Right for the Wrong Reasons

Common Bad Arguments for the Correct Answer to the Monty Hall Problem

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Abstract: The answer to the Monty Hall Problem that many people—including some well-trained mathematicians—initially give is incorrect. Nonetheless, there is little controversy among mathematicians and philosophers about what the correct answer is. However, many different arguments have been given for this answer. Although Bayes's Theorem is the gold standard for carrying out probabilistic inferences, many mathematicians and philosophers try to give shorter and more intuitive arguments for the correct answer to the Monty Hall Problem. Unfortunately, as we argue in this paper, an unconscionably large number of these shortcut arguments involve bad reasoning. Thus, many people end up believing "the right answer for a wrong reason." Moreover, since these arguments only yield the correct answer in a very restricted range of cases, people learn techniques that lead to false conclusions when they are applied to many other probabilistic inference problems. In this paper, we identify three distinct bad arguments for switching in the Monty Hall Problem that are commonly given by quite reputable sources. We show that these arguments yield incorrect answers when applied to slight variations of the Monty Hall Problem. And we identify exactly where each argument goes wrong. We argue that it would be much better to simply teach people to ask how likely the evidence is given each of the hypotheses.

Keywords: Bad Arguments, Bayes's Theorem, Counterexamples, Monty Hall Problem, Probabilistic Inferences.

> *If you want to think like a mathematician here, you don't just care about finding the answer. You care about developing general problem solving tools and techniques.*
> — 3Blue1Brown

> *Rigorous teachers complain that the incomplete demonstration is an intellectual fraud.*
> — Roy Sorensen

1. Introduction

There are many questions about the world that we are uncertain about, such as what the weather will be like tomorrow or who will win the next election. And it is rare that we can find evidence that definitively settles the matter. But we can often at least reduce our uncertainty on the basis of evidence. Unfortunately, humans are notoriously bad at such reasoning (see, e.g., Casscells 1978, Titelbaum 2022, 170). One example that has received a lot of attention in the popular imagination is the Monty Hall Problem.[1] The intuitive answer that most people initially give to this puzzle is incorrect. Even many well-trained mathematicians have vehemently defended this wrong answer (see Rosenhouse 2009, 24-25).[2]

Given the controversy that it has generated, numerous mathematicians and philosophers have undertaken to explain the reasoning that yields the correct answer to the Monty Hall Problem. The gold standard for carrying out probabilistic inferences is, of course, Bayes's Theorem (see, e.g., Weisberg 2019, ch. 8, Titelbaum 2022, ch. 4). However, many quite reputable sources have tried to give shorter and more intuitive arguments for the correct answer to the Monty Hall Problem. Unfortunately, as we argue below, an unconscionably large number of these *shortcut* arguments (e.g., Bruce 2001, Gardner 2001, Clark 2002, Devlin 2003, Sorensen 2003, Winkler 2004, Nihous 2009, Rosenthal 2009, Martin 2011, Kahn Academy 2012, Champkin 2013, Li 2013, Pynes 2013, Talwalkar 2013, Goldberg 2014, Gessell 2015, Hájek and Hitchcock 2016, Brilliant 2018, Huemer 2018, Stewart 2019, Cook 2020, Adams 2022, Bollobás 2022, Titelbaum 2022, University of Illinois 2023, Bellos 2024) involve bad reasoning.[3] To use Terrence Horgan's (1995, 219) nice phrase, many well-trained mathematicians and philosophers continue to defend "the right answer for a wrong reason." Thus, to use Roy Sorensen's (2016b, 133) nice phrase, they lure *us* into "following the wrong path to the right conclusion."

The problem with these shortcut arguments is that they fail to take into account all of the relevant evidence in the Monty Hall Problem. In order to carry out probabilistic inferences correctly, it is critical to ask how likely our total evidence is given each of the various

[1] See Rosenhouse 2009, ch. 1 for a history of the Monty Hall Problem. It has even been featured in films, such as *21*, and on television series, such as *Brooklyn Nine-Nine*, *Numb3rs*, and *Survivor*.
[2] Even Paul Erdös reportedly got it wrong (see Rosenhouse 2009, 54-55, Bollobás 2022, 182).
[3] Admittedly, the shortcut arguments given in film and on television tend to be even worse.

hypotheses. This question is asked implicitly in Bayes's Theorem. And some mathematicians and philosophers (e.g., Bradley and Fitelson 2003, Rosenthal 2008, Rosenhouse 2009, Pawitan and Lee 2024) make it very explicit in their discussions of the Monty Hall Problem. However, the shortcut arguments do not ask this question. As a result, even though they give the correct answer in the Monty Hall Problem, they give the wrong answer in many other probabilistic inference problems. So, people can easily end up with false beliefs if they apply such reasoning more broadly.

In sections 2 and 3, we describe the Monty Hall Problem and offer good (and fairly intuitive) reasoning that leads to the correct answer (the **Favoring Procedure**). In sections 4 and 5, we give a common argument for the correct answer (the **Wi-Phi Probability Concentration** argument) and a counterexample to it (the **Random Monty** variation) that a few critics have proposed. We also identify exactly where the argument goes wrong. In section 6, we give two other common arguments for the correct answer (the **Strengthened Probability Concentration** argument and the **Wi-Phi Probability Swap** argument) which avoid the **Random Monty** counterexample. However, in section 7, we show that there are other counterexamples to all three arguments (the **Lazy Monty** and **Unequal Monty** variations) and we identify exactly where the arguments go wrong. Finally, in section 8, we show that the bad reasoning embodied in these arguments can lead to errors in a wide variety of probabilistic inference problems.

2. The Monty Hall Problem

The Monty Hall Problem was first posed by Steve Selvin (1975) in a mathematics journal.[4] But its most famous incarnation was when it appeared in Marylin vos Savant's (1990) widely-read magazine column (distributed in many weekend newspapers):

The Monty Hall Problem:

> "Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?"

As it stands, this statement of the puzzle (from vos Savant's column as posed by Craig F. Whitaker) is a bit ambiguous. First, while it specifies that there are three possible locations for the car, it does not say anything about how likely each of those locations is to conceal the car. Second, while it specifies that the host Monty opens a door—different from the door that you initially chose—and reveals a goat, it does not say that Monty was guaranteed to do so. Third, even if we assume that Monty must open a door, must not open the door that you initially chose, *and* must not reveal the car, this statement does not specify how Monty chooses when he still has a choice about which door to open.[5] Finally, while it specifies that, after he opens a door and

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[4] The Monty Hall Problem is structurally equivalent to the older Three Prisoners Problem (see Morgan et al. 1991, 284, Gardner 2001, 283, Rosenhouse 2009, 14).

[5] Even if he is not allowed to open the door that you initially chose, Monty can always open a door to reveal a goat. Since there are two goats, there is guaranteed to be a goat behind at least one of the two remaining doors. And if you initially chose the door with the car behind it, Monty has a choice about which door to open.

reveals a goat, Monty gives you the opportunity to switch to the last remaining door, it does not say that Monty was guaranteed to do so.[6]

Moreover, we get different answers to the question of whether you should stick or switch depending on how these ambiguities are resolved (see Bar-Hillel 1989, Nickerson 1996).

However, the ambiguity of this statement of the Monty Hall Problem is not a serious difficulty. Logic and probability puzzles are supposed to be fun. It would be tedious to always have to state them in a formal language (such as first-order predicate logic). Thus, puzzles are often presented in ways that fail to eliminate absolutely all ambiguity. Even so, it is usually clear how we are intended to resolve these ambiguities (see, e.g., Devlin 2010). For example, it is reasonable to assume that the events described in the puzzle occur every time that the game is played. Namely, we can assume that Monty always opens a door, that it is never the door that you initially chose, that there is always a goat behind it, and that Monty always gives you the opportunity to switch.[7] In addition, whenever we are not told that different possibilities have different probabilities, the default is to assume that they are equally likely. Thus, we can assume that the car is initially equally likely to be behind each of the three doors. Also, we can assume that, when he has a choice about which door to open, Monty chooses at random (i.e., each door is equally likely to be chosen).[8]

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[6] Maybe Monty only gives you this opportunity when the car is behind the door that you initially chose (see Rosenhouse 2009, 113).

[7] After all, since we are explicitly told that Monty knows where the car is, it is reasonable to assume that he uses this knowledge. Of course, we shouldn't assume that you initially choose the very same door each time or that Monty opens the very same door to reveal a goat. The doors are clearly just numbered in the statement of the puzzle for the sake of convenience.

[8] For example, if he has a choice between two doors, Monty might flip a coin. Of course, he would need to conceal from the contestant that he is flipping a coin. Learning whether or not Monty has a choice might reveal information about the location of the car.

Once we make these assumptions, there is a unique answer to the Monty Hall Problem. And this is the puzzle that most people are trying to solve.[9]

3. The Correct Answer to the Monty Hall Problem

The intuitive answer to the Monty Hall Problem is that it doesn't matter whether you stick or switch. Once Monty opens a door and reveals a goat, two possible locations for the car remain. Those two locations started out equally likely. And it seems that they are still equally likely after Monty opens a door and reveals a goat. While you learn that the car is not behind the door that Monty opens, it doesn't seem that you learn anything that distinguishes between the two remaining doors. After all, Monty was guaranteed to be able to reveal a goat no matter where the car is.

Moreover, there is a pretty sensible sounding procedure that yields the result that the car is equally likely to be in each of the two remaining locations. This procedure can be found in the episode of TED Ed—a collection of fun, informative, and largely reliable educational videos—on the "Frog Riddle" (Abbott 2016). This procedure can be applied to almost any probabilistic inference problem, including the Monty Hall Problem.

The TED Ed Procedure:

Step 1: Assign initial probabilities to each hypothesis in a set of mutually exclusive and jointly exhaustive hypotheses.

[9] As vos Savant (1991, 347) notes, "nearly all of my critics understood the intended scenario, and few raised questions of ambiguity" (see also Bar-Hillel 1989, 354).

In the Monty Hall Problem, at the outset, there are three possible locations for the car (door #1, door #2, and door #3) and they are equally likely. Thus, since the probabilities have to add up to 1, each of these hypotheses has an initial probability of 1/3.

Step 2: Eliminate any hypothesis that is inconsistent with the evidence.

In the Monty Hall Problem, the evidence (viz., Monty opening door #3 and revealing a goat) rules out the hypothesis that the car is behind door #3.

Step 3: Normalize the probabilities of the remaining hypotheses (so that they stay in the same ratios but add up to 1).

In the Monty Hall Problem, each of the remaining hypotheses has a probability of 1/3. So, they both get adjusted up to 1/2. Thus, it doesn't matter whether you stick or switch.

However, the intuitive answer to the Monty Hall Problem is incorrect. And the **TED Ed Procedure** involves bad reasoning.[10] Namely, it is not guaranteed to take into account *everything* that the evidence tells us. Basically, using this procedure, we run the risk of violating Rudolf Carnap's (1947, 138) "Principle of Total Evidence." In the Monty Hall Problem, the evidence doesn't just eliminate one of the hypotheses. The evidence also favors one of the two remaining hypotheses over the other. We can see this by asking ourselves how likely the

[10] The **TED Ed Procedure** yields the wrong answer in the Frog Riddle as well (see, e.g., Jongerius 2017, Volpi 2019).

evidence is given each of the hypotheses (see, e.g., Bradley and Fitelson 2003, Rosenthal 2008, 6, Rosenhouse 2009, 82-84, Pawitan and Lee 2024, 243-44).

If the car were behind door #1 (the door that you initially chose), there is a 50% chance that Monty would open door #3 and reveal a goat. This is because Monty has a choice about whether to open door #2 or door #3 and he chooses at random. If the car were behind door #2, there is a 100% chance that Monty would open door #3 and reveal a goat. This is because Monty has no choice about which door to open (since he is not allowed to open the door that you initially chose). If the car were behind door #3, there is, of course, a 0% chance that Monty would open door #3 and reveal a goat.

Since door #1 and door #2 started out equally likely, and since the evidence favors door #2 over door #1, once Monty opens door #3 and reveals a goat, the car is more likely to be behind door #2 than door #1. So, you should definitely switch to door #2.

We will call this the **Favoring Procedure**. In general, if the hypothesis that the evidence favors was at least as likely as any other hypothesis to start with, then that hypothesis is more likely than the other hypotheses given the evidence.[11] (And of course, if the evidence does not favor one hypothesis over another and those hypotheses were equally likely to start with, then those hypotheses are equally likely given the evidence.) Admittedly, this procedure does not tell us *how much* more likely that hypothesis is. But statements of the Monty Hall Problem (including vos Savant 1990) typically only ask whether the car is more likely to be behind the door that you initially chose or the last remaining door.

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[11] For example, suppose that we start off thinking that it is equally likely that the urn in front of us is predominantly filled with red balls or that it is predominantly filled with green balls. If we pick out a ball at random and it is red, it is now clearly more likely that the urn is predominantly red than that it is predominantly green.

Unlike Bayes's Theorem, the **Favoring Procedure** provides a very intuitive argument for switching. In addition, the **Favoring Procedure** explains where people go wrong when they arrive at the intuitive answer that it doesn't matter whether you stick or switch.[12] People tend to assume that the evidence (viz., Monty opening door #3 and revealing a goat) does not distinguish between the two remaining doors. The **Favoring Procedure** shows that this assumption is false.[13]

Now, there are some similar puzzles where we do get the correct answer by using the **TED Ed Procedure**. For example, consider a case (the **Random Monty** variation) that is exactly like the original puzzle except that Monty *always* chooses at random which door—of the two that you did not initially choose—to open.[14] Thus, while Monty actually opens a door and reveals a goat, he might have opened a door and revealed the car. As in the original puzzle, the **TED Ed Procedure** yields the result that, after Monty opens door #3 and reveals a goat, each of the remaining hypotheses has a probability of 1/2. Thus, it doesn't matter whether you stick or switch.

We can see that this is the correct answer in the **Random Monty** variation by using the **Favoring Procedure**. In **Random Monty**, the evidence *does not* favor any of the remaining hypotheses. Since he always chooses at random which door to open, Monty is just as likely to

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[12] Thus, even though the **Favoring Procedure** is not significantly more formal than the reasoning that leads to the intuitive answer, it is clear that we should trust the former over the latter.

[13] Basically, people are mistakenly applying the principle of indifference. See Weisberg 2019, sec. 18.2, Titelbaum 2022, 145-49 for discussions of the principle of indifference.

[14] As noted above, in the original puzzle, when you initially chose the door with the car behind it, Monty chooses at random which of the other two doors to open. But in **Random Monty**, Monty chooses at random which of the other two doors to open regardless of whether or not you initially chose the door with the car behind it. See Rosenhouse 2009, 57-75 for an analysis of this sort of variation.

open door #3 and reveal a goat if the car is behind door #1 as he is to open door #3 and reveal a goat if the car is behind door #2 (a 50% chance in each case). So, since door #1 and door #2 started out equally likely, if Monty opens door #3 and reveals a goat, the car is still just as likely to be behind door #1 (the door that you initially chose) as it is to be behind door #2.

However, even though the **TED Ed Procedure** yields the correct answer in **Random Monty**, we end up with "the right answer for a wrong reason." The **TED Ed Procedure** still involves bad reasoning. In particular, even though it yields the correct probabilities for the three possible locations for the car in this case, we never verified that the evidence does not favor any of the remaining hypotheses. And as Merrilee Salmon (1995, 70) notes, "the conclusion of a fallacious argument *might* be true, but the premisses of the argument are not good reasons to believe that this is so."

The **Favoring Procedure** establishes that you should switch in the Monty Hall Problem. But if we do want to determine exactly how much more likely the car is to be behind door #2 than door #1, we just need to revise the **TED Ed Procedure** slightly. In particular, in the **Revised TED Ed Procedure**, we simply replace **Step 2** with:

> **Step 2*:** Multiply the probability of each hypothesis by the likelihood of observing the evidence if that hypothesis is true.

In the Monty Hall Problem, the new probability that the car is behind door #1 is $1/6 = 1/3\times1/2$. The new probability that the car is behind door #2 is $1/3 = 1/3\times1$. And the new probability that the car is behind door #3 is $0 = 1/3\times0$. (Thus, the hypothesis that the car is behind door #3 gets eliminated). And once we normalize these probabilities in **Step 3**, we find that the probability that the car is behind door #1 is 1/3 and the probability that the car is behind door #2 is 2/3.[15]

Thus, the correct answer to the Monty Hall Problem is that the car is twice as likely to be behind the door that Monty did not open rather than the door that you initially chose. So, you should definitely switch to the last remaining door.

The **Revised TED Ed Procedure** is equivalent to using Bayes's Theorem.[16] Of course, Bayes's Theorem is not very intuitive to many people. Hopefully, the **Revised TED Ed Procedure** is somewhat more intuitive.

4. The Wi-Phi Probability Concentration Argument for Switching

Although there have been exceptions in the past, the vast majority of mathematicians and philosophers now agree that switching is better than sticking in the Monty Hall Problem. But

[15] The **Revised TED Ed Procedure** is a generalization of what Rosenthal (2008, 6) calls the "proportionality principle." It gives the correct answer for **Random Monty** as well. But we leave that as an exercise for the reader.

[16] See Cross 2000, 322, Devlin 2005, Rosenhouse 2009, ch. 3, Hájek and Hitchcock 2016, 15-16, Brilliant 2018, Huemer 2018, 3 for direct applications of Bayes's Theorem to the Monty Hall Problem. To see that the **Revised TED Ed Procedure** is equivalent to applying Bayes's Theorem, let C_i for $1 \leq i \leq 3$ denote the event that the car is behind door i. According to Bayes's Theorem, $\mathrm{P}(C_i|E) = \frac{\mathrm{P}(E|C_i)\times\mathrm{P}(C_i)}{\mathrm{P}(E|C_1)\times\mathrm{P}(C_1)+\mathrm{P}(E|C_2)\times\mathrm{P}(C_2)+\mathrm{P}(E|C_3)\times\mathrm{P}(C_3)}$. **Step 2*** of the **Revised TED Ed Procedure** is the same as calculating the numerator of this formula (for each of the three doors). And **Step 3** is the same as dividing by the denominator. There are other procedures that are also equivalent to applying Bayes's Theorem. For example, we can take into account our total evidence by constructing a tree diagram of the Monty Hall Problem (see Weisberg 2019, ch. 1). Alternatively, we *can* take into account our total evidence simply by eliminating hypotheses that are inconsistent with the evidence as long as we are careful to use a rich enough partition of hypotheses. For example, for each possible location and for each possible piece of evidence, we might include their conjunction (e.g., "The car is behind door #1 and Monty opens door #3 to reveal a goat") as a hypothesis (see Horgan 1995, 214-15, Titelbaum 2022, 177-82). But all of these procedures involve weighting the prior probabilities of the hypotheses regarding the location of the car by the likelihood of the evidence given these hypotheses (since $\mathrm{P}(C_i \& E) = \mathrm{P}(C_i) \times \mathrm{P}(E|C_i)$) just like in **Step 2*** of the **Revised TED Ed Procedure**.

instead of resorting to Bayes's Theorem, mathematicians and philosophers often try to give a more intuitive explanation of this answer. Their shortcut arguments for switching tend to pay more attention to the evidence than the **TED Ed Procedure**. But as we argue below, they often still don't pay *enough* attention to the evidence. In particular, they don't ask the critical question of how likely the evidence is given each of the hypotheses. Thus, they end up with "the right answer for a wrong reason." A notable example is an episode of Wi-Phi—another collection of fun, informative, and largely reliable educational videos—on "The Monty Hall Problem" (Gessell 2015).

Since the probabilities of the hypotheses always have to add up to 1, the 1/3 probability that was initially assigned to the door that Monty opens has to go somewhere. In the **TED Ed Procedure**, it gets split evenly between the two remaining hypotheses. But there are clear differences between those two hypotheses. Most notably, one hypothesis is about a door that Monty could have opened and the other is about a door that Monty could not have opened (viz., the door that you initially chose). So, maybe the 1/3 probability should not get split evenly between them?

Wi-Phi takes this difference between the two hypotheses into account. According to Wi-Phi, since Monty could not have opened it, you learn nothing about the door that you initially chose.[17] So, *none* of the 1/3 probability should go to the hypothesis that the car is behind that door. And the only other place that it can go is to the hypothesis that the car is behind the last remaining door.

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[17] You clearly learn *something* when Monty opens a door and reveals a goat. For example, you learn that the car is not behind that particular door. But you do not seem to learn anything that is relevant to whether the car is behind the door that you initially chose.

The Wi-Phi Probability Concentration argument:

> There is a 1/3 chance that the car is behind the door that you initially chose. So, there is a 2/3 chance that the car is behind one of the two remaining doors. Monty is not allowed to open the door that you initially chose. So, you learn nothing about that door when Monty opens one of the other doors and reveals a goat. So, there is still a 1/3 chance that the car is behind that door. Thus, the remaining 2/3 chance gets concentrated on the last remaining door. So, you should switch to the last remaining door when Monty opens a door and reveals a goat.

Several other mathematicians and philosophers (e.g., Martin 2011, 122, Kahn Academy 2012, Goldberg 2014, Adams 2022, 20, Bellos 2024, 127) also give this sort of argument.[18] (See Appendix 1 for quotes.) This argument yields the correct answer to the Monty Hall Problem. But as we discuss in the following section, it is a bad argument.

5. A Counterexample to the Wi-Phi Probability Concentration Argument

One complaint that could be raised about the **Wi-Phi Probability Concentration** argument is that it is *not sufficiently formal*. Because it is aimed at an audience that may not have experience or facility with formal methods, the reasoning in the **Wi-Phi Probability Concentration** argument is somewhat informal. However, informal proofs are typically thought by mathematicians and philosophers to be less reliable than formal proofs (see Tanswell 2024, 6). So, we shouldn't be surprised if errors are sometimes made when we reason informally.

[18] Snyder (2018, 24) uses the same sort of reasoning to analyze an extension of the Monty Hall Problem in which there are four doors, Monty opens two doors to reveal two goats, and after opening each of these doors, he gives you the opportunity to switch.

Indeed, people might not have mistakenly concluded that it doesn't matter whether you stick or switch if they had endeavored to reason more formally (e.g., by using Bayes's Theorem)

But informality by itself is not why the **Wi-Phi Probability Concentration** argument is a bad argument. Informal arguments can be—and often are—good arguments. The **Favoring Procedure** is just one example. Indeed, this level of informality is quite common in mathematics. As Fenner Tanswell (2024, 11) notes, "most proofs produced by mathematicians are not formal" (see also Fallis 2003, 49-50, Andersen 2020, 233). While many philosophers do claim that arguments in mathematics are only good if they are *formalizable*, arguments in mathematics don't have to actually be *formalized* in order to be good arguments (see Fallis 2003, 62, Tanswell 2024, ch. 3).[19]

The **Wi-Phi Probability Concentration** argument is a bad argument because it is *invalid*. In fact, a few mathematicians and philosophers (e.g., Horgan 1995, 213, Cross 2000, 321, Devlin 2005, Rosenthal 2008, 5) have already pointed out (in some cases, decades before the video even appeared) that there is a *counterexample* to it.[20] Even though it yields the correct

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[19] Someone might also worry that, unlike the **Favoring Procedure** (see section 3 above), the **Wi-Phi Probability Concentration** argument fails to explain where people go wrong when they conclude that it doesn't matter whether you stick or switch. In other words, it arguably leaves us with a dialectical standoff between the intuitive answer and the correct answer. However, this argument does purport to give positive reasons for why the two remaining doors are not equally likely (i.e., for why the principle of indifference is not applicable). In any event, even if it were the case that the **Wi-Phi Probability Concentration** argument fails to explain where the reasoning that leads to the intuitive answer goes wrong, it is not clear that this would make it a bad argument. A good argument does not have to be what John Pollock (1986, 38-39) calls an "undermining defeater" for any argument that reaches a conflicting conclusion. It just has to be a "rebutting defeater." Whenever two arguments reach conflicting conclusions, we can usually find a flaw in one argument or the other simply by examining them more carefully. (Of course, there may be exceptional cases where we are not able to resolve the issue in this way (see De Millo et al. 1979, 272).)

[20] Moser and Mulder (1994, 119) also criticize this sort of argument, but without offering a counterexample. They point out that, initially, there is also a 2/3 chance that the car is behind the door that you initially chose or behind the door that Monty will end up opening. So, when

answer in the original puzzle, this argument yields the wrong answer when it is applied to a slight variation of the Monty Hall Problem.[21] While the premises of the argument (i.e., the assumptions to which the argument appeals) are true in the variation, the conclusion is false. As Horgan (1995, 213) puts it, "the argument-form is clearly fallacious, because in some decision situations it generates blatantly mistaken conclusions."

Note that the **Wi-Phi Probability Concentration** argument makes use of the fact that Monty is not allowed to open the door that you initially chose. And it makes use of the fact that Monty *actually* opens a door and reveals a goat. But note that it does not make use of the fact that Monty is not allowed to open a door and reveal the car.

Since this argument does not appeal to the fact that Monty is required to open a door to reveal a goat, it is also applicable to cases where Monty might open a door and reveal the car. However, in such variations of the original puzzle, it is not necessarily the case that you should switch doors after Monty opens a door and reveals a goat. For example, in **Random Monty**, there is a 1/3 chance that the car is behind the door that you initially chose, Monty is not allowed to open the door that you initially chose, and Monty opens one of the other doors and reveals a goat. Thus, the **Wi-Phi Probability Concentration** argument yields the answer that it is better to switch in this case as well as in the original puzzle. But as discussed above, in **Random**

Monty actually opens that door and reveals a goat, why doesn't this 2/3 chance get concentrated on the door that you initially chose? But of course, proponents of the **Wi-Phi Probability Concentration** argument have a plausible response to this objection. Namely, you can't learn anything about the door that you initially chose because Monty could not have opened it.

[21] Such variations are generated by resolving the ambiguities in the statement of the puzzle in different ways.

Monty, the car is just as likely to be behind the door that you initially chose as it is to be behind the last remaining door. So, it is not better to switch.[22]

Furthermore, we can identify exactly where the argument goes wrong. The **Wi-Phi Probability Concentration** argument concludes that the probability that the car is behind the door that you initially chose does not change when Monty opens another door and reveals a goat *on the grounds* that Monty cannot open that door. Now, as can be verified using Bayes's Theorem or the **Revised TED Ed Procedure**, it is true in the original puzzle that the probability that the car is behind the door that you initially chose does not change. But it is not simply because Monty cannot open that door. Even when Monty cannot open the door that you initially chose, the probability that the car is behind that door might change, as it does in the **Random Monty** variation. Thus, the argument has what Don Fallis (2003, 51) calls an "inferential gap." The claim that the probability that the car is behind the door that you initially chose does not change when Monty opens another door and reveals a goat is what Sorensen (2016a, 250) calls "a paralemma" which is "an invalid intermediate step."[23]

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[22] The **Random Monty** variation shows that you can learn about the door that you initially chose even if Monty is not allowed to open it. For example, if Monty opens a door and reveals a car, you know for sure that the car is not behind the door that you initially chose. Now, if Monty opens a door and reveals a goat, you don't know for sure what is behind the door that you initially chose. But if one possible result lowers the probability that it is a car, the other possible result raises the probability that it is a car.

[23] As Sorensen (2016a, 250) notes, "a paralemma can be true. It might entail a conclusion that is entailed by the premises." Indeed, the truth of this particular paralemma might help to explain why so many mathematicians and philosophers have mistakenly endorsed the **Wi-Phi Probability Concentration** argument. According to Sorensen (2016b, 133), "paralemmic reasoning within sound arguments tends to pass unnoticed. The truth of the conclusion and the validity of the argument conceal the invalidity of the reasoner's derivation … The presumption is that if the conclusion is correct, then the sub-conclusions were correct." Interestingly, we argue below that, although he does not endorse the **Wi-Phi Probability Concentration** argument, Sorensen (2003) himself has engaged in paralemmic reasoning with respect to the Monty Hall Problem.

5.1 A Possible Defense of the Wi-Phi Probability Concentration Argument

Nevertheless, it might be suggested that the **Wi-Phi Probability Concentration** argument is not a bad argument. It is merely incomplete as it stands. Just as it would be tedious to have to make all of the constraints explicit when stating a puzzle, it would be tedious to have to make all of the details explicit when giving an argument for the answer. Indeed, mathematicians and philosophers often leave out some of the details of their arguments (see Andersen 2020). In other words, they often leave what Fallis (2003, 53) calls an "enthymematic gap." And unlike inferential gaps, enthymematic gaps are perfectly acceptable as long as it is clear to the intended audience how they are to be filled in.

In particular, proponents of the **Wi-Phi Probability Concentration** argument might claim that it is understood that this argument is not intended to apply to the **Random Monty** variation. It is only intended to apply to the original puzzle. As discussed in section 2 above, we are entitled to assume that various constraints are in place in the Monty Hall Problem even though they are not stated explicitly. The idea here is that we are entitled to assume that these same constraints are in place in the **Wi-Phi Probability Concentration** argument even though they are not stated explicitly. And it does follow from these constraints that the probability that the car is behind the door that you initially chose does not change when Monty opens another door and reveals a goat.

So, the **Wi-Phi Probability Concentration** argument could simply be incomplete rather than invalid. We cannot definitively eliminate this possibility. However, there are several reasons to think that this is not a plausible interpretation of what's really going on.[24] Basically, it is overly charitable to the proponents of the **Wi-Phi Probability Concentration** argument.

First, the gap in the argument is *extremely* large. In addition to failing to state the constraints of the original puzzle to which it allegedly appeals, the argument does not give any indication how to get from those constraints to the desired conclusion. Even when a conclusion follows from the premises, simply stating the premises followed by the conclusion is not much of an argument when a lot of intermediate steps are left out (see Fallis 2003, 48-49, Boghossian 2014, 6). Moreover, in this case, it is not clear how to fill the gap without simply using Bayes's Theorem or something equivalent. And if we have to go to that trouble, the shortcut argument is not much of a shortcut.

Second, on its face, the **Wi-Phi Probability Concentration** argument certainly appears to be fallacious. It suggests falsely that something is sufficient grounds for believing something else when it is not. In particular, as discussed above, it suggests that we can infer directly from **(a)** the fact that Monty cannot open the door that you initially chose that **(b)** the probability that the car is behind that door does not change. In order for the argument to be incomplete rather than invalid, the "So" in "So, you learn nothing about that door when Monty opens one of the other doors and reveals a goat" has to be understood in an extremely expansive way. It cannot just mean that the claim follows from the previous step in the argument (viz., that Monty cannot open that door). It has to mean that the claim follows from the previous step plus a whole suite of unstated assumptions and argumentative moves that are not alluded to at all.

[24] In fact, the previous critics of the **Wi-Phi Probability Concentration** argument (such as Horgan 1995) do not even consider this possible interpretation. They simply dismiss the argument as invalid on the basis of a counterexample.

Finally, even if the mathematicians and philosophers who give the **Wi-Phi Probability Concentration** argument do understand the "So" in this expansive way *and* have worked through these unstated argumentative moves to verify that this probability does not change, it is unlikely that their audience has. (Recall that the intended audience for this argument is not necessarily mathematically sophisticated.) Thus, their audience is almost certainly unjustified in *their* belief that this probability does not change and, thus, in their belief that switching is better. For knowledge and (doxastic) justification, what matters is the path that you actually take to a conclusion, not merely whether there is a good path (see Turri 2010, 313).

6. Two Additional Arguments for Switching

There are two other shortcut arguments for switching that avoid the **Random Monty** counterexample. First, several mathematicians and philosophers (e.g., Gardner 2001, 283, Clark 2002, 114-15, Devlin 2003, Sorensen 2003, 225, Rosenthal 2009, 36, Champkin 2013, 33, Pynes 2013, 36-37) have given a *strengthened* version of the **Wi-Phi Probability Concentration** argument. (See Appendix 2 for quotes.) These mathematicians and philosophers still contend that, in the Monty Hall Problem, you get no information about the door that you initially chose. But it is not just because Monty could not open it. It is because you know ahead of time that he is going to open another door and reveal a goat. Basically, if that information were going to affect the probability that the car is behind the door that you initially chose, it would have affected it before he opened this other door.

The Strengthened Probability Concentration argument:

There is a 1/3 chance that the car is behind the door that you initially chose. So, there is a 2/3 chance that the car is behind one of the two remaining doors. You *know* that Monty will open one of those doors to reveal a goat. (You just do not know which one it will be.) So, you learn nothing about the door that you initially chose when Monty opens one of the other doors and reveals a goat. So, there is still a 1/3 chance that the car is behind that door. Thus, the remaining 2/3 chance gets concentrated on the last remaining door. So, you should switch to the last remaining door when Monty opens a door and reveals a goat.

In addition, although Wi-Phi does not give the **Strengthened Probability Concentration** argument, it does give a second argument for switching that also avoids the **Random Monty** counterexample.

The Wi-Phi Probability Swap argument:

There is a 1/3 chance that the car is behind the door that you initially chose. So, 1/3 of the times that you play the game, you will win by sticking with that door. And 2/3 of the times that you play, you will lose by sticking. However, if you switch to the door that Monty did not open, you will lose all of the times that you would have won by sticking. And you will win all of the times that you would have lost by sticking. (In those cases, there is a car behind one of the two remaining doors and a goat behind the other. So, when Monty opens one of those doors and reveals a goat, you will win by switching to the other door.) So, you should switch to the last remaining door when Monty opens a door and reveals a goat.

In addition to Wi-Phi, several other mathematicians and philosophers (e.g., Bruce 2001, 112, Winkler 2004, 34, Nihous 2009, 94, Kahn Academy 2012, Li 2013, Pynes 2013, 42, Talwalkar 2013, 56, Hájek and Hitchcock 2016, 16, Brilliant 2018, Huemer 2018, 4-5, Stewart 2019, 67, Cook 2020, 84, Bollobás 2022, 181, Titelbaum 2022, 181, University of Illinois 2023) also give this sort of argument. (See Appendix 3 for quotes.)

Random Monty is not a counterexample to the **Strengthened Probability Concentration** argument or to the **Wi-Phi Probability Swap** argument. The **Strengthened Probability Concentration** argument clearly does not apply to **Random Monty**. With respect to the **Wi-Phi Probability Swap** argument, note that, as in the original puzzle, 1/3 of the times that you play the game in **Random Monty**, you will win by sticking with the door that you initially chose. But it is not true that, if you switch to the door that Monty did not open, you will win all of the times that you would have lost by sticking. In half of those cases, Monty will have opened a door to reveal a car (since Monty always chooses at random which door, of the two that you did not initially choose, to open). And if Monty opens a door and reveals a car, you lose whether you stick or switch. Thus, if you switch to the door that Monty did not open, you will still win only 1/3 of the times that you play the game.

But even though they avoid the **Random Monty** counterexample, the **Strengthened Probability Concentration** argument and the **Wi-Phi Probability Swap** argument are bad arguments. Indeed, as we argue in the following section, even if we assume that Monty is required to reveal a goat, there are at least two types of counterexamples to these two arguments. And it is worth noting that they are counterexamples to the **Wi-Phi Probability Concentration** argument as well.

7. Two Counterexamples to All Three Arguments for Switching

There are many variations of the Monty Hall Problem where it is not better to switch. Not all such variations are counterexamples because these three arguments tacitly put some constraints on Monty's procedure. For example, consider a case (the **Mean Monty** variation) that is exactly like the original puzzle except that Monty opens a door to reveal a goat *only if* the car is behind the door that you initially chose.[25] It is clear that these three arguments don't apply to **Mean Monty** (i.e., that they appeal to some assumptions that are not true in this variation). The **Wi-Phi Probability Concentration** argument does not apply because you clearly *do* learn something about the door that you initially chose when Monty opens one of the other doors to reveal a goat. The **Strengthened Probability Concentration** argument does not apply because you *do not* know that Monty will open one of the other doors to reveal a goat. Finally, the **Wi-Phi Probability Swap** argument does not apply because you clearly *never* win by switching when Monty opens a door to reveal a goat. However, there *are* variations that are counterexamples to these arguments.

Note that these arguments do not make use of the fact that Monty chooses at random when he has a choice about which door to open to reveal a goat. This is clear in the case of both versions of the **Probability Concentration** argument. And with respect to the **Wi-Phi Probability Swap** argument, note that, even if Monty has a bias toward one of the two remaining doors (say, the higher-numbered door), it is still true that, if you switch to the door

[25] If the car is not behind the door that you initially chose, you have to decide whether to switch without any door having been opened for you. Rosenhouse (2009, 113) discusses a similar variation where "Malevolent Monty" only gives you an opportunity to switch when your initial choice was correct.

that Monty did not open, you will win all of the times that you would have lost by sticking (i.e., two-thirds of the time).[26]

Since these arguments do not appeal to the fact that Monty chooses at random when he has a choice, they are also applicable to cases where Monty has a bias toward one of the two remaining doors. However, in such variations of the original puzzle, it is not necessarily the case that you should switch doors after Monty opens a door and reveals a goat. For example, consider a case (the **Lazy Monty** variation) that is exactly like the original puzzle except that Monty is very lazy and the latch on door #2 is difficult to open. Thus, Monty doesn't open door #2 unless he absolutely has to (viz., when it is the only door he can open to reveal a goat).[27] Also, suppose that you initially choose door #1. In this case, Monty is just as likely to open door #3 to reveal a goat if the car is behind door #1 as he is to open door #3 to reveal a goat if the car is behind door #2. So, since door #1 and door #2 started out equally likely, if Monty opens door #3 to reveal a goat, the car is still just as likely to be behind door #1 (the door that you initially chose) as it is to be behind door #2.[28]

In addition, note that these arguments appeal to the fact that the probability that the car is behind the door that you initially chose is 1/3. And it follows from this that the probability that

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[26] As in the original puzzle, these wins will be equally distributed between the two doors. But unlike in the original puzzle, the losses from switching will not be equally distributed between the two doors. Most of these losses will come when Monty opens the door toward which he is biased.

[27] See Rosenhouse 2009, 83-84 for an analysis of this sort of variation. Cross (2000, 321) and Rosenthal (2008, 5) do mention that it is a counterexample to what we are calling the **Strengthened Probability Concentration** argument. But ironically, although Rosenthal (2008, 5) calls this argument a "shaky solution" to the Monty Hall Problem, Rosenthal (2009, 36) later endorses it without any reservation.

[28] Note that the best strategy depends on *which door* Monty opens. If Monty opens door #2 and reveals a goat, you do better by switching. In fact, you are *guaranteed* to win the car. But if he opens door #3 and reveals a goat, it makes no difference whether you stick or switch.

the car is behind one of the two remaining doors is 2/3. And these arguments do make use of this fact. But these arguments *do not* make use of the fact that this 2/3 probability is split evenly between the two remaining doors.[29] This is clear in the case of both versions of the **Probability Concentration** argument. And with respect to the **Wi-Phi Probability Swap** argument, note that, even if the 2/3 probability is not split evenly, it is still true that, if you switch to the door that Monty did not open, you will win all of the times that you would have lost by sticking (i.e., two-thirds of the time).[30]

Since these arguments do not appeal to the fact that the 2/3 probability is split evenly, they are also applicable to cases where the 2/3 probability is not split evenly. However, in such variations of the original puzzle, it is not necessarily the case that you should switch doors after Monty opens a door and reveals a goat. For example, consider a case (the **Unequal Monty** variation) that is exactly like the original puzzle except that the probabilities that the car is behind each of the three doors are 1/3, 2/15, 8/15, respectively.[31] Also, suppose that you initially choose door #1.[32] As in the original puzzle, Monty is twice as likely to open door #3 to reveal a goat if the car is behind door #2 than he is to open door #3 to reveal a goat if the car is behind door #1. But in this case, the car is initially *two-and-a-half* times more likely to be behind door #1 than to be behind door #2. So, all things considered, if Monty opens door #3 and reveals a

[29] Some of the people who offer these arguments do mention that the three hypotheses are equiprobable. But they only use this fact to establish that the probability that the car is behind the door that you initially chose is 1/3.

[30] Unlike in the original puzzle, these wins will not be equally distributed between the two doors. Most of these wins will come when Monty opens the door that the car is less likely to be behind.

[31] See Rosenhouse 2009, 78-80 for an analysis of this sort of variation.

[32] Note that these three arguments are only applicable to those instances of the Unequal Monty variation in which the contestant picks the door that has a 1/3 probability of concealing the car. But it is also worth noting that, if the contestant picks a door at random, it is still the case that the contestant will win two-thirds of the time by switching.

goat, the car is still more likely to be behind door #1 (the door that you initially chose) than it is to be behind door #2.[33,34,35]

Moreover, in addition to finding counterexamples to them, we can identify exactly where the two arguments go wrong. First, the **Strengthened Probability Concentration** argument concludes that the probability that the car is behind the door that you initially chose does not change when Monty opens another door and reveals a goat *on the grounds* that you already knew that Monty would open another door to reveal a goat. Now, it is true in the original puzzle that the probability that the car is behind the door that you initially chose does not change. But it is not simply because you already knew that Monty would open another door to reveal a goat.

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[33] Again, the best strategy depends on which door Monty opens. If Monty opens door #2 and reveals a goat, you do better by switching. But if he opens door #3 and reveals a goat, you do better by sticking. Basically, switching in that case would be to ignore the base rate. See Weisberg 2019, sec. 8.5, Titelbaum 2022, 169-73 for discussions of the base-rate fallacy. It should be noted that Bradley and Fitelson (2003) analyze the Monty Hall Problem without making any assumptions about the prior probabilities. But they only show that, as long as there is some chance that Monty will open either door when he has a choice, the evidence favors the hypothesis that the car is behind the last remaining door over the hypothesis that the car is behind the door that you initially chose. They do not claim that it is better to switch to the last remaining door. Yudi Pawitan and Youngjo Lee (2024, 244) also analyze the Monty Hall Problem without making any assumptions about the prior probabilities. However, they *do* claim incorrectly that it still follows that switching is better.

[34] We used the **Favoring Procedure** to confirm that the car is just as likely to be behind the door that you initially chose in **Lazy Monty**. In order to confirm that the car is more likely to be behind the door that you initially chose in **Unequal Monty**, we have to balance the influence of the initial probabilities against the degree to which the evidence favors the remaining door. This requires using Bayes's Theorem and not just the **Favoring Procedure**.

[35] Another common explanation for why switching is the best strategy in the Monty Hall Problem involves increasing the number of doors (e.g., to 100). Once you have chosen a door, Monty opens all of the other doors, except one (say, door #37), to reveal goats (see vos Savant 1991, Clark 2002, 115, Devlin 2003, Goldberg 2014, Brilliant 2018). In this case, it seems very clear that you should switch. However, the argument given for switching in the 100-door case is typically a version of one or the other of the **Probability Concentration** arguments. So, it is subject to the same sort of counterexamples. (Of course, the car does have to be extremely unlikely to be behind door #37 at the outset—with a chance of less than 1 in 10,000—in the **Unequal Monty** counterexample for the 100-door case.)

Even if you already knew that Monty would open another door to reveal a goat, the probability that the car is behind the door that you initially chose might change, as it does in the **Lazy Monty** and **Unequal Monty** variations.

Second, the **Wi-Phi Probability Swap** argument concludes that switching is better in this particular instance *on the grounds* that you will win more often in the long run by switching whenever you play the game. Now, it is true in the original puzzle that switching is better. But it is not simply because you will win more often in the long run by switching. Even when you will win more often in the long run by switching whenever you play the game, switching might not be better in this particular instance, as it is not in the **Lazy Monty** and **Unequal Monty** variations.[36] In these variations, even though the chance of winning if you always switch is still 2/3, the chance of winning if you switch *when Monty opens door #3 and reveals a goat* is less than 2/3.

Finally, just like with the **Wi-Phi Probability Concentration** argument, it might be suggested that these two arguments are simply incomplete rather than invalid. However, there are reasons—*the very same* reasons discussed in section 5.1 above—to think that this is not a plausible interpretation of what's really going on. Admittedly, as we discuss in sections 7.2 and 7.3 below, there are ways to fill the gap in the **Wi-Phi Probability Swap** argument that do not require simply using Bayes's Theorem or something equivalent. But the gap in both arguments is still large. And most importantly, on their face, both arguments appear to have "invalid intermediate steps."

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[36] As discussed in section 7.3 below, there is a *restricted* set of plays of the game (viz., those plays in which Monty opens door #3 after the contestant initially chooses door #1) such that, if you will win more often on these plays of the game by switching, it does follow that switching is better in this instance. But the long run that the **Wi-Phi Probability Swap** argument literally refers to is simply playing the game multiple times.

7.1 Rosenhouse on Cases Where the Initial Probabilities are Unequal

Interestingly, the mathematician Jason Rosenhouse (2009, 79) claims to prove that, even if the car is not equally likely to be behind each of the three doors, Monty opening a door to reveal a goat does not change the probability that the car is behind the door that you initially chose.[37] He says that "our calculation shows that the probability of our initial choice does not change when Monty reveals an empty door." However, the **Unequal Monty** variation shows that this claim is false. You *can* learn something about the door that you initially chose even if you know ahead of time that Monty is going to open another door and reveal a goat.

Rosenhouse makes this false claim along the way to arguing that, when the car is not equally likely to be behind each of the three doors, the best strategy is to initially choose the door that the car is least likely to be behind and then switch when Monty opens a door and reveals a goat. Even though his argument for it is unsound (as it relies on a false lemma), *this* claim is true. (See Appendix 4 for a proof.) Thus, when you initially choose the door with probability 1/3 in **Unequal Monty**, you are not adopting the best possible strategy. You should have chosen the door with probability 2/15 (and then switched when Monty opens a door and reveals a goat). But even if you have made a mistake, we should still be able to ask what you should do now. And the **Strengthened Probability Concentration** argument and the **Wi-Phi Probability Swap** argument incorrectly suggest that you should switch to the last remaining door.

7.2 Bolstering the Wi-Phi Probability Swap Argument

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[37] It is true that, if we take the *average* of the various new probabilities that could result when Monty opens a door to reveal a goat, it is the same as the prior probability that the car is behind the door that you initially chose. But that is a different matter.

Despite the counterexamples, there is something to be said for the **Wi-Phi Probability Swap** argument. As Morgan et al. (1991, 285) point out, it is a good argument for switching *if* you have to make your decision about what to do *before* Monty opens a door and reveals a goat. None of the variations of the Monty Hall Problem are counterexamples to this claim. But the puzzle clearly asks what you should do *after* Monty opens a door and reveals a goat. And the **Lazy Monty** and **Unequal Monty** variations show that it may not be better to switch at that point.[38]

However, given the fact that the **Wi-Phi Probability Swap** argument is a good argument for switching if you have to decide what to do *before* Monty opens a door, we can turn it into a good argument for switching even if you get to decide what to do *after* Monty opens a door. In the original puzzle, but not in the **Lazy Monty** and **Unequal Monty** variations, the two doors that Monty could open are *antecedently indistinguishable*. That is, the car is initially equally likely to be behind each of them and Monty chooses at random whenever he has a choice about which of them to open. If the doors are antecedently indistinguishable, you will get essentially the same information regardless of which one of them is opened—just with door #2 and door #3 transposed. Thus, if switching is best if they open door #2, it will also be best if they open door #3, and vice versa. So, you might as well decide ahead of time what to do. And we already know from the **Wi-Phi Probability Swap** argument that, if you have to decide ahead of time, you should switch.

[38] In other words, **Wi-Phi Probability Swap** argument is a perfectly good argument for a weaker conclusion. But the desired conclusion does not follow from this weaker conclusion. In a similar vein, Sorensen (2016b, 136) discusses an argument that purports to show that the area of a parallelogram is always base times height, but it only shows that this holds for some parallelograms.

With this bolstering, we rule out the **Lazy Monty** and **Unequal Monty** counterexamples. Thus, the **Bolstered Probability Swap** argument is a good argument for switching in the Monty Hall Problem. We do not end up with "the right answer for a wrong reason."[39]

Even so, it is worth noting that the **Bolstered Probability Swap** argument has fairly limited applicability. Explicitly paying attention to the evidence yields a much more general procedure for carrying out probabilistic inferences. Even if we do not want to go as far as using Bayes's Theorem or the **Revised TED Ed Procedure**, we can handle a much broader range of cases just by asking ourselves how likely the evidence is given each of the hypotheses (i.e., by using the **Favoring Procedure**). Note that we can ask this question regardless of what bias Monty might have when he has a choice about which door to open. Also, we do not have to assume that the car was initially equally likely to be in each of the three locations. As long as the car was initially at least as likely to be behind the door that the evidence favors, it is clear whether you should stick or switch.[40] Indeed, the broader applicability of simply asking about the likelihood of the evidence given the hypotheses is part of what makes it such a good explanation for why switching is the right thing to do in the original puzzle.

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[39] It could be suggested that proponents of the **Wi-Phi Probability Swap** argument really have in mind the **Bolstered Probability Swap** argument. In that case, their argument only leaves an enthymematic gap rather than an inferential gap. But as noted above, this interpretation of their argument is not very plausible. For example, their argument leaves a large gap. Indeed, it doesn't make any reference at all to the symmetry of the two unpicked doors. (In fact, we don't know of anyone other than ourselves who has explicitly put forward the **Bolstered Probability Swap** argument.)

[40] There are cases, such as the **Unequal Monty** variation, where the evidence favors one door, but the car was initially more likely to be behind the other door. But as Bradley and Fitelson (2003) show, as long as there is some chance that Monty will open either door when he has a choice, the evidence favors the hypothesis that the car is behind the last remaining door over the hypothesis that the car is behind the door that you initially chose. So, the only time that we will need to resort to Bayes's Theorem or the **Revised TED Ed Procedure** is when the door that you initially chose was initially more likely to conceal the car than the last remaining door.

7.3 Running Simulations

Finally, it should be noted that there is another common case for switching that is closely related to the **Wi-Phi Probability Swap** argument. Namely, it has often been suggested (e.g., by vos Savant 1990, Martin 2011, 123, Clark 2002, 116, Titelbaum 2022, 181) that, if we are not convinced that we should switch after Monty opens a door and reveals a goat, we should run a simulation. That is, we should play the game described in the Monty Hall Problem a few hundred times, keeping track of how often we win if we stick with our initial choice, and keeping track of how often we win if we switch. And such simulations almost always result in sticking winning about a third of the time and switching winning about two-thirds of the time. Basically, this is a real-life implementation of the **Wi-Phi Probability Swap** argument.

Simulations can be a very good way to establish scientific and mathematical truths (see Winsberg 2019). But before trusting the results of a simulation, we do have to be careful that we are doing the *right* simulation to solve the problem at hand.[41] In particular, in order for the results of a simulation to be a convincing case for switching in the Monty Hall Problem, the case needs to be made that the winning percentages in the long-run bear on what you should do in a single play of the game.[42] Now, contra Paul Moser and Hudson Mulder (1994), single-case probabilities do not diverge from long-run frequencies (see Horgan 1995). But we still have to be sure that we are talking about the right long-run frequencies. As the **Lazy Monty** and **Unequal Monty** counterexamples show, even though the long-run frequency of winning if you always switch is 2/3, the long-run frequency of winning if you switch when Monty opens door

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[41] See Winsberg 2019, sec. 4.3 for a survey of the literature on the verification and validation of simulations.

[42] See Rosenhouse 2009, ch. 7 for a survey of the philosophical literature on this point.

#3 and reveals a goat might differ from 2/3. One way that we can insure that we are talking about the right long-run frequencies is by only simulating plays of the game where you initially choose the same door and Monty then opens the same door (see Morgan et al. 1991, 285).[43] But people don't tend to do this.[44] Alternatively, much as we had to do to bolster the **Wi-Phi Probability Swap** argument itself, we could confirm that the two doors that you do not initially choose are always antecedently indistinguishable *and* explain why that matters (see Horgan 1995, 219). But people don't tend to do this either.

8. The Wi-Phi Arguments Mislead

Both versions of the **Probability Concentration** argument and the **Wi-Phi Probability Swap** argument are bad arguments. As a result, people can end up with an unjustified belief that they should switch in the Monty Hall Problem. And that is a problem. Also, as a result, much like TED Ed, Wi-Phi wastes an opportunity to teach people the right way to carry out probabilistic inferences.[45] And that is also a problem. But despite these problems, people still end up with a *true* belief that they should switch in the Monty Hall Problem.

Philosophers work to identify fallacious argument forms, such as affirming the consequent, because they commonly arise and trip people up (see Salmon 1995, 87-89). Like

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[43] See Appendix 5 for a Python simulation of the **Unequal Monty** variation. (The code is inspired by Snyder 2018, 24-25.) As in the original puzzle, sticking wins about a third of the time and switching wins about two-thirds of the time. But if we just focus on trials where you initially choose door #1 and Monty then opens door #3, sticking wins more often than switching.

[44] It could be suggested that proponents of the **Wi-Phi Probability Swap** argument really have in mind a *restricted* set of plays of the game. But again, this interpretation of their argument is not very plausible. For example, their argument still leaves a very large gap. Indeed, it doesn't make any reference at all to a restricted set of plays (or say anything about why the long-run frequency of winning in this restricted set of plays might be 2/3).

[45] Even though the **Bolstered Probability Swap** argument is a good argument, given its limited applicability, it arguably would still have been a wasted opportunity if Wi-Phi had presented it.

many other fallacious argument forms, the **TED Ed Procedure** is epistemically dangerous because it is applicable to all sorts of scenarios where it yields wrong answers. But it might be suggested that the Wi-Phi arguments, in contrast, do not create much of a risk of people actually being *misled*.

The Wi-Phi arguments are only directly applicable to Monty Hall Problem-like scenarios. Moreover, as long as the hypotheses are indistinguishable prior to the evidence, the Wi-Phi arguments give the correct answer. And how often do we face Monty Hall Problem-like scenarios where the hypotheses are not antecedently indistinguishable? However, the same sort of bad reasoning that we find in the Wi-Phi arguments can actually lead to false conclusions in many scenarios involving probabilistic inference.

If it seems intuitive that we don't get any information about a hypothesis, the **Probability Concentration** arguments advise us to keep the probability of that hypothesis fixed.[46] But here is a mundane scenario where this sort of reasoning results in a mistake: Suppose that you don't know whether your keys are in your pockets, in your backpack, or out in your car. But you are too tired to go out to search the car. If you don't find your keys in your pockets, that doesn't mean that only the probability that the keys are in your backpack goes up. The probability that the keys are out in your car also goes up even though you could not have searched that location.[47]

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[46] People don't seem to be particular good at intuiting when evidence does or does not change the probability of a hypothesis. It is true that you do not get any new information about the door that you initially chose in the Monty Hall Problem. But contrary to the **Wi-Phi Probability Concentration** argument, it is not simply because Monty could not open that door. And contrary to the **Strengthened Probability Concentration** argument, it is not simply because you knew that he would open another door to reveal a goat.

[47] It is harder to find examples where reasoning along *exactly* the same lines as the **Strengthened Probability Concentration** argument gives the wrong result. David Lewis's (2001, 174) halfer argument in the Sleeping Beauty Problem *might* be an example. He assumes that, since Sleeping

If a strategy always leads to more wins than losses, the **Wi-Phi Probability Swap** argument advises us to follow that strategy regardless of the specific information that we get. But here is a mundane scenario where this sort of reasoning results in a mistake: Suppose that the champion wins the vast majority of his boxing matches. So, you will win much more often if you bet on the champion than if you bet against him. However, the champion almost always loses if he is under the weather. Now, since he usually feeling fine, you should definitely bet on the champion if you have to make your decision before getting the results of the pre-fight physical examination. But that doesn't mean that you should bet on the champion if you learn that he did not get a clean bill of health. You would be ignoring relevant evidence if you still bet on him in that case.

9. Conclusion

There are at least three distinct bad arguments for switching in the Monty Hall Problem that are still circulating widely and that come from reputable sources.[48] Thus, many people end up believing "the right answer for a wrong reason." Moreover, since these arguments only yield the correct answer in a very restricted range of cases, people learn techniques that lead to false conclusions when they are applied to many other probabilistic inference problems. So, instead of, for example, promulgating the questionable shortcut of holding fixed the probability of

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Beauty knew that she was going to be awakened, she gets no information that changes the probability of the coin having come up heads.

[48] There are, of course, other bad arguments for switching in the Monty Hall Problem (see Morgan et al. 1991, 284-85). But they are not nearly as common.

hypotheses that we seem to learn nothing about, it would be much better to simply teach people to ask how likely the evidence is given each of the hypotheses.[49]

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11. Appendices

NB: As discussed in the main text, some of these authors may have in mind unstated details that would make their arguments valid. But what they *explicitly* say is consistent with the invalid arguments that we identify here.

1. Versions of the Wi-Phi Probability Concentration Argument:

Adams (2022, 20) …
"You should switch … The probability that you picked right initially is one third … So the probability that door 1 is right is one third. But then Hall purposely had them open a door that was a wrong choice, that had a concrete wall behind it. Whatever door you pick, there is always another door with a concrete wall behind it that he can have opened. But that doesn't change the probability you picked right. So if you stay with your current door, there's only a one third chance you're right. But now there is only one other remaining unopened door, door number 2. If you switch to it, the only other remaining possibility, there is a two third chance that it's right."

Bellos (2024, 127) …
"At the beginning of the game … there is a combined two-in-three chance that the car is *not* behind door 1. Once Monty reveals the goat behind door 2, the chance that the car is *not* behind door 1 is still two in three, so the chance that the car *is* behind door 3 rises to two in three."

Gessell (2015) …
"Let's divide the three doors into two different groups … The only door in group YOURS is the one you choose originally. And the other two doors are in OTHERS. Before any doors are opened, it's easy to see that group YOURS has a 1 in 3 chance of winning, while group OTHERS has a 2 in 3 chance, divided equally between the two doors in that group. But when the host opens the second door, you've learned something. And what you've learned is that the prize isn't behind one of the two doors in OTHERS. But what happens to the 1 in 3 chance that belonged to the open door? … [It] can't go to the door in the YOURS group because you haven't learned anything about the door in YOURS. You've only learned something about the doors in the OTHERS group. So, the entire 1 in 3 chance from the open door gets reassigned to the remaining closed door in OTHERS. The chance of winning with the door in the YOURS group is still only 1 in 3."

Goldberg (2014) …
"There is a one-third chance that the car is behind the door you picked initially. That means there must be a two-thirds chance—much greater, twice as big—that the car is somewhere else. And since we know that somewhere else cannot be door number two—because Monty showed us that—it's gotta be over here. So, this is what you should choose. You should switch. Twice as likely to have the car behind the door that you didn't pick as the door that you did … the initial two-thirds chance that the car was behind door number 2 and door number 3 got concentrated behind the door that Monty did not open."

Khan Academy (2012) …
"Are you better off switching to whatever curtain is left? … When you first make your initial pick, there's a 1/3 chance that it's there, and there's a 2/3 chance that it's in one of the other two doors. And they're going to empty out one of them. So, when you switch, you essentially are capturing that 2/3 probability."

Martin (2011, 122) …

"Switching is a far better strategy. Here's why. It's 1/3 likely that door 3, the one you picked, had the car, and you'd lose it if you switched. But it's 2/3 likely that door 3, which you picked, had a goat, and the car was behind 1 or 2—but Monty opened 2 to show the goat there, so it's 2/3 likely to be behind 1. So that means switching is 2/3 likely to get you that car."

2. Versions of the Strengthened Probability Concentration Argument:

Champkin (2013, 33) …
"The key insight is that no matter which door you select, Monty can always show you a goat behind one of the remaining doors. His action provides no new information about your selection but does provide information about the other two doors."

Clark (2002, 114-15) …
"When the contestant first picks a door the chance that it has the prize is 1/3. She knows that the host will be able to open a door concealing no prize, since at least one of the other doors must be a loser. Hence she learns nothing new which is relevant to the probability that she has already chosen the winning door: that remains at 1/3. Since if she swaps she will not choose the door the host has just revealed to be a loser, the opportunity to swap is equivalent to the opportunity of opening both the other doors instead of the one she has picked, which clearly doubles her chances of winning."

Devlin (2003) …
"By opening his door, Monty is saying to the contestant "There are two doors you did not choose, and the probability that the prize is behind one of them is 2/3. I'll help you by using my knowledge of where the prize is to open one of those two doors to show you that it does not hide the prize. You can now take advantage of this additional information. Your choice of door A has a chance of 1 in 3 of being the winner. I have not changed that. But by eliminating door C, I have shown you that the probability that door B hides the prize is 2 in 3" … when Monty opens door C, the attractive 2/3 odds that the prize is behind door B or C are shifted to door B alone."

Gardner (2001, 283) …
"Now suppose, that after the guest's selection is voiced, Monty Hall, who knows what is behind each door, opens one door to disclose a goat … the probability the guest had chosen the correct door … remains 1/3. Because Monty can always open a door with a goat, his opening such a door conveys no new information that alters the 1/3 probability."

Pynes (2013, 36-37) …
"No matter what door the contestant initially picks, Monty will always tease reveal, a non-selected door that doesn't have a prize behind it. … Imagine instead of Monty revealing a door, he offers you both of the doors you didn't pick. You know a priori one of those doors cannot have the prize behind it. So, when Monty reveals one of the unpicked doors that doesn't have a prize behind it, you haven't learned any new information about the set of doors you didn't pick. If you wanted the two doors you didn't choose before Monty tease reveals (and you do), then you should want to switch after the reveal—it's the same deal. The odds of winning are 2/3 if you switch because you only had a 1/3 chance of being right in your initial door choice."

Rosenthal (2009, 36) …
"Note that the strategy of sticking will only succeed if your original guess happened to be correct, which had probability 1/3. And since we knew the Host was going to open some door not containing the car, observing this doesn't change the probability 1/3 that we were right in the first place."

Sorensen (2003, 225) …
"Monty's revelation that door 2 has a goat cannot raise the probability that door 1 has the prize because you already knew that Monty was going to either reveal 2 as a loser or reveal 3 as a loser. … [Thus] … the probability that the prize is behind door 3 rises to 2/3 because the probability of door 1 winning is not affected."

3. Versions of the Wi-Phi Probability Swap Argument:

Bollobás (2022, 181) …
"The contestant … should definitely switch. To see this, let us call the door chosen by the contestant door A. First, suppose that the contestant guesses correctly, and door A hides the car. The probability of this is 1/3. Hence, by staying with his choice, his probability of winning the car is 1/3. Second suppose that the contestant guesses incorrectly, and a goat is behind door A. The probability of this is 2/3. By swapping, the contestant wins the car with probability 2/3."

Brilliant (2018) …
"In two out of three cases, you win the car by changing your selection after one of the doors is revealed. This is because there is a greater probability that you choose a door with a goat behind it in the first go, and then Monty is guaranteed to reveal that one of the other doors has a goat behind it. Hence, by changing your option, you double your probability of winning."

Bruce (2001, 112) …
"If your policy is to stick with your original choice, you win only one time in three. But two times out of three you will pick an empty box. In that case, I have no choice. I am constrained to open the only empty one of the other two. Then changing your choice guarantees you a win two-thirds of the time."

Cook (2020, 84) …
"If Door One hides the car, you lose by switching. If Door One does not hide the car, then by switching, you guarantee yourself a win because the remaining door that hides a goat is necessarily eliminated by the host. So really, the probability that you will land on the car by switching is equal to the probability that Door One does *not* have the car behind it. That probability is 2/3."

Gessell (2015) …
"If you adopt the switching strategy, the only way you can lose the game is if you choose the door with the prize behind it on the first try. But the odds of choosing the door with the prize behind it on the first try are just 1 out of 3. This means that the odds of winning the game by switching are 2 out of 3."

Hájek and Hitchcock (2016, 16) …
“Here’s the simplest way to see that switching is advantageous. Suppose that you commit to a strategy of ‘sticking’ or ‘switching’ before you choose a door. It is easy to see that if you pursue a strategy of sticking, you will win the prize just in case your first guess is correct. For instance, if you initially choose door , and then stick with door 1 when offered a chance to switch, then you will win just in case the prize is behind door 1. Thus a strategy of sticking has a one in three probability of success. By contrast, switching will win you the prize just in case your initial choice was incorrect. For example, suppose you choose door 1, and the prize is behind door 2. Monty Hall will now show you what’s behind door 3, and switching to door 2 will win you the prize. Parallel reasoning applies if the prize is behind door 3. So a strategy of switching will win the prize in two cases out of three.”

Huemer (2018, 4-5) …
“Why does the prize have a 2/3 probability of being behind door B? … Suppose Monty runs the game 300 times. Each time, the location of the good prize is randomly selected from among the three doors. We would expect that in about 100 of these games, the contestant’s initial guess is correct, that is, the first door they pick has the prize behind it. The other 200 times, the initial guess is wrong. Therefore, if the contestants always stick with their initial guess, then 100 of the 300 will win the real prize, and 200 will receive goats … Now, on the other hand, suppose that the contestants always switch doors. Then the 100 contestants who initially picked the correct door will lose, as they give up that door. But the other 200, the ones who initially picked wrong, will all switch doors. And they will all switch to the *correct* door, since the correct door will be the only remaining door, after rejecting the door they initially picked and the goat door that Monty just opened. So the “switch doors” strategy wins 2/3 of the time, whereas the “stick with your door” strategy wins only 1/3 of the time.”

Khan Academy (2012) …
“Are you better off switching to whatever curtain is left? … There’s three doors. The prize is equally likely to be behind any one of them. … One has the outcome that you desire. The probability of winning will be 1/3 if you don’t switch. … [But] if you picked one of the wrong doors, they’re going to have to show the other wrong door. And so if you switch, you’re going to end up on the right answer. So, what is the probability of winning if you always switch? Well, it’s going to be the probability that you initially picked wrong. … There’s two out of the three ways to initially pick wrong. So you actually have a 2/3 chance of winning.”

Li (2013) …
“If you stay with your first choice, you win only if your first choice happened to be the right one. And that is the case with probability one-third. So … the probability of winning given the strategy of staying with your first choice is one-third … If your first choice happened to be the right door, then switching away from that choice will always lose … that happens with probability one-third. But the rest of the time, with probability two-thirds, your first choice will be wrong … if your initial pick was wrong, then the prize is behind one of the two other doors. Your friend has to open one of the doors. But he can’t open the door that has the prize behind it. So, he has to open the other bad door, leaving the good door with the prize behind it as the one that you can switch to. And so, by switching, you will win in this scenario … so … the probability of winning if you switch is two-thirds.”

Nihous (2009, 94) …
"While switching would be a systematic loss if the initial pick was the winning site (probability 1/3), it would guarantee a win if the initial pick was one of the two losing sites (probability 2/3). Hence, the chance of winning by switching is 2/3, i.e., twice as great as the initial probability!"

Pynes (2013, 42) …
"In 1/3 of the cases she initially picks the prize door and wins when she stays. In the other 2/3 of the cases where she didn't initially pick the prize door, she wins when she switches. So the best strategy to win the prize is to pick a door and then switch to the non-tease reveal door when given the opportunity to switch."

Stewart (2019, 67) …
"Since another door has been eliminated, the conditional probability that the car is behind a door, given that this is not the one you chose, is 1 – 1/3 = 2/3, because there's only one such door, and we've just seen that two times out of three your door is the wrong one. Therefore, two times out of three, changing to the other door wins the car."

Talwalkar (2013, 56) …
"Note that if you stay, you only win if the initial door you picked had the grand prize. This is a 1/3 chance. On the other hand, if you switch, you would win if the grand prize were in either of the doors you did not pick. That's a 2/3 chance."

Titelbaum (2022, 181) …
"When the contestant originally selected her door, she was 1/3 confident that the prize was behind it and 2/3 confident that the prize was somewhere else. If her initial pick was correct, she claims the prize just in case she sticks with that pick. But if her initial selection was wrong, she wins by switching to the other remaining closed door, because it must contain the prize. So there's a 1/3 chance that sticking is the winning strategy, and a 2/3 chance that switching will earn her the prize. Clearly switching is a better idea!"

University of Illinois (2023) …
"When you first pick your door, the probability of picking the winning door is 1/3 … if you pick Door 1 and don't change your door when Door 1 is winning -- you win! If you pick Door 1 and don't change your door and Door 2 is winning -- you lose! You can see all possibilities in this table. So overall, you will win 3/9 or 1/3 times in this scenario … if you pick Door 1 originally and change your door when Door 1 is winning -- you lose! If you pick Door 1 and change your door and Door 2 is winning -- you win! You can see all possibilities in this table. So overall, you will win 6/9 or 2/3 times in this scenario."

Winkler (2004, 34) …
"Of course, she should switch. If the game is played 300 times, the right door will be chosen *initially* about 100 of those times; the other 200 games will be won by the contestant who switches!"

4. The Best Strategy in Cases Where the Initial Probabilities are Unequal:

Theorem: Let C_i and M_j for $1 \leq i, j \leq 3$ and $i \neq j$ denote the events that the car is behind door i and that Monty opens door j, respectively. And let P(C_i) = p_i. When the probabilities are $p_1 > p_2 > p_3$, the best strategy is to pick door #3 and switch.

Proof:

Suppose that you pick door #3. P(C_3| M_1) = p_3/(p_3 + 2p_2). This is less than $\frac{1}{2}$ because p_3 is less than 2p_2 by assumption. P(C_3| M_2) = p_3/(p_3 + 2p_1). This is also less than $\frac{1}{2}$ because p_3 is less than 2p_1 by assumption. So, if you pick door #3, it is better to switch rather than stay regardless of which door Monty opens (and regardless of what the precise values of p_1, p_2, and p_3 are). And if you pick door #3 and switch, you clearly win the car with an overall probability of 1 - p_3.

But what if you pick another door initially, say, door #1? There are three possibilities. First, it might be better to stay no matter what door Monty opens. Second, it might be better to switch no matter what door Monty opens. Third, it might be better to stay if Monty opens one door and better to switch if he opens the other door.

In the first case, you win with an overall probability of p_1. But this is less than 1 - p_3, which is equal to p_1 + p_2. In the second case, you win with an overall probability of 1 - p_1. But this is less than 1 - p_3, since p_3 is less than p_1 by assumption. In the third case, you win with an overall probability that is a weighted average of p_1 and 1 - p_1. In other words, you win with an overall probability that is between p_1 and 1 - p_1, which is guaranteed to be less than 1 - p_3 by the arguments in the preceding cases.

So, if you pick door #1, you are guaranteed to do worse than picking door #3 and switching. The argument for door #2 is exactly the same (except that the first case is not possible since it is never better to stay if Monty opens door #3).

5. Python Simulation of the Unequal Monty variation:

```
import numpy as np

NUM_TRIALS = 100000

wins_stick = 0
wins_switch = 0

rel_trials = 0
rel_wins_stick = 0
rel_wins_switch = 0

for t in range(NUM_TRIALS):
```

```
    #Initialize array of closed doors
    closed_doors = np.array([1, 2, 3])

    #Determine winning door
    winning_door = np.random.choice(closed_doors, p=[1/3, 2/15, 8/15])

    #Make initial choice of door
    initial_door = np.random.choice(closed_doors)

    #Monty chooses a door to open
    door_to_open = np.random.choice(np.setdiff1d(closed_doors, [winning_door, initial_door]))

    #Update the array of closed doors
    closed_doors = np.setdiff1d(closed_doors, door_to_open)

    #Check whether this is a relevant trial
    #That is, check that you pick door #1 and Monty opens door #3 and reveals a goat)
    if initial_door == 1 and door_to_open == 3:
        rel_trials += 1

    #Stick strategy
    final_door = initial_door
    if final_door == winning_door:
        wins_stick += 1
        if initial_door == 1 and door_to_open == 3:
            rel_wins_stick += 1

    #Switch strategy
    final_door = np.random.choice(np.setdiff1d(closed_doors, initial_door))
    if final_door == winning_door:
        wins_switch += 1
        if initial_door == 1 and door_to_open == 3:
            rel_wins_switch += 1

#Print results
print('{:15s}: {:7d} /{:7d} ({:.4f})'.format('stick', wins_stick, NUM_TRIALS,
1.0*wins_stick/NUM_TRIALS))
print('{:15s}: {:7d} /{:7d} ({:.4f})'.format('switch', wins_switch, NUM_TRIALS,
1.0*wins_switch/NUM_TRIALS))
print('{:15s}: {:7d} /{:7d} ({:.4f})'.format('relevant stick', rel_wins_stick, rel_trials,
1.0*rel_wins_stick/rel_trials))
print('{:15s}: {:7d} /{:7d} ({:.4f})'.format('relevant switch', rel_wins_switch, rel_trials,
1.0*rel_wins_switch/rel_trials))
```

Sample Output:

```
stick          :   33228 / 100000 (0.3323)
switch         :   66772 / 100000 (0.6677)
relevant stick :    5494 /   9875 (0.5564)
relevant switch:    4381 /   9875 (0.4436)
```