

From Spacetime to Fields and Back: Elementary Domains and their Philosophical Roots

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Abstract In his *Field theory of elementary domains and particles* (1968, co-authored with KATAYAMA Yasuhisa), YUKAWA Hideki presents his programme for reconstructing particle physics based on the concept of *elementary domains*. Rooted in the ongoing debates surrounding relativity and quantum physics, his project pursues a twofold objective: to study particle interactions while circumventing the divergence problem of quantum field theory, and to provide a unified framework for the recently discovered particle theories. This approach ultimately results in a question about the nature of spacetime. Yukawa's answer is an audacious assumption about spacetime continuity: an elementary domain is a minimal region of spacetime with peculiar geometrical properties. In this contribution, I argue that Yukawa's approach exemplifies a specific account of unification in physics, based on formal derivation and spatial intuition. The quantum nature of these domains reduces the entire particle content to geometrical properties, transforming the physical vacuum into a state of spacetime itself. As such, Yukawa faces the problem of illustrating how non-geometrical properties can emerge from underlying spacetime structures. I discuss how the philosophy of NISHIDA Kitarō might have provided the theoretical and philosophical background for the development of Yukawa's physical theory. Ultimately, Yukawa's attempt shows how the reconstruction of a unified particle picture presents a genuine challenge to both Eastern and Western Philosophy of Physics.

Key words: Yukawa Hideki; Elementary domains; Unification; Particle theories; Quantum spacetime.

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1 Introduction

In 1968, nineteen years after receiving the Nobel Prize for his theory of mesons, YUKAWA Hideki published a two-part paper in collaboration with KATAYAMA Yasuhisa and UMEMURA Isao. In this work, the authors developed a novel proposal to address the proliferation of theories for newly discovered particles that had emerged over the preceding forty years. Yukawa's key insight consisted in developing a theory of particles through a reconceptualisation of spacetime geometry. This not only aimed to provide a unified framework for particle physics but also sought to resolve the problem of divergences that threatened the consistency of standard quantum field theory (QFT).

The introduction of *elementary domains* as the foundation of the theory was deeply influenced by philosophical insights that Yukawa might have drawn from his conversations with the philosopher NISHIDA Kitarō.¹ Nishida's philosophy might have inspired the new geometric approach of the physicist, complementing earlier physical attempts. Yukawa's theory is indeed rooted in the physics of the first half of the 20th century, but also pushes some of its assumptions in new directions.² For instance, Takabayasi (1991) notes a shift in Yukawa's work beginning in 1942, marked by a more direct engagement with the divergence problem and the challenge of formulating a theory of elementary particles. He also highlights Yukawa's attraction to Nishida's philosophy, while acknowledging that their work never fully aligned with the tenets of the Kyoto School.

Given this background, it is worthwhile to examine Yukawa's theory of elementary domains in detail, not only from a physical perspective (as a theory of elementary particles), but also as an example of philosophically-oriented construction. The central argument presented in this contribution is that Yukawa's theory exemplifies a specific account of unification, based on the formal derivation of the relevant properties of particle species from a modified spacetime geometry. This geometric modification not only enhances spatial intelligibility, but also reduces the entire particle content to geometrical properties. Consequently, Yukawa is confronted with the challenge of demonstrating how seemingly non-geometrical properties derive from spacetime properties. Whether or not his theory successfully addresses this challenge remains an open question.

This contribution is structured as follows. Section 2 situates Yukawa's work within the broader context of early 20th-century physical research. In this period, the divergence problem and the rapid development of particle physics prompted efforts to develop a unified account encompassing all particle species and their interactions.

¹ Yukawa explicitly mentions Nishida in his autobiography. Yukawa was indeed a colleague of his son Sotohiko under TAMAKI Kajuro, and later attended Nishida's lectures in Kyoto and met him in person: see Yukawa (1982: 164–166). Nishida himself mentions Yukawa on several occasions. For example, see *NKZ* XXIII, 360, or a reference to the physicist's work in *NKZ* XIX, 362.

² Brown (1985) and Darrigol (1988) present the discovery of the theory of mesons as a case of both innovation and continuation of traditional instances. As I will argue in this paper, the same applies for the theory of elementary domains. This is evidenced by the references to previous work summarised in Sections 2-3.

Section 3 argues that Yukawa’s non-local field theory emerges directly from this scientific environment and represents an initial attempt at such unification. This framework, along with its 1950s’ extensions, is contrasted with an alternative approach based on the role of special-relativistic causality principles. The latter, which involves modifying the structure of spacetime itself, is the approach Yukawa ultimately pursued in 1968.

Section 4 reconstructs the conceptual structure of Yukawa’s theory of elementary domains and interprets it as a unified framework for particle physics resulting from a profound reconceptualisation of spacetime. Yukawa reduces the particle content to a peculiar state of the elementary domain it occupies, and so particle properties to geometrical properties. I argue that this strategy allows Yukawa to achieve a *formal unification* of different particle theories, all of which ontologically depend on the underlying elementary domains. To explain the creation of particle content, Yukawa then has to assign quantum nature to those elementary domains, characterising their states as a superposition of complementary states, namely *empty* and *occupied*.

Finally, Section 5 emphasises the similarities between Yukawa’s elementary domains and Nishida’s philosophy of *basho*. The term *basho*, variously translated as *place*, *locus*, or *field*, among other terms, indicates a concrete situation, placedness or *implacement*³ that grounds and envelops any duality or complementary elements. In this sense, the *basho* cannot exist without these oppositions, which are its own determinations. The *basho* determines its properties through a dynamical process of self-mirroring or self-determination. The resonance with Yukawa’s theory becomes apparent in the treatment of the domain as a unity that transcends and underlies the opposition of the complementary quantum states. I concur with Takabayasi that Yukawa was likely inspired by Nishida in developing a novel unified framework. However, I argue that Yukawa’s theory should not be regarded as a “physical model” of Nishida’s philosophy, insofar as it is insufficient to address the challenge of understanding the mechanism of particle production from geometric properties. By extension, Yukawa also raises the broader issue of how to formulate an adequate physical description of this mechanism.

2 Divergences and Unification

During the 1930s, the development of QFT posed a formidable challenge to physicists: understanding the divergence of the self-energy.⁴ These unphysical divergences were initially attributed to the point-like nature of the source particles, suggesting a profound connection between quantum physics and the structure of spacetime.

³ For this translation, see Krummel and Nagatomo (2012: 5).

⁴ The *self-energy* of a particle is the energy of that particle in interaction with the rest of the system, for example, with a field. The divergence is a mathematical prediction of infinite energy of the particle in microscopic regimes, which is ultimately unphysical. In order to salvage the consistency of QFT, one possible solution was to introduce a *regulator*, that is to say, a bound on the energy of the particle.

Since point-like behaviour was considered incompatible with field-mediated interactions, several models attempted to circumvent these divergences by introducing a regulator as an inherent property of the source particles. For instance, Dirac (1938) proposed the electron's radius as a natural regulator for quantum electrodynamics. Consequently, the traditional point-like picture of the electron (and by extension, of other particles) was reconceptualised to incorporate spatial extension. The electron came to be understood as a small spherical body with a distributed charge, "held together" by a cohesive force distinct from electromagnetism.⁵

This revision of the standard conception of particles aligned well with the concurrent development of particle physics. New experimental discoveries presented physicists with a broader spectrum of available particles and a variety of theories to describe them. At that time, the notion of elementary particle, as a minimal constituent of physical reality, implied a degree of permanence. These immutable entities could combine and rearrange to form new particles; thus, particle instability was generally interpreted as evidence of a complex internal structure.⁶ However, no existing theory could fully explain these interaction processes or provide a unified account of particle complexity. The nature of some particles also remained unclear. For instance, mesons were observed both in atomic nuclei and in cosmic rays.⁷ Yet any claim of identical behaviour in both contexts required direct experimental confirmation.⁸

The parallel development of particle physics and the unexpected divergences of QFT became significant driving forces behind the pursuit of unification. This unification project aimed to develop a comprehensive framework capable of accounting for particle behaviour and interactions. For example, Yukawa (1942: 390–391) emphasised the significance of meson theory in advancing research of such a unified understanding. A successful unified theory would have to reconcile the hierarchical complexity of particles with the point-like or extended nature of their properties. Yukawa and his contemporaries recognised that addressing these issues posed a fundamental challenge to the prevailing geometric conception of quantum particles, an insight informed by their direct engagement with the divergence problem. Not only could particles be understood either as extended, non-decomposable entities or as decomposable aggregates held together by cohesive forces; it was also possible that the entire "preconceived framework"⁹ would need to be revised in order to overcome the stagnation of the discipline in addressing these problems.

⁵ On the relation between divergence problem and extension, see Hagar (2014). See also Yukawa's (1942: 396–397) comment on the method of cohesive forces.

⁶ See Darrigol (1988).

⁷ Dating these observations is complicated. For instance, while recalling the Shelter Island Conference of June 1947, Oppenheimer (1966: 57) admits that "for ten years we had misread the particles." I refer to this paper for an overview of the history of the discovery of the mesons.

⁸ See Yukawa (1942: 389–390).

⁹ Yukawa (1973: 183).

3 From Unification to Spacetime

In the 1950s, the discovery of new unstable particles strongly suggested that any future theory would need to go beyond the scope of existing QFT. The objective was to incorporate the entire “particle zoo” while maintaining internal consistency. A trivial addition of new particles was not a viable solution. Indeed, the introduction of new particles required new degrees of freedom to describe them.¹⁰ However, the “mechanical picture” of fields already permitted an infinite number of degrees of freedom. In the case of an electromagnetic field, each degree of freedom could acquire excitation energy, and the sum of these excitations would diverge. Geometrical constraints, such as assigning finite radii for particles, were proposed as a means of salvaging the theory by effectively eliminating some of the infinitely many degrees of freedom.¹¹

In line with this proposal, Yukawa (1950a,b) introduced a theory of scalar fields that depend on *two* spacetime points.¹² This field, henceforth U , is characterised by its external coordinates (i.e., the position of its centre of mass) and its internal coordinates (i.e., the relative distances between the base points), along with their respective canonical conjugates.¹³ Departing from the standard view, which treats fields as locally defined at single points in spacetime, Yukawa introduced the concept of *non-locality*, whereby the field is defined over, and depends on, *pairs* of distinct spacetime points. Consequently, its state cannot be decomposed into two independent values, each corresponding to one of the base points.¹⁴ The U field evolves as a harmonic oscillator: its configurations are described as combinations of vibrational modes, with each mode labelled by four quantum numbers¹⁵ for each direction of vibration.

Yukawa demonstrated that the non-local field U represents a system of identical particles with mass m , radius λ and spin 0. In this sense, the theory provides a unified framework capable of reproducing *several* particle theories upon fixing the values of m and λ . Furthermore, it predicts a direct relation between a particle’s radius and its

¹⁰ The term *degrees of freedom of a system* is used to describe the parameters that are allowed to vary and can be adjusted in order to describe the system’s state. In this context, the introduction of new degrees of freedom would have entailed the introduction of new tunable parameters for each particle, which would have implied the loss of the capacity to identify recurrent properties or behaviour.

¹¹ See Yukawa (1942: 410–411).

¹² Mathematically, a *scalar field* associates a real number to each point of a spacetime region.

¹³ The *canonical conjugates* are defined as pairs of variables with specific duality properties. In the context of classical mechanics, one possible set of canonical conjugates includes position and momentum. In Yukawa’s theory, the conjugate variables of the external coordinates are the momentum coordinates of the centre of mass. The interpretation of the conjugates of the internal variables is arguably more challenging.

¹⁴ More precisely, non-separability implied that each state was a combination of external and internal coordinates, and so the two contributions could not be isolated.

¹⁵ A *quantum number* is a quantity that specifies the possible states of a system. Tuples of quantum numbers characterise the system and are invariant throughout its evolution.

mass. Finally, in the zero-radius limit, the theory reduces to an ordinary, local field theory (provided that the rest mass is non-zero).

Takabayasi (1965) extended Yukawa's idea by allowing the particle state to depend on *four* spacetime points. This extension suggested that the symmetries of the system should be given a geometrical interpretation, grounded in the dimensionality of physical space and the structure of the four-point configuration. In this formulation, the non-local field is represented as a deformable, extended object embedded in Minkowski spacetime, with symmetries expressed as constraints on its allowable shape and motion. However, this solution threatened to reintroduce the need for a cohesive force. Hara and Gotō (1968) addressed this issue by modelling elementary particles as purely extended systems, rather than as assemblies of points. This solution enabled the extension of the unified framework to include fermions (i.e., half-integer spin particles) and permitted the derivation of particle–antiparticle states without invoking Dirac's hole theory.¹⁶

All these models, however, implied a violation of relativistic causality: within the region occupied by the extended particle, signals could, in principle, propagate faster than light. Yet relativistic causality is a fundamental spacetime-dependent constraint, one that all particles are expected to respect. Therefore, any spatial extension assigned to particles had to be compatible with the causal structure of relativistic spacetime.

This line of reasoning gave rise to a second possible strategy for unification. It was inspired by the new understanding of the relationship between fields and spacetime that emerged from the analysis of divergences in QFT. Originating in the work of Heisenberg and Pauli (1929), this proposal derived the concept of extended particle from a modification of the spacetime structure itself. In contrast to non-local theories, which still presumed the continuity of spacetime in the sense of infinite divisibility, this new strategy posited that spacetime is partitioned into regions of fixed size. In the field-theoretic picture, fields are defined on these regions, which act as natural regulators that prevent divergence. In the particle picture, particles are extended objects whose size corresponds to the dimension of the region they occupy.

Should these proposals seek unification, it would be necessary to specify a mechanism by which spatiotemporal properties determine, or at least constrain, the variety of possible particle theories. For instance, Takano (1967a,b) suggested that the appearance of different particles, each associated with distinct vibrational modes, could be explained by appealing to fluctuations in the background spacetime. According to this view, a spacetime fluctuation represents an elementary particle, and the diversity of particles is due to the multiplicity of fluctuation modes. These fluctuations are governed by probability distributions that mimic the behaviour of quantum effects at microscopic scales. The distributions, in turn, define the internal wavefunctions of elementary particles that occupy these fluctuation regions.

¹⁶ For an accessible introduction to Dirac's hole theory, see Wright (2016).

4 Elementary Domains as a Theory of Unification

In 1968, Yukawa, in collaboration with Katayama Yasuhisa and Umemura Isao, published a two-part paper outlining the development of his novel theory of elementary domains.¹⁷ Elementary domains were presented as a promising alternative framework for reconstructing standard QFT. This theory stands out as a case of unification based on a profound reflection on the role of space and time. In fact, Yukawa (1942: 410) writes:

I believe that the problem of the continuity of space and time would be the most difficult and probably the last problem. The discontinuity of matter and phenomena introduced by quantum theory, has not been directly contradictory to the continuity of time and space which is needed for the description of objects [...]. Of course it [sc. QFT] contains contradictions – moreover internal inconsistency – but we cannot easily clarify how they are related with the assumption of continuity of space and time.

Here, Yukawa suggests that a reconceptualisation of our spatial and temporal frameworks may open the path towards a unified theory of particles. Such a theory would require two key elements:

1. the derivation of all particle properties from the internal structure of the objects postulated by the new theory, and
2. the elimination of divergences from the formalism.

If unification is to be achieved through a revision of the assumption of spacetime continuity, then the central question becomes that of finding a substitute spacetime model. Yukawa (1973: 175) recognises that any specific definition of *spatiotemporal extension* necessarily restricts the theory to a limited subset of the results that particle physics, in principle, is capable of producing. For instance, most of the earlier proposals discussed in Section 3 (including Yukawa's own non-local theory) modelled extended particles by analogy with molecular structures. In these models, distinct simpler parts interact in specific ways to determine the system's overall dynamics. However, the *molecule-based conception of extension* is inadequate. Firstly, the mass of the components must be fixed from the outset, rather than derived within the theory. Secondly, such model fails to satisfy the symmetries required for unifying various particle theories. Even the addition of new degrees of freedom, such as in Takabayasi's tetrahedron model, ultimately falls short in addressing these theoretical demands.

4.1 The Argument for the Elementary Domains

The inadequacy of molecule-based extension leaves open an alternative, which instead draws on the classical dynamics of continuous bodies as its source of anal-

¹⁷ Yukawa himself provides insightful interpretations of different parts of the theory, which have been taken into consideration in the following reconstruction. For these references, see Yukawa (1942: 410–413; 1973: 166–184).

ogy. Inspired by the work of Hara and Gotō, this proposal conceptualises extended elementary particles as continuous, classical, deformable bodies. The *continuous-body-based conception of extension* is more promising, as it avoids both artificial geometric constructions and the difficulties associated with deriving the symmetries of hadrons.

Upon quantisation, each extended particle is assigned a ground state, the vacuum. This is conceived as a diffused medium in which no particle is present.¹⁸ Particle content is then created through excitations of this vacuum state.

In the opening section of the paper, Katayama and Yukawa (1968) postulate that

(Ext) A particle is a quantum mechanical object extended in space and time.

If this postulate holds, then particles must occupy a certain spacetime region entirely and all at once. This region is indeed *defined* as one that can be entirely occupied by an extended particle. Owing to the continuous-body analogy, the particle cannot be further decomposed into sub-constituents localized at different points within that region, as would be required by the standard picture. Instead, decomposability becomes contingent upon the continuity of the underlying space and time.

Yukawa and Katayama also postulate that

(Sp) Space cannot be divided to infinity.¹⁹

This postulate arises from efforts to avoid divergences in the theory by rejecting the assumption of spacetime continuity. The constituent regions of space are called *domains*. Each domain D has volume V_D and can be occupied by a particle. Given two such domains, the commutation relations between the particles they contain depend on whether the domains intersect or are disjoint. In the limit where domains shrink to points, one recovers the standard point-like description.

Each domain is occupied by extended particles, and its wavefunction²⁰ is obtained by averaging pointwise values over the domain's volume. Each pair of wavefunctions is associated with a pair of creation and annihilation operators for the extended particles that occupy the domain at a given time t . In particular,

(Occ) A particle uniformly occupies a domain D all at once at any instant.

Consequently, the extended particle is identified with the domain it occupies, and its vacuum state becomes the ground state of that domain. The specific region that can be uniformly occupied by a particle is termed an *elementary domain*.

In summary, the derivation of an elementary domain proceeds from an examination of the notion of extension, here identified with the continuous-body-based conception, and rests on three independent assumptions: **(Sp)**, **(Ext)** and **(Occ)**. Schematically, the argument goes as follows:

¹⁸ Properly speaking, the vacuum is the state of maximum annihilation, i.e., of particle number zero.

¹⁹ The assumption is not initially extended to time due to practical reasons. For an overview of Yukawa's conception of space and time, see Yukawa (1973: 144–149; 156–158).

²⁰ The *wavefunction* of a system is a mathematical representation of its state. In the case of a quantum system, this function is complex-valued and provides the probability amplitude for a specific state to be obtained.

1. Space cannot be divided to infinity [(Sp)].
2. There is a (minimal) extended region of space, the (elementary) domain [from 1].
3. A particle is an extended object in space [(Ext)].
4. There is a domain that contains each particle [from 2,3].
5. A particle uniformly occupies the domain all at once at any instant [(Occ)].
6. Therefore, the particle is *geometrically* identified with the (elementary) domain it occupies [from 4,5].

It should be noted that, at this stage, the identification is primarily geometrical. The further step of identifying the particle's vacuum state with that of the domain itself entails a reduction to geometry that requires further examination.

Yukawa himself alludes to this reduction in his essay *The Wayside Inn for Wayfaring Elementary Particles* (1964). Here, he portrays elementary particles as wayfarers, each occupying a room, or region, or domain of space. In this metaphor, space thus offers seats (*loci*) for matter. Yukawa identifies particles entirely by the seat they occupy, as it is the room that determines the type of particle and its properties. A particle may arrive at or depart from the inn, corresponding to its creation or annihilation in a particular *locus*. Time, in turn, is represented by the sequence of changes in seat occupancy.²¹ While continuous space offers an infinite number of such seats, (Sp) implies that particles residing in spatial *loci* possess a minimal extension. Moreover, each seat is defined by the number of particles it can contain: as it has finite extension, there is an upper bound on the number of possible occupants.

By definition, an elementary domain is a spatial region that cannot be further subdivided. In this context, the term *elementary* implies *minimality*: no elementary domain can be further decomposed into smaller regions, each entirely occupied by a single particle without residual extension beyond its boundary. To illustrate, suppose an elementary domain is occupied by a particle. If the particle does not exhaust the domain's volume, then, by (Occ), it uniformly occupies a proper subregion. Consequently, the elementary domain could be decomposed into two parts: one occupied by the particle and another left empty. This implies a violation of elementarity. In other words, the elementary domain is introduced as a means to "extend, in a sense, the atomistic point of view to the physical space itself."²² In the language of the wayside inn metaphor, localising a particle *within* the elementary domain, i.e., in a seat of smaller size, contradicts the principle of elementarity and is therefore physically meaningless.

Furthermore, Katayama and Yukawa note (though without elaboration) that elementary domains are impenetrable. One possible interpretation of this remark may

²¹ The elementary domains have a finite size, and thus the dynamics must admit finite displacements. The evolution of the wavefunction is then described in terms of *finite difference equations*. In fact, due to the (Sp) assumption, changes occurring in a fraction of the minimum temporal increment have no physical meaning in the theory. In other words, these conceivable phenomena are irrelevant at the scale of the elementary domain. The domains are extended, with linear dimension of the order of a fundamental length, λ . Consequently, the domains are distinguishable only if their centres are separated in space by a distance comparable with λ . In other words, *the degree of distinguishability in space and time is contingent upon the spacetime distance*.

²² Katayama and Yukawa (1968: 12). For other senses of elementarity, see Darrigol (1988: 253-254).

be drawn from the analogy with elastic collisions between spheres, governed by fermionic relations. Just as such spheres neither break nor merge into each other, elementary domains likewise do not overlap or coalesce to form a new domain. Rather, they interact by exhibiting analogous dynamical behaviour.²³

4.2 The Wavefunction of a Domain and its Evolution

In the wayside inn metaphor, the wayfarers are wholly identified by the seats they occupy. In physical terms, particles are not distinguished by names or intrinsic labels, but solely by the elementary domain they occupy. In this sense, the properties of particle kinds are reduced to the geometrical features of the spacetime regions they fill. The rest of Katayama and Yukawa's paper attempts to elucidate this insight and demonstrate its physical realisation.

4.2.1 The Geometry of Elementary Domains

It is important to recall that the argument for the elementary domains concludes with a form of geometric reduction. This reduction implies that a relation must exist between the vacuum state of a particle and the domain. In particular, Yukawa, inspired by Takano's notion of spacetime fluctuations, proposes assigning a vacuum state to each elementary domain, by analogy with the particle picture. The domain's vacuum is no longer conceived as an extended medium of fluctuating fields, but rather as the state of an extended spatiotemporal object. It is anticipated that the domain's geometric properties, such as shape, size, and orientation, will constrain both its vacuum state and its excited states. However, although the state of the elementary domain depends on its geometry, the precise geometrical properties that are relevant remain unspecified.

Yukawa describes the elementary domain as an ellipsoid, characterised by ten parameters: three coordinates for its centre, three Euler angles for its orientation, three components of its momentum tensor, and one temporal coordinate. Admissible domain shapes are constrained by Lorentz-invariance, which demands that their geometrical symmetries reproduce those of relativistic spacetime.

Traditionally, particle properties and geometric properties have been treated as distinct degrees of freedom. The challenge of coupling these two sets of features has been a fundamental obstacle for unification attempts grounded in modifications of the underlying geometry. The standard approach among proponents of a unified particle theory is encapsulated by the following assumption:

(UEP) Each kind of particle possesses a distinctive structure that can be expressed upon the selection of an appropriate spacetime framework,

²³ For further details, cf. Katayama, Umemura and Yukawa (1968: 43).

notably the one defined by relativity. Consequently, such theories have pursued unification by attempting to derive particle properties from conjectures about internal structure. Among these properties, some have been regarded as especially fundamental, insofar as they allow for consistent identification of the particle type throughout dynamical evolution. In particular, quantum numbers, whose values remain invariant under time evolution, have traditionally been considered adequate identifiers of the different particle species.²⁴

The identification of particle types by tuples of quantum numbers suggests a novel approach to unification: deriving all physically admissible combinations of quantum numbers from geometric properties. The ellipsoidal shape of the elementary domain constrains the admissible tuples of quantum numbers to those associated with known particle theories. In fact, the shape provides sufficient degrees of freedom to derive the relevant parameters that distinguish particle species, while simultaneously ensuring the spatial intelligibility of the construction. In this way, Yukawa achieves a formal unification of the various (hadronic)²⁵ particle theories through a unified geometrical account of spatial domains. The geometric structure of the domain is sufficient to derive the parameters that constrain the admissible states of the families of particles that occupy the elementary domains.

However, two important issues remain unresolved. First, Yukawa does not demonstrate that particle theories can be entirely reduced to the theory of elementary domains in overlapping domains of applicability. Consequently, particle interactions across different energy scales may not be geometrically reproduced. Second, it is unclear whether the mapping from geometrical properties to particle properties is surjective, or whether it is merely an analogy. In other words, it is possible that certain particle properties lack a geometric counterpart. Properties such as isospin and strangeness, in particular, appear to challenge the scope of this geometric reduction.²⁶

4.2.2 The Geometric Derivation of the Particle Content

The preceding construction motivates the kinematics of the elementary domains but does not account for their dynamics. In particular, it does not explain how a domain transitions from an unoccupied state (its vacuum) to an occupied one. Yukawa's picture of the occupation of the elementary domains suggests the following principle:

(CA) The divisibility of space constrains the creation and annihilation of the extended particles that occupy the elementary domains.

²⁴ It is worthwhile noting that the identification of families of particles by invariant quantities is fairly common in physics. For a more modern example based on symmetries, see Castellani (1998).

²⁵ *Hadrons* are subatomic particles constituted by quarks or groups of quarks and antiquarks. Baryons are hadrons composed of an odd number of quarks, whereas mesons are composed of an even number of quarks and antiquarks. Katayama and Yukawa's (1968: 2) theory of elementary domains is an attempt to develop a unified theory of hadrons.

²⁶ Isospin and strangeness are two quantum numbers employed in the study of the strong interaction.

If this holds, then the entire space must be partitioned into elementary domains, regardless of whether any given domain is currently occupied. Particle creation is then described as an excitation of the domain's vacuum state, whereas annihilation corresponds to the suppression of an exciton.²⁷ The excitation of the ground state produces the particle content of the world, with each particle type determined by the geometry of the underlying domain.

This gives rise to a twofold situation. First, the existence of elementary domains is a precondition for the existence of particles: particles can only be created in elementary domains. Second, elementary domains remain well-defined even in the absence of occupying particles. Consequently, there is an asymmetry between the conditions for the definability of the particle content and those of particles. As a consequence, particles *ontologically depend* on elementary domains, but not vice versa.

Nevertheless, while elementary domains possess geometrical properties, Yukawa's proposal requires more than a kinematical correlation between domain geometry and particle type. In order to be developed into a physical theory, it must provide a mechanism by which particle properties are produced from geometrical structures. The key to this problem lies in the contrast between classical regions and elementary domains. A classical spatial region is either empty or occupied, but never both at the same time. In contrast, an elementary domain is a *quantum* object. In the quantum regime, the existence and non-existence of an occupant at a certain time cannot be discriminated in the same way as in the classical regime. The elementary domain is in a superposition of being both *empty* and *occupied*.

The quantum nature of the elementary domain allows for particle creation: the vacuum can fluctuate. When a domain receives sufficient energy, its vacuum state is excited, and a particle is created. In other words, *empty* and *occupied* constitute a pair of complementary properties of the quantum state of the domain. Both properties hold in superposition without contradiction, despite being mutually exclusive. Upon excitation, one of the two properties is realised. Neither *empty* nor *occupied* exhausts the description of the state of the domain.²⁸

5 Elementary Domains and their Philosophical Underpinnings

Yukawa's appeal to quantum indeterminacy²⁹ establishes the elementary domain as a paradigmatic example of a quantum spacetime region. Takano's spacetime model is now integrated with a probabilistic interpretation of the quantum field theoretic

²⁷ An *exciton* is a bound state comprising an electron and an electron hole. It represents an excitation that is capable of transporting energy without the transfer of electric charge.

²⁸ Cf. Bohr (1927; 1949).

²⁹ In *Tabibito* (1982), Yukawa recalls the different stages of his exposure to quantum mechanics. First mediated by TANABE Hajime and Fritz REICHE, he then encountered the "new quantum mechanics" in Max BORN's *The Mechanics of the Atom*, and Erwin SCHRÖDINGER's wave mechanics once it was introduced in Japan in 1926.

wavefunction.³⁰ In this framework, the field introduces a probability distribution over its possible realisations on three-dimensional closed hypersurfaces in spacetime. Similarly, the wavefunction of an elementary domain yields a probability distribution over its particle occupation states, which are realised upon excitation.

Nevertheless, this framework leaves the mechanism of particle creation incomplete. While it accounts for the possibility of transition from vacuum to occupation, it lacks a dynamical “trigger” to initiate such transition.

One possible suggestion comes from Yukawa’s intellectual biography. In *Tabibito*, he explicitly acknowledges the formative influence of the philosophy of Nishida Kitarō on the development of both his physical and philosophical ideas. In a famous passage, he writes: “In ancient times, philosophy and theoretical physics were one and the same; today they are quite apart, but when I was conversing with Professor Nishida, they seemed to come closer together again.”³¹ Yukawa recounts attending Nishida’s lectures in Kyoto and engaging in several conversations with him. The theory of elementary domains undoubtedly echoes aspects of Nishida’s concept of *basho*. Certain pivotal ideas in Nishida’s philosophy suggest intriguing parallels with Yukawa’s treatment of space in his theory of elementary domains. However, the analogy is not without limitations, especially when applied to physics.

I argue that, although Yukawa’s theory bears a conceptual affinity with Nishida’s philosophy of *basho*, it cannot be properly regarded as a “physical model” of it, since it fails to express a fundamental, irreducible feature of the *basho* in physical terms. Consequently, I examine two alternative candidate mechanisms for initiating the process of particle creation. I conclude that they both fall short in key respects.

5.1 Nishida and Yukawa

There are profound parallels between Nishida’s philosophy of *basho* and the concept of spacetime as constituted by elementary domains. As quantum objects, elementary domains are characterised by a pair of complementary (i.e., mutually exclusive yet jointly exhaustive) properties. Consequently, the elementary domain does not depend on specifying one side of this complementarity pair alone; rather, it represents an underlying unity that transcends their opposition. When excitons are added or eliminated, this complementary pair is resolved into one of two possible states: either *empty* or *occupied*.

Tremblay (2018) emphasises the influence of relativity and quantum mechanics on Nishida’s late philosophical thought. Within Bohr’s interpretation, the observer plays a necessary role in the classical description of the quantum regime. For Nishida, this implies that the subject (indeed, the observer) cannot be isolated from the rest of the world it inhabits; rather, the subject is situated within the world and emerges from its internal dynamics. In this sense, subject and world form a complementary pair.

³⁰ See Yukawa (1942: 391).

³¹ Yukawa (1982: 165).

They are opposed to one another, and yet the opposition can be enveloped by a more encompassing unity. More specifically, Berland (this volume) draws a parallel between Nishida's notion of *contradictory self-identity* and Yukawa's complementary pair of states of the elementary domain. He proposes that Yukawa's theory should be regarded as an attempt to address ontological levels where contradictory properties are ascribed to a single structure (the elementary domain) capable of encompassing the contradiction.

These analogies between Nishida's *basho* and the elementary domain suggest that Yukawa may have drawn inspiration from Nishida's philosophy in developing his new unified theory of particle physics. The concept of *basho* appears aligned with earlier physical attempts and supports the idea that physical properties might emerge from geometric properties. Nevertheless, this analogy alone does not explain the mechanism that causes the production of the particle content within the elementary domain.

In Nishida's theory of *basho*, self-mirroring is always an act of *will*. Will unfolds the complementary pairs from the unity of the *basho* in the process of self-mirroring. As such, it provides an answer to the question of what initiates the distinction process and the ascription of properties to the individuals (akin to the subsumption of a grammatical subject within a predicate in logic). If the elementary domain is analogous to the *basho*, Yukawa may have intended to propose this mechanism as a physical model of particle creation. By physical model, I here mean a mathematical construction, motivated by philosophical insight, that receives precise physical meaning as the initiator of the creation process of particle content. Put differently, the dynamics of the *basho* must provide a model that: (i) can be expressed in mathematical language; (ii) admits a physical interpretation as a mechanism underlying the physical entities and processes of the theory; (iii) successfully addresses open issues of this theory, such as the missing trigger for particle creation.

However, it remains unclear what self-determination means *physically* for an elementary domain. The elementary domain is a region of spacetime with peculiar properties. How would such a theory describe anything resembling the role of will in the self-mirroring of the *basho*? In particular, the question is: how can an elementary domain, that is, a spacetime region, *self-determine*?

Nishida was well aware of the connection between spacetime and human knowledge. He proposed understanding space and time "as a universal form of the expression of will."³² Their co-emergence requires accounting for human knowledge, since multiple wills interact in constituting the world. Similarly, Bridgmanian operationalism insists that every physical concept be defined via operations or measurements of its corresponding quantity. Consequently, it introduces an observer capable of performing these measurements at the appropriate scale. However, both operationalism and Nishida's emphasis on cognition appear incompatible with a theory of fundamental spacetime. No act of observation can access the relevant physical scales at which elementary domains constitute the fundamental geometry of the world. This is not merely due to technological limitations, but because theoretical results prohibit

³² *What Lies Behind Physical Phenomena* (1924), in Brink (2021: 26).

any well-posed description of observations at such scales. In this regime, appeals to operationalism or observers become questionable.

5.2 Harmony and Structure

If the trigger for particle creation cannot resemble the functioning of cognition and will as in Nishida's philosophy, one possible solution is to locate its initiation "outside" the isolated elementary domain. In other words, the trigger might be identified with the specific *relational* dynamics between domains. Such a relational perspective aligns more naturally with the physical understanding that any process of particle creation results from the interaction between two systems, as evidenced in particle scattering or correlations with an external environment.

In the case of elementary domains, the dematerialisation of particle physics entails reconceptualising interactions in terms of correlated behaviours among the elementary domains themselves. While Katayama, Umemura and Yukawa (1968) briefly allude to the potential development of a theory of interactions between elementary domains at the conclusion of their paper, they do not elaborate further. Their note suggests that the correlations between domains follow a kind of *pre-established harmony*. Two domains are said to interact when their states exhibit correlated behaviour, such that changes in one state correspond systematically to changes in the other. Notably, direct contact between the domains is not required to manifest such correlation; rather, relativistic causality merely constrains the propagation of these correlation in spacetime. Consequently, one might argue that particle creation in one domain is triggered by a change in the state of correlated domains. Put differently, explaining the production of particle content ultimately involves referencing the global spacetime picture as tiled by elementary domains.

However, while a pre-established harmony could serve as a plausible trigger for particle creation through excitation of the domain's vacuum state, it also raises profound epistemological concerns. Although this approach "saves the phenomena," it does not offer a physical (other than metaphysical) explanation for *why* such correlations arise. In particular, the approach postulates a metaphysical basis for a correlation, but fails to articulate it in precise physical terms that would enhance our understanding of how the correlation operates or how to predict its outcomes. One might argue that there are instances in which such an explanation coexists with the concept of pre-established harmony. Nevertheless, in these cases, it is the empirical understanding of such cases, prior to any metaphysical grounding in harmony, that truly guarantees their physical explainability. Moreover, a pre-established harmony between elementary domains would not allow for the prediction of future states of correlated domains in the absence of additional empirical knowledge, something that is, in fact, impossible at those energy scales.

An alternative approach, which shares similar underlying ideas, postulates the existence of a *structure* that correlates elementary domains. The use of structure-related terminology is preferable to that of pre-established harmony, as it demands

a more precise characterisation of the correlations involved. Provided the requisite information is available, the notion of structure offers an explanation for the interactions between the elementary domains (the *relata* of the structure) and supports predictive power. However, the absence of a fully developed interaction theory for elementary domains ultimately undermines this option. The required details are still lacking.

Furthermore, the adoption of structural terminology invites additional philosophical scrutiny, particularly from the perspective of Nishida's philosophy. Structures are, by definition, networks of relations and *relata*. This raises the question of whether the structure has the same ontological status as its *relata*. In other words, is the structure itself a more fundamental *basho* of forms than the elementary domains it correlates? This is a profound philosophical question requiring further exploration. However, the lack of a comprehensive interaction theory for elementary domains makes such investigation exceedingly difficult.

In conclusion, the central challenge of Yukawa's theory of elementary domains lies in reconciling physics with its philosophical foundations, both in Eastern philosophy and, potentially, in the Western philosophy of science. While alternative approaches exist, they all encounter the same fundamental issues when applied to the theory of elementary domains. The primary difficulty lies in *understanding* the mechanism underlying reality. A secondary but related issue is the physical *description* of these mechanisms, hindered by the limited expressive power of the theoretical language. In this case, the physical theory of elementary domains is still at an early stage of development.

Philosophy can offer valuable insights and suggest viable paths towards the clarification and completion of a physical theory. However, unless these insights can be articulated within a rigorous physical and mathematical framework, the theory remains, in an important sense, incomplete. That is, it lacks the conceptual and technical resources to fully describe the phenomena within its intended domain. Therefore, further inquiry is necessary to (hopefully) bridge the gap between philosophy and physics.

6 Conclusion

The reconstruction of Yukawa's theory of elementary domains highlights the intricate relationship between two distinct layers. The first layer pertains to the physical problem of unification, which became increasingly relevant with the rapid discovery of new particles. The second concerns the philosophical problem of making sense of complementary predicates of an underlying unity, an idea which Yukawa likely inherited from Nishida. This philosophical problem is well integrated into the physical one, as it stimulates a reconceptualisation and the search for new solutions. However, the physical framework ultimately fails to capture the full depth of insight provided by philosophy. It remains unclear how certain aspects of the dynamics of *basho* can be meaningfully interpreted within a *physical* theory such as Yukawa's, and thus

contribute to a complete physical description of reality at the intended scale. In this regard, Yukawa's theory is not a "mere physical model" of Nishida's philosophy of *basho*.

Yukawa's theory was significantly shaped by the debates and philosophical perspectives surrounding the role of renormalisation techniques during the 1960s, particularly in relation to the geometry of spacetime and the development of particle physics. Despite its historical and philosophical interest as an instance of the intersection between physics and philosophy, the theory is, from a modern perspective, ultimately unsuccessful.³³ This is evident from the limited resonance it had at the time, and from the radically different trajectory particle physics subsequently followed. For instance, a more successful unified approach to hadronic physics had already been proposed in 1961 by Murray GELL-MANN. His "eightfold way" treated symmetries as features of the *internal space* of particles, rather than as transformations of spacetime regions. In this framework, different hadrons corresponded to distinct irreducible representations of the unitary group $SU(3)$.

In light of Yukawa's attempt, it is worth pointing out that today we do have a unified framework for particle theory: the Standard Model. This model classifies the properties of the different families of elementary particles and builds a theory of interaction using modern QFT. However, the Standard Model is notoriously incomplete, failing to explain several of its more enigmatic features. One such feature is the mass gap between high-temperature, massless quarks and gluons, and lower-temperature, massive hadronic bound states resulting from confinement.

The focus of unification has since shifted towards reconciling QFT with general relativity at high-energy scales. It is widely believed that a deeper understanding of how fields couple to gravity will provide solutions to deep problems in the foundations of physics and lead to a more comprehensive unified framework. Some approaches to unification attempt to quantise spacetime itself. These approaches, primarily based on general relativity, often struggle to adequately represent field-theoretic phenomena at the relevant scales. Conversely, algebraic approaches avoid spacetime geometry altogether, favouring special "quantum" configurations of fields. This introduces opposite challenges: the recovery of spacetime geometry and the risk of empirical incoherence.³⁴

Both families of approaches rely on deep philosophical intuitions to guide the construction of new physical theories, especially in the absence of empirical data at such high-energy scales. In this respect, the intertwined lines of philosophical speculation and physical elaboration, exemplified by Yukawa's work, remain active. Yukawa's theory thus continues to serve as a rich and engaging testing ground for both traditional and contemporary issues in the philosophy of science.

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³³ However, there have been proposals to recover its main ideas in anti-de Sitter spacetime: see Aouda et al. (2018).

³⁴ For a definition and examination of the problem of *empirical incoherence*, see Huggett and Wüthrich (2013).

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