Prolegomenon to an epistemology of orbifolds

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Abstract

The field of the epistemology of spacetime interrogates how, if at all, we can come to know about the various constituent parts of spacetime. Historically in this field, the bulk of attention has rested on the epistemology of the *metrical* structure of spacetime; the question of the epistemological significance of various sub-metrical components of spacetime, e.g. topological structure, has up to this point been substantially neglected. In this article, we propose to rectify this situation, focussing in particular on exploring the epistemological and empirical upshots of generalising standard manifold topology to that of a orbifold. We demonstrate that there are various possible empirical signatures of orbifold structure, and various tests which could in principle be implemented in order to test whether one inhabits an orbifold. Along the way, we highlight certain other philosophically significant features of orbifolds—for example, the significance of the 'mirror' orbifold for Kantian discussions of incongruent counterparts.

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1 Introduction

In the philosophy of physics, it's usually assumed—taking the lead from our best current theory of space and time, namely general relativity (GR)—that spacetime is a (3+1)-dimensional Lorentzian manifold which is Hausdorff, paracompact, etc. At the same time, in contemporary physics one can identify other candidate mathematical structures for spacetime. One example of such candidate structures are orbifolds—which are, roughly, topological spaces which locally resemble finite group quotients of Euclidean space. For instance, physicists have recently become interested in studying black holes on orbifolds and their distinct physics (Nitta and Uzawa 2021). What's more, orbifolds appear in some solutions to the Einstein equations—for instance, in the case of Euclidean signature solutions (Crisafio et al. 2025). Moreover, orbifolds turn out to be of great importance in, e.g., string theory (Dixon et al. 1986; Dixon et al. 1985; Kraniotis 2000), where they are useful in, e.g., recovering realistic phenomenology via compactification, as well as in M-theory (Cheung et al. 2022; Krause 2000). Other examples include orbifolds in some of the 'Grand Unified Theories' (GUTs) (McReynolds 2005).

Yet, despite of the physical relevance of orbifolds in GR and beyond, they remain as yet wholly unstudied by philosophers. This fact is striking once one recalls the vast amount of philosophical work on spacetime theories set on manifolds (for which Earman (1989) is, arguably, the *locus classicus*). In this article, we aim to rectify this situation, and choose to focus in particular on the *epistemology* of orbifolds: what would it take for us to ascertain that we inhabit a world with a spacetime structure set on an orbifold? This is a particularly fecund angle from which to approach the philosophy of orbifolds, both due to the motivation which comes from physics—for instance, given their wide presence in string theory and other theories of quantum gravity, one can expect that the study of their empirical consequences might pave the way for novel approaches to empirical testing of such theories—and from philosophy—as it fits into the broader discussion of the epistemology of spacetime, which in recent years has undergone something of a resurgence—see Dewar et al. (2022) for a review.

Although much discussion in contemporary philosophy of space and time regards how we might gain empirical access to the *metric field* of spacetime manifolds, it is just as legitimate to probe the empirical significance of (a) sub-metrical constituents such as projective and conformal structures (on which see Adlam et al. (2025)), or (b) topological aspects of spacetime, such as its being a Hausdorff manifold (see Luc and Placek (2020)), or (c) its having certain differential structure (see Brans and Randall (1992)). Since, as we'll see, the difference between orbifolds and manifolds lies already at the level of topology and differential structure, in this article we continue in the spirit of (b) and (c), by asking whether there would be any empirical consequences of relaxing the manifold structure to the structure of an orbifold. To repeat, then, the central question of this paper is this: *what would it take to ascertain that we inhabit a world with the spacetime structure of an orbifold?*

The structure of this article is as follows. In §2, we introduce the mathematics of orbifolds, and in §3 we distinguish them from manifolds. In §4, we explore the ways in which the specific topological structure of an orbifold might manifest itself empirically. In §5, we consider in more detail the philosophical upshots of this work.

2 Orbifolds as generalisations of manifolds

Topological manifolds $\mathcal{M}$ are topological spaces such that a neighbourhood of each point is homeomorphic to $\mathbb{R}^n$, and in that sense they are sometimes considered to be 'locally Euclidean'. Usually it is possible to add to them some differential structure which allows one to perform calculus on them, thereby obtaining a differentiable manifold. Whether or not it is possible to define a differential structure on a manifold depends upon certain topological properties, such as its dimensionality.[1,2]

To a differentiable manifold one can further add a metrical structure g which gives an account of distances and angles, and which thereby yields a metric manifold $(\mathcal{M}, g)$.[3] This latter class is of particular interest in the context of spacetime theories, as, for example, a manifold with a Lorentzian metric is usually taken to represent spacetime in GR.[4]

In this article, unless stated otherwise, by 'manifold' we always mean a topological manifold. Notably, such manifolds do not behave well under algebraic operations. Consider quotienting—namely, acting on a manifold by a group action. For our purposes, we can take a quotient to collect similar elements of the initial group using an equivalence relation. Applying quotienting to a manifold typically does not result in another manifold. For instance, quotienting a plane with a group of reflections will result in a half-plane with an edge, which is not a manifold. However, typically the pathologies which arise from quotienting arise only at a small subset of points, sometimes even just a single point. Thus, the structure obtained by quotienting a manifold might well remain sufficiently 'manifold-like' to be of physical interest.

Orbifolds are candidate such structures. The notion of an orbifold was introduced originally by Satake (1956), and was popularised subsequently by Thurston (1980). Neighbourhoods of each point of an orbifold $\mathcal{O}$ are homeomorphic not to $\mathbb{R}^n$ (as in the case of a

[1]Note that manifolds that do not admit differential structure are rarely relevant to discussions in physics. In particular, all manifolds of dimensions $n = 1, 2, 3$ admit differential structure. Known examples of higher-dimensional manifolds which do not admit differential structure include an E^8 manifold, discovered by Freedman (1982) and a K^{4n+2} Kervaire manifold (Kervaire 1960). Manifolds discussed in this article always admit differential structures.

[2]Observe that while whether a topological manifold admits a differential structure depends on its topology, the type of a differential structure that it admits is not *uniquely* determined by topology. For instance, as showed by Kervaire and Milnor (1963), a 7-sphere admits 28 smooth structures (i.e., there exists 28 diffeomorphism classes of differentiable 7-manifolds homeomorphic to S^7).

[3]Typically, the differential structure on $\mathcal{M}$ is suppressed in this notation.

[4]See e.g. Wald (1984) for details.

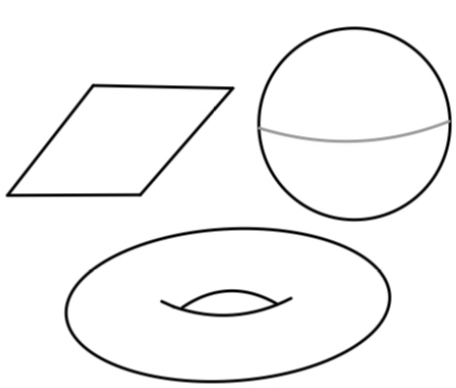

Figure 1: Examples of 2-dimensional manifolds: $\mathbb{R}^2$ (a plane), $\mathbb{S}^2$ (a sphere), and $\mathbb{T}^2$ (a torus).

Figure 2: Examples of 2-dimensional orbifolds: coxeter (boundaries), stellate (cones), and hat (cones and boundaries).

standard topological manifold), but rather to a finite group G quotient of $\mathbb{R}^n$, so $\mathbb{R}^n/G$. In this sense, it is sometimes said in the physics literature that orbifolds are locally finite group quotients of Euclidean space.[5] In this way, an orbifold can be considered to be an obvious generalisation of a manifold, and in a trivial sense all manifolds are also orbifolds (i.e. when the group G is a trivial group). Thus, from now on, whenever we speak of orbifolds, we are countenancing orbifolds that are not manifolds (unless indicated otherwise).

The generalisations to orbifolds of transition functions on a manifold are 'embeddings', which include the action of G on charts—see Appendix A for further details on the mathematics of orbifolds.

Examples of manifolds are presented in Figure 1, and examples of orbifolds are presented in Figure 2; precise definitions of both manifolds and of orbifolds are provided in Appendix A. Given that orbifolds are generalizations of manifolds, and that manifolds have garnered extensive interest in philosophical literature, it is useful to identify the differences between manifolds and orbifolds; this will be our next task.

3 Differences between manifolds and orbifolds

A commonly noted difference between manifolds and orbifolds in the literature is that orbifolds can, in general, have singularities arising from quotient constructions, whereas manifolds (of course) cannot. We discuss these singularities in §3.1. We suggest, however, that there exist also two other interesting properties that set the orbifolds apart from manifolds—namely, a possibility of local non-orientability of orbifolds (§3.2), and a possibility to localise certain properties of orbifolds (§3.3). These differences will turn out to be of significance in §4, when we turn to the empirical consequences of specific orbifold structures.

[5]Cf. Wang and Wang (2016, p. 286).

3.1 Singularities

3.1.1 Singular structure of orbifolds

The best-known difference between orbifolds and manifolds is that only the former can exhibit singularities arising from quotienting. For instance, already Satake (1957, p. 468) in one of his first papers introducing the notion of an orbifold (at that time called by him a 'V-manifold') speaks about 'singular points'.

Depending on the properties of the group action, quotienting a manifold by a group can result either in another manifold (if no singularities occur) or in an orbifold that is not a manifold (if quotienting leads to singularities). In particular, singularities occur if the group action is not free at some point in the original manifold, giving rise to an orbifold.[6] To illustrate this difference, consider the following two examples, with the same underlying set, but different quotienting groups.

Example 3.1. Consider a manifold $\mathcal{M} = \mathbb{R}^2$ and a group $G = \mathbb{Z} \times \mathbb{Z}$ with an action $\cdot$ which acts on $\mathcal{M}$ by $(p, q) \cdot (x, y) = (x + p, y + q)$, where $(p, q) \in \mathcal{M}$ and $(x, y) \in G$. It is easy to see that the structure $\mathcal{M}/G$ is a torus.

Therefore, in this case, quotienting a manifold yields another manifold, namely a torus. It is still a manifold because the action is free. However, in general, quotienting leads to singularities—as in the next example:

Example 3.2. Consider a manifold $\mathcal{M} = \mathbb{R}^2$ and a group $G = \mathbb{Z}/3$, which acts on $\mathcal{M}$ by $n \in \mathbb{Z}/3$ rotating every point of this manifold by $2\pi n/3$. A structure $\mathcal{M}/G$ obtained in such a way is cone-like.

Clearly, the cone is not a manifold because the neighbourhood of its vertex—that is, the point $(0, 0)$—is not homeomorphic to $\mathbb{R}^n$, but rather to $\mathbb{R}^n/G$, where G does not act freely.[7] Instead, the cone is an orbifold.

So far, we have simply followed the mathematics literature in referring to 'singularities' on orbifolds, but we have not yet clarified what exactly is meant by these orbifold singularities—i.e., in what sense they are singular and at which level of structure they arise: topological, metrical, affine, or differential. Philosophy offers insights into various types of singularities. However, before we turn to a summary of these, let us first examine the level at which singularities arise in an orbifold structure when they result from a group action that is not free.

Consider again the example of a cone and its singularity at the vertex. *Prima facie*, this singularity is at the level of differential structure. Indeed, one cannot define a manifold-style differential structure, which should be understood as a maximal smooth atlas $\mathcal{A} = \{(U_\alpha, \varphi_\alpha)\}_{\alpha \in A}$. This is because, at the point of singularity of an orbifold, there is no chart $(U_\alpha, \varphi_\alpha)$! Instead, there exists a chart $(U_\alpha, G_\alpha, \varphi_\alpha)$, where the action of a group G_α is not free (which leads to the singularity). This is precisely the singularity of this orbifold—that is, a point of which chart maps into $\mathbb{R}^n$ quotiented by a non-trivial group G_i. However, the form of a chart—should it be $(U_\alpha, \varphi_\alpha)$ or $(U_\alpha, G_\alpha, \varphi_\alpha)$—is a *topological* property! Thus, even

[6] For rigorous definitions of a free action, see Appendix A.

[7] Thus, it violates point (2) from the definition of a manifold in Appendix A.

though one could argue that orbifold singularities arise at the level of differential structure in the specific sense that orbifolds do not admit differential structure as can be defined on manifolds, the reason for this fact lies at the level of topology.[8]

3.1.2 Classifying orbifold singularities

Now we can turn to the question of where orbifold singularities fit into the understanding and classification of singularities familiar from the philosophy of physics literature. Given that the field has as yet neglected to study orbifolds, it's no surprise that singularities are usually discussed by philosophers in the context of manifolds. However, given that orbifolds are generalisations of manifolds, one can expect that the categories proposed by philosophers to classify manifold singularities might also be relevant in the discussion of orbifold singularities.

The philosophy of singularities is a developing field, one of the central questions of which concerns how singularities should be defined. In discussions regarding singularities of spacetime manifolds, philosophers have distinguished between (Curiel 1999):[9]

1. 'incomplete paths' spacetimes,
2. 'missing points' spacetimes,
3. 'curvature blowups' spacetimes, and
4. 'non-standard singular' spacetimes.

(These categories are not mutually exclusive.) In order to determine into which of these categories orbifold singularities fall, one must first be clear on the level of structure at which each type of manifold singularity arises. This is a challenging task for several reasons. First, as noted earlier, there is no consensus on how to define singularities, including definitions within each category (Curiel et al. 2025). Second, the debate over manifold singularities is formulated within the philosophy of general relativity, which means that it typically concerns Lorentzian manifolds—and these are, of course, metrical manifolds. Below, we present a tentative—yet sufficient for the purposes of our discussion of orbifold singularities—classification of these singularities with respect to the various levels of manifold structure that they, in general, concern.

Given that both 'paths' and manifold points belong to topological structure, it seems that singularities in classes (1) and (2) can be understood on a purely topological level.[10] An example of the first kind of singularity is for manifolds with boundary, in which there are paths that 'go into' the boundary.[11] For instance, consider a manifold $\mathbb{R}^3$ with a path crossing a plane $x = 0$; then we remove the subspace $x \leq 0$, leading to the path's being incomplete.

[8] For a discussion of a conic singularity at the level of metrical structure, see Norton (2025).

[9] For further recent philosophical discussion of singularities, see e.g. Cudek (2025).

[10] Note that paths are normally understood in a topological sense, i.e. a path is a continuous function from a closed interval into this space.

[11] For particular examples of such singularities and for more extensive discussion on them, see e.g. Penrose (1965) and Hawking and Penrose (1970). Note that some attempts to provide a rigorous definition of path-incomplete spacetimes include the notion of geodesics, or invoke the distinction between timelike and spacelike paths—meaning that they require affine/metrical structure rather than pure topology. Here we follow the most general understanding of this type of singularity.

An example of the second kind of singularity understood in a topological sense would be taking the same manifold $\mathbb{R}^3$ and then removing a point at the coordinates $x = y = z = 0$. Obviously, similar kinds of topological singularities can appear on orbifolds that either have incomplete paths or missing points obtained in the same way.[12]

Singularities of type (3) have to do with affine structure and as such are relevant to topological discussion about orbifolds only insofar as topological facts about a manifold are not independent of the affine/geometrical properties thereon. And the class (4) of 'non-standard singularities' is, in the nomenclature of Curiel (1999), an umbrella term for singularities that do not fall into previous categories—these include, for example, divergences of time derivatives of such quantities as the Hubble parameter.

In orbifolds, we are confronted with the possibility of a new kind of topological singularity: those which are the consequence of quotienting and which *ipso facto* do not involve *removing* points. For example, a vertex singularity of a cone obtained by quotienting $\mathbb{R}^2$ is not related to removing a point, but rather to modifying the topological structure around this point—in that, around this point, an orbifold is not locally Euclidean, but rather homeomorphic to Euclidean space quotiented by a non-trivial finite group.

Into which category do the quotienting singularities of orbifolds fall—that is, at which level of structure are they defined? It seems that they are neither (1) 'incomplete path' singularities—for there are no paths in our story; nor (2) 'missing point' singularities—for these do not entail that any points, lines, or surfaces are missing from the underlying topological manifold; nor (3) curvature blow-up singularities, which require affine structure, while quotienting singularities arise from the topology. Thus, by a process of elimination, it seems appropriate to classify orbifold singularities as a new type of (4) non-standard singularity.

Of course, one may argue that this conclusion is hardly surprising, given that the classification given above was developed originally for a special type of manifold—that is, Lorentzian manifolds—rather than more general structures, such as purely topological manifolds or orbifolds. Nonetheless, given that this is the only discussion of spacetime singularities available in the philosophical literature, and given that physicists speak of orbifold 'singularities', it is helpful to discuss this notion within the context of the existing classification of singularities for Lorentzian manifolds.

Two further points should be made here. First, Earman (1989, pp. 40–44) argued famously against understanding spacetime singularities literally as points 'missing' from a topological manifold. Instead, he considered them as being related to geodesics, therefore also rejecting the other type of singularity classified above as topological—that is, incomplete path singularities. Given this, orbifold singularities should be of particular interest, since they are the only candidates for topological singularities in the contemporary literature.

Second, one should further note that orbifold singularities are 'weak' in the sense that they do not entirely destroy either level of spacetime structure. Even though they affect topological structure, one could argue that they only destroy manifold topology—in that they lead to points that are homeomorphic to $\mathbb{R}^n/G_i$ rather than just $\mathbb{R}^n$. However, at the same

[12]Invocation of singularities such as 'missing points' might be argued to be *ad hoc*, in the sense (loosely speaking) that there is no good physical reason why a spacetime should be lacking such points. Cf. a similar discussion of the Principle of Sufficient Reason applied to spacetime properties and a claim that spacetime is 'as large as it can be' (Earman 1995; Manchak 2017).

time, one could argue that they just introduce a new topology—an orbifold topology—which simply involves homeomorphism to $\mathbb{R}^n/G_i$, where G_i is a non-trivial group. In this sense, these orbifold singularities are only singular when looked at from a manifold perspective, but they are not singular when we look at them as orbifolds. The same applies to the differential structure; even though one cannot define a differential structure on an orbifold in the same way in which it is defined on a manifold, there are alternative ways to construct a differentiable orbifold structure (see, for example, Watts (2017)). Nonetheless, in the rest of this article, we stick to the conventional nomenclature of 'orbifold singularities', while acknowledging their 'weak' character.

3.2 Local orientability

One of the properties which can characterise topological manifolds is their orientability. Roughly, orientability describes whether it is possible to determine the parity (or 'handedness' or 'chirality') of objects (indeed such as hands) on the entire topological space.[13] Many familiar manifolds are orientable, for example $\mathbb{R}^n$ or a sphere, but among the non-orientable manifolds one can enumerate the Möbius band, the Klein bottle, and the real projective plane.

In order to develop further understanding of orientability, consider two 2-dimensional manifolds, namely surfaces: a plane and a Möbius band. Now, consider two pairs of vectors, such that each pair has the same initial point at each surface and the vectors are perpendicular to these surfaces, but they are 'facing opposite directions'. In the first case, there are two different classes of vectors: one is pointing 'up' and another one is pointing 'down'. In the latter case, one can at first think that the same holds: it is possible to, at a given point, draw two vectors, out which one is pointing 'up', and another one is pointing 'down.' This, however, is merely an illusion. For, if we 'move'[14] one of this vectors around a closed loop across the entire length of the Möbius band, it turns out that it overlaps with the other one—so they both in fact belong to the same class of vectors. This kind of orientability for manifolds is defined on a topological level.[15] It is also the kind of orientability that is usually discussed in the very few philosophical works which discuss orientability at all, e.g. Earman (1991), Nerlich (1991), Sklar (1974), and Walker (1991).

As argued by Bielińska (2021) and Read and Bielińska (2022), on a topological level one can think of orientability in two ways: namely, local and global orientability. Global orientability is a property of an entire manifold, and is the kind of orientability that is usually discussed in the literature and has been mentioned above; for example, a Möbius band is non-orientable in a global sense.[16] Hence, we follow the convention; whenever we

[13] Such objects as hands are known in the philosophical literature as 'incongruent counterparts', starting with Immanuel Kant and his essay "On the First Ground of the Distinction of Regions in Space" (1768). For a debate on incongruent counterparts in Euclidean spaces, see for example the papers in the collection by van Cleve and Frederick (1991).

[14] For a rigorous understanding of this notion see Bielińska (2021) and Read and Bielińska (2022).

[15] There is another sense of orientability, defined on manifolds with metric, that is not relevant from the perspective of this article. For more details see Read and Bielińska (2022).

[16] For a precise understanding of global spacetime properties see Manchak (2020) and for its application to orientability see Read and Bielińska (2022).

say 'orientability' in this article, we refer to global orientability. Such orientability can be defined in a number of equivalent ways, for example:[17,18]

Definition 1 (orientability of a manifold)**.** *A manifold $\mathcal{M}$ is* orientable *iff all transition functions are orientation-preserving.*

There is, however, also a sense in which one can think about the *local* orientability of manifolds. As already mentioned above, manifolds are widely described as being 'locally Euclidean', in the sense that the neighbourhood of each point of a manifold is homeomorphic to $\mathbb{R}^n$, so that one can think of a local notion of orientability as orientability restricted to a chart, which maps from such a neighbourhood. This brings us to the following observation:

Remark 3.1. All manifolds are trivially locally orientable, as the neighbourhoods of each of their points are homeomorphic to $\mathbb{R}^n$, which is orientable (i.e. all transition functions on $\mathbb{R}^n$ are orientation-preserving).

Thanks to this property of manifolds, we can see why it is the case that even on non-orientable manifolds there are *locally* two kinds of handedness. For example, someone living on a Klein bottle (a non-orientable 3-dimensional manifold) still has two hands that cannot be superimposed locally—that they could map one hand to the other by traversing the Klein bottle globally doesn't detract from this. The global non-orientability of this manifold will be revealed only if such a person takes two incongruent gloves, sends one on a journey around the manifold that follows one of the orientation-reversing loops, and it comes back with reversed orientation.

Examining the notion of orientability in the case of orbifolds requires a little more care and caution. Global orientability—to which we again refer as just 'orientability'—is defined in a very similar way to the case of manifolds:

Definition 2 (orientability of an orbifold)**.** *An orbifold $\mathcal{O}$ is* orientable *iff all embeddings are orientation-preserving.*

Note that this is the same as in Definition 1 of the orientability of a manifold, but with transition functions replaced with embeddings. Part of the reason to move from transition functions to embeddings is to accommodate group actions as well; generalising in this way then invites the following two remarks:[19]

Remark 3.2. Group actions are embeddings. If at least one of the group actions is orientation-reversing, then the orbifold is not orientable.

[17] According to another common definition, a n-dimensional differentiable manifold is orientable iff the exists a non-vanishing n-form on this manifold. For more definitions of orientability and proofs of their equivalence, see Bielińska (2021). For a rigorous definition of an orientation-preserving transformation, see Appendix A.2.

[18] Note that this applies to those topological manifolds which admit a differential structure, because such a structure is necessary in order to have transition functions. One can extend this to topological manifolds by talking about *possible* transition functions. Rigorously, one could write: a topological manifold $\mathcal{M}$ is orientable iff all *possible* transition functions that belong to the domain of a differential structure that this manifold admits are orientation-preserving.

[19] See Appendix A for formal definitions of transition functions and embeddings.

Remark 3.3. An orbifold with singularities can still be orientable—e.g., the cone orbifold, for which it is still the case that all embeddings are orientation-preserving.

Just as for manifolds, it is also possible to define local orientability of orbifolds. Let (U_i, G_i, φ_i) be a chart of an orbifold, where $U_i \subset \mathcal{O}$ is an open set, $\varphi_i : U_i \to \varphi_i[U_i] \subseteq \mathbb{R}^n$ is a homeomorphism into a connected open set, and G_i is a finite group acting smoothly and effectively (see Appendix A) on U_i. Then, one can talk about local orientability at a certain point of an orbifold in the following way:

Definition 3 (local orientability of an orbifold)**.** *An orbifold $\mathcal{O}$ is* locally orientable *at a point $p \in U_i$ iff there exists a chart (U_i, G_i, φ_i), such that G_i acts by orientation-preserving transformations.*

In this definition it is not *any* embedding function which should be orientable, but rather a specific kind of embedding—namely, a group action. Note that this definition is satisfied trivially for manifolds, but not for orbifolds; this is because for manifolds the group is trivial, and hence is orientation-preserving. This observation is in line with Remark 3.1. Further, we observe that:

Remark 3.4. In the physics literature on orbifolds (see e.g. Caramello (2022) or Richardson and Stanhope (2020)), the most common definition of local orientability is different to the one above. According to it, an orbifold $\mathcal{O}$ is *locally orientable* iff there exits an atlas $\mathcal{A} = \{(U_i, G_i, \varphi_i)\}$, such that each G_i acts by orientation-preserving transformation.

Note that this is essentially what we define as local orientability above, but quantified over all charts in a given atlas. In fact, the choice of the word 'local' is questionable, given that it requires that *all* groups in a certain atlas act by orientation-preserving transformations. Therefore we mention this definition here for completeness, but will not engage with it further in what follows.

Remark 3.5. An orbifold that is (globally) non-orientable can be either locally orientable or locally non-orientable. However, an orbifold that is locally non-orientable in the neighbourhood of at least one point is always (globally) non-orientable, as in this case there exists at least one embedding (that is, in this case, a group action) that is not orientation-preserving.

Finally, let us observe that:

Remark 3.6. Local non-orientability can occur only at orbifold singularities, i.e. where the group action G_i is not free. However, not all orbifold singularities lead to non-orientability (for instance, a cone from Example 3.2).

This is because neighbourhoods of all other points of an orbifold—i.e. those in which it group action is free—are homeomorphic to $\mathbb{R}^n$, so an orbifold at these points is locally orientable. Only at points whose neighbourhoods are homeomorphic to $\mathbb{R}^n/G_i$ (with a nontrivial group G_i) can local non-orientability occur—when G_i is orientation-reversing, as per Definition 3.

All these considerations deliver the following observation regarding the differences between manifolds and orbifolds:

Fact 1. *Both manifolds and orbifolds can be (globally) non-orientable, but all manifolds are trivially locally orientable, whereas orbifolds in general are not locally orientable.*

3.3 Localisability

To close this section, we move to discussing the third notable topological feature of orbifolds, which has to do with the localisability of certain properties defined upon them. In our discussion, these include the localisability of the singularities of an orbifold and the localisability of regions (i.e. sets of points) of non-orientability in certain cases.

First, let us observe that, at the level of group theory, this property of orbifolds is related to the concept of a *fixed point*:

Definition 4 (fixed points)**.** *Let G be a group acting smoothly on a manifold M. A point $p \in M$ is called a* fixed point *of the group action if there exists a nontrivial element $g \in G$, such that $g \cdot p = p$.*

Fixed points can form hypersurfaces. From a more philosophical perspective, these fixed points on an orbifold are localisable in that they arise at particular points (or hypersurfaces) as a result of group action on an orbifold at these points. Note that this is a very different type of localisability than one discussed in the literature on singularities on spacetime manifolds in GR—first and foremost because it does not require any affine or metrical structure. Thus, localisability here is already defined at the topological level—and only then can it further lead to localisability at the level of affine structure or metric. For instance, topological non-orientability at a certain point of a topological orbifold would result in, once one adds metric to this orbifold, metrical non-orientability (cf. Read and Bielińska (2022)).

One can in general localise singularities on orbifolds. To get a feel for this, note that after quotienting a manifold with a group, the resulting orbifold is usually still locally homeomorphic to $\mathbb{R}^n$ in certain regions (i.e., neighbourhoods of certain points are homeomorphic to $\mathbb{R}^n$). Consider the cone orbifold of Example 3.2. Even though quotienting in this case results in an orbifold with a singularity, this singularity is confined to one point—specifically, to the point around which the group of rotations acts, which is the vertex of the cone. Thus, even though the entire orbifold is described as $\mathcal{M}/G$, and, according to the convention that we adopted earlier from the literature we say that it is 'locally $\mathbb{R}^n/G$' (as opposite to being locally $\mathbb{R}^n$ as in the case of manifolds), there is also a sense in which one can say that it is locally $\mathbb{R}^n/G$ at the vertex (more precisely, the vertex is homeomorphic to $\mathbb{R}^n/G$) and it is locally $\mathbb{R}^n$ for the rest of the cone.

Note that in the case of orbifolds one can also localise regions of orientability and non-orientability—which, in fact, follows from Remark 3.6 and the paragraph above. Crucially, however, as noted by Read and Bielińska (2022), this is not possible in the case of manifolds. Consider a Möbius band, which is a non-orientable manifold. It is impossible to localise the non-orientability of this manifold, in the sense that it is impossible to attribute the non-orientability to some specific region of the manifold. What is important, even if the apparent 'region' of non-orientability is small compared with the apparent 'region' of orientability (consider, for example, a large sphere with a tiny Möbius band attached to it in such a way that they constitute one manifold) is that it is still the *entire* manifold which is non-orientable (i.e., there does not exist a smooth n-form on this manifold). Orientability is, thus, a global property of manifolds, in the sense of Manchak (2020).

For orbifolds, by contrast, one can in fact localise regions of non-orientabilty if it is the case that these regions arise from quotienting. For example, consider an orbifold that consists in a half-plane $\mathbb{R}^2_+$, such that its edge is a result of quotienting by a group G that acts in an orientation-reversing way. Then one can say that this orbifold is orientable at all points save those located on the edge.[20] This being said, note that one still cannot localise regions of non-orientability of orbifolds if they arise in a different way than from quotienting. For example, a Möbius band with a point removed is an orbifold, but one cannot localise the region of its non-orientability. We codify this in the following fact:

Fact 2. *It is possible to localise some properties of orbifolds, such as singularities and regions of non-orientability, if they arise from quotienting. It is not possible to localise regions of non-orientability of manifolds.*

4 Empirical significance of orbifolds

Having introduced the mathematics of orbifolds, we turn now to an assessment of the empirical significance of orbifold structures. We begin with topological considerations (§§4.1–4.3), before adding metrical structure and material structure (§§4.4–4.5). Each time we add further structure, we ask: are there now new ways in which to detect empirically the orbifold structure?

4.1 Handedness on orbifolds

In §3.2, we mentioned the notion of orientability using the example of hands and we saw that chiral objects such as hands can serve in litmus tests for orientability. Accordingly, in this subsection we explore whether examining the property of handedness can help with determining whether our spacetime has an orbifold structure. This comes down to two questions:

(A) Can one tell via considerations of handedness whether one is living on an orbifold or on a manifold?

(B) Can one identify via considerations of handedness the kind of orbifold on which one is living? E.g., is it globally or locally orientable?

Regarding the second question of (B), note that there are three kinds of orbifolds, classified with respect to orientability: (1) globally and locally orientable, (2) globally non-orientable and locally orientable, and (3) globally non-orientable and locally non-orientable. Note that there are no orbifolds that are locally non-orientable but globally orientable (cf. Remark 3.5). Both orbifolds and manifolds can be globally non-orientable, but only orbifolds can be locally non-orientable (cf. Fact 2). Hence, one way to address question (B) is to focus on tests of for local orientability.

4.1.1 Epistemology of global orientability

In globally orientable orbifolds, including of course locally orientable manifolds, there is a well-defined notion of handedness. For discussion of tests determining whether a spacetime

[20] The same is also true of the 'mirror' orbifold, introduced in Example 4.2 below.

is globally orientable, see Bielińska (2021) and Read and Bielińska (2022).[21] For example, in the vein of experiments explored by Hadley (2002), those authors consider hypothetical experiments that involve transporting n-ads of orthogonal vectors along closed loops on a spacetime manifold, for example tetrads of orthogonal vectors in (3+1)–Lorentzian manifolds that consist in a clock (a timelike vector) and a hand (a triad of orthogonal spacelike vectors), appropriately arranged.[22] There is also some possible indirect evidence for global spacetime orientability, such as an existence of fermions in QFT allegedly requiring existence of orthogonal tetrads (though see Pitts (2012) for pushback), parity violation, quantum electrodynamic fluctuations, or the Cosmic Microwave Background. These tests, originally formulated for manifolds, can be generalised easily to orbifolds. If spacetime is globally orientable, then it is also locally orientable. Some of these tests can therefore help with answering question (B). However, given that both manifolds and more generally orbifolds can be both globally and locally orientable, these tests cannot help with answering (A), unless there exist other ways in which to determine difference between these spacetimes, such as encountering the orbifold singularities (see §4.3).

Second, consider orbifolds that are globally non-orientable but locally orientable. A good example of such orbifolds include all kinds of non-orientable manifolds since, as we have argued earlier, all manifolds are locally orientable. A famous example is the Möbius band:

Example 4.1 (Möbius band)**.** Consider $\mathcal{O} = \mathbb{T}^2$ and $G = \mathrm{Sym}_2$, where $\mathbb{T}^2$ is a 2-dimensional torus and Sym_2 is the group of permutations of the set of elements. It results in a Möbius band with a boundary.

Bielińska (2021) and Read and Bielińska (2022) point out some important limitations of experiments concerning global orientability. To give one example: consider an experiment in which one sends a tetrad of orthogonal vectors in (3+1)-dimensional Lorentzian manifolds that consist in a clock (a timelike vector) and a hand (a triad of orthogonal spacelike vectors) to travel around closed loops on an orbifold. It is not guaranteed that one would happen to send such a probe along an orientation-reversing loop. For instance, on a Möbius band, there are numerous closed loops that are orientation-preserving! One might in fact not reach such an orientation-reversing loop for various reasons—for instance, one might simply not happen to find it! Testing global orientability of orbifolds inherits these limitations in carrying out experiments.

However, in the case of orbifolds, there is a new possibility of testing global orientability that is not available for manifolds, which follows from Remark 3.5. Since every manifold that is locally non-orientable is also globally non-orientable, one can test local non-orientability of an orbifold, and—if it is detected—conclude that the orbifold is also globally non-orientable.

[21] Note that topological orientability discussed in this paper is referred to as a 'manifold orientability' by Read and Bielińska (2022), as opposed to space and time orientability defined at the level of the metric, which is beyond the scope of this paper. There are, however, theorems linking both kinds of orientability (e.g., if a (3+1)-dimensional Lorentzian manifold is both time and space orientable, then it is also manifold orientable), which open new possibilities of indirect tests for manifold orientability—for example, by jointly testing space and time orientability.

[22] Although this and some tests which we will mention later presuppose metrical structure (in the sense that they presuppose a time/space distinction), one can infer that a conjunction of time- and space-orientability requires topological orientability—see footnote 21.

At the same time, one should note that such a type of experiment cannot detect any type of global non-orientability, but rather only that which arises from orientation-reversibility of an embedding of quotient groups. Furthermore, as in the case of manifolds, one cannot confirm experimentally that a spacetime is globally orientable because of the limitations set out above. Nonetheless, a possibility of testing global orientability through local orientability remains interesting, as it is unique to orbifolds.

4.1.2 Epistemology of local orientability

Local orientability has particular epistemic significance, because it accounts for why one can still take objects to have handedness—left or right—on globally non-orientable spacetimes. The reason for this is that locally these spaces are homeomorphic either to $\mathbb{R}^n$ (for manifolds) or to $\mathbb{R}^n/G$ (for orbifolds), where G is a group that does not inhibit the existence of two distinct equivalence classes of n-ads of orthogonal vectors.

Further, a handedness-related property that distinguishes orbifolds from manifolds is local orientability, for it can fail only for orbifolds (cf. Fact 1). This is because going into a locally non-orientable region makes sense in the case of an orbifold, but not in the case of a manifold. Consider the following example:

Example 4.2 (mirror reflection)**.** Consider $\mathcal{O} = \mathbb{R}^3$ and $G = \mathbb{Z}_2$ acting on $\mathcal{O}$ by reflection along the plane $x = 0$. It results in a positive hyperplane:

$$\mathcal{O}/G = \mathbb{R}^3/\mathbb{Z}_2 = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x\}.$$

In this example, we obtain an orbifold in which points with $x \neq 0$ have trivial isotropy, namely their neighbourhoods are homeomorphic to $\mathbb{R}^3$, while points on a plane given by $x = 0$ have a non-trivial isotropy—their neighbourhoods are homeomorphic to $\mathbb{R}^3/\mathbb{Z}_2$.

What would life be like on such an orbifold? Consider first someone living on this orbifold far from $x = 0$, such that they cannot see this plane. For such an observer, this orbifold will be indistinguishable from a manifold $\mathbb{R}^3$. Thus, locally one can distinguish between 'left' and 'right' hands. What happens, though, as our observer approaches the hyperplane $x = 0$? One may be tempted to think about this scenario as looking at one's reflection in the mirror. This intuition is partially correct, for it is precisely what the plane $x = 0$ is doing: that is, it 'reflects'. Contrary to a traditional mirror, however, in such an orbifold—being an underlying spacetime structure—one can walk into the 'mirror region'—here, $x = 0$. What would an observer experience when they arrive at $x = 0$? There the orbifold is locally homeomorphic to $\mathbb{R}^2/\mathbb{Z}_2$, where the group acts by an orientation-reversing operation. Thus, it is a region of local non-orientability, which means that there is no standard of 'leftness' and 'rightness'—in other words, the very notion of handedness ceases to make sense.

What happens when an object makes its way out of a non-orientable region? In the absence of some narrative about the dynamics of the material constituents of the hand, the answer to this question is simply underdetermined. (Here, there are connections with the 'dynamical approach' to spacetime theories associated with Brown (2005) and Brown and Pooley (2001): does spacetime have this orbifold structure by virtue of the behaviour of material bodies such as hands, or vice versa? Although in our view discussions of the dynamical approach and spacetime topology are underexplored—though see Menon (2019) and Norton (2008) for exceptions—we won't pursue these questions further here.)

What would constitute an appropriate test for local orbifold orientability? First, we can say with certainty that even if we live on an orbifold containing locally non-orientable regions, we do not currently live in one of those regions. This is because, if we did live in such a region, we would not have incongruent hands! In fact, we probably wouldn't have even developed the concepts of 'left' and 'right'. A more extensive philosophical discussion of handedness in locally non-orientable regions is presented in §5.2.

It is still possible, though, that we live on an orbifold containing locally non-orientable regions—though we do not (exclusively) live in such a region. To find such regions, one would need to use a probe that has handedness, such as a tetrad of orthogonal vectors. Such a hand should be sent on a journey around various regions of spacetime. For instance, in Example 4.2, in order to detect the non-orientable region, it is sufficient to send such a probe to $x = 0$, bring it back to the positive hyperplane, and observe that it has reversed its handedness. (Again, though, whether the hand does in fact change its handedness will depend upon the details of its dynamics.)

Nonetheless, there are still some significant epistemic limitations to testing local non-orientability of orbifolds—some of them already familiar from the discussion of the epistemology of global orientability on manifolds. For instance, it is still possible that the region of local non-orientability is located very far from our spatiotemporal location and we simply won't find it within a reasonable time. Furthermore, when considering orbifolds with metric fields, it is possible that a region of local non-orientability is hidden behind e.g. an event horizon, complicating the possibility of securing empirical access to it.

4.2 Hearing the shape of an orbifold

So far, our considerations about testing orientability of orbifolds were based on analysing consequences of their definition, such as the fact that the quotient group can in principle be orientation-reversing (in the sense discussed in the previous subsection). However, in the literature there have also been interrogations of the empirical significance of various properties of orbifolds, including their local orientability, from a different perspective—such as 'hearing' them.

In 1966, Mark Kac published an article titled 'Can One Hear the Shape of a Drum?' (Kac 1966), in which he demonstrated that the shape of a drumhead determines the frequencies at which it can vibrate. This can be achieved thanks to the Helmholtz equation, which allows one to calculate the possible frequencies of oscillation of a given space, since these are the eigenvalues of the Laplacian in the space. Kac's paper gained tremendous popularity and soon after its publication mathematicians started to investigate in earnest the extent to which it is in general possible to 'hear' the shape of a given shape/surface, i.e. to determine its shape via the spectrum of its Laplacian frequencies.

The idea of linking the shape of a surface with eigenvalues is explored within the field known as 'spectral shape analysis', and relies on the spectrum of the Laplace–Beltrami operator to compare and analyze shapes. The Laplace–Beltrami operator of some function

f is the divergence of the gradient,

$$\Delta f := \mathrm{div}(\nabla f). \tag{1}$$

In spectral components the Laplace–Beltrami operator can be computed by solving the Helmholtz equation,

$$\Delta \varphi_i + \lambda_i \varphi_i = 0, \tag{2}$$

where φ_i are modes (solutions) and λ_i are the corresponding eigenvalues. Since eigenfunctions describe normal modes of vibrating systems, mathematicians say that one can 'hear' the shape. 'Hearing' is, however, not the only interpretation. For example, eigenvalues of a Hamiltonian operator are stationary states of the quantum mechanical system, each with a corresponding admissible energy state of the physical system. For example, for the sphere, the eigenfunctions are the spherical harmonics. Other shapes for which one can obtain eigenfunctions analytically are rectangles, tori, cylinders, and disks.[23]

As pointed out by Chiang (1990), the study of spectral shape analysis can be extended to orbifolds. A natural question which arises in light of this, then, is whether it is possible to determine empirically, for example by 'hearing', whether one lives on an orbifold or a manifold, based on the eigenfunctions given by the Helmholtz equation. In order to explore this question, one should first investigate the extent to which the spectra of orbifolds carry information about their properties. In answer to this, it has been proven by Farsi (2001) that the spectrum determines the dimension and volume of an orbifold.[24] Moreover, more recent results show that one can 'hear' the local orientability of an orbifold, understood as in Definition 3. More precisely, orbifolds with at least one coordinate chart with an orientation-reversing embedding are not isospectral to orbifolds in with all coordinate charts have orientation-preserving embeddings (Richardson and Stanhope 2020).

Does this mean, then, that one can *uniquely* identify an orbifold exclusively on the basis of its spectrum? The answer is 'no'. For example, it has been demonstrated by Shams et al. (2006) that isospectral orbifolds can have topologically distinct singular sets, which makes them different.

Given these results it seems, however, that if one can determine the spectrum of the Helmholtz equation of one's space, then it is possible to deduce that, for example, one lives on a locally non-orientable orbifold rather than on a manifold; second, one can determine the dimension of this orbifold; third, one can determine the volume of this orbifold. Then, finally, since orbifolds that are not locally orientable cannot be globally orientable, one can also determine whether an orbifold is globally orientable. Note, however, that in such a way one can only confirm that an orbifold is globally non-orientable; if it is locally orientable, it can be still either globally orientable or globally non-orientable. One should note, moreover, that it is unclear how one could actually 'hear' the various properties of an orbifold, i.e. how to ascertain the spectrum—obviously, spacetimes do not vibrate in the same way in which, say, a drum, does.

[23]To be clear, though: obtaining such a spectrum is not always possible—for a counterexample see Gordon et al. (1992), and for constraints necessary to obtain a one-to-one relation between space shape and the spectrum of its Laplacian frequencies see Hezari and Zelditch (2010).

[24]This is an extension of the Weyl's law of spectral theory from manifolds to orbifolds.

4.3 Singularities on orbifolds

One of the primary differences between manifolds and orbifolds—as we have discussed in §3.1—is that there is a new kind of topological singularity on orbifolds that does not involve removing points or 'cutting' paths: namely, singularities that arise from quotienting, such as the singularity at the vertex of a cone. What we wish to look into now is the extent to which these orbifold singularities might have empirical significance.

Even thought there exists literature on the epistemology of spacetime singularities in GR that could afford a starting point to our considerations on orbifold singularities, this literature is not concerned with the bare topological structure of a manifold, but rather requires the existence of e.g. affine structure, or metrical or material fields. The story concerning the epistemology of such singularities with additional, non-topological, structure is usually told as follows: Bob, who is far away from a black hole and observes Alice falling into it, sees that Alice slows down and eventually appears to freeze at the horizon, never quite crossing it. However, Alice, in her own frame, does cross the event horizon in finite time and, at first, does not experience anything abnormal—for the black hole is locally smooth. One significant change, however, is that after crossing, Alice cannot turn back; instead, she will continue falling into the singularity. Eventually, tidal forces become enormous and tear Alice apart as she approaches the singularity. Furthermore, if one incorporates quantum theory into this picture, some new effects can occur—most famously, Hawking radiation.[25]

When it comes to the study of orbifold singularities—contrary to the story about epistemology of black holes outlined above—we begin our considerations with the topological level. In order to understand what happens to Alice once she encounters an orbifold singularity that results from quotienting, let's consider an example of the 'new kind' of singularities of orbifolds, such as the cone from Example 3.2.

Plausibly, probes that test topological singularities should be able to distinguish between continuous or non-continuous lines or other hypersurfaces. For instance, Maudlin (2012, p. 6) illustrates topological structure with a pencil: if it is possible to draw a line on a hypersurface without removing the tip of the pencil from this hypersurface, then it has a topological structure. Perhaps an even better way of testing spacetime topology—i.e. such that it does not presuppose existence of spacetime on which one can 'draw'—would be to use a membrane as a probe. An experiment could involve then using a membrane with some figure that consists in continuous lines or hypersurfaces and observing whether the topology of this figure is destroyed as the membrane moves through the orbifold. If any of these hypersurfaces 'tears'—i.e. if there occurs a discontinuity—then it means that we have encountered a topological singularity.

It should be acknowledged, however, that it is unclear how to conduct such an experiment in physical spacetime—which, after all, has all other levels of structure, and is filled with matter, which behaviour is governed by some dynamical equations. For example, how to

[25]Interesting and more scientific 'direct' ways of observing black holes (which do not require throwing Alice into a black hole to be torn apart by tidal forces!) have recently been suggested by Doboszewski and Lehmkuhl (2023), in which observers examine the exterior of a black hole candidate by shooting lasers toward the black hole to test whether their paths overlap with null geodesics of the Kerr geometry. Such a test, however, requires a distinction of null geodesics, which requires more than the bare topological structure of a manifold.

determine whether, if such a 'tear' occurs, it is a result of a topological singularity rather than some dynamical effects?

Crucially, however, it should be noted that even though a membrane would be a good probe for testing topological structure of spacetime orbifold, it could only detect the missing-point style of singularities (understood on a topological level, as outlined in §3.1)—but, as we have argued, orbifold singularities that arise from quotienting are not singularities in this sense! In particular, the vertex of a cone is not 'missing' from this orbifold. Rather, it is 'singular' in that its neighbourhood has a different topology than the rest of the entire orbifold, as it is homeomorphic to $\mathbb{R}^n/G$, where in this case $G = \mathbb{Z}_3$ (but G could be any other quotient group).

We know, then, that one cannot discover orbifold singularities by using a membrane with a continuous shape painted on it. Is it possible to test such singularities at all? Here, one should distinguish between two types of orbifold singularities: those which are associated with local non-orientability and those which are not. In the former case, given that local non-orientability can occur only at singular points (cf. Remark 3.6), one can use tests developed before that involve hand-like probes (cf. §4.1.2) to discover such singularities. Thus, let us focus on empirical tests for singularities that preserve local orientability, such as a vertex of a cone.

These considerations bring us back to the question of what would Alice see when entering a vertex of a cone. Since no point is missing on this orbifold, Alice can enter the vertex following a continuous path. What she may observe, though, is that the space around her no longer resembles $\mathbb{R}^n$, but rather has a different topology of $\mathbb{R}^n/G$. How would such a topological structure manifest itself empirically? In other words, how can Alice tell that the space around her is no longer Euclidean? While it could be detected using metrical structure—for instance, she could draw a small circle of a radius r around the vertex and discover that its circumference is smaller than $2\pi r$—how could it be detected at a purely topological level?

Prima facie, it seems that, at least in such cases as a singularity of a vertex of a cone, it cannot. This is because the topological properties which could be used for designing empirical tests for the presence of orbifold singularities are preserved at that point—these tests include the possibility of local triangulability (which could be tested by trying to cover a space with simplices, such as triangles or tetrahedra).

But perhaps this verdict is too swift. An orbifold property which we can probe is *local simple connectedness*. A topological space is simply connected just in case every loop can be continuously shrunk down to a point, and if any two points can be connected by a continuous path (this latter condition is that of path-connectedness). For instance, a torus is not simply connected because not every loop can shrunk to a point. Further, a topological space is locally simply connected just in case for every point of that space, every neighbourhood of this point is simply connected. In other words, a loop in every neighbourhood of each point of such a space can be shrunk to a point within this neighbourhood. For instance, $\mathbb{R}/\{0\}$ is not locally simply connected.

Now, $\mathbb{R}^2$ is obviously both simply connected and locally simply connected. This property can be tested for an orbifold, which in this case is also a manifold, by drawing circles on stretched membranes and seeing whether they will collapse to a point (obviously, that would require an infinitely stretchable membranes). What happens, though, if we quotient $\mathbb{R}^2$ by a group $G = \mathbb{Z}/3$, which acts by rotating every point by $2\pi n/3$, where $n \in \mathbb{Z}/3$? One can define a covering space on this orbifold as a regular projection map $p : \mathbb{R}^2 \to \mathbb{R}^2/\mathbb{Z}_3$ of degree 3 everywhere except of the vertex—it is the so-called 'branching covering'. Loops around the vertex lift to paths in $\mathbb{R}^2$ that only close after traversing the loop three times. Therefore, such loops cannot be contracted within any neighborhood of the vertex. As a result, the vertex is not locally simply connected: every neighborhood of the singular point contains non-contractible loops.

Thus, in this specific case of a cone, it seems that one could detect a singularity by taking a stretched membrane on and drawing a circle on it. If, after lifting it to the covering space, it closes after one round of traversing the loop, then we are not at the vertex. However, if it remains open—and requires three such rounds, then we are at the singular point. It can be further verified by releasing a membrane and observing that the shape will not contract to a point. Notably, such tests that involve local simple connectedness would work also for other orbifolds that are such familiar manifolds as tori or Möbius bands with vertex-style singularities—for, even though they are not simply connected, they are locally simply connected, so a singularity can be detected.

However, there are some limitations to such experiments. First, it is unclear how to perform 'lifting' a membrane to the covering space, and whether it is possible to observe that a loop does not close after a single round just at the level of the orbifold rather than its covering space.

Further, such a test would work only for a vertex-style singularities—that is, singularities that arise from rotations. It won't detect other types of singularities, such as the those that results from reflections—think of the mirror example and a singularity at $x = 0$. This is because, although the covering of this orbifold is also branched, it doesn't affect simple connectivity, since there are no rotations around a singularity as in an example of a cone. One could note, though, that in the mirror case the singularity can be detected by testing the failure of local orientability.

Thus, we have designed experiments that allow to detect all types of singularities that arise from the most familiar quotient groups, that is rotations, reflections, and combinations of thereof (e.g. a dihedral group) just on a topological level. One should note that this is a complete list of groups which lead to 2-dimensional orbifolds, in which case we have (1) a cyclic group (leads to cone points), (2) reflection group (leads to mirror boundaries), (3) dihedral group, and a (4) trivial group (doesn't lead to singularities and results in another manifold). In the future, it would be interesting to evaluate other groups for higher-dimensional orbifolds, such as tetrahedral, iscosahedral, or wallpaper groups.

4.4 Field theory on an orbifold

In what follows, we will assume that the orbifold under consideration is imbued with metrical structure. In this subsection, we consider whether—and if so the extent to which—an orbifold topology can be revealed via various fields defined on that orbifold. This question is in fact not novel in the context of spacetime physics; here we present two such examples, which overall motivate an affirmative answer to the question of whether an orbifold topology can have any detectable empirical signatures.

This first example on this front pertains to 'orbifold black holes'—on which see Nitta and Uzawa (2021) and references therein. The idea behind an orbifold black hole is straightforward: one considers various Lorentzian metrics on orbifolds (e.g., a 'Gibbons–Hawking metric' on $\mathbb{C}^2/\mathbb{Z}_2$), the orbifold singularities often coinciding with e.g. black hole singularities, and studies the physics of said spacetime via consideration of *inter alia* freely-falling observers, charges, etc. What work such as Nitta and Uzawa (2021) reveals is that there is in fact a diverse range of physics associated with these solutions—for example, the motion of freely-falling observers around these extremal black holes is studied and compared with the Reissner–Nordström case.[26]

The second example which we consider here has to do with QED on an orbifold. Bessa and Rebouças (2020), Bessa et al. (2020), and Lemos et al. (2021, 2023) write Minkowski spacetime as $\mathcal{M}_4 = \mathbb{R} \times M_3$, and then note that while M_3 is usually taken to be $\mathbb{E}^3$, in fact it can be any one of seventeen possible quotient manifolds (associated with the seventeen wallpaper groups) $M_3 = \mathbb{E}^3/\Gamma$, where Γ is a discrete group acting freely on $\mathbb{E}^3$. As these authors write, "In a manifold with periodic boundary conditions only certain modes of fields can exist. Thus, a nontrivial topology may leave its mark on the expectation values of local physical quantities" (Bessa et al. 2020, p. 2). Indeed, these authors show that these topological differences can manifest themselves in terms of QED correlation functions—the work was already discussed in the philosophical literature by Read and Bielińska (2022).

What we wish to emphasise here is that this quotienting operation yields orbifolds—for example, one can consider precisely the mirror orbifold $\mathbb{E}^3/\mathbb{Z}_2$ introduced above (Bessa et al. 2020, p. 7). Therefore, if spacetime does indeed have the structure of an orbifold, what this work demonstrates is that this should be evident in the structure of the electromagnetic interactions occurring upon it! As Lemos et al. (2023) write, "Topology is a global property of manifolds that requires consideration of the entire manifold. However, if local physics is brought into the scene, it can play a key role in the local access to the topological properties of spacetime."

4.5 Quantum gravity and string theory

Dixon et al. (1985) show that compactification of the heterotic string on an orbifold background spacetime can yield "more or less realistic" phenomenology.[27] The exact way in which this compactification occurs can yield distinct phenomenology—so, if this phenomenology were observed, it would constitute an empirical manifestation of orbifold topology. (Going the other way—i.e., inferring specific string-theoretic and orbifold-based origins for this phenomenology is much more challenging, and there might be a wide variety of distinct potential UV sources of this phenomenology.)

It's also by now well-known that different string compactifications can yield different inflationary/dark energy cosmological models,[28] and can also have distinct signatures in

[26]It remains open whether any of this physics would *in practice* allow us to determine whether our universe is structured as an orbifold—for example, even if one can "conclude that the observer along the free-fall geodesic can across [*sic*] the horizon due to C^2 extension traversing there", this is not obviously of any practical help to us, as Earthbound observers.

[27]Albeit not exactly—of course, the problem of string compactification to yield Standard Model phenomenology in the IR remains unsolved.

[28]For a philosophical problem of underdetermination of theory by evidence arising from this, see Ferreira

(say) the CMB—see Cicoli et al. (2024) for a recent review. Therefore—albeit again in a much less direct way!—if some theory of quantum gravity such as string theory makes use of orbifolds, then this may well (e.g. after compactification) have empirically detectable consequences downstream.

How should one think about epistemology of orbifolds in the context of string theory? For example, as suggested before, in general relativity orbifolds may be taken to represent spacetime, and thus in earlier discussions we considered the epistemology of various properties of orbifolds—such as local properties like local orientability—in terms of travelling to different regions of the orbifold and testing those properties there. However, in the context of string theory we do not have quite the same notion of spacetime (see Huggett and Wüthrich (2025)). Moreover, there arise questions regarding how one should think about orbifold singularities—e.g., whether they are breakdowns of the theory or whether they can be resolved and, if so, what additional assumptions need to be introduced (see Witten (2002) on conic singularities; for more general considerations see Aspinwall (1996)). This raises the question of whether, and to what extent, earlier considerations concerning orbifold singularities remain adequate when we think of orbifolds not as spacetimes, but rather in the context of string theory. In this article, which introduces the notion of orbifolds into the philosophical literature, we leave these questions open, while signalling the need for further analysis of orbifolds in the context of string theory.

5 Philosophical reflections

In this closing section, we pick up two philosophical themes relating to our above work on orbifolds: (a) issues regarding the underdetermination of global spacetime structure based upon local observations (§5.1), and (b) the implications of orbifolds (in particular, the mirror orbifold $\mathbb{R}^3/\mathbb{Z}_2$) for Kantian arguments regarding handedness (§5.2).

5.1 Philosophical arguments against exotic topological structure

To the extent that there *are* empirical signatures of non-standard topological structures, e.g. the orbifold structures which have been our focus in this article, and to the extent that those empirical signatures pick out such exotic topologies *uniquely*, there is clearly an empirical basis on which to maintain that the spatiotemporal structure of our world indeed has these topological features.

However, even supposing—contrary to what we've seen in this article—that there are *no* empirical consequences of such exotic topologies, can we affirm that they are not *bona fide* features of reality? The answer to this question is far from obvious, for—as stressed by Butterfield (2014), Manchak (2011), and Norton (2011)—there is apparently no *empirical* basis on which to prefer inductive extrapolations based upon such-and-such topological features than those based upon some other such features. On exactly this topic, Manchak (2011, p. 418) writes that

et al. (2025) and Wolf and Read (2026).

[w]e have proceeded under the rather basic assumptions that space-time is temporally orientable, the manifold is Hausdorff, and so on. But, do we know that all physically reasonable spacetimes possess these properties? It seems that we do not—a construction similar to the one used above shows this.

Now, authors such as Maudlin (2007) have given *a priori*, transcendental arguments to the effect that that topological features of spacetime such as temporal orientability are necessary for physical reasoning about the world. However, given such considerations regarding induction, it is far from clear that this is in fact the case; doubly so given the apparent coherence of physics in temporally non-orientable spacetimes, on which see again Lemos et al. (2023).

5.2 Reflections on the mirror

Since Kant, there has been a great deal of philosophical discussion regarding the following question: could a hand in an otherwise empty universe be determinately a left hand or a right hand, given that *ex hypothesi* there are no other bodies in the universe with respect to which to verify any given answer to this question? Of course, this is the reflection symmetry version of the classic Leibnizian 'static shift' and 'kinematic shift' arguments, according to which since there is (apparently) no way of verifying, respectively, the absolute position or velocity of a body satisfying Newtonian mechanics in a Newtonian spacetime structure, this must in fact be a 'distinction without a difference', and bodies must have absolute positions/velocities, but not determinate such positions/velocities.

We should pause here before going further. In response to our last point above, one could ask: but once one moves to the framework of Galilean rather than Newtonian spacetime in response to the kinematic shift argument (recall: unlike Newtonian spacetime, Galilean spacetime does not come equipped with a standard of absolute rest), doesn't one render the notion of absolute velocity *meaningless*? To say this would, in fact, be too fast—for the move from Newtonian to Galilean spacetime in light of the kinematic shift argument can in fact be understood as an example of what Dewar (2019) calls *sophistication*: a situation in which, when one has symmetry-related models of some theory (in this case: models of Newtonian gravity related by a kinematic shift), one 'forgets' about structure in those models such that they are isomorphic, without (to speak loosely) changing the *number* of models in the theory. (Sophistication is to be constructed with *reduction*, in which case when faced with a class of symmetry-related models of some theory, one constructs some new theory with some *unique* model associated with the original class of symmetry-related models—for further details here, see March and Read (2025) and Martens and Read (2020).) As a result of this, as argued by Cheng and Read (2021), bodies in Galilean spacetime can still have absolute velocities, albeit not *determinate* absolute velocities.[29]

Now let's return to the case of hands in an otherwise empty universe. There, a by-now mainstream line in the literature is that such hands have handedness, without being *determinately* left or right. This makes sense, since two possible parity-related Newtonian models are isomorphic, so in fact the situation is already exactly analogous to the Leibnizian static shift, in response to which a 'sophisticated' (i.e., anti-haecceitist) substantivalist can

[29]We grant that this might be controversial. But here we simply follow the lead of Cheng and Read (2021), given the arguments presented therein.

maintain that bodies have absolute positions, albeit not determinate absolute positions. This line about the hands is, in other words, completely consistent with the lessons of sophistication (although note that, unlike the case of the kinematic shift, the symmetry-related models were *already* isomorphic, so no mathematical reformulation of the models was necessary—cf. Martens and Read (2020), Møller-Nielsen (2017), and Read and Møller-Nielsen (2020)).

But now suppose that, instead of taking this line, one instead *quotients* the manifold of the original models, to arrive at the above-discussed 'mirror' orbifold, $\mathbb{R}^4/\mathbb{Z}_2$ (Example 4.2). In that case, as we've seen above, for the region of the 'mirror' singularity it can be argued that very notion of handedness ceases to make sense, insofar as one has quotiented by $\mathbb{Z}_2$, which undergirds one's ability to articulate the difference between left and right hands. As a result, making *this* move is an example of *reduction* in the presence of symmetry-related models, rather than sophistication.

That one can move to the mirror orbifold as a way of implementing a reduction-based response to the philosophical issues of parity symmetry raised by Kant's thought experiment of the hands is interesting. That said, one might wish to push back here against the plausibility of this move, for two reasons. First, the sophisticated response seems perfectly adequate in this case—why be motivated to demand more? And second, the mirror orbifold is of course *locally* orientable (and homeomorphic to $\mathbb{R}^4$) away from the $x = 0$ singularity—so it's not obvious that making this move even gives one everything which one is after in this case, for the original Kantian questions and issues would seem thereby to re-arise in such regions of the mirror orbifold.

Acknowledgments

We are grateful to Frank Cudek and Tushar Menon for helpful feedback on a previous draft of this article. We also thank Nick Huggett, Eleanor March, and John Norton for valuable discussions.

A Mathematical appendix

In this appendix, we review some definitions to do with (a) manifolds and orbifolds (§A.1), (b) orientability (§A.2), and (c) connectedness (§A.3).

A.1 Manifolds and orbifolds

There are many perspectives from which one can introduce the concept of an orbifold. Here, we do this by contrasting with manifolds, since the latter are already familiar in the philosophy of physics.[30] To emphasise the parallels between manifolds and orbifolds, we introduce corresponding concepts, such as chart or atlas, first for manifolds, and then for orbifolds.

[30]References to the alternative accounts can be found in Caramello (2022, pp. 1–2).

Definition 5 (chart of a manifold). *A chart of a manifold M is a tuple (U, φ), where $U \subset \mathcal{M}$ is an open subset of $\mathcal{M}$ and φ is a homeomorphism from U to an open subset $\tilde{U} := \varphi[U] \subset \mathbb{R}^n$.*

To speak about orbifolds, one needs to introduce the notion of a group action. Let e be the identity element of the group G, that is $eg = ge = g$ for any $g \in G$. Then:

Definition 6 (group action). *An action of a group G on a set X is a map $\mu : G \times X \to X$, where for all $x \in X$ and $g \in G$ we have that $\mu(e, x) = x$ and $\mu(g, \mu(h, x)) = \mu(gh, x)$.*

A group action is *effective* when its kernel is trivial. Then a chart of an orbifold is defined as follows:

Definition 7 (chart of an orbifold). *A chart of an orbifold $\mathcal{O}$ for an open set $U \subset \mathcal{O}$ is a triple (U, G, φ), where $U \subset \mathcal{O}$ is an open subset, G is a finite group of maps acting smoothly and effectively on U, and $\varphi : \tilde{U} \to \mathcal{O}$ is a G-invariant map that induces homeomorphism from U to $\tilde{U}/G := \varphi[U] \subset \mathbb{R}^n$.*

Next, two charts of a manifold can be compared using a transition function.

Definition 8 (transition function). *Consider two charts of a manifold: (U_i, φ_i) and (U_j, φ_j) which have a non-empty intersection, i.e. $U_i \cap U_j \neq \emptyset$. The transition function $\varphi_{ij} : \varphi_i[U_i \cap U_j] \subset \varphi_i[U_i] \subset \mathbb{R}^n \to \varphi_j[U_i \cap U_j] \subset \varphi_j[U_j] \subset \mathbb{R}^n$ is defined as*

$$\varphi_{ij} = \varphi_j \circ \varphi_i^{-1}.$$

In a similar way, one can define equivalents of transition functions between charts of an orbifold, known as embeddings:[31]

Definition 9 (embedding). *Consider two charts of an orbifold: (U_i, G_i, φ_i) and (U_j, G_j, φ_j) which have a non-empty intersection, i.e. $U_i \cap U_j \neq \emptyset$. An embedding $\varphi_{ij} : \varphi_i[U_i \cap U_j] \subset \varphi_i[U_i] \subset \mathbb{R}^n \to \varphi_j[U_i \cap U_j] \subset \varphi_j[U_j] \subset \mathbb{R}^n$ is defined as:*

$$\lambda_{ij} = \varphi_j \circ \varphi_i^{-1}.$$

In particular, group actions g_i are also examples of embeddings on an orbifold, i.e. when we consider charts $(U_i, G_i, \varphi_i \circ g_i)$ and (U_i, G_i, φ_i).

Further, one can define an atlas on a manifold and an atlas of an orbifold in the following way:

Definition 10 (atlas of a manifold). *An atlas of a manifold $\mathcal{M}$ is an indexed family of charts $\{(U_i, \varphi_i) : i \in I\}$ which covers $\mathcal{M}$, i.e. $\bigcup_{i \in I} U_i = \mathcal{M}$.*

Definition 11 (atlas of an orbifold). *An an atlas of an orbifold O is an indexed family of charts $\{(U_i, G_i, \varphi_i) : i \in I\}$ which covers $\mathcal{O}$, i.e. $\bigcup_{i \in I} U_i = \mathcal{O}$.*

Finally, we can define a topological manifold:

[31] The terminology of 'embeddings' is due to Satake (1956, 1957).

Definition 12 (topological manifold). *A topological manifold $\mathcal{M}$ of dimension n is a set of points together with a collection of neighbourhoods $\{U_i\}$ satisfying the following axioms:*

1. $\bigcup_i \{U_i\} = \mathcal{M}$*, i.e. the subsets* $\{U_i\}$ *cover* $\mathcal{M}$*;*
2. *For each i, there exists a homeomorphism $\varphi_i : U_i \to \varphi_i[U_i]$, where $\varphi_i[U_i]$ is an open subset of $\mathbb{R}^n$;*
3. *If any two sets U_i and U_j overlap, i.e. $U_i \cap U_j \neq \emptyset$, then*
$$\varphi_j \circ \varphi_i^{-1} : \varphi_i[U_i \cap U_j] \longrightarrow \varphi_j[U_i \cap U_j]$$
is a homeomorphism between open subsets of $\mathbb{R}^n$;
4. *(Hausdorff condition) For any two points $p_i, p_j \in \mathcal{M}$, there exist charts (U_i, φ_i) and (U_j, φ_j) such that*
$$p_i \in U_i, \quad p_j \in U_j, \quad \text{and} \quad U_i \cap U_j = \emptyset.$$

Note that one can define a differentiable manifold by strengthening condition (3) in order to require that functions $\varphi_j \circ \varphi_i^{-1}$ be smooth.

An orbifold can be defined analogously as follows:

Definition 13 (an orbifold). *An n–dimensional smooth orbifold $\mathcal{O}$ is a Hausdorff paracompact topological space together with an equivalence class of n-dimensional orbifold atlases for this topological space.*

All parallels between definitions of manifolds and orbifolds discussed above lead to a conclusion that all manifolds are also orbifolds (but not *vice versa*); in this sense, orbifolds are a generalisation of manifolds.

A.2 Orientability

In order to formally introduce a notion of orientability for both manifolds and orbifolds, one needs first to define orientation-preserving and orientation-reversing transformations:

Definition 14 (orientation-preserving/reversing transformation). *A transformation F is orientation-preserving iff $\det(F) < 0$. Otherwise it is orientation-reversing.*

This notion can be used to define orientability of manifolds, in which case these transformations are transition functions between manifold charts, and orbifolds and their charts, where these transformations are embeddings. Three equivalent definitions of orientability of manifolds have been introduced and discussed by Read and Bielińska (2022).

An example of (globally) non-orientable orbifold is presented in Figure 3 and an example of a (globally) orientable orbifolds is presented in Figure 4.

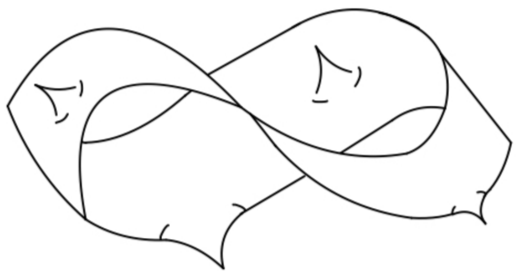

Figure 3: An example of an (globally) non-orientable orbifold.

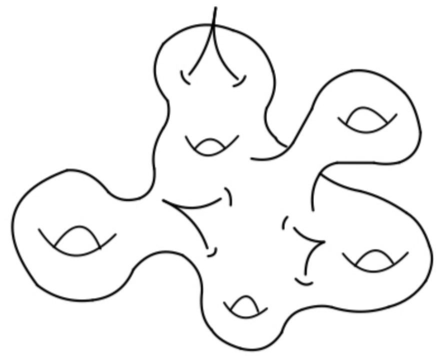

Figure 4: An example of an (globally) orientable orbifold.

A.3 Connectedness

Finally, we recall some notions of connectedness for topological spaces.

Definition 15 (path connectedness)**.** *A topological space X is said to be* path connected *iff for any two points $x_0, x_1 \in X$, there exists a continuous map*

$$p : [0,1] \to X$$

such that

$$p(0) = x_0 \quad \textit{and} \quad p(1) = x_1.$$

In other words, a topological space X—such as a topological orbifold or, more specifically, a manifold—is path connected iff any two points in this space can be joined by a continuous path lying entirely in this topological space.

Definition 16 (simple connectedness)**.** *A topological space X is* simply connected *if and only if it is path connected, and whenever*

$$p : [0,1] \to X \quad \textit{and} \quad q : [0,1] \to X$$

are two paths (that is, continuous maps) with the same start and endpoint

$$p(0) = q(0) \quad \textit{and} \quad p(1) = q(1),$$

then p can be continuously deformed into q while keeping both endpoints fixed.

Intuitively, a topological space X is simply connected if one can always shrink a loop smoothly to a simple point without leaving this space. For instance, a torus is not simply connected, but the plane $\mathbb{R}^2$ is simply connected.

Definition 17 (local simple connectedness)**.** *A topological space X is* locally simply connected *if for every point $x \in X$ and every open neighbourhood U of x, there exists an open neighbourhood V of x such that*

$$x \in V \subseteq U,$$

and V is simply connected.

In other words, a topological space is locally simple connected iff each point of X has arbitrarily small neighbourhoods that are simply connected.

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