The Probabilistic No-Miracle Argument

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1. Introduction

In the debate between scientific realists and empiricists, Mario Alai is undoubtedly one of the ablest defenders of the realist position (see, e.g., Alai 2014a, 2014b, 2021). Not only does his work excel in depth and clarity, but as a true researcher Mario is always willing to engage in constructive dialogue. It is in this spirit of constructive thought and cooperation that I would like to make some critical remarks about scientific realism, in particular regarding attempts to formalize and strengthen the so-called no-miracle argument via probability theory.

It is an undeniable fact that the natural sciences are enormously successful in predicting new phenomena. How can we explain this success? One natural explanation is that science correctly identifies the mechanisms behind observable phenomena; that successful scientific theories represent the world as it really is, not only at the level of what is observable, but also at the submicroscopic level. That this is indeed so is the core idea of scientific realism.

But the reasoning is not uncontroversial. A counterargument is that all our information about the world comes from observation and that our theoretical accounts of the processes responsible for what we observe are hypothetical. It does not seem unreasonable to assume that our own creativity, and the conceptual tools that we happen to have at our disposal, play a key role in the formulation of hypotheses; and that we should expect that countless hypotheses, many of which may never be formulated by humans, can accommodate the same empirical data. If that is the case, it seems unlikely that whatever hypothesis chosen will hit the bullseye of truth. According to this line of thinking, it is only justified to accept the truth of theories as far as observable consequences are concerned; the rest is hypothetical. This is the empiricist position.

In scientific practice, both attitudes are common, often even in the same individuals. As Einstein (1949) famously noted, the working scientist is an opportunistic pragmatist in methodological and epistemological matters. Although the realist and empiricist attitudes pull in different directions, they can both underlie groundbreaking research.

But such a pragmatic attitude does not fit well within epistemology, which is about reasoning concerning what we *should* believe. In the latter context, realist philosophers of science have put forward the *no-miracle argument*. The no-miracle argument begins by noting that the amazing predictive and explanatory successes of established scientific theories are self-evident if these theories are true, or close to the truth in their essence. On the other hand, it would be nothing short of a miracle if theories had this enormous success despite being untrue. Since it is undesirable to invoke miracles, we must conclude that successful theories are true or approximately true.

The no-miracle argument is an example of abduction or inference to the best explanation. As such, it can be criticised: Abduction is not logically convincing because it commits the fallacy of the converse (affirming the consequent). Furthermore, one may wonder whether the assumption that truth is the best explanation for empirical success does not betray a realist bias. Moreover, even if it is conceded that truth-based explanations are an ideal we should strive for, it is not clear why our current theories should meet this ideal---it may be that only future theories with radically different structures will be able to provide the desired true explanations (Devitt 2020, Dieks 2023). Indeed, a consensus seems to be developing that the no-miracle argument as formulated above has a limited scope: while it reassures realists, it cannot refute the empiricist position (Psillos 2011a, 2011b, Forbes 2017). The result is a stalemate, one in which scientific realism and empiricism represent two admittedly different but both consistent positions.

However, it is possible to give the no-miracle argument a probabilistic spin, making some objections less urgent. In particular, the accusation of logical error can be circumvented by conceiving of the argument as an example of Bayesian reasoning. It is this probabilistic argument that we will now discuss, to see if it can break the deadlock between realism and empiricism.

2. The No-Miracle Argument and Probability

In probabilistic terms, the premises of the no-miracle argument are, first, that if a scientific theory is true (or almost true on the relevant points), then the probability of it being successful in predicting new observable phenomena is one (or almost one). Second, if a theory is false, making a true prediction is like miraculously finding a needle in a haystack purely by luck. Therefore, the conditional probability of the theory being successful given that it is false is very small, ε say. So, if $P(S|T)$ and $P(S|\neg T)$ stand for the probabilities of success in the cases of a true and false theory, respectively, we have $P(S|T) = 1$ and $P(S|\neg T) = \varepsilon$. Repeating the steps of the original no-miracle reasoning now yields the reasoning: $\{P(S|T) = 1 \;\&\; P(S|\neg T) = \varepsilon \;\&\; S\} \to P(T|S) \approx 1$.

In words: If the probability of a successful prediction S by a theory, given that it is true, is one; and if the probability of success is very small given that the theory is false ($\neg T$); and S turns out to obtain; then the probability that the theory is true must be close to one. This implies that after successful verification of novel predictions of a theory, we can be almost certain of the theory's truth.

However, this form of the probabilistic no-miracle argument commits a fallacy, related to the logical fallacy in the original argument (Howson 2000, Magnus and Callender 2003). In the statistical literature the mistake in question is known as the *base-rate fallacy*. What needs to be considered, but was forgotten in the above argument, is that there may be reasons to believe a priori that it is highly unlikely that the theory in question is true; that the *prior probability* of the theory being true, *P(T)*, might be very small or even zero. Updating on the evidence of successful predictions in this case need not result in a posterior probability $P(T|S)$ sufficiently high to warrant belief in the theory's truth. This is clear from Bayes' formula:

$$P(T|S) = \frac{P(S|T)P(T)}{P(S|T)P(T) + P(S|\neg T)P(\neg T)}$$

If the prior probability vanishes, $P(T) = 0$, no updating on whatever evidence will lead to a non-zero posterior $P(T|S)$.

Now, a determined empiricist might argue that the a priori probability that we happen to have formulated the true theory among the infinitely many logically possible hypotheses compatible with the data is zero. Such an empiricist will be unimpressed by the probabilistic no-miracle argument.

One might object that it is dogmatic to set $P(T) = 0$, as this leaves no room for change of opinion when new empirical evidence comes in. It would be more in the spirit of empiricism to be open to all possibilities and assign a positive value to $P(T)$, even if this value is infinitesimal.

So, consider a less fanatic empiricist, one who feels that there is a finite though very small probability that a proposed theory happens to be true, $P(T) = \delta$. In this case we find:

$$P(T|S) = \frac{\delta}{\delta + P(S|\neg T)(1-\delta)}$$

It follows that there is a range of values of δ such that the posterior probability $P(T|S)$ remains very small: if $\delta \ll P(S|\neg T)$, then $P(T|S) \ll 1$. So, if there is no lower bound on δ, our liberal empiricist can still resist the probabilistic no-miracle argument.

To turn the probabilistic no-miracle argument into an effective realist weapon it is therefore needed to make a case that the prior probability cannot be made arbitrarily small. Importantly, this argument must be convincing by empiricist lights---realists are confident anyway that $P(T|S)$ can quickly grow to values close to one.

One strategy[1] is to argue that if scientists have done their best to find successful alternatives to a theory, but to no avail, this may be construed as evidence that serious alternatives do not exist or are limited in number; let us say that there will probably not be more than M of them. This could be considered a reason to take the prior probability as not smaller than $1/M$. A variation on this theme is to look at the history of the relevant research field. If one scrutinizes forgotten publications, old correspondence between scientists, etc., it will turn out that at any stage in the history of science many theoretical ideas were proposed, of which a small number has stayed alive. Suppose that on average a number N_s of the number of all proposed theoretical schemes N_p proved acceptable. Then, one idea is to set $P(T) = N_s/N_p$.

However, there are several problems with this approach and similar ones. First, science is not really concerned with systematically investigating how many viable alternatives there are to successful theories; rather, it is concerned with finding one theory that is empirically successful, using that theory and developing it further. Even if there are historical cases where alternatives were systematically sought and no or only a few options were found, the implications are unclear. Failure to find alternatives may be due to the difficulty of the problem, the limited capabilities of human scientists (a point often overlooked), Kuhnian conservatism limiting the conceptual tools used, and so on; and why would the one theory that was found be true instead of merely empirically successful? These are exactly the kind of considerations that motivate the empiricist's reluctance to accept theories as true in the first place, so nothing is achieved that would convince the empiricist.

Mario Alai has proposed[2] a different strategy to argue that the prior probability $P(T)$ cannot be arbitrarily small. He suggests that we have direct evidence for the truth of many theoretical hypotheses because we can verify them using instruments whose reliability has been tested by unaided observation within the domain of the observable and whose reliability in the unobservable domain can be guaranteed by inductive extrapolation. For example, X-rays[3] of macroscopic objects are reliable because we can directly check that they accurately represent their objects. By extrapolation we can then conclude---so the argument goes---that X-rays of, e.g., single atoms (such pictures have been taken recently) accurately represent the atoms in question. Mario Alai estimates the number of theoretical principles whose truth can be established in this semi-empirical way to be at least two per cent of all hypotheses and therefore suggests that $P(T)$ should be equal to a number of this order of magnitude (0.02). Once such a fixed finite value of the prior probability is accepted, Bayesian updating on empirical success according to the scheme discussed above (but see the next section for criticisms of this scheme!) quickly leads to values of $P(T|S)$ justifying belief in the theory's truth.

The argument seems plausible, especially when applied to simple instruments like optical microscopes. From an empiricist point of view, however, it is moot on several grounds. First, inductive extrapolation from the observable to the unobservable is not recognised as valid by empiricists, and the X-ray example illustrates why: X-rays of individual atoms cannot be understood as direct images and their interpretation requires familiarity with quantum mechanics. Second, even if it were true that two per cent of all theoretical hypotheses could be considered true because of the possibility of almost direct empirical verification, it is unclear why it would follow that hypotheses that cannot be checked by observation have a two per cent probability of being true.

3. A Fundamental Problem for the No-Miracle Argument

So, the probabilistic no-miracle argument has problems in the details, and this may be enough to comfort the empiricist. However, there is a more fundamental problem that affects both the probabilistic and non-probabilistic versions of the argument. The intuitive appeal of the no-miracle argument is due to the strong contrast between the obviousness of success in the case where the theory is true, and the miraculous nature of success when the theory is false. The sense that this asymmetry is important seems reasonable, and in this light the empiricist defence that the prior probability $P(T)$

[1] See: Dawid, Hartmann, Sprenger 2015; Sprenger 2016; Dawid, Hartmann 2018.

[2] Personal communication.

[3] This example is mine; Mario Alai discusses microscopes.

could be very small may seem artificial. Indeed, Leah Henderson (2017) comments that this empiricist response would only be convincing if the theories being considered were random choices from an infinity of theories compatible with the data. But the theories proposed in science are not random inventions; they are the result of diligent applications of the scientific method. Leah Henderson is right that the role of the scientific method should not be overlooked. But while she argues that involving the scientific method weakens the empiricist position, we will argue that from an empiricist point of view, considering the role of the scientific method exposes a flaw in the no-miracle argument.

According to realist philosophers of science, the scientific method is *truth-conducive* (Alai 2014a, 2014b). The method is likely to generate truth, which clearly is a great help in countering empiricist arguments. How does the scientific method achieve this remarkable feat? According to realist analysis (Alai 2014a, p. 300), the essence of the scientific method is that scientists formulate profound and general hypotheses whose implications go beyond the phenomena, while respecting empirical constraints. In doing so, scientists make assumptions, e.g. that nature is simple, symmetrical, etc., and formulate hypotheses about unobservable natural systems through analogy, abduction and inductive extrapolation of what has been observed.

What is striking in this characterisation is that there is no reference to truth at all. Of course, a scientist with realist beliefs will pursue truth when applying this method, but it is not clear from the description why the method should approach this goal. There is only talk of hypotheses, assumptions, analogies, etc., but none of these gives a guarantee of approaching the truth, at least not according to logic and not in empiricist eyes. Therefore, the claim that the scientific method is truth-conducive must itself be considered a hypothesis, one that realists add to the description of the method. It is clearly a hypothesis rejected by empiricists.

In fact, there is nothing in the above description of the scientific method that is not congenial to scientists with empiricist beliefs[4]. But, of course, the empiricist will not endorse the claim that this method leads to truth. The empiricist simply adds that the scientific method works, that it leads to the formulation of theories that are empirically adequate. This means that the empiricist expects that theories generated by the scientific method will lead to successful new predictions, regardless of whether these theories are true or not. Therefore, from the empiricist perspective the central idea of the no-miracle argument, that it would be miraculous if an untrue theory had empirical success, does not hold water. The empiricist believes that the world possesses a structure such that the scientific method works and can boast that this metaphysical commitment is weaker than that of the realist.

For the probabilistic version of the no-miracle argument the consequence is that in Bayes' formula the method M that generated the theory should be included:

$$P(T|S,M) = \frac{P(S|T,M)P(T,M)}{P(S|T,M)P(T,M) + P(S|\neg T,M)P(\neg T,M)}$$

According to the empiricist credo, the success of theories is due to M, so that $P(S|T,M) = P(S|\neg T,M)$. This implies $P(T|S,M) = P(T,M)$.

Therefore, the success of a theory does not add anything to the probability that it is true. This success does, of course, increase our confidence that the theory works.

4. Conclusions

It turns out that the no-miracle argument, in its probabilistic formulation no less than in its original version, is effective only when assumptions are made that are not endorsed by empiricists. The probabilistic no-miracle argument thus fails to break the deadlock between empiricists and realists.

Realists assume that the scientific method leads us to pieces of approximate truth that are future-proof. For empiricists, on the other hand, it is an open possibility that things are more complicated: that the theories we have formulated to explain the phenomenal patterns we observe are not truthful. It could, for instance, be that they are *effective* theories, arising from more basic theories as limiting cases that are only valid under special circumstances. Such limiting cases, in which new behaviour

[4] Except perhaps for the part about extrapolation from what is observable, which seems out of step with modern natural science, from both a realist and empiricist point of view.

emerges, often have characteristics completely different from those typical of more basic theories. In the natural sciences and their philosophy, this concept of emergence has received much attention in recent decades. For example, it is now widely accepted that our familiar ontology of stable objects governed by classical "laws" represents a limiting case that differs drastically in ontology and principles from what is supposed to be valid at more fundamental levels. Therefore, classical descriptions cannot be considered true in the sense of representing fundamental insights that survive theory change, even approximately (Dieks 2023). Emergence can be repeated at successive levels of description, leaving it open whether there exists a deepest level of ultimate truth. This picture of emergent behaviour offers one option for making scientific success comprehensible without appealing to theoretical truth.

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