Half Gauge-Dependent, Half Gauge-Invariant: The Aharonov–Bohm Effect Reconsidered

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Abstract

The standard semiclassical treatment of the Aharonov–Bohm (AB) effect, which predicts no observable phase shift for open electron paths in field-free regions due to gauge dependence, is shown to be incomplete. By fully quantizing both the probe electron and the electromagnetic source within quantum electrodynamics (QED), we derive an effective action that decomposes the AB phase into two equal contributions: a gauge-dependent probe phase from the source's potential acting on the electron, and a gauge-invariant source phase arising from the electron's quantum backreaction on the source via virtual-photon exchange. For open paths, where the probe phase vanishes under gauge choice, the source phase persists as a measurable part of the closed-loop value, isolated through a polarization–magnetization decomposition that separates its invariant bulk from a gauge-dependent boundary term. This uncovers a leading-order QED correction to nonrelativistic quantum mechanics, resolves longstanding debates on the AB phase's origin and locality, and refines gauge principles by revealing hidden gauge invariants in formally gauge-dependent quantities. While challenging to test directly, a conceptual scheme involving phase transfer to auxiliary qubits illustrates its potential observability, emphasizing its novelty beyond conventional closed-loop paradigms.

1 Introduction

The Aharonov–Bohm (AB) effect stands as one of the most profound illustrations of quantum mechanics' departure from classical intuition. It reveals that charged particles can acquire measurable phase shifts due to electromagnetic potentials in regions where the electric and magnetic fields vanish entirely—regions inaccessible to classical forces [1, 8]. In standard semiclassical treatments, the effect's gauge invariance for closed paths follows from Stokes' theorem in field-free regions. For open paths, however, the phase is gauge-dependent and thus unobservable, leading textbooks to conclude that no measurable AB effect exists beyond closed-loop configurations.

This semiclassical perspective, while successful for describing closed-path experiments, rests on a key approximation: treating the electromagnetic source (e.g., solenoid currents or capacitor charges) as classical. Quantizing the source introduces quantum backreaction effects mediated by virtual photons, which have sparked significant debate regarding the physical origin of the AB phase and the role of electromagnetic potentials. Vaidman [9] argued that, when the source is quantized, the phase shift can be fully explained without invoking potentials, attributing it instead to the local action of the probe electron's electromagnetic field on the source. This interpretation was contested by Aharonov et al. [2], who maintained that potentials remain essential, with the debate continuing in Vaidman's reply [10]. Pearle and Rizzi [6, 7] provided a rigorous quantum mechanical treatment, demonstrating that the phase arises symmetrically from reciprocal interactions: it can be viewed as originating from either the source potential acting on the probe or the probe potential acting on the source. More recently, Marletto and Vedral [5] proposed a local explanation via entanglement between the probe charge and the quantized electromagnetic field, claiming the phase is generated locally like other quantum phases and observable even for non-closed paths. However, closer examination reveals that their formulation relies on an energy-based description that is inconsistent with QED's minimal coupling and gauge principles [3].

This paper resolves these controversies through a rigorous quantum electrodynamics (QED) derivation that fully quantizes both probe and source. We demonstrate that the total AB phase decomposes symmetrically into two equal parts: a gauge-dependent "forward" phase acquired by the probe in the source's potential, and a "backreaction" phase acquired by the source due to the probe's potential. For closed paths, reciprocity yields the full gauge-invariant phase. For open paths, the forward phase vanishes under suitable gauge choices, but the backreaction phase persists, with its gauge-invariant bulk component—extracted via a polarization–magnetization decomposition—constituting a measurable half-phase. This finding uncovers a novel leading-order QED correction to nonrelativistic quantum mechanics, challenges conventional views on gauge dependence, and refines gauge principles by revealing hidden gauge invariants in formally gauge-dependent quantities. While direct detection poses experimental challenges, a conceptual phase-transfer scheme illustrates its potential observability.

The paper is structured as follows. Section 2 reviews the standard semiclassical treatment and its prediction of a null open-path phase. Section 3 presents the full quantum derivation, with detailed path-integral and Hamiltonian formulations establishing the symmetric "1/2 + 1/2" phase split. Section 4 analyzes the gauge properties and physical interpretation of the source phase via polarization–magnetization decomposition, clarifying bulk (invariant) and boundary (dependent) components. Section 5 explores broader implications for gauge theory, including evidence from ordinary quantum mechanics, refinements to gauge invariance, resolution of interpretive conflicts, and a proposed universal principle allowing gauge-dependent quantities to harbor hidden observables. Section 6 outlines a conceptual experimental scheme to detect the half-source phase, acknowledging its illustrative nature and current impracticality. Finally, the conclusions summarize the findings, emphasizing resolution of locality debates and the necessity of source quantization in topological quantum effects.

2 Standard Semiclassical Treatment

The AB effect is traditionally analyzed within a semiclassical framework in which the electromagnetic field is treated as a classical external background, governed by Maxwell's equations and sourced by prescribed classical currents or charges. The quantum probe particle—a charged particle such as an electron—evolves according to the Schrödinger equation (or its relativistic counterpart) in the presence of this classical four-potential $A_\mu = (A_0, -\mathbf{A})$. This approximation has been remarkably successful in describing all confirmed AB experiments to date, which universally employ closed interferometer loops enclosing a confined electromagnetic flux.

In the nonrelativistic limit (appropriate for typical electron interferometry experiments), the Schrödinger equation for a particle of charge e and mass m in an electromagnetic potential reads (in units where $\hbar = c = 1$):

$$i\frac{\partial \psi}{\partial t} = \frac{1}{2m}\left(-i\nabla - e\mathbf{A}\right)^2 \psi + eA_0\psi. \tag{1}$$

In regions where the electric and magnetic fields vanish ($\mathbf{E} = -\nabla A_0 - \partial_t \mathbf{A} = 0$, $\mathbf{B} = \nabla \times \mathbf{A} = 0$), classical forces are absent, yet the potentials A_μ are generally nonzero due to topology or confinement of the fields (e.g., flux inside a solenoid).

To extract the phase acquired by the wave function, we employ the WKB (semiclassical) approximation, valid when the de Broglie wavelength is short compared to variations in the potential and the action is large. We write $\psi(\mathbf{x}, t) = R(\mathbf{x}, t) \exp(iS(\mathbf{x}, t))$ and expand in powers of $\hbar$. At leading order, the phase S satisfies the Hamilton–Jacobi equation modified by minimal coupling. Along a classical trajectory $\gamma : \mathbf{x}(t)$ from initial time t_i to final time t_f, the accumulated phase is

$$\Delta\phi[\gamma] = \int_\gamma \left(m\mathbf{v} \cdot d\mathbf{x} - e\mathbf{A} \cdot d\mathbf{x} + eA_0 \, dt \right), \tag{2}$$

where $\mathbf{v} = d\mathbf{x}/dt$. The first term is the dynamical (kinetic) phase, while the second and third constitute the electromagnetic contribution:

$$S[\gamma] = \int_\gamma \left(eA_0 \, dt - e\mathbf{A} \cdot d\mathbf{x} \right) = e \int_\gamma A_\mu \, dx^\mu. \tag{3}$$

This is precisely the line integral of the four-potential along the spacetime path γ. Under a gauge transformation $A_\mu \to A_\mu + \partial_\mu \chi$, where $\chi(x)$ is an arbitrary scalar function, the phase changes by

$$\delta S[\gamma] = e \int_\gamma \partial_\mu \chi \, dx^\mu = e \left[\chi(t_f, \mathbf{x}_f) - \chi(t_i, \mathbf{x}_i) \right]. \tag{4}$$

The variation depends only on the values of χ at the endpoints of the path.

For a *closed* path in spacetime (returning to the same point, $t_i = t_f$, $\mathbf{x}_i = \mathbf{x}_f$), the boundary terms cancel, yielding $\delta S = 0$. The phase is therefore gauge-invariant. In a simply connected field-free region surrounding a confined magnetic flux $\Phi = \oint \mathbf{A} \cdot d\mathbf{l}$, Stokes' theorem ensures that the line integral depends only on the enclosed flux:

$$\oint_\gamma \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \Phi, \tag{5}$$

where S is any surface bounded by the closed spatial loop. Thus, for two paths γ_L and γ_R forming a closed interferometer loop enclosing flux Φ, the relative phase is

$$\Delta\phi = e\Phi, \tag{6}$$

independent of path details and gauge choice. This gauge-invariant, topological phase is the hallmark of the magnetic AB effect and has been spectacularly confirmed in experiments such as those by Tonomura et al. [8].

For *open* paths with distinct endpoints ($\mathbf{x}_i \neq \mathbf{x}_f$ or $t_i \neq t_f$), the boundary term in Eq. (4) is generally nonzero. In a field-free but not necessarily simply connected region, one can always choose a gauge in which $\mathbf{A} = \nabla \chi$ and $A_0 = -\partial_t \chi$ locally along the paths (since $\mathbf{B} = 0$, $\mathbf{E} = 0$). In this gauge, the phase $S[\gamma]$ becomes a pure gauge artifact:

$$S[\gamma] = e \left[\chi(t_f, \mathbf{x}_f) - \chi(t_i, \mathbf{x}_i) \right], \tag{7}$$

which can be absorbed into an unobservable redefinition of the overall phase of the wave function at the endpoints. Consequently, no measurable interference effect arises from the potentials alone for open paths.

This conclusion—that observable AB phases exist only for closed paths—has become textbook orthodoxy and is supported by all existing experiments, which invariably use closed interferometer geometries (e.g., electron biprisms, ring interferometers, or toroidal magnets). The semiclassical treatment thus predicts a *null observable AB phase for open paths* in field-free regions.

While this framework accurately describes current experiments, it rests on a crucial approximation: treating the electromagnetic source as strictly classical. No quantum backreaction from the probe onto the source is included, and the potentials are imposed externally without responding to the probe's own field. As we will show in the next section, quantizing both the source and the field reveals a subtle but significant correction: a backreaction phase that survives gauge transformations even for open probe paths, challenging the standard conclusion that no measurable effect exists beyond closed loops.

3 Full Quantum Treatment

The semiclassical treatment of the AB effect, which treats the electromagnetic source (e.g., solenoid currents or capacitor charges) as classical, successfully accounts for the observed phase shift in closed-path interferometry experiments. However, it provides no insight into the physical origin of the phase when the source is quantized. To address these limitations and resolve ongoing debates, we now turn to a fully quantum treatment within QED.

In this approach, both the probe particle (typically an electron) and the electromagnetic source are treated as quantum systems, interacting through the quantized electromagnetic field via the exchange of virtual photons. This symmetric treatment reveals that the total AB phase arises from two reciprocal contributions of equal magnitude: a "forward" phase acquired by the probe due to the source-generated potential, and a "backreaction" phase acquired by the source due to the probe-generated potential. For closed probe paths, these contributions add coherently to yield the full gauge-invariant phase $\Delta\phi = e\Phi$. For open paths, the forward phase becomes gauge-dependent and can be gauged away in field-free regions, but the backreaction phase persists, with its gauge-invariant component—extracted via a polarization–magnetization decomposition—corresponding to half the closed-path value (in the limiting case where the open paths lie arbitrarily close to the overlapping region of a closed configuration). This symmetric "1/2 + 1/2" decomposition emerges naturally from integrating out the photon field, producing cross terms in the effective action that reflect the reciprocity of electromagnetic interactions. It confirms and extends earlier insights into source quantization [6, 7, 9] while providing a rigorous resolution to questions of gauge dependence and observability for open paths.

The derivation proceeds in two complementary formulations. First, the path-integral approach yields the effective action for the matter fields after Gaussian integration over the photons, evaluated semiclassically along perturbed trajectories. Second, the Hamiltonian approach, via Dyson series expansion and Wick contractions in the interaction picture, reproduces the identical effective action. These formulations are shown to be equivalent, reinforcing the robustness of the phase split and its gauge properties.

3.1 Path Integral Formulation

In this subsection, we derive the symmetric decomposition of the AB phase using the path-integral formulation of QED. This covariant framework allows exact integration over the electromagnetic field, yielding an effective action for the probe and source matter fields mediated by virtual-photon exchange. In the nonrelativistic, weak-coupling regime characteristic of the AB effect, this effective action is dominated by semiclassical saddle-point configurations corresponding to well-localized wave packets following the semiclassical trajectories determined by the reciprocal electromagnetic interaction. Evaluation along these trajectories naturally produces two equal cross terms: one representing the phase acquired by the probe in the potential generated by the source ($S_{p\leftarrow s}$), and the other representing the backreaction phase acquired by the source in the potential generated by the probe ($S_{s\leftarrow p}$).

3.1.1 The Full QED Path Integral

We begin with the full QED path integral for the system comprising a probe particle (e.g., an electron), a macroscopic source (e.g., a solenoid with current), and the quantized electromagnetic field A_μ. The generating functional is

$$Z = \int \mathcal{D}A\,\mathcal{D}\psi_p\,\mathcal{D}\psi_s\,\exp\left(iS[A,\psi_p,\psi_s]\right), \tag{8}$$

where the action S is given by

$$S[A,\psi_p,\psi_s] = S_{\text{field}}[A] + S_m[\psi_p] + S_m[\psi_s] - \int d^4x\,A_\mu(x)\left(j_p^\mu(x) + j_s^\mu(x)\right), \tag{9}$$

where

$$S_{\text{field}}[A] = -\frac{1}{4}\int d^4x\,F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{10}$$

and $S_m[\psi_{p,s}]$ are the free matter actions for the probe and source, and $j^\mu_{p,s}$ are the conserved ($\partial_\mu j^\mu = 0$) four-currents associated with each. Varying S with respect to A_μ gives the Maxwell equation:

$$\partial^\nu F_{\mu\nu} = j_\mu, \tag{11}$$

subject to a gauge condition (e.g., Coulomb gauge $\nabla\cdot\mathbf{A} = 0$). The solution for A_μ is:

$$A_\mu(x) = \int d^4y\,\mathcal{D}_{\mu\nu}(x-y)j^\nu(y), \tag{12}$$

where $\mathcal{D}_{\mu\nu}(x-y)$ is the photon Feynman propagator in the chosen gauge.

3.1.2 Integrating Out the Photon Field

Since the action is quadratic in A_μ, the functional integral over the electromagnetic field can be performed exactly. We first fix the gauge to remove redundant degrees of freedom. The gauge-fixed photon action becomes

$$S_{\text{gf}}[A] = \int d^4x\left[-\frac{1}{2}A_\mu\mathcal{O}^{\mu\nu}A_\nu\right], \tag{13}$$

where $\mathcal{O}^{\mu\nu}$ is an invertible differential operator (e.g., $\mathcal{O}^{\mu\nu} = \eta^{\mu\nu}\Box - (1-\xi^{-1})\partial^\mu\partial^\nu$ in the Lorenz gauge). The generating functional then factorizes as

$$Z = \int \mathcal{D}\psi_p\,\mathcal{D}\psi_s\,\exp\left(iS_m[\psi_p] + iS_m[\psi_s]\right)\int \mathcal{D}A\,\exp\left(iS_{\text{gf}}[A] - i\int d^4x\,A_\mu(j_p^\mu + j_s^\mu)\right). \tag{14}$$

The Gaussian integral over A_μ yields

$$\int \mathcal{D}A\,e^{iS_{\text{gf}}[A] - i\int A_\mu j^\mu} = \exp\left(-\frac{i}{2}\int d^4x d^4y\,j^\mu(x)\mathcal{D}_{\mu\nu}(x-y)j^\nu(y)\right), \tag{15}$$

where $\mathcal{D}_{\mu\nu}(x-y)$ is the photon Feynman propagator (the inverse of $\mathcal{O}^{\mu\nu}$) in the chosen gauge. The full generating functional thus reduces to an effective theory for the matter fields alone:

$$Z = \int \mathcal{D}\psi_p \, \mathcal{D}\psi_s \, \exp\left(iS_m[\psi_p] + iS_m[\psi_s] + iS_{\text{eff}}[j_p, j_s]\right), \tag{16}$$

with the effective action

$$S_{\text{eff}}[j_p, j_s] = -\frac{1}{2}\int d^4x d^4y \, \left(j_p^\mu(x) + j_s^\mu(x)\right) \mathcal{D}_{\mu\nu}(x-y) \left(j_p^\nu(y) + j_s^\nu(y)\right). \tag{17}$$

3.1.3 Decomposition into Forward and Backreaction Contributions

Expanding Eq. (17) gives three types of terms: self-interactions of the probe ($j_p \mathcal{D} j_p$), self-interactions of the source ($j_s \mathcal{D} j_s$), and cross terms ($j_p \mathcal{D} j_s$ and $j_s \mathcal{D} j_p$). The self-interactions renormalize the masses and energies of the probe and source, but do not contribute to the interference phase in the AB setup. The cross terms, however, are responsible for the AB phase.

$$\begin{aligned} S_{\text{cross}} &= -\frac{1}{2}\int d^4x d^4y \, j_p^\mu(x) \mathcal{D}_{\mu\nu}(x-y) j_s^\nu(y) - \frac{1}{2}\int d^4x d^4y \, j_s^\mu(x) \mathcal{D}_{\mu\nu}(x-y) j_p^\nu(y) \\ &= S_{p\leftarrow s} + S_{s\leftarrow p}, \end{aligned} \tag{18}$$

where we define

$$S_{p\leftarrow s} = -\frac{1}{2}\int d^4x d^4y \, j_p^\mu(x) \mathcal{D}_{\mu\nu}(x-y) j_s^\nu(y) \tag{19}$$

as the phase acquired by the probe due to the source's field, and

$$S_{s\leftarrow p} = -\frac{1}{2}\int d^4x d^4y \, j_s^\mu(x) \mathcal{D}_{\mu\nu}(x-y) j_p^\nu(y) \tag{20}$$

as the phase acquired by the source due to the probe's field (backreaction). The factor of 1/2 in each term is crucial and arises from the symmetric treatment of both currents in the path integral; it ensures that the total phase for closed paths equals the full AB phase $e\Phi$, as shown below.

3.1.4 Evaluation on Semiclassical Trajectories

In the nonrelativistic, weak-coupling limit relevant to the AB effect, the integral over $\mathcal{D}\psi_p \mathcal{D}\psi_s$ in Eq. (16) is dominated by semiclassical saddle-point configurations corresponding to the well-localized wave packets of the probe and source, which follow the semiclassical trajectories determined by the reciprocal electromagnetic interaction. The corresponding classical currents $j_p^\mu(x)$ and $j_s^\mu(x)$ are c-number functions along these perturbed trajectories.

Inserting these currents into Eq. (18) and using the explicit form of the photon propagator yields

$$S_{p\leftarrow s} = -\frac{1}{2}\int d^4x \, j_p^\mu(x) A_\mu^{(s)}(x), \tag{21}$$

$$S_{s\leftarrow p} = -\frac{1}{2}\int d^4x \, j_s^\mu(x) A_\mu^{(p)}(x), \tag{22}$$

where $A_\mu^{(s)}(x) = \int d^4y \, \mathcal{D}_{\mu\nu}(x-y) j_s^\nu(y)$ is the potential generated by the source, and $A_\mu^{(p)}(x) = \int d^4y \, \mathcal{D}_{\mu\nu}(x-y) j_p^\nu(y)$ is the potential generated by the probe. The factor 1/2 appears because the interaction is mediated by a virtual photon exchanged between the two currents; a naive semi-classical calculation would incorrectly use the full $A_\mu^{(s)}$ for the probe phase, missing the symmetric split.

For an interferometer with two probe paths γ_L and γ_R, the phase difference is

$$\Delta\phi_{\text{AB}} = \Delta S_{p\leftarrow s} + \Delta S_{s\leftarrow p}, \tag{23}$$

where Δ denotes the difference between the two paths. In the closed-loop case, Stokes' theorem ensures that $\Delta S_{p\leftarrow s} = e\Phi$, and reciprocity (due to the symmetry of the propagator) gives $\Delta S_{s\leftarrow p} = e\Phi$, so that the total phase is $2 \times (e\Phi/2) = e\Phi$, reproducing the full AB phase.

3.2 The Hamiltonian Formulation

In this subsection, we derive the AB phase using the Hamiltonian formalism of QED. This approach complements the path-integral treatment by working directly in the operator framework, where the probe particle, the electromagnetic source, and the photon field are all treated as fully quantum degrees of freedom. This Hamiltonian derivation confirms the robustness of the symmetric phase split, while highlighting the operator-origin of the factor of 1/2 in each cross term.

3.2.1 The Full QED Hamiltonian

The full QED Hamiltonian is

$$\hat{H} = \hat{H}_p + \hat{H}_s + \hat{H}_{\text{field}} + \hat{H}_{\text{int}}, \tag{24}$$

where $\hat{H}_p$ and $\hat{H}_s$ are the free Hamiltonians for the probe particle and source, respectively, $\hat{H}_{\text{field}}$ is the free photon Hamiltonian, describing the quantized electromagnetic field, $\hat{H}_{\text{int}}$ is the interaction term, coupling the conserved currents $\hat{j}_p^\mu$ (probe) and $\hat{j}_s^\mu$ (source) to the quantized four-potential $\hat{A}_\mu$:

$$\hat{H}_{\text{int}} = -\int d^3\mathbf{x} \left[\hat{j}_p^\mu(\mathbf{x}) + \hat{j}_s^\mu(\mathbf{x})\right] \hat{A}_\mu(\mathbf{x}). \tag{25}$$

This minimal-coupling form ensures $U(1)$ gauge invariance and physically represents the exchange of virtual photons between the probe and source, which is the microscopic origin of the AB effect.

3.2.2 Interaction Picture and Time-Evolution Operator

We work in the interaction picture, where operators evolve under the free Hamiltonian $\hat{H}_0 = \hat{H}_p + \hat{H}_s + \hat{H}_{\text{field}}$:

$$\hat{O}_I(t) = e^{i\hat{H}_0 t}\hat{O}(0)e^{-i\hat{H}_0 t}. \tag{26}$$

The time-evolution operator in this picture is the time-ordered exponential:

$$\hat{U}_I(t) = \mathcal{T}\exp\Big[-i\int_0^t dt'\, \hat{H}_{\text{int,I}}(t')\Big], \tag{27}$$

where $\mathcal{T}$ denotes time-ordering, and $\hat{H}_{\text{int,I}}(t)$ is the interaction Hamiltonian in the interaction picture. This operator evolves the initial state $|\Psi(0)\rangle = |\psi_p\rangle|\psi_s\rangle|0_{\text{ph}}\rangle$ (probe and source in specified matter states, photons in the vacuum) to the final state $|\Psi(t)\rangle = \hat{U}_I(t)|\Psi(0)\rangle$.

Since the AB effect is mediated by virtual photons without creating or annihilating real photons, the photon field remains in its vacuum state. We thus compute the photon-vacuum matrix element:

$$\hat{U}_{\text{eff}}(t) = \langle 0_{\text{ph}}|\hat{U}_I(t)|0_{\text{ph}}\rangle, \tag{28}$$

which acts as an effective evolution operator for the probe and source degrees of freedom alone.

3.2.3 Perturbative Expansion and Wick Contractions

We expand $\hat{U}_I(t)$ perturbatively in the coupling constant e (or equivalently, $\alpha = e^2/4\pi \approx 1/137$, which is small in the nonrelativistic AB regime):

$$\hat{U}_I(t) = 1 - i\int_0^t dt_1\, \hat{H}_{\rm int,I}(t_1) - \frac{1}{2}\int_0^t dt_1 dt_2\, \mathcal{T}\big[\hat{H}_{\rm int,I}(t_1)\hat{H}_{\rm int,I}(t_2)\big] + \mathcal{O}(e^3). \tag{29}$$

The first-order term vanishes in the photon vacuum:

$$\langle 0_{\rm ph}|\hat{H}_{\rm int,I}(t_1)|0_{\rm ph}\rangle = 0, \tag{30}$$

because the vacuum expectation value of $\hat{A}_\mu$ is zero. The AB phase emerges at second order, corresponding to single virtual-photon exchange.

Substituting the form of $\hat{H}_{\rm int,I}(t) = -\int d^3x\,(\hat{j}_p^\mu + \hat{j}_s^\mu)_I(x,t)\hat{A}_{\mu,I}(x,t)$ into the second-order term gives:

$$\begin{aligned}\langle 0_{\rm ph}|\hat{U}_I^{(2)}(t)|0_{\rm ph}\rangle = -\frac{1}{2}\int_0^t dt_1 dt_2 \int d^3x_1 d^3x_2\, \langle 0_{\rm ph}|\mathcal{T}\Big[(\hat{j}_p^\mu + \hat{j}_s^\mu)_I(x_1,t_1)\hat{A}_{\mu,I}(x_1,t_1)\\ \times (\hat{j}_p^\nu + \hat{j}_s^\nu)_I(x_2,t_2)\hat{A}_{\nu,I}(x_2,t_2)\Big]|0_{\rm ph}\rangle.\end{aligned} \tag{31}$$

The factor of 1/2 originates from the Dyson series expansion of the time-ordered exponential. Applying Wick's theorem to the photon operators in the vacuum expectation value:

$$\langle 0_{\rm ph}|\mathcal{T}[\hat{A}_{\mu,I}(x_1,t_1)\hat{A}_{\nu,I}(x_2,t_2)]|0_{\rm ph}\rangle = i\mathcal{D}_{\mu\nu}(x_1 - x_2, t_1 - t_2), \tag{32}$$

where $\mathcal{D}_{\mu\nu}$ is the Feynman photon propagator in the choosen gauge. This contraction—a fundamental feature of quantum field theory—encodes the propagation of virtual photons between the currents and has no direct classical counterpart. After contraction and integrating over spacetime, the expression simplifies to:

$$\langle 0_{\rm ph}|\hat{U}_I^{(2)}(t)|0_{\rm ph}\rangle = -\frac{i}{2}\int d^4x d^4y\, (\hat{j}_p^\mu + \hat{j}_s^\mu)(x)\mathcal{D}_{\mu\nu}(x-y)(\hat{j}_p^\nu + \hat{j}_s^\nu)(y). \tag{33}$$

Since $\hat{H}_{\rm int}$ is linear in $\hat{A}_\mu$, higher even-order terms in the expansion resum exactly via the Gaussian property of the photon field, and thus we obtain

$$\hat{U}_{\rm eff}(t) = \langle 0_{\rm ph}|\hat{U}_I(t)|0_{\rm ph}\rangle = \exp\Big[iS_{\rm eff}[\hat{j}_p, \hat{j}_s]\Big], \tag{34}$$

with the effective action

$$S_{\rm eff}[\hat{j}_p, \hat{j}_s] = -\frac{1}{2}\int d^4x d^4y\, (\hat{j}_p^\mu + \hat{j}_s^\mu)(x)\mathcal{D}_{\mu\nu}(x-y)(\hat{j}_p^\nu + \hat{j}_s^\nu)(y). \tag{35}$$

3.2.4 Evaluation on Semiclassical Trajectories and the AB Phase

For well-localized wave packets following semiclassical paths (valid in the nonrelativistic AB regime), we replace the current operators with their expectation values (c-number functions):

$$\hat{j}_p^\mu(x) \to j_p^\mu(x) = \langle\psi_p|\hat{j}_p^\mu(x)|\psi_p\rangle, \quad \hat{j}_s^\mu(x) \to j_s^\mu(x) = \langle\psi_s|\hat{j}_s^\mu(x)|\psi_s\rangle. \tag{36}$$

The matrix element of the effective evolution operator then becomes

$$\langle\psi_p\psi_s|\hat{U}_{\rm eff}(t)|\psi_p\psi_s\rangle = \exp\big[iS_{\rm eff}[j_p, j_s]\big], \tag{37}$$

which is the transition amplitude for the system to remain in its initial matter state after virtual-photon exchange—the exact scenario of the AB effect. The real part of S_{eff} yields the phase shift, while the imaginary part (if present) describes decoherence, which is negligible for typical AB setups.

Expanding S_{eff} yields self-interaction terms $S_{\text{self}}[j_p]$ and $S_{\text{self}}[j_s]$ (which do not contribute to the AB phase) and cross terms mediating the probe–source interaction:

$$\begin{aligned} S_{\text{cross}} &= -\frac{1}{2}\int d^4x d^4y\, j_p^\mu(x)\mathcal{D}_{\mu\nu}(x-y)j_s^\nu(y) - \frac{1}{2}\int d^4x d^4y\, j_s^\mu(x)\mathcal{D}_{\mu\nu}(x-y)j_p^\nu(y) \\ &= S_{p\leftarrow s} + S_{s\leftarrow p}. \end{aligned} \tag{38}$$

Physically, $S_{p\leftarrow s}$ represents the phase the probe acquires from the source's potentials, while $S_{s\leftarrow p}$ is the phase the source acquires from the probe's backreaction. Evaluating S_{cross} along the probe and source worldlines reproduces the AB phase. Due to the symmetry of the propagator $\mathcal{D}_{\mu\nu}(x-y) = \mathcal{D}_{\nu\mu}(y-x)$, reciprocity holds ($S_{p\leftarrow s} = S_{s\leftarrow p}$) for closed paths, yielding the full AB phase $\Delta\phi = e\Phi$.

3.2.5 Comparison with Naive Semiclassical Approaches and Double-Counting

A common pitfall in the full quantum treatment of the AB effect is the inadvertent double-counting of the single virtual-photon exchange that mediates the interaction. This error arises when one attempts to construct a symmetric description by naively combining two separate semiclassical calculations, rather than deriving the interaction from the unified quantum field theory.

The naive semiclassical reasoning proceeds in three steps. First, solve the classical Maxwell equations for the source current j_s^μ to obtain a classical potential $A_\mu^{(s)}$. Second, compute the phase acquired by the probe along its trajectory as the line integral $S_{p\leftarrow s}^{(\text{naive})} = \int j_p^\mu A_\mu^{(s)} d^4x$. Third, invoking reciprocity, symmetrically add a corresponding phase for the source due to the probe's field: $S_{s\leftarrow p}^{(\text{naive})} = \int j_s^\mu A_\mu^{(p)} d^4x$, where $A_\mu^{(p)}$ is the potential generated by the probe current. The total phase would then be $S_{\text{total}}^{(\text{naive})} = S_{p\leftarrow s}^{(\text{naive})} + S_{s\leftarrow p}^{(\text{naive})} = 2\int j_p^\mu \mathcal{D}_{\mu\nu} j_s^\nu d^4x$, where we have expressed the potentials in terms of the classical photon propagator $\mathcal{D}_{\mu\nu}$.

This result overcounts the true interaction by a factor of two. In the full QED derivation, whether via the path integral or the Hamiltonian formulation, the effective action arising from a single virtual-photon exchange contains precisely one cross term, which is symmetrically split into two equal contributions of magnitude $\frac{1}{2}\int j_p^\mu \mathcal{D}_{\mu\nu} j_s^\nu d^4x$ each. The factor of $\frac{1}{2}$ is not optional; it emerges from the Gaussian integration over the photon field (path integral) or from the Dyson series expansion and Wick contractions (Hamiltonian). This factor ensures that the total phase for a closed loop equals the experimentally observed AB phase $e\Phi$, not twice that value.

The double-counting error stems from treating the forward and backreaction processes as independent classical interactions. In reality, the probe and source interact through a single quantum event—the exchange of a virtual photon—which entangles their states and yields a combined phase shift that cannot be decomposed into two separately computed classical phases without overcounting. The symmetric $1/2 + 1/2$ split reflects the fact that the interaction energy is shared equally between the two systems, not that each experiences a full independent phase shift.

Attempting to compute $S_{s\leftarrow p}$ separately by evolving the source in the probe's field would repeat the same mistake, effectively inserting a second virtual-photon exchange where there is only one. Only the complete QED treatment, which integrates out the photon field at the outset, correctly captures the quantum nature of the interaction and yields the proper normalization. This distinction is crucial for understanding why the semiclassical approximation—while successful for closed loops—fails to reveal the existence of the observable half-source phase in open-path configurations.

3.3 Summary

The full quantum treatment of the AB effect, developed within QED by quantizing both the probe particle and the electromagnetic source, reveals a symmetric structure absent in semiclassical analyses. Integrating out the photon field—whether via the path-integral Gaussian functional or the Hamiltonian Dyson series and Wick contractions—yields an effective action whose cross terms decompose into two equal contributions: a forward phase $S_{p\leftarrow s}$ acquired by the probe in the source potential, and a backreaction phase $S_{s\leftarrow p}$ acquired by the source in the probe potential. The factor of 1/2 in each term, enforced by the quantum treatment of virtual-photon exchange, ensures that the total phase for closed paths reproduces the observed gauge-invariant value $e\Phi$. The two formulations—path integral and Hamiltonian—are mathematically equivalent, confirming the robustness of this "1/2 + 1/2" decomposition. This equivalence underscores that the symmetric decomposition is not an artifact of a particular method but a fundamental consequence of the quantum nature of the interaction.

4 Gauge Properties of $S_{p\leftarrow s}$ and $S_{s\leftarrow p}$

The symmetric "1/2 + 1/2" decomposition of the AB phase into a forward probe contribution $S_{p\leftarrow s}$ and a backreaction source contribution $S_{s\leftarrow p}$ raises immediate questions about their behaviour under $U(1)$ gauge transformations. In the standard semiclassical treatment, the probe phase is gauge-dependent for open paths and gauge-invariant for closed paths, while no source phase is considered. Here, with both contributions present and derived from full QED, a careful analysis of their individual gauge properties is essential to determine which parts of the total phase are physically observable.

We show that $S_{p\leftarrow s}$ transforms in the familiar way: it is gauge-invariant for closed probe paths but acquires a boundary term $e[\chi(t_f, \mathbf{x}_f) - \chi(t_i, \mathbf{x}_i)]$ for open paths, rendering it unobservable in field-free regions under appropriate gauge choices. In contrast, the source phase $S_{s\leftarrow p}$ exhibits more subtle behaviour. Although formally gauge-dependent due to current conservation and integration-by-parts boundary terms, a polarization–magnetization (P–M) decomposition reveals that it contains a substantial gauge-invariant component—the bulk interaction energy between the source's intrinsic polarization/magnetization and the probe-induced fields. This invariant bulk term is the physically observable core of the source phase and persists regardless of whether the probe paths are open or closed. The following subsections establish these properties rigorously, first for the probe phase, then for the source phase, culminating in the P–M decomposition that isolates its maximum gauge-invariant content.

4.1 Gauge Properties of $S_{p\leftarrow s}$

The probe phase $S_{p\leftarrow s}$ is defined as half the interaction action felt by the probe:

$$S_{p\leftarrow s} = -\frac{1}{2}\int d^4x j_p^\mu(x) A_\mu^{(s)}(x). \tag{39}$$

Under an infinitesimal gauge transformation $A_\mu^{(s)} \to A_\mu^{(s)} + \partial_\mu\chi$, its variation is

$$\delta S_{p\leftarrow s} = -\frac{1}{2}\int d^4x j_p^\mu(x)\partial_\mu\chi(x). \tag{40}$$

The probe's current is localized along its worldline γ:

$$j_p^\mu(x) = e\int_{\tau_i}^{\tau_f} d\tau\, u^\mu(\tau)\, \delta^{(4)}\big(x - x_p(\tau)\big), \tag{41}$$

where $x_p(\tau)$ describes the probe trajectory and $u^\mu(\tau) = dx_p^\mu/d\tau$ is the four-velocity. Substituting the current into the variation formula:

$$\delta S_{p\leftarrow s} = -\frac{1}{2}e \int d^4x \int_{\tau_i}^{\tau_f} d\tau\, u^\mu(\tau)\, \delta^{(4)}\big(x - x_p(\tau)\big)\partial_\mu\chi(x). \tag{42}$$

Interchanging the order of integration and evaluating the delta function:

$$\delta S_{p\leftarrow s} = -\frac{1}{2}e \int_{\tau_i}^{\tau_f} d\tau\, u^\mu(\tau)\partial_\mu\chi(x_p(\tau)). \tag{43}$$

Recognizing that the directional derivative along the worldline satisfies $u^\mu\partial_\mu\chi = \frac{d\chi}{d\tau}$, we obtain:

$$\delta S_{p\leftarrow s} = -\frac{1}{2}e \int_{\tau_i}^{\tau_f} d\tau\, \frac{d\chi}{d\tau} = e\left[\chi(\tau_f) - \chi(\tau_i)\right]. \tag{44}$$

Thus, $S_{p\leftarrow s}$ acquires a boundary term proportional to the difference of χ at the endpoints. For closed paths ($\tau_i = \tau_f$ at the same spacetime point), the boundary terms cancel and $\delta S_{p\leftarrow s} = 0$, rendering the phase gauge-invariant. For open paths with distinct endpoints, the variation is generally nonzero, making $S_{p\leftarrow s}$ gauge-dependent and, in field-free regions, unobservable under suitable gauge choices where the potential is pure gauge.

4.2 Gauge Properties of $S_{s\leftarrow p}$

The gauge transformation properties of the source phase $S_{s\leftarrow p}$ are crucial for determining its physical observability. In the standard semiclassical treatment, only the probe phase is considered, and its gauge dependence for open paths renders it unobservable. However, in the full QED treatment, the source phase exhibits more subtle behavior under gauge transformations.

4.2.1 Gauge Dependence of the Source Phase $S_{s\leftarrow p}$

The source phase $S_{p\leftarrow s}$ is defined as half the interaction action felt by the source:

$$S_{s\leftarrow p} = -\frac{1}{2}\int d^4x\, j_s^\mu(x) A_\mu^{(p)}(x), \tag{45}$$

Consider an infinitesimal gauge transformation:

$$A_\mu^{(p)}(x) \to A_\mu^{(p)}(x) + \partial_\mu\chi(x), \tag{46}$$

where $\chi(x)$ is an arbitrary scalar function. Under this transformation, $S_{s\leftarrow p}$ changes by

$$\delta S_{s\leftarrow p} = -\frac{1}{2}\int d^4x\, j_s^\mu(x)\partial_\mu\chi(x). \tag{47}$$

To analyze this variation, we integrate by parts:

$$\delta S_{s\leftarrow p} = -\frac{1}{2}\int d^4x\, \left[\partial_\mu\left(j_s^\mu(x)\chi(x)\right) - \left(\partial_\mu j_s^\mu(x)\right)\chi(x)\right] \tag{48}$$

$$= -\frac{1}{2}\oint_{\partial\mathcal{V}} d\Sigma_\mu\, j_s^\mu(x)\chi(x) + \frac{1}{2}\int d^4x\, \left(\partial_\mu j_s^\mu(x)\right)\chi(x), \tag{49}$$

where $\partial\mathcal{V}$ denotes the boundary of the spacetime region $\mathcal{V}$, and $d\Sigma_\mu$ is the oriented surface element. The second term vanishes due to current conservation $\partial_\mu j_s^\mu = 0$, a fundamental property of the source current. The variation thus reduces to a pure boundary term:

$$\delta S_{s\leftarrow p} = -\frac{1}{2}\oint_{\partial\mathcal{V}} d\Sigma_\mu \, j_s^\mu(x)\chi(x). \tag{50}$$

The boundary $\partial\mathcal{V}$ consists of three parts: the initial time slice Σ_{t_i}, the final time slice Σ_{t_f}, and the spatial boundary at infinity Σ_∞. For a localized source with compact support, the current vanishes at spatial infinity, so the contribution from Σ_∞ is zero. The remaining contributions are

$$\delta S_{s\leftarrow p} = -\frac{1}{2}\left(\int_{\Sigma_{t_f}} d^3x\, j_s^0\chi - \int_{\Sigma_{t_i}} d^3x\, j_s^0\chi\right), \tag{51}$$

where we have used $d\Sigma_\mu = (d^3x, \mathbf{0})$ on constant-time hypersurfaces, with the minus sign at t_i accounting for the opposite orientation.

This result reveals two important features. The first feature is formal gauge dependence. The variation $\delta S_{s\leftarrow p}$ is generally nonzero, indicating that $S_{s\leftarrow p}$ is not gauge-invariant, regardless of whether the particle paths are open or closed. Unlike the probe phase, where gauge dependence appears as a difference of χ at the endpoints of the probe trajectory, here it appears as a difference of integrated charge density $\rho_s = j_s^0$ weighted by χ at initial and final times. The second feature is path dependence in interferometry. In an AB interferometer, the source current j_s^μ is perturbed differently by the probe electron taking the left (L) versus right (R) path. Consequently, the boundary terms for the two paths are not equal: $\delta S^L_{s\leftarrow p} \neq \delta S^R_{s\leftarrow p}$. The phase difference $\Delta S_{s\leftarrow p} = S^L_{s\leftarrow p} - S^R_{s\leftarrow p}$ therefore transforms as

$$\delta(\Delta S_{s\leftarrow p}) = -\frac{1}{2}\int d^3x \left[\left(\rho_s^L - \rho_s^R\right)\chi\right]_{t_i}^{t_f}, \tag{52}$$

which is generally nonzero due to the path-dependent charge densities $\rho_s^{L,R}$ induced by backreaction.

This formal gauge dependence might suggest that $S_{s\leftarrow p}$ is unobservable. However, as shown in the next subsection through the polarization–magnetization decomposition, $S_{s\leftarrow p}$ can be separated into two distinct parts:

$$S_{s\leftarrow p} = S_{\text{source,bulk}} + S_{\text{source,bdry}}, \tag{53}$$

where the bulk term $S_{\text{source,bulk}}$ is gauge-invariant and represents the physical interaction energy between the source's polarization/magnetization and the probe-induced fields, while the boundary term $S_{\text{source,bdry}}$ absorbs the gauge dependence. It is this gauge-invariant bulk component that constitutes the observable half-source phase for open paths. Thus, while the total $S_{s\leftarrow p}$ is formally gauge-dependent, it contains a physically meaningful, gauge-invariant core that can be isolated and measured—a key insight that refines our understanding of gauge principles in quantum field theory.

4.2.2 P–M Decomposition of the Source Phase in Electromagnetism

Consider a macroscopic source with a conserved four-current density $J_s^\mu(x)$ satisfying $\partial_\mu J_s^\mu = 0$. In a topologically trivial region, any conserved current can be written as the divergence of an antisymmetric tensor field $M_s^{\mu\nu}(x)$:

$$J_s^\mu = \partial_\nu M_s^{\nu\mu}, \qquad M_s^{\mu\nu} = -M_s^{\nu\mu}. \tag{54}$$

In three-dimensional notation the components of $M_s^{\mu\nu}$ are related to the electric polarization $\mathbf{P}_s$ and the magnetization $\mathbf{M}_s$ by

$$M_s^{0i} = P_s^{\,i}, \qquad M_s^{ij} = \epsilon^{ijk} M_k^s. \tag{55}$$

Substituting (54) in the source phase (45) and integrating by parts gives

$$S_{s\leftarrow p} = -\frac{1}{2}\int d^4x\, (\partial_\nu M_s^{\nu\mu}) A_\mu^p = -\frac{1}{2}\int d^4x\, \partial_\nu\big(M_s^{\nu\mu} A_\mu^p\big) + \frac{1}{2}\int d^4x\, M_s^{\nu\mu}\partial_\nu A_\mu^p. \tag{56}$$

Because $M_s^{\nu\mu}$ is antisymmetric, the second term can be rewritten with the field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$:

$$\frac{1}{2}\int d^4x\, M_s^{\nu\mu}\partial_\nu A_\mu^p = \frac{1}{4}\int d^4x\, M_s^{\nu\mu} F_{\nu\mu}^p. \tag{57}$$

We therefore define the *bulk term* and the *boundary term* as

$$S_{\text{source,bulk}} \equiv \frac{1}{4}\int d^4x\, M_s^{\nu\mu} F_{\nu\mu}^p, \qquad S_{\text{source,bdry}} \equiv -\frac{1}{2}\int d^4x\, \partial_\nu\big(M_s^{\nu\mu} A_\mu^p\big), \tag{58}$$

so that

$$S_{s\leftarrow p} = S_{\text{source,bulk}} + S_{\text{source,bdry}}. \tag{59}$$

The boundary term is a total divergence. By the generalised Stokes theorem it equals an integral over the boundary $\partial\mathcal{V}$ of the four-dimensional region $\mathcal{V}$:

$$S_{\text{source,bdry}} = -\frac{1}{2}\oint_{\partial\mathcal{V}} d\Sigma_\nu\, M_s^{\nu\mu} A_\mu^p, \tag{60}$$

where $d\Sigma_\nu$ is the oriented surface element on $\partial\mathcal{V}$. Take $\mathcal{V}$ to be the whole spacetime. Its boundary consists of the spacelike hypersurface Σ_{t_i} at the initial time t_i,the spacelike hypersurface Σ_{t_f} at the final time t_f, and the timelike hypersurface Σ_∞ at spatial infinity. If the source has compact spatial support and the fields decay sufficiently fast at infinity, the contribution from Σ_∞ vanishes. Equation (60) then reduces to

$$S_{\text{source,bdry}} = -\frac{1}{2}\int_{\Sigma_{t_f}} d\Sigma_\nu\, M_s^{\nu\mu} A_\mu^p + \frac{1}{2}\int_{\Sigma_{t_i}} d\Sigma_\nu\, M_s^{\nu\mu} A_\mu^p. \tag{61}$$

On a constant-time hypersurface the surface element is $d\Sigma_\nu = (d^3x, \mathbf{0})$; hence

$$S_{\text{source,bdry}} = -\frac{1}{2}\int d^3x\, \Big[M_s^{0\mu} A_\mu^p\Big]_{t_i}^{t_f} = -\frac{1}{2}\int d^3x\, \Big[M_s^{00} A_0^p + M^{0i} A_i^p\Big]_{t_i}^{t_f}. \tag{62}$$

Because $M^{\mu\nu}$ is antisymmetric, $M_s^{00} = 0$ and $M_s^{0i} = P_s^{\,i}$ by (55). Therefore

$$S_{\text{source,bdry}} = -\frac{1}{2}\int d^3x\, \Big[\mathbf{P}_s(x)\cdot\mathbf{A}_p(x)\Big]_{t_i}^{t_f}. \tag{63}$$

Note that for the particle which is in a region where $F_{\mu\nu} = 0$ in the AB effect, the bulk term vanishes identically outside the solenoid. Thus, while both the source and the particle admit a formal P–M decomposition, the physical content of the decomposition is different: for the source the bulk term represents a measurable AB phase, whereas for the particle the physics resides entirely in the boundary term.

4.3 Physical Interpretation

The P–M decomposition of the source phase provides a clear physical understanding of how the formally gauge-dependent interaction $S_{s\leftarrow p}$ can contain both gauge-invariant and gauge-dependent contributions. This decomposition separates the source phase into a bulk term, which captures the intrinsic, measurable interaction with the electromagnetic fields, and a boundary term, which encodes the gauge freedom at the temporal boundaries of the interaction. Below, we elaborate on the physical meaning of each term and discuss their implications in the context of the AB effect.

4.3.1 The Bulk Term: Gauge-Invariant Interaction Energy

The bulk term is given by

$$S_{\text{source,bulk}} = \frac{1}{4}\int d^4x\, M_s^{\nu\mu} F^p_{\nu\mu} = \frac{1}{2}\int d^4x\, \big(\mathbf{P}_s \cdot \mathbf{E}_p - \mathbf{M}_s \cdot \mathbf{B}_p\big), \tag{64}$$

where we have used the relations in Eq. (55) to express it in terms of the electric polarization $\mathbf{P}_s$ and magnetization $\mathbf{M}_s$ of the source, and the electric and magnetic fields $\mathbf{E}_p$ and $\mathbf{B}_p$ of the probe.

Physically, this term represents the energy of interaction between the polarized or magnetized source material and the local electromagnetic fields generated by the probe particle. It is explicitly gauge-invariant because it depends only on the field strengths $F_{\mu\nu}$, which are unaffected by gauge transformations. For a magnetic AB effect (e.g., a solenoid), the dominant contribution comes from $\mathbf{M}_s \cdot \mathbf{B}_p$, where $\mathbf{M}_s$ is the magnetization of the solenoid current, and $\mathbf{B}_p$ is the weak magnetic field induced by the probe's virtual-photon exchange. Similarly, in the electric AB effect, $\mathbf{P}_s \cdot \mathbf{E}_p$ captures the interaction with the induced electric field on capacitor plates.

This bulk term is the "physical core" of the source phase: it quantifies the quantum backreaction of the probe on the source, leading to a measurable phase shift. Importantly, it survives even for open probe paths, where the forward probe phase $S_{p\leftarrow s}$ is gauge-dependent and unobservable. The bulk term thus constitutes the leading-order correction from QED to the semiclassical AB phase, manifesting as half the total phase in the limiting case where the open paths lie arbitrarily close to the overlapping region of a closed configuration.

4.3.2 The Boundary Term: Gauge Freedom and Backreaction Effects

The boundary term, as derived in Eq. (63), depends explicitly on the vector potential $\mathbf{A}_p$ o the probe evaluated at the initial (t_i) and final (t_f) times, integrated over the source's polarization $\mathbf{P}_s$. Under a gauge transformation $\mathbf{A}_p \to \mathbf{A}_p + \nabla\chi$, it shifts by

$$\delta S_{\text{source,bdry}} = \int d^3x \left[\mathbf{P}_s \cdot \nabla\chi\right]_{t_i}^{t_f} = \int d^3x \left[\rho_s \cdot \chi\right]_{t_i}^{t_f}, \tag{65}$$

which generally does not vanish, confirming its gauge dependence and consistent with earlier analysis (52).

Physically, the boundary term arises from the temporal edges of the interaction and reflects the incomplete cancellation of surface contributions in the integration by parts. In a fully isolated system with no net charge flow across boundaries, one might expect this term to vanish or cancel in phase differences. However, in the AB effect, the probe electron's backreaction perturbs the source's polarization $\mathbf{P}_s$ slightly, making $\mathbf{P}_s$ path-dependent. For the two arms of an interferometer (L and R), the boundary terms are

$$S^{L/R}_{\text{source,bdry}} = \int d^3x \left[\mathbf{P}_s^{L/R}(x) \cdot \mathbf{A}_p(x)\right]_{t_i}^{t_f}, \tag{66}$$

where $\mathbf{P}_s^L$ and $\mathbf{P}_s^R$ differ due to the distinct backreactions from each path. Consequently, the phase difference $\Delta S_{\text{source,bdry}} = S_{\text{source,bdry}}^L - S_{\text{source,bdry}}^R$ is nonzero and remains gauge-dependent, even for closed electron paths.

To illustrate this, consider the Tonomura et al. experiment [8], which used electron interferometry around a toroidal magnet to confirm the AB phase shift $\Delta\phi = e\Phi$. In this setup, the electron paths form a closed loop around the flux, and the total observed phase matches the semiclassical prediction. However, according to our QED analysis, this total phase is the sum of the probe contribution and the source contribution. The source phase includes the gauge-invariant bulk term (half the total phase) and the boundary term. If the boundary term were zero or gauge-invariant, the total source phase would be fully observable and invariant. But in Tonomura's experiment, the source (toroidal magnet) is macroscopic and experiences a tiny quantum backreaction from the electron, perturbing its polarization and current distribution differently for each interferometer arm. This path-dependent perturbation ensures that the boundary term does not cancel in the phase difference, introducing a gauge-dependent component. This analysis underscores that even in landmark experiments like Tonomura's, the boundary term is nonzero and gauge-dependent, which is crucial for understanding the implications for gauge principles discussed in the next section.

4.3.3 Summary

The P–M decomposition systematically extracts the maximum gauge-invariant content from the QED-derived source phase $S_{s\leftarrow p}$. By isolating the bulk term, which depends solely on the gauge-invariant field strengths $F_{\nu\mu}$ and the source's intrinsic polarization/magnetization, we identify the physically observable part of what is otherwise a formally gauge-dependent quantity.

Note that the decomposition is formally non-unique due to mathematical ambiguities in the antisymmetric tensor $M^{\mu\nu}$ (the equation $J^\mu = \partial_\nu M^{\nu\mu}$ mathematically admits gauge-like transformations), but in the physical setting of a localized macroscopic source—such as a solenoid or magnetized material in the AB effect—the tensor $M^{\mu\nu}$ is uniquely determined by the standard definition of magnetization $\mathbf{M}$ as the magnetic dipole moment density per unit volume (and analogously for $\mathbf{P}$). This definition, rooted in the microscopic averaging of atomic currents or spins, fixes $M^{\mu\nu}$ uniquely within the material volume, with compact support and bound currents matching the physical source distribution. This yields a unique bulk term, independent of formal ambiguities.

In the context of the AB effect, this maximum gauge-invariant content corresponds to half the total phase for closed paths, arising purely from the quantum backreaction. It establishes that QED provides a well-defined way to decompose the source phase into observable (bulk) and unobservable (boundary) parts, resolving apparent paradoxes about gauge dependence in open-path scenarios. This interpretation not only clarifies the physical origin of the half-source phase but also highlights the role of quantum field theory in revealing hidden gauge-invariant observables within gauge-dependent formalisms.

5 Implications for Gauge Principles

The full quantum treatment of the AB effect, as derived in previous sections, reveals a nuanced structure in the phase contributions: the total phase decomposes into a gauge-dependent probe term $S_{p\leftarrow s}$ and a source term $S_{s\leftarrow p}$ that, while formally gauge-dependent, contains a physically observable, gauge-invariant bulk component via the P–M decomposition. This finding challenges and refines conventional wisdom in gauge theories, where gauge-dependent quantities are typically dismissed as unphysical. Here, we explore the broader implications of this result for the principles of gauge invariance, drawing connections to the experimental confirmation in setups like the Tonomura experiment [8] and proposing a new foundational principle for identifying observables in quantum gauge theories.

5.1 Evidence from Standard Quantum Mechanical Treatment

An early indication of the refined gauge principle proposed in this work—that gauge-dependent quantities can contain intrinsic, gauge-invariant observables—emerges already within the standard semiclassical treatment of the AB effect in nonrelativistic quantum mechanics, before any quantization of the electromagnetic source or invocation of quantum field theory. In typical electron interferometry, the total observable phase shift between the two arms comprises two distinct contributions: the topological AB phase, arising from the line integral of the vector potential $\mathbf{A}$ along the paths, and the dynamical kinetic (or path-length) phase, stemming from differences in travel time, velocity, or classical action due to path geometry or asymmetries.

In the WKB or semiclassical approximation, the phase accumulated along a trajectory γ is $S[\gamma] = \int_\gamma (m\mathbf{v} \cdot d\mathbf{x} - e\mathbf{A} \cdot d\mathbf{x})$ (in nonrelativistic limit). The interference phase difference thus decomposes as $\Delta\phi_{\text{total}} = \Delta\phi_{\text{AB}} + \Delta\phi_{\text{kin}}$, where $\Delta\phi_{\text{AB}} = e \oint \mathbf{A} \cdot d\mathbf{x}$ is the AB contribution and $\Delta\phi_{\text{kin}}$ reflects the gauge-independent kinetic action. For closed paths enclosing flux Φ, Stokes' theorem renders $\Delta\phi_{\text{AB}} = e\Phi$ gauge-invariant and measurable. The kinetic phase $\Delta\phi_{\text{kin}}$ is always gauge-invariant, depending only on the classical trajectories and energies. Crucially, for open paths with distinct endpoints, the AB term becomes explicitly gauge-dependent: under $\mathbf{A} \to \mathbf{A} + \nabla\chi$, it shifts by $e[\chi(\mathbf{x}_f) - \chi(\mathbf{x}_i)]$, rendering it unobservable in field-free regions where $\mathbf{A} = \nabla\chi$. Standard textbooks conclude that no measurable AB phase exists for open trajectories. Yet the kinetic phase $\Delta\phi_{\text{kin}}$ remains fully gauge-invariant and observable (via nonlocal correlation measurements)—even when the AB contribution can be gauged away entirely.

This standard quantum-mechanical scenario provides a direct analogue to the QED-derived structure discussed in later subsections. Just as the formally gauge-dependent semiclassical action contains a separable, gauge-invariant kinetic component that is physically measurable along open paths, the gauge-dependent source phase $S_{s\leftarrow p}$ in full QED decomposes (via the P–M formalism) into an invariant bulk term—representing the observable backreaction energy—and a dependent boundary term. In both cases, gauge dependence of the overall expression does not preclude the existence of hidden gauge-invariant observables that contribute to physical interference patterns. This continuity from ordinary quantum mechanics to quantum field theory suggests that the principle is a fundamental feature of gauge-coupled dynamics, not merely an artifact of source quantization, and motivates the refinements and generalizations developed below.

5.2 Refinement of the Gauge Invariance Principle

In standard gauge theories, including QED, the principle of gauge invariance dictates that physical observables must be invariant under local gauge transformations. Gauge-dependent objects, such as the vector potential A_μ or the action functionals involving it, are regarded as mathematical artifacts without direct physical meaning. This view underpins the interpretation of the AB effect in semiclassical treatments, where only closed-path phases are considered measurable due to their gauge invariance.

However, our QED analysis demonstrates that the formally gauge-dependent source phase $S_{s\leftarrow p}$ encodes a nonzero, gauge-invariant contribution through its bulk term $S_{\text{source,bulk}}$, which represents the interaction energy between the source's polarization/magnetization and the probe-induced fields. This bulk term survives for both closed and open probe paths, contributing half the total AB phase in closed-loop configurations. The boundary term, meanwhile, captures the residual gauge freedom but does not eliminate the observable content.

This decomposition implies a refinement to the traditional gauge principle: gauge-dependent quantities are not inherently unphysical but may contain extractable gauge-invariant components that correspond to measurable effects. In the AB context, the gauge-invariant bulk arises from quantum backreaction via virtual-photon exchange, a feature absent in semiclassical models. This hidden invariance explains why experiments like Tonomura et al.'s [8], which measure the total closed-loop phase, align with semiclassical predictions while implicitly including the source backreaction through its gauge-invariant core.

Building on these insights, we propose a new principle that generalizes beyond the AB effect:

> *Gauge-dependent quantities in quantum gauge theories may contain intrinsic, gauge-invariant components that encode physically observable effects. These components can be isolated through appropriate decompositions, such as bulk-boundary splits, revealing hidden dynamics that are measurable even in regimes where formal gauge dependence persists.*

This principle carries several important implications. It extends naturally to non-Abelian gauge theories, such as QCD and the Standard Model, where analogous decompositions might reveal physical observables hidden within formally gauge-dependent objects like Wilson lines or source interactions, potentially shedding new light on phenomena such as confinement or electroweak symmetry breaking. Methodologically, it encourages systematic exploration of gauge-dependent expressions—through techniques such as integration by parts, topological classifications, or other decompositions—to uncover invariant subsets that encode genuine physical content. Philosophically, it suggests that gauge freedom is not simply a redundancy to be eliminated, but a rich structure that can veil real physics, requiring careful analysis to separate observable effects from mere artifacts.

5.3 Resolution of Apparent Conflicts with QED Interpretation

The presence of a gauge-invariant observable within a gauge-dependent formalism appears paradoxical at first glance. Standard QED emphasizes that matrix elements or expectation values must be gauge-invariant for observability, yet $S_{s\leftarrow p}$ is not inherently invariant. The P–M decomposition resolves this by isolating the bulk term, which depends solely on gauge-invariant field strengths $F_{\mu\nu}$, from the boundary term, which absorbs the gauge ambiguity.

This resolution indicates that the standard interpretation of QED is incomplete. In particular, QED's operator structure permits gauge-dependent quantities to contain genuine physical information, which can be extracted through decompositions such as the polarization–magnetization formalism. This is especially significant in systems with quantized sources, where backreaction effects emerge as gauge-invariant energies. Although formal gauge dependence often indicates underlying non-uniqueness, operational observables—such as interference fringes in AB experiments—may arise from invariant subsets within these quantities. In the proposed open-path scenarios (Sec. 6), measuring the half-source phase would directly access this invariant bulk, affirming its physical reality. Furthermore, in Tonomura's toroidal magnet experiment, the observed total phase $\Delta\phi = e\Phi$ incorporates both probe and source contributions. Our analysis reveals that the experiment effectively captures the gauge-invariant bulk of the source phase, thereby reconciling full QED predictions with observations without relying on the classical-source approximation.

If future open-path experiments detect the predicted half-phase, it would further validate this refined interpretation, highlighting QED's ability to predict observables beyond naive gauge-invariant operators.

6 Experimental Proposal: Detecting the Half-Source AB Phase

To clarify the physical meaning and potential observability of the half-source contribution in the AB effect, we outline a schematic experimental scenario in which the phase associated with source backreaction is coherently transferred from a probe electron to auxiliary quantum degrees of freedom. The essential purpose of this proposal is conceptual rather than practical: it demonstrates, in principle, how a gauge-invariant source phase associated with open probe paths could be rendered observable. We emphasize from the outset that such an experiment lies beyond current technological capabilities.[1] In particular, it would require the coherent annihilation of an electron while preserving its path superposition and transferring the relative phase to stationary quantum systems, a task that exceeds the present state of the art in both electron interferometry and hybrid quantum platforms. Nevertheless, the construction serves as a useful Gedankenexperiment, sharpening the distinction between probe and source phases in a fully quantized treatment of electromagnetism.

6.1 Setup: Open-Path Electron Interferometer with Confined Flux

Consider a Mach–Zehnder-type electron interferometer in which a localized magnetic flux Φ is confined within a shielded region, such as a nanoscale solenoid or toroidal magnet, placed between the two arms of the interferometer. The shielding ensures that the magnetic field vanishes everywhere along the electron trajectories, while the vector potential remains nontrivial. The electron is prepared in an equal superposition of the left and right paths,

$$|\Psi_e(0)\rangle = \frac{1}{\sqrt{2}}\big(|L\rangle + |R\rangle\big), \tag{67}$$

where $|L\rangle$ and $|R\rangle$ denote states localized along the corresponding arms.

In the conventional closed-loop configuration, recombination of the two paths yields the standard AB phase difference $\Delta\phi_{\mathrm{AB}} = e\Phi$. In contrast, we consider an explicitly open-path geometry in which the electron paths do not form a closed loop. In such a configuration, the forward phase accumulated by the electron due to the external vector potential becomes gauge-dependent and therefore unobservable. By contrast, the phase arising from the backreaction of the electron on the quantized source remains observable via its gauge invariant bulk component. For open paths sufficiently close to the overlapping region of the interferometer, this backreaction phase approaches one-half of the closed-loop AB phase, providing a well-defined and isolated contribution associated solely with the source dynamics.

6.2 Conceptual Phase Transfer to Auxiliary Qubits

To access this source-induced phase, we coherently transfer the electron's path superposition to auxiliary quantum systems positioned along each arm of the interferometer. These auxiliary systems may be modeled as two-level qubits initially prepared in their ground states. The total initial state may be written as

$$|\Psi_{\mathrm{total}}(0)\rangle = \frac{1}{\sqrt{2}}\big(|L\rangle + |R\rangle\big) \otimes |g\rangle_L |g\rangle_R. \tag{68}$$

At the output of the interferometer, the electron is assumed to be annihilated in a path-dependent manner while exciting the corresponding qubit,

$$|L\rangle|g\rangle_L \to |0\rangle|e\rangle_L, \qquad |R\rangle|g\rangle_R \to |0\rangle|e\rangle_R, \tag{69}$$

[1] By comparison, a similar experimental proposal for the gravitational AB effect is more feasible [4].

where $|0\rangle$ denotes the electron vacuum state. If this transfer could be implemented coherently, the final state of the qubits would take the form

$$|\Psi_{\text{final}}\rangle = \frac{1}{\sqrt{2}}\Big(|e\rangle_L|g\rangle_R + e^{i\Delta\phi_{\text{AB}}/2}\,|g\rangle_L|e\rangle_R\Big), \tag{70}$$

with the relative phase encoding the gauge-invariant source backreaction contribution.

This step constitutes the central technical obstacle of the proposal. Coherently annihilating an electron while preserving phase information and mapping it onto localized qubits would require exquisite control over timing, coupling strength, and environmental decoherence. Although similar ideas exist in quantum optics and cavity QED, no known mechanism currently allows such a process to be realized for free or weakly bound electrons.

6.3 Extracting the Half-Source Phase

Assuming that the phase transfer could be achieved, the encoded half-source phase could be extracted using standard qubit control and measurement techniques. After applying local single-qubit rotations of the form $U_L(\theta) = \exp(-i\theta\sigma_y/2)$ and $U_R(\phi) = \exp(-i\phi\sigma_y/2)$, projective measurements in the $\{|g\rangle, |e\rangle\}$ basis would allow reconstruction of joint probabilities. In particular, the probability of finding both qubits in the excited state would be

$$P_{ee}(\theta,\phi) = \frac{1}{4}\Big[1 - \cos\theta\cos\phi + \sin\theta\sin\phi\cos(\Delta\phi_{\text{AB}}/2)\Big]. \tag{71}$$

Choosing $\theta = \phi = \pi/2$ maximizes sensitivity to the relative phase, yielding

$$P_{ee}(\pi/2,\pi/2) = \frac{1}{4}\Big[1 + \cos(\Delta\phi_{\text{AB}}/2)\Big]. \tag{72}$$

While such correlation measurements are routine in quantum information experiments, integrating them with electron interferometry would introduce severe additional challenges, including synchronization, phase stability, and noise arising from the annihilation process.

6.4 Why the Half-Source Phase Is Genuinely New

The effect isolated here—the emergence of a gauge-invariant source backreaction phase for open probe paths—represents a genuinely new aspect of the AB phenomenon that is absent from standard treatments. Virtually all textbook discussions and experimental realizations of the AB effect rely on two simplifying assumptions: the electromagnetic source is treated classically, and the probe paths form a closed loop. Under these conditions, reciprocity in virtual-photon exchange enforces equality between the forward probe phase and the source backreaction phase. As a result, a semiclassical calculation that retains only the probe phase in an external potential accidentally reproduces the correct total phase, since the omitted source contribution is numerically identical and experimentally inseparable.

The novelty arises when these assumptions are relaxed. In a fully quantized description with open probe paths, the forward probe phase becomes gauge-dependent and thus physically meaningless, whereas the bulk component of the source backreaction phase remains gauge invariant and, in suitable geometries, constitutes the entire observable phase shift. In the near-overlap regime, this phase approaches one-half of the familiar closed-loop AB phase, revealing a leading-order quantum-field-theoretic effect with no classical analog. Previous discussions of quantized sources have focused primarily on reinterpretations of standard closed-loop experiments and on debates concerning locality, without identifying this open-path isolation of a distinct, gauge-invariant half-phase.

Experimental confirmation of such an effect—should future technologies permit access to source-sensitive observables in non-closed geometries—would establish the half-source phase as a universal manifestation of electromagnetic backreaction in quantum mechanics, extending the conceptual scope of the AB effect beyond its traditional semiclassical interpretation.

6.5 Summary

In summary, this section has presented a conceptual experimental scheme illustrating how a gauge-invariant half-source AB phase could, in principle, be isolated and detected. The proposal relies on an open-path interferometric geometry, a phase-preserving transfer from an electron to auxiliary qubits, and nonlocal correlation measurements to extract the source contribution. Although impractical with current technology, the construction demonstrates that the half-source phase is not merely a formal artifact but a well-defined physical quantity whose observability is constrained only by experimental capability, not by principle.

7 Conclusions

This work establishes, through a rigorous QED treatment, that the AB phase decomposes symmetrically into two contributions of equal magnitude: a gauge-dependent forward phase acquired by the probe particle in the source's electromagnetic potential, and a backreaction phase acquired by the source due to the probe's potential via virtual-photon exchange. This "1/2 + 1/2" structure arises naturally from integrating out the quantized photon field, yielding cross terms that reflect the reciprocity of electromagnetic interactions.

For closed probe paths, reciprocity enforces equality between the two halves, reproducing the full gauge-invariant AB phase familiar from semiclassical analyses and experiments. For open paths, however, the forward probe phase becomes gauge-dependent and unobservable in field-free regions, while the source backreaction phase—isolated via the polarization–magnetization (P–M) decomposition—reveals a gauge-invariant bulk component corresponding to the interaction energy between the source's intrinsic polarization/magnetization and the probe-induced fields. This invariant bulk term persists as a measurable half-phase, constituting a genuine leading-order QED correction to nonrelativistic quantum mechanics that has no semiclassical counterpart.

The P–M decomposition systematically extracts this maximum gauge-invariant content from the formally gauge-dependent source phase, separating the observable bulk (backreaction energy) from the unobservable boundary term. This not only resolves gauge paradoxes for open paths but also refines foundational principles of gauge theory: formally gauge-dependent quantities can harbor hidden gauge-invariant observables, a feature with analogues even in ordinary quantum mechanics (e.g., gauge-invariant kinetic phases amid dependent potential contributions).

These findings clarify longstanding debates on the AB effect's origin and locality. Although direct experimental verification remains challenging, the conceptual phase-transfer scheme outlined illustrates the potential observability of the isolated half-source phase in open-path configurations. Future advances in hybrid quantum systems may enable tests of this prediction, probing subtle electromagnetic backreaction effects. Ultimately, this analysis bridges semiclassical intuition with quantum field theory, underscoring the physical reality encoded in electromagnetic potentials and urging deeper consideration of backreaction and hidden invariants in gauge-dependent formalisms across quantum theories.

References

[1] Aharonov, Y. & Bohm, D. Significance of electromagnetic potentials in the quantum theory. *Phys. Rev.* 115, 485 (1959).

[2] Y. Aharonov, E. Cohen, and D. Rohrlich. Comment on 'On the role of potentials in the Aharonov-Bohm effect'. *Phys. Rev. A* **91**, 052123 (2015).

[3] Gao, S. Comment on "Aharonov-Bohm Phase Is Locally Generated Like All Other Quantum Phases". *Phys. Rev. Lett.* 135, 098901 (2025).

[4] Gao, S. A New Approach for Testing the Quantum Nature of Gravity. Preprint (2025). https://doi.org/10.5281/zenodo.17557495.

[5] C. Marletto and V. Vedral. Aharonov-Bohm phase is locally generated like all other quantum phases. *Phys. Rev. Lett.* **125**, 040401 (2020).

[6] P. Pearle and A. Rizzi. Quantum Mechanical Inclusion of the Source in the Aharonov–Bohm Effects. *Phys. Rev. A* 95, 052123 (2017).

[7] P. Pearle and A. Rizzi. Quantized vector potential and alternative views of the magnetic Aharonov-Bohm phase shift. *Phys. Rev. A* 95, 052124 (2017).

[8] Tonomura, A. et al. Evidence for Aharonov–Bohm effect with magnetic field completely shielded from electron wave. *Phys. Rev. Lett.* 56, 792 (1986).

[9] Vaidman, L. Role of potentials in the Aharonov–Bohm effect. *Phys. Rev. A* 86, 040101(R) (2012).

[10] Vaidman, L. Reply to "Comment on 'Role of potentials in the Aharonov-Bohm effect'". *Phys. Rev. A* 92, 026102 (2015).