Whence the Desire to Close the Universe?

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Abstract. The spatial geometry of the universe is widely accepted as flat, based on precision cosmological measurements obtained in the 2000s. In the absence of definitive empirical evidence prior to this period, the geometry of the universe during the 1970s and 1980s was essentially unknown. Yet, one finds within the relevant literature claims suggesting a strong preference for a universe which is "just closed", based on philosophical and other "non-experimental" reasons. The main aim of this article is to identify these reasons and assess the extent to which philosophical reasoning influenced the early interpretation of cosmological density estimates and the development of the dark matter hypothesis. Building on groundwork laid by de Swart (2020), this article expands the historical narrative by: (a) arguing that consensus on the geometry of the universe during that era was more fragmented than historically portrayed, (b) providing an in-depth analysis of Wheeler's motivation for a closed universe based on his relativity notebooks, (c) uncovering and tracing the history of a lesser-known Machian argument for flat geometry by Dennis Sciama, and (d) analysing the fine-tuning arguments arising from Robert Dicke's "coincidence problem". Ultimately, the study provides a nuanced perspective on how philosophical considerations continued to steer cosmological inquiries even at a period when the discipline was consolidating its identity as a precision empirical science.

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1. Introduction

In a seminal article on early dark matter theory, pioneering cosmologists Jeremiah Ostriker, James Peebles, and Amos Yahil open their discussion with the following observation:

> There are reasons, increasing in number and quality, to believe that the masses of ordinary galaxies may have been underestimated by a factor of 10 or more. (Ostriker et al. 1974)

Shortly afterwards, they add this rather notable remark:

> If we increase the estimated mass of each galaxy by a factor well in excess of 10, we increase this ratio by the same amount and conclude that observations may be consistent with a Universe which is "just closed" ($\Omega = 1$) – a conclusion believed strongly by some (cf. Wheeler 1973) *for essentially nonexperimental reasons* (*ibid.*, emphasis added).

The authors then proceed to compile various independent dynamical mass estimates for spiral galaxies and show that the inferred mass of these galaxies from all methods keeps rising approximately linearly with radius from ~ 2 kpc to ~ 1 Mpc. This result clearly implies the presence of extremely extended "halos" of gravitating matter well beyond the visible disk, supporting the conclusion that most of the mass in and around galaxies is dark. Discussing the cosmological implications of their results, the authors note that the calculated masses of spiral galaxies correspond to a local mean mass density of $\Omega \approx 0.2$, indicating that the additional mass required to "close the universe" remains unaccounted for. In the absence of empirical data capable of definitively determining the shape of the universe—data that would only become available several years later (e.g. Spergel et al. 2003)—the authors close by alluding to "well-known" existing "cosmological arguments which can be given for either a low- or high-density Universe" (*ibid. p.L4*). They briefly mention two such arguments: the first, based on Big-Bang nucleosynthesis via deuterium (Rogerson and York 1973), suggesting a very low density parameter ($\Omega \sim 0.01$), and the second, based on large structure formation and gravitational instability (Peebles 1974), suggesting a density parameter $\Omega \sim 1$ and hence a (nearly) flat universe.

Taken literally, Ostriker, Peebles and Yahil's result of $\Omega_{galaxies} \geq 0.2$ implied an open Universe with negative curvature. However, what the authors emphasize in their striking opening lines is that their dynamical estimates push the inferred

Ω closer to the critical value of $\Omega \sim 1$, favoured by many physicists (e.g. Wheeler) for "essentially nonexperimental reasons" and that the remaining matter is to be found outside galaxies.

The central aim of the present article is to identify and analyse these non-experimental reasons that shaped cosmologists' preference for certain models over others in the 1980s, and to assess the extent to which philosophical considerations informed their reasoning. In doing so, we show how the interplay between philosophical arguments and empirical developments of the time significantly influenced scientific practice at a time when cosmology was acquiring a new reputation as a precise, observation-driven discipline. Although some arguments for the geometry of the Universe (and thus its matter content) were grounded in empirical data, we shall see that, during the 1970s and 1980s, the community had no consensus on the exact geometry of the universe, and that some of these arguments were deeply coloured by explicitly philosophical considerations.

An important first step in this direction has been made by de Swart (2020), who presents a compelling argument showing how the widespread preference for additional matter to 'close the universe' in the 1970s largely motivated the establishment of dark matter on a cosmological level. His narrative is focused on the landmark papers by Ostriker et al. (1974) and Einasto et al. (1974) in which the authors argue that the total mass of galaxies is significantly underestimated and that additional mass-energy is required to achieve a total density of $\Omega \geq 1$. According to de Swart (2020, p.268), this preference for a closed geometry was widely shared by many cosmologists at the time and it was closely related to a Machian conception of general relativity (GR) of which, as we shall see, John Archibald Wheeler (1962; 1973) was a prime advocate. In de Swart's historical account, the realization that the observed mass density at the time appeared to be much less than the required critical density to close the universe essentially made the observational anomalies of missing mass in clusters and flat rotation curves of stars in galaxies relevant, giving birth to the concept of non-baryonic dark matter as we know it today. The core idea was that the missing additional matter required for a closed universe was the same 'dark matter' required to explain the velocities of galaxies and the flat rotation curves of stars.

Our analysis expands on de Swart's work by making four novel contributions. First, we clarify that the consensus for a closed geometry was perhaps not so profound as implied by de Swart, and that opinions regarding the geometry of the universe in the 1970s and 1980s were rather divided (Section 2). Nonetheless, as rightly noted by de Swart, there was at the time a strong consensus that the mean mass density of baryonic matter was insufficient to account for the available observations and that additional matter was required, regardless of the actual shape of the universe. Second, we discuss the well-known Machian argument for a closed universe riginating with Einstein and later developed and promoted by Wheeler (Section 3.1) which was based on a need to eliminate cosmic boundary conditions as well as the convergence of the inertial influences coming from distant matter.

Third, we bring to the surface a lesser-known Machian argument due especially to Dennis Sciama, which nevertheless leads to a different conclusion, namely a preference for a universe with flat geometry (Section 3.2). Whereas de Swart (2020) characterises this argument as "similar reasoning" to Einstein and Wheeler's argument for the closure, we find, on the contrary, that despite also being based upon considerations of Mach's principle, this argument differs wildly in its methodology. Finally, we also present and discuss a markedly different argument based on fine-tuning considerations and the 'coincidence problem' due to Robert Dicke, which, as will be shown, was the predecessor of the well-known flatness problem in cosmology (Section 4).

Further notable novel results, presented in Section 3, are the following: (a) We respond to a question raised by Blum and Brill (2020) concerning the possibility that Wheeler may have been influenced by Sciama (1953a) in his inclusion of a brief derivation of the relation[1]

$$\frac{G}{c^2}\sum_i \frac{m_i}{r_i} \sim 1 \tag{1}$$

in a talk in Tokyo in the Autumn of 1953. We show that arguments of this kind were already more widely known at the time and suggest that Wheeler may have been influenced by Hermann Weyl in this regard. (b) We uncover an argument by Hermann Bondi which clarifies why relation (1) was taken by some (including Sciama) to imply a universe of critical density. (c) We trace this Machian programme of Sciama and his colleagues through the late 1960s and early 1970s, and clarify the reasons for its eventual abandonment by these cosmologists. Apart from uncovering the historical influence of the various arguments from Mach's principle, we also make assessments concerning their strength.

[1]Here m_i and r_i are the masses of the universe and their 'distance' from a given point; here, and throughout this paper, G is the strength of gravitational coupling and c is the speed of light. See sections 3.1.2 and 3.2 for in-depth discussion of this and similar relations.

With respect to sources, in our discussion of Wheeler's promotion of the universe's closure we will be citing extensively from his *relativity notebooks*.[2] Our discussion of Dennis Sciama's engagement with Mach's principle is informed by close reading of his doctoral thesis Sciama (1953b) which has not previously been cited in published work and was recovered by Jonathan Fay from the University of Cambridge archives.[3]

Taken together, the analyses presented here reveal the diverse motivations behind the scientific exploration of cosmic geometry, which, among other consequences, supported the eventual establishment of dark matter as a cornerstone of modern cosmology. We demonstrate how non-empirical considerations, such as prior commitments to certain appealing hypotheses and scepticism towards fine-tuning, guided the development of cosmological models in the absence of definitive observational data. By examining these arguments in detail, we aim to illuminate the intellectual and historical context in which philosophical reasoning influenced the scientific decisions of cosmologists in the 1970s and 1980s, a period when cosmology was striving to establish itself as a precise and empirically driven field.

2. Dark Matter and Geometry

Before we delve into our main analysis, it is useful to clarify the relationship between the geometry of space and the energy-matter density of the universe to appreciate the strong connection between dark matter and the shape of the universe. Consider the standard Friedmann equation, with no cosmological constant:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \tag{2}$$

where $H^2 \equiv (\dot{a}/a)^2$ is the Hubble parameter, a is the scale factor of the universe describing how physical distances change with time, k is the curvature term, ρ is the matter-energy density of the universe, and G and c are constants representing the gravitational constant and the speed of light respectively. It can be easily seen that for a given H, there is a special value of the matter density:

$$\rho_c(t) = \frac{3H^2}{8\pi G} \tag{3}$$

[2]We will be citing especially from Wheeler's first relativity notebook, which we will be abbreviating as WR1 following the citing convention of Blum and Brill (2020). We also in some instances cite from the second relativity notebook as well as the seventh, which will be abbreviated WR2 and WR7 respectively. These notebooks are to be found in the John Wheeler Papers, held at the American Philosophical Society in Philadelphia, Section V, Volumes 39, 40 and 54.

[3]We are thankful to the family of Dennis Sciama for allowing us to order a digitalisation of the manuscript.

that renders the geometry of a Friedmann–Lemaître–Robertson–Walker (FLRW) universe flat ($k = 0$). This is the so-called critical density ρ_c and corresponds to the amount of matter necessary to provide enough gravitational attraction to balance the expansion of the universe. If the actual mean matter density of the universe is less than the critical density, then the universe has a negative curvature and expands forever. If it is equal, then the universe is flat and 'balanced'. If it is larger, then it is 'closed', and matter eventually halts and reverses the expansion. These possibilities are typically expressed in terms of the density parameter $\Omega(t) = \rho/\rho_c$ which is a dimensionless quantity describing the ratio between the actual density of the universe and the critical density. Depending on the actual matter density of the universe (and regardless of the type of matter), the density parameter can be (i) $0 < \Omega < 1$, (ii) $\Omega = 1$, or (iii) $\Omega > 1$ corresponding to a hyperbolic, flat, and spherical universe respectively. Hence, amongst other things, the Friedmann equation indicates how the amount of matter in an FLRW universe relates to its geometry via the curvature term.

As noted in the introduction, de Swart (2020) provides a convincing argument showing how a widespread preference for a closed universe in the 1970s and 1980s played an instrumental role in the establishment of the dark matter hypothesis. It should be noted however, that the terminology used by de Swart in terms of a closed geometry is potentially ambiguous, since, as we shall soon see, some prominent cosmologists of the time were in favour of a flat rather than a closed geometry. The widespread desire to 'close the universe' to which de Swart refers, should be understood as a requirement for a total mass-energy density corresponding to $\Omega \geq 1$, i.e. a value of ρ_0 which is *at least equal* to the critical density and therefore eliminates the possibility of an open universe.

The philosophical arguments for closed and flat cosmological models will be presented in detail in later sections. In the present section, our aim is to show that some cosmologists were in fact in favour of an open universe, and that during the 1970s and 1980s there was no clear consensus on the exact geometry of the universe. Nonetheless, we align with de Swart in emphasising that by the early 1980s most cosmologists were convinced that, regardless of the actual geometry, an additional component of non-baryonic matter had to exist, and that this conviction was sufficient for the establishment of non-baryonic dark matter.

Assessing the precise degree of preference for different cosmological models within the scientific community in the decades preceding the establishment of dark matter is challenging. Nevertheless, it should be noted that the literature of the time contains claims from cosmologists who endorsed the existence of additional non-baryonic matter while simultaneously advocating for an open cosmological model. For instance, in a letter published in *Nature*, Lindley (1981, p.391) begins by noting that 'The feeling among cosmologists is that the universe is open, with Ω perhaps in the range 0.1 – 0.5' and then proceeds to discuss the transition from quantum to classical gravitational behaviour near the initial cosmic singularity and propose a physical process 'whose action leads naturally to a Universe which is open' (*ibid.*). An earlier example comes from Chiu (1967) who considers a set of astronomical observations from the cosmic microwave background (CMB) radiation and the intergalactic density of hydrogen to conclude that 'Our Universe can be described by an open cosmological model.' (p.1) and that 'it is unlikely that [it is] described by a closed model.' (p.3). The most characteristic example, however, comes from a rather influential paper of the time by Gott et al. (1974), who clearly express their preference for an open universe invoking arguments from simplicity and unification: 'Open models [...] explain simply and readily a wide variety of observations, from the abundance of deuterium to the value of the Hubble constant. If a closed model is to fit the observations, a number of additional (apparently ad hoc) processes must be called into play, which mimic in a complicated fashion the same result one would obtain from a simple open model' (p.553).

Interestingly, in this paper the authors close their discussion by considering the possible sources of mass in a purportedly closed universe, noting: 'Finally, let us discuss where the missing mass can be hiding if it is demanded on *theological or other grounds* that $\Omega \geq 1$' (Gott et al. 1974, p.550, emphasis added). The precise meaning of 'theological or other grounds' is not made explicit in the text, but a plausible interpretation is that the authors were alluding to the fact that the available arguments for a closed universe at the time were not empirically based. Rather, as will be shown in the following sections, these arguments were largely grounded in aesthetic, philosophical, and metaphysical considerations and were thus seen by some as 'theological' in nature.

de Swart (2020, p.275) also cites this passage to support his view that there was a widespread 'religious-like' belief in a closed universe amongst many cosmologists of the time. We do not dispute de Swart's reading. Our aim is to emphasise, however, that there also existed a substantial group of cosmologists who supported an open universe while nonetheless accepting the possible necessity of additional non-baryonic matter. In other words, although there was no clear consensus on the geometry of the universe—and hence the actual matter density—most cosmologists were convinced, based on empirical arguments, that the observed luminous (baryonic) mass was insufficient and that additional dark (non-baryonic) mass must be present.

The reasons for this consensus are well known and have been widely addressed in the literature on the history and philosophy of dark matter.[4] We therefore do not repeat them here. For completeness, we shall only mention that the most important arguments in favour of additional matter were mainly based on various estimations of the actual mean mass density at the time from different methods (e.g. Field 1972), the calculation of baryonic density from primordial abundances of helium and deuterium (e.g. Schramm and Wagoner 1977), and the conception of the structure formation problem with respect to data from the cosmic microwave background radiation (Peebles 1965, 1966, 1968). The common feature of these arguments was that the matter density inferred from various sources of light and primordial abundances of baryonic elements could not account for the available data. By the early 1980s, dark matter as we know it today, i.e. as a (family of) non-baryonic primordial particle(s) at a cosmological scale, had gradually gained widespread acceptance within the scientific community. The exact percentage of the contribution of this non-baryonic component to the total mass density of the universe however—and hence the geometry of the universe—was essentially unknown.

Nevertheless, a number of cosmologists were strongly motivated by independent philosophical arguments in favour of an—at least—closed universe, that is, a universe which is either closed ($\Omega > 1$) or flat ($\Omega = 1$). In the following three sections, we trace the origins of these preferences by presenting what we consider to be the three most important philosophical arguments regarding the geometry of space in the twentieth century, prior to any experimental determination of the curvature of the universe. As will be shown, two of these arguments are based on Mach's principle but nonetheless follow entirely different trajectories of reasoning. The third argument involves an independent line of thought based on fine-tuning considerations.

[4]See, for instance, Sanders (2010); de Swart et al. (2017); Bertone and Hooper (2018); Peebles (2020); Antoniou (2025).

3. Leveraging Mach's principle

The earliest argument for a particular cosmic geometry in the context of general relativity was put forwards by Albert Einstein (1917[1952]). Einstein's argument is based on his attempts to implement Mach's principle in general relativity and concludes that this is only possible if the universe is *closed*.

It is well recognised that one of the key motivations for Einstein's development of GR was the perceived lack of conformity with Mach's principle of classical mechanics, however, whether GR was successful in this regard, and indeed, how one should even define Mach's principle, remains contentious.[5]

Indeed, in the proceedings from the 1993 Tübingen conference on Mach's principle, Barbour and Pfister (1995, p.530) identify 21 distinct possible meanings of "Mach's principle" which are discussed at various points throughout the volume. These include: a general appeal to the principle of sufficient reason, some kind of extension of the principle of special relativity (SR) to accelerations, a hypothetical inductive interaction for inertial forces, and a requirement for spatial closure to eliminate cosmic boundary conditions (as we will discuss in this section). Some of these definitions are explicitly *cosmological* in their scope whereas others do not immediately appear to be so. For our purposes in this article, we will avoid getting caught up in these controversies and adopt the view expressed by Fay (2024) as our standard account.

What we can say with some clarity however is that at the core of what has been called "Mach's principle" there is a *hypothesis* about the origin of inertia, which in a more or less vague manner implies the *unity of the universe*, and thereby connects the physics of everyday systems to their place within this undivided whole. It is essential to this form of reasoning that it is possible to consider the totality of the material world in some sense as a single object, which (importantly) lacks any physical reality outside of itself, or with respect to which it might be referred.

In these more abstract and general terms, Machian thinking has often been generalised beyond the specific domain which Mach discussed.[6] In this way, it becomes fair to describe as "Machian" various realisations that the universe as a whole should have no particular *size*, *position, time, velocity* or overall *angular velocity* on the basis that these spatio-temporal specifications can only be defined between two

[5]See for instance Hoefer (1994); Norton (1995); Barbour (1995); Fay (2024); DiSalle (2002); Staley (2021); Shi (2025) for various views on Mach's principle and its relationship to general relativity.
[6]See especially Julian Barbour's work for this; for instance Barbour (2010, 2001, 1981).

or more real things. These considerations bring "Mach's principle" into close association with certain "relativity principles", and this is often how Mach's principle has been understood. However it is important to note that as a mere requirement for the relativity of spatio-temporal quantities, Mach's principle is essentially agnostic to the empirical content of physical theories (Fay 2024); something else must be added in order to consider Mach's principle as a meaningful guide to the development of physical theories.

The key ingredient in question is what Misner et al. (1973, p.547) call the "democratic principle" that not only is inertial motion relationally defined, but that all of the massive bodies of the universe play a proportional part in this relational determination (specifically a part proportional to their mass). Various "Machian" models have tried to implement this relativisation of inertia both before and after Einstein's work (and in many cases independently of it).[7] What Einstein's work has done the most to influence, however, was the view that the relativisation of inertia should be brought into relation with gravitational theory. This was made clear by Einstein's focused investigation of what became the various forms of the "principle of equivalence" which indicated a profound connection between inertia and gravitation. The first indication of this connection, however, was by Friedlaender and Friedlaender (1896) and preceded Einstein's equivalence principle by over a decade.

3.1. **The Einstein-Wheeler closed universe.** To understand how Einstein and Wheeler both came to associate Mach's principle with the closure of the universe, we have to look closely both at the historical context of their ideas: How did Einstein's winding path towards GR lead him to favour the closed universe model? What was the context through which Wheeler got interested in GR and how did this lead him to follow in Einstein's footsteps? In the following two subsections we shall answer each one of these questions respectively.

3.1.1. *Einstein.* It is often forgotten just how central was the role of Mach's principle in Einstein's initial development of general relativity. Two principles motivated and guided Einstein in his development of his theory: the equivalence principle and Mach's principle, which were seen as complementary to one-another.[8] Whereas the equivalence principle suggests that gravity and inertia ought fundamentally to be unified such that gravitationally affected motion in one frame may be interpreted as

[7]See for context the contributions of Friedlaender and Friedlaender (1896); Föppl et al. (1904); Hofmann (1904); Reissner (1914, 1915); Schrödinger (1925); Sciama (1953a); Treder (1972); Assis (1989); Barbour (1974, 1975); Lynden-Bell (1967), to name a few. Many of these are discussed in Barbour and Pfister (1995) and Fay and Braun (2025).

[8]See Einstein (1996, p.228) for his explicit admission of these motivations.

inertial motion in another, Mach's principle suggests that, like gravity, inertia has its origin in mass-mass interactions. The two together thus harmoniously pointed towards a theory in which the inertia of a mass would be defined with respect to other masses, and the medium through which the relativisation of inertia would be accomplished would be the gravitational field.

After Einstein completed the field equations in November 1915 (Einstein 1915), he initially hoped his theory would easily vindicate the 'relativity of inertia,' but it soon became clear that certain non-Machian features—especially vacuum solutions and the need for boundary conditions that effectively re-introduce inertial structure 'at infinity'—could not easily be eliminated. Einstein spent 1916–1918 trying to secure a more fully Machian framework by restricting admissible solutions, discussing these issues most of all with Willem de Sitter (Hoefer 1994). Einstein at first suggests that the theory will be made Machian if the components of the metric at infinity are either 0 or ∞ such that they are immune to coordinate changes, however around December 1916, possibly prompted by objections from de Sitter, Einstein became convinced that this strategy is no longer viable and suggested instead to remove the need for boundary conditions altogether which would be achieved if the universe is of overall positive curvature such that it is spatially closed.[9] His argument would be published in Einstein (1917) where he also famously introduces his cosmological constant Λ in order to ensure that his closed universe with non-zero mass density does not collapse upon itself. In 1918, Einstein adds a section on cosmology to the new edition of his popular booklet "Über die spezielle und allgemeine Relativitätstheorie: Gemeinverständlich" (Einstein 1918) in which he argues that our universe should be spatially closed provided that there is an average matter density greater than zero everywhere in space (Einstein 2015):

> If we are to have in the universe an average density of matter which differs from zero, however small may be that difference, then the universe cannot be quasi-Euclidean. On the contrary, the results of calculation indicate that if matter be distributed uniformly, the universe would necessarily be spherical (or elliptical). Since in reality the detailed distribution of matter is not uniform, the real universe will deviate in individual parts from the spherical, i.e. the universe will be quasi-spherical. But it will be necessarily finite. In fact,

[9]See Hoefer (1994, p.311-312) for a discussion of Einstein's rejection of his initial *divergent-boundary conditions* approach and the role de Sitter may have played in this.

> the theory supplies us with a simple connection between the space-expanse of the universe and the average density of matter in it

All this of course is prior to the development of experimental cosmology and evidence of Hubble expansion, which would undermine the argument since the critical density will no longer be zero.

In some later texts Einstein emphasised the connection with Mach's principle. For instance, in his 1934 collection of essays *Meine Weltbild* (Einstein 1934b), he endorses the connection between Mach's principle and the closed universe (Einstein 1934a):

> In my opinion the general theory of relativity can only solve [the Mach problem] satisfactorily if it regards the world as spatially self-enclosed. The mathematical results of the theory force one to this view, if one believes that the mean density of ponderable matter in the world possesses some ultimate value, however small.[10]

In summary, Einstein endorsed the spatially closed spherical universe model for two separate reasons: (1) the necessity, as he saw it, to ensure that boundary conditions at infinity are eliminated; and (2) working prior to knowledge of the Hubble expansion, Einstein believed that a universe with a finite mean density, no matter how small, should be spatially closed.

As we will see, both of these motivations would be undermined by advances in experimental cosmology, and many of Einstein's contemporaries would reject his arguments. Nonetheless, the notion that the universe should be closed on grounds of Mach's principle survived well into the 1980s, and this has a lot to do with the influence of John Wheeler on a new generation of relativists in the renaissance period of general relativity. In the next section, we will examine the development of Wheeler's thoughts on this issue through the lens of his relativity notebooks, focussing in particular on the first notebook which includes the notes Wheeler recorded as he began teaching his first class on general relativity at Princeton in 1953.

3.1.2. *Wheeler.* Like Einstein, Wheeler's interest in gravitation was intimately bound up with the Machian programme from the start. The reason for this was that over the course of the 1940s, Wheeler had collaborated with his student Richard Feynman to develop an action-at-a-distance (AAD) reformulation of electrodynamics involving a combination of advanced and retarded potentials propagating from

[10]We know also that this particular passage influenced Wheeler as he quotes from it in the epigraph to his section on Mach's principle in the influential textbook *Gravitation* Misner et al. (1973), which was based on the lecture series he taught at Princeton on general relativity.

particles in both directions time-wise (Wheeler and Feynman 1945). As Blum and Brill (2020) explain, for Wheeler this was part of a programme to reconsider physics from the perspective of a particle-first ontology in which all electromagnetic radiation must have a source and a destination (an "absorber"). It was in this context that Wheeler began turning his attention towards gravitation in the late 1940s, hoping that he would be able to recover general relativity in a similar manner through an AAD framework. Wheeler struggled to develop this theory, and gradually came to the conclusion that he should focus on studying general relativity on its own terms, and naturally, Mach's principle was his way in as it provided the perfect analogue of the absorber condition in AAD electromagnetism. Since there is no better way to learn a subject than to teach it, Wheeler persuaded the physics department at Princeton to let him teach their first course on GR. This was an important moment for the renaissance of GR as many of Wheeler's students in this period would go on to become significant relativists.

The notebook entries discussing his first classes teaching of the course evidence Wheeler's evolving knowledge of general relativity. Of particular interest are Wheeler's notes from a class dated to the 31st of March 1953 concerning the interpretation of the Schwarzschild solution. The issue is raised that the solution seems to contradict Mach's principle since it is Minkowski at infinity, meaning that a test particle infinitely far away from the central mass will still have inertial properties despite there not being any other matter with respect to which these inertial properties would be defined. Wheeler uses this issue to explain to his class that the field equations alone are insufficient as a mathematical realisation of Mach's philosophy and that implicitly, the Schwarzschild solution "corresponds to a many mass world—point mass, plus masses at large distances" (WR1, p.103). Wheeler then also claims that "there is a one-to-one correspondence between mass distribution and metric only when space closes up on itself", a point which he will at various times later on remind himself to search for a proof of.[11] He then recounts introducing the idea of the closed universe:

> Describe how many such masses allow a closing up of the space on itself. Connection between mass density and curvature qualitatively of form $\frac{1}{\text{rad of curv}} \sim \rho_m$.

[11]See WR1 (p.105) in which he highlights this question, adding a note in the margin: "Very important question of principle."

At this point, it seems as though what Wheeler has in mind is the original Einstein universe in which any positive non-zero density "ρ_m" will lead to closure. Interestingly, it is his students that raise the objection: "Question raised in class whether mass density enough to permit open or closed universe, in view of expansion rate" (WR1, p.104). Wheeler then responds that this must mean that the density must be assumed to be above the critical threshold emphasising that

> closure so fundamental to whole Mach idea that in present state of knowledge think of density value as having to yield precedence to Mach principle.

Interestingly, Wheeler then adds another argument in favour of the closure of the universe on grounds of Mach's principle: "if think of inertial properties of a particle in a given region of space as coming from summation over all other regions of space, à la $1/r^2$, that system should have to close up to allow convergence. Recall presence of sum in acceleration analysis of inertia" (WR1, p.104).

It is not clear that Wheeler is right about this however, since if the integration is assumed to be performed over the past light cone for instance, one would need to justify why the mass content of the universe is not integrated over infinitely many times. The condition that an integral over the mass of the universe must converge might better be leveraged as an argument for the universe's temporal horizon rather than an argument for its spatial closure. Wheeler does show some awareness that there may be further issues here, writing a note in the margin: "Investigate whether light ever gets to cross universe (closed boundary conditions)". Later on in the notebook Wheeler also begins to flirt with the question of whether temporal closure should be required in addition to or perhaps instead of spatial closure;[12] the clearest passage expressing this is (WR1, p.134):

> We have been concerned[13] how to formulate the condition that the universe close up in such a way, or with such completeness, either in space or in time or in both, that the connection between distribution of matter and metric shall become unique.

As his teaching of the course went on, Wheeler took the chance when possible to consult with various colleagues including Hermann Weyl, Eugene Wigner and John von Neumann. On the 13th of April, Wheeler recounts asking von Neumann

[12]Mentions of possible temporal closure can be found in p.108, p.122 and p.134 of the first relativity notebook.

[13]Here the word "concerned" replaces the original word "worried" which Wheeler decided to cross out.

about the question of uniqueness between the metric and matter distribution in a spatially closed universe, noting that von Neumann “had no feel for uniqueness question” (WR1, p.120). In general, Wheeler received pushback from his colleagues who raised various issues with the viability of Mach’s idea in GR. Weyl appears to have been the colleague with whom Wheeler discussed the issues the most, indeed Weyl’s comments were much more substantive than those of other colleagues which has a lot to do with the fact that in 1924 Weyl had written directly about the status of the Machian programme in the aftermath of the theory of general relativity in a charming dialogue set between fictional philosophically inclined physicists “Petrus” and “Paulus”. This dialogue is possibly one of the most perceptive analyses of the issues that Mach’s principle faced in the aftermath of general relativity, and already contained all the answers that Weyl needed to give Wheeler. The main problem that Weyl had identified in the dialogue was that GR, by undermining the classical notion of a rigid reference system, poses serious difficulties for the central presupposition of Mach’s proposal, that is, the possibility of unmediated spatio-temporal relations at a distance (Weyl 1924):[14]

> according to the general theory of relativity, the concept of relative motion between several separate bodies is just as untenable as the concept of the absolute motion of a single body.

Nonetheless, the dialogue as a whole is highly sympathetic towards Mach’s principle, acknowledging its appeal and emphasising it’s centrality to the motivations for the development of GR.

Wheeler’s first discussion with Weyl concerning Mach’s principle takes place on the 2nd of April 1953, here Wheeler recounts proposing to Weyl that he support “Einstein’s view that metric and distribution of matter only then have a one-to-one relationship when it is demanded that solutions $R_{ik} = 0$ or T_{ik} close up.” Weyl, Wheeler reports, is “inclined to regard this view as completely unconvincing.” Some days later, on April 14th 1953, Wheeler has Weyl round to dinner and makes a note following their chat (WR1, p.125): “Doesn’t believe that Ein. theory = Mach’s prin. See Petrus - Paulus in die Naturwiss,” which refers to Weyl’s 1924 dialogue. On April 18th (WR1, p.129), Wheeler shows that he has read the dialogue taking note of two of the main points that Weyl raised: (1) the impossibility of defining unmediated relative motion at a distance, and (2) that Mach’s principle is “pure

[14]Translation of: “nach der allgemeinen Relativitiitstheorie der Begriff der relativen Bewegung mehrerer getrennter Korper gegeneinander ebensowenig haltbar ist wie der der absoluten Bewegung eines einzigen.”

speculation until we can observe the universe as a whole." Despite the pushback that Wheeler received from Weyl, Wigner and von Neumann, he nonetheless nonetheless continues to defend what he saw as Einstein's position that the closure of the universe will resolve his difficulties (WR1, p.121): "I am inclined to agree with Einstein, + disagree with Weyl + von N. + Wigner but haven't yet seen way to do a proof".

It is well documented that Wheeler had a deep admiration for Weyl. Evidence of this can be found in the address he prepared for the *Internationaler Hermann-Weyl-Kongress*—held in celebration of the centenary of Weyl's birth—in which Wheeler writes (Wheeler 1986): "If I had to come up with a single word to characterize Hermann Weyl, [...] it would be that old-fashioned word, so rarely heard in our day, 'nobility.' I use it here [...] also in the sense of showing exceptional vision."[15]

Concerning the issue at hand however, it appears that Wheeler was quite profoundly influenced by Weyl's dialogue on Mach's principle specifically. Wheeler's book, along with Thorne and Misner, *Gravitation*—which represents the culmination of the knowledge accrued over the course of Wheeler's teaching of general relativity at Princeton—specifically cites Weyl's 1924 dialogue—which is described as "delightful"—along with works by Dennis Sciama as further reading (Misner et al. 1973, §21.12, p.543). Although as we saw, he had already located and read Weyl's dialogue in April 1953, Wheeler was careful to keep a stained cutout from a letter he received from Weyl dated to May of that year in which Weyl details exactly where to find the dialogue in question (WR2, p.24).

It is interesting to remark here too that it is quite plausible that it was his reading of Weyl's dialogue that inspired Wheeler to adopt the same format for a pivotal talk he gave in the autumn of 1953 in Tokyo. Wheeler's Tokyo talk is cast as a dialogue between two heroic figures from Japanese history: "Saigo Takamori" and "Sugawara no Michizane", mirroring the traditionally biblical names "Petrus" and "Paulus" chosen by Weyl for his dialogue.

Wheeler's Tokyo talk, which was on the problems of elementary particle physics, and the associated "Tokyo program" (WR2, p.35), brought together many of the ideas that Wheeler had been toying with that year and previously, and represented a certain shift in Wheeler's thought from focus on a particle based

[15]See (Furlan 2020) for some discussion of Weyl's influence on Wheeler.

ontology (especially in the 1940s) to a field-first approach.[16] It is plausible that Weyl directly influenced Wheeler in this shift too, especially given that Weyl's dialogue follows a very similar conceptual progression, coming to terms with the necessity of a field-first picture of nature, and thus moving away somewhat from the Machian view.

Wheeler's Tokyo talk involves a brief discussion of Mach's principle, during which Wheeler makes a new argument, not related to the Einstein closed-universe idea. Wheeler considers a simple AAD gravity model with an interaction potential representing inertia in addition to the Newtonian and analogous to the long-distance Liénard-Wiechert field of an accelerating charge in electrodynamics (Blum and Brill 2020). Wheeler's simple expression gives a *relative inertia* + *gravity* model characterised by the equation of motion:

$$\frac{Gm_1m_2}{r^2} - \sum_k \frac{Gm_1am_k}{c^2r_k} = 0, \tag{4}$$

which reduces to the normal Newtonian equation of motion provided that:

$$\frac{G}{c^2}\sum_k \frac{m_k}{r_k} = 1, \tag{5}$$

where the sum extends over all of the masses of the universe m_k which are at distances r_k from the mass m_1. This curious equation (5) has long been associated with Mach's principle, and has a winding history of being forgotten and rediscovered independently (sometimes with slightly different constants of proportionality), including by Schrödinger (1925).[17] As we will see shortly, Wheeler's argument is remarkably similar to the derivation made by Dennis Sciama in his article "On the origin of inertia" (Sciama 1953a) published earlier that same year, although Sciama was working with a simplified vector potential gravitational theory also based on some analogies with electromagnetism. Blum and Brill (2020) have discussed the striking similarity to Sciama's paper, noting that:

> It is unclear whether Wheeler knew of Sciama's argument and merely rephrased it in AAD terms, or whether he had found it independently in his attempts at constructing an AAD version of gravity, building on an AAD formulation of electrodynamics.

Since Sciama's paper was not discussed in Wheeler's talk or in the lead up to it in the notebooks, it seems most likely that we are witnessing a convergence of

[16] See Blum and Brill (2020) for a deep dive into Wheeler's Tokyo programme as well as the English translation of his talk in Tokyo (which had formerly only been published in Japanese translation).
[17] See Fay and Braun (2025) for a thorough overview of many of the derivations.

thought, however there is one source that we know Wheeler had read that may have influenced him to derive this expression, namely, Weyl's dialogue. At the start of the second half of Weyl's dialogue, Petrus recounts the "well known result of Thirring" (namely from Thirring (1918)) that a "rotating hollow sphere H (representing the celestial sphere)" produces forces analogous to centrifugal forces on a test body at it's centre (Weyl 1924). Petrus then explains that:[18]

> one can explain, in Machian terms, the centrifugal force and the flattening of the Earth as effects of the starry heavens rotating around the stationary Earth, if one assumes that the mean distance of the stars is as large as the gravitational radius of their total mass.

By the "gravitational radius" Weyl is referring to what we would call the Schwarzschild radius $r_s = \frac{2GM}{c^2}$. Petrus's argument therefore implies that this cosmological Machian criterion might be satisfied provided that:

$$c^2 \sim \frac{GM_u}{R_u}, \tag{6}$$

with M_u and R_u as the mass and radius of the universe respectively, which is an expression that is essentially equivalent to that derived by Wheeler, and which indeed Wheeler would continue to endorse later on.[19] In fact, in the 1973 textbook *Gravitation*, Wheeler includes a retelling of Thirring's argument concluding with the same result as Weyl, that the expression: "$\sum_{\text{far away stars}} \frac{m_{\text{star}}}{r_{\text{star}}} \sim \frac{m_{\text{universe}}}{r_{\text{universe}}}$, must be of order unity" (with units chosen such that $G = c = 1$), and adds a remark that this result may be consistent with his preference for closed universe models: "Just such a relation of approximate identity between the mass content of the universe and its radius at the phase of maximum expansion is a characteristic feature of the Friedmann model and other simple models of a closed universe" (Misner et al. 1973, p.548, eq.21.160).[20] Although it is plausible that Weyl influenced Wheeler in this regard, it is also perfectly possible that Wheeler arrived at it independently.[21]

[18]Translation of "Man wird danach in Machscher Weise die Zentrifugalkraft, die Abplattung der Erde als eine Wirkung des um die ruhende Erde sich drehenden Sternenhimmels erklären können, wenn man annimmt, daß die mittlere Entfernung der Sterne so groß ist wie der Gravitationsradius ihrer Gesamtmasse."

[19]Note that Schrödinger (1925) derives a result of the same order the following year in a classical model using a similar method to Wheeler, and this is often characterised as an original discovery (Schrödinger 1995; Assis and Assis 1999). However it should be noted that Schrödinger's paper actually cites Weyl's dialogue in the introduction, so it is possible that Schrödinger too caught wind of it from there.

[20]It is safe to assume that Wheeler would have written the parts of the textbook relevant to Mach's principle given that Wheeler was much more involved with this topic than Misner or Thorne; moreover that section of the book strongly reflects Wheeler's sensibilities on the issue.

[21]Note for instance that Wheeler's remark about the sum over the inertial influence of matter needing to converge based on the "presence of sum in acceleration analysis of inertia" is given in

What is important to understand about the last result is that the Thirring-Weyl type argument is quite different in nature from the Einstein-Wheeler argument for the closed universe. The logic of the argument is of a totally different nature even though both appear to proceed from a prior commitment to Mach's principle. As we will see in the next section, Sciama (as mentioned), developed this argument more clearly, however he was inclined to regard it as an endorsement of a flat universe of critical density, rather than a spatially closed one.

As we have seen, many of the questions concerning Mach's principle that Wheeler raised in his early relativity notebooks were vague and inconclusive, the arguments used to favour the closed universe model did not seem to hold up to scrutiny. Several years later, we see that the situation remains much the same. In his 7th relativity notebook, covering the period from 1959-1960, while sat on a train to Syracuse, Wheeler writes an interesting passage reflecting on a seminar titled "Comments on Mach's principle" he had recently given which was attended by various notable figures including Roger Penrose (WR7, p.150). Wheeler recounts that there was "much discussion [following his talk of] whether retarded or simultaneous potentials" should be used, indicating that even this basic consideration has not been decisively resolved by 1959. Among the points listed requiring further examination, Wheeler includes: the question of the closure of the universe, the meaning of $1/r$ in the potential, whether the potential is *retarded* and what is the mass-energy term that should be taken as the source. Additionally, Wheeler reports that Penrose raised questions both concerning whether inertia at a point should be determined by the interior of the light cone or merely the light cone surface, and concerning the technicalities of temporal closure. Overall, Wheeler's notes following his talk appear to show little evidence of conceptual progress on the issues since 1953. The idea of the closed cosmos, which had been initiated by Einstein in the context of a universe thought to be eternal, roughly homogeneous and non-expanding, continued to endure in association with Mach's principle, although the arguments in favour of this concept appear to have received little clarification.

3.2. **Sciama's lost promise.**

3.2.1. *Secret hopes.* Dennis Sciama's initial and most lasting contribution to the literature on Mach's principle comes from his article "On the origin of inertia"

p.104 of his notebook and precedes his reading of Weyl's dialogue. It is plausible that this kind of reasoning refers to a simple calculation similar to that which Wheeler presented in Tokyo. What can be said with certainty however is that this kind of argument did not originate with Sciama (1953a) but was already somewhat more widely recognised.

(Sciama 1953a), published in 1953, which is incidentally the same year that, as we saw, Wheeler developed many of his own views on the topic while teaching general relativity at Princeton. Sciama came at the topic from a somewhat different angle to Wheeler; he considered it a failure of general relativity that it sustained non-Machian solutions such as empty Minkowski space which lacks matter but in which a test-particle nonetheless has inertial properties, and thought therefore that an alternative theory should be developed. In a brief correspondence with Einstein spanning from December 1950 to January 1951, Sciama expressed some criticisms he had of general relativity and in particular the general-covariance-based approach which he saw as physically vacuous, instead advocating for a strong sense of invariance (which he calls "form invariance" rather than "value invariance," relating the former to the invariance we see in SR and the latter to that of GR which he was critical of) applied only to the conformal subgroup of the diffeomorphism group.[22] In 1955, two weeks before Einstein's death, Sciama would travel to Princeton to meet him. In an interview with Spencer Weart (Weart and Sciama 1978), Sciama recounts that he started the conversation by telling Einstein: "I've come to talk about Mach's principle and I've come to defend your former self against your later self," to which Einstein is reported to have responded: "Ho, ho, ho, that is gut, Ja!"

Sciama's 1953 article is an edited version of the third chapter of his PhD thesis of the same name (Sciama 1953b). In the second chapter of his thesis, while addressing the failure of general relativity to implement Mach's principle, Sciama comments on Einstein's argument for the closed universe in a passage discussing Einstein's introduction of the cosmological constant Λ (Sciama 1953b, p.29):

> The point of this modification was that the new equations have a solution corresponding to a closed space (Einstein universe), so that no difficulty arises with boundary conditions at infinity. This space is full of matter and Mach's principle is satisfied, so that Einstein originally thought that he had found a complete solution to the problem of inertia. However, De Sitter (1917) pointed out that the modified equations still had a solution corresponding to empty space (de Sitter universe), so that they are still not completely consistent with Mach's principle. Furthermore, the discovery of systematic redshift in the spectra of distant nebulae a few years later showed that the

[22]Dennis W. Sciama to Albert Einstein, 11 January 1951, EA 20-470, Albert Einstein Archives, Hebrew University of Jerusalem.

> static Einstein universe was in conflict with observation. Einstein therefore gave up all hope of satisfying Mach's principle and withdrew the term, for which he now saw no justification.

Sciama struggled to create a rank-2 tensorial theory that would implement Mach's principle, although he ultimately regarded that this was necessary due to "kinematical considerations" (Sciama 1953b, p.33). Concerning the intended rank-2 theory, Sciama remarks that "the use of such a potential leads to rather involved mathematics which tends to obscure the physical significance of the theory," he therefore opted to start out by constructing a vector-potential theory as a proof-of-concept model which works by explicit analogy with Maxwellian electromagnetism and captures the physically significant results of the intended theory. Unlike general relativity, Sciama's theory is inherently cosmological and incorporates Mach's principle by construction in its very foundation, linking the strength of gravity directly to the large scale structure of the cosmos. Gravitational and inertial forces are generated by scalar and vector potentials analogous to those used in electromagnetism:

$$\Phi = -\int_V \frac{\rho}{r} dV, \qquad \bar{A} = -\frac{1}{c}\int_V \frac{\bar{v}\rho}{r} dV. \tag{7}$$

Instead of the charge and current density of surrounding matter as sources of Φ and $\bar{A}$, the sources of these gravito-inertial potentials are now the mass ρ and momentum $\rho\bar{v}$ densities of the relevant surrounding matter. Since, unlike charge, mass is always positive, distant matter is no longer neutral and the integration must be performed over the entire observable universe V, where r is the distance of the matter density being integrated from the relevant point at which we are calculating the potentials. From this simple model, Sciama was able to derive both inertial and gravitational effects simultaneously, however, the correct centrifugal and Coriolis forces were only produced if the strength G of this gravito-inertial coupling is given by the expression:

$$U = c^2 \tag{8}$$

where $U = -G\Phi$ is the gravitational potential of the universe. In particular, the Coriolis field is generated by the gravito-magnetic field $\bar{A}$ of a rotating universe, and the centrifugal field is generated by a relativistic modification of Φ in a rotating frame (Sciama 1953a).[23] Therefore, according to the model, inertia can only be explained entirely in terms of the gravitational influence of distant matter if this equation holds exactly. Unlike the results of Wheeler and Weyl based on Thirring's

[23] Note that Sciama's 'toy-model' is not intended as a complete theory and borrows results from special relativity without being a rigorous relativistic field theory.

considerations as we saw previously, this is not an order-of-magnitude estimate but a precise condition. Assuming a smoothed-out model of homogeneous and isotropic universe of density ρ expanding with a simple Hubble law $\mathbf{v} = \mathbf{r}/\tau$ where $\mathbf{v}$ is the velocity of matter at distance $\mathbf{r}$ and τ is a constant, and adding that "matter receding with velocity greater than that of light makes no contribution to the potential," Sciama was able to integrate Φ giving the condition $2\pi G\rho\tau^2 = 1$, which is conspicuously similar to the critical density condition in Friedmann models (which differs by a factor of 4/3).[24]

Reflecting on the consequences of equation 8, Sciama (1953b, p.40) notes that it implies that the total energy (inertial and gravitational) of a particle at rest in the universe is zero, in particular, the energy that will be needed for a given particle of mass m_p to escape the gravitational well due to the rest of the matter in the cosmos ($m_p G\Phi$) is exactly equal to its rest energy ($m_p c^2$), which was an attractive result for Sciama particularly since at this time he was working in the context of the Steady State model of cosmology, and this zero-energy condition would allow for matter creation without violating energy conservation. Incidentally, although there is no evidence that Sciama was aware of this at the time, the same condition had been proposed in 1939 by Pascal Jordan (Jordan 1939), based precisely on this requirement that the total energy contribution of any given mass to the universe is zero, but entirely independently of considerations of Mach's principle.[25] At one point in Wheeler's notebooks he expressed Einstein's notion of a closed space as a result which is much more profound than what Mach's principle initially seems to suggest (WR1, p.104): "Mach principle, say it leads to something much deeper than showed on surface — idea of closed space." We might say that for Sciama the *deeper* result that Mach's principle leads to is the idea of the zero-energy universe; although it is clear that Sciama did not evangelise for his result as vigorously as Wheeler who had the authority of Einstein's writings to appeal to.

Whereas in the 1950s, Sciama was admired by his friends and colleagues for his openness and enthusiasm about the steady state model and Mach's principle,

[24]Note that Sciama's simple model here is based upon basic assumptions that differ substantially from those underlying the Friedmann equation. We will see shortly, based on remarks by Bondi (1995) why there is a more robust case for a connection to the critical density condition.

[25]Jordan had been led to consider variable-G cosmologies due to Dirac's observation of the famous "large number" coincidences (Dirac 1938), rather than a study of Mach's principle. We thank Alexander Blum for alerting us to Jordan's paper in connection with Sciama's result.

having notably inspired his close friend Roger Penrose to switch from pure mathematics to astrophysics and cosmology,[26] the latter part of his career seems to have involved a progressive demoralisation on those issues of early interest. Sciama underwent a shift from his status as a young pioneer advancing his own bold ideas to a mentor figure who is now primarily remembered for having supervised the doctorates of a long list of astrophysicists and cosmologists who subsequently became very prominent in their fields.[27] Despite his enthusiastic advocacy in favour of steady state cosmology in his early career, Sciama was rather quick to accept the fall of the model once enough evidence had mounted against it, especially after one of his students, Martin Rees, had played a key role interpreting radio-astronomical data on the CMB which provided evidence against the model in the mid 1960s. Sciama later acknowledged that the demise of the steady state theory was, in a certain sense, a disappointment—particularly for those physicists who were drawn to it for its conceptual boldness. In an interview with Spencer Weart in 1978, he remarked that he had often been struck by a contrast in attitudes toward the theory, and that he had tended to associate sympathy for steady state with a more imaginative cast of mind (Weart and Sciama 1978):

> I often tried to correlate the psychology of people with whether they supported or decried the Steady State theory or were just neutral, I'm not sure I ever got very far doing that, I couldn't help thinking that on the whole the more imaginative people seemed to like the theory, not necessarily think it was true, but at least were sympathetic.

Despite the decline of the Steady State theory, for quite some time longer, Sciama continued to investigate whether Mach's principle could provide a guiding light towards the selection of cosmological models. Later in the interview with Weart, Sciama indicated that his gravitational-inertial energy balance expression derived

[26]Penrose has often recalled how Dennis Sciama (quite successfully) attempted to interest him in foundational topics in physics, including Mach's principle. In an interview conducted by Jonathan Fay, Penrose describes travelling with Sciama from Cambridge to Stratford to attend a Shakespeare performance. As they drove along "windy roads with sort of cliffs on the side," Penrose recalls that Sciama would remark, as the car turned and its occupants were thrown sideways, "That's the action of the fixed stars ... that was very much Dennis' point of view" (Interview with Roger Penrose, 18 July 2023).

[27]These include: George Ellis (1964), Stephen Hawking (1966), Brandon Carter (1967), Martin Rees (1967), Gary Gibbons (1973), James Binney (1975), John D. Barrow (1977), Philip Candelas (1977), David Deutsch (1978), Adrian Melott (1981) and Antony Valentini (1992). For more on Sciama's influence on a generation of relativists, see the collection Ellis et al. (1993) which was written in celebration of Sciama's 65th birthday.

in 1953 might correspond to the critical density condition of the Einstein-de Sitter model (Weart and Sciama 1978):[28]

> You see, if you take these Newtonian analogues of Milne and McCrea, the Einstein-de Sitter model is the one where the total energy of the universe is zero, the kinetic energy and the negative gravitational potential energy just balancing. Well, if you think that kinetic energy manifesting inertia is due to gravitation, then you might intuit that the most Machian way of having one made by the other would be if there's equal amount of energy, which would give you uniquely the Einstein-de Sitter model, I still have a secret hope that that might turn out so, but it may well not.

This connection to the Einstein-de Sitter model is expressed more explicitly by Hermann Bondi is his brief talk in Tübingen (Bondi 1995, p.475). Here he discusses the quantity $4\pi\rho G\tau^2$ which a Machian condition such as Sciama's eq. 8 fixes to be constant over the course of cosmological evolution:

> If you say that this quantity, which of course is a pure number, has a particular value, that tells you something either about the mean density of the universe, about its time scale, or about the constant of gravitation (which can be regarded as the ratio of gravitational to inertial forces). You can read it as you wish, but in almost any way you read it, it suggests something which is very Machian: a link between the universe and a local quantity like the constant of gravitation. [...] It is constant in only two cosmologies that I know of: the steady-state theory and the Einstein–de Sitter model. For, since the time scale τ is effectively $R/\dot{R}$,
>
> $$4\pi G\rho\tau^2 = \frac{4\pi G\rho R^2}{\dot{R}^2}.$$
>
> In a Friedmann universe with negligible pressure $\rho \sim R^{-3}$:
>
> $$4\pi G\rho\tau^2 \sim \frac{1}{R\dot{R}^2}.$$
>
> The only case where this is a constant is if R varies like $t^{2/3}$, which is the Einstein–de Sitter model.

[28] Note that de Swart (2020) characterises this passage as evidence that Sciama "had a different preference [to Wheeler] but similar reasoning for favouring a universe with a specific density." However, as we have seen, the reasoning involved is altogether different, despite both arguments being based on considerations of Mach's principle.

Bondi and Samuel (1997) would later list the condition $4\pi\rho G\tau^2 \sim 1$ as one of 10 possible formulations of Mach's principle. We may note here that the quantity $4\pi\rho G\tau^2$ ought also to be held constant according to the argument which Wheeler endorsed drawing on the results of Thirring (1918), which would undermine Wheeler's claim in Misner et al. (1973, §21.12) that this relation is compatible with "the Friedmann model and other simple models of a closed universe."[29]

3.2.2. *Decline of Sciama's Machian programme.* Unfortunately, this line of reasoning was not developed very far in Sciama's career, and this was largely due to subsequent research on Mach's principle carried out by Sciama's colleagues. In his 1953 paper, Sciama had promised a follow-up article, referred to as "II" which would implement the same physical ideas in a tensorial theory capable of reproducing the empirical successes of general relativity. The difficulty that Sciama faced with this task was made clear by the fact that the promised paper only arrived 16 years after the first, and moreover failed to deliver the results that so many had appreciated in the original paper.

The second paper (Sciama et al. 1969) came out of a collaboration with Peter Waylen and Robert Gilman (a former student of Wheeler who had been influenced by Sciama's 1953 article) in July of 1969. At this point, Sciama had just published a book titled *The Physical Foundations of General Relativity* (Sciama 1969)—in April of that year—in which he endorses the Machian status of general relativity and discusses how Mach's principle and the equivalence principle played foundational roles in the development of the theory.

It is not clear exactly when this happened, but by this point Sciama is no longer bothered by the objection he had had in 1950 to Einstein's use of general covariance and seems to have undergone a process of reconciliation—or perhaps of compromise—with the growing community of relativists that he was involved in mentoring. The aim of Sciama et al. (1969) was to rewrite the Einstein field equations in integral form, and this required using much more sophisticated and conceptually challenging mathematical tools involving *Green's functions* which Sciama's collaborators helped him navigate, a development which Sciama lamented given that it opposed his personal preference for technical simplicity and clarity of thought.[30]

[29]Wheeler implicitly acknowledges in saying that the relation only holds for closed Friedmann models "at the phase of maximum expansion" (Misner et al. 1973, p.548), however the reason why it should only hold here and not at other points in time is not given.

[30]See this expressed in the interview with Weart (Weart and Sciama 1978): "I'm afraid so, but I feel forced by the physics to be that complicated." Note also that around this time, other researchers were independently developing similar approaches to the implementation of Mach's principle based upon Green's functions, including: AlTsuler (1967); Lynden-Bell (1967).

Following the development of this toolkit, Gilman wrote a follow-up paper (Gilman 1970) applying these methods to Mach's principle and cosmological models in which he determined that "many (and probably all) nonempty relativistic Robertson-Walker models with $\Lambda = 0$ and $0 \leq p \leq \rho c^2$ are acceptable Machian cosmologies." The original result ($U = c^2$) that had been present in Sciama's 1953 paper was nowhere to be found, and it seemed that Mach's principle could no longer be leveraged in a meaningful way for the selection of cosmological models.

By the mid 1970s, Sciama as well as his Machian colleague Derek Raine were saying similar things. At the Colloque International CNRS No. 220 (Ondes et radiations gravitationnelles) held at the *Institut Henri Poincaré* in Paris in June 1973, Sciama gave a talk reporting the latest results of his and colleagues' work on the integral formulation of general relativity (Sciama 1974). In the subsequent Q&A, the French physicist Olivier Costa de Beauregard, who clearly had some prior familiarity with Sciama's work asked him:[31] "In this approach, do you arrive at a precise statement regarding the value of GM/c^2L?" (where GM/c^2L refers to the quantity which as we have seen numerous sources have suggested should be of order unity on the grounds of Mach's principle). Sciama responds:

> Unfortunately not. All Robertson-Walker models turn out to be Machian. The reason is that the Green's function adjusts itself whatever the value of the present mean density of matter in the universe, to give compatibility with M_1 and M_2

where M_1 and M_2 refer to two "Mach conditions" discussed in the paper. After this paper, Sciama no longer published on the topic of Mach's principle. In the mid 1970s he appears to pivot towards a focus on dark matter, black holes and other topics in astronomy. Interestingly, it was just around this time, in 1974, that Sciama would be responsible for green-lighting the publication of a short paper by Julian Barbour in *Nature* (Barbour 1974) which would greatly benefit Barbour's career, enabling him to collaborate with Bruno Bertotti and subsequently become the world's leading voice on 'Mach's principle', a role arguably previously held by Sciama.[32]

[31] Translation of: "Dans cette approche, aboutissez-vous a un énoncé précis concernant la valeur de GM/c^2L?"

[32] Note that in a follow-up paper, Barbour (1975) uses a simple classical model, similar to one 60 years prior by Reissner (1915), from which he derives a relation given as $GM/Rc^2 \sim 1$ (with M and R representing the mass and radius of the universe), which he associated with the "cosmic coincidences" of Dirac (1938). Later on however, Barbour (1995) would argue that general relativity is a "perfectly Machian theory" on the basis of his collaborative work with Bruno Bertotti (Barbour and Bertotti 1982) in which the possible relative configuration of masses of classical

Sciama did not attend the 1993 Tübingen conference on Mach's principle (co-organised by Barbour), although his 1953 work was cited at various points by participants. Raine's article in the conference proceedings mentions Sciama's "toy model" of inertia and summarises the predicament in which the research programme culminated (Raine 1995):

> Mach's Principle as originally conceived was supposed to supply a reason for the existence of the matter in the universe, in the quantity found, through the relation $G\rho\tau^2 = 1$ (i.e., an $\Omega = 1$ universe). In fact, we find that any FRW universe fulfills the conditions, so we need a different explanation [...] Most of the competing explanations (inflation, anthropy) also require the universe to be reasonably isotropic. This leaves precious little for Mach's Principle to explain! In this case Mach's Principle is simply true by accident.

That same year, Sciama would publish *Modern Cosmology and the Dark Matter Problem* (Sciama 1993), which was in part an update of a previous book *Modern Cosmology* from 1971 (Sciama 1971). Despite transferring much of the material from the first book, Sciama entirely removed any mention of Mach's principle from his new publication.[33]

While Gilman's results seemed to have dissuaded Sciama and Raine from further pursuing the Machian programme, interestingly, de Beauregard was not affected in the same way. In 1999 (the year of Sciama's death), in defiance of the pessimistic results of his British and American colleagues, de Beauregard wrote a short article on Mach's principle in which he endorsed Sciama's original $U = c^2$ result, connecting it both to Einstein's equivalence principle and the unification of mass and energy (de Beauregard 2000):

> Compare the formulas $W_g := Um_g$ and $W_i := c^2 m_i$, the first expressing the gravitational potential energy of a test particle of passive gravitational mass m_g and the second the energy equivalent to its inertial mass m_i. The equality $m_g = m_i$ was inferred from a thought experiment by Galileo. Assuming the equality $W_g = W_i$ is an updated form of Mach's conjecture. The equality $U = c^2$ then follows.

mechanics is replaced with the "superspace" of possible Riemannian 3-geometries. In this context the condition $GM/Rc^2 \sim 1$ no longer has any relevance.

[33] For instance, chapter 8 of Sciama (1971) survives almost intact in section 3.2 of Sciama (1993) apart from the introductory section and the final section on "cosmological coincidences" (i.e. the Dirac large number coincidences) which were the two sections in which Mach's principle had been discussed.

> This means that inertial mass is induced via gravitational potential energy, expressing an equivalence of the two Einstein equivalences. In other words, assertion of the equivalence of mass and energy, and the equivalence of inertial and passive gravitational mass, are equivalent if Mach's principle is true.

de Beauregard's perspective on this issue is quite intriguing, however the paper is not widely cited and this view does not seem to have penetrated into mainstream cosmology or physics in general.

Both de Beauregard (2000) and Woodward and Mahood (1999) (among others) endorse the view that Sciama's key contribution to the story concerning Mach's principle was to introduce the equivalence $U = c^2$. In Woodward and Mahood's paper it is claimed in addition to this that in the *linearised* (Maxwell-like) context: "Sciama shows that, up to factors of the order of unity, U must be equal to $-GM/R$, M and R are the mass and radius of the observable universe, and the density of matter is about 10^{-29} g/cm^3—that is, about "critical" (or closure) density." Since at the time, supernova data suggested that the density should be below the critical threshold, this was seen as a potential problem and Woodward and Mahood appeal to the results of Gilman (1970) and subsequent development by Raine (1981) to argue that general relativity "dictates that inertia is gravitationally induced irrespective of whether or not cosmic matter density is critical," without realising that this result entirely undermines the initial result $U = c^2$ that they had started out by endorsing. Some others such as Vigoureux et al. (2003) on the other hand have continued to speculate along the lines of Bondi (1995), arguing that Sciama's $U = c^2$ result leads to a strong condition on cosmology that can provide a concrete approach to interpreting and resolving various cosmological questions including the horizon problem, the homogeneity of the CMB and the issue of the universe's flatness.

In recent times, the leading physicist who seems to be actively endorsing the view that Sciama's $U = c^2$ is equivalent to the critical density condition is C.S. Unnikrishnan (Unnikrishnan 2022, p.216-7), whose views concerning both special and general relativity appear to closely mirror those expressed in Bondi's short talk at Tübingen.[34]

[34]In his talk, Bondi discussed both the association between Mach's principle and the critical density condition, as well as a critique of the universality of special relativity on the basis that "there is at every point of the universe a preferred velocity, namely, that from which the universe appears most isotropic" (Bondi 1995). Both of these points are found to be core tenets of Unnikrishnan's proposed paradigm of "cosmic relativity" (Unnikrishnan 2022).

However Unnikrishnan's proposals have so far been largely ignored by the broader astrophysics and cosmology research communities.

3.2.3. *Summary.* In summary, the second class of arguments from Mach's principle we have seen here involves the derivation of a relation of the form $G\rho\tau^2 \sim 1$. Expressions of this sort are widespread in the literature and have been derived in many contexts. In the context of general relativity, a simple (yet methodologically questionable) derivation involves fitting the Lense-Thirring frame dragging of a rotating universe to inertial forces in a rotating frame, or in a gravito-electromagnetism analogue model it involves fitting gravito-magnetic effects of the rotating universe to the Coriolis force of classical mechanics. Expressions of this form were recognised at least as far back as Weyl (1924), but also derived widely, including by Schrödinger, Barbour, Wheeler and many others.[35]

In Sciama's imperfect but physically illuminating 'toy-model', the relation comes out as $U = c^2$ which is associated with a zero-energy universe. For Sciama and Raine at least this was meant to uniquely select a critical density universe, however their efforts, along with Gilman and others to transfer Mach's principle into the conceptual mould of general relativity wound up undermining this hope. Bondi (1995) has proposed that in a broader sense the Machian condition is that the quantity $4\pi G\rho\tau^2 \sim 1$ should be a constant over cosmological time, which is indeed a feature of classical Machian models in general. Bondi points out that among Robertson-Walker models this picks out the critical density, although he also added that experimental evidence did not favour it. These ideas continue to be considered by some researchers, although they have not had a substantial impact on mainstream cosmology since the 1990s.

4. DICKE'S COINCIDENCES AND THE FLATNESS PROBLEM

The third and final philosophical argument for the geometry of the universe in the 1970s and 1980s comes from a fine-tuning problem associated mainly with Robert Dicke, the supervisor and long-term collaborator of James Peebles at Princeton. As is well known, Robert Dicke was deeply interested in incorporating Mach's principle into gravitational theory, which led him to develop his famous scalar-tensor theory with Carl Brans (Brans and Dicke 1961). A central motivation behind the Brans-Dicke theory was their concern that Einstein's general relativity was not truly Machian, since it admits "empty solutions", that is, spacetimes in

[35] See for instance Assis (1989); Treder (1972), or for a very early version of this relation in a slightly different form see Reissner (1915).

which geometry and inertia exist without the presence of matter. Given that Mach's principle holds that inertial properties must be determined by the presence and distribution of matter in the universe, Brans and Dicke took these vacuum solutions to indicate a fundamental shortcoming of general relativity. On their view, any adequate theory of gravity should preclude such matter-free yet fully determinate inertial structures.

Nevertheless, in the 1961 paper, Brans and Dicke do not express any explicit preference for the geometry of the universe. Rather, they are mainly concerned with developing a new scalar-tensor theory of gravity that better incorporates Mach's principle by ensuring that matter uniquely determines the geometry of space. Dicke's preferences only appear about a decade later in his first discussions of the 'coincidence problem'. Although not widely discussed in the literature of the time, this issue was, according to Peebles (1993, p.365), 'part of the standard lore for at least a decade' within the discussions of Dicke's research group at Princeton, until it was first published in a series of lecture notes by Dicke (1970).[36] Dicke's coincidence arguments served as the precursor to the well-known flatness problem which led, amongst other things, to the development of the theory of inflation by Alan Guth.[37] The underlying idea is that if the curvature term or the cosmological constant term in the Friedmann equation are taken to be zero at the early universe, then galaxies, the solar system and therefore we, as observers, begin to form right after the very special epoch at which the deceleration of the expanding universe turns into an acceleration, i.e. when the energy-mass density ceases to be the dominant term.

For a more concrete example, consider the Friedmann equation in terms of the different fractional contributions to the expansion rate:

$$\left(\frac{\dot{a}}{a}\right)^2 = \Omega_r H_0^2 \left(\frac{a_0}{a(t)}\right)^4 \left[1 + \frac{\Omega_m}{\Omega_r}\left(\frac{a}{a_0}\right) + \frac{\Omega_k}{\Omega_r}\left(\frac{a}{a_0}\right)^2 + \frac{\Omega_\Lambda}{\Omega_r}\left(\frac{a}{a_0}\right)^4\right] \tag{9}$$

[36]cf. also (Peebles 2020, p.59): 'This coincidence argument may have occurred to many who did not bother to publish, because it was not at all clear what to make of it. I remember its discussions in the early 1960s in meetings of Dicke's Gravity Research Group.'

[37]Dicke mentions in an oral interview to the American Institute that Alan Guth, who was a postdoc at Cornell at the time, attended one of his lectures on these strange coincidences (Lightman and Dicke 1988). This fact is also reported by Alan Guth in his short autobiography for the 2014 Kavli Prize in Astrophysics: 'On Monday (November 13 [1978]) I happened into a lecture given by Bob Dicke, a well-known cosmologist from Princeton, who described a feature of the conventional big bang theory called the flatness problem. The problem concerned the extreme amount of fine tuning that is needed to make the conventional big bang theory work. [...] I did not understand the calculations behind what Dicke was saying, but I was very impressed by the conclusion, and tucked it away in the back of my brain.'(Guth 2014).

Here, the parameter $\Omega_r \sim 10^{-4}$ is the fractional contribution to the present expansion rate by the thermal radiation and massless neutrinos and Ω_m is the matter density parameter, which based on observations is about 0.1. At the epoch of nucleosynthesis where light elements begin to form, i.e. at redshift $z = a_0/a \sim 10^{10}$, we may consider two possible scenarios: one in which there is no cosmological constant and $\Omega_k = 0.9$, and one in which there is no space curvature and the cosmological constant contributes $\Omega_\Lambda = 0.9$ to the present rate of expansion. In both cases, the contributions from the curvature ($\sim 10^{-16}$) and Λ ($\sim 10^{-36}$) to the expansion rate at the time of nucleosynthesis are practically negligible compared to the contribution of the mass density ($\sim 10^{-7}$). However, in both cases, it is a curious coincidence that galaxies, stars and observers begin to form at the very special epoch where the curvature and cosmological constant, which would have been extremely small at the epoch of nucleosynthesis, become the dominant contribution to the expansion rate. If both terms are taken to be non-zero at the epoch of nucleosynthesis then the situation becomes even more puzzling since in that case one of the terms would dominate the expansion much earlier leading to either an open or closed universe or an accelerated expansion at times that do not match the observational data from the early universe.

In other words, the fine-tuning worry occurs from the fact that we seem to live in a special epoch when one of these two terms (Ω_k and Ω_Λ) which had an almost negligible effect in the early universe, have just recently grown enough to dominate the expansion rate. For Dicke and his collaborators this was a 'suspicious coincidence'. This coincidence, however, can be avoided if both space curvature and Λ are negligibly small, corresponding to an Einstein-de Sitter universe with flat geometry. In this case, the universe evolves from radiation-dominated to matter-dominated and the mass density remains constant at a value very near the critical density (Ω_m=1) throughout the entire cosmic history.

The well-known flatness problem in cosmology is a special case of these curious coincidences. This issue first appears in writing in a series of published lectures (Dicke 1970), in which Dicke presents an early version of the flatness problem, without however suggesting a solution:

> [H]ow did the initial explosion become started with such precision, the outward radial motion became so *finely adjusted* as to enable the various parts of the Universe to fly apart while continuously slowing the rate of expansion? There seems to be no fundamental theoretical reason for such a fine balance. If the fireball had expanded only 0.1

> per cent faster, the present rate of the expansion would have been 3×10^3 times as great. Had the initial expansion rate been 0.1 per cent less and the Universe would have expanded to only 3×10^{-6} of its present radius before collapsing. [...] No stars could have formed in such a Universe, for it would not have existed long enough to form stars. (Dicke 1970, p.62, emphasis added)

A later formulation of the problem appears in a co-authored chapter by Dicke and Peebles (1979) in an edited collection of essays on general relativity. In this chapter, written in a rather jaunty and conversational style, Dicke and Peebles consider a number of 'enigmas' in cosmology, including the flatness problem, and discuss possible solutions. The flatness problem in this context arises from a basic consideration of the Friedmann equation with $\Lambda = 0$. Dicke and Peebles begin by noting that the (then) present relative values of the curvature term and the mass density are poorly known 'because the mean mass density, ρ, is so uncertain' (p.506). They then note that tracing the expansion back in time, at the time of nucleosynthesis, we find that the mass term is 14 orders of magnitude larger than the curvature term which means that the expansion rate has been fine-tuned to agree with the mass density to a great accuracy. Their conclusion is that this precise initial balance of the effective kinetic energy of expansion and the gravitational potential energy must apply to each separate part, otherwise the inhomogeneities would produce large and irregular space curvature leading to black holes of all sizes and essentially a chaotic universe which would look much different from the one we observe. In other words, these considerations show that, regardless of the (unknown) mass density at the time, it must have been arbitrarily close to the critical value at the time of the Big Bang.

Dicke and Peebles then proceed to note that this model represents an Einstein-de Sitter universe (with $k = \Lambda = 0$) which, according to them, was motivated back in 1932 merely because of its simplicity, in the absence of any observational data.[38] Interestingly, the authors find the argument from simplicity rather weak, and conclude their short discussion of the flatness problem by noting that the observed large-scale clumping of matter in the universe requires that the balance between expansion and gravity in the early universe was extremely accurate

[38]cf. Einstein and de Sitter (1932, p.214) '...we must conclude that at the present time it is possible to represent the facts without assuming a curvature of three-dimensional space. The curvature is, however, essentially determinable, and an increase in the precision of the data derived from observations will enable us in the future to fix its sign and to determine its value'.

though not exact, leaving open the possibilities of both a closed and an open—but nevertheless nearly flat–universe.[39]

It is not clear whether Ostriker, Peebles and Yahil (1974) had these complications in mind when referring to some cosmologists' preference for a universe which is "just closed" due to "non-experimental" reasons, although their immediate reference to Wheeler (1973) suggests that this was probably not the case. Nevertheless, in later work, Peebles (1984) presented a model for dark matter and the structure of galaxies and star clusters based on a set of 'particularly simple and attractive assumptions' (p.470). The first of the two assumptions which are relevant to our discussion, was that the cosmological constant is negligibly small since according to Peebles 'if Λ did play an important role it would be a coincidence that the mass density associated with Λ is comparable to the present density of matter, which is not attractive' (*ibid.*). The second was that the density parameter is $\Omega = 1$, i.e. that we live in a flat universe, with Peebles noting that the motivation for this assumption is the same as for the previous one, citing Dicke and Peebles (1979). It is therefore evident that, at least for Peebles, the preference for additional matter corresponding to a flat geometry was mainly driven by his (and Dicke's) reluctance to accept that galaxies—and the observers within them–coincidentally came into existence at a special point in cosmic history.

Nonetheless, it should be noted that Peebles' philosophical preference for a flat universe was also supported by his argument from gravitational instabilities (Peebles 1974), alluded to in the Ostriker et al. (1974) paper. The central point of the argument is that the amount of gravitational growth available depends strongly on the total matter density (Ω), and a low-density universe does not allow the growth of structures observed at the time. The argument was based on the idea that the structure in the universe formed due to the growth of small initial density fluctuations due to gravity, an idea that goes back to Lifshitz (1946). In order for these initial fluctuations to be consistent with the—then observed—perfectly smooth distribution of the cosmic microwave background radiation they had to be extremely small, and thus insufficient to produce the observed growth. To amplify these fluctuations into galaxies, clusters, superclusters, etc., one needs maximal

[39]In a recent review of the history of the flatness problem, Helbig (2021, p.6) rightly notes that the Einstein–de Sitter model is best understood merely as a practical and simple suggestion by the authors and that there is no clear indication in the article that Einstein or de Sitter were indeed in favour of a flat universe due to its simplicity. We agree, however, what is important for the purposes of our discussion is that Dicke and Peebles were not convinced by the argument from simplicity although it is, in their own words, 'attractive'. The flatness of the universe was for them a necessary condition to explain the current structure of the universe.

gravitational growth, and this maximal growth happens only when the universe is nearly flat, i.e. when $\Omega \sim 1$.

5. Discussion - Conclusions

The establishment of dark matter as a non-baryonic primordial particle in the 1980s was the culmination of a complex process that lasted several years. This process involved the observation of anomalies at galactic and cosmological scales, theoretical advancements in understanding physical processes across early and late cosmic history, and philosophical considerations by prominent cosmologists of the time. In this article, we argued that a critical milestone in the acceptance of dark matter as we know it today was the realization that baryonic matter constituted significantly less than the critical density, and that the observed large-scale structure formation and the temperature spectrum of the cosmic microwave background radiation could not be explained by the gravitational collapse of a photon-baryon fluid in the early universe. Alongside these scientific developments, the philosophically motivated preference of many cosmologists for an 'at least closed' geometry (i.e., a flat or closed universe with $\Omega \geq 1$) laid the groundwork for hypothesizing the existence of a non-luminous form of matter. This additional matter, which by far exceeded the amount of baryonic matter, also provided a potential explanation for the observed mass discrepancies in galaxies and galaxy clusters.

As we have seen, in the absence of direct observational data about the geometry of the universe, cosmologists of the time relied on philosophical and non-experimental reasoning to propose different models of the cosmos. This article has identified and presented three such arguments: (a) Einstein and Wheeler's Machian argument for a closed universe, (b) Sciama and Bondi's Machian argument for a flat universe, and (c) Dicke and Peebles' coincidence argument for a flat universe. Despite their differing motivations and methodologies, all three arguments converge on the conclusion that additional mass is required to achieve or exceed the critical density, aligning with the idea of non-baryonic dark matter as a necessary component of the universe.

The first two arguments we examined both came from considerations of Mach's principle, although they differ wildly in their methodology. In the first place, the argument for the universe's spatial closure originated in Einstein's struggle to find appropriate boundary conditions following his publication of the field equations. Einstein became convinced that appropriate boundary conditions could not be found and elected to resolve the issue by eliminating the need for these altogether by requiring that the universe is spatially closed. Einstein was also led to this quite naturally since he was working under the assumption of a non-expanding cosmos for which any density $\rho > 0$ would lead to closure. Einstein's views were adopted by Wheeler whose interest in Mach's principle was conditioned by his former work on AAD models of electromagnetism and gravity. Wheeler added an argument that spatial closure is needed to ensure that inertial influences of other masses on a body converge; at various times he considered extending the requirement to (or replacing it with) temporal closure, although he did not convincingly resolve the issues with this line of argument, which, in the context of a big bang cosmology, might be better construed as a requirement for the universe's temporal horizon.

The second class of arguments from Mach's principle follows from an entirely independent line of reasoning. Whether by reasoning from a consideration of the Lense-Thirring effect or from classical electromagnetic analogue models, numerous physicists have derived expressions of the form $GM_U/R_u \sim c^2$ (or similar expressions) by identifying induced influences with inertial forces. What is important to note, however, is that Mach's principle applied in this way leads to a fixed relation, regardless of the particular constant of proportionality (which may vary depending on the methodology, but which Sciama found may give the appealing result $U = c^2$). Based either on heuristic arguments by Dennis Sciama and others, or on the more explicit argument by Hermann Bondi, the resultant constancy of $G\rho\tau^2$ picks out the Einstein-de Sitter model for a critical density cosmos, and there is some evidence that at least for a handful of cosmologists (such as Sciama, Raine and their associates) this was a relevant consideration in the 1970s and 1980s. Although Wheeler claimed that the Machian result $GM_U/R_u \sim c^2$ supports his preference for closed universe models, his argument is not convincing and it is more likely that the two Machian arguments are mutually exclusive.

Finally, the third argument due to Dicke and Peebles, takes a markedly different approach, originating from their reluctance to accept that galaxy and star formation occurred at a very special epoch when the deceleration of the expanding universe turns into an acceleration. From a philosophical perspective, Dicke's coincidence argument is best understood as an attempt to define what should count as an acceptable cosmological explanation at a time when empirical evidence for spatial geometry was still inconclusive. What Dicke and his collaborators found 'suspicious' was not merely that the curvature or the cosmological constant were extremely small in the early universe, but that they started to become dynamically significant at the exact epoch where galaxies, and observers within them, begin to form. Put differently, their worry implicitly assumes that this temporal coincidence between structure formation and the emergence of curvature (or Λ) requires a convincing explanation that goes beyond brute contingency.

Whether such coincidence constitutes a genuine explanatory problem depends on one's stance towards anthropic reasoning in cosmology. On a weak reading of the anthropic principle, any observation is necessarily conditioned on our existence as observers and we should therefore not be surprised to find ourselves in a universe compatible with that fact. However, what Dicke shows is that the existence of observers does not require the specific and spurious temporal coincidence to which he draws attention: observers could also exist in cosmologies where curvature and Λ never become dynamically important. The weak anthropic principle alone does not therefore trivialize or dissolve the coincidence. The anthropic condition is satisfied in both types of universes, and thus, the coincidence cannot be dismissed as a selection effect.

Nevertheless, even if one accepts that Dicke's worry is not trivial from the point of view of a (weak) anthropic reasoning, we submit that his argument should be seen as revealing a deeper methodological tension rather than as a decisive justification for the geometry of the universe. At its core, Dicke's reasoning relies on the sensitivity of cosmological evolution to the initial conditions of the universe and the strong philosophical intuition that theories requiring some form of fine-tuning of their parameters are less satisfactory. However, as already highlighted in philosophical discussions of inflationary cosmology (Earman and Mosterin 1999; McCoy 2015), moving from this sensitivity to initial conditions to claims about the "implausibility" of non-flat universes implicitly presupposes some kind of probabilistic assessment of the space of cosmological models. Such an assessment was neither formally articulated, nor supported by a well-defined measure in Dicke's writings.

The later development of the theory of inflation by Alan Guth can be seen as providing exactly this sort of framework. As is well known, inflation dynamically drives Ω towards unity, rendering spatial flatness a generic attractor behaviour across a wide class of initial states, as opposed to a fine-tuned requirement for the initial state. Moreover, in many inflationary models curvature and Λ-like contributions naturally become negligible until very late times, mitigating in this way the spurious coincidence that Dicke regarded as problematic. Nevertheless, it is important to note that these developments, although directly inspired by Dicke, address his fine-tuning concerns by introducing new dynamical, probabilistic, and high-energy theoretical structure that lies outside the conceptual space in which Dicke's original coincidence argument was formulated. Dicke's coincidence argument is thus best read as a philosophically informed and heuristic principle in a data-limited context: it expresses a methodological preference for cosmological models in which the large-scale properties of the universe emerge as dynamically stable outcomes across a broad class of initial conditions, rather than depending on initial parameters that must be adjusted with extreme precision in order to yield a universe resembling the one we observe. The theory of inflation retrospectively reinforced this argument by offering a physical mechanism addressing and responding to the concerns articulated by Dicke. In this sense, Dicke's reasoning is philosophically significant not because it conclusively establishes spatial flatness, but because it illustrates how concerns about fine-tuning, explanation, and anthropic reasoning partially shaped the methodological landscape within which cosmologists evaluated cosmological models, even at a time when cosmology was striving to become a precise and data-driven scientific discipline.

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