continuous, deterministic evolution and discontinuous, stochastic measurement outcomes is then interpreted as a physical inconsistency rather than as a mismatch between descriptive levels.

On the model-invariance view, by contrast, stochasticity is not something that demands a special dynamical explanation at the point of measurement. It emerges inevitably once the theory is coarse-grained relative to empirically accessible states and observables. From this standpoint, the appearance of collapse reflects a change in descriptive regime rather than a physical interruption of an underlying unitary process. What varies across descriptions is not the empirical content, but the representational resources used to encode it.

The measurement problem is therefore not eliminated, but reclassified. It becomes a question about how different representational frameworks—unitary or stochastic, deterministic or indeterministic—encode the same invariant empirical transition structure not at a single fixed level of description, but stably under changes of coarse-graining. The focus remains on empirically verifiable and model-invariant structure rather than a postulated microdynamics with a universal range of validity, and there is no longer any need to invoke either fundamental collapse mechanisms or hidden ontological substrates. What remains is the task of identifying which features of quantum theory are invariant across admissible models, and which arise only within particular representational gauges.

4 Discussion

It belongs to the nature of science that observed regularities exist relative to an idealised empirical regime where a small number of variables may be controlled while all other degrees of freedom may be safely neglected. This is not a limitation of experimental procedure, but a structural feature of what counts as empirical support. To bring this out explicitly in the mathematical formalism of a theory has the huge philosophical payoff that we can affirm that structure unreservedly within the relevant regime, without requiring it to extend to other regimes. It has the further advantage of providing us with a catalogue of possibilities for investigating how that structure extends—or is modified—when the regime is altered.

This general strategy is empirically led, fallibilist in the sense that empirical agreement can never be taken for granted, but avoids a naïve, excessively purgatory fallibilism that insists that a successful theory is falsified as soon as its domain of applicability is delimited. A theory can be exactly true within its idealised domain without being universal.

There is a sense, then, in which all successful scientific theories are effective theories rather than truly fundamental and exhaustive descriptions of the real world. Nevertheless, according to this view, there is also a sense in which the structure that they study captures robust and perhaps even exact features of reality at appropriately idealised levels. The danger then lies in thinking that surplus features of their representational machinery are ontic, when this contradicts the method by which they were constructed in the first place.

Quotienting over gauge freedoms that relate empirically adequate but operationally equivalent representations of a physical theory is a well-established procedure in physics. The real novelty here consists in suggesting that this should be applied to an existing level of empirical access even in cases where refinements of that level are available in principle. Doing so demarcates clearly between what has been empirically established and what is hypothetical, and gives clear mathematical structure to distinct possible routes of empirical refinement. The examples in this paper suggest that any level of empirical refinement may leave decisive modal questions unanswered: even if the regime described is modally stable, its decisive modal possibilities are generally governed by regimes involving quantum or chaotic stochasticity, which are not.

The question "why?" is not univocal in the philosophy of physics. A first sense is nomological: given a specified dynamical model and auxiliary conditions, why does a particular event or regularity occur? A second sense is structural and connects directly with model-invariant descriptors: why are certain regularities robust across changes of representation, coarse-graining, or modelling choices? The present paper is concerned primarily with this structural sense, articulated in terms of empirically motivated model-invariant constraints. A third sense is metaphysical: why does reality instantiate laws or structures of this kind at all? Nothing in the empirical success of a model, by itself, settles this further question. Where multiple inequivalent modal completions remain compatible with the same invariant empirical structure at a given level of access, the appropriate conclusion is not that the world is thereby shown to be determined or undetermined beyond that empirical horizon, but that the decisive modalities modulating between those extremes are not fixed by the current theory—even in principle, and regardless of the empirical fate of any proposed refinements. This is a diagnostic fact, not a failure of the theory.

Where model-invariant structure thus leaves open whether decisive modal transitions are stochastic or deterministic in origin, it is more accurate to say that this distinction is simply not fixed by the empirically accessible dynamical structure itself. In such cases, the observed regularities constrain but do not determine the modal character of the underlying processes in any empirically verified sense.

Phenomena generally described as involving agentic freedom fall under the same category: they are conditioned by structural regularities, but not exhaustively determined by them. The present paper does not pursue the broader anthropological implications of this shift in perspective, but it is nonetheless worth noting that a closely related point was emphasised by Viktor Frankl in his metaclinical work (Frankl 1949), where he argued that a philosophically adequate view of the *existentia* of the human person requires resisting reductions that overinterpret scientific regularities as exhaustive determinants of human action. This may help to clarify where and in what way the activity of hypothetical modelling necessarily goes beyond the *essentia* of the empirically established facts.

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