Tractatus Quanticus

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Abstract

This is a re-editing, which takes quantum mechanics into account, of Wittgenstein's famous Tractatus. The operation has a playful side in the form, but is a serious attempt to capture possible philosophical implications of the Relational Interpretation of Quantum Mechanics, and formalize the naturalistic third-way between realism and instrumentalism explored by this interpretation.

Keywords: *Relational Quantum Mechanics; Tractatus Logico-Philosophicus; Quantum Foundations; Philosophy of Physics.*

This text will perhaps only be understood by those who have themselves already thought the thoughts which are expressed in it, or similar thoughts. Its meaning could be summed up somewhat as follows: whatever can be said at all, can be said clearly by any speaker who is part of the world that is spoken about, and who speaks from its perspective.
Of course others far better than us have already been around here. But we feel some details have been left out, which quantum phenomena have brought into sharp light, and these bring further clarity.
Whoever understands us recognizes again the following propositions as senseless. After climbing through these, over them so to speak, will kick away the ladder, once more.
Thanks to this futility, we feel shamelessly entitled to parrot a Master. Inspired not by his preaching but rather by his example (and not in small part by its later reflections), we have written all this, as we actually think that not to remain silent, even whereof one cannot speak, is not such a bad idea after all, maybe only, as we learned from him, to keep the warning alive that there are questions that have no meaning.
What we hold onto is not knowledge of an absolute order, but the fleeting grasp of a perspective on perspectives. Hence this is at most just a manifesto for a picture of the world suggested by quantum mechanics. Each proposition attempts to show a possible way of formulating questions about nature in a manner coherent with what we have recently learned about it. A manner aware of its own partiality. Good philosophy, seems to us, dismantles idols, rather than creating them, and, like a gentle friend, reminds us with indirect hints, over and over again, that the answer to the riddle is that there is no riddle.

Synthesis

1. The world is everything that is the case from some perspective.
2. My knowledge about the world is a particular kind of perspective.
3. Perspectives themselves are facts, when considered from another perspective.
4. Facts can be expressed as values of variables, which are ways systems interact.
5. Perspectives are transparent to each other, because they are facts.

6. There is nothing wrong with circularity.

7. What we do not have information about, we must pass over in silence.

Short

1. **The world is everything that is the case from some perspective.**

   1.1. Everything that is the case, is the case from some perspective.

   1.2. Something that is the case from a perspective is a fact, from (or in) a perspective.

   1.3. Our perspectives are special cases of the physical notion of perspective.

   1.4. Perspectives are not about minds. They are about physics, like reference systems in relativistic physics are about physics, not about minds.

   1.5. No perspective stands out above the rest.

2. **My knowledge about the world is a particular kind of perspective.**[1]

   2.1. My knowledge[2] of the world is the information I have or can have about the world.

   2.2. Information has degrees of reliability.[3]

   2.3. A fact in my perspective is knowledge about which I (reasonably) estimate no uncertainty.

   2.4. Degrees of reliability assigned to facts evolve; perspective can change accordingly.

   2.5. To have uncertain information about a fact means to have the information that by gathering more information the fact will be ascertained with a known likelihood.

   2.6. If I reliably expect that one out of N mutually exclusive alternatives a will be the case, then I can talk about a set of alternative possible facts.

   2.7. Facts in a perspective can be represented in logical spaces. They can be actual, or have probability.

   2.8. My knowledge about the world does not create the world; it is determined by the world and it refers to it.

3. **Perspectives themselves are facts, when considered from another perspective.**

   3.1. A perspective is embodied in a set of facts, when considered from any other perspective (including itself).

   3.2. If knowing b determines which a is a fact, then a is fact with respect to b, or in the perspective of b.

   3.3. If $p(a, b)$ is the joint probability for two sets of possible facts $A = \{a\}$ and $B = \{b\}$, then b embodies a perspective in which the probabilities of a are $p_b(a) = p(a|b) = p(a, b)/p(b)$.

   3.4. To know *what* B knows about A is not the same as knowing *that* B knows about A.

   3.5. The mutual information $I_{A:B}$ quantifies the information on one variable that can be obtained from the other.

[1] Here, "particular kind" denotes a concrete instantiation of what is assumed to be universal. The world is constituted by particular perspectives, whereas a universal is merely the sum of these particulars. A whole that can always be extended by adding a further particular.

[2] We use the term "knowledge" in the Wittgenstein (1953)'s sense §116, where knowing is embedded in practices and does not imply access to a total view or a metaphysical picture.

[3] Cfr. De Finetti (1989).

3.6. When the relative information $I_{A|B}$ is maximal, knowing b determines which a is a fact, and a is a fact relative to b.[4]

3.7. All things stand only in relation to a perspective, and with a shift of perspective, what once appeared as fact may be absent or incompatible with other facts from another perspective.

3.8. A subject is a (case of a) perspective and defines a world.

3.9. From a perspective distinct from mine, I, a subject, am a physical process, namely a set of facts.

3.10. Knowledge is physically underpinned by relative information between variables of two systems: the subject and the object of knowledge.

3.11. The world for me is the world I know or can know, and my knowledge is a physical part of this world.

4. **Facts can be expressed as values of variables, which are ways systems interact.**

4.1. Properties are facts expressed by values a of a variable A. The fact that A has value a is expressed as $A = a$.

4.2. Systems are collections of variables that admit approximate closed dynamical descriptions.

4.3. Variables are ways systems interact and manifest themselves in interactions.

4.4. Physics is *modal*: the general form of dynamical laws is: *if* a *then* b with a certain reliability. Dynamical laws assign values to conditional probabilities $p(b|a)$: likelihood of b given a.

4.5. Causation is the aspect of modality that can support intervention.

4.6. My world is coloured by my interaction with it.[5]

4.7. Variables in a given perspective may have no value.[6]

4.8. In general, not all variables of a system can simultaneously have a value.

4.9. The mathematical theory allowing physical laws to be expressed associates elements A of a non-commutative algebra to the variables A of system.[7]

4.10. Mathematical laws associated to a system relate variables — hence facts — relative to the same system, within the same perspective.

4.11. A variable having a value in a perspective is an *atomic fact*.

4.12. Combinations of systems are systems.

4.13. Since variables express the way a system affects another in interactions, dynamical laws, which are about variables, regard interactions between systems.[8]

4.14. The information about a system A in a perspective B changes when A and B interact. In the perspective B, this is an update of information. In any other perspective, this is an evolution of the mutual information that is continuous in time.[9]

4.15. The maximum information available about a system is achieved when a maximal set A of commuting variables has values that are facts.

4.16. In a perspective B, the facts making up the world are values of sets of variables A. From a different perspective, the information that B has is embodied in values of the commuting set of variables defining B and the fact that these are correlated to the A's.

[4] Di Biagio and Rovelli (2025).
[5] Price (2007).
[6] This aspect of the world was not known before the discovery of quantum phenomena.
[7] This is not true if we disregard quantum phenomena, as Wittgenstein did.
[8] Hence dynamical laws viewed as pertaining to a single system, such as a Schrödinger equation in isolation, are intrinsically incomplete.
[9] The mis-named "measurement problem" is this mismatch of perspectives.

4.17. I, as carrier of thought and knowledge, am a set of values of a set of variables that commute.[10]

5. **Perspectives are transparent to each other because they are facts.**

5.1. An act of measurement is a special case of an ordinary physical interaction.

5.2. A system B can interact with a system A in many different manners. Accordingly, A may admit different layers of descriptions, all relative to B.

5.3. A perspective is accessible from another perspective, because it is simply the value of a family of (commuting) variables.

5.4. The self is the reflective point of a knowledge that includes information about itself.[11]

5.5. Value attribution, such as $A = a$, is a component of a language.

5.6. The riddle of consciousness is the result of mistakenly assuming that a perspective is not a physical fact for any other perspective.

6. **There is nothing wrong in circularity.**

6.1. Understanding perspective from other perspectives leads to circularity.

6.2. Escaping circularity is not physical possibile, is logically incoherent and has no interest. It is the left over of an outdated metaphysics that limps in trying to make sense of new knowledge about the real world.

6.3. The world of any perspective has a blind spot: the physical variables that embody this very perspective.

6.4. Physics itself is in a perspective.

6.5. Within any perspective, circularity is broken indexically by that very perspective.[12]

6.6. Information is always incomplete.

6.7. The limits of my information are the limits of my world.

6.8. The world is open.

6.9. We can represent facts in a perspective with a proposition.

6.10. All this is a useful conceptual structure compatible with current knowledge. Concepts change with learning. Any conceptual structure to frame reality is provisional.

7. **What we do not have information about, we must pass over in silence.**

Long

1. **The world is everything that is the case from some perspective.**

1.1. Everything that is the case, is the case from some perspective.

1.2. Something that is the case from a perspective is a fact, from (or in) a perspective.

1.2.1. Each fact is a fact from a perspective.

1.2.1.1. The question whether there are facts outside a perspective has no meaning.

[10] My personal knowledge is a physical configuration of my brain.
[11] Ismael (2006)
[12] Ismael (2006)

1.2.2. A world is a collection of facts, not things.

1.2.3. If you remove facts from a world, nothing remains.

1.3. Our perspectives are special cases of the physical notion of perspective.

1.3.1. Perspectives are not specific to us, biological systems, societies, agents, computing systems, or similar.

1.3.2. A perspective, in general, is not associated with mental or similar capacities.

1.3.3. What is commonly called "an observer" is associated with a perspective, but not vice versa: the expression "observer" is commonly associated with restricted cases of a general perspective.

1.4. Perspectives are not about minds. They are about physics, like reference systems in relativistic physics are about physics, not about minds.

1.4.1. At this point, an attentive reader might be led to question the absoluteness of physical rules; this issue will be addressed in 6.4.,as we must climb the ladder one rung at a time in order to fully grasp the lesson proposed by physics.

1.5. No perspective stands out above the rest.

1.5.1. The reality of which we speak is the partial reality from within a perspective.

1.5.2. The reality of which we speak can never be the total reality of all possible perspectives.[13]

1.5.2.1. Reality consists in states of affairs,[14] but states of affairs do not exist outside a perspective.

1.5.2.2. The question of whether a perspective-independent ("absolute") reality exists, is a question without meaning.

2. **My knowledge about the world is a particular kind of perspective.**

2.1. My knowledge of the world is based on information I have or can have about the world.

2.1.1. My (evolving) information about the world is my access to the world.

2.1.1.1. My perspective is my access to the world.

2.1.1.2. My (evolving) information traces my perspective on the world.

2.1.2. I can think of the world as empty, but this emptiness would still be counted as information I have about it.

2.1.3. I can think about parts of the world I do not have information about, but their possibility is still information I have or can have.

2.2. Information has degrees of reliability.[15]

2.2.1. Degrees of reliability can be quantified: $p(a) \in [0,1]$ quantifies my expectation that a possible fact a is going to be the case if I acquire information about it. $p(a) = 1$ indicates certainty that a is the case, and $p(a) = 0$ indicates certainty that a is not the case.

2.3. A fact in my perspective is knowledge about which I (reasonably) estimate no uncertainty.

2.3.1. A fact from my perspective is an a such that $p(a) \approx 1$ and indicates (an almost) certainty that a is the case.

2.3.1.1. "There is a cup on the table I am looking at" is an example of such a fact.

[13] Cfr. Laozi and Ivanhoe (2003), 1.1.
[14] As expressed by Wittgenstein (1922) proposition 2.01.
[15] Cfr. De Finetti (1989).

2.3.1.2. What I know with (sufficient) certainty constitutes a fact relative in my perspective.

2.4. Degrees of reliability assigned to facts evolve; perspective can change accordingly.

2.4.1. a is a fact if I expect that new information will confirm it.

2.4.1.1. Once something is established as a fact, observing it again yields no additional information.

2.4.1.2. A fact is what we call an element of physical reality.[16]

2.5. To have uncertain information about a fact means to have the information that by gathering more information the fact will be ascertained with a known likelihood.

2.5.1. An a such that $0 < p(a) < 1$ is an event about which I have information that lacks certainty.

2.6. If I reliably expect that one out of N mutually exclusive alternatives a will be the case, then I can talk about a set of alternative possible facts.

2.6.1. If reliably expect that one out of N mutually exclusive alternatives a to be the case, then $\sum_a p(a) = 1$.

2.6.2. The uncertainty in the probability $p(a)$ can be quantified by $H(p) = -\sum_a p(a) \ln p(a)$, which is called the Shannon's entropy of $p(a)$.[17]

2.6.3. If I know which of N alternatives is a fact, I have an information $I(N) = \ln N$, Shannon's first definition of amount of information.[18]

2.6.4. The amount of information in a distribution $p(a)$ over N alternatives can be quantified by $I(p) = I(N) - H(p)$. This vanishes if I do not know anything about which a is realized ($p(a) = constant$), and is maximal ($\ln N$) if I know which a is realized ($p(a)$ peaked on a single value).

2.6.4.1. $p(a)$ expresses physical possibilities. No metaphysical possibility here, only physics.

2.7. Facts in a perspective can be represented in logical spaces. They can be actual, or have probability.

2.7.1. Logical spaces are not the world. They are ways to represent perspectives.

2.7.2. A logical space is the totality of all logically possible states of affairs.

2.7.2.1. A logical space is a set of possible a's and possible $p(a)$'s

2.7.2.2. What lies outside logical space is not false but meaningless.

2.7.2.2.1. To fall outside logical spaces is not negated, it is outside possibility.

2.7.3. The selection of a logical space depends on the perspective. No logical space is privileged.

2.7.4. A fuzzy logical space[19] can account for the situation when we are unable to predicate facts with $p(a) = 1$.

2.8. My knowledge about the world does not create the world; it is determined by the world and it refers to it.

2.8.1. A perspective is not intended as a Dutch book.

[16] Cfr. Einstein et al. (1935), where an element of reality is defined in a very similar manner, as something I reliably expect to be true if observed.
[17] Shannon (1948)
[18] Shannon (1948)
[19] Zadeh (1965)

2.8.1.1. Facts a and their probabilities $p(a)$ form the physics of the world. They are well-defined also in the absence of credences or agents.

2.8.1.2. There is nothing in the physical explanation of the world to gamble on.

3. **Perspectives themselves are facts, when considered from another perspective.**

3.1. A perspective is embodied in a set of facts, when considered from any other perspective (including itself).

3.1.1. Here "Embodied" is not meant in the human or cognitive sense, but in a physical one: it applies to any physical object or system (as in 1.3.1..)

3.1.2. Perspectives are facts in two distinct senses: as the ensemble of actual facts *in* the perspective itself, and as the possible facts *embodying* the perspective, when seen in other perspectives.

3.2. If knowing b determines which a is a fact, then a is fact with respect to b, or in the perspective embodied in b.

3.3. If $p(a, b)$ is the joint probability for two sets of possible facts $A = \{a\}$ and $B = \{b\}$, then b embodies a perspective in which the probabilities of a are $p_b(a) = p(a|b) = p(a, b)/p(b)$.

3.3.1. a is a fact relative to b, or is a fact in the perspective b, if $p_b(a) = 1$.

3.3.2. Notice that a can be a fact relative to b (namely $p_b(a) = 1$) whether or not it is a fact relative to me, to which the probabilities $p(a, b)$ refer (that is, whether or not $p_a(a) = 1$). Indeed, a is a fact relative to b if it is a fact, but also if I have no certainty about a but I could get it by looking at B. (Namely $I_{A:B}$ is maximal, see 3.5..)

3.4. To know *what* B knows about A is not the same as knowing *that* B knows about A.

3.4.1. The second is a fact about B and the a's; the first is an additional fact about B.

3.4.2. The second is knowledge of a correlation; the first about its terms.

3.4.3. If we can get information on a set of facts $A = \{a\}$ by observing facts in a set $B = \{b\}$, then B embodies a perspective on A.

3.4.3.1. The above is the case when A and B are correlated: knowledge of one informs about the other.

3.4.3.2. If $p(a, b)$ is the probability for the couple (a, b) to be the case, then the marginals $p_A(a) = \sum_b p(a, b)$ and $p_B(b) = \sum_a p(a, b)$ are the respective probabilities for a to be the case and for b to be the case, independently from one another; and *the* amount of correlation in $p(a, b)$ is measured by the mutual information $I_{A:B} = I(p) - I(p_A) - I(p_B)$.

3.5. The mutual information $I_{A:B}$ quantifies the information on one variable that can be obtained from the other.

3.5.1. By measuring b, the average increase in information about a, is given precisely by $I_{A:B}$.[20]

3.5.2. Mutual information quantifies the reduction in uncertainty about one random fact provided by knowledge of another.

3.5.2.1. Mutual information is a form of relative information.

3.6. When the relative information $I_{A|B}$ is maximal, knowing b determines which a is a fact, and a is a fact relative to b.[21]

[20] Di Biagio and Rovelli (2025)
[21] Di Biagio and Rovelli (2025)

3.6.1. This use of "fact" is consistent with 2.3 but refers to knowledge about the perspective B from a different perspective.

3.6.2. A fact about a system in a perspective is accounted for, in any other perspective, either as a fact or, if not a fact in this other perspective, as relative information.

3.6.3. What we are grasping here is the physical embodiment of perspectives, seen from other perspectives.

3.7. All things stand only in relation to a perspective, and with a shift of perspective, what once appeared as fact may be absent or incompatible with other facts from another perspective.

3.7.1. All facts are relative facts.

3.8. A subject is a (case of a) perspective and defines a world, as said in 1.3..

3.9. From a perspective distinct from mine, I, a subject, am a physical process, namely a set of facts.

3.9.1. I, as carrier of thought and knowledge that instantiate a special case of perspective, am a set of facts both from my own perspective and in other perspectives. These facts concern my physical brain and its correlations with the else.

3.9.2. Knowledge is physically embodied. There is no knowledge that is not embodied.

3.9.3. My knowledge of the world is a set of facts regarding the physical process I am, and correlations with the rest of the world.[22]

3.10. Knowledge is physically underpinned by relative information between variables of two systems: the subject and the object of knowledge.

3.10.1. Knowledge can appear as a perspective for the subject and as correlation from an external perspective.

3.10.2. In physical interactions and more in general in time, knowledge either is acquired or lost.

3.10.2.1. Nothing different from physical interactions and time evolution happens in an agent or object of knowledge when knowledge is acquired or lost.

3.10.2.2. Experience is a physical phenomenon, in which knowledge changes.

3.10.2.3. Phenomenology is same, not distinct, from physics. The two are just different renderings of the same reality.

3.10.2.4. The relation between experience and physical reality has been an enduring riddle in philosophy. Conceiving them as distinct a priori, rather than continuous, is the mistake: they are the same phenomenon seen in different perspectives. The answer to the riddle is that there is no riddle.

3.10.3. In the classical limit, the concept of information acquires an epistemic dimension: the observer's –possibly partial– knowledge about the objective perspective-independent state of affairs.

3.10.3.1. The sharp distinction between facts and information about them makes only sense assuming absolute states of facts (see 4.8.4.3.).

3.10.3.2. In the quantum world, mutual information is always a measure of physical correlations.

3.10.3.2.1. The von Neumann's entropy of a density matrix ρ quantifies physical correlation.

3.10.3.2.1.1. By contrast, the measure introduced in 3.10.3.2 quanti-

[22]The emphasis on this point is novel with respect to Wittgenstein (1922). Knowledge and language are themselves physical facts, they can be also understood as such, and doing so clarifies.

fies entanglement correlations between subsystems, rather than epistemic knowledge.

3.10.3.3. We can recover of an epistemic notion of information in the sense of comparing –say– limited information on a system available in a perspective, with richer information about the same system available in –say– our own perspective.

3.10.3.3.1. For example when we quantify our knowledge of a quantum system, through the von Neumann entropy of the global state, we are adopting once again an epistemic notion of information.

3.10.4. Meaning is evolutionarily — or biologically — relevant information.[23]

3.10.4.1. Physics and biology together dispel the apparent magic in meaning.

3.10.4.2. Physics clarifies the circular relation between knowledge as an epistemic concept and physical information in the physical world.

3.10.4.3. The term "information" refers both to the gain in knowledge of a subject and to the correlations between systems. The latter is what we measure with relative information. The two are the same from different perspectives.

3.10.4.3.1. Information is not a primitive ontological ingredient; it is a structural feature of the correlations between systems.[24]

3.11. The world for me is the world I know or can know, and my knowledge is part of this world.

3.11.1. There is nothing wrong in self-consistent circularity (see 6, below).

4. **Facts can be expressed as values of variables, which are ways systems interact.**

4.1. Properties are facts expressed by values a of a variable A. The fact that A has value a is expressed as $A = a$.

4.2. Systems are collections of variables that admit approximate closed dynamical descriptions (see 4.3.1.).

4.3. Variables are ways systems interact and manifest themselves in interactions.

4.3.1. Values of a variable A of a system label manners in which a different system can be affected in a certain interaction with the first system.

4.3.1.1. As facts are relative, so variables are the manners for a system to manifest itself in interacting with other systems.

4.3.1.2. Measurements are cases of interactions leaving long-lasting traces, which increase the correlation between two systems.

4.3.1.2.1. These traces are not rare; they are not, like *Dasein*[25], specific of humans. They are generic effects of physical interaction upon systems.

4.3.1.3. The question "what is the value of a variable A of a system?" Formulated without (explicitly or implicitly) a perspective, is as meaningless as the question "what is the velocity of an object?" formulated without (explicitly or implicitly) a reference.

4.3.1.3.1. To ask about a property of a system apart from any perspective is to pose a question that has no meaning. Whatever is the case, is so from a perspective. Thinking otherwise is a misleading metaphysical assumption.

[23] Rovelli (2018) and Arcas (2025).
[24] This marks a departure from the information-theoretic interpretation proposed by Bub and Pitowsky (2010).
[25] (Heidegger, 1962).

4.3.1. A dynamical description is a mathematical model of how values of variables change and are manifested. This is also called a "dynamical law".

4.3.2. Physics is *modal*: the general form of dynamical laws is: *if a then b* with a certain reliability. That is, dynamical laws assign values to $p(b|a)$: likelihood of b given a.

4.3.2.1. Physics is not descriptive. It does not state or prescribes what *is* the case. It states what *may* be the case if something else is the case. It restricts the space of possibilities; it does not fix one single possibility.

4.3.2.2. Modality is not a signal of antirealism. It is the expression of the regularities we have found in the real world.

4.4. Variables in a given perspective may have no value.[26]

4.4.1. If a variable A has no value and $p(a) + p(a') = 1$, then it may be that there is a c such that $p(c \mid a)p(a) + p(c \mid a')p(a') \neq p(c)$. This is called *interference*.[27]

4.4.1.1. The above is not a contradiction, because it may be impossible to ascertain $p(c)$ and $p(a)$ independently (see below). Hence $p(a)$ and $p(c)$ refer to two different situations.

4.4.1.2. In this case, we say that a and a' are "in quantum superposition".

4.4.1.2.1. The term comes from quantum phenomena, where it is the case that a system may exist in a superposition of states.

4.4.2. $A = a$ can be a fact in a perspective, and not be a fact in another perspective.

4.4.2.1. $A = a$ can be a fact in our perspective and not be a fact in another perspective.

4.4.2.1.1. The possibility in 4.3.2.1 is realized, for instance, if in the other perspective we are outselves a term "in a quantum superposition".

4.4.2.1.2. If a is a term in a quantum superposition in a perspective, a is not a fact in that perspective.

4.4.2.1.3. The point 4.3.2.2. is what forbids us to take facts that we observe as absolute, beyond our perspective. These are the limits of naïve realism.

4.5. In general, not all variables of a system can simultaneously have a value.

4.5.1. Two variables that can always have values together are said to *commute*.

4.5.1.1. Systems generically have variables that do not commute.

4.5.1.2. If two variables A and B commute, we can always obtain new information about A without affecting the information we have about B. Not so if they do not commute, because in this case since they may not have values together, getting information about one can render the information we had about the other unreliable.[28]

4.6. The mathematical theory allowing physical laws to be expressed associates elements A of a non-commutative algebra to the variables A of system.[29]

4.6.1. A non-commutative algebra is a set where addition and multiplication are defined but, in general, $AB \neq BA$. This reflects the fact that, in getting information about A and B, the order in which we do so matters.

[26] This aspect of the world was not known before the discovery of quantum phenomena.
[27] This characterizes quantum phenomena (quantum interference).
[28] Heisenberg's uncertainty.
[29] This is not true if we disregard quantum phenomena, as Wittgenstein did.

4.6.2. The eigenvalues of an algebra element A are the possible values a that the variable A can take.[30]

4.6.3. The facts that embody a perspective are represented by simultaneous values of variables. Therefore a perspective is defined by an *abelian subalgebra* of the algebra of variables. An abelian subalgebra is a linear subset of an algebra formed by elements that commute (such that $AB = BA$).[31]

4.6.3.1. Information is carried by values of abelian subalgebras.

4.6.3.2. A perspective is given by the facts and the probabilities relative to the values of the variables of an abelian subalgebra of a system.

4.6.4. In a given perspective, the "state" of another system is the information about that system available in that perspective.

4.6.5. The state of a system relative to a perspective is given by a *linear* functional ρ (a "quantum state") on the algebra of the variables of this system. Given the spectral decomposition $A = \sum_a a\,\Pi_a$, of a variable A, the probability $p(a)$ is given by $p(a) = \rho(\Pi_a)$. For two commuting variables A, B, $p(a, b) = \rho(\Pi_a \Pi_b)$. This shows that the quantum formalism includes the precise mathematical tools for defining relative facts, as detailed in 3.6..

4.7. Mathematical laws associated to a system relate variables — hence facts — relative to the same system, within the same perspective.

4.7.1. Mathematical laws limit how values of variables can change together.

4.7.1.1. They make the world partially predictable and intelligible.

4.7.2. A world has a logical form made by a family of possible relative facts $a, b, \ldots$ and likelihoods $p(a)$ given by conditionals $p(a|b)$. This allows any sufficiently competent agent to have expectations about what new knowledge could bring.

4.8. A variable having a value in a perspective is an *atomic fact*.

4.8.1. An atomic fact is the simplest entity.

4.8.2. An atomic fact involves always a variable and a perspective.

4.8.2.1. This is the most fundamental way to have an atomic fact; nothing can be made in a less structured way.

4.8.2.2. We say that systems and in particular objects (which are bundles of properties[32]) exist, insofar as they are manifest in a perspective (a relation). This is a decisive break from the older conception of things.[33]

4.8.3. An atomic fact is the possibility of the value of a variable occurring in a perspective.[34]

4.8.4. An atomic fact a is accounted for as an eigenvalue of an algebra element A.

4.8.4.1. The totality of actual atomic facts in a perspective is the world from that perspective.

4.8.4.1.1. We can distinguish two kinds of atomic facts, following the lesson learned from the Master.

4.8.4.1.2. *Sachverhalte* are the atomic facts as defined in 4.7.2., the possible configurations within logical space.

[30] The eigenvalues of an algebra element A are the real numbers a for which $(A - a\,\mathbf{1})$ has no inverse.
[31] Di Biagio and Rovelli (2025).
[32] (Simons, 1994).
[33] See Wittgenstein (1922), §3.25.
[34] See Wittgenstein (1922), §2.0123.

4.8.4.1.3. *Tatsachen* are the *actual atomic facts* that make the world always a set of facts that are the case (see 1.2.)

4.8.4.2. All perspectives are themselves in a space made of possible atomic facts. I can think of this space as empty, but not of a perspective without this space.

4.8.4.3. It would then be impossible to draw up a picture of the world without a perspective .[35]

4.9. Combinations of systems are systems.

4.9.1. Facts regarding combined systems may sometimes be determined by facts regarding the individual systems that compose them. But not always so.[36]

4.10. Since variables express the way a system affects another in interactions, dynamical laws, which are about variables, regard interactions between systems.[37]

4.10.1. Dynamics regards relations.

4.10.2. In the non-relativistic limit, dynamical laws can be expressed as evolution in a preferred time variable.

4.11. The information about a system A in a perspective B changes when A and B interact. In the perspective B, this is an update of information. In any other perspective, this is an evolution of the mutual information that is continuous in time.[38]

4.12. The maximum information available about a system is achieved when a maximal set A of commuting variables has values that are facts.

4.12.1. When this is the case, Shannon entropy is minimal.

4.12.2. The maximum information available about a system is insufficient to determine the further information that the system can provide, because values of variables that do not commute with the known ones are not facts. Interactions with these variables bring about genuinely new information. This does not increase the maximum information about the system because it renders previous information irrelevant: some facts cease to be facts.[39]

4.12.2.1. A fact ceases to be a fact once its probability $p(a)$ becomes less than one. This can happen after a new interaction, or simply as time passes.

4.12.3. A perspective may be understood as the information concerning facts relative to a system, which in a particular can be (but needs not to be) a complex observer.

4.13. In a perspective B, the facts making up the world are values of sets of variables A. From a different perspective, the information that B has is embodied in values of the commuting set of variables defining B and the fact that these are correlated to the A's.

4.14. I, as carrier of thought and knowledge, am a set of values of a set of variables that commute.[40]

5. **Perspectives are transparent to one another because they are facts.**

5.1. An act of measurement is a special case of an ordinary physical interaction.

[35] See Wittgenstein (1922), §2.0212.

[36] Failure of this is called entanglement. It does not happen if we disregard quantum phenomena.

[37] Hence dynamical laws viewed as pertaining to a single system, such as a Schrödinger equation in isolation, are intrinsically incomplete.

[38] The mis-named "measurement problem" is this mismatch of perspectives.

[39] The two postulates in axiomatic formulation of RQM, Rovelli (1996) are (i) system's relevant information is limited, but (ii) new information can always be acquired.

[40] My personal knowledge is a physical configuration of my brain.

5.1.1. What I know or can know about the events a and b is given by their joint probability $p(a, b)$. From this, the perspective of b on a is defined by the conditional probabilities $p(a|b)$.

5.1.2. Knowledge, as stated in 3.10. is physically underpinned by a physical configuration correlated with the part of the world it concerns. Because it is physical, it is accessible from other perspectives through interaction. No perspective lies beyond the physical.

5.2. A system B can interact with a system A in many different manners. Accordingly, A may admit different layers of descriptions, all relative to B.

5.2.1. The same subject can have information about the same system organized in different layers.

5.2.2. Layers in the world are approximate descriptions of the same set of facts[41] reflecting different modes of physical interaction.

5.2.2.1. For instance, "The forest has grown" and "New trees are born near the old ones" refer to two layers of description of the same events.

5.2.2.2. Different layers define different logical spaces and may, though not necessarily, require different logics.

5.2.3. The world admits different layers of description.

5.2.4. Layers are distinct and have diverse efficacy, but are never in contradiction. Nature is coherent.

5.3. A perspective can be accessible from another perspective, because it is simply (embodied in) the value of a family of (commuting) variables.

5.4. My perspective is my world.

5.4.1. Humans are systems that embody perspectives, but are peculiar on many grounds.

5.4.2. My own perspective is a limit of my world. The variables where its information is stored are accessible by any other perspective.

5.5. The self is the reflective point of a knowledge that includes information about itself.[42]

5.5.1. Knowledge about ourselves is always vastly incomplete.[43]

5.5.2. Values of variables, including those forming a perspective, hence knowledge, hence the self, are physical phenomena.

5.5.3. Perspectives are physical facts: there is no fact that is not also a physical fact. Yet, most phenomena are far better described using the language of special layers, because as physical facts they are obscure. Appropriate language makes them clear and understandable.

5.5.3.1. Thought is a physical phenomenon; it can in principle be described as a set of facts.[44] Ethics is a physical phenomenon; it can be also be in principle described as a set of facts. But they are both phenomena better described in appropriate layers as in 5.2..

5.5.3.1.1. That each layer admits its own logical space does not divide the world into distinct phenomena.

5.5.3.2. The transparency of a perspective to another does not need to be realized at the physical level. It works more easily at higher levels, with the associated

[41] For instance: physical, chemical, biological, functional, psychological, social, ...
[42] Ismael (2006)
[43] (Spinoza, 2020).
[44] (Spinoza, 2020). This is also what neurosciences aim to do.

uncertainty. This is called "communicating".[45]

5.6. Value attribution, such as $A = a$, is a component of a language.

5.6.1. A language can express the knowledge in a perspective.

5.6.2. A language is a set of enunciations.

5.6.2.1. Enunciations are propositions expressed according to the rules of the logic that underlies the chosen perspective.

5.6.3. An enunciation in a language is itself a fact in a world. It is not outside it.

5.6.4. No enunciation exists unless it is enunciated.

5.6.4.1. There is no un-embodied language.[46]

5.6.4.2. A world lives in a logical space, and any logical proposition is a fact in a world.

5.6.5. A proposition shows the logical form of a perspective.

5.6.5.1. No unique logical form can be assumed; the form of logic itself depends on the layer under consideration.

5.6.6. The general form of physical knowledge can be expressed both in laws of the form $p(a|b)$ and in the structure of logical spaces.

5.7. The riddle of consciousness is the result of mistakenly assuming that a perspective is not a physical fact for any other perspective.

5.7.1. The hard problem of consciousness stems from assuming, not from observing, that there are facts that are not physical facts. All facts are simply facts in a perspective.

5.7.1.1. There are no physical facts outside a perspective, nor perspectives which are not physical facts.

5.7.1.2. It is not the first-person perspective that is mysterious. It is the absolute third-person perspective that is non-existing.

6. **There is nothing wrong in circularity.**

6.1. Understanding perspective from other perspectives leads to circularity (as in 3., above).

6.1.1. Circularity in understanding occurs when some phenomenon is accounted for by assuming the phenomenon itself (perspectives can be accounted for from other perspectives).

6.1.2. Searching for an escape from circularity by regressing to larger and larger perspectives leads to an infinite regress.

6.1.3. Escaping circularity is a request for more than relative information: it is a search for un-embodied information. But the idea of un-embodied information is a metaphysical dream (dreamed by embodied creatures).

6.2. Escaping circularity is not physical possibile, is logically incoherent and has no interest. It is the left over of an outdated metaphysics that limps in trying to make sense of new knowledge about the real world.

6.3. The world of any perspective has a blind spot: the physical variables that embody this very perspective.

6.3.1. These same variables can be accounted for from another perspective. But there is no way to account for all perspectives, because of 1.4.2, 3.10. and 5.4. A world is only such with respect to something physical, when quantum phenomena are not disregarded. This

[45] Watson (2003), 17.

[46] This is the the central difference from Wittgenstein (1922).

gives rise to the circularity of making sense of perspective, from other perspectives, as correlations.

6.3.2. Circularity appears to be a problem in logical analysis only when trying to make claims outside any perspective. Such claims cannot be made by any embodied speaker.

6.3.2.1. On logical analysis, circularity is already employed in set theory through the theorem stating that "every graph has a unique decoration."[47] In this way, it becomes possible to define a graph in terms of itself, something forbidden in Zermelo–Frankel set theory. This is useful circularity.

6.4. Physics itself is in a perspective.

6.4.1. In physics, we use variables, functions and rules, and mistakenly consider these absolute. This gives rise to the wrong idea of a series of absolute rules outside any perspective. But the rules, in fact, are part of our knowledge, and as such perspectival themselves.

6.5. Within any perspective, circularity is broken indexically by that very perspective.

6.5.1. The idea of a single fundamental perspective from which everything would flow is flawed.

6.5.2. It is the idea of a universal foundation of reality that is flawed.

6.6. Information is always incomplete

6.6.1. It is always possible for a system to acquire new information.[48]

6.6.1.1. Quantum non-predictability renders information permanently incomplete.

6.6.1.2. Interactions between systems change correlations and mutual information rendering information widespread and ever-changing.

6.6.1.3. The emergence of new correlations yields new information and changing perspectives.

6.6.1.4. The world cannot be described from a closed standpoint.

6.6.1.5. It is never the case that a new interaction fails to yield information. When we speak of "no new information," this is only because we have forgotten that the observer's knowledge is itself a perspective.

6.6.2. Information is always partial because it is incomplete and perspectival.

6.7. The limits of my information are the limits of my world.

6.7.1. The limits of my world move continuously, because I learn and my information changes.

6.7.2. My knowledge is always incomplete.

6.8. The world is open.

6.8.1. My world is my perspective.

6.8.2. We know with confidence that reality is wider than the information we have about it. Lack of information does not imply knowledge of absence. Nor does it implies knowledge about what is not known.

6.8.3. To think that we can have a picture of all facts from all possible perspectives is childish.

6.8.4. Our ignorance of what we do not know and could learn is not lack of knowledge in the sense of Shannon's theory, because we do not have knowledge of a fixed space of possibilities for what we don't know.

6.9. We can represent facts in a perspective with a proposition.

[47] Aczel (1988) chapter 6.

[48] This is the second postulate in axiomatic formulation of RQM.

6.9.1. A set of propositions is a model.

6.9.2. A model is not reality. It is a shard of reality.

6.9.2.1. Looking through a model is to see reality from a point of view, like from a part of a mirror.

6.9.2.1.1. The mirror is never whole.

6.9.2.1.2. It is not ignorance that makes the mirror incomplete, but the perspectival nature of reality.

6.9.3. The idea of the ensemble formed by the totality of all perspectives, and of the facts within them, does not belong to reality, for it is not contained within any perspective.[49]

6.9.3.1. Quantum phenomena reveal what should have been evident all along: since any knower is part of reality, all knowledge is necessarily perspectival.

6.9.3.2. Quantum phenomena only block a wrong metaphysical jump that is not granted by anything but seemed to allowed by classical physics: the idea of accessing a completely perspective independent reality.

6.9.4. By reality we may indicate the content of any perspective, or the idea of all perspectives, because these can be the case from a perspective, but not the collection of all perspectives, which is not something that is the case from any perspective.

6.9.5. The totality of the perspectives cannot be a perspective because it lacks embodiment.

6.9.6. The ghost of an absolute unaccessible reality is useless, because it plays no role for us except for troubling us for nothing.

6.10. All this is a useful conceptual structure compatible with current knowledge. Concepts change with learning. Any conceptual structure to frame reality is provisional.

6.10.1. Hence all of the above is just a perspective on perspectives, and as such partial. It is not a fundamental view.[50]

6.10.2. Anything posed as ultimately fundamental, or ultimately primary, is uninteresting.

7. **What we do not have information about, we must pass over in silence.**

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[49] The emphasis on this point is connected to Wittgenstein (1922) (Proposition 5.6), where the author reflects on the limits of language: the boundary between what can be said and what can only be shown.

[50] (Siderits, 2022) MMK.13.8.

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