

# A Heptalemma for Quantum Mechanics

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We present a seven-pronged no-go result for quantum mechanics: a “heptalemma”. It shows that seven initially plausible theses about physical reality are jointly inconsistent with the predictions of quantum mechanics, while any six are jointly consistent. We must then decide which theses to retain and which to give up. Since different interpretations of quantum mechanics entail different responses to the heptalemma, we get a novel taxonomy of such interpretations. Beyond the application to quantum mechanics, the heptalemma offers a general diagnostic criterion for determining whether a given scientific domain should count as classical or not, and if not, how it departs from classicality.

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## 1. Introduction

It is widely appreciated that quantum mechanics gives us a picture of physical reality that goes against some key tenets of a classical physical worldview. Over the years, a series of paradoxes and no-go results, including the Einstein-Podolsky-Rosen (EPR) paradox (Einstein *et al.* 1935), Bell’s theorem (Bell 1964, 1976), the Kochen-Specker theorem (Kochen and Specker 1967, Budroni *et al.* 2022), and the thought experiment of “Wigner’s friend” (Wigner 1961, Schmid *et al.* 2023), have helped physicists to identify the ways in which quantum-mechanical predictions are in tension with various traditional or intuitive assumptions about the nature of the physical world.<sup>1</sup> But although those tensions are well known, there is still no consensus on how to respond to them. What picture of reality best fits quantum mechanics?

In this paper, we will present a new no-go result that we hope will shed further light on this question. We will state seven initially plausible theses that may all be viewed as

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<sup>1</sup>Indeed, deriving no-go theorems for quantum mechanics is, by now, a broader tradition. For a comprehensive review, see Leifer (2014).

key ingredients of a pre-quantum-mechanical worldview – the sort of worldview Einstein might have subscribed to – but which are jointly inconsistent with the predictions of quantum mechanics. Since any six of the seven theses are jointly consistent, our result admits seven distinct “escape routes” (strictly speaking, families of escape routes), depending on which thesis we reject. The result therefore forces us to decide which theses to retain and which not. We call this a “heptalemma for quantum mechanics”. Different interpretations of quantum mechanics can be shown to correspond to different “horns” of this heptalemma, and so we get a novel taxonomy of interpretations of quantum mechanics.

Formally, the heptalemma can be viewed as an extension of Bell’s theorem, obtained via fine-graining some of Bell’s assumptions. It therefore stands in the tradition of earlier such refinements in the literature (notably Jarrett 1984, where one of Bell’s assumptions, locality, is split into two assumptions, parameter independence and outcome independence). We do something similar with Bell’s assumption of realism (sometimes also known as outcome definiteness or counterfactual definiteness), but our philosophical ambition goes further. The goal is to offer a detailed map of different ways in which the quantum-mechanical picture of reality departs from the classical one.

The paper is structured as follows. In Section 2, we briefly review Bell’s theorem as a background to our analysis. In Section 3, we state the seven theses and show how they lead to a heptalemma. In Section 4, we discuss the seven logically possible escape routes, and in Section 5, we say more about where three particularly challenging interpretations of quantum mechanics – Copenhagen, Everett, and QBism – fit into the resulting picture. In Section 6, we explain how the heptalemma may be used as a criterion for determining which scientific domains should count as classical and which not, and for diagnosing different departures from classicality more broadly. In Section 7, finally, we conclude.

## 2. Bell’s theorem

Bell’s theorem can be motivated by reference to the famous Einstein-Podolsky-Rosen paradox (EPR) paradox (Einstein *et al.* 1935). Recall that, in quantum mechanics, it is impossible to measure an individual particle’s position and momentum simultaneously with perfect precision, by Heisenberg’s uncertainty principle. Several of the founders of quantum mechanics, notably Bohr and Heisenberg, took this and other aspects of the new theory to suggest that there is no fact of the matter about a particle’s position or momentum before or unless we perform a measurement. (Perhaps the particle’s position or momentum will become “real” only in the act of measurement or is only a meaningful

quantity with respect to the particular experimental arrangement necessary to elicit a value.) Einstein, Podolsky, and Rosen presented a thought experiment that challenges this “no fact” assumption. Suppose we send two maximally entangled particles in opposite directions and let each of them travel very far, in principle even for light-years. The entanglement of the two particles implies that if an observer were to measure the position of one of them, they would be able to predict the outcome of a position measurement of the other with perfect precision. Similarly, if an observer were to measure the momentum of one of the particles, they would be able to predict the outcome of a momentum measurement of the other equally precisely. On the assumption that physics admits no “spooky action at a distance”, especially no transmission of information faster than the speed of light, there is no way any measurement performed on the first particle could instantaneously affect the second particle. It should make no difference to the second particle whether the observer measures the position or the momentum of the first particle. Einstein, Podolsky, and Rosen concluded that the perfect predictability of the measurement outcome for the second particle can only be explained if there are some pre-existing facts about what those measurement outcomes would be.<sup>2</sup> This goes against Bohr’s and Heisenberg’s “no fact” assumption. Einstein, Podolsky, and Rosen took this to suggest that quantum mechanics is incomplete. It seems there must be some hidden variables by which the measurement outcomes have been determined in advance.<sup>3</sup>

This is where Bell’s theorem comes into the picture. It shows that the assumption of hidden variables would not resolve the EPR paradox if we retain the other assumptions of the thought experiment. Rather, we face a general no-go result. Suppose we make the following assumptions about a physical system:

**Realism:** Any observer’s measurement outcomes correspond to objective facts about the system over which probabilities are well-defined.

**Locality:** Measurements performed in one place do not instantaneously bring about effects in a distant place.

**Measurement independence:** Measurement choices (e.g., the settings of a measurement apparatus) are probabilistically independent of each other and of any states of the system that are being measured. (Informally, observers can freely choose their measurement settings, independently of one another and of the system in question.)

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<sup>2</sup>Here they invoke the principle that “[i]f, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity” (Einstein *et al.* 1935, p. 777).

<sup>3</sup>The EPR paradox is often misrepresented. For an exceptionally clear exposition, we recommend Stacey (2018).

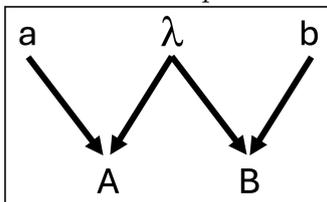
Bell’s theorem shows that these three assumptions are jointly inconsistent with the predictions of quantum mechanics. More precisely, Bell’s theorem establishes the following:

- If a system meets these three assumptions, then the system will satisfy certain constraints on the correlations between the outcomes of different measurements that we could perform on that system – the so-called Bell inequalities; and
- quantum mechanics predicts violations of those inequalities.

One can think of the Bell inequalities as expressing upper bounds on the correlations that can occur in a system satisfying the three assumptions. Slightly more technically, each of the terms of a particular Bell inequality known as the CHSH (Clauser-Horne-Shimony-Holt) inequality corresponds to a different pair of measurements that could be performed on the given system (Clauser *et al.* 1969).<sup>4</sup> Under Bell’s assumptions, the expectations of four distinct such measurement pairs combined in a particular way are bounded above by a value that quantum-mechanical predictions can violate. And so it follows that quantum-mechanical systems cannot satisfy all of realism, locality, and measurement independence. At least one of these assumptions must be false.

To illustrate this result, let us represent something like the EPR setting with the help of a simple model. Suppose that two particles are sent in opposite directions (“left” and “right”), starting from a common source, and suppose that one observer, Alice, performs a measurement on the left particle, while another observer, Bob, performs a measurement on the right particle. The setup includes five variables,  $\lambda, A, B, a, b$ , where  $\lambda$  stands for the state of the common source,  $a$  and  $b$  stand for Alice’s and Bob’s measurement settings (broadly analogous to the choice between measuring position or measuring momentum), and  $A$  and  $B$  stand for Alice’s and Bob’s measurement outcomes. Bell’s assumptions entail that the probabilistic dependencies between the five variables are as shown by the directed acyclic graph in Figure 1, where arrows represent probabilistic dependencies between variables. In particular, because of locality, Alice’s measurement outcome,  $A$ ,

Figure 1: Bell’s assumptions illustrated



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<sup>4</sup>The satisfaction of the CHSH inequality, in turn, is equivalent to the existence of a single joint probability distribution for all measurement outcomes (Fine 1982).

depends on the state of the common source ( $\lambda$ ) and her chosen measurement setting ( $a$ ), but not on anything that happens on Bob’s side (such as  $b$  or  $B$ ). Similarly, Bob’s measurement outcome,  $B$ , depends on the state of the common source ( $\lambda$ ) and his measurement setting ( $b$ ), but not on anything that happens on Alice’s side (such as  $a$  or  $A$ ). Finally, because of measurement independence, Alice’s and Bob’s measurement settings,  $a$  and  $b$ , are independent from each other and from the common source of the particles ( $\lambda$ ).<sup>5</sup> This dependency structure can be shown to satisfy the CHSH inequality, as explained in detail in the appendix. Of course, there will be some correlations in the present system, but in the simple model of Figure 1, these satisfy the relevant constraints. For example, there is a correlation between  $A$  and  $B$ , since they are both dependent on  $\lambda$ . At the same time,  $A$  and  $B$  will be conditionally independent, given their common causes.<sup>6</sup> The crux is that quantum mechanics predicts breaches of the implied constraints on the correlations, and so the given model is inconsistent with quantum mechanics. Since the model accurately represents the implications of Bell’s assumptions, at least one of the three assumptions must be false.

It is important to note that there is nothing incoherent about Bell’s assumptions *by themselves*. In a classical mechanical system, say a system of billiard balls that behave according to the laws of classical physics, the assumptions can be unproblematically met. There could be a classical physical system that is accurately represented by the setup of Figure 1. What Bell’s theorem establishes is that the three assumptions are jointly inconsistent with quantum mechanics. In effect, quantum mechanics implies that reality behaves non-classically.

Bell’s theorem is useful not only for diagnosing how quantum mechanics departs from the classical picture of reality, but also for generating a taxonomy of different interpretations of quantum mechanics. These can be classified in terms of which of Bell’s assumptions they give up. Let us briefly illustrate this by reference to a few examples. One class of interpretations gives up the assumption of locality. The de Broglie-Bohm interpretation (Goldstein 2025), for instance, takes this route, by postulating that quantum-mechanical systems behave deterministically based on the *global* configuration of underlying particle positions. Giving up locality, however, amounts to admitting what Einstein called “spooky action at a distance”. A second class of interpretations gives up measurement independence. So-called superdeterministic interpretations fall into this

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<sup>5</sup>Readers might wonder how this model invokes Bell’s assumption of realism. That assumption allows us to postulate that there are objective facts about the five variables and their probabilistic dependencies.

<sup>6</sup>We can think of the directed acyclic graph in Figure 1 as being associated with a Bayesian network. Locality and measurement independence can then be viewed as implications of the parental Markov condition applied to the given network structure. Generally, in a Bayesian network satisfying the parental Markov condition, any variable is independent of its non-descendants, given its parents (Pearl 2000).

class. They postulate that there are dependencies between measurement choices and the states of the system that are being measured: different observers' measurements are correlated due to underlying initial conditions. This kind of interpretation is sometimes characterized as denying that observers have free will with respect to the choice of measurement settings – an implication that some scholars see as a feature and others as a bug. A third class of interpretations, finally, gives up realism as it is defined in Bell's theorem. This is exemplified by the kind of view associated with Bohr and Heisenberg, often called the Copenhagen interpretation, though there are different variants of it. As already noted, Bohr and Heisenberg thought that there is no fact of the matter about certain properties of a physical system until these are explicitly measured. However, a very diverse range of theories is usually grouped together under the umbrella of the denial of Bell's realism assumption, and as we will see, there are many distinct ways in which one might relax that assumption. Accordingly, we will show that one can arrive at a more fine-grained taxonomy.<sup>7</sup>

### 3. A heptalemma

We will show that quantum mechanics faces a seven-pronged no-go result that can be viewed as a refinement of Bell's theorem. Specifically, there are seven initially plausible theses about physical reality which are jointly inconsistent with the predictions of quantum mechanics and of which any six are consistent with those predictions.<sup>8</sup>

Our first step is to unpack the formulation of realism in Bell's theorem. We propose that this actually expresses the conjunction of four theses, which are jointly sufficient for realism in the original sense used in Bell's theorem. We will first state the four theses and then make some further explanatory comments.

**Measurement realism:** Any observer's measurement outcomes correspond to facts over which there are well-defined probabilities, where there is no presumption whether

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<sup>7</sup>Importantly, of course, the literature already contains several more nuanced no-go results in the vicinity of Bell's theorem. For example, in a similar spirit to Bell's theorem, Greenberger *et al.* (1989) derive a deterministic contradiction between local realism and quantum predictions for three or more particles; Hardy's paradox (Hardy 1992) shows a contradiction between local realism and quantum predictions using a single set of events; and Leggett's theorem (Leggett 2003) cuts locality and realism along different seams, ruling out a different class of hidden-variable models if a stronger realism condition is adopted. Subject to a preparation independence assumption, the Pusey-Barrett-Rudolph theorem (Pusey *et al.* 2012) rules out *purely* epistemic interpretations of the wavefunction. The Colbeck-Renner theorem (Colbeck and Renner 2011) shows, under assumptions of measurement independence and no-signaling, that no hidden variable extension of quantum theory can improve outcome predictions.

<sup>8</sup>While some of our theses are inherited from Bell's theorem, others are variants of related theses from the philosophical literature, especially in two philosophical no-go results in List (2025) and Fine (2005), respectively. We will say more about the connections with that literature below.

the facts are objective or subjective, absolute or relative, coherent or incoherent.

**Non-relationalism:** Facts, including all to which any observer’s measurement outcomes correspond, are non-relational: when they obtain, they obtain *simpliciter*, not relative to something else.

**Non-fragmentation:** Any possible world is coherent, i.e., the total collection of facts that hold at that world can be jointly instantiated.

**One world:** Reality is exhausted by one objective world, which can be defined as the total collection of facts that hold at that world.

The first thesis – measurement realism – asserts only a very undemanding form of realism. It says that any observer’s measurement outcomes correspond to *some* aspects of reality – some “facts” – while still leaving open the precise nature of those facts. In particular, measurement realism says nothing about whether measurement outcomes correspond to objective or subjective facts, to absolute or relative ones, to jointly coherent or incoherent ones. So, as far as measurement realism is concerned, the postulated facts could be facts in an extraordinarily weak sense: they could be subjective rather than objective, relative rather than absolute, for instance relative to an observer or some other physical system, and they could even be incoherent rather than coherent. The next three theses then successively constrain the nature of the postulated facts, demanding that they must be absolute (not relative), jointly coherent (not possibly incoherent), and objective (not subjective). By way of contrast, Bell’s original assumption of realism already presupposes all of these constraints.

In more detail, non-relationalism asserts that any fact is of the “absolute” form “such and such is the case”, not of the “relative” form “such and such is the case, relative to something else”. As Wittgenstein noted in his *Tractatus*, a fact is something that is the case. Examples of facts are the following: water is  $H_2O$ ; a hydrogen atom contains one proton;  $2 + 2 = 4$ ; humans live on Earth. By accepting these as facts, we take them to hold *simpliciter*, not relative to something else. For example, humans live on Earth, full stop, not just relative to something else. If facts were relational, facts could only be properly specified as ordered pairs of the form  $\langle A, B \rangle$ , where  $A$  is something that is the case relative to  $B$ . Thus facts would always be *dyadic*. On the standard view according to which facts are non-relational, by contrast, any fact is *monadic*: a fact is something ( $A$ ) that is the case *simpliciter*; no relativization to anything else (any  $B$ ) is needed.

Non-fragmentation then states that any possible world – i.e., any way a world could be – is coherent. We should think of any world – whether actual or possible – as a coherent and complete collection of facts. Equivalently, the total collection of facts that

obtain at any given world can be coherently coinstantiated. This also goes back to one of Wittgenstein’s famous ideas from his *Tractatus*, namely the idea that “[t]he world is everything that is the case”, which he further clarifies by saying “[t]he world is the totality of facts” and “the totality of facts determines both what is the case, and also all that is not the case” (Wittgenstein 1922, 1, 1.1, 1.12). The background assumption is, of course, that the totality of facts making up the world is coherent. Otherwise we would not be dealing with a *possible* world. The assumption of non-fragmentation is so widely accepted that it is seldom even explicitly acknowledged. After all, the idea of coherent worlds is central to science, logic, and philosophy.

The thesis of one world, finally, asserts a central tenet of an objectivist worldview: reality is exhausted by one objective world, not multiple “subjective” worlds. It is not the case that different observers occupy their own distinct subjective worlds. This assumption is also almost unanimously accepted throughout science and philosophy. As we will see, even the Everett interpretation of quantum mechanics, which is sometimes (perhaps misleadingly) described as a “many worlds” interpretation, can be viewed as retaining the assumption of one world.

What Bell’s theorem shows, in effect, is that any system satisfying these four theses together with locality and measurement independence must satisfy Bell inequalities. And since quantum-mechanical systems can violate Bell inequalities, we can infer that the conjunction of the four theses conflicts with locality and measurement independence. Strictly speaking, however, the conflict arises only in conjunction with a further thesis, which is usually presupposed tacitly:

**Non-solipsism:** Reality admits more than one observer.

If there were only one observer, we would not even be able to construct the kinds of scenarios in which different observers could each choose their measurement settings independently and in which the issue of Bell-inequality-violating correlations could arise.

Combining the foregoing theses we get the following no-go result:

**Heptalemma for quantum mechanics:** Given the predictions of quantum mechanics, the following seven theses are mutually inconsistent: measurement realism, non-relationalism, non-fragmentation, one world, locality, measurement independence, and non-solipsism. Any subset of these theses is mutually consistent.

As already noted, the easiest way to derive this result is via the observation that four of our theses jointly imply realism in Bell’s original sense, and that Bell’s original theorem presupposes non-solipsism. When we further add locality and measurement independence, the no-go result then follows from Bell’s theorem. Like Bell’s original

theorem, the present result establishes that the relevant theses are inconsistent *with the predictions of quantum mechanics*. The seven theses *by themselves* are mutually consistent. We will return to the significance of this point in Section 6.

Interestingly, our no-go result can also be seen as a quantum-mechanical analogue of some recent no-go results from the philosophical literature on first-personal facts. Examples of first-personal facts are “I am having such-and-such experiences” or “I occupy such-and-such position in the world”. First-personal facts are non-objective insofar as they are non-invariant under changes in the subjective perspective from which they hold. Whether it is true that I am having such-and-such experiences obviously depends on who I am and which perspective I occupy. There are at least two no-go results concerning first-personal facts in the recent philosophical literature. One, called a “quadrilemma for theories of consciousness” (List 2025), asserts that realism about first-personal facts is jointly inconsistent with non-solipsism, non-fragmentation, and one world, where these theses are essentially the same as in the present heptalemma. This “quadrilemma” presupposes that first-personal facts are non-relational, and so it implicitly relies on the thesis of non-relationalism as well.<sup>9</sup> Another related no-go result, due to Fine (2005), asserts a conflict between realism about first-personal facts and three other theses called “first-personal neutrality” (which broadly plays the role of non-solipsism), “absolutism” (broadly equivalent to non-relationalism), and “coherence” (broadly equivalent to non-fragmentation). Fine’s result can be interpreted either as presupposing one world or as treating the combination of non-fragmentation and one world as a single thesis.

The analogy between our heptalemma and those philosophical no-go results goes as follows. Suppose we accept locality and measurement independence in Bell’s theorem. Then, given the predictions of quantum mechanics, we must reject Bell’s original assumption of realism. Suppose, however, we would still like to retain measurement realism in the very undemanding sense we have introduced, i.e., we would still like to postulate some “facts” – however minimalistically understood – that correspond to each observer’s measurement outcomes. And suppose further that we do not want to go along the solipsistic route of assuming that reality admits only a single observer. Then something about the postulated facts must be non-classical, i.e., distinct from how Bell or Einstein, Podolsky, and Rosen thought about those facts. Specifically, the postulated facts must either be of a relational sort, or they must fail to generate a coherent totality, or the total collection of facts must be able to vary from observer to observer. In the first of these cases, we have a breach of non-relationalism; in the second, a breach of

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<sup>9</sup>According to realism about first-personal facts, it is not merely the case that “relative to Christian’s perspective, I am having Christian’s experiences”, but “I am having Christian’s experiences” *simpliciter*.

non-fragmentation; and in the last, a breach of one world. This structurally resembles the above-mentioned no-go results for first-personal facts. To see that any six of our seven theses are jointly consistent with the predictions of quantum mechanics, we now turn to the possible escape routes from the inconsistency.

#### 4. Escape routes from the heptalemma

While the seven theses of the heptalemma are jointly inconsistent with the predictions of quantum mechanics, any six can be jointly satisfied. In other words, there are seven fundamentally distinct ways to “escape” the heptalemma, depending on which thesis one rejects. These seven escape routes correspond to different interpretations of quantum mechanics, or more precisely different *families* of interpretations. Which theses an interpretation rejects and which it salvages tells us something about its priorities in the negotiation of an updated scientific worldview. In this section we discuss each of the possible escape routes.

##### 4.1. Rejecting locality

As we have seen, one reaction to Bell’s theorem is to reject locality, i.e., to allow measurements performed in one place to instantaneously bring about effects in a distant place or to stand in correlations that cannot be accounted for by purely local mechanisms. This, of course, remains a consistent escape route from the heptalemma as well.

The idea that reality admits only local influences is deeply ingrained in modern physics and philosophy. Physics, however, did not consistently embrace the idea of locality until the late 19th century, and only with Einstein did strict relativistic locality become foundational. For example, Newton’s principle of universal gravitation violates locality, allowing for “action at a distance”. But even then, the intuition of locality was pervasive; Newton himself was dissatisfied with the non-locality in his theory (Berkovitz 2007). In a letter to Richard Bentley, he wrote:

“That gravity should be innate[,] inherent and [essential] to matter so that one body may act upon another at a distance through a vacuum without the mediation of any thing else by and through which their action or force [may] be conveyed from one to another is to me so great an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it.” (Newton 1692/3, also quoted in Berkovitz 2007)

Arguably, part of Einstein’s triumph with his relativity theories lies in the physical codification of this intuition of locality. In philosophy, similarly, locality is a widely

held assumption. In Hall’s influential analysis of the concept of causation as production, locality is treated as a key defining condition of causal production (Hall 2004).

A relaxation of locality therefore comes with substantial metaphysical cost. In its most clearcut form, it means admitting what Einstein called “spooky action at a distance”. While he did not live to see Bell’s theorem, it seems likely that Einstein would have entertained almost any other option. Indeed, he argued that rejecting locality imperiled the entire enterprise of science, writing that “[t]he complete suspension of this basic principle would make impossible the idea of (quasi-) closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us” (Einstein 1948, p. 322, quoted and translated in Howard 1985, p. 188). While many do not share these instincts, locality is certainly not abandoned lightly.

Moreover, any rejection of locality must be done with care as quantum mechanics does not permit *signaling*, the transmission of information faster than the speed of light. In other words, *even if* there is non-locality in a physical system, this property cannot be leveraged to enable superluminal communication. We would thus require a form of non-locality that carefully constrains the permissible action at a distance. The onus seems to be on proponents of non-locality to explain why there is non-locality and yet no signalling.

That said, several interpretations of quantum mechanics do reject locality. First and most familiar is the de Broglie-Bohm interpretation, already mentioned in Section 2. It postulates particles with individual velocities that instantaneously depend on the full configuration of every other, no matter how distant, in a manner derived from the physical wavefunction. Another family of interpretations that reject locality are the so-called generalized collapse models, including those developed by Ghirardi, Rimini, Weber and others (Ghirardi *et al.* 1986). These interpretations posit a stochastic element in the dynamical evolution of the wavefunction, such that its non-local collapses, including those upon measurement, may be given a more familiar dynamical explanation. The so-called transactional interpretation (Cramer 1986), meanwhile, explains quantum correlations in terms of waves that propagate both forward and backwards in time. The wavefunction realism of Ney (2021), in some ways more parsimoniously, proposes that wavefunctions are physically real and physically collapse. While this collapse is non-local from the perspective of our usual three-dimensional space, wavefunctions are not taken to *inhabit* this three-dimensional space, but ultimately *give rise* to it: the (approximate) locality of our experience is supposed to be an emergent phenomenon. Finally, an interpretation recently developed by Barandes (2025) takes quantum mechanics to be a kind of “indivisible” stochastic process theory. The idea here is that the predictions of quantum

mechanics are consistent with a non-Markovian stochastic process theory, where hidden variables are non-local, but, of course, signaling is still prohibited. There are certainly more non-local interpretations – this list is not an exhaustive catalog. The point is that the escape route of rejecting locality is not only consistent, but popular as well.

## 4.2. Rejecting measurement independence

A second escape route that we already mentioned in relation to Bell’s theorem is to reject measurement independence. This would permit dependencies between Alice’s and Bob’s measurement choices as well as between the choices of either and the system being measured. However, if Alice’s or Bob’s choices depend on the choices of the other or on the system itself, the sense in which these can be said to be *choices* at all is called into question. This challenge is not easy to rebut, since the rejection of measurement independence explicitly allows for the possibility that the system in question influences the supposed choices of measurements made upon it. As we also mentioned above, denial of measurement independence is sometimes associated with a denial that experimenters possess free will as far as their choices of measurement settings are concerned. It can of course be contested whether a lack of measurement independence really implies a lack of free will and, if so, whether this speaks against the present escape route. Nonetheless, measurement independence may be rejected in the heptalemma, leaving behind six mutually consistent theses.

Taken to its extreme, one could even reject measurement independence *deterministically*, such that the choices of Alice and Bob are determined *with certainty* by the physical state of the system. It is perhaps unsurprising that there are physicists and philosophers who prefer interpretations of this kind, known as superdeterministic interpretations, as this intuition amounts to a supreme dedication to physical determinism. A recent view of this type is the cellular automaton interpretation of ’t Hooft (2016). Here it is imagined that physical reality is fundamentally discrete, local, and deterministic and that this state of affairs is best understood by analogy with models of computation like cellular automata. Everything that happens in reality, including any proposed Bell scenarios and choices of the agents involved, is determined by the initial state of the universe. Another variant of superdeterminism is discussed by Hossenfelder and Palmer (2020) and does not insist on a discrete, computational analogy. For an approach that rejects measurement independence without postulating determinism, see, for example, the statistical contextual interpretation (Kupczynski 2025).

### 4.3. Rejecting non-solipsism

Rejecting non-solipsism is the first of five novel escape routes that the heptalemma allows us to discuss. This route supposes that reality *does not* admit more than one observer, i.e., there is *at most* one real observer. This is a consistent, albeit rather extreme, way out because it cuts off the thought experiment before it can even begin. If there is only one observer, we cannot actually have Alice *and* Bob.

While rejecting non-solipsism is a logically possible escape route, it does not appear that anyone seriously entertains it as a reasonable response to the challenges of quantum mechanics.<sup>10</sup> That said, we note that QBism (Fuchs and Schack 2013, Fuchs and Stacey 2018) has been accused of being a solipsistic view (e.g., Norsen 2014). This accusation is derived from QBism’s suggestion that the quantum formalism is a “single-user theory”, i.e., that when it is used, it concerns the actions and experiences of a single agent, the user. While this reading of the formalism only *requires* one observer, QBists are certainly not supposing that there actually is only one. Rather, the quantum formalism, according to QBism, may be adopted by *any* observer to help her navigate the consequences of her actions. A solipsist could interpret quantum mechanics in a QBist way, but QBism itself does not entail solipsism at all. We say more about QBism in Section 5.

### 4.4. Rejecting measurement realism

Rejecting measurement realism is the second novel escape route and the first that can be viewed as a refinement of one of the existing routes out of Bell’s theorem. As Bell’s realism combines the theses of measurement realism, non-relationalism, non-fragmentation, and one world, rejection of any of these four corresponds to rejecting Bell’s realism. However, specifically rejecting one of these finer-grained theses enables us to draw more precise distinctions and to compare different reasons why realism in Bell’s original sense might not hold.

One readily sees that there are two ways to reject measurement realism. The first is to claim that measurement outcomes do not correspond to facts in *any* sense. It is not that facts about measurement outcomes are subjective rather than objective, dyadic rather than monadic, or fail to be coherent in the standard sense. They are simply not facts *at all*. Measurement outcomes are not something that is the case, no matter how

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<sup>10</sup>Convivial Solipsism (Zwirn 2000) and the solipsistic hidden variable model of Nikolić (2012) both postulate multiple observers and are therefore not true solipsisms under our definition. The latter can be converted into a true solipsism by simply supposing that only one observer exists. As presented, however, objective reality is associated to each conscious observer, and so Nikolić’s model is actually a rejection of the one-world postulate.

loosely you interpret the condition.

The second way to reject measurement realism supposes that outcomes *are* facts in one or more of the relevant senses, but that there are no well-defined probabilities over them. An absurd caricature of this second option could even claim that measurement outcomes are objective facts in one coherent world, yet the quantum formalism does not give us anything that is interpretable as probabilities over those facts. But rejecting measurement realism in this way would have to be accompanied by some account of why quantum mechanics is correct even if the numbers it spits out are not interpretable as probabilities at all.

Both ways of rejecting measurement realism have precedent. Quantum Darwinism (Zurek 2009) is a modern example of a rejection of the first kind. The idea is to try to solve the quantum measurement problem by absorbing the concept of measurement into quantum dynamics. Towards this, it is argued that the phenomenon of decoherence allows one to understand measurement outcomes as quasi-stable aspects of the physical unitary evolution. Zurek writes: “This transition from uncertainty (initial presence of many branches – potential for multiple outcomes) to certainty (once a sufficiently long branch fragment becomes known) accounts for perception of ‘collapse’” (2009, p. 185). Thus, while we have the perception of stable measurement outcomes, there is no associated fact at the lowest level. We will argue in Section 5.2 that the Everett interpretation (Wallace 2012, Vaidman 2021), also known as “the many-worlds interpretation” or “the relative state formulation”, is best understood, despite these names, as rejecting measurement realism in the second way. Briefly, because *all* outcomes factually occur, there is no sense in which they occur with well-defined probabilities. Although interpretationally quite far from Everett, one can also make the case that some variants of the Copenhagen interpretation take this route. However, as we elaborate in Section 5.1, Copenhagen is not just one view; while all versions robustly reject Bell’s realism, it does not appear that all versions cleanly reject only one of the four theses that constitute it. Finally, one might also read into QBism an inclination towards rejecting measurement realism in this way, but as we argue in Section 5.3, this is not what the conceptual thrust of QBism most clearly points to.

#### **4.5. Rejecting non-relationalism**

If one wishes to preserve the relatively undemanding notion of realism articulated by measurement realism, one can still deny one of the three additional constraints that together entail Bell’s original realism. The first of these is non-relationalism. Rejecting non-relationalism means admitting a nonstandard relational character for at least some

facts. This is a revision of the nature of a fact itself – it is not the case that when a fact obtains, it obtains *simpliciter*. Rejecting non-relationalism means that at least some facts are dyadic in nature. On this view, a fact is not something that simply is the case, but may be something that is the case only relative to something else. Generically, any fact is of the form “A is the case, relative to B”, and so any collection of facts is a collection of ordered pairs of the form  $\langle A, B \rangle$ . This move can disable the derivation of Bell inequalities by massively enlarging the sample space. Outcomes are no longer of the “absolute” or “monadic” form “such and such outcome occurred”, but only of the “relational” or “dyadic” form “such and such outcome occurred, relative to such and such relativization parameter”, where the latter could be an observer or a system.

The best-known interpretation which takes this route is aptly named Relational Quantum Mechanics (RQM) (Rovelli 1996, 2025). In RQM, the state of a quantum system is inherently relational, i.e., the state *is* the relation between observer and system, or even more generally between system and system. In this view, any physical system can have a quantum state relative to any other – so, there are no stipulations on what could count as an “observer” – and there are no states in a non-relational sense. This commitment to relationalism has been explicitly spelled out by Di Biagio and Rovelli (2021). They write: “Facts happen at every interaction, but they are not absolute: they are relative to the systems involved in the interaction” (p. 29). The authors go on to develop an understanding of what they call stable facts, which can be understood as facts whose relativity can be mostly ignored. Although facts are intrinsically dyadic, of the form  $\langle A, B \rangle$ , which can be read “A is the case relative to B”, a stable fact is one that is invariant under changes in the second component; i.e.,  $\langle A, B \rangle$  is a stable fact if and only if it continues to hold whenever B is replaced by any other  $B'$ . RQM correctly identifies itself as a realist view, moreover, explicitly adhering to the one-non-fragmented-world picture when it describes itself as

“[r]ealist in the sense that it is not about agents, beliefs, observers, or experiences: it is about real facts of the world and relative probabilities of their occurrence. The ontology is relational, in the sense that it is based on facts established at interactions and are labelled by physical contexts.” (Di Biagio and Rovelli 2021, p. 30)

Thus RQM retains measurement realism, non-fragmentation, and one world. There is also room to define variants of RQM with different degrees of commitment to relationalism (see, e.g., Calosi and Riedel 2024, Stacey 2021).

Some other interpretations appear to reject non-relationalism too, though they differ in what exactly they take outcomes or facts to be relative to, for instance whether the

relativization can be to any system or merely to an observer. For example, Brukner (2018, p. 7) notes that a possible consequence of his no-go result is that “there cannot be facts of the world per se, but only relative to an observer.” Healey’s pragmatic interpretation (2012) takes a different route in considering measurement outcomes to be non-relational, yet quantum states are agent-relative.

#### 4.6. Rejecting non-fragmentation

Another way to retain measurement realism while giving up Bell’s classical realism is to reject non-fragmentation. Recall that non-fragmentation asserts that any possible world is coherent in the sense that all facts that obtain at it can indeed be jointly instantiated. Another gloss on the same idea is that a world is a *coherent totality*. To reject this is to admit that while there might be locally coherent subcollections of all the facts that hold at a given world, one cannot demand that every fact is compatible with every other. This move allows one to escape the heptalemma because it allows for the possibility that in some circumstances, like the Bell scenario, there simply cannot be a coherent joint probability distribution. At most certain subsets of the relevant measurement outcomes can be jointly assigned coherent probabilities.

As we have mentioned, however, non-fragmentation is central to much of science, logic, and philosophy. This is so much the case that it is rarely even mentioned as an explicit assumption. Accordingly, a rejection of non-fragmentation comes, *prima facie*, with considerable cost. Before committing to such a route, many would want to have a sensible fragmentalist alternative in hand. Indeed, there has been some work to develop fragmentalist views in philosophy, including in recent debates about consciousness and first-person facts (see, e.g., Fine 2005, Lipman 2023, cf. Merlo 2016). Merlo (2016, p. 324), for instance, writes (without endorsing the position): “one could take all points of view to be on a par vis-a-vis truth *simpliciter* by treating them as different ‘fragments’ of an overall incoherent totality of facts”.

Fragmentalist views have been more readily adopted in the foundations of physics so far. Within the Copenhagen family of interpretations, Bohr’s complementarity pushes us in this direction. The quantum logic tradition (Wilce 2024) postulates that facts form an orthomodular lattice, but not a Boolean algebra. Bub and Pitowsky (2010) similarly point to non-Boolean event structures, emphasizing that while there is one objective world, it does not support a classical, global fact structure. With less emphasis on modifications to logic, the consistent histories approach (Griffiths 2024) proposes that the principal distinction of quantum mechanics lies in the availability of multiple mutually-incompatible modes of description, so-called frameworks, such that no single globally

coherent totality of facts can obtain. Abramsky and Brandenburger (2011) provide a formalization of a fragmentalist reading of contextuality with sheaf theory.

#### 4.7. Rejecting one world

If one wishes to retain measurement realism, non-relationalism, and non-fragmentation (as well as locality, measurement independence, and non-solipsism), one could take the final escape route from the heptalemma, which is to reject the thesis of one world. Rejecting this would be to deny that reality is exhausted by one objective world. There must be more worlds, more “seats” for facts. We would have to give up the assumption that “reality” and “the world” are synonymous, since reality would consist of several distinct worlds, which are in some sense parallel to each other but equally real. Each world would be the total collection of facts that hold at that world. Any such world can then be both coherent and non-relational. And if there is more than one world, these could index different probability measures, so that we no longer have the single joint probability distribution of Bell’s theorem, which gives rise to the no-go result.

A natural way to reject one world is to postulate *subjective* worlds, perhaps one for each observer. These could be worlds in which genuinely subjective or first-personal facts reside. A subjective world could be defined as what philosophers and logicians call a “centered world”, i.e., as an ordered pair consisting of an objective world and a subject or agent around which it is conceptually “centered”. Or it could be understood in some primitive or basic sense, according to which the notion of a subjective world is fundamental and the notion of an objective world is merely emergent, arising from invariances across subjective worlds. For example, any fact that is invariant under changes in the observer could be deemed objective. This many-worlds perspective has recently been discussed by one of the authors, though in relation to consciousness rather than quantum mechanics (List 2023). Earlier philosophical accounts that reject the assumption of one world, in a variety of different ways, include Goodman’s view that there is a plurality of actual worlds (1978, 1984; cf. Declos 2019), Vacariu’s account of “epistemologically different worlds” (2005, 2016), who also proposes its application to quantum mechanics, Honderich’s proposal that there are “subjective physical worlds” (2014), and Gabriel’s critique of the notion of a single world as a totality of everything (2015).

The view in quantum mechanics that comes closest to an outright rejection of one world is QBism. In fact, we will argue below that more explicitly aligning with this picture is one of two generic directions in which QBism might develop in the future.

## 5. Where do Copenhagen, Everett, and QBism fit?

Since some interpretations of quantum mechanics seem harder to categorize in terms of our heptalemma than others, we now comment on three particularly salient cases.

### 5.1. Copenhagen

Strictly speaking, there is no unique Copenhagen interpretation, but only a family of overlapping views with a shared intellectual history going back to work done in Copenhagen in the 1920s (Faye 2024). When one speaks of *the* Copenhagen interpretation, one could be thinking of the views of any of a substantial list of physicists including Bohr, Heisenberg, Born, Pauli, Peierls, Landau, Rosenfeld, von Weizsäcker, and Peres. One could also have in mind a somewhat vague interpretational gloss that is broadly inspired by these and other authors' ideas. This makes “the” Copenhagen interpretation harder to pin down in our heptalemma as well. Notwithstanding this diversity, it remains a coherent enough family to robustly reject Bell's realism. Moreover, any set of ideas that is recognizably Copenhagen will certainly wish to preserve measurement independence and non-solipsism. Preservation of locality is almost universal as well, although there are some exceptions, notably Peres (1993).

Precisely how Copenhagen rejects Bell's realism does not appear to be uniform across the family, but we can still narrow it down significantly. First, and most straightforwardly, no Copengener seems to have entertained the idea that there could be more than the one objective world. While Pauli, for instance, takes issue with the notion of a “detached observer”, he does not go so far as to say that outcomes are personal and don't obtain in an objective, public world.<sup>11</sup> Second, it seems that Copenhagen does not reject non-relationalism, either. While Bohr emphasized that measurement quantities are only meaningful relative to the complete experimental arrangement, this is best viewed either as a somewhat operationalist attitude or as a form of relativism about description, but not as a relational account of facts of the kind now endorsed by Rovelli and others. Measurement outcomes, for Bohr, are always distinctly permanent marks on photographic plates or instances of irreversible amplification in large, classical measurement instruments such that the results are objectively available for anyone to verify.

Thus, the way in which Copenhagen rejects Bell's realism appears to be either by rejecting measurement realism entirely or by rejecting non-fragmentation. Both readings are defensible and have been argued for in the past. Almost all varieties of Copenhagen

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<sup>11</sup>The reader is encouraged to peruse a correspondence between Pauli and Bohr, discussed by Fuchs (2017a, Section 3).

emphasize that the quantum state is not an objective property of the system, and is instead a mathematical tool for thinking about it. As this does not appear to be a claim of subjectivity or relativism, this stance can be read as a rejection of measurement realism through the denial that there are well-defined probabilities associated to measurement outcomes. This is often identified as the salient anti-realist feature of the Copenhagen interpretation, see, e.g., Maudlin (2019). On the other hand, the lesson that Bohr and others draw from the complementarity principle is one in which a single picture does not exist that unites outcomes that could be obtained in different experimental contexts (Bohr 1963). This kind of reading seems to reject non-fragmentation.

## 5.2. Everett

The Everett interpretation is popularly known as “the many-worlds interpretation” and was named “the relative state formulation” by its inventor. From the perspective of the heptalemma, however, we suggest it should be viewed neither as a many-worlds interpretation nor as a relational one. Rather, it arguably upholds both one world and non-relationalism. The supposed “many worlds” of the Everett interpretation are just branches of the universal wavefunction evolution, and this straightforwardly fits into a single world, albeit one that is richer than conventionally assumed (namely, branching-tree-like). Chalmers (1996, p. 347) describes the Everett interpretation as a “one-big-world interpretation”: “There is only one world, but it has more in it than we might have thought” (1996, p. 347). Similarly, while speaking of a particular measurement outcome is only meaningful relative to the branch in which it occurs, the ontology of this view is that *all* possible outcomes actually do occur. Although we find ourselves in one branch, all branches are equally real. All of these outcomes factually obtain, *simpliciter*, in one coherent world. The label “multiverse” arguably describes this idea reasonably well, as long as it is understood that the multiverse in its entirety is a single world in the philosophical sense, not many.

In the taxonomy of our heptalemma, we think the Everettians are best understood as rejecting measurement realism. Since all possible outcomes equally occur in the Everettian multiverse, albeit in different branches, there isn’t a fact of the matter as to which particular outcome actually results from a given measurement. At most, Everettians could say that *all* of those outcomes are facts. But then there is no single outcome for which one can have well-defined probabilities. Indeed, probabilities are not properly part of Everettianism. On this picture, Everettians reject measurement realism in the second way discussed above. What is real according to the Everett interpretation is the universal wavefunction and its unitary evolution. As in Quantum Darwinism,

measurement is an add-on, which is – strictly speaking – not part of the physics except insofar as one should address how measurement can be understood to be part of the physical dynamics. Making contact with probabilities and with the Born rule is a research project for Everettians and has been approached from a variety of perspectives including decision theory (Deutsch 1999, Wallace 2012) and so-called self-locating probability (Saunders 1998, Sebens and Carroll 2018).

Could one also squeeze the Everett interpretation into different horns of our heptalemma, by understanding it in a slightly different way? One might say, for instance, that it gives up non-relationalism by making facts relative to branches. But the Everett interpretation is not normally understood as being committed to an ontology of dyadic facts in the way RQM is. Alternatively, one might say that the Everett interpretation postulates branch-specific facts, which are best viewed as *indexical* facts, akin to facts about where I am currently located or what time it is now. Assuming I find myself in a particular branch, it is an indexical fact that my measurement outcome is such and such. One might then say that each branch corresponds to a different total collection of indexical facts: a different “indexical world”. The Everett interpretation could thus be viewed as rejecting one world, after all, by postulating many indexical worlds. However, we suspect most Everettians would not want to “reify” those indexical worlds or view them as primitives of their physical ontology. Indeed, as Wallace (2012) has argued, the multiverse is an emergent phenomenon; it is not ontologically fundamental.

### 5.3. QBism

QBism starts out by taking a radically anti-objectivist stance towards quantum theory. At its most basic, it treats the quantum formalism as an aid to an agent’s decision-making. This sharply contrasts with interpretations that suppose that quantum mechanics seeks to describe the objective world. QBism asserts that quantum states and measurements do not represent anything that transcends the user’s unique situation, but that they capture the user’s capacity for action, physical embodiment, and any beliefs the user holds. On this picture, the objects of quantum mechanics are personal judgments that help the user to navigate the consequences of his or her actions (where a consequence is understood to be one’s personal experience of a measurement outcome). Moreover, QBists take themselves to interpret quantum theory *normatively*, namely as providing checks on the consistency of an agent’s beliefs, similar to checking whether a set of credences form a coherent set of probabilistic beliefs.

This much, however, is only part of the story. The greater part of QBism’s ambition, and where it remains a work in progress, is the articulation of lessons about the nature of

reality. QBism can thus also be viewed as an ontological project. In particular, it asks, what is it about reality that makes *this* reasoning structure, this particular mathematical addition to a physics-agnostic decision theory, the right means for navigation. So, what is QBism’s take on our seven theses?

We noted above how QBism responds to accusations of solipsism. Only a caricatured radical single-user version of QBism would be solipsistic. Moreover, while it denies talk of hidden variables, QBism accepts measurement independence insofar as it affirms that the quantum-mechanical formalism can be used to help agents manage the consequences of their *freely chosen* actions. Locality, meanwhile, becomes almost tautological because, in QBism, the quantum formalism never describes physical influences between distant events; it is a tool for an agent to manage her own possible future experiences, and these always occur along her worldline, never at spacelike separations.<sup>12</sup>

These affirmations require QBism to reject Bell’s realism, which it naturally does. Here, the heptalemma’s refinement of Bell’s realism brings more clarity to QBism’s conceptual location. The first thing to highlight is that QBism strongly affirms non-relationalism, allowing us to formalize the often-neglected distinction between QBism and interpretations that admit relational facts, such as relational quantum mechanics.<sup>13</sup> Fuchs, for instance, is clear that he rejects any kind of relationalism when he writes:

“Experience is a far richer notion than a perspective on some pre-existing thing. As well it is far more than a simple relation among pre-existing things. Lived experience has an autonomy that neither of these notions capture.”  
(Fuchs 2023, p. 128)

But what about the other three theses that jointly entail Bell’s realism? Does QBism affirm measurement realism as we have stated it in the heptalemma? While there is some subtlety, we think QBism is not only consistent with measurement realism, but should be read as robustly affirming it. There is no doubt that it regards measurement outcomes as factual in the sense that outcomes are something that is the case. Outcomes are intrinsically *personal*, but they are not illusions, relational, or approximate notions. The facts in question may be first-personal, but they are facts no less.<sup>14</sup> Indeed, QBism

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<sup>12</sup>In this context, the equivalent of non-locality would have to be detected *within* the agent’s mesh of beliefs in the form of a lack of independence of her probability assignments for the outcomes of her future actions on a distant system from her *freely chosen* actions on a nearby system.

<sup>13</sup>Pienaar (2021) and Barzegar and Oriti (2024) both describe QBism as relational, but this appears to stem from lacking the distinctions we draw in the heptalemma. As we have emphasized, a genuine rejection of non-relationalism entails a reconstrual of the notion of a fact as dyadic rather than monadic. RQM explicitly endorses this, but we do not take QBism to do the same.

<sup>14</sup>Importantly, they are of the usual monadic form: such and such is the case; they are not merely of the relational form: such and such is the case relative to such and such.

looks to measurement for a grounding of the notion of *participatory realism* (Fuchs 2017b). A recent paper further foregrounds QBism’s measurement realism, and situates it as pursuing the diametric opposite of most approaches to the quantum measurement problem, by deriving quantum dynamics from consistent reasoning about measurement (DeBroda *et al.* 2024). In other words, rather than being somehow part of the dynamics, measurement is the *most* real thing according to QBism.

The subtlety comes from the second sense in which measurement realism can be rejected, which corresponds to the denial that there are well-defined probabilities for outcomes. QBism holds that there are well-defined probabilities, but it treats them, like measurement outcomes, as personal or subjective. QBists tend to view probabilities as degrees of belief for the consequences of one’s actions and even espouse a particularly radical form of subjective Bayesianism that identifies probabilities with betting dispositions. It is here that QBism might seem most anti-realist.

However, we suggest that such a purely epistemic or practical stance on probabilities is not strictly *mandated* by the rest of QBism’s commitments, at least according to our heptalemma. Our analysis shows that there is no conflict, in principle, between a subjectivist interpretation of probabilities and a realist one, in the weak sense of measurement realism. The relevant probabilities can be both factual and subjective.

We think that the most straightforward version of QBism is one that rejects one world while retaining measurement realism. Different observers’ measurements will then correspond to facts over which probabilities are well defined; it just so happens that the totality of facts and probability assignments can vary from observer to observer. Each observer is associated with his or her own observer-specific world (in analogy to the first-personally centered worlds discussed in List 2023). The denial that reality is exhausted by one objective world would be consistent with the above-mentioned philosophical critiques of one world (Goodman 1978, 1984; Vacariu 2005, 2016; Gabriel 2015). Mermin, especially, seems to take this route:

“The fact is that *my* science has a subject (me) as well as an object (my world). *Your* science has a subject (you) as well as an object (your world). *Alice’s* science has a subject (she) as well as an object (her world). I make the same point three times to underline both the plurality of subjects, and the plurality of worlds that each of us constructs on the basis of our own individual experience.” (Mermin 2019, p. 5)

Fuchs also seems to support a rejection of one world when he writes:

“[T]he lesson of quantum theory is that *experience happens*, and the refine-

ment of the notion that comes from Wigner’s thought experiment is that there *is a sense* in which those experiences need not live in a single universe.” (Fuchs 2023, p. 125)

Although both seem to be saying that the experiences of different participants inhabit distinct worlds, our heptalemma allows us to pinpoint more precisely what this view amounts to.

On the other hand, QBists also routinely speak of *the* world – an unfinished, yet singular, world constantly coming into being, where “[e]ach quantum measurement creates something new in the universe that is above and beyond the agent’s relation to the quantum system they are acting upon” (Fuchs 2023, p. 128). This intuition deprecates pre-existent subjective worlds just as much as a pre-existent objective one. The idea is that reality does not take place in any such containers, but is actively constituted by parts that always maintain some autonomy, as in James’s republican banquet (James 1882). Indeed, QBism feels the pull of fragmentation. For instance, Fuchs (2007) makes the analogy to Escher’s paintings of impossible objects. This reading locates the personalism of QBism not in separate worlds, but in the sense that there are local parts of the world which do not admit a global coherence. It seems, then, that rather than rejecting one world, QBists could be saying that the single world we live in is not a coherent totality.

If both of these are plausible readings, then, for now, there are two distinct versions of QBism, in addition to the implausible solipsistic version. The first version, which we may label “Pluriverse QBism”<sup>15</sup>, rejects one world and takes different agents to inhabit or experience genuinely distinct subjective worlds that overlap in suitable ways. The second version of QBism, which we may label “Fragmentalist QBism”, locates all observers in a single world, fundamentally fragmented along suitable notions of personalist fault lines.

These options enjoy a clearcut formal difference. The formal machinery that would be needed to precisify each approach would look very different. The pluriverse approach would postulate many subjective worlds and many subjective probability measures, where each of them individually behaves in the standard way, i.e., each subjective world is a complete and consistent collection of facts that hold at that world, and each subjective probability measure is a coherent assignment of probabilities to all elements of the relevant algebra of events. By contrast, the fragmentalist approach would not postulate any coherent totalities at all. We would be dealing with a single world/reality

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<sup>15</sup>It is ironic that the word “multiverse” is already associated with the Everett interpretation, as that interpretation manifestly affirms the one world thesis. In our treatment, if any interpretation deserved the term “many worlds”, it would be the version of QBism that rejects one world. On the other hand, “pluriverse” is uniquely appropriate for QBism due to its intellectual debts to William James.

that is an incoherent totality and of which only fragments are coherent. Moreover, we wouldn't have a globally coherent probability assignment but only locally coherent subassignments. Pluriverse QBism might be easier to develop in some ways, as it can probably leverage more from existing philosophical literature. On the other hand, Fragmentalist QBism might be relatively more at home among the variety of fragmentalist constructions that have appeared in physics. Or, finally, it might turn out that one or the other version of QBism is ultimately incompatible with other convictions once considered closely enough.

## 6. The heptalemma as a diagnostic criterion

As noted, the seven theses are not jointly inconsistent by themselves, but only together with the predictions of quantum mechanics. If physical reality were classical – i.e., in line with a pre-quantum-mechanical worldview – there would be no conflict between them; they could be jointly satisfied. Had Einstein's theories of relativity been the final word, in some sense, we would be in a position to accept all of them. This observation suggests that the heptalemma may also be put to another use: the seven theses give us a helpful diagnostic criterion for determining whether a given scientific domain (as represented by a particular scientific theory) should count as classical or not, and if not, in which sense the given domain behaves non-classically.

For example, suppose we focus solely on a sufficiently macroscopic (and non-quantum) subdomain of physics, or on a particular domain within geology, or on a particular domain of the biomedical sciences. Then it is quite plausible to think that we can consistently embrace all seven theses (and we suspect that researchers in those fields do so, at least implicitly): measurement outcomes correspond to real facts, and there are well-defined probabilities over them; facts are non-relational; the total collection of facts is coherent; there is one objective world; everything is local; we have measurement independence; and we can have as many observers as we like. The domain of quantum mechanics stands out insofar as it is not like this. Whether quantum mechanics is the only – or the main – example of a non-classical domain in the sciences is a matter for debate. Some philosophical accounts of consciousness, but by no means all, suggest that the study of consciousness might be another non-classical domain. Recall, in particular, the theories that postulate first-person facts mentioned earlier.

Formally, the criterion for classicality is the following:

**A criterion for classicality:** The domain of facts represented by a particular scientific theory is classical if and only if the theory in question is jointly consistent with

all of measurement realism, non-relationalism, non-fragmentation, one world, locality, measurement independence, and non-solipsism.

If a domain fails this test, this domain counts as non-classical, and we can use the heptalemma to pinpoint the precise sense in which it is non-classical, depending on the horn of the heptalemma that we think best fits that domain. The ongoing debates about the interpretation of quantum mechanics can be viewed as debates about this question.

## 7. Concluding remarks

We have presented a heptalemma for quantum mechanics: a seven-pronged no-go result. One of its upshots is a novel taxonomy of interpretations of quantum mechanics, where they are characterized in terms of which horn of the heptalemma they take. Table 1 summarizes this taxonomy.

Table 1: Interpretations of quantum mechanics and what they reject

Interpretation	Measurement realism	Non-relationalism	Non-fragmentation	One world	Locality	Measurement independence	Non-solipsism
de Broglie-Bohm					<b>X</b>		
Collapse models					<b>X</b>		
Wavefunction realism					<b>X</b>		
Transactional					<b>X</b>		
Indivisible stochastic					<b>X</b>		
Superdeterminism						<b>X</b>	
Cellular automaton						<b>X</b>	
Statistical Contextual						<b>X</b>	
Copenhagen	<b>X</b>						
Everett	<b>X</b>						
Quantum Darwinism	<b>X</b>						
Relational		<b>X</b>					
Pragmatic		<b>X</b>					
Brukner		<b>X</b>					
Quantum logic			<b>X</b>				
Bub-Pitowsky			<b>X</b>				
Consistent Hhistories			<b>X</b>				
Sheaf contextual			<b>X</b>				
Fragmentalist QBism			<b>X</b>				
Pluriverse QBism				<b>X</b>			
Radical single user							<b>X</b>

In this table, we only indicate which thesis each interpretation primarily rejects,

leaving open its stand on the remaining six theses. There may be some debate about whether some interpretations reject more than one thesis, and there may not always be a clearcut answer, insofar as some interpretations are best viewed as families of interpretations that come in different variants. The logic of the heptalemma, however, forces each interpretation to reject only one of the seven theses. So, in principle, there could be a consistent variant of each interpretation that rejects solely the indicated thesis and accepts all others. Finally, like Bell’s original theorem, the heptalemma could be refined further, by splitting some of the seven theses into further conjuncts. The result would be an even more fine-grained taxonomy. We hope, however, to have struck a balance between informativeness and parsimony.

In the long run, of course, getting a clear lay of the land obtains its full value not merely in telling us where it is possible to go, but, hopefully, in helping us make a choice on which route to take. Someday the challenges that quantum mechanics posed to the scientific worldview might be taught as history, rather than a state of affairs that has now persisted for over a century. Perhaps then a new challenge will be just around the corner.

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### Appendix: Bell’s theorem

In this appendix we present a pedagogical derivation of Bell’s theorem. The following does not trace one particular source, though many have appeared. One can refer to Myrvold *et al.* (2024) for further details.

The Bell scenario concerns two spacelike separated agents Alice and Bob, each equipped with a measurement device. A source produces two signals, sending one to Alice’s device and one to Bob’s. Each agent chooses a measurement setting  $a$  and  $b$  and subsequently registers an outcome  $A$  and  $B$ .

Bell’s theorem establishes that quantum theory is not compatible with the conjunction of the following theses:

**Realism:** There exists a single joint probability space such that all measurement outcomes  $A$  and  $B$  are probabilistically determined by the values of the measurement settings  $a, b$  and a “hidden variable”  $\lambda$ .

**Locality:** Measurements performed in one place do not instantaneously bring about effects in a distant place. So, the outcome  $B$  does not depend on the setting  $a$  or the outcome  $A$ ; likewise, the outcome  $A$  does not depend on the setting  $b$  or the outcome  $B$ .

**Measurement independence:** Measurement choices are statistically independent of each other and of the system being measured. This ensures that the experimenters’ choices are not pre-determined by the physical scenario.

For simplicity we assume that the probability space is discrete, so as to avoid the potentially subtle machinery of measure theory, but this comes with no conceptual loss.

Realism ensures that there are objective facts about the five variables  $\lambda$ ,  $A$ ,  $B$ ,  $a$ ,  $b$  and their probabilistic dependencies. The dependency structure entailed by locality and measurement independence then results in a factorization of the joint probability for  $A$  and  $B$  given the measurement choices  $a, b$ . Let us spell this out. First, the objective reality of  $\lambda$  ensures this joint probability can be expressed as a single marginalization over the possible hidden variable values,

$$P(A, B|a, b) = \sum_{\lambda} P(A, B, \lambda|a, b) = \sum_{\lambda} P(\lambda|a, b)P(A, B|a, b, \lambda), \quad (1)$$

where the second equality follows from the product rule. Locality implies the independence of  $A$  and  $B$  from settings or outcomes on the other side

$$P(A|a, b, B, \lambda) = P(A|a, \lambda) \quad \text{and} \quad P(B|a, b, A, \lambda) = P(B|b, \lambda), \quad (2)$$

so that, after another use of the product rule, we see that

$$P(A, B|a, b, \lambda) = \begin{cases} P(A|a, b, \lambda)P(B|a, b, A, \lambda) \\ P(A|a, b, B, \lambda)P(B|a, b, \lambda) \end{cases} = P(A|a, \lambda)P(B|b, \lambda). \quad (3)$$

Measurement independence establishes that the probability of  $\lambda$  cannot depend on the measurement settings  $a, b$  the experimenters choose,

$$P(\lambda|a, b) = P(\lambda). \quad (4)$$

These observations together imply the factorization

$$P(A, B|a, b) = \sum_{\lambda} P(\lambda)P(A|a, \lambda)P(B|b, \lambda). \quad (5)$$

The factorized form (5) is then used to derive so-called Bell inequalities, which may be understood as bounds on the strength of measurement outcome correlations compatible with Bell's theses. Here we present the well-known CHSH inequality (Clauser *et al.* 1969).

Suppose Alice and Bob each choose between two measurement settings, which (in a slight abuse of notation<sup>16</sup>) we denote  $a, a'$  on Alice's side and  $b, b'$  on Bob's side, and that each measurement has only two outcomes  $A, B \in \{\pm 1\}$ . For  $x \in \{a, a'\}$  and  $y \in \{b, b'\}$ ,

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<sup>16</sup>Here,  $a$  and  $a'$  are the possible values for the variable  $a$ , and  $b$  and  $b'$  are those for the variable  $b$ .

define the correlation, the expected product of the outcomes,

$$E(x, y) := \sum_{A, B = \pm 1} A B P(A, B|x, y), \quad (6)$$

and the conditional expectations

$$\bar{A}(x, \lambda) := \sum_{A = \pm 1} A P(A|x, \lambda), \quad \bar{B}(y, \lambda) := \sum_{B = \pm 1} B P(B|y, \lambda). \quad (7)$$

Substituting (5) into (6), we see that Bell's assumptions imply that the correlation is the averaged product of the expectations

$$E(x, y) = \sum_{\lambda} P(\lambda) \bar{A}(x, \lambda) \bar{B}(y, \lambda). \quad (8)$$

For simplicity, we first assume that  $\lambda$  and the measurement setting deterministically determine the measurement outcomes, that is, that the conditional probabilities take only the values  $P(A|x, \lambda) \in \{0, 1\}$  and  $P(B|y, \lambda) \in \{0, 1\}$ , so the conditional expectations are equal to the outcome values and

$$A(a, \lambda), A(a', \lambda), B(b, \lambda), B(b', \lambda) \in \{\pm 1\}, \quad (9)$$

so that the correlation function in this case is

$$E(x, y) = \sum_{\lambda} P(\lambda) A(x, \lambda) B(y, \lambda). \quad (10)$$

Now we define the CHSH combination, which relates the correlations associated with four different experiments,

$$S := E(a, b) + E(a, b') + E(a', b) - E(a', b'). \quad (11)$$

For a fixed  $\lambda$  define

$$S(\lambda) := A(a, \lambda)B(b, \lambda) + A(a, \lambda)B(b', \lambda) + A(a', \lambda)B(b, \lambda) - A(a', \lambda)B(b', \lambda). \quad (12)$$

Factor  $S(\lambda)$  as

$$S(\lambda) = A(a, \lambda)[B(b, \lambda) + B(b', \lambda)] + A(a', \lambda)[B(b, \lambda) - B(b', \lambda)]. \quad (13)$$

Since  $B(b, \lambda), B(b', \lambda) \in \{\pm 1\}$ , there are two cases:

1. If  $B(b, \lambda) = B(b', \lambda)$ , then  $B(b, \lambda) + B(b', \lambda) = \pm 2$  and  $B(b, \lambda) - B(b', \lambda) = 0$ , so  $S(\lambda) = \pm 2A(a, \lambda)$ .
2. If  $B(b, \lambda) = -B(b', \lambda)$ , then  $B(b, \lambda) + B(b', \lambda) = 0$  and  $B(b, \lambda) - B(b', \lambda) = \pm 2$ , so  $S(\lambda) = \pm 2A(a', \lambda)$ .

In all cases  $|S(\lambda)| = 2$ . Averaging over  $\lambda$  with  $P(\lambda)$  gives

$$|S| = \left| \sum_{\lambda} P(\lambda) S(\lambda) \right| \leq \sum_{\lambda} P(\lambda) |S(\lambda)| = 2. \quad (14)$$

Therefore, for any *deterministic* theory satisfying the Bell theses,

$$|E(a, b) + E(a, b') + E(a', b) - E(a', b')| \leq 2, \quad (15)$$

which is the CHSH inequality.

This proof straightforwardly extends to the stochastic case where we place no restrictions on the conditional probabilities. Replace the values in equation (13) with their conditional means  $\bar{A}(x, \lambda)$  and  $\bar{B}(y, \lambda)$ . Then  $|\bar{A}(y, \lambda)| \leq 1$  implies

$$|S(\lambda)| \leq |\bar{B}(b, \lambda) + \bar{B}(b', \lambda)| + |\bar{B}(b, \lambda) - \bar{B}(b', \lambda)|. \quad (16)$$

Now for two real numbers  $x, y$  with  $|x| \leq 1, |y| \leq 1$ , one has  $|x + y| + |x - y| \leq 2$ .<sup>17</sup> Applying this with  $x = \bar{B}(b, \lambda)$  and  $y = \bar{B}(b', \lambda)$  yields  $|S(\lambda)| \leq 2$  as before.

So, subject to the Bell assumptions, measurement outcome correlations may not exceed the CHSH inequality bound of 2. However, quantum mechanics predicts a violation of this bound up to a maximal value of  $2\sqrt{2}$ , as we now show.

Suppose Alice and Bob each receive a two-level quantum system and each chooses between two possible quantum measurements with outcome values of  $\pm 1$ . This setting can be realized as measurements  $\hat{a} = \vec{a} \cdot \vec{\sigma}$ ,  $\hat{a}' = \vec{a}' \cdot \vec{\sigma}$ ,  $\hat{b} = \vec{b} \cdot \vec{\sigma}$ ,  $\hat{b}' = \vec{b}' \cdot \vec{\sigma}$ , where  $\vec{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$  is the vector of Pauli matrices and  $\vec{a}, \vec{a}', \vec{b}, \vec{b}'$  are unit vectors in  $\mathbb{R}^3$ , so that Alice's and Bob's measurement choices can each be considered choices between two directions in space along which to measure spin. Given a pure quantum state  $|\psi\rangle$  on the bipartite system, the correlation of measurement outcomes, say with measurement settings  $a$  and  $b$ , is  $E(a, b) = \langle \psi | \hat{a} \otimes \hat{b} | \psi \rangle$ . Consider the two-qubit singlet state

$$|\psi^-\rangle := \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

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<sup>17</sup>If  $x$  and  $y$  have the same sign, then  $|x + y| = |x| + |y|$  and  $|x - y| = ||x| - |y||$  and it follows that  $|x + y| + |x - y| = 2 \max(|x|, |y|) \leq 2$ . If they have opposite signs, the roles reverse, but the bound is the same.

For spin measurements, the singlet correlations are

$$\langle \psi^- | (\vec{u} \cdot \vec{\sigma}) \otimes (\vec{v} \cdot \vec{\sigma}) | \psi^- \rangle = -\vec{u} \cdot \vec{v}.$$

Thus, for this state,

$$S = -(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b}' + \vec{a}' \cdot \vec{b} - \vec{a}' \cdot \vec{b}').$$

Choosing the four unit vectors to be

$$\vec{a} = (1, 0), \quad \vec{a}' = (0, 1), \quad \vec{b} = \frac{1}{\sqrt{2}}(1, 1), \quad \vec{b}' = \frac{1}{\sqrt{2}}(1, -1),$$

we have

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{b}' = \vec{a}' \cdot \vec{b} = \frac{1}{\sqrt{2}}, \quad \vec{a}' \cdot \vec{b}' = -\frac{1}{\sqrt{2}},$$

so

$$S = -\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right) = -\frac{4}{\sqrt{2}} = -2\sqrt{2}.$$

Therefore  $|S| = 2\sqrt{2}$ , which exceeds the bound predicted by Bell's assumptions. The fact that this violation is maximal is known as Tsirelson's bound (Cirel'son 1980) and follows from operator norm arguments that we omit here.

To summarize, the Bell assumptions predict that measurement outcome correlations cannot exceed the CHSH inequality bound of 2. Quantum mechanics predicts a violation of this bound. Indeed, violations of Bell inequalities have been conclusively demonstrated experimentally as well. Consequently, at least one of Bell's assumptions must be denied.