The Aharonov-Bohm Phase is Gauge Dependent, But Its Gauge-Invariant Core is Observable

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Abstract

Contrary to long-standing textbook doctrine, we demonstrate that the Aharonov-Bohm (AB) phase for a closed interferometric loop — the relative phase governing interference — becomes gauge dependent when backreaction of the test charge on the electromagnetic source is included in a fully quantum treatment. The standard Stokes-theorem argument fails because different interfering paths become entangled with distinct field configurations, permitting branch-dependent gauge transformations. We prove that the total phase decomposes into (1) a gauge-invariant core tied to the fixed background flux, which fully accounts for the observed interference shift, and (2) a purely gauge-dependent backreaction term that can be eliminated by an appropriate gauge choice and carries no observable consequences. This decomposition follows from a general framework for observables in gauge theories, which we establish from a foundational axiom that purely gauge-dependent quantities have no observational consequences. From this axiom, we derive theorems showing that any gauge-dependent quantity contains an extractable gauge-invariant core that constitutes its measurable content, while the remaining gauge-dependent component is physically irrelevant. This framework resolves the apparent paradox between formal gauge dependence and robust experimental observations, and offers a conceptually sharper understanding of observables in quantum electrodynamics, non-abelian gauge theories, general relativity, and other gauge-based theories.

1 Introduction

The Aharonov-Bohm (AB) effect stands as one of the most profound and conceptually challenging phenomena in modern physics [3, 1, 11, 10, 2]. For over six decades, it has been celebrated as a paradigmatic demonstration of gauge invariance in quantum theory. The standard interpretation is elegant and compelling: in the AB setup, a charged particle's wave function splits into two paths γ_1 and γ_2 that together form a closed loop $\Gamma = \gamma_1 - \gamma_2$ around a magnetic flux. The relative phase $\Delta\phi = e \oint_\Gamma A_\mu dx^\mu$ can be expressed via Stokes' theorem as $\Delta\phi = e \int_\Sigma F_{\mu\nu} d\sigma^{\mu\nu}$, where $F_{\mu\nu}$ is the gauge-invariant field strength. This argument has convinced generations of physicists that closed-loop phases in gauge theories are necessarily gauge invariant.

However, this reasoning contains a critical hidden assumption: that the electromagnetic field configuration is independent of the charged particle's path. In the standard AB setup, the solenoid current is treated as fixed and unaffected by the electron's motion. While this approximation is excellent for most experimental situations, it represents an incomplete quantum treatment. When one considers the complete quantum electrodynamical system—including the backreaction of the electron's electromagnetic field on the solenoid—a startling result emerges: *the AB phase for the closed loop becomes gauge dependent*.

This surprising finding challenges what has been considered a fundamental principle of quantum gauge theories. The resolution of this apparent paradox requires a careful distinction between two fundamentally different physical regimes:

1. The standard AB effect with a fixed classical background field, where all paths experience the same electromagnetic configuration.
2. The complete quantum treatment with dynamical sources, where different electron paths induce different responses in the solenoid, creating a quantum superposition of distinct field configurations.

In this paper, we provide a comprehensive analysis of both regimes, establishing a new framework for understanding observables in gauge theories. We begin by reviewing the standard AB effect and proving the gauge invariance of the closed-loop phase in the fixed-background approximation (Section 2). Section 3 introduces backreaction effects and demonstrates how the quantum superposition of field configurations leads to gauge dependence of the phase through branch-dependent gauge transformations. Section 4 clarifies the distinction between classical gauge transformations and proper quantum gauge transformations, showing that both approaches confirm the gauge dependence when backreaction is included.

The core of our analysis is presented in Section 5, where we resolve the apparent contradiction between formal gauge dependence and experimental observations. We establish a foundational axiom that purely gauge-dependent quantities have no observational consequences, from which we derive theorems demonstrating that any gauge-dependent quantity decomposes into a gauge-invariant core (the observable content) and a gauge-dependent remainder (unobservable). For the AB effect, this yields the decomposition $\Delta\phi = \Delta\phi_0 + \Delta(\delta\phi)$, where $\Delta\phi_0$ is the gauge-invariant phase associated with fixed background flux and $\Delta(\delta\phi)$ is the gauge-dependent backreaction contribution. This explains why experiments measure only the flux-proportional interference shift while the total phase is formally gauge dependent.

Section 6 extends this framework to other gauge-invariant phenomena, including the Aharonov-Casher effect, non-abelian AB effects, and gravitational AB effects. Section 7 examines why this gauge dependence has not been widely recognized, discussing historical, conceptual, and experimental factors. Finally, Section 8 summarizes our findings and discusses implications for gauge theories and future research directions.

Our analysis reveals a profound insight: gauge invariance in quantum theory is not an absolute principle but rather depends on the physical context. When fields become dynamical and entangled with matter degrees of freedom, the standard gauge invariance arguments require careful reexamination. The resulting framework provides a clearer understanding of what is truly measurable in quantum gauge theories.

2 Standard Aharonov-Bohm Effect

The standard magnetic AB effect considers an electron moving in the presence of an infinite solenoid carrying a constant magnetic flux Φ. The solenoid is assumed to be perfectly shielded such that the magnetic field $\mathbf{B}$ vanishes outside the solenoid, while the vector potential $\mathbf{A}$ does not. The electron's wave function splits into two paths γ_1 and γ_2 that form a closed loop $\Gamma = \gamma_1 - \gamma_2$ enclosing the solenoid. The AB phase difference between the two paths is:

$$\Delta\phi = e \oint_\Gamma A^s_\mu dx^\mu = e \left(\int_{\gamma_1} A^s_\mu dx^\mu - \int_{\gamma_2} A^s_\mu dx^\mu \right), \tag{1}$$

where A^s_μ is the vector potential due to the solenoid current j^μ_s, which is assumed fixed and unaffected by the electron.

Applying Stokes' theorem to convert the line integral to a surface integral yields:

$$\Delta\phi = e \oint_\Gamma A^s_\mu dx^\mu = e \int_\Sigma F^s_{\mu\nu} d\sigma^{\mu\nu}, \tag{2}$$

where Σ is any surface bounded by Γ, and $F^s_{\mu\nu} = \partial_\mu A^s_\nu - \partial_\nu A^s_\mu$ is the electromagnetic field strength. Since $F^s_{\mu\nu}$ is gauge invariant, the phase difference $\Delta\phi$ is gauge invariant. An alternative proof considers the effect of a gauge transformation $A_\mu \to A_\mu + \partial_\mu \Lambda$ on the phase. Suppose the two paths γ_1 and γ_2 connect the same source point S to the same detector point D. The phase along each individual path transforms as:

$$\phi_i \to \phi_i + e[\Lambda(D) - \Lambda(S)], \quad i = 1, 2. \tag{3}$$

Consequently, the relative phase transforms as:

$$\Delta\phi \to \Delta\phi + e\left([\Lambda(D) - \Lambda(S)] - [\Lambda(D) - \Lambda(S)]\right) = \Delta\phi. \tag{4}$$

Thus, the AB phase difference—and hence the interference pattern—remains invariant under gauge transformations.

Both proofs rely on the critical assumption that the solenoid current j^μ_s is unaffected by the electron's presence, and thus the electromagnetic field configuration A^s_μ is fixed and identical for both paths. This assumption corresponds to treating the electromagnetic field as a classical background field rather than a dynamical quantum entity. While this approximation is excellent for most experimental setups, it represents an incomplete quantum treatment. The full quantum electrodynamical system includes backreaction: the electron's electromagnetic field perturbs the solenoid current, which in turn affects the total electromagnetic field. This leads to the surprising results explored in the next section.

3 Aharonov-Bohm Effect with Back-Reaction

When backreaction is included, the electromagnetic field becomes *quantum dynamical*. Different electron paths induce different responses in the solenoid, leading to entanglement between the electron's path and the electromagnetic field state. The complete quantum state is:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\gamma_1\rangle |\text{sol}_1\rangle |A^{(1)}_\mu\rangle + |\gamma_2\rangle |\text{sol}_2\rangle |A^{(2)}_\mu\rangle \right), \tag{5}$$

where $|\gamma_i\rangle$ represents the electron along path γ_i, $|\text{sol}_i\rangle$ represents the solenoid state responding to path γ_i, and $|A_\mu^{(i)}\rangle$ represents the electromagnetic field configuration associated with path γ_i. Crucially, $|A_\mu^{(1)}\rangle$ and $|A_\mu^{(2)}\rangle$ are *different physical field configurations*—they are distinct quantum states of the electromagnetic field. This represents a quantum superposition of field configurations, not merely different paths through the same field.

In the quantum superposition regime, the notion of gauge transformation must be reconsidered. Because $A_\mu^{(1)}$ and $A_\mu^{(2)}$ are distinct field configurations in the superposition, they can be transformed independently. This leads to branch-dependent gauge transformations, namely applying different $\Lambda_i(x)$ to each branch:

$$|\Psi\rangle \to \frac{1}{\sqrt{2}}\left(e^{i\Lambda_1}|\gamma_1\rangle|\text{sol}_1\rangle|A_\mu^{(1)} + \partial_\mu\Lambda_1\rangle + e^{i\Lambda_2}|\gamma_2\rangle|\text{sol}_2\rangle|A_\mu^{(2)} + \partial_\mu\Lambda_2\rangle\right), \tag{6}$$

where Λ_1 and Λ_2 are independent functions. Mathematically, branch-dependent transformations are allowed because $A_\mu^{(1)}$ and $A_\mu^{(2)}$ are different points in the configuration space $\mathcal{A}$. The gauge group acts independently on each configuration. Physically, this corresponds to using different gauge conventions to describe different physical situations—just as different experimental setups may use different coordinate systems.

Now consider the electron traveling from source S to detector D along two paths γ_1 and γ_2. The phase accumulated along each path in the presence of its associated field configuration is:

$$\phi_i = e\int_{\gamma_i} A_\mu^{(i)} dx^\mu. \tag{7}$$

Under a branch-dependent gauge transformation:

$$A_\mu^{(i)} \to A_\mu^{(i)} + \partial_\mu\Lambda_i, \tag{8}$$

the phase transforms as:

$$\phi_i \to \phi_i + e\left[\Lambda_i(D) - \Lambda_i(S)\right], \tag{9}$$

and the AB phase difference transforms as:

$$\Delta\phi \to \Delta\phi + e\left[(\Lambda_1(D) - \Lambda_2(D)) - (\Lambda_1(S) - \Lambda_2(S))\right]. \tag{10}$$

The key observation is that if Λ_1 and Λ_2 are independent, then $\Lambda_1(D) - \Lambda_2(D)$ and $\Lambda_1(S) - \Lambda_2(S)$ can be chosen arbitrarily. In particular, we can choose Λ_1 and Λ_2 such that:

$$(\Lambda_1(D) - \Lambda_2(D)) - (\Lambda_1(S) - \Lambda_2(S)) \neq 0. \tag{11}$$

This demonstrates that the AB phase shift for a closed loop becomes gauge dependent when backreaction creates a quantum superposition of field configurations.

Note that the standard Stokes argument fails because the two interfering paths couple to distinct gauge potentials, $A_\mu^{(1)}$ and $A_\mu^{(2)}$. Their phase difference $\int_{\gamma_1} A_\mu^{(1)} dx^\mu - \int_{\gamma_2} A_\mu^{(2)} dx^\mu$ cannot be expressed as a closed-loop integral of any single 1-form, so no common surface flux exists to which Stokes' theorem could apply. Consequently, the gauge-invariance proof based on converting the line integral into a flux integral breaks down.

4 Quantum Gauge Transformations in QED

The analysis in Section 3 demonstrated that the AB phase becomes gauge dependent when backreaction is included, using a branch-dependent gauge transformation argument. However, to establish this result rigorously within quantum field theory (QFT), we must clarify what constitutes a valid gauge transformation in the quantum context. Classical gauge transformations, while intuitive and useful in semiclassical contexts, are mathematically inconsistent in full quantum electrodynamics. The correct implementation of gauge symmetry in QFT involves changes of gauge-fixing conditions in the path integral, which transform the photon propagator in a specific, translationally invariant manner. This section provides a systematic treatment of gauge transformations in QED, distinguishes between classical and quantum implementations, and demonstrates how proper quantum gauge transformations confirm the gauge dependence of the AB phase in the presence of backreaction.

4.1 Inconsistency of Classical Gauge Transformations in QFT

Classical gauge transformations of the form $A_\mu(x) \to A_\mu(x) + \partial_\mu \Lambda(x)$ with an arbitrary c-number function $\Lambda(x)$ are fundamental in classical electrodynamics, where they represent a redundancy in the mathematical description of the same physical electromagnetic field. However, when one attempts to apply such transformations directly in QFT, significant inconsistencies emerge.

In QED, the photon propagator $D_{\mu\nu}(x-y) = \langle 0|T A_\mu(x) A_\nu(y)|0\rangle$ is derived from a gauge-fixed action, typically using a covariant gauge condition such as $\partial_\mu A^\mu = 0$ with gauge-fixing parameter ξ. A naive application of a classical gauge shift to the quantum propagator yields:

$$D'_{\mu\nu}(x,y) = D_{\mu\nu}(x-y) + \partial^x_\mu \partial^y_\nu [\Lambda(x)\Lambda(y)]. \tag{12}$$

This transformed propagator violates the fundamental principle of translational invariance (unless Λ is constant), as it depends separately on x and y rather than their difference $x-y$. Moreover, it shifts the expectation value $\langle A_\mu \rangle$ from zero to $\partial_\mu \Lambda$. This breaks the consistency of the gauge-fixed quantum action and does not correspond to a symmetry of the quantized theory.

The fundamental issue is that classical gauge transformations treat the gauge function $\Lambda(x)$ as an external c-number field, whereas in QFT, gauge transformations must preserve the structure of correlation functions and respect the gauge-fixing conditions that define the quantum theory. Thus, classical gauge transformations are not valid symmetry operations in QFT; they instead correspond to adding external classical potentials rather than changing the description of the same quantum system.

4.2 Proper Quantum Gauge Transformations

The correct implementation of gauge symmetry in QFT is through changes of gauge-fixing conditions in the path integral formulation. In covariant gauges, the gauge-fixed Lagrangian includes a term $-\frac{1}{2\xi}(\partial_\mu A^\mu)^2$, where ξ is the gauge-fixing parameter. Changing from one gauge condition to another (e.g., from ξ to ξ') transforms the photon propagator in a specific, translationally invariant manner.

In momentum space, the transformation takes the form:

$$\tilde{D}'_{\mu\nu}(k) = \tilde{D}_{\mu\nu}(k) + \frac{-i}{k^2 + i\epsilon}(\xi - \xi')\frac{k_\mu k_\nu}{k^2}. \tag{13}$$

This can be written in the general form:

$$\tilde{D}'_{\mu\nu}(k) = \tilde{D}_{\mu\nu}(k) + k_\mu k_\nu \tilde{F}(k), \tag{14}$$

where $\tilde{F}(k)$ is a function of k^2. In position space, this becomes:

$$D'_{\mu\nu}(x - y) = D_{\mu\nu}(x - y) + \partial^x_\mu \partial^y_\nu F(x - y), \tag{15}$$

with $F(x - y)$ translationally invariant. This transformation preserves the structure of the theory and represents a genuine change of gauge in the quantum context, leaving all physical observables invariant when applied consistently throughout the calculation.

The key distinction from classical gauge transformations is that quantum gauge transformations maintain translational invariance and respect the gauge-fixed structure of the path integral. They represent proper symmetry operations that connect equivalent descriptions of the same quantum system, whereas classical gauge transformations do not.

4.3 Relationship Between Classical and Quantum Transformations

Despite their formal inconsistency in QFT, classical gauge transformations can be understood as a restricted subclass of proper quantum transformations under specific conditions. This explains why classical reasoning often yields correct results in semiclassical approximations.

If we consider a classical gauge function $\Lambda(x)$ that is a linear functional of the current:

$$\Lambda(x) = \int d^4y \partial^y_\nu F(x - y) j^\nu(y), \tag{16}$$

then the classical shift $A_\mu \to A_\mu + \partial_\mu \Lambda$ produces the same propagator transformation as the quantum gauge-fixing change in Eq. (15). In this case, the classical transformation happens to preserve the structure of correlation functions, making it compatible with the quantum theory.

This relationship clarifies why the semiclassical treatment of the AB effect, which uses classical gauge transformations, correctly predicts gauge invariance for closed loops in the fixed-background approximation. However, when backreaction is included and different paths become entangled with distinct field configurations, the simple classical reasoning breaks down. The effective gauge function becomes path-dependent through its functional dependence on the source currents, leading to the gauge dependence identified in Section 3.

Thus, while classical gauge transformations can serve as a useful heuristic in certain limits, they must be replaced by proper quantum gauge transformations when analyzing the full quantum system with dynamical fields and backreaction.

4.4 Confirming Gauge Dependence with Quantum Gauge Transformations

We now provide an explicit demonstration that the AB phase for a closed loop becomes gauge dependent when backreaction is included, using the framework of proper quantum gauge transformations. The crucial insight is that the corresponding classical gauge function $\Lambda(x)$ becomes path-dependent through its functional dependence on the source currents.

The interaction term contributing to the phase along path γ is

$$\phi[\gamma] = e \int d^4x \int d^4y \, j_e^\mu[\gamma](x) \, D_{\mu\nu}(x-y) \, j_s^\nu[\gamma](y). \tag{17}$$

Under a proper quantum gauge transformation of the photon propagator

$$D_{\mu\nu}(x-y) \;\longrightarrow\; D_{\mu\nu}(x-y) + \partial_\mu^x \partial_\nu^y F(x-y), \tag{18}$$

the phase acquires the additional term

$$\Delta\phi[\gamma] = e \int d^4x \int d^4y \, j_e^\mu[\gamma](x) \big(\partial_\mu^x \partial_\nu^y F(x-y)\big) j_s^\nu[\gamma](y). \tag{19}$$

Using the definition of the effective gauge function already introduced in the previous subsection,

$$\Lambda_\gamma(x) = \int d^4y \, \partial_\nu^y F(x-y) \, j_s^\nu[\gamma](y), \tag{20}$$

we can immediately rewrite the double integral as

$$\Delta\phi[\gamma] = e \int d^4x \, j_e^\mu[\gamma](x) \, \partial_\mu^x \Lambda_\gamma(x). \tag{21}$$

This is the same as the classical expression. For a point-like electron, we obtain

$$\Delta\phi[\gamma] = e\big[\Lambda_\gamma(D) - \Lambda_\gamma(S)\big], \tag{22}$$

where S and D are the source and detector points/regions. For a closed loop containing two paths γ_1 and γ_2, the AB phase changes by

$$\delta(\Delta\phi_{\rm AB}) = \Delta\phi[\gamma_1] - \Delta\phi[\gamma_2] = e\Big[\big(\Lambda_{\gamma_1}(D) - \Lambda_{\gamma_1}(S)\big) - \big(\Lambda_{\gamma_2}(D) - \Lambda_{\gamma_2}(S)\big)\Big]. \tag{23}$$

Because backreaction makes the induced solenoid current path-dependent, $j_s^\nu[\gamma_1] \neq j_s^\nu[\gamma_2]$, and therefore the effective gauge functions are different:

$$\Lambda_{\gamma_1}(x) \;\neq\; \Lambda_{\gamma_2}(x) \quad \text{for } \gamma_1 \neq \gamma_2. \tag{24}$$

As a result, the boundary terms no longer cancel and

$$\delta(\Delta\phi_{\rm AB}) \neq 0. \tag{25}$$

Moreover, since $F(x-y)$ can be chosen freely (within the class of translationally invariant functions), this shift can be made arbitrarily large.

4.5 Summary

The quantum electrodynamical treatment given in this Section provides a more fundamental and explicit proof of gauge dependence of the AB phase compared to the branch-dependent argument in Section 3. While the branch-dependent approach physically motivates the possibility of independent gauge choices on different paths, the present calculation rigorously derives the non-invariance from the structure of the gauge-fixed propagator transformation and the resulting current-dependent $\Lambda_\gamma(x)$.

Concretely speaking, proper quantum gauge transformations in QED are implemented globally: a single, translationally invariant function $F(x-y)$ modifies the photon propagator throughout the theory.[1] The path dependence necessary to produce a non-vanishing relative shift $\delta(\Delta\phi_{\mathrm{AB}})$ arises naturally from the path-dependent induced currents $\delta j_s^\nu[\gamma]$ themselves, which lead to different effective gauge functions $\Lambda_\gamma(x)$ even when the same $F(x-y)$ is used for the entire system. This provides a rigorous and consistent demonstration within the QFT framework that the relative AB phase is genuinely gauge dependent when backreaction is included.

5 Resolving the Paradox: A Framework for Observables in Gauge Theories

The previous analysis of the AB effect with backreaction presents a significant conceptual challenge. On the one hand, the analysis in Sections 3 and 4 rigorously demonstrates that the AB phase accumulated for a closed loop becomes gauge dependent in the full quantum electrodynamical treatment with dynamical sources. On the other hand, decades of experimental observations consistently measure interference shifts proportional to the enclosed magnetic flux Φ_0, which are universally interpreted as gauge-invariant phenomena. This apparent contradiction between theoretical formalism and experimental reality requires careful resolution.

The paradox arises from conflicting interpretations of what constitutes an observable in gauge theories. The standard textbook treatment emphasizes that closed-loop phases must be gauge invariant by Stokes' theorem, but this argument implicitly assumes a fixed background field configuration. When this assumption is relaxed to include backreaction effects, the mathematical structure changes fundamentally: different interfering paths become entangled with distinct field configurations, permitting branch-dependent gauge transformations that render the total phase gauge dependent. Yet experiments continue to measure unambiguous, reproducible results. This tension motivates a deeper examination of the relationship between gauge dependence and observability in quantum gauge theories.

5.1 Foundational Axiom: Unobservability of Pure Gauge Terms

At the heart of all gauge theories lies a fundamental principle: gauge transformations represent redundant descriptions of the same physical reality. Different gauge choices correspond to different coordinate systems or mathematical representations of identical

[1] Note that the use of a single, global $F(x-y)$ is required because proper quantum gauge transformations act on the entire gauge-fixed path integral and photon propagator, preserving translational invariance and the structure of the theory across all field configurations and branches of the superposition.

physical configurations. This redundancy is not merely a mathematical convenience but a defining feature of gauge theories that enables their formulation while maintaining physical consistency. From this foundational principle follows a crucial axiom that governs the interpretation of all gauge-dependent quantities:

> **Axiom (Observational Insignificance of Pure Gauge)**: Any quantity that transforms non-trivially under gauge transformations and possesses no gauge-invariant content cannot have observational consequences. If such a quantity could affect measurements, then physical predictions would depend on arbitrary gauge choices, violating the redundancy principle that defines gauge theories.

This axiom represents the minimal condition necessary for gauge theories to describe physical reality rather than mathematical artifacts. It is implicitly assumed in all consistent formulations of gauge theories, from classical electrodynamics to quantum field theory and general relativity. The axiom does not claim that gauge-dependent quantities are mathematically ill-defined or physically meaningless; rather, it asserts that their physical significance is necessarily mediated through their relationship to gauge-invariant components. Any observable effect must ultimately be expressible in terms of quantities that remain unchanged under gauge transformations.

The justification for this axiom is both pragmatic and fundamental. Pragmatically, if physical predictions depended on gauge choices, experiments would yield inconsistent results when different researchers used different gauges to analyze the same phenomenon. Fundamentally, gauge symmetry represents a redundancy in our mathematical description of reality, not a symmetry of the physical world itself. Observable quantities must therefore be invariant under such redundancies, just as measurable distances in geometry must be independent of coordinate system choices.

5.2 Derived Theorem from the Foundational Axiom

From this foundational axiom, we derive the central theorem that establishes the relationship between gauge dependence and observability. This theorem provides the logical framework for resolving the AB paradox and for understanding observables in gauge theories more generally.

Theorem 1 (Invariant Core as Observable Content)**.** *For any quantity Q_{total} that can be decomposed as $Q_{total} = Q_{inv} + Q_{gauge}$, where Q_{inv} is gauge-invariant and Q_{gauge} is purely gauge-dependent, the observable value of Q_{total} is exactly Q_{inv}.*

Proof. Let M be any physical measurement. By the foundational axiom, the purely gauge-dependent quantity Q_{gauge} has no observational consequences. Therefore, adding Q_{gauge} to Q_{inv} cannot change the measurement outcome:

$$M(Q_{\text{inv}} + Q_{\text{gauge}}) = M(Q_{\text{inv}}).$$

Since $Q_{\text{total}} = Q_{\text{inv}} + Q_{\text{gauge}}$, this means $M(Q_{\text{total}}) = M(Q_{\text{inv}})$. Hence, every measurement yields the same result for Q_{total} as for Q_{inv} alone, proving that the observable content of Q_{total} is Q_{inv}. □

This theorem provides the crucial link between formally gauge-dependent expressions and their observable content. It demonstrates that when a quantity can be separated into invariant and purely gauge-dependent parts, only the invariant part contributes to physical measurements.

A straightforward application of this theorem occurs in the standard semiclassical treatment of the AB effect (without backaction). Consider two open paths γ_1 and γ_2 with distinct endpoints, as may occur during an electron interferometer. The total phase difference between the two paths comprises two distinct contributions: the AB phase, arising from the line integral of the vector potential $\mathbf{A}$ along the paths, and the kinetic phase, stemming from differences in travel velocity. In the WKB approximation, the total phase accumulated along a trajectory γ is given by the action

$$\phi_{\text{total}} = \int_\gamma \left(\frac{1}{2} m \mathbf{v}^2 + e\mathbf{A} \cdot \mathbf{v} \right) dt. \tag{26}$$

The phase difference thus decomposes as

$$\Delta\phi_{\text{total}} = \Delta\phi_{\text{AB}} + \Delta\phi_{\text{kin}}, \tag{27}$$

where $\Delta\phi_{\text{AB}} = e \left(\int_{\gamma_1} \mathbf{A} \cdot d\mathbf{x} - \int_{\gamma_2} \mathbf{A} \cdot d\mathbf{x} \right)$ and $\Delta\phi_{\text{kin}} = \int_{\gamma_1} \frac{1}{2} m v^2 dt - \int_{\gamma_2} \frac{1}{2} m v^2 dt$. Under a gauge transformation $\mathbf{A} \to \mathbf{A} + \nabla\chi$, the AB term shifts by $e[\chi(\mathbf{x}_1) - \chi(\mathbf{x}_2)]$ (where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the end points of the two paths, respectively), rendering it gauge-dependent, while the kinetic phase $\Delta\phi_{\text{kin}}$ is gauge-invariant. By Theorem 1, the observable part of $\Delta\phi_{\text{total}}$ is $\Delta\phi_{\text{kin}}$. This explains why, for open paths, the AB phase is not observable, yet the kinetic phase remains fully observable through nonlocal correlation measurements [5, 6, 7].

5.3 Application to the AB Effect with Backreaction

With the general framework established, we now apply it to resolve the specific paradox in the AB effect with backreaction.

For the AB effect with backreaction, the total phase accumulated along a path γ can be expressed in terms of the interaction between the electron current and the total electromagnetic field. Using the linearity of the interaction and the separation of the solenoid current into fixed background and induced components, we obtain:

$$\phi[\gamma] = e \int d^4x \int d^4y \, j_e^\mu[\gamma](x) D_{\mu\nu}(x-y) \left[j_{s0}^\nu(y) + \delta j_s^\nu[\gamma](y) \right], \tag{28}$$

where j_{s0}^ν is the fixed background solenoid current (independent of the electron path), and $\delta j_s^\nu[\gamma]$ is the backreaction current induced by the electron's electromagnetic field, which depends on the specific path γ. This naturally decomposes into two components:

$$\phi[\gamma] = \underbrace{e \int d^4x \int d^4y \, j_e^\mu[\gamma](x) D_{\mu\nu}(x-y) j_{s0}^\nu(y)}_{\phi_0[\gamma]} + \underbrace{e \int d^4x \int d^4y \, j_e^\mu[\gamma](x) D_{\mu\nu}(x-y) \delta j_s^\nu[\gamma](y)}_{\delta\phi[\gamma]}. \tag{29}$$

The first term, $\phi_0[\gamma]$, depends only on the fixed background current j_{s0}^ν. The second term, $\delta\phi[\gamma]$, involves the path-dependent induced current $\delta j_s^\nu[\gamma]$.

We now consider the *relative phase* between two paths γ_1 and γ_2 forming a closed loop $\Gamma = \gamma_1 - \gamma_2$. The relative phase governing the interference pattern is:

$$\Delta\phi_{\text{total}} = \phi[\gamma_1] - \phi[\gamma_2] = \Delta\phi_0 + \Delta(\delta\phi), \tag{30}$$

where $\Delta\phi_0 = \phi_0[\gamma_1] - \phi_0[\gamma_2]$ and $\Delta(\delta\phi) = \delta\phi[\gamma_1] - \delta\phi[\gamma_2]$. As established in previous sections, the relative phase $\Delta\phi_0$ is manifestly gauge invariant, and $\Delta(\delta\phi)$ is purely gauge dependent, carrying no invariant physical content. Thus, the total relative phase can be separated into a gauge-invariant core $\Delta\phi_0$ (proportional to the background magnetic flux) and a gauge-dependent backreaction term $\Delta(\delta\phi)$.

The decomposition (30) precisely fits the structure of Theorem 1, with $\Delta\phi_0$ as the gauge-invariant core and $\Delta(\delta\phi)$ as the purely gauge-dependent remainder. Applying Theorem 1, we conclude that the observable interference shift is exactly $\Delta\phi_0 = e\Phi_0$, where Φ_0 is the magnetic flux enclosed by the loop $\gamma_1 - \gamma_2$. The backreaction contribution $\delta(\Delta\phi)$ is purely gauge-dependent and therefore unobservable.

This resolves the apparent paradox completely. The formal gauge dependence of the total phase $\Delta\phi_{\text{total}}$ is real but physically irrelevant: it affects only the unobservable component $\delta(\Delta\phi)$. Experimental measurements necessarily probe only the gauge-invariant core $\Delta\phi_0$, which remains robust and unambiguous regardless of gauge choices or backreaction effects.

5.4 Philosophical and Conceptual Implications

The decomposition framework developed in this section reveals a profound rethinking of how observables emerge in gauge theories. Rather than being primitive entities defined *ab initio* as gauge-invariant objects, observables are shown to arise naturally as the invariant cores of more general gauge-dependent quantities. This represents a significant conceptual shift: instead of viewing gauge invariance as a rigid constraint to be imposed on all physical quantities, we now recognize it as a structural feature that emerges through careful decomposition of gauge-dependent quantities.

This perspective resolves a longstanding tension in the interpretation of gauge theories. Traditional approaches often treat gauge-dependent quantities as physically meaningless, requiring purification through elaborate constructions of gauge-invariant observables. Our framework demonstrates that gauge-dependent quantities are not deficient but rather contain within them both physical content (the gauge-invariant core) and mathematical redundancy (the gauge-dependent remainder). The observable physics is not something external to these quantities but is encoded within them, waiting to be extracted through appropriate decomposition.

This decomposition approach also clarifies the relationship between mathematical description and physical measurement in gauge theories. When we have a gauge-dependent quantity, we do not have something unphysical; rather, we have a complete mathematical description that contains both physical information and representational redundancy. The process of measurement effectively projects out the invariant core, discarding the gauge-dependent remainder. This projection is not arbitrary but follows from the fundamental axiom of gauge theories and the nature of physical interactions.

Finally, this understanding provides practical guidance for theoretical physics. When faced with a gauge-dependent quantity, we need not abandon it as unphysical. Instead, we can systematically decompose it to extract its invariant core, which will correspond to observable effects. The AB effect, long viewed as a paradigm of gauge invariance, thus serves as a gateway to a more nuanced understanding of gauge principles in quantum theory. This approach is particularly valuable in QFT and general relativity, where gauge (or coordinate) dependence is ubiquitous. By embracing gauge dependence as a feature rather than a bug, we gain access to powerful mathematical tools while maintaining clear connections to observable physics.

6 Extensions to Other Gauge-Invariant Phenomena

The discovery that the AB phase becomes gauge dependent when backreaction is included raises important questions about other gauge-invariant phenomena. In this section, we identify and analyze phenomena that share the structural characteristics necessary for similar revisions.

6.1 Criteria for AB-Type Revisions

The gauge dependence mechanism identified in the AB effect relies on a specific structural feature: the observable phase is a relative phase between quantum-superposed branches (physically distinct interfering paths), and backreaction entangles those branches with distinct field configurations. This entanglement permits branch-dependent gauge transformations (independent gauge choices Λ_i on each branch), rendering the total relative phase gauge-dependent while the experimentally relevant contribution decomposes into a gauge-invariant core and an unobservable, gaugable-away backreaction term.

We therefore adopt the following strict criterion for an AB-type revision:

> AB-type gauge dependence arises if and only if a phase is a relative phase between quantum-superposed branches and the gauge transformation required to compare those branches is branch-dependent due to backreaction.

Only phenomena satisfying this condition exhibit the same direct form of gauge dependence as analyzed for the AB effect. Other gauge/geometric invariants may still be influenced by backreaction and entanglement, but typically preserve invariance for single closed contours or single-branch evolutions.

6.2 Phenomena Exhibiting Similar Gauge Dependence

The following cases share the essential interfering-branch structure of the AB effect and thus require the same revision.

6.2.1 The Aharonov-Casher Effect

The Aharonov-Casher (AC) effect is the direct dual of the AB effect: a neutral particle with magnetic moment interferes after traversing two paths around a line of electric charge. The phase shift is a relative phase between the superposed paths. When the charge line is treated as dynamical (full quantum treatment), backreaction from the particle's magnetic field perturbs the source, entangling the two paths with distinct electric field configurations. This permits branch-dependent gauge transformations on the scalar potential, making the total relative phase gauge-dependent. The decomposition approach applies analogously: the observable interference is determined by a gauge-invariant core proportional to the enclosed electric flux, with the backreaction term unobservable.

6.2.2 Non-Abelian AB Effects

In fundamental Yang–Mills theories such as QCD, the analog of the AB phase for a closed loop Γ is encoded in the Wilson loop operator

$$W[\Gamma] = \mathrm{Tr}\,\mathcal{P}\exp\Big(ig\oint_\Gamma A^a_\mu(x)T_a dx^\mu\Big), \tag{31}$$

where $A^a_\mu T_a$ is a Lie-algebra–valued connection, g is the coupling constant, and T_a are the generators of the gauge group. In the fixed-background approximation, the gauge field is taken to be a classical configuration $A^a_{0\mu}$. The AB phase is then given directly by the phase of the classical Wilson loop

$$W_0[\Gamma] = \mathrm{Tr}\,\mathcal{P}\exp\Big(ig\oint_\Gamma A^a_{0\mu}(x)T_a dx^\mu\Big), \tag{32}$$

which is a gauge-invariant complex number.

When backreaction is included, the gauge field becomes dynamical and the two interfering paths become entangled with distinct field configurations. This permits branch-dependent gauge transformations—independent gauge transformations on each path. The observable quantity in the full quantum treatment is the expectation value of the Wilson loop operator:

$$\langle W[\Gamma]\rangle := \Big\langle \mathrm{Tr}\,\mathcal{P}\exp\Big(ig\oint_\Gamma A^a_\mu(x)T_a dx^\mu\Big)\Big\rangle, \tag{33}$$

where the expectation value is taken over the quantum gauge field, including backreaction.

Following the decomposition framework, we write $\langle W[\Gamma]\rangle$ in polar form:

$$\langle W[\Gamma]\rangle = \big|\langle W[\Gamma]\rangle\big|\,\exp\big(i\Phi_{\mathrm{eff}}[\Gamma]\big), \tag{34}$$

where $\Phi_{\mathrm{eff}}[\Gamma]$ is the effective (total) phase. This phase decomposes additively as:

$$\Phi_{\mathrm{eff}}[\Gamma] = \Phi_{\mathrm{core}}[\Gamma] + \Phi_{\mathrm{gauge}}[\Gamma].$$

The gauge-invariant core is defined as

$$\Phi_{\mathrm{core}}[\Gamma] := \arg\big(W_0[\Gamma]\big) = \arg\Big(\mathrm{Tr}\,\mathcal{P}\exp\Big(ig\oint_\Gamma A^a_{0\mu}T_a dx^\mu\Big)\Big). \tag{35}$$

This is precisely the phase of the fixed-background Wilson loop. It is manifestly gauge-invariant and reduces to the standard non-Abelian AB phase. It constitutes the observable interference shift. The gauge-dependent backreaction contribution is defined as

$$\Phi_{\mathrm{gauge}}[\Gamma] := \arg\Big(\frac{\langle \mathrm{Tr}[U(\gamma_2)^{-1}U(\gamma_1)]\rangle_A}{W_0[\Gamma]}\Big). \tag{36}$$

This term encodes all effects of branch-dependent backreaction. It transforms non-trivially under gauge transformations and can be shifted arbitrarily by gauge choice. By the foundational axiom, it carries no observational consequences and is unobservable.

Thus, the measurable interference pattern is determined solely by the gauge-invariant core $\Phi_{\mathrm{core}}[\Gamma]$, which corresponds to the AB phase computed from the fixed background flux. The gauge-dependent term $\Phi_{\mathrm{gauge}}[\Gamma]$ is unobservable and can be gauged away. This decomposition provides a non-Abelian generalization of the result derived for the electromagnetic AB effect.

In analog non-Abelian systems—such as condensed matter, cold atoms, or photonic lattices that simulate non-Abelian gauge fields—the "gauge field" is typically externally imposed and non-dynamical. It represents a fixed classical parameter in the effective Hamiltonian and does not respond dynamically to the particle's motion. There is no backreaction, no entanglement between the particle's path and the gauge configuration, and therefore no possibility of branch-dependent gauge transformations. Consequently, the standard picture of a gauge-invariant phase determined by the Wilson loop remains valid without revision.

6.2.3 Gravitational AB Effect

The gravitational AB (GAB) effect refers to a quantum phase shift acquired by particles traveling along different paths in a gravitational field, even in regions where the gravitational force or spacetime curvature vanishes locally along the trajectories. The GAB phase may arise from the scalar gravitational potential or from the enclosed gravitomagnetic flux generated by rotating masses.

In the standard semiclassical treatment—where the gravitational source is treated as fixed and classical, and the metric is an unperturbed background—the GAB phase is diffeomorphism-invariant. The observable interference shift is determined by coordinate-independent quantities: the integrated proper-time difference for the scalar case, or the holonomy associated with the gravitomagnetic connection for the rotating case.

However, when backreaction is considered in a fully dynamical treatment of gravity (e.g., perturbative quantum gravity), the test particle's stress-energy perturbs the metric. In an interferometric superposition, the different paths could in principle become entangled with distinct metric configurations, allowing branch-dependent diffeomorphisms (independent coordinate choices on each branch). This would render the total relative phase formally diffeomorphism-dependent. Following the framework developed for the electromagnetic AB effect, the phase would decompose into a diffeomorphism-invariant core, proportional to the enclosed gravitational potential difference (scalar) or gravitomagnetic flux (vector) of the fixed background, and a purely diffeomorphism-dependent backreaction term that carries no observational content and can be eliminated by an appropriate coordinate choice. The observable interference would then be governed solely by the invariant core.

6.3 Summary

To summarize, the AB-type gauge dependence mechanism is tied to the interference of physically distinct quantum paths entangled with distinct field configurations. The AC effect, non-abelian AB variants, and gravitational AB effect fully satisfy this criterion. The broader principle—that formally gauge-dependent quantities often contain observable gauge-invariant cores—remains applicable across gauge theories, even when backreaction does not induce branch-dependent gauge transformations.

7 Why This Gauge Dependence Remained Unrecognized

The result that the total closed-loop AB phase becomes gauge dependent in a fully quantum electrodynamical treatment with dynamical sources and backreaction—while decomposing into an observable gauge-invariant core and an unobservable gauge-dependent backreaction term—may appear surprising in light of the long-standing textbook consensus on gauge invariance. After all, quantum entanglement, backreaction, and the subtleties of gauge transformations in QFT are well-established concepts. Yet this specific implication for closed-loop phases has remained largely unemphasized in the literature for over six decades. In this section, we examine the historical, conceptual, experimental, and theoretical reasons for this persistent oversight.

7.1 Historical and Conceptual Reliance on Semiclassical Approximations

The original proposal by Aharonov and Bohm in 1959 [1] (and the earlier independent derivation by Ehrenberg and Siday in 1949 [3]) was framed within a semiclassical context: the electromagnetic source (e.g., solenoid) is treated as a fixed classical background with infinite mass or rigidity, so that the probe particle exerts no backreaction on it. In this limit, all interfering paths experience the identical field configuration, and Stokes' theorem applies directly, converting the line integral of the vector potential to a surface integral of the gauge-invariant field strength. Gauge invariance for closed loops follows immediately, and the phase shift is proportional to the enclosed magnetic flux—a result that matches experimental observations perfectly.

This semiclassical idealization became the standard paradigm in textbooks and review articles. The approximation is extraordinarily accurate for macroscopic setups, where backreaction effects are suppressed by enormous factors. As a consequence, the need to consider full quantization of the source, with its attendant entanglement between electron paths and distinct field configurations, was rarely pursued. The conceptual shift to branch-dependent gauge transformations in superpositions of field states was not necessary for practical calculations, so it remained unexplored in mainstream treatments.

7.2 Theoretical Dogma and Focus on Gauge Invariance as a Principle

Gauge invariance is one of the most cherished principles in modern physics, underpinning the construction of all fundamental gauge theories (QED, QCD, electroweak theory, etc.). Textbooks and foundational works emphasize that physical observables must be gauge invariant, with the AB effect often cited as the paradigmatic demonstration that closed-loop phases are invariant via Stokes' theorem. This created a strong conceptual bias: questioning invariance for closed loops felt like challenging a bedrock principle without compelling reason.

When critiques of the standard picture appeared, they typically focused on open paths or on locality debates [12, 9, 4]. Hayashi [8] explicitly demonstrated gauge dependence of the AB phase in a QED framework but concluded that invariance holds for closed loops. These works advanced the discussion but did not extend the analysis to closed loops under full backreaction with entanglement and superposition of field configurations—the key step that reveals the decomposition into invariant core and dependent term.

The distinction between classical gauge shifts (inconsistent in QFT) and proper quantum gauge-fixing changes (Section 4) further obscured the issue. Many analyses remained within semiclassical or approximate QED frameworks that implicitly retained fixed sources, missing the path-dependent effective gauge functions $\Lambda_\gamma(x)$ that cause boundary terms to fail to cancel in the relative phase.

7.3 Delayed Recognition of Entanglement in Dynamical Gauge Fields

While particle entanglement has been intensively studied since the 1980s (e.g., Bell tests), the entanglement of probe degrees of freedom with dynamical gauge field configurations received less attention until the 2000s–2010s, driven by advances in quantum optics, condensed matter analogs, and holographic duality. In standard QED pedagogy, the electromagnetic field is often treated as external or in the vacuum state, so the possibility of path-dependent superpositions of field states (enabling branch-dependent gauges) was not foregrounded in AB discussions.

Only with the rise of quantum technologies and renewed interest in foundational issues (e.g., the role of potentials, locality in gauge theories) has the full quantum treatment gained traction. Recent works and related critiques highlight gauge dependence in QED frameworks, but the synthesis into a general principle—of gauge-dependent quantities containing observable invariant cores—has been rare until now.

In summary, the oversight stems from a powerful combination of excellent approximations that match experiment, the sanctity of gauge invariance as a guiding principle, the practical invisibility of backreaction in traditional setups, and the historical focus on fixed-background treatments. The present analysis, by explicitly incorporating backreaction, entanglement, and the decomposition approach, reveals a subtler structure that resolves apparent paradoxes while preserving agreement with observations. This perspective invites a reevaluation of gauge invariance in quantum gauge theories not as an absolute rule, but as context-dependent—holding rigorously only when sources are effectively classical.

8 Conclusion and Future Directions

In this work, we have systematically analyzed the gauge transformation properties of the AB phase when backreaction effects are included in a fully quantum treatment. Our investigation reveals that the conventional wisdom regarding gauge invariance of closed-loop phases requires significant revision, and establishes a new framework for understanding observables in gauge theories.

The key findings of our analysis can be summarized as follows. First, when backreaction is properly accounted for in quantum electrodynamics, different electron paths become entangled with distinct electromagnetic field configurations. This entanglement permits branch-dependent gauge transformations that render the total AB phase for a closed loop genuinely gauge dependent. This result challenges the textbook claim that closed-loop phases are necessarily gauge invariant via Stokes' theorem, demonstrating instead that this invariance holds only in the fixed-background approximation where all paths experience identical field configurations.

Second, we have established a rigorous framework for understanding observables in gauge theories. Beginning from the fundamental axiom that purely gauge-dependent quantities cannot have observational consequences, we derived theorems showing that any gauge-dependent quantity decomposes into a gauge-invariant core (which constitutes its observable content) and a gauge-dependent remainder (which is unobservable and can be eliminated by gauge choice). For the AB effect, this decomposition takes the specific form $\Delta\phi = \Delta\phi_0 + \Delta(\delta\phi)$, where $\Delta\phi_0$ is the gauge-invariant phase associated with fixed background flux and $\Delta(\delta\phi)$ is the gauge-dependent backreaction contribution.

Third, this decomposition resolves the apparent paradox between the formal gauge dependence of the total phase and the experimentally observed flux-proportional interference shifts. Experiments necessarily measure only the gauge-invariant core $\Delta\phi_0$, while the gauge-dependent backreaction term $\Delta(\delta\phi)$ represents an unobservable mathematical artifact. The robust experimental confirmation of AB interference thus provides empirical support for the gauge-invariant core while remaining insensitive to the gauge-dependent remainder.

The implications of this work extend well beyond the specific context of the AB effect. Our decomposition framework provides a general approach for extracting physical observables from gauge-dependent formalisms across various domains of theoretical physics. In QFT, the approach clarifies how observables emerge from gauge-dependent correlation functions and Wilson lines. It provides conceptual clarity for interpreting quantities that are formally gauge dependent yet yield unambiguous physical predictions when their invariant cores are properly identified. In general relativity and gravitational physics, the framework offers insight into the nature of observables in diffeomorphism-invariant theories. Coordinate-dependent quantities can be understood as containing invariant cores that correspond to measurable effects, with coordinate artifacts playing the role of gauge-dependent remainders.

Philosophically, this work challenges the traditional view that gauge dependence is merely a mathematical nuisance to be eliminated. Instead, gauge dependence serves a constructive role in revealing the separation between invariant cores and gauge-dependent remainders. This perspective shifts the focus from insisting on gauge invariance as an absolute requirement to identifying invariant structures through physically motivated decompositions.

Several promising directions for future research emerge from this work. Experimental tests could be designed to probe the decomposition more directly, particularly in regimes where backreaction effects become non-negligible. Theoretical extensions could apply the decomposition framework to non-abelian gauge theories like QCD, where similar issues arise in the interpretation of Wilson loops and confinement. Applications to quantum gravity could explore how observable spacetime structure emerges from gauge-dependent (diffeomorphism-dependent) descriptions. Foundational investigations could further explore the relationship between gauge symmetry, entanglement, and the emergence of observables in quantum theories.

In conclusion, our analysis demonstrates that gauge invariance in quantum theory is more nuanced than traditionally assumed. The AB effect, long celebrated as a paradigm of gauge invariance, actually reveals the limitations of this principle when systems are treated fully quantum mechanically. By recognizing that gauge-dependent quantities contain within them invariant cores that encode observable physics, we gain a more sophisticated understanding of what is truly measurable in gauge theories. This perspective not only resolves specific paradoxes but also enriches our understanding of how mathematical structures in theoretical physics encode physical reality, providing a clearer path forward for interpreting observables in gauge theories across the spectrum of modern physics.

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