# Impossibility of Gauge-Invariant Explanations of the Aharonov–Bohm Effect

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**Abstract**

We present a rigorous no-go theorem that excludes *all* gauge-invariant dynamical explanations of the generalized, time-dependent Aharonov–Bohm (AB) effect. In this generalization, the AB phase is determined by the time-averaged magnetic flux enclosed by the particle's path, $\Delta\phi_{\rm AB} = \frac{e}{\hbar} \cdot \frac{1}{T} \int_0^T \Phi(t)\, dt$. Using a carefully designed protocol in which a constant flux $\Phi$ persists only for a finite interval $\Delta t$ after wavepacket splitting and is switched to zero before detection, we demonstrate that during the phase-generating period all gauge-invariant observables of the electron remain identical to those in a zero-flux experiment. Relativistic causality further prohibits backreaction from the source. Consequently, no gauge-invariant quantity—whether local or nonlocal, field-based or source-mediated—can account for the observed phase, which depends solely on flux history. We also critically examine Healey's holonomy interpretation and show that it, too, fails to explain the time-dependent AB effect, as its static formalism lacks the dynamical structure needed to capture temporal averaging. This result establishes the inescapable physical reality of gauge potentials in quantum theory and clarifies the foundational status of the AB effect as a genuine manifestation of potentials rather than fields.

## 1 Introduction

The Aharonov–Bohm (AB) effect stands as one of the most profound and conceptually challenging phenomena in modern physics. In its magnetic form, a charged particle acquires a measurable phase shift determined by the magnetic flux it encloses, even while traveling entirely through regions of vanishing magnetic field [1, 2, 3, 4, 10, 13, 14, 17]. This result clearly demonstrates the physical significance of electromagnetic potentials in quantum mechanics, challenging the classical notion that only gauge-invariant fields can have observable consequences.

While the static AB effect is well-established, its generalization to time-dependent fluxes has been a subject of continued investigation and debate [15, 16, 19]. Recent work has shown that for time-varying potentials, the AB phase shift is given by the *time-averaged* enclosed magnetic flux:

$$\Delta\phi_{\rm AB} = \frac{e}{\hbar} \cdot \frac{1}{T} \int_0^T \Phi(t)\, dt, \tag{1}$$

where $T$ is the total travel time from emission to detection [6]. This generalized effect reveals a hybrid dependence on both gauge potentials and induced electric fields, yet preserves the core nonlocal character of the original AB phenomenon.

A persistent interpretational challenge has been the attempt to explain the AB effect using only gauge-invariant quantities—whether local electromagnetic fields [5, 11], nonlocal holonomies [9], backreaction with the source [18], or intrinsic electron properties—thereby circumventing the need to ascribe physical reality to gauge potentials. While such approaches may appear attractive, they ultimately fail to account for the global, history-dependent nature of the phase shift, particularly in time-dependent settings.

In this work, we present a rigorous no-go theorem that excludes *all* gauge-invariant dynamical explanations of the generalized AB effect, including Healey's holonomy interpretation. Our analysis centers on a carefully designed time-dependent protocol: after the electron wavepacket is split, a constant flux $\Phi$ is maintained for a finite duration $\Delta t$, after which the flux is switched to zero well before detection. The key observation is that during the primary phase-generating interval $[0, \Delta t]$, all gauge-invariant observables of the electron are *identical* to those in a zero-flux experiment. Consequently, these quantities cannot encode the flux history required to produce the observed AB phase, which depends solely on the time-averaged flux $\Phi\Delta t/T$. By combining this insight with constraints from relativistic causality and the experimental tunability of the kinetic phase, we establish that no gauge-invariant quantity—whether local, nonlocal, or source-mediated—can account for the AB phase. This result reinforces the indispensable physical role of gauge potentials in quantum theory and clarifies the foundational status of the AB effect as a manifestation of potentials rather than fields.

The paper is organized as follows. Section 2 reviews the generalized AB phase for time-dependent fluxes and clarifies the distinction between the pure AB phase and the kinetic phase. Section 3 introduces the time-dependent protocol and derives the history-dependent AB phase. Section 4 analyzes possible gauge-invariant contributions—electromagnetic fields, source interactions, and electron observables—and shows why each fails. Section 5 states and proves the no-go theorem. Section 6 contrasts our approach with traditional static AB arguments, highlighting how the time-dependent protocol closes key interpretational loopholes. Section 7 examines Healey's holonomy interpretation and demonstrates its inability to account for the time-dependent AB phase. Finally, Section 8 concludes with implications for the ontology of gauge theories and the foundational status of potentials.

## 2 The Time-Dependent Aharonov–Bohm Phase

While the original AB effect concerns a static magnetic flux, realistic experimental implementations and theoretical extensions naturally involve time-dependent fluxes. When the enclosed magnetic flux $\Phi(t)$ varies during the time the electron wave packets propagate along the two arms, the accumulated phase is no longer simply $e\Phi/\hbar$. Instead, recent analyses show that the pure AB phase contribution becomes proportional to the *time-averaged* enclosed flux [6].

This section summarizes the key result for the generalized, time-dependent AB phase, focusing on the physically relevant AB component, as opposed to the total observed phase shift, which also includes a kinetic contribution from induced electric fields.

### 2.1 The Time-Averaged Flux Formula

In the path-integral formulation of quantum mechanics, the phase acquired by a charged particle along a path $\gamma$ in the presence of an electromagnetic potential is given by the line integral

$$\phi[\gamma] = \frac{e}{\hbar} \int_\gamma \mathbf{A}(\mathbf{x}(t), t) \cdot d\mathbf{x} - \frac{e}{\hbar} \int_\gamma \phi(\mathbf{x}(t), t)\, dt. \tag{2}$$

For interference experiments, the physically observable quantity is the relative phase between two paths $\gamma_1$ and $\gamma_2$ connecting the same initial and final points.

In the standard (static) magnetic AB geometry, the scalar potential $\phi = 0$ and the vector potential $\mathbf{A}$ is time-independent, so the phase difference reduces to the familiar gauge-invariant expression

$$\Delta\phi_{\mathrm{AB}} = \frac{e}{\hbar}\oint_C \mathbf{A}\cdot d\mathbf{l} = \frac{e}{\hbar}\Phi, \tag{3}$$

where $C = \gamma_1 - \gamma_2$ is the closed loop formed by the two arms, and $\Phi$ is the magnetic flux through any surface bounded by $C$. However, when $\mathbf{A}(\mathbf{x}, t)$ is explicitly time-dependent, the phase becomes path *and* history dependent. The line integral must be evaluated along the actual spacetime trajectory of each arm [6].

Consider the AB geometry with two symmetric arms forming circular arcs of radius $R$ around a long solenoid. The AB contribution to the relative phase is

$$\begin{aligned}\Delta\phi_{\mathrm{AB}} &= \frac{e}{\hbar}\int_0^T \Big[\mathbf{A}(\mathbf{x}_1(t), t)\cdot\mathbf{v}_1(t) - \mathbf{A}(\mathbf{x}_2(t), t)\cdot\mathbf{v}_2(t)\Big]\,dt \\ &= \frac{e}{\hbar}\cdot\frac{1}{T}\int_0^T \Phi(t)\,dt.\end{aligned} \tag{4}$$

where $\mathbf{x}_{1,2}(t)$ and $\mathbf{v}_{1,2}(t)$ are the instantaneous positions and velocities along each path, and $T$ is the total transit time from splitting to recombination [6]. This is the central result: for circular paths, the pure AB phase shift is proportional to the *time average* of the enclosed flux over the entire propagation interval $[0, T]$.[1]

The factor $1/T$ arises because each infinitesimal time interval $dt$ contributes an instantaneous phase increment proportional to the instantaneous flux $\Phi(t)$, and these increments add coherently along the trajectory. The total phase is therefore sensitive to how long each value of flux was "experienced" by the wave packets.

## 2.2 Separating the AB and Kinetic Contributions

Importantly, the *total* observed phase difference at the detector generally contains an additional contribution from the induced electric field $\mathbf{E}_{\mathrm{ind}} = -\partial\mathbf{A}/\partial t$ (Faraday's law). When the flux changes, $\mathbf{E}_{\mathrm{ind}}$ exerts a tangential force on the electron, altering its mechanical angular momentum and velocity along each path. These velocity asymmetries persist and produce a differential *kinetic* phase

$$\Delta\phi_{\mathrm{kin}} = \frac{1}{\hbar}\int_0^T \Big[\frac{1}{2}mv_1^2(t) - \frac{1}{2}mv_2^2(t)\Big]\,dt, \tag{5}$$

which depends on the *changes* in flux rather than its time average.

Detailed WKB calculations show that, for circular paths and a flux that starts at $\Phi(0)$ and ends at some final value, the *total* phase shift (AB + kinetic) is

$$\Delta\phi_{\mathrm{tot}} = \frac{e}{\hbar}\Phi(0), \tag{6}$$

[1] In the quasistatic regime where the magnetic fields outside the solonoid is omittable, the enclosed flux is the flux inside the solonoid. In particular, when the time dependence of the flux is linear such as in the later protocol, the magnetic fields outside the solonoid is exactly zero. By contrast, when the magnetic fields outside the solonoid is not omittable, the enclosed flux is through the actual electron paths [6].

independent of the detailed time history [6]. The kinetic phase exactly compensates for deviations from the initial flux, so that only the initial flux appears in the final interference pattern when the paths are circular and symmetric.

However, the *pure AB contribution* $\Delta\phi_{\rm AB}$ remains the time-averaged form (4). It is this term—the part that cannot be eliminated by local mechanical adjustments and that depends on the global flux history—that plays the central role in our no-go argument. The kinetic phase, being generated by an electric force, is gauge-invariant and can be manipulated or cancelled by external fields; the AB phase cannot.

### 2.3 Physical Meaning and Generality

The generalized AB effect has a hybrid character: the total phase retains a potential-like contribution (the flux holonomy integrated over time), yet it is influenced by induced electric fields that produce measurable mechanical effects along the paths. For non-circular paths or strong time dependence, additional geometric and dynamical corrections appear, but the essential history-dependence of the AB piece persists.

This generalized AB phase is the quantity our protocol aims to isolate and explain. As we will show, its dependence on the *duration* for which a given flux was present—rather than on any local field or instantaneous value at detection—cannot be accounted for by any purely gauge-invariant dynamical mechanism.

## 3 A Causal Protocol to Isolate the AB Phase

We define the flux function $\Phi(t)$ for our protocol explicitly (see Fig. 1):

$$\Phi(t) = \begin{cases} \Phi & \text{for } t < 0 \quad \text{(initial constant flux)} \\ \Phi & \text{for } 0 \le t \le \Delta t \quad \text{(post-split constant flux)} \\ \Phi\left(1 - \frac{t-\Delta t}{\tau}\right) & \text{for } \Delta t < t < \Delta t + \tau \quad \text{(linear switch-off)} \\ 0 & \text{for } t \ge \Delta t + \tau \quad \text{(zero flux).} \end{cases} \tag{7}$$

The electron's journey from source (S) to detector (D) takes total time $T$, with $T \gg \Delta t \gg \tau$. The critical causal constraint is that the turning-on time of the solonoid is shorter than the light-crossing time $L/c$ between the spatial region of the electron paths and the flux source: $\Delta t + \tau < L/c$. This ensures no signal can propagate between the electron and the source during the turning-on period, forbidding backreaction.

Substituting $\Phi(t)$ from Eq. (7) into the generalized AB formula Eq. (4) yields the AB phase:

$$\Delta\phi_{\rm AB} = \frac{e}{\hbar} \cdot \frac{1}{T} \left( \int_0^{\Delta t} \Phi \, dt + \int_{\Delta t}^{\Delta t + \tau} \Phi(t) \, dt \right) \approx \frac{\Delta t}{T} \frac{e\Phi}{\hbar}. \tag{8}$$

The approximation holds because $\tau \ll \Delta t$, making the contribution during the switching interval negligible. The phase is finite, determined by the product of the initial flux $\Phi$ and the fraction of the total travel time $\Delta t/T$ during which that flux was present after the split.

This result is crucial: the observed AB phase depends on a *history* (the flux averaged over time) and not on any instantaneous value of the field or potential at the moment of interference (when $\Phi(T) = 0$).

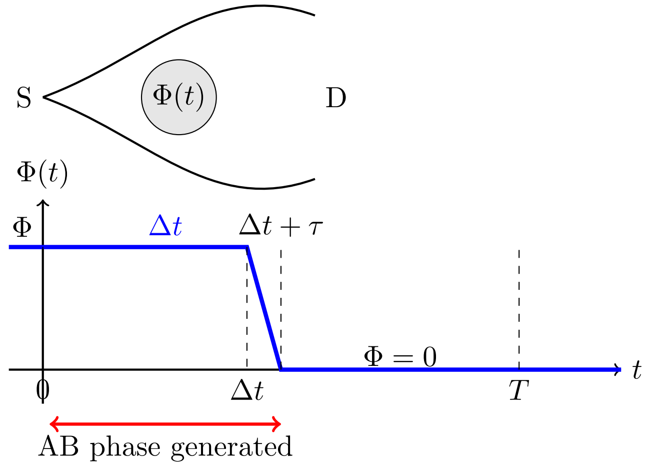


Figure 1: The time-dependent AB protocol. Top: Spatial paths $\gamma_1, \gamma_2$ enclosing the flux region. Bottom: Timeline of flux $\Phi(t)$. The AB phase is generated during the intervals $[0, \Delta t]$ (red arrows). During $[0, \Delta t]$, the flux is constant at $\Phi$; during $[\Delta t + \tau, T]$, the flux is zero. The turning-on period $\Delta t + \tau$ is causally disconnected from the source. The flux is zero at the interference time $T$.

# 4 Why Gauge-Invariant Explanations Fail

## 4.1 Why Fields Cannot Explain the Phase

During the switching interval $\Delta t < t < \Delta t + \tau$, the changing flux induces an electric field $\mathbf{E}_{\text{ind}}$ via Faraday's law, $\oint_C \mathbf{E}_{\text{ind}} \cdot d\mathbf{l} = -d\Phi/dt$. This field does work on the electron, imparting differential angular momentum kicks to the two paths. These kicks establish velocity asymmetries that contribute to the kinetic phase.

Following the WKB treatment from the generalized AB theory [6], the angular momentum changes during switching are:

$$\Delta L_1 = -\frac{e}{2\pi}[\Phi(\Delta t + \tau) - \Phi(\Delta t)] = \frac{e}{2\pi}\Phi, \tag{9}$$

$$\Delta L_2 = +\frac{e}{2\pi}[\Phi(\Delta t + \tau) - \Phi(\Delta t)] = -\frac{e}{2\pi}\Phi. \tag{10}$$

These angular momentum changes persist throughout the subsequent free propagation interval $[\Delta t + \tau, T]$, leading to different angular velocities for circular paths with radius $R$:

$$\omega_1(t) = \omega_1(0) + \frac{e\Phi}{2\pi m R^2}, \tag{11}$$

$$\omega_2(t) = \omega_2(0) - \frac{e\Phi}{2\pi m R^2} \quad \text{for } t > \Delta t + \tau. \tag{12}$$

The total phase shift decomposes as $\Delta\phi_{\text{tot}} = \Delta\phi_{\text{AB}} + \Delta\phi_{\text{kin}}$, where $\Delta\phi_{\text{AB}} \approx \frac{e}{\hbar}\Phi\frac{\Delta t}{T}$ (time-averaged flux), and $\Delta\phi_{\text{kin}} \approx \frac{e}{\hbar}\Phi\left(1 - \frac{\Delta t}{T}\right)$ (from persistent velocity asymmetry).

The fundamental differences between these two phases are essential for understanding the physical origin of the AB phase. The AB phase is tied to the gauge potential $\mathbf{A}$ via the line integral $\oint \mathbf{A} \cdot d\mathbf{l}$, while the kinetic phase originates from the force $\mathbf{F} = e\mathbf{E}_{\text{ind}}$ via work done on the electron. The AB phase is gauge-invariant only for closed paths, whereas the kinetic phase is gauge-invariant for both open and closed paths, as it depends only on $\mathbf{E}$ and mechanical quantities. In particular, the kinetic phase can be completely cancelled by applying appropriate external forces that exactly counteract the induced electric field's effects.

### 4.2 The Insensitivity of Electron Observables

During the AB phase-generating interval $[0, \Delta t]$, the magnetic field $\mathbf{B}$ is zero in the region where the electron moves. Crucially, before the wave packets overlap at the detector, each path $\gamma_i$ $(i = 1, 2)$ traverses a spatial region that is simply connected and does not enclose the solenoid. This allows us to choose a gauge transformation $\chi(\mathbf{x}, t)$ such that the transformed vector potential $\mathbf{A}' = \mathbf{A} + \nabla\chi$ satisfies $\mathbf{A}' = 0$ everywhere along both paths during this interval.

In this gauge, the Hamiltonian reduces to that of a free particle, $H = \hat{\mathbf{p}}^2/(2m)$, along both paths. Consequently, the Schrödinger evolution of the electron's wave function along each path is identical to that of a free particle. Since gauge-invariant observables are independent of the gauge choice, they must take exactly the same values along each path as they would in a zero-flux experiment. In other words, all gauge-invariant observables of the electron—including its probability density $\rho(\mathbf{x}, t)$ and current density $\mathbf{j}(\mathbf{x}, t)$—evolve exactly as they would if $\Phi = 0$. The constant flux $\Phi$ leaves *no imprint* on these gauge-invariant quantities.

During the final free propagation interval $[\Delta t + \tau, T]$, the flux is zero, and while the electron's motion differs from the zero-flux case due to persistent velocity changes established during switching, these differences contributes only to the kinetic phase, not to the AB phase.

### 4.3 Experimental Isolation of the AB Phase

The different properties of these phases allow experimental isolation of $\Delta\phi_{\text{AB}}$. Since the kinetic phase is gauge-invariant and force-generated, it can be cancelled by external forces that modify the electron's motion without affecting the AB phase. Moreover, the kinetic phase $\Delta\phi_{\text{kin}}$ can also be made to equal $2n\pi$ (for integer $n$) by adjusting experimental parameters such as the flux $\Phi$ and the total travel time $T$. When $\Delta\phi_{\text{kin}} = 2n\pi$, the observable interference pattern is determined *solely* by the AB phase:

$$\Delta\phi_{\text{obs}} = \Delta\phi_{\text{AB}} \approx \frac{\Delta t}{T}\frac{e\Phi}{\hbar}. \tag{13}$$

In this case, any explanation of the observed interference shift must account for $\Delta\phi_{\text{AB}}$ alone.

### 4.4 Causal Exclusion of Source-Mediated Effects

During the switching period $\tau$, the condition $\Delta t + \tau < L/c$ ensures the electron and the flux source are causally disconnected. Any proposed mechanism requiring a backreaction of the electron's state on the source to generate the phase would require superluminal signaling, violating relativistic causality. Therefore, explanations relying on source-electron entanglement or backreaction are categorically excluded [18].

## 5 A Rigorous No-Go Theorem

**Theorem 1** (No Gauge-Invariant Dynamical Explanation for the Generalized AB Effect)**.** *Consider the time-dependent magnetic AB effect as defined by the protocol in Section 3, with $\Delta t + \tau < L/c$ and $T \gg \Delta t \gg \tau$. The AB phase $\Delta\phi_{\text{AB}} = \frac{e}{\hbar} \cdot \frac{1}{T} \int_0^T \Phi(t)dt$ cannot be accounted for by any gauge-invariant dynamical quantity associated with:*

1. *The local electromagnetic fields interacting with the electron.*

*2. Any backreaction or correlation between the electron and the flux source.*

*3. Any gauge-invariant property of the electron's state during its propagation from source to detector.*

*Proof.* The proof follows from the causal and physical separation of the identified phase contributions and the behavior of gauge-invariant observables.

1. **Electromagnetic Fields**: During $[0, \Delta t]$ (when $\Delta\phi_{\text{AB}}$ is generated), $\mathbf{B} = 0$ and $\mathbf{E} = 0$ at the electron's position. The sole gauge-invariant field contribution is the kinetic phase $\Delta\phi_{\text{kin}}$ from the induced $\mathbf{E}$-field during switching and the persistent velocity changes thereafter. This phase is numerically and physically distinct from $\Delta\phi_{\text{AB}}$. Fields cannot explain $\Delta\phi_{\text{AB}}$.

2. **Source Backreaction**: By construction, $\Delta t + \tau < L/c$. This condition ensures the electron and the flux source are causally disconnected. Any possible influence or correlation from the source is forbidden by relativistic causality.

3. **Electron Observables**: During the phase-generating interval $[0, \Delta t]$, all gauge-invariant observables of the electron are identical to those in a zero-flux experiment. They are therefore independent of the flux $\Phi$ and cannot contain the information about the flux duty cycle $\Delta t/T$ that determines $\Delta\phi_{\text{AB}}$. During the post-switching interval $[\Delta t + \tau, T]$, while gauge-invariant observables differ from the zero-flux case, these differences contributes only to the kinetic phase. Therefore, no gauge-invariant property of the electron's state during propagation can explain the AB phase.

4. **Experimental Isolation**: When $\Delta\phi_{\text{kin}} = 2n\pi$, the observable interference pattern is determined solely by $\Delta\phi_{\text{AB}}$. Any explanation of the observed interference shift must account for $\Delta\phi_{\text{AB}}$.

Since all possible categories of gauge-invariant dynamical explanations are invalidated, the theorem holds.

□

# 6 Beyond the Static Case: Closing the Loopholes

A traditional no-go argument against gauge-invariant explanations can be formulated within the framework of the *standard*, static magnetic AB effect. This argument, however, suffers from critical limitations that are overcome by the time-dependent protocol analyzed in this work. Examining this contrast clarifies the necessity and strength of our generalized approach.

## 6.1 The Static AB Argument and Its Limits

Consider the standard magnetic AB setup: an electron wavepacket is split to encircle an infinite solenoid carrying a *constant, time-independent* magnetic flux $\Phi$. The electron propagates along two paths $\gamma_1$ and $\gamma_2$ in a region where the magnetic field $\mathbf{B}$ is identically zero. The observed phase shift upon interference is $\Delta\phi_{\text{AB}} = e\Phi/\hbar$.

The core of the standard argument proceeds as follows. In the region of space where the electron moves (i.e., outside the solenoid) before interference, the magnetic field vanishes, $\mathbf{B} = 0$. Consequently, in this simply-connected region, one can choose a gauge where the vector potential is also identically zero: $\mathbf{A} \equiv 0$. In this particular gauge, the Hamiltonian governing the electron's evolution reduces to that of a free particle, $H = \hat{\mathbf{p}}^2/2m$.

Therefore, in this gauge, *all gauge-invariant observables* of the electron—including its probability density $\rho(\mathbf{x}, t)$ and current density $\mathbf{j}(\mathbf{x}, t)$—evolve exactly as they would in a completely field-free experiment with $\Phi = 0$. Since these observables are, by definition, independent of the chosen gauge, their values and evolution must be the same in *any* gauge. We are thus led to a stark conclusion: prior to the moment of interference, all gauge-invariant properties of the electron's quantum state are *independent* of the enclosed flux $\Phi$.

Yet, at the screen, the interference pattern depends decisively on $\Phi$. The phase shift $\Delta\phi_{\mathrm{AB}}$ appears discontinuously at recombination without having been encoded in any gauge-invariant property of the electron during its propagation. This presents a direct contradiction for any theory claiming that the AB phase arises from, or is mediated by, such gauge-invariant dynamical quantities of the electron.

## 6.2 Two Critical Loopholes

Despite its conceptual appeal, this argument contains two significant and often-overlooked loopholes that prevent it from being a definitive no-go theorem.

### 6.2.1 Loophole I: Influence of the Flux Source

The standard argument focuses exclusively on the electron and the electromagnetic field in the region external to the solenoid. It does not, and cannot, categorically exclude the possibility that the phase information is somehow mediated by the *flux source itself*. One could speculate, for instance, that the electron and the source become quantum-entangled, and there is a backreaction of the electron's state on the source that subsequently influences the phase [18]. The static setup provides no mechanism to rule out such correlations or influences between the electron and the source, as they are not forbidden by any dynamical principle in a time-independent scenario.

### 6.2.2 Loophole II: Instantaneous Influence at Interference

A second, more subtle loophole concerns the moment of interference itself. The standard argument establishes that gauge-invariant electron observables *during propagation* are flux-independent. However, it does not preclude the possibility that the presence of the flux $\Phi$ exerts an *instantaneous influence* directly at the point and time of wavefunction overlap. One could imagine a mechanism whereby the flux, through its associated potential or field, acts directly on the interference process without leaving a trace on the electron's prior gauge-invariant state. In the static case, the flux $\Phi$ is present both during propagation *and* at the instant of interference ($t = T$), making it impossible to disentangle historical from instantaneous effects.

## 6.3 How the Time-Dependent Protocol Closes Both Loopholes

The time-dependent protocol analyzed in this paper is specifically designed to close these two loopholes, transforming a suggestive argument into a rigorous no-go theorem.

### 6.3.1 Closing Loophole I: Relativistic Causality

In our protocol, the flux is switched from $\Phi$ to 0 after a short interval $\Delta t + \tau$. Crucially, we enforce the condition $\Delta t + \tau < L/c$, where $L$ is the minimum electron-source distance. This guarantees *causal disconnection* between the electron and the flux source during the period when the source's flux is not zero. By relativistic causality, no correlation or backreaction can be established between the two during this period. Therefore, any explanation relying on the source to encode phase information via entanglement is categorically excluded.

#### 6.3.2 Closing Loophole II: Flux History vs. Instantaneous Value

The most decisive feature of our protocol is that at the moment of interference ($t = T$), the magnetic flux in the solenoid is *zero*: $\Phi(T) = 0$. Any explanation based on an instantaneous influence of the flux at the detector would predict a null result, since the instantaneous flux is zero. However, the observed phase is non-zero and determined by the *time-averaged flux*: $\Delta\phi_{\text{AB}} \approx (e/\hbar)(\Delta t/T)\Phi$. This phase is a *memory effect* of the flux history, specifically the finite period $\Delta t$ during which the flux was constant after the split. This cleanly separates the history-dependent AB phase from any hypothetical effect contingent on the instantaneous value of the flux at interference.

### 6.4 From Suggestive Argument to Rigorous Theorem

Thus, the standard magnetic AB effect argument provides the essential insight that the AB phase is not reflected in the electron's local, gauge-invariant dynamics. Our time-dependent generalization fortifies this insight into a robust theorem by:

1. Incorporating *relativistic causality* to eliminate the source as a potential mediator (closing Loophole I).
2. Leveraging the *history-dependence* of the generalized AB phase to eliminate explanations based on instantaneous action at interference (closing Loophole II).

This progression from the standard to the generalized framework not only strengthens the no-go conclusion but also deepens our understanding of the AB effect.

## 7 Why Holonomy Interpretations Cannot Explain the Time-Dependent AB Effect

The AB effect has inspired various philosophical interpretations aimed at preserving gauge invariance while explaining the observed phase shift without ascribing physical reality to the gauge potentials. One prominent such view is Healey's holonomy interpretation [9], which posits that the fundamental physical entities in gauge theories are gauge-invariant holonomies associated with closed loops, rather than the local gauge potentials or fields. This section examines the relevance of Healey's interpretation to the time-dependent protocol analyzed in this paper and demonstrates that it cannot account for the generalized AB phase in the proposed experiment.

### 7.1 The Holonomy Interpretation in Static Settings

In his book *Gauging What's Real: The Conceptual Foundations of Contemporary Gauge Theories* [9] and related works [7, 8], Healey argues that the true ontology of gauge theories consists of nonlocal, gauge-invariant holonomies. For electromagnetism, the holonomy along a closed loop $C$ is defined as

$$H(C) = \exp\left(i\frac{e}{\hbar}\oint_C \mathbf{A}\cdot d\mathbf{l}\right) = \exp\left(i\frac{e}{\hbar}\Phi\right), \tag{14}$$

where $\Phi$ is the magnetic flux enclosed by $C$. These holonomies are intrinsic relational properties of the theory, attached to loops rather than points in space, and they fully capture the physical content of the gauge field without requiring potentials as real entities.

In the static AB effect, Healey explains the phase shift $\Delta\phi_{\mathrm{AB}} = e\Phi/\hbar$ as arising from the holonomy of the closed loop formed by the two interfering paths of the electron. This holonomy acts nonlocally on the electron's wave function, producing the AB relative phase. Healey emphasizes that this avoids action-at-a-distance while preserving gauge invariance, drawing analogies to quantum nonseparability in entangled systems.

### 7.2 Applying Holonomy Reasoning to the Time-Dependent Case

The time-dependent protocol introduced in Section 3 features a flux $\Phi(t)$ that is constant at $\Phi$ for $0 \leq t \leq \Delta t$ after wave-packet splitting, then linearly switched off over a short interval $\tau \ll \Delta t$, and zero thereafter until detection at time $T \gg \Delta t$. The resulting AB phase is history-dependent:

$$\Delta\phi_{\mathrm{AB}} \approx \frac{\Delta t}{T}\frac{e\Phi}{\hbar}, \tag{15}$$

determined by the time-averaged flux, with the flux vanishing at the moment of interference ($\Phi(T) = 0$).

Healey's holonomy interpretation is relevant because it offers a gauge-invariant, nonlocal explanation of AB-like effects without invoking potentials. If extended to time-dependent cases, one might attempt to interpret the phase as arising from a time-averaged or effective holonomy over the electron's spacetime path, preserving gauge invariance while accounting for the history dependence. This would potentially provide a counterexample to the no-go theorem's claim that *all* gauge-invariant explanations fail.

However, as shown below, Healey's framework—developed primarily for static and semiclassical regimes—cannot consistently explain the protocol without either reintroducing potentials or violating its core commitments to gauge-invariant holonomies as the sole physical entities.

### 7.3 Why Holonomies Fail in Time-Dependent Regimes

We prove that Healey's holonomy interpretation fails to account for $\Delta\phi_{\mathrm{AB}}$ in the proposed protocol.

**Theorem 2** (Impossibility of Holonomy Explanation for the Time-Dependent Protocol)**.** *Healey's holonomy interpretation cannot explain the observed AB phase in the time-dependent protocol of Section 3.*

*Proof.* The proof appeals to the structure of Healey's ontology, the idealizations it requires, and the history-dependent features of the protocol.

**Step 1: Holonomies are fundamentally spatial and static.** Healey's holonomies are defined for closed spatial loops in a fixed, multiply-connected topology (e.g., $\mathbb{R}^3$ minus an infinite solenoid), assuming static fields [9]. The holonomy $H(C)$ depends only on the instantaneous flux $\Phi$ through $C$ at a given time. No intrinsic dynamical evolution or time-averaging mechanism is built into the ontology.

**Step 2: The observed phase is history-dependent and requires temporal averaging.** The phase $\Delta\phi_{\mathrm{AB}}$ depends on the time integral $\int_0^T \Phi(t)\,dt$, not on the instantaneous flux at any particular time. At detection ($t = T$), $\Phi(T) = 0$, so the instantaneous holonomy is trivial ($H(C,T) = 1$). Any explanation must therefore encode the flux history $[\Phi(t)]_{0\leq t\leq\Delta t}$ into the phase without relying on local fields or potentials.

**Step 3: No mechanism exists in Healey's framework for temporal averaging of holonomies.** To explain $\Delta\phi_{\mathrm{AB}}$, one would need an effective or averaged holonomy over the electron's spacetime trajectory. However, Healey's holonomies are spatial (3D loops), not spatiotemporal (4D). Extending to time-dependent fluxes requires a 4D formulation (e.g., via the 4-dimensional Stokes theorem), but Healey does not provide such an extension. Critically, Nounou [12] argues that Healey's interpretation relies on unsustainable idealizations (infinite-duration fields, quasistatic approximations) that break down when time dependence is introduced. When these idealizations are removed, the holonomy cannot consistently produce a history-dependent phase.

**Step 4: Gauge-invariant observables remain unchanged during the phase-generating interval.** During $[0, \Delta t]$—the interval contributing most to $\Delta\phi_{\mathrm{AB}}$—all gauge-invariant observables of the electron are identical to those in a zero-flux experiment (Section 4.3). If the holonomy were responsible, it must somehow imprint the nonzero flux history on the electron's state during this interval. But no such imprint occurs in any gauge-invariant quantity. This contradicts the assumption that Healey's holonomy interpretation offers a gauge-invariant explanation of AB-like effects.

**Step 5: Experimental isolation of $\Delta\phi_{\mathrm{AB}}$ eliminates residual loopholes.** When the kinetic phase $\Delta\phi_{\mathrm{kin}}$ is tuned to $2n\pi$ (Section 4.4), the interference pattern depends solely on $\Delta\phi_{\mathrm{AB}}$. At $t = T$, the holonomy is trivial, yet a nonzero phase persists. This cleanly separates historical from instantaneous effects, closing any possibility of instantaneous holonomy action at interference (analogous to Loophole II in the static case, Section 6).

Thus, Healey's holonomy interpretation cannot explain the generalized AB effect.

□

### 7.4 Implications for Gauge Theories

This limitation highlights a broader challenge for holonomy-based interpretations in time-dependent regimes: they excel in static, ideal cases but struggle with genuine dynamics, finite durations, and history dependence. The time-dependent protocol therefore strengthens the case that gauge potentials possess irreducible physical significance in quantum electrodynamics, beyond what nonlocal holonomies can provide.

## 8 Conclusion

We have presented a rigorous no-go theorem that excludes all gauge-invariant explanations of the generalized AB effect. The time-dependent AB protocol provides a decisive test for the physical reality of gauge potentials. By engineering a flux history where $\Phi(t)$ is non-zero only during a finite interval $\Delta t$ after wavepacket splitting and vanishes completely at detection, we have isolated the AB phase as a pure memory effect: $\Delta\phi_{\mathrm{AB}} \approx (e/\hbar)(\Delta t/T)\Phi$. This temporal structure reveals three fundamental barriers to gauge-invariant explanations.

First, during the phase-generating interval $[0, \Delta t]$, all gauge-invariant observables of the electron are identical to those in a zero-flux experiment. These quantities are independent of the flux $\Phi$ and cannot record the history necessary to produce the AB phase. Second, relativistic causality, enforced by the condition $\Delta t + \tau < L/c$, eliminates any possibility of source-mediated effects or backreaction during the critical period when the phase accumulates. Third, the experimental separability of the AB phase from the kinetic phase allows for a clean observation where only the former contributes to the measured interference pattern.

These constraints collectively invalidate all proposed gauge-invariant mechanisms. Local electromagnetic fields cannot account for the phase, as they vanish both during the primary generation interval and at detection. Non-local field configurations or topological effects tied to the instantaneous flux at interference are equally inadequate, since $\Phi(T) = 0$. Attempts to attribute the effect to source-electron correlations fail the test of relativistic causality. Even Healey's holonomy interpretation, designed as a gauge-invariant alternative, cannot accommodate the temporal averaging essential to the generalized AB effect, revealing its limitations beyond static regimes.

The implications extend beyond the interpretation of a single quantum phenomenon. Our no-go theorem demonstrates that gauge potentials play an irreducible role in quantum theory—they are not mere mathematical conveniences but possess direct physical significance. Any formulation of electrodynamics—or gauge theories more generally—that seeks to describe all phenomena solely through gauge-invariant dynamics is fundamentally incomplete. The AB effect thus stands not as an exotic curiosity, but as a fundamental manifestation of gauge structure in quantum mechanics.

Looking forward, this work suggests several directions. The protocol could be implemented experimentally to test the predictions directly. The framework may also apply to other gauge theories, where similar history-dependent effects might reveal the physical significance of non-Abelian gauge potentials in QCD and metric fields in general relativity. Philosophically, the result strengthens the case for a realist interpretation of gauge potentials, while challenging purely relational or holonomic approaches to gauge theory ontology.

In summary, we have established through rigorous argument that gauge-invariant explanations of the AB effect are categorically impossible in time-dependent settings. The phase shift emerges not from fields, forces, or correlations, but from the history of the gauge potential itself—a clear testament to the physical reality of potentials in quantum theory.

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