Historical Debates over the Physical Reality of the Wave Function

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Abstract

This paper provides a detailed historical account of early debates over wave-function realism, the modern term for the view that the wave function of quantum theory is physically real. As this paper will show, the idea of physical waves associated with particles had its roots in work by Einstein and de Broglie, who both originally thought of these waves as propagating in three-dimensional physical space. De Broglie quickly turned this wave-particle duality into an early pilot-wave theory, on which a particle's associated "phase wave" piloted or guided the particle along its trajectory. Schrödinger built on de Broglie's phase-wave hypothesis to provide a comprehensive account of the nascent quantum theory. However, Schrödinger's new "undulatory mechanics" came at the cost of replacing de Broglie's phase waves propagating in physical space with a wave function propagating in a system's abstract configuration space. The present work will argue that this move from three-dimensional physical space to a many-dimensional configuration space was a key reason why the founders of quantum theory uniformly abandoned the physical reality of the wave function. This paper will further clarify that de Broglie introduced two distinct pilot-wave theories, and will then argue that it was Bohm's rediscovery of the second of these two pilot-wave theories over two decades later, as well as Bohm's vociferous defense of wave-function realism, that were responsible for resurrecting the idea of an ontological wave function. This idea ended up playing a central role in Everett's development of the many-worlds interpretation.

1 Introduction

In many pedagogical and formal treatments of quantum theory today, the wave function plays a central, starring role. One is then confronted with an obvious question: is the wave function physically real? That is, is the wave function ontological?

Before proceeding, it will be important to disambiguate the term 'wave function.' On the Dirac-von Neumann formulation of quantum theory (Dirac 1930, von Neumann 1932), there is an important distinction to be made between, on the one hand, an abstract quantum state and, on

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the other hand, a configuration-space wave function. In the research literature, the term 'wave function,' without qualification, is sometimes used to refer to the former, and sometimes to the latter.

On the Dirac-von Neumann axioms, an abstract quantum state is represented by a vector $|\Psi\rangle$ in (or, more generally, an operator ρ on) a Hilbert space. By contrast, a configuration-space wave function $\Psi(q)$ is defined as the set of complex-valued components of a state vector $|\Psi\rangle$ with respect to a specific orthonormal basis for the Hilbert space, where the members $|q\rangle$ of that orthonormal basis are indexed or labeled by a suitable set of configurations q making up a configuration space:[1]

$$|\Psi\rangle = \int dq\, \Psi(q)|q\rangle, \qquad \Psi(q) = \langle q|\Psi\rangle. \tag{1}$$

The present work will be focused primarily on configuration-space wave functions $\Psi(q)$.

The view that configuration-space wave functions are physically real or ontological—or at least directly represent something physically real or ontological—has come to be known as 'wave-function realism.' In recent decades, wave-function realism has generated a tremendous amount of scholarship and controversy (Albert 1996; Lewis 2004; Ney, Albert 2013; Myrvold 2015; Chen 2019; Wallace 2020; Ney 2021; Ney 2023).

The purpose of the present work is not to wade into these contemporary debates, but to trace their origins to discussions between the founders of quantum theory. In particular, this paper will argue for the following key points:

- The original wave-particle duality introduced by Albert Einstein in 1905 and further developed by Louis de Broglie from 1923 to 1926 involved waves propagating in three-dimensional physical space.

- On de Broglie's "phase wave" theory, the waves that accompanied material particles also determined the motion of those particles according to an early form of a 'guiding equation.' Even then, de Broglie questioned the physical reality of his phase waves because, on his theory, they propagated with superluminal phase velocity.

- Erwin Schrödinger, in his foundational papers on "undulatory"[2] or "wave mechanics" in 1926 and 1927, moved de Broglie's waves from three-dimensional physical space to the much more abstract, many-dimensional configuration space of the system in question. Schrödinger immediately tried to grapple with the physical reality of his newly introduced wave function, to the point of introducing a proto-Everettian ("many worlds") interpretation. Schrödinger quickly settled on the nuanced view that his wave function was an ontological object whose observable effect was to determine distributions of electric charge in three-dimensional physical space.

[1] Technically speaking, if the configuration space is continuous, then the basis vectors $|q\rangle$ will not be normalizable and will not actually belong to the Hilbert space, but will instead be defined in terms of a suitable rigged Hilbert space. These technicalities will not be important for the purposes of this paper.

[2] "Undulatory" comes from the Latin root *unda*, which means "wave," and is also the source of the French word "onde" for "wave."

- In many letters to colleagues in 1926 and 1927, including in his famous "God does not play dice" letter to Max Born in 1926, Einstein repeatedly and vocally expressed his opposition to the physicality of waves in a many-dimensional configuration space.

- Born's 1926 statistical interpretation of the wave function complicated discussions over the physicality of the configuration-space wave function. On this statistical interpretation, the modulus-square of the wave function gave the probability density with which a measurement of the system's configuration would yield a specific result.

- Starting with a presentation to the fifth Solvay Conference in 1927, and building on the work of Schrödinger and Born, de Broglie developed two distinct pilot-wave theories on which quantum-mechanical waves piloted or guided particles along their trajectories in three-dimensional physical space.

- In a 1927 paper, de Broglie gave a detailed presentation of the first of these two pilot-wave theories, which eventually became known as his "double-solution theory." This pilot-wave theory was based on two conceptually distinct kinds of waves that both satisfied the same wave equation. The first kind of wave conjecturally featured solitonic singularities or local maxima that de Broglie identified with material particles. The second kind of wave realized Born's statistical interpretation. De Broglie found the mathematics of this double-solution theory to be quite complicated, and he eventually put it aside.

- In a lecture in 1928, Schrödinger publicly disavowed his earlier view that his wave function in a configuration space should be understood to be physically real.

- De Broglie presented a second, conceptually distinct pilot-wave theory in a 1930 book. This second pilot-wave theory involved only one kind of wave—Schrödinger's wave function. On this theory, the wave function piloted or guided material particles along their trajectories according to a precise and general guiding equation, where de Broglie treated those material particles as a separate form of ontology rather than as solitonic singularities derived from the wave function itself. In the later chapters of that book, de Broglie attempted to make sense of this pilot-wave theory in the multi-particle case, in which the configuration space was many-dimensional. De Broglie decided that a wave function propagating in such a many-dimensional space could only be treated as abstract and fictitious, and not as ontological.

- As a result, after completing his 1930 book, de Broglie abandoned both of his pilot-wave theories for several decades, only returning to his double-solution theory after David Bohm contacted him in 1951 to share his own pilot-wave theory with de Broglie.

- A denial of the physical reality of Schrödinger's wave function was one of the few things that the major figures in the development of quantum theory—Einstein, de Broglie, Schrödinger, Born, Heisenberg, Bohr, Pauli, and Dirac—all agreed on.

- Bohm introduced the first comprehensive treatment of decoherence in the later chapters of his 1951 textbook on quantum theory, a contribution to the progress of physics that has not been sufficiently recognized.[3]

- Bohm then developed what he initially thought was an original pilot-wave theory from 1951 to 1952, eventually publishing a pair of foundational papers laying out the theory in 1952. The first of these two papers focused primarily on the axiomatic construction of Bohm's pilot-wave theory, and the second paper used decoherence to explain how his pilot-wave theory provided a means of resolving the measurement problem.

- Bohm's pilot-wave theory was identical to de Broglie's second pilot-wave theory, as Bohm acknowledged explicitly in correspondence with Wolfgang Pauli in 1951.

- One of Bohm's major contributions was using decoherence to solve old problems with de Broglie's version of the theory.

- Another of Bohm's major contributions was to breathe new life into what is now called 'wave-function realism' by arguing that the pilot wave should be regarded as physically real or ontological, despite the fact that it propagated in a many-dimensional configuration space. Bohm mentioned this claim in only a limited way in his two 1952 papers introducing his pilot-wave theory, but made the case far more extensively and robustly in written correspondence with Pauli in 1951.

- Bohm's defense of wave-function realism inspired others to take wave functions, and even more abstract quantum states, to be physically real entities. In particular, Hugh Everett III was clearly aware of Bohm's pivotal development of the theory of decoherence, and wrote in detail about Bohm's work in his unpublished, extended 1956 dissertation. Going farther than Bohm, Everett took the quantum state of the universe, treated now as an element of a Hilbert space, to be the sole ontology of nature. Everett was also greatly influenced by Schrödinger's writings in the early 1950s, in which Schrödinger attempted to resurrect his realism about the wave function and cast doubt on the need for probability axioms.

2 The History of Waves in Quantum Theory

2.1 De Broglie's Phase Waves

In the autumn of 1923, Louis de Broglie wrote a paper titled "*Ondes et Quanta*" ("Waves and Quanta," de Broglie 1923a) that was inspired both by Albert Einstein's 1905 introduction of quanta

[3] Most references credit Dieter Zeh and Wojciech Zurek for the discovery of decoherence, including the Stanford Encyclopedia of Philosophy's entry "The Role of Decoherence in Quantum Mechanics" (Bacciagaluppi 2025). Indeed, the front matter in 2025 book by Zurek himself says "Wojciech Hubert Zurek is credited with developing the theory of decoherence" (Zurek 2025). Until February 2024, the Wikipedia entry "Quantum decoherence" did not mention Bohm's pivotal contributions (Wikipedia 2026). A marvelous exception is a 2004 paper by Brown and Wallace (Brown, Wallace 2004, Footnote 9).

of light (Einstein 1905) and also by an apparent conflict between special relativity and quantum theory.

On the one hand, relative to a fixed observer in some inertial reference frame, a microscopic object moving at a speed v, and thus traveling at a fraction $\beta = v/c$ of the speed of light c, should exhibit time dilation. As a consequence, any internal periodic process inside the object with proper frequency ν_0 should appear to decrease to a *lower* frequency

$$\nu_1 = \nu_0\sqrt{1-\beta^2} < \nu_0. \tag{2}$$

On the other hand, the proper mass-energy $E_0 = m_0c^2$ of the microscopic object should be related to some sort of proper frequency ν_0 according to Planck's quantum formula $E_0 = h\nu_0$, where h is Planck's constant. If the object were in motion at a fraction $\beta = v/c$ of the speed of light, then the mass-energy E_0 should Lorentz-transform to the *larger* value $m_0c^2/\sqrt{1-\beta^2} > m_0c^2$, and thus the proper frequency ν_0 should Lorentz-transform to the correspondingly *higher* frequency

$$\nu = \nu_0/\sqrt{1-\beta^2} > \nu_0. \tag{3}$$

To de Broglie, this disparity between the lower value (2) of the internal periodic frequency ν_1 and the higher value (3) of the Planck frequency ν suggested that the Planck frequency did not refer to a periodic process internal to the microscopic object. De Broglie posited instead that the Planck frequency referred to some kind of wave associated with the object, propagating along with the object in three-dimensional physical space. If this wave shared the same proper frequency ν_0 as some internal periodic process, then because of the discrepancy $\nu > \nu_1$, these two frequencies could only *remain* synchronized or in harmony for nonzero speeds $\beta \neq 0$ if the wave had a faster phase velocity than the physical velocity of the microscopic object. Intuitively speaking, the wave's higher phase velocity could then 'spread out' the peaks and valleys of the wave to keep them in harmony with the peaks and valleys of the object's internal periodic process.

A short calculation showed that in order to maintain phase harmony ("*théorème de l'harmonie des phases*" or "*l'accord des phases*"), the wave would need a phase velocity of $c/\beta = c^2/v > c$, meaning that the wave needed to have a phase velocity faster than light. De Broglie concluded that this "phase wave" ("*onde de phase*") could not transport energy, and was therefore a "fictitious wave" ("*onde fictive*"):

> Suppose now that at the time $t = 0$, the moving object coincides in space with a wave of frequency ν defined above and propagating in the same direction as it with the speed c/β. This wave, with a speed greater than c, cannot correspond to transport of energy; we will consider it only as a fictitious wave associated with the motion of the object. [De Broglie 1923a, in literal English translation from the original French by the author of the present work]

Nonetheless, de Broglie used these propagating phase waves to try to account for various quantization formulas that were known from early work on quantum theory.

In a follow-up paper in the same year, titled "*Quanta de Lumière, Diffraction et Interférences*" ("Quanta of Light, Diffraction and Interference," de Broglie 1923b), de Broglie also argued that on this phase-wave hypothesis, each particle of matter moved at a velocity equal to the *group* velocity of its associated wave packet, and along a trajectory that matched the geometric ray of that associated wave packet. Indeed, in that second paper, de Broglie had already begun to use the language of "guiding" ("*guidant*") for his phase waves:

> We therefore conceive of the phase wave as guiding the displacements of energy, and this is what can permit the synthesis of undulations and quanta. [De Broglie 1923b, in literal English translation from the original French by the author of the present work]

Both in that paper, and in a subsequent paper written in English also in the same year, de Broglie referred to his phase waves as "non-material" ("*non matèrielle*"), again due to their superluminal propagation speeds (de Broglie 1923b, 1923c).

De Broglie expanded on these ideas in his 1924 doctoral dissertation, titled "*Recherches sur la théorie des Quanta*" ("Research on the Quantum Theory," de Broglie 1924), for which he would go on to win the 1929 Nobel Prize in Physics. In Chapter 2, Section V of de Broglie's dissertation, he presented an early version of a 'guiding equation' in the form

$$O_i = \frac{1}{h} J_i, \tag{4}$$

where, following de Broglie's notation, i is a Lorentz index.

On the right-hand side of (4), J_i was the object's "world vector" ("*vecteur d'Univers*"),

$$J_i = m_0 c u_i + e\varphi_i. \tag{5}$$

This formula involved the object's proper or rest mass m_0, the speed of light c, and the object's dimensionless four-velocity $u_i = dx_i/ds$, where $ds = \sqrt{c^2dt^2 - dx^2 - dy^2 - dz^2}$ denoted infinitesimal increments along the object's worldline. This formula for J_i also included the object's electric charge e and the electromagnetic gauge potentials φ_i (not to be confused with the notation φ for phase functions to be introduced shortly).

On the left-hand side of (4), O_i was the phase wave's four-dimensional "wave world vector" ("*le vecteur Onde d'Univers*"), given in terms of the phase function $\varphi(x, y, z, t)$ of the phase wave according to the differential relationship

$$d\varphi = 2\pi \sum_i O_i dx^i. \tag{6}$$

As a consequence, the spatial components $\mathbf{O} = (O_x, O_y, O_z)$ of O_i are given by the gradient of φ, up to a reciprocal factor of 2π:

$$\mathbf{O} = \frac{1}{2\pi} \nabla\varphi. \tag{7}$$

In the non-relativistic limit $v \ll c$, and in the absence of electromagnetic fields, one has $c\mathbf{u} \approx \mathbf{v}$, where $\mathbf{v}$ is the particle's ordinary velocity, and $\varphi_i = 0$. The spatial parts of the guiding equation (4) therefore reduce to

$$\mathbf{v} = \frac{1}{m_0}\left(\frac{h}{2\pi}\right)\nabla\varphi. \tag{8}$$

Here the quantity appearing in parentheses, $h/2\pi$, would now be called the reduced Planck constant $\hbar$ ('h-bar'), a notation introduced by Paul Dirac a few years later (Dirac 1928).

The equation (8) will turn out to be closely related to the guiding equation of a pilot-wave theory. However, it is worth keeping in mind that when de Broglie wrote his dissertation, his phase-wave theory only applied in the geometrical-optics limit, meaning for very short wavelengths, and therefore under circumstances in which he could directly relate Maupertuis's principle for the paths of material particles to Fermat's principle for the paths of light rays. At that point in his work, de Broglie had not yet clearly specified a detailed physical relationship between particles of matter and his phase waves more generally.

One saw the first steps toward describing such a physical relationship in some of de Broglie's writings starting in 1926. For example, in a 1926 paper titled "*Les Principes de la Nouvelle Mécanique Ondulatoire*" ("The Principles of the New Wave Mechanics," de Broglie 1926), in Section 10 ("*Le point matériel*"), de Broglie wrote that in the geometrical-optics limit, a particle of matter could be identified with "a very pronounced maximum" ("*un maximum très pronuncé*") in its associated phase wave. He then added:

> The rays of the associated wave are the possible trajectories of the moving body. But in a *given* motion, one of the rays is of particular physical importance, namely, the ray which is actually described by the moving body. The associated wave, in so far as we have imagined it up to the present, does not enable us to understand what distinguishes this ray from the others. In order to get some light on this point it is necessary, as I pointed out in my thesis, to consider not only a monochromatic associated wave, but a group of associated waves with frequencies very close to each other. [De Broglie 1926, translated from the French to English by W. M. Deans, emphasis in the original]

Later in that 1926 paper, de Broglie briefly returned to this wave-derived conception of particles of matter, referring to each now as a "point singularity" ("*singularité ponctuelle*") in its associated phase wave, but then also casting doubt on the idea outside the geometrical-optics limit:

> If in this case there still exists in every group of waves a *point*-singularity worthy of the name of material particle, we can easily conceive that each of the groups of waves is propagated according to a law like (59), where the coefficients depend on the position at that time of the singularities in the other groups; but if, in this kind of motion, there are no longer any well-defined material particles, the question becomes obscure. [Ibid., emphasis in the original]

2.2 Schrödinger's Wave Function

In early 1926, motivated in part by de Broglie's phase-wave hypothesis, and also by classical Hamilton-Jacobi theory, Erwin Schrödinger introduced a new kind of wave $\psi(q,t)$. This was his famous "wave function" ("*Wellenfunktion*"), where q labels an abstract point in the system's classical configuration space, and where t, as usual, is the time.

Schrödinger laid out the foundations of his new theory of "undulatory" or "wave mechanics" in a series of four German-language papers, all sharing the title "*Quantisierung als Eigenwertproblem*" ("Quantization as an Eigenvalue Problem," Schrödinger 1926a–d). In the first of the four *Eigenwertproblem* papers, Schrödinger began with the restricted Hamilton-Jacobi equation for a system with classical Hamiltonian H and total energy E:

$$H\left(q, \frac{\partial S}{\partial q}\right) = E. \tag{9}$$

Then Schrödinger introduced ψ according to

$$S = K \lg \psi, \tag{10}$$

where lg was his notation for the natural logarithm, and where K was a constant with units of action. Schrödinger would eventually identify K as

$$K = \frac{h}{2\pi}, \tag{11}$$

which, again, as in (8), would now be called the reduced Planck constant $\hbar$.

According to Schrödinger's new undulatory theory, the wave function evolved with time according to what is now known as the configuration-space Schrödinger equation, which Schrödinger originally wrote down as his eq. $(4'')$ in the fourth *Eigenwertproblem* paper (Schrödinger 1926d):[4]

$$\Delta\psi - \frac{8\pi^2}{h^2} V\psi \mp \frac{4\pi i}{h}\frac{\partial \psi}{\partial t} = 0. \tag{12}$$

In this equation, Δ was a second-order differential operator acting on the system's configuration space that included the inertial masses of the various particles, and V was an arbitrary potential function on the configuration space. The ambiguous sign in the third term reflected the freedom to choose either ψ or its complex conjugate $\bar{\psi}$ as the function satisfying the equation. In terms of the reduced Planck constant $\hbar = h/2\pi$, this equation takes the more familiar form

$$\mp i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2}\Delta\psi + V\psi, \tag{13}$$

where the positive sign convention is generally used today. Schrödinger would go on to share the 1933 Nobel Prize in Physics for this work, alongside Dirac.

[4]See also eq. (32) of Schrödinger (1926e).

An immediate question was the physical status of the wave function. Schrödinger wavered on this question. In the first *Eigenwertproblem* paper, Schrödinger wrote:[5]

> It is, of course, strongly suggested that we should try to connect the function ψ with some *vibration process* in the atom, which would more nearly approach reality than the electronic orbits, the real existence of which is being very much questioned today. [Schrödinger 1926a, translated from the German to English by Shearer and Deans, emphasis in the original]

In the second *Eigenwertproblem* paper, Schrödinger referred explicitly to ψ "as a physical quantity" ("*physikalische Größe*"), and added that, as a consequence of its physicality, "the function ψ must be single-valued, finite, and continuous throughout configuration space" (Schrödinger 1926b).

As was transparently evident from the Schrödinger equation (13), Schrödinger's wave function was complex-valued, rather than real-valued, as Schrödinger complained in a letter to Hendrik Lorentz, dated June 6, 1926:

> What is unpleasant here, and indeed directly to be objected to, is the use of complex numbers. ψ is surely fundamentally a real function [...]." [Schrödinger 1963, translated from the German to English by M. J. Klein]

In the fourth *Eigenwertproblem* paper, Schrödinger wrote:

> Meantime, there is no doubt a certain crudeness in the use of a *complex* wave function. If it were unavoidable *in principle*, and not merely a facilitation of the calculation, this would mean that there are in principle *two* wave functions, which must be used *together* in order to obtain information on the state of the system. This somewhat unacceptable inference admits, I believe, of the very much more congenial interpretation that the state of the system is given by a real function and its time-derivative. Our inability to give more accurate information about this is intimately connected with the fact that, in the pair of equations ($4''$), we have before us only the *substitute*—extraordinarily convenient for the calculation, to be sure—for a real wave equation of probably the fourth order, which, however, I have not succeeded in forming for the non-conservative case. [Schrödinger 1926d, translated from the German to English, emphasis in the original]

Schrödinger was also troubled by the fact that his wave function $\psi(q,t)$ was generically defined not in three-dimensional physical space, but in the abstract, many-dimensional configuration space, or "q-space," of the system in question. Hendrik Lorentz raised this concern in a letter that he sent to Schrödinger, dated May 27, 1926:

> If I had to choose now between your wave mechanics and the matrix mechanics [of Born, Heisenberg, and Jordan], I would give the preference to the former, because of its

[5] All English translations of Schrödinger's foundational papers on wave mechanics here are due to J. F. Shearer and W. M. Deans (1928).

greater intuitive clarity, so long as one only has to deal with the three coordinates x, y, z. If, however, there are more degrees of freedom, then I cannot interpret the waves and vibrations physically, and I must therefore decide in favor of matrix mechanics. But your way of thinking has the advantage for this case too that it brings us closer to the real solution of the equations; the eigenvalue problem is the same in principle for a higher dimensional q-space as it is for three dimensional space. [Schrödinger 1963, translated from the German to English by M. J. Klein]

Schrödinger himself thought extensively about this issue as well. In a separate paper published in 1926, titled "*Über das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen*" ("On the Relationship Between Heisenberg-Born-Jordan Quantum Mechanics and My Own," Schrödinger 1926f), Schrödinger wrote:

> The difficulty in regard to the problem of *several* electrons, which mainly lies in the fact that ψ is a function in *configuration space*, not in real space, must be mentioned. [Schrödinger 1926f, translated from the German to English, emphasis in the original]

One saw further concern from Schrödinger in a paper that he published in English at the end of 1926, titled "An Undulatory Theory of the Mechanics of Atoms and Molecules":

> This [the quantity $\psi\bar{\psi}$] of course, like ψ itself, is in the general case a function of the generalized coordinates $q_1, \ldots, q_N$ and the time,—not a function of ordinary space and time as in ordinary wave-problems. This raises some difficulty in attaching a physical meaning to the wave-function. [Schrödinger 1926e]

In his June 6, 1926 reply to the May 27 letter from Lorentz mentioned above, Schrödinger expressed a more elaborate interpretation of his configuration-space wave function:

> You mention the difficulty of projecting the waves in q-space, when there are more than three coordinates, into ordinary three dimensional space and of interpreting them physically there. I have been very sensitive to this difficulty for a long time but believe that I have now overcome it. I believe (and I have worked it out at the end of the third article) that the physical meaning belongs not to the quantity itself but rather to a *quadratic* function of it. *There* I chose the real part of $\psi\bar{\psi}$, where ψ is taken to be complex in the obvious way (for criticism, see below) and the bar denotes the complex conjugate. *Now* I want to choose more simply $\psi\bar{\psi}$, that is, the square of the absolute value of the quantity ψ. If we now have to deal with N particles, then $\psi\bar{\psi}$ (just as ψ itself) is a function of $3N$ variables or, as I want to say, of N three dimensional spaces, $R_1, R_2, \ldots, R_N$. Now first let R_1 be identified with the real space and integrate $\psi\bar{\psi}$ over $R_2, \ldots, R_N$; second, identify R_2 with the real space and integrate over $R_1, R_3, \ldots, R_N$; and so on. The N individual results are to be added after they have been multiplied by certain constants which characterize the particles (their charges, according to the former theory). I consider the result to be the electric charge density

> in real space. In this manner one obtains for an atom with many electrons exactly what Born-Heisenberg-Jordan designate as the transition probability, with the new and plausible meaning "component of the electric moment" (strictly speaking *that* partial moment which oscillates with the *emission* frequency in question). [Schrödinger 1963, translated from the German to English by M. J. Klein, emphasis in the original]

One finds the clearest and most extensive statement of Schrödinger's early views on his wave function in the fourth *Eigenwertproblem* paper from 1926. This statement is likewise worth quoting in full:

> This rule is now equivalent to the following conception, which allows the true meaning of ψ to stand out more clearly. $\psi\bar{\psi}$ is a kind of *weight-function* in the system's configuration space. The *wave-mechanical* configuration of the system is a *superposition* of many, strictly speaking of *all*, point-mechanical configurations kinematically possible. Thus, each point-mechanical configuration contributes to the true wave-mechanical configuration with a certain *weight*, which is given precisely by $\psi\bar{\psi}$. If we like paradoxes, we may say that the system exists, as it were, simultaneously in all the positions kinematically imaginable, but not "equally strongly" in all.[6] In macroscopic motions, the weight-function is practically concentrated in a small region of positions, which are practically indistinguishable. The centre of gravity of this region in configuration space travels over distances which are macroscopically perceptible. In problems of microscopic motions, we are in any case interested *also*, and in certain cases even *mainly*, in the varying *distribution* over the region.
>
> This new interpretation may shock us at first glance, since we have often previously spoken in such an intuitive concrete way of the "ψ-vibrations" as though of something quite real. But there is something tangibly real behind the present conception also, namely, the very real electrodynamically effective fluctuations of the electric space-density. The ψ-function is to do no more and no less than permit of the totality of these fluctuations being mastered and surveyed mathematically by a single partial differential equation. We have repeatedly called attention to the fact that the ψ-function itself cannot and may not be interpreted directly in terms of three-dimensional space—however much the one-electron problem tends to mislead us on this point—because it is in general a function in configuration space, not real space. [Schrödinger 1926d, translated from the German to English, emphasis in the original]

Schrödinger's configuration-space wave function did not sit well with Albert Einstein. In a series of letters to Hendrik Lorentz, Paul Ehrenfest, and Arnold Sommerfeld through 1926 and 1927 (Howard 1990), Einstein expressed his dismay and his disbelief repeatedly:

- "Schrödinger's conception of the quantum rules makes a great impression on me; it seems to me to be a bit of reality, however unclear the sense of waves in n-dimensional q-space remains." [In a letter dated May 1, 1926, to Lorentz]

[6] Note the proto-Everettian "many worlds" interpretation here. The present work will return to the connection between Schrödinger and Everett in Section 3.

- "Schrödinger's works are wonderful—but even so one nevertheless hardly comes closer to a real understanding. The field in a many-dimensional coordinate space does not smell like something real." [In a letter dated June 18, 1926, to Ehrenfest]

- "The method of Schrödinger seems indeed more correctly conceived than that of Heisenberg, and yet it is hard to place a function in coordinate space and view it as an equivalent for a motion. But if one could succeed in doing something similar in four-dimensional space, then it would be more satisfying." [In a letter dated June 22, 1926, to Lorentz again]

- "Of the new attempts to obtain a deeper formulation of the quantum laws, that by Schrödinger pleases me most. If only the undulatory fields introduced there could be transplanted from the n-dimensional coordinate space to the 3 or 4 dimensional! The Heisenberg-Dirac theories compel my admiration, but to me they don't smell like reality." [In a letter dated August 21, 1926, to Sommerfeld]

- "Admiringly—mistrustfully I stand opposed to quantum mechanics. I do not understand Dirac at all in points of detail (Compton effect) . . . . Schrödinger is, in the beginning, very captivating. But the waves in n-dimensional coordinate space are indigestible, as well as the absence of any understanding of the frequency of the emitted light." [In a letter dated August 28, 1926, to Ehrenfest again]

- "The quantum theory has been completely Schrödingerized and has much practical success from that. But this can nevertheless not be the description of a real process. It is a mystery." [In a letter dated February 16, 1927, to Lorentz again]

Of all these letters, the most famous was surely Einstein's "God does not play dice" letter to Max Born, dated December 4, 1926:[7]

> Quantum mechanics is very awe-inspiring. But an inner voice tells me that this is not the real Jacob after all.[8] The theory delivers a lot, but to the secrets of the Old One it brings us hardly any closer. Anyway I am convinced that *He* does not roll dice. Waves in $3n$-dimensional space[9] whose speed through potential energy (e.g. rubber bands) is regulated... [Translated from the German to English by the author of the present work, emphasis in the original]

[7] In the original German:

> Die Quantenmechanik ist sehr achtung-gebietend. Aber eine innere Stimme sagt mir, daß das doch nicht der wahre Jakob ist. Die Theorie liefert viel, aber dem Geheimnis des Alten bringt sie uns kaum näher. Jedenfalls bin ich überzeugt, daß *der* nicht würfelt. Wellen im $3n$-dimensionalen Raum, deren Geschwindigkeit durch potentielle Energie (z. B. Gummibänder) reguliert wird...

[8] *Der Wahre Jacob* was the title of a German satirical magazine in circulation at various points from 1879 to 1933. Its title may have been inspired by the Biblical story of Jacob and Esau.

[9] In the canonical English translation by Irene Born (1971), the n is missing in "$3n$-dimensional space" ("$3n$-*dimensionalen Raum*"), obscuring Einstein's concerns about the physical reality of waves in a many-dimensional configuration space.

De Broglie was not any more amenable to the idea of ontological wave functions propagating in configuration spaces than Einstein was. In the 1926 paper discussed earlier, "*Les Principes de la Nouvelle Mécanique Ondulatoire*," de Broglie explicitly mentioned Schrödinger's recent work:

> But what happens if the dynamical phenomenon runs its course in a domain of the order of a wave-length, as in the case of the atom? Schrödinger thinks that then we can no longer speak of a material particle describing a trajectory. [...] This is a difficult problem, which we shall meet with in various forms farther on. [De Broglie 1926, translated from the French to English by W. M. Deans]

De Broglie then described Schrödinger's view, on which each ensemble of N particles should be described by a wave function propagating in the system's $3N$-dimensional configuration space, rather than in three-dimensional physical space:

> Schrödinger has leanings toward this latter opinion, and, in order to establish the dynamics of systems, he generalizes my idea of phase-waves: with the motion of the system of N moving bodies he associates a wave in the space of $3N$ dimensions imagined by the classical theories in order to represent the motion of the whole system by the displacement of a single representative point. This wave associated with the system of N particles would thus be a function of $3N$ space-coordinates and of the time. [Ibid.]

Continuing, de Broglie expressly rejected the ontological reality of Schrödinger's wave function:

> Up to the present I have been unable to accept this point of view; for me the associated waves have physical reality and must be expressed as functions of the three space-co-ordinates and of the time. I cannot dwell further on this difficult question; it is obvious, from the above, that the wave dynamics of systems does not seem to be solidly established as yet. [Ibid.]

At the end of a series of lectures on wave mechanics that Schrödinger delivered in English at the Royal Institution in London in March 1928, he recapitulated his 1926 interpretation of the wave function in Section 15 ("The physical meaning of the generalized ψ-function"):

> Perhaps the latter conclusions are obscured by the fact that we have hitherto avoided putting forward any definite assumption as to the physical interpretation of the function $\psi(q_1, q_2, \ldots, q_n, t)$ relating to a system whose configuration in terms of ordinary mechanics is described by the generalized co-ordinates $q_1, q_2, \ldots, q_n$. This interpretation *is* a very delicate question. As an obvious generalization of the procedure of spreading out the electronic charge according to a relative density function $\psi\bar{\psi}$ (which furnished satisfactory results in the one-electron problem; see section 5), the following view would present itself in the case of a general mechanical system: the real natural system does not behave like the picture which ordinary mechanics forms of it (e.g., a system of point-charges in a definite configuration), but rather behaves like what would be the result

of spreading out the system, described by $q_1, \dots, q_n$, *throughout its configuration-space* in accordance with a relative density function $\psi\bar{\psi}$. This would mean that, if the ordinary mechanical picture is to be made use of at all, the actual system behaves like the ordinary mechanical picture, present in all its possible configurations at the same time, though "stronger" in some of them than in others.[10] [Schrödinger 1928, emphasis in the original]

However, Schrödinger then immediately described having had a change of heart:

> I maintained this view for some time. The fact that it proves *very* useful can be seen from the one-electron problem (see section 5). *No* other interpretation of the ψ-function is capable of making us *understand* the large amount of information which the constants a_{kl} furnish about the intensity and polarization of the radiation. Yet this way of putting the matter is surely not quite satisfactory. For what does the expression "to behave like" mean in the preceding sentences? The "behavior" of the ψ-function, i.e. its development in time, is governed by nothing like the laws of classical mechanics; it is governed by the wave-equation.— [Ibid., emphasis in the original]

Schrödinger then mentioned Max Born's statistical interpretation, which Born had included in a paper that he had originally submitted for publication on June 25, 1926 (Born 1926):[11]

> An obvious *statistical* interpretation of the ψ-function has been put forward, viz. that it does not relate to a single system at all but to an assemblage of systems, $\psi\bar{\psi}$ determining the fraction of the systems which happen to be in a definite configuration. This view is a little unsatisfactory, since it offers no explanation whatever why the quantities a_{kl} yield all the information which they do yield. [Ibid., emphasis in the original][12]

It is worth also briefly mentioning Paul Dirac's early views on the wave function, about which he said very little. An exception was the first chapter of the first edition of his 1930 textbook,

[10]Notice again the proto-Everettian "many worlds" interpretation that Schrödinger presented here.

[11]As is well known, in the main text of the paper, Born erroneously identified the statistical probability with the wave function itself. In a footnote that he added during the correction process, he wrote that, on further reflection, it was not the wave function itself, but its (modulus) square, that should be interpreted as the relevant probability. Born would go on to win the 1954 Nobel Prize in Physics for this realization.

[12]In the following portion of that lecture, Schrödinger also foreshadowed John Bell's 1990 article "Against 'Measurement' " (Bell 1990) with a criticism of the vague language of measurement:

> In connexion with the statistical interpretation it has been said that to any physical quantity which would have a definite physical meaning and be in principle (*principiell*) measurable according to the classical picture of the atom, there belong definite proper values (just as e.g. the proper values E_k, belong to the energy); and it has been said that the result of measuring such a quantity will always be one or the other of these proper values, but never anything intermediate. It seems to me that this statement contains a rather vague conception, namely that of *measuring* a quantity (e.g. energy or moment of momentum), which relates to the classical picture of the atom, i.e. to an obviously wrong one. [Schrödinger 1928, emphasis in the original]

Principles of Quantum Mechanics, in which Dirac took an explicitly instrumentalist view that dismissed regarding the wave function as ontological:[13]

> To obtain a consistent theory of light which shall include interference and diffraction phenomena, we must consider the photons as being controlled by waves, in some way which cannot be understood from the point of view of ordinary mechanics. This intimate connexion between waves and particles is of very great generality in the new quantum mechanics. It occurs not only in the case of light. *All* particles are connected in this way with waves, which control them and give rise to interference and diffraction phenomena under suitable conditions. The influence of the waves on the motion of the particles is less noticeable the more massive the particles and only in the case of photons, the lightest of all particles, is it easily demonstrated.
>
> The waves and particles should be regarded as two abstractions which are useful for describing the same physical reality. One must not picture this reality as containing both the waves and particles together and try to construct a mechanism, acting according to classical laws, which shall correctly describe their connexion and account for the motion of the particles. Any such attempt would be quite opposed to the principles by which modern physics advances. What quantum mechanics does is to try to formulate the underlying laws in such a way that one can determine from them without ambiguity what will happen under any given experimental conditions. It would be useless and meaningless to attempt to go more deeply into the relations between waves and particles than is required for this purpose.
>
> [...]
>
> The reader may, perhaps, feel that we have not really solved the difficulty of the conflict between the waves and the corpuscles, but have merely talked about it in a certain way and, by using some of the concepts of waves and some of corpuscles, have arrived at a formal account of the phenomena, which does not really tell us anything that we did not know before. The difficulty of the conflict between the waves and corpuscles is, however, actually solved as soon as one can give an unambiguous answer to any experimental question. *The only object of theoretical physics is to calculate results that can be compared with experiment*, and it is quite unnecessary that any satisfying description of the whole course of the phenomena should be given. [Dirac 1930, emphasis in the original]

2.3 De Broglie's Pilot-Wave Theory

De Broglie sought a notion of particles that made sense even beyond the geometrical-optics limit. In a 1927 paper, titled "*La Mécanique Ondulatoire et la Structure Atomique de la Matière et du Rayonnement*" ("Wave Mechanics and the Atomic Structure of Matter and Radiation," de Broglie

[13] Although Dirac rewrote this part of his book in later editions, he did not present a conceptually or philosophically different view in those later editions.

1927), de Broglie initiated a different research direction based on what he called "the principle of the double solution" ("*principe de la double solution*"). This new approach featured a *pair* of waves, both propagating in three-dimensional physical space, and that both satisfied the same appropriately specified wave equation. One of these waves was intended to have a singularity in it representing a particle of matter, and the other was continuous and represented the flow of probability. As a matter of history, it was this paper that first featured the guiding equation in its more modern-looking form (8), as de Broglie's eq. (26′),

$$\overrightarrow{v_M} = \frac{1}{m_0}\overrightarrow{\mathrm{grad}\varphi_1}, \tag{14}$$

where $\overrightarrow{v_M}$ was the velocity of the matter particle, m_0 was its inertial mass, and φ_1 was the phase function of de Broglie's singularity-bearing wave. De Broglie would put his double-solution theory aside for many years, before returning to it in the 1950s, a story that he retold in the very first article published in the newly founded journal *Foundations of Physics* in 1970, titled "The Reinterpretation of Wave Mechanics" (de Broglie 1970).

In the meantime, de Broglie laid out a conceptually distinct pilot-wave theory in his 1930 book *Introduction à l'Étude de la Mécanique Ondulatoire* (*An Introduction to the Study of Wave Mechanics*, de Broglie 1930). De Broglie's 1930 book made no mention of his double-solution theory, and only glancingly mentioned his earlier supposition that particles of matter were singularities in phase waves.

In the introduction to the book, de Broglie laid out his detailed views on his understanding of wave-particle duality. He began by describing why he could not support the notion that particles were merely wave packets:[14]

> But what does this duality of waves and particles mean? This is a very difficult question and one which is still far from being very clearly elucidated.
>
> The simplest idea is that which Schrödinger put forward at the beginning of his work, viz. that the particle or the electron is constituted by a group of waves; it is a "wave packet." We have seen that this can be maintained so long as mechanical phenomena are considered which are in harmony with the old dynamics, that is to say in the new language, phenomena in which the propagation of the associated wave obeys the laws of geometrical optics. Unfortunately, when we pass to the domain proper of the new theory it appears scarcely possible to support this idea which is so attractive on account of its simplicity. In an experiment like that of the diffraction of an electron by a crystal the wave packet would be completely dispersed and destroyed; as a result no particles would be found in the diffracted bundles. In other words, if they were simple wave packets the particles would have no stable existence. [De Broglie 1930, General Introduction]

De Broglie then clarified his earlier view that particles could be regarded as singularities in his phase waves, before explaining why he no longer found that view tenable, either, and would not mention it again in the book:

[14] All English translations of de Broglie's 1930 book here are due to H. T. Flint (1930).

> If it appears impossible to maintain Schrödinger's view in all its consequences, neither is it easy to develop another opinion with which the author has for a long time associated himself and according to which the particle is a singularity in a wave phenomenon. In the special case of the uniform motion of a particle it is possible to find a solution of the wave equation showing a moving singularity and capable of representing the particle. But it is very difficult to make the generalisation to the case of non-uniform motion, and there are serious objections to this point of view; we shall not discuss this difficulty any further in this volume. [Ibid.]

De Broglie then turned to his new pilot-wave theory, which he first explicitly presented at the fifth Solvay conference in 1927, and which featured a pilot wave that guided his particles ("*On peut donc concevoir le corpuscule comme guidé par l'onde qui joue le rôle d'onde-pilote*"):

> The author has also made another suggestion which is published in his report to the Fifth Solvay Congress. We have seen that we must always associate a wave with a particle, and so the idea which is in best agreement with the older views of physics is to consider the wave as a reality and as occupying a certain region of space, while the particle is regarded as a material point having a definite position in the wave. This is the basis of the suggestion. Since it is necessary, as we have said above, that the intensity of the wave should be proportional at each point to the probability of the occurrence of the particle at that point, we must attempt to connect the motion of the particle with the propagation of the wave so that this relation is automatically realised in every case. It is, in fact, actually possible to establish a connection between the motion of the particle and the propagation of the wave, so that if at an initial instant the intensity of the wave measures the required probability the same is true at all later instants. We may thus suppose that the particle is guided by the wave which plays the part of a pilot-wave. This view permits of an interesting visualization of the corpuscular motion in wave mechanics without too wide a departure from classical ideas. [Ibid.]

However, de Broglie then previewed his concerns with his pilot-wave idea:

> Unfortunately, we encounter very serious objections to this view also, and these will be pointed out in the course of the book. It is not possible to regard the theory of the pilot-wave as satisfactory. Nevertheless, since the equations on which this theory rests are sound, we may preserve some of its consequences by giving to it a modified form in agreement with ideas developed independently by Kennard. Instead of speaking of the motion and of the trajectory of the particles, we speak of the motion and of the trajectory of the "elements of probability" and in this way the difficulties noted are avoided. [Ibid., with reference to a paper by Kennard (1928)]

De Broglie then pointed out that the 'Copenhagen' interpretation associated with Heisenberg and Bohr rejected an ontological view of the wave function:

> Finally, there is a fourth point of view developed by Heisenberg and Bohr which is most favoured at present. This point of view is a little disconcerting at first sight, but yet it appears to contain a large body of truth. According to this view, the wave does not represent a physical phenomenon taking place in a region of space; rather it is simply a symbolic representation of what we know about the particle.

Putting all these concerns aside, in Section IX.V ("*La théorie de l'onde-pilote*"), de Broglie posited that the wave function itself, obeying a wave equation like the Schrödinger equation (12), acted as a pilot wave that determined the velocities of de Broglie's particles according to a guiding equation that generalized (8) and (14):

> If now we still wish to retain the classical conception of the particle in the domain proper of the new mechanics, that is to say, outside the approximation to geometrical optics, we naturally wish to maintain the identity of the particle with one of the probability elements and to represent the state of affairs by imagining on the one hand the wave, and on the other the particle to be localised in space, and we connect the motion of the particle with the propagation of the wave by the relation:[15]
>
> $$\vec{v} = -\frac{1}{m}\mathrm{grad}\varphi \qquad \text{[de Broglie's eq. (IX.38)]}, \tag{15}$$
>
> where φ is the phase of the wave defined by [de Broglie's] (32). The velocity of the particle is thus determined at each instant if we know the initial position, and thus its path also is determined. Moreover, from the formulae of the first paragraph, if we know the form of the associated ψ-wave and if we know that initially the probability of occurrence of the particle at a point is equal to the intensity of the wave at the point, it will be so automatically at every succeeding instant; thus the principle of interference will be satisfied. We may describe this theory as the pilot-wave theory, because we imagine the wave as guiding the motion of the particle. [De Broglie 1930, Chapter IX, Section 5]

Despite occasional comments to the contrary in the research literature (for example, Dewdney et al. 1992), de Broglie generalized his pilot-wave theory to the multi-particle case in his 1930 book, as laid out in his Chapters XIV ("Wave Mechanics of Systems of Particles") and XV ("The Interpretation of the Wave Associated with the Motion of a System"). In particular, in addressing the multi-particle case, de Broglie began referring explicitly to the system's configuration space ("*l'espace de configuration*"), which Flint translated into English as "generalised space":

> Here, as in the case of the single particle, we might be tempted to develop a pilot-wave theory, at the same time maintaining that the particles of the system have a definite position in space and that, consequently, the representative point has also a definite

[15] Note that de Broglie chose the negative sign convention in his version of the Schrödinger equation (12), which de Broglie wrote as $\Delta\Psi - (8\pi^2 m/h^2)F\Psi = (4\pi i m/h)\partial\Psi/\partial t$ in his eq. (IX.1). This sign convention led to an additional minus sign in his guiding equation $\vec{v} = -(1/m)\mathrm{grad}\varphi$ in his eqs. (IX.5) and (IX.38).

position at each instant in the generalised [i.e., configuration] space. We should then admit that the representative point associated with a ψ-wave has the motion defined by (15) [de Broglie's guiding equation], coinciding always with one of the probability elements. [De Broglie 1930, Chapter XV, Section 4]

Even as he was introducing his pilot-wave theory, however, de Broglie was already expressing doubts about the reality of the pilot wave as propagating in a configuration space, in contrast to waves propagating in ordinary space ("*l'espace ordinaire*"):

> Moreover, we have, in agreement with Schrödinger's work, considered wave propagation in the fictitious generalised [i.e., configuration] space. It has not yet been possible to connect this propagation of a single wave in a fictitious space with that of one or several waves in ordinary space.
>
> This impossibility seems to strengthen the view that no physical reality is to be attached to the associated wave, but that it is simply a symbolic representation of probability. [De Broglie 1930, Chapter XIV ("Wave Mechanics of Systems of Particles"), Section 4]

De Broglie returned to these concerns about multi-particle systems later in the book:

> Here, as in the case of the single particle, we might be tempted to develop a pilot-wave theory, at the same time maintaining that the particles of the system have a definite position in space and that, consequently, the representative point has also a definite position at each instant in the generalised [configuration] space. [...] Unfortunately all the difficulties arise once more which we discovered in the pilot-wave theory of a [single] particle, and it is even more difficult to consider the theory as offering an actual physical picture of the phenomena because of the abstract and fictitious character of the propagation of a wave in generalised [configuration] space. [De Broglie 1930, Chapter XV ("The Interpretation of the Wave Associated with the Motion of a System"), Section 4]

De Broglie echoed these concerns many years later, in a letter to Wolfgang Pauli dated February 10, 1952:

> I became persuaded, upon reflection, of the ineffectiveness of the pilot-wave theory as I had presented it to the Solvay Conference because it has the particle "guided" by a wave ψ that is merely the representation of a probability and whose fictitious character is evident (propagation in configuration space, in particular). [Pauli 1996, parenthetical statement in the original, in literal English translation from the original French by the author of the present work]

To see how de Broglie's construction worked in more modern notation for a system of N particles with configuration q consisting of $n = 3N$ degrees of freedom $q_1, \dots, q_n$, it will be useful to write the Schrödinger equation for the configuration-space wave function $\Psi(q,t)$ in the more schematic form[16]

$$i\,\partial_t\Psi = -\Delta\Psi + V\Psi, \tag{16}$$

where $\partial_t = \partial/\partial t$ is the partial derivative with respect to the time t, and where factors of $\hbar$ have been suppressed by a suitable choice of measurement units. Suppose that the second-order differential operator Δ appearing in the Schrödinger equation (16) is expressible in the schematic form

$$\Delta = \sum_{i,j} \frac{1}{2}\partial_i(\mu_{ij}\partial_j), \tag{17}$$

where $\mu_{ij} = \mu_{ji}$ is a symmetric array of real-valued functions of the coordinates (assumed for simplicity to have unit determinant), and where $\partial_i = \partial/\partial q_i$ and $\partial_j = \partial/\partial q_j$ are differential operators acting along the directions of the respective coordinates q_i and q_j. Writing the complex-valued wave function in polar form in terms of a real-valued radial function $R(q,t)$ and a real-valued phase function $\theta(q,t)$,

$$\Psi(q,t) = R(q,t)e^{i\theta(q,t)}, \tag{18}$$

one can define a guiding equation to consist of the statement that the velocities $\dot{Q}_i(t)$ of the particles are given by the gradients $\partial_j\theta$ of the phase function, evaluated at the system's actual instantaneous configuration $Q(t) = (Q_1(t),\ldots,Q_n(t))$ at the time t:[17]

$$\dot{Q}_i(t) = \sum_j \mu_{ij}\partial_j\theta\Big|_{Q(t)}, \tag{19}$$

where dots denote time derivatives. Anticipating the Born rule, which states that the probability density $\rho(q,t)$ for the system to be found in its configuration q at the time t should be given by the modulus-square of the wave function (Born 1926),

$$\rho(q,t) = \Psi(q,t)\overline{\Psi(q,t)} = |\Psi(q,t)|^2 = R^2(q,t), \tag{20}$$

where bars denote complex-conjugation, it follows from a simple calculation that the following continuity equation holds:

$$\partial_t\rho = -\sum_i \partial_i J_i. \tag{21}$$

Here $J_i(q,t)$ is a probability current density given by

$$J_i = \rho\sum_j \mu_{ij}\partial_j\theta. \tag{22}$$

[16]Note again that de Broglie chose the negative sign convention in his version of the Schrödinger equation (12), leading to an additional minus sign in his own version of the guiding equation in his 1930 book.

[17]Compare this equation for $\dot{q}_i$ with de Broglie's eq. (XV.15), which he wrote as $q_i' = -\sum_k \mu^{ik}\partial\varphi/\partial q_k$ (de Broglie 1930).

One can then re-express the guiding equation (19) as the statement that the velocity $\dot{Q}_i(t)$ is equal to the ratio of the corresponding probability current density J_i and the probability density ρ, evaluated at the system's actual instantaneous configuration $Q(t) = (Q_1(t), \ldots, Q_n(t))$ at the time t:

$$\dot{Q}_i(t) = \left.\frac{J_i}{\rho}\right|_{Q(t)} = \frac{J_i(Q(t), t)}{\rho(Q(t), t)}. \tag{23}$$

There are never divide-by-zero problems here because the system's configuration $Q(t)$ has, by definition, zero probability of being located in a region of its configuration space where the probability density ρ is zero. If the system gets close to a region of its configuration space with very small probability density, then its velocities will become highly unstable, driving the system away from that region.

Collectively, these equations ensure a crucial feature of the pilot-wave theory, known today as 'equivariance,' and which de Broglie originally called "the principle of interference" ("*le principe des interférences*"). Equivariance is the feature that if $\rho(q, t_0) = |\Psi(q, t_0)|^2$ is the correct probability density at an initial time t_0, a condition known today as the 'quantum equilibrium hypothesis,' then $\rho(q, t) = |\Psi(q, t)|^2$ will continue to be the correct probability density at all later times t.

2.4 Bohmian Mechanics

In 1951, David Bohm published a new textbook on quantum mechanics, titled simply *Quantum Theory* (Bohm 1951). Chapter 22, "Quantum Theory of the Measurement Process," presented a new approach to the measurement problem. In particular, in Section 22.8, "Destruction of Interference in the Process of Measurement," Bohm developed an early version of the theory of decoherence. However, in conversations shortly thereafter, Albert Einstein persuaded Bohm that decoherence alone was insufficient for solving the measurement problem.

In 1952, Bohm independently introduced essentially the same pilot-wave theory as in de Broglie's 1930 book, in a pair of papers both titled "A Suggested Interpretation of the Quantum Theory in terms of 'Hidden Variables' " (Bohm 1952a,b). In the acknowledgments section of the first of these two papers, Bohm wrote, simply, "The author wishes to thank Dr. Einstein for several interesting and stimulating discussions." (Bohm did not include an acknowledgments section in his second "Suggested Interpretation" paper.)

At Wolfgang Pauli's prompting,[18] Bohm clarified de Broglie's priority in the first "Suggested Interpretation" paper:

> After this article was completed, the author's attention was called to similar proposals for an alternative interpretation of the quantum theory made by de Broglie in 1926, but later given up by him partly as a result of certain criticisms made by Pauli and partly because of additional objections raised by de Broglie himself. [Bohm 1952a]

Bohm conceded de Broglie's priority explicitly in a letter to Pauli in the summer of 1951:

[18] From a letter to Pauli dated November 20, 1951: "I have changed the introduction of my paper so as to give due credit to de Broglie, and have stated that he gave up the theory too soon (as suggested in your letter)." (Pauli 1996)

> With regard to de Broglie, I am ready to admit that he thought of the idea first. However, the essential point is not the credit for who got this idea first. The fact is that de Broglie came to the erroneous conclusion that the idea does not work. My main contention is that de Broglie did not carry his ideas to a logical conclusion, and that if he had, he would have seen that they led to precisely the same results as are required experimentally and as are obtained from the usual interpretations. [Pauli 1996]

In a subsequent letter to Pauli in October of 1951, Bohm revisited the question of priority once again, with a bit of additional metaphorical flair:

> You refer to this interpretation as de Broglie's. It is true that he suggested it first, but he gave it up because he came to the erroneous conclusion that it does not work. The essentially new point that I have added is to show in detail (especially by working out the theory of measurements in paper II) that this interpretation leads to all of the results of the usual interpretation. [...] If one man finds a diamond and then throws it away because he falsely concludes that it is a valueless stone, and if this stone is later found by another man who recognizes its true value, would you not say that the stone belongs to the second man? I think the same applies to this interpretation of the quantum theory. [Ibid.]

Bohm's work on decoherence ended up being crucial for clearing up various subtleties in the pilot-wave theory's approach to resolving the measurement problem, as Bohm explained, in particular, in Section 2 and in Appendix B of the second of his 1952 "Suggested Interpretation" papers:

> As we shall show in Appendix B of Paper II, however, all of the objections of de Broglie and Pauli could have been met if only de Broglie had carried his ideas to their logical conclusion. The essential new step in doing this is to apply our interpretation in the theory of the measurement process itself as well as in the description of the observer system. [Bohm 1952a]

Today this pilot-wave theory is called "the de Broglie-Bohm theory," or "Bohmian mechanics."

Prompted by Einstein and Pauli (Drezet 2023), Bohm contacted de Broglie in 1951 to discuss his theory. De Broglie quickly published a short article that same year, titled "*Remarques sur la Théorie de l'Onde Pilote*" ("Remarks on the Pilot-Wave Theory, de Broglie 1951), explaining why he had abandoned the pilot-wave theory laid out in his 1930 *Introduction à l'Étude* book. De Broglie wrote:

> On the other hand, the pilot-wave theory only veritably attained its aim, which is a return to an interpretation of wave mechanics conforming with classical conceptions, if the wave Ψ, from which the quantum potential is derived, is a "physical reality" capable of acting on the movement of the particle or the system, for if it is merely a representation of the probabilities, as is generally admitted today, the movement defined by the pilot-wave theory would depend on possibilities that are not realized, which is

> paradoxical and takes us completely away from classical conceptions. However, it seems impossible to consider the wave Ψ as a physical reality. It is, in effect, represented by an essentially complex function and, in the case of general systems, it propagates in configuration space, which is clearly abstract and fictitious ["*l'espace de configuration qui est visiblement abstrait et fictif*"]. [De Broglie 1951, in literal English translation from the original French by the author of the present work]

De Broglie did not mince words, concluding:

> These considerations and others like them seem to be absolutely opposed to attributing to the wave Ψ the character of physical reality, and this still seems to me to be the strongest objection against the pilot-wave theory. [...] In summary, the interpretation of wave mechanics by the pilot-wave theory, to which Mr. Bohm's theory brings attention, still seems to me to encounter insurmountable difficulties, principally for the reason of the impossibility of attributing to the wave Ψ a physical reality or of admitting that the motion of a particle is determined by possible motions that are not realized. [Ibid.]

Despite his negative view toward his pilot-wave theory, de Broglie used the final parts of this article to explain why the pilot-wave theory evaded von Neumann's famous no-go theorem (von Neumann 1932) against the existence of "hidden parameters" or "hidden variables," which Bohm took to refer to his particles. Bohm also addressed von Neumann's no-go theorem himself in a letter to Pauli in October 1951:[19]

> Let us now discuss von Neumann's proof that quantum theory is inconsistent with hidden causal parameters. His proof involves the demonstration that no "dispersionless" states can exist in the quantum theory, so that no *single* distribution of hidden parameters could possibly determine the results of all experiments (including for example, the measurements of momentum and position). However, von Neumann implicitly assumes that the hidden variables are only in the observed system and not in the measuring apparatus. On the other hand, in my interpretation, the hidden variables are in *both* the measuring apparatus and the observed system. Moreover, since different apparatus is needed to measure momentum and position, the actual results in each respective type of measurement are determined by different distributions of hidden parameters. Thus, von Neumann's proof is irrelevant to my interpretation.
>
> It appears that von Neumann has agreed that my interpretation is logically consistent and leads to all of the results of the usual interpretation. (This I am told by some people.) Also, he came to a talk given by me and did not raise any objections. [Pauli 1996, emphasis in the original]

Bohm took a strong stand on the ontology of the pilot wave, despite its living in a many-dimensional configuration space. In the first of his 1952 "Suggested Interpretation" papers, received

[19] Famously, Grete Hermann and John Bell also found loopholes in von Neumann's no-go theorem (Hermann 1935; Bell 1966; Crull, Bacciagaluppi 2016).

by the journal *Physical Review* on July 5, 1951, Bohm consistently referred to the wave function as a "ψ-field," in analogy with the electromagnetic field (Bohm 1952a). He also wrote:

> Since the force on a particle now depends on a function of the absolute value, $R(\mathbf{x})$, of the wave function, $\psi(\mathbf{x})$, evaluated at the actual location of the particle, we have effectively been led to regard the wave function of an individual electron as a mathematical representation of an objectively real field. [Ibid.]

However, Bohm had little to say in his two "Suggested Interpretation" papers about the ontological meaning of the wave function in the case of more than one particle, except to say:

> In the two-body problem, the system is described therefore by a six-dimensional Schroedinger wave and by a six-dimensional trajectory, specifying the actual location of each of the two particles. [Ibid.]

Bohm's second "Suggested Interpretation" paper, also received by the journal *Physical Review* on July 5, 1951, concluded with these words:

> Nevertheless, evidence for the existence of individual atoms was ultimately discovered by people who took the atomic hypothesis seriously enough to suppose that individual atoms might actually exist, even though no one had yet observed them. We may have here, perhaps, a close analogy to the usual interpretation of the quantum theory, which avoids considering the possibility that the wave function of an individual system may represent objective reality, because we cannot observe it with the aid of existing experiments and theories. [Bohm 1952b]

In a letter in English to Bohm dated December 3, 1951, Pauli wrote "I think that the fundamental difference of this ψ-function from classical fields is the fact that the former is in a polydimensional space and not in ordinary space-time" (Pauli 1996). To this statement, Bohm wrote back the following reply:

> Since you admit the logical consistency of my point of view, and since you cannot give any arguments showing that it is wrong, it seems to me that your desire to hold on to the usual interpretation can have only one justification; namely, the positivist principle of not postulating constructs that do not correspond to things that can not be observed. This is exactly the principle which caused Mach to reject the reality of atoms, for example, since no one in his day knew how to observe them. As for your objections about the polydimensional character of the ψ function, I have shown in my previous letter that they lead to no inconsistencies and that they provide a perfectly good set of concepts. There is no reason why some aspects of reality should not be polydimensional. This merely means a closer unity between distant systems than we had previously been led to suppose. After all, we must not expect the world at the atomic level to be a precise copy of our large scale experience (as proponents of the usual interpretation

> are so fond of saying). Rather than accept a perfectly logical and definite concept of polydimensional reality that leads to the right results in all known cases, and opens up new mathematical possibilities, you prefer the much more outlandish idea that there is no way to conceive of reality at all at the atomic level. Instead you are willing to restrict your conceptions to results that can be *observed* at the long scale level, even though more detached conceptions are available, which show at least, never [sic] the production of these results *might* be understood causally and continuously. It is hard for me to understand such a point of view on the part of a person who claims that he is not a positivist. [Ibid., emphasis in the original]

Bohm described his formulation of the pilot-wave theory to Einstein, but it does not appear that Einstein found the theory fully convincing. In a 1952 letter to Born, dated May 12, 1952, Einstein wrote:

> Have you noticed that Bohm believes (as de Broglie did, by the way, 25 years ago) that he is able to interpret the quantum theory in deterministic terms? That way seems too cheap to me. But you, of course, can judge this better than I. [Einstein, Born 1971]

Although Pauli was cordial in his letters to Bohm, Pauli was much more caustic in German-language letters to his colleagues. For example, in a letter in German to Markus Fierz, dated January 6, 1952, Pauli wrote:

> He [Bohm] writes me letters like a sectarian priest ["*Sektenpfaff*"], to *convert* me – and specifically to the old pilot-wave theory of de Broglie (1926/27), which he has revived. I did suggest to Bohm that we temporarily suspend our correspondence until he has new results to report; but that has not helped, as letters from him come almost daily, often with penalty postage (he apparently has an unconscious wish to punish me).
>
> [...]
>
> I now fear that Bohm will also find a considerable number of followers among young people[...]
>
> There is naturally no helping a fool ["*Narren*"] like Bohm; but do you not think *that such an argument*, to younger students who want to find their orientation, would make an impression? Can you think of anything the fools could say against it? (There is also the danger that – if I simply remain silent – Bohm will spread that I have, "except for philosophical prejudices," nothing to object against his "theory.") [Pauli 1996, emphasis in the original, in literal English translation from the original German by the author of the present work]

3 Conclusion

These historical debates over the ontological status of the wave function have had a substantial legacy. Indeed, just a few years after Bohm's 1952 "Suggested Interpretation" papers, Hugh Everett

III introduced his own new interpretation of quantum theory (Everett 1956, 1957), an interpretation that took Bohm's ontological stance toward the wave function so seriously that Everett decided that the "universal wave function" was the *only* ontology needed for the universe. Everett himself made that claim quite clear in describing the benefits of his interpretation in his unpublished long-form dissertation in 1956:

> One is thus free to build a conceptual model of the universe, which postulates only the existence of a universal wave function which obeys a linear wave equation. [Everett 1956, Section VI]

Everett reiterated this view in the shorter published version of his interpretation in 1957:

> The wave function is taken as the basic physical entity with *no a priori interpretation.* Interpretation only comes *after* an investigation of the logical structure of the theory. [Everett 1957, emphasis in the original]

Everett's theory, of course, is now widely known as the "many worlds" interpretation, a name later given to the theory by Bryce DeWitt (Everett, DeWitt, Graham 1973).

It is worth highlighting how Everett described Bohm's theory near the end of that long-form dissertation. Notice in the following passage how Everett expressed no qualms whatsoever with Bohm's wave-function realism, and also how Everett made clear that his own theory took the wave function to be the sole ontology of nature:

> Bohm considers ψ to be a real force field acting on a particle which always has a well-defined position and momentum (which are the hidden variables of this theory). The ψ-field satisfying Schrödinger's equation is pictured as somewhat analogous to the electromagnetic field satisfying Maxwell's equations, although for systems of n particles the ψ-field is in a $3n$-dimensional space. With this theory Bohm succeeds in showing that in all actual cases of measurement the best predictions that can be made are those of the usual theory, so that no experiments could ever rule out his interpretation in favor of the ordinary theory. Our main criticism of this view is on the grounds of simplicity – if one desires to hold the view that ψ is a real field then the associated particle is superfluous since, as we have endeavored to illustrate, the pure wave theory is itself satisfactory. [Everett 1956, Section VI.c]

The timing of Everett's work could hardly have been a coincidence—Everett started graduate school in physics at Princeton in 1953. Bohm had until recently been an assistant professor at Princeton, and when Everett arrived, Bohm's textbook and his "Suggested Interpretation" papers were still brand-new. Bohm's 1951 textbook and his two 1952 "Suggested Interpretation" papers were the first three cited works in Everett's 137-page long-form dissertation, which included fewer than two dozen citations altogether. In fact, Bohm's textbook was the *only* pedagogical textbook on quantum mechanics cited by Everett in his dissertation, and Everett's detailed footnotes made clear that he had carefully read Bohm's later chapters on the measurement process and Bohm's original treatment of what would now be called decoherence, all of which played prominent roles in Everett's dissertation.

Schrödinger's writings also clearly had an influence on Everett. Here is another way in which Everett described his interpretation in his long-form dissertation:

> e. *The wave interpretation.* This is the position proposed in the present thesis, in which the wave function itself is held to be the fundamental entity, obeying at all times a deterministic wave equation.
>
> This view also corresponds most closely with that held by Schrodinger. [Everett 1956, Section VI.e]

The last sentence in this passage included a footnote to the first of a pair of 1952 papers by Schrödinger titled "Are There Quantum Jumps?" (Schrödinger 1952a,b). In these two papers, Schrödinger returned to arguing that the waves of quantum theory were physically real, expressed the view that perhaps only waves were needed for a physical description of nature, and explicitly questioned the need for separate probability postulates:

> After this has been recognised, is the probability scheme any longer needed? Has the idea of the mysterious sudden leaps of single electrons not become gratituitous? Is it expedient? The waves are there anyhow, and we are not at a loss to prove it. [Schrödinger 1952a]

How did Schrödinger assuage his earlier concerns about the physical reality of waves propagating in a many-dimensional configuration space? In the second of his two 1952 papers, Schrödinger cited an earlier 1950 paper, titled "What is an Elementary Particle?" (Schrödinger 1950) in which he argued that second quantization had provided an escape hatch:

> A method of dealing with the problem of many particles was indicated in 1926 by the present writer. The method uses waves in many-dimensional space, in a manifold of $3N$ dimensions, N being the number of particles. Deeper insight led to its improvement. The step leading to this improvement is of momentous significance. The many-dimensional treatment has been superseded by so-called second quantization, which is mathematically equivalent to uniting into one three-dimensional formulation the cases $N = 0, 1, 2, 3, ...$ (to infinity) of the many-dimensional treatment. This highly ingenious device includes the so-called new statistics, with which we shall have to deal below in much simpler terms. It is the only precise formulation of the views now held, and the one that is always used. [Ibid.]

It would be an understatement to say that this claim did not settle the controversy over the physical reality of the wave function or quantum state. After all, for second-quantized systems, such as those used in quantum field theory, the quantum state does not go away or become less abstract. At least for bosonic fields, the quantum state retains a configuration-space realization that is now defined in a configuration space whose dimension is continuously infinite.

Debates over the ontological nature of the wave function continue to this day, inspiring approaches to quantum foundations like the Harrigan-Spekkens "ontological models" framework (Harrigan, Spekkens 2010). One of the most prominent recent theorems in quantum foundations, due to Matthew Pusey, Jonathan Barrett, and Terry Rudolph, and known widely as the 'PBR theorem,' was also aimed squarely at the question of the ontological status of the wave function. The original 2012 PBR paper opened its introduction with words that should sound very familiar by this point:

> At the heart of much debate concerning quantum theory lies the quantum state. Does the wavefunction correspond directly to some kind of physical wave? If so, it is an odd kind of wave, as it is defined on an abstract configuration space, rather than the three-dimensional space in which we live. [Pusey, Barrett, Rudolph 2012]

Due to limitations of space, nothing more will be said about these contemporary developments in the present work.

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