

# Quantum Entanglement, Internality and Dependence

Author: Matías Daniel Pasqualini

## Abstract

The metaphysics of quantum entanglement has been a subject of interest among philosophers of physics in recent decades. Entanglement is commonly described as a relation that does not depend on the intrinsic properties of its relata. This feature has led some authors to propose that the quantum reality is fundamentally relational and/or holistic. Moreover, it has been employed to support various influential metaphysical perspectives within the metaphysics of science, including structuralism, monism, and, recently, coherentism. This paper advocates a non-reductionist approach to internal relations, drawing on Fine's analysis of propositions involving essential properties. Assuming the pervasiveness of quantum entanglement, it is argued that treating it as an internal relation is the most compelling option. Under this interpretation, entanglement can be accommodated within different metaphysical frameworks: (1) as a fundamental internal relation, it aligns with structuralism; (2) as a derivative internal relation, it is compatible with monism; and (3) as a relation of dependence, it fits with coherentism.

## Keywords

Coherentism; Essential properties; Internal relations; Monism; Quantum entanglement; Structuralism.

## 1. Introduction

The metaphysical nature of quantum entanglement has attracted the attention of philosophers of physics over the past few decades. Generally, quantum entanglement has been characterized as a relation that does not supervene on the intrinsic properties of its relata. This feature has led some authors to propose that, according to quantum mechanics, physical reality is fundamentally relational and/or holistic (Teller, 1986; Howard, 1989; Healey, 1991; Esfeld, 2004). Moreover, entanglement has been invoked to support some innovative metaphysical frameworks, such as ontic structural realism (French, 2006, 2010; Ladyman et al., 2007),

monism (Schaffer, 2010a, 2010b), and, recently, coherentism (Calosi and Morganti, 2021). From now on, the term 'structuralism' will be used to refer to ontic structural realism.

At first sight, the non-supervenience of entanglement prompts an analysis of its metaphysical nature in terms of external relations, understood as non-supervenient according to Lewis's (1986) standard characterization. The aim of this article is to show that such an analysis is neither the only possible nor the most appropriate one. Assuming that quantum entanglement is a widespread phenomenon, it will be proposed that analyzing entanglement in terms of internal relations better accounts for its metaphysics. To this end, a non-reductionist approach to internal relations is proposed, allowing them to be non-supervenient and interpreting internality in terms of essence. This approach is formalized using Fine's logic of essence (1995b). Finally, it is shown that quantum entanglement, as an internal relation, can naturally fit into various metaphysical frameworks: (1) as a fundamental internal relation, it aligns with structuralism; (2) as a derivative internal relation, it aligns with monism; (3) as an internal relation equivalent to symmetric dependence, it aligns with coherentism.

The structure of the article is as follows. In Section (2), some preliminary aspects of the physics and metaphysics of quantum entanglement are introduced. Section (3) is dedicated to the metaphysics of relations and ontological dependence. Subsection (3.1) discusses the ontological status of internal relations and proposes an analysis in terms of essence. Subsection (3.2) offers various analyses of the notion of ontological dependence. Section (4) addresses the metaphysical nature of quantum entanglement. Subsection (4.1) discusses the inadequacy of treating entanglement as an external relation. Subsection (4.2) argues in favor of entanglement as an internal relation, and after introducing some physical and mereological assumptions, it shows how entanglement as an internal relation aligns with structuralism, monism, and coherentism.

## 2. Preliminaries on Quantum Entanglement

We now turn to a preliminary examination of the physics and metaphysics of entanglement. Consider the simplest case of quantum entanglement: two fermions,  $x_1$  and  $x_2$  in the singlet state:

$$|0\rangle_{x_1x_2} = \frac{1}{\sqrt{2}}|\uparrow\rangle_{x_1} \otimes |\downarrow\rangle_{x_2} - \frac{1}{\sqrt{2}}|\downarrow\rangle_{x_1} \otimes |\uparrow\rangle_{x_2} \quad (1)$$

The vector  $|0\rangle$  represents the singlet state of the composite system  $x_1x_2$ , while  $|\uparrow\rangle$  and  $|\downarrow\rangle$  are, respectively, the spin-up and spin-down eigenstates of fermions  $x_1$  and  $x_2$ . The state  $|0\rangle$  indicates that when a spin measurement is performed in a given direction (e.g.,  $z$ ), there is a 0.5 probability of finding  $x_1$  spin-up and  $x_2$  spin-down and a 0.5 probability of finding  $x_1$  spin-down and  $x_2$  spin-up. This state of the composite system is such that each fermion does not have an independent state vector; instead, the spin eigenstates of one fermion are correlated with those of the other. This arises from the fact that the state  $|0\rangle$  is non-separable; that is, it is not a tensor product of two spin eigenstates but rather a superposition of two product states, namely  $|\uparrow\rangle_{x_1} \otimes |\downarrow\rangle_{x_2}$  and  $|\downarrow\rangle_{x_1} \otimes |\uparrow\rangle_{x_2}$ . Thus, the singlet state is considered a non-separable or entangled state. This example illustrates that, when discussing cases of entanglement, the non-separability of a state is typically regarded as a sufficient condition for it to be entangled. Some subtleties regarding this will be considered soon.

To move from the mere quantum formalism to metaphysical considerations, it is necessary to assume an interpretative postulate that assigns properties to the objects to which quantum mechanics refers. The so-called eigenstate-eigenvalue link (EEL) is the most popular in interpretations of quantum mechanics. (EEL) establishes that the object corresponding to the system  $x$  has the property  $P$ , corresponding to the eigenvalue  $p$ , iff the state  $|\psi\rangle_x$  of  $x$  is the eigenvector  $|p\rangle$  corresponding to that eigenvalue. Formally:

$$(|\psi\rangle_x = |p\rangle) \leftrightarrow Px \tag{2}$$

With  $O|p\rangle = p|p\rangle$ , where  $p$  is one of the eigenvalues and  $|p\rangle$  one of the eigenvectors corresponding to a certain observable represented by operator  $O$ . In the case of two fermions in the singlet state, it follows from (EEL) that the composite system  $x_1x_2$  has a property corresponding to the eigenstate  $|0\rangle$ , with eigenvalue zero of the total spin operator, whereas each fermion does not have properties corresponding to spin eigenstates. However, the fermions are related in such a way that it is possible to specify a set of connections between their possible properties, corresponding to the spin correlations. That is, the relation between the

entangled fermions is such that, if a measurement is performed and one fermion is found to be spin-up then the other will necessarily be spin-down:

$$R^{\text{sym}}_{x_1x_2} \equiv (Ux_1 \leftrightarrow Dx_2) \wedge (Dx_1 \leftrightarrow Ux_2) \quad (3)$$

Here,  $R^{\text{sym}}_{x_1x_2}$  represents the relation between entangled fermions;  $U$  and  $D$  are predicates corresponding respectively to the spin-up and spin-down properties; and  $Ux_1 \leftrightarrow Dx_2$ ,  $Dx_1 \leftrightarrow Ux_2$  denote connections that could be given a modal character, in the sense that, for instance, if  $Ux_1$  is obtained, then *necessarily*  $Dx_2$  is the case. At this point, the characterization of entanglement has shifted from being a property of the state of the composite system to a relation between its subsystems. The peculiarity of entanglement lies in the fact that if  $R^{\text{sym}}_{x_1x_2}$  is the case, not only are  $Ux_1 \wedge Dx_1$  and  $Ux_2 \wedge Dx_2$  ruled out, but  $x_1$  and  $x_2$  cannot have spin properties, at least as long as the subsystems involved remain entangled and (EEL) is assumed. Note that, in equation (3),  $Ux_1 \leftrightarrow Dx_2$  and  $Dx_1 \leftrightarrow Ux_2$  are just conditional statements, so  $R^{\text{sym}}_{x_1x_2}$  does not entail that either  $Ux_1$ ,  $Dx_1$ ,  $Ux_2$  or  $Dx_2$  are the case. This corresponds to the fact that it is not possible to assign states to the subsystems from which the state of the composite system can be derived. Hence, duplicating each relatum along with all its non-relational properties and spatiotemporal relations is insufficient to reproduce the entanglement relation. That is,  $R^{\text{sym}}_{x_1x_2}$  cannot supervene on  $Ux_1$ ,  $Dx_1$ ,  $Ux_2$  and  $Dx_2$  nor on any other properties of the subsystems.

This surprising feature of quantum entanglement has led some authors in past decades to suggest that the reality referred to by quantum mechanics is fundamentally relational and/or holistic. Teller (1986) was likely the first to characterize entanglement as a non-supervenient relation. According to him, entanglement relations are inherent—that is, fundamental—relations, which give rise to a form of relational holism. Similarly, Esfeld (2004) points out that there are properties of the whole that indicate the way the parts are related to one another. In the example of two fermions in a singlet state presented above, Esfeld's insight

means that the property of the composite system corresponding to the eigenstate  $|0\rangle$  is somehow associated with the entanglement relation  $R^{\text{sym}}_{x_1, x_2}$  that holds between the subsystems.

Now we mention some characteristics of quantum entanglement that must be taken into account in the following sections. First, entanglement is an irreflexive and symmetric relation. Since the properties of a single system are necessarily correlated, entanglement cannot be reflexive without becoming trivial, as everything would be entangled with itself. Therefore, entanglement should be regarded as an irreflexive relation. Additionally, entanglement is not a one-way relation but rather a symmetric one. If a quantum system  $x_1$  is entangled with  $x_2$ , then  $x_2$  must necessarily be entangled with  $x_1$ . This holds true not only when the quantum systems are indistinguishable but also in cases where they are not.

Second, what has been said about entanglement in bipartite systems (such as the case of two fermions in the singlet state considered earlier) can be extended to multipartite cases, with certain precautions. When dealing with multipartite cases, it is necessary to distinguish between genuine entanglement and entanglement resulting from the application of the symmetrization postulate (see Ghirardi et al., 2002). In other words, mere non-separability of the state does not guarantee genuine entanglement. Here, genuine entanglement is defined as that which violates Bell inequalities. According to Bell's theorem (1964), statistical correlations must satisfy certain inequalities if there exist local hidden variables determining the outcomes. Empirical research has shown that these inequalities are not satisfied in cases of genuine quantum entanglement (Aspect et al., 1982). Moreover, it is possible to obtain non-separable states that satisfy Bell inequalities through the application of the symmetrization postulate, which is mandatory in the case of indistinguishable particles. While genuine entanglement usually presupposes prior interactions between entangled systems, indistinguishable particles may not have interacted yet still be in a non-separable state.

Third, a peculiarity arises even in the case of genuine entanglement. Bigaj (2012) provides an example where a composite system of three subsystems  $x_1$ ,  $x_2$ , and  $x_3$  is in a non-separable state, yet  $x_1$  and  $x_3$  are not entangled with each other.

$$|\psi\rangle_{x_1x_2x_3} = \frac{1}{2} \left( |0\rangle_{x_1} |1\rangle_{x_2} |2\rangle_{x_3} + |0\rangle_{x_1} |3\rangle_{x_2} |0\rangle_{x_3} + |1\rangle_{x_1} |0\rangle_{x_2} |2\rangle_{x_3} + |1\rangle_{x_1} |2\rangle_{x_2} |0\rangle_{x_3} \right) \quad (4)$$

Here,  $|0\rangle$ ,  $|1\rangle$ ,  $|2\rangle$  and  $|3\rangle$  span the state space of each subsystem (with  $\otimes$  omitted for simplicity). As can be directly read from the state in equation (4),  $x_1$  can only take values (0) and (1),  $x_3$  can only take values (0) and (2), and  $x_2$  can take any possible value (0), (1), (2), and (3). This state shows that:

- (a) The value (0) of  $x_1$  correlates with values (1) and (3) of  $x_2$ ; and the value (1) of  $x_1$  with (0) and (2) of  $x_2$ .
- (b) The value (0) of  $x_3$  correlates with values (2) and (3) of  $x_2$ ; and the value (2) of  $x_3$  with (0) and (1) of  $x_2$ .
- (c) Among the possible values of  $x_1$  and  $x_3$ , any combination is admissible.

Point (c) indicates that there are no statistical correlations between  $x_1$  and  $x_3$ . Therefore,  $x_1$  and  $x_3$  are not entangled. A metaphysical lesson about entanglement can be drawn from this fact: if  $Rx_1x_2 \wedge Rx_2x_3$  does not imply  $Rx_1x_3$  (where  $R$  is the entanglement relation), then the entanglement relation is not transitive.

Fourth, while entanglement is relative to the partition or tensor product structure adopted (Earman, 2015), it is not relative to observables. If a state, expressed as a linear combination of eigenstates of a certain observable, is entangled, it is not possible to choose a basis corresponding to another observable and, keeping the partition fixed, rewrite the state as a tensor product. This follows from the fact that it is not possible to assign a state vector to an entangled subsystem (technically, its state is an improper mixture). As a result, quantum systems are not only entangled with respect to specific bases, such as the spin basis, but are entangled tout court. In a recent study, Cinti et al. (2022) show that entanglement cannot be reduced to a set of multiple relations relative to observables. For instance, if the systems have spin and position observables, entanglement cannot be decomposed, without loss of information, into an entanglement relation connecting the spins and another connecting the positions. From this study, it follows that entanglement must be considered a single multigrade relation. That is, it can involve any number of relata but cannot be decomposed into a series of relations of lower arity (since entanglement is not transitive) or into a set of

relations indexed to observables. This point is important because it helps clarify a skeptical doubt raised by Calosi and Morganti (2021) against structuralism (a matter to be addressed in Subsection 4.2.2).

In conclusion, this section arrives at a preliminary characterization of quantum entanglement as a non-supervenient, irreflexive, symmetric, non-transitive, multigrade, and single relation. These are metaphysical aspects of entanglement that can be considered determined by physics itself, with the sole addition of the eigenstate-eigenvalue link. In what follows, other aspects of the metaphysics of entanglement that admit adaptation to different metaphysical frameworks will be studied. This leads us to examine certain general conditions of the metaphysics of relations and ontological dependence.

### 3. Some Aspects of the Metaphysics of Relations and Ontological Dependence

#### 3.1. Internal Relations as Essential Polyadic Features

Relations can be characterized and classified according to various criteria (see MacBride, 2020). For instance, relations may be assessed based on whether they are fundamental or derivative, or, in other words, whether they are non-supervenient or supervenient. According to Lewis (1986), a relation that supervenes on the intrinsic properties of its relata is considered internal, whereas a relation that does not supervene on them is considered external. Armstrong's (1978) definition is similar: a relation necessitated by the intrinsic natures of its relata is deemed internal. Cleland (1984) proposed a distinction between weakly non-supervenient relations and strongly non-supervenient relations. According to Cleland, a dyadic relation  $R$  between  $x_1$  and  $x_2$  is strongly supervenient on the intrinsic property  $P$  iff:

- (1) It is not possible for  $Rx_1x_2$  to hold without  $x_1$  and  $x_2$  instantiating  $P$ .
- (2) From  $Rx_1x_2$ , it follows that  $x_1$  and  $x_2$  instantiate  $P$ , and from  $P_i x_1$  and  $P_j x_2$ , it follows that  $Rx_1x_2$ .

Here,  $P_i$  and  $P_j$  represent distinct instances of  $P$ . According to Cleland, the relation  $R$  is strongly non-supervenient if both conditions (1) and (2) are unmet, whereas it is weakly non-supervenient if only condition (2) is unmet (see also French and Krause, 2006, 165). It is worth noting that while authors such as Teller (1986) and Esfeld (2004) have used Cleland's notion to account for the metaphysics of entanglement, Cleland

originally proposed this notion to account for relations typically considered external, such as the relation of spatial distance.

As can be observed, internal relations have generally been regarded as supervenient and therefore reducible to non-relational properties, while only external relations are seen as non-supervenient or fundamental. This characterization likely stems from a prior commitment to a reductionist view of internal relations. This view aligns with metaphysical frameworks that can be described as particularist, which are based on particular facts or individual objects (what structuralists might term "object-oriented metaphysics"). In these frameworks, relations are rarely included in fundamental ontology, except for a minimal set of recognized external relations. For instance, in Humean supervenience (Lewis, 1986), only spatiotemporal relations are accepted as fundamental.

However, tracing further back in history, one finds alternative definitions of internal relations. For example, according to Moore (1919), a relation is internal if it follows from the existence of its relata, a view that crucially resembles the modal account of essence (see Fine, 1994). Notably, Moore's definition allows internal relations to be considered at least as fundamental as the non-relational properties of their relata, since internal relations here derive solely from the existence of the relata and not from their non-relational properties. The difference between Moore's definition and those of Armstrong and Lewis lies in the latter belonging to a generation of metaphysicians for whom the debate on the status of internal relations in favor of reductionism had already been settled. By contrast, Moore's more moderate position reflects his belonging to a generation of metaphysicians for whom the debate between particularism and holism (or between particularism and monistic holism) and the fundamental or derivative status of internal relations was still ongoing.

This historical issue was reintroduced in recent discussions by Schaffer (2010a, 2010b), who is recognized for his defense of a moderate form of monism that will be considered in Subsection (4.2.3). Schaffer's reflections on the late 19th and early 20th-century debate between neo-Hegelians (advocates of monistic holism) and early analytic philosophers (advocates of particularism) help clarify why a reductionist stance toward internal relations was adopted. If internal relations, understood as essential relations in Moore's sense,

are merely derivative and not fundamental, then particularist metaphysics prevails over holistic metaphysical frameworks. Indeed, there is nothing fundamental that essentially ties one object to another. However, if internal relations, as essential relations, are allowed to be at least as fundamental as the non-relational properties of their relata, then internally related objects would be fundamentally integrated wholes, thereby blurring the particularist view (more details on this line of reasoning are provided in Subsection 4.2.3).

Taking into account these conceptual and historical underpinnings, and in contrast to the settled definition of internal relations as merely supervenient, we define them in this paper as those that are essential to the relata—that is, relations on which their existence and identity depend. This definition allows us to regard internal relations as essential polyadic features of the relata, which could be either fundamental or, as typically considered, supervenient upon their intrinsic properties. This contrasts with external relations, which cannot be considered essential. This approach to assessing relations—focusing on their essential or non-essential character rather than their fundamental or derivative status—allows fundamental internal relations to exist, thus resolving the tension between entanglement's non-supervenience and internality. Clearly, this challenges the "reductionist view of internal relations" assumed in particularist metaphysical frameworks, while precisely enabling the characterization of entanglement—and other families of relations—as both internal and non-supervenient.

In what follows, we present our non-reductionist analysis of internal relations as essential polyadic features. As proposed, the focus shifts away from whether internal relations are fundamental or supervenient and instead centers on their essential character. As said before, the essential properties of an individual have been traditionally interpreted as those necessarily following from its existence. However, according to Fine, propositions involving essential predicates cannot be reduced to merely modal propositions. Nonetheless, modal propositions are necessary conditions for essential ones. Thus, the fact that a certain property is part of the essence of  $x$  can be expressed in modal-existential terms as follows (Fine, 1994):

$$P^{\text{ess}}x \rightarrow \Box(Ex \rightarrow \exists P(Px)) \quad (5)$$

Here,  $P^{\text{ess}}$  is an essential monadic predicate, and  $E$  is the existential predicate, which can be defined in terms of the existential quantifier and identity as  $Ex \equiv \exists z(z = x)$ . Equation (5) indicates that if  $P$  is an essential property of  $x$ , then it is necessary that, if  $x$  exists, there exists a property  $P$  such that  $Px$ . To define  $P^{\text{ess}}x$  by establishing equivalence, it is necessary to use Fine's indexed modal operator ( $\Box_x$ ), introduced in his logic of essence (1995b), which signifies "it is true in virtue of the essence of  $x$  that". Thus, we obtain:

$$P^{\text{ess}}x \equiv \Box_x (Ex \rightarrow \exists P(Px)) \quad (6)$$

Equation (6) means that  $P$  is an essential property of  $x$  iff it is true in virtue of the essence of  $x$  that, if  $x$  exists, then there exists a property  $P$  such that  $Px$ . The expression  $\Box_x (Ex \rightarrow \exists P(Px))$  in equation (6) can be simplified as  $\Box_x Px$ , which should be read as: "it is true in virtue of the essence of  $x$  that  $Px$ ." Thus,  $P^{\text{ess}}x$  meaning " $x$  is essentially  $P$ ," or " $P$  is an essential property of  $x$ ," can be formalized simply as  $\Box_x Px$ . That is:

$$P^{\text{ess}}x \equiv \Box_x Px \quad (7)$$

Following this line of reasoning, for a rigid essential dyadic predicate, that is, for an one-way rigid internal relation  $R^{\text{int.rig}}$ , we propose the following analysis:

$$R^{\text{int.rig}}x_1x_2 \equiv \Box_{x_1} Rx_1x_2 \equiv \Box_{x_1} (Ex_1 \rightarrow \exists R(Rx_1x_2)) \quad (8)$$

That is,  $x_1$  is internally and rigidly  $R$ -related to  $x_2$  iff it is true, in virtue of the essence of  $x_1$ , that from the existence of  $x_1$ , it follows that there exists a relation  $R$  such that  $Rx_1x_2$ .  $R$  on the right cannot be just any relation but must be one that holds in virtue of the essence of  $x_1$ ; otherwise, the condition would be trivially met. Of course, we are assuming here that  $x_1$  and  $x_2$  are distinct entities, so  $R$  cannot be identity. If we allowed identity to take the place of  $R$ , then everything would be internally related to itself, which is not the intended meaning of an internal relation. Clearly this is not the case for entanglement, which is an irreflexive relation, as discussed in Section (2).

In case the reference for  $x_2$  is not rigid but generic—namely, some  $x_2$  of a certain sort specified by predicate  $F$  (see Correia, 2008)—we obtain a one-way generic internal relation  $R^{\text{int.gen}}_{x_1x_2}$ , which Calosi and Morganti (2021, see their equation 7) analyze as  $\Box_{x_1} \exists x_2 (Fx_2 \wedge Rx_1x_2)$ . We further elaborate its analysis as follows:

$$R^{\text{int.gen}}_{x_1x_2} \equiv \Box_{x_1} (\exists x_2 (Fx_2) \wedge (Ex_1 \rightarrow \exists R(Rx_1x_2))) \quad (9)$$

That is,  $x_1$  is internally  $R$ -related to a generic  $x_2$  iff it is true, in virtue of the essence of  $x_1$ , that there exists some  $x_2$  such that  $x_2$  is  $F$  and, from the existence of  $x_1$ , it follows that there exists a relation  $R$  such that  $Rx_1x_2$ .

For a symmetric rigid internal relation—that is, for a relation  $R^{\text{int.rig.sym}}_{x_1x_2}$  obtained as the conjunction of  $R^{\text{int.rig}}_{x_1x_2}$  (equation 8) and its converse—the proposed analysis is:

$$R^{\text{int.rig.sym}}_{x_1x_2} \equiv \Box_{x_1x_2} Rx_1x_2 \equiv \Box_{x_1x_2} (Ex_1 \wedge Ex_2 \rightarrow \exists R^{\text{sym}}(Rx_1x_2)) \quad (10)$$

That is,  $x_1$  and  $x_2$  are internally, rigidly and symmetrically  $R$ -related iff it is true, in virtue of the essences of  $x_1$  and  $x_2$ , that from the existences of  $x_1$  and  $x_2$ , it follows that there exists a symmetric relation  $R$  such that  $Rx_1x_2$ . In equation (10), as can be observed, the existence of  $R$  follows, in virtue of the essences of both  $x_1$  and  $x_2$ , from the existence of its two relata. Henceforth,  $\Box_{x_1x_2} Rx_1x_2$  will be taken as the simplest formal expression to represent the fact that  $x_1$  and  $x_2$  are internally and rigidly related by the symmetric relation  $R$ .

Finally, it is worth noting that a symmetric generic internal relation  $R^{\text{int.gen.sym}}_{x_1x_2}$  can be construed by allowing one or both relata to belong to a certain kind rather than being rigidly fixed. For instance, if the reference for  $x_1$  is rigid while that for  $x_2$  is generic, we obtain the following analysis:

$$R^{\text{int.gen}_2\text{sym}}_{x_1x_2} \equiv \Box_{x_1} \exists x_2 (Fx_2) \wedge \Box_{x_1x_2} (Ex_1 \wedge Ex_2 \rightarrow \exists R^{\text{sym}}(Rx_1x_2)) \quad (11)$$

That is,  $x_1$  and  $x_2$  are internally and symmetrically  $R$ -related—with rigid reference for  $x_1$  and generic for  $x_2$ —iff it is true, in virtue of the essence of  $x_1$ , that there exists some  $x_2$  such that  $x_2$  is  $F$  and, in virtue of the

essences of  $x_1$  and  $x_2$ , that from the existences of  $x_1$  and  $x_2$ , it follows that there exists a symmetric relation  $R$  such that  $Rx_1x_2$ .

In this way, a non-reductionist analysis of different types of internal relations is achieved, where internality is associated not with a supposed supervenient character but with its essential character, as proposed in the above definition. Interestingly, essential relations—when allowed to be fundamental—can be taken to indicate the existence of a joint essence pertaining to a fundamental whole, rather than the mere cooperation of separate essences. This is precisely why such relations can be termed ‘internal’: they hold between different internal parts of a single whole. This point is taken up in Section (4), where the above analysis will prove useful for arguing in favor of considering entanglement as an internal relation within different metaphysical frameworks. With this foundation in place, we now turn to the notion of ontological dependence.

### 3.2. The Notion of Ontological Dependence

The metaphysical frameworks mentioned earlier are crucially distinguished by the structure of ontological dependence relations they posit. In structuralism, the fundamental entities are relations, and particular objects depend on them; in monism, the fundamental entity is the whole, and particular objects depend on it; in coherentism, particular objects symmetrically depend on one another. In particularist or object-oriented metaphysics, particular objects are the fundamental entities, with their internal relations and the whole they constitute being dependent entities.

Now, we outline several characteristics of this notion. First, ontological dependence is a relation whose relata are typically concrete particular objects. Nonetheless, it has also been proposed that the relata may belong to different ontological categories—for example, particulars and the relations between them. This is the case in structuralism, where concrete particulars ontologically depend on the relations between them. Conversely, in particularism, internal relations depend on the concrete particulars they relate. Second, most metaphysicians regard ontological dependence as an asymmetric relation. This is the case in structuralism, monism, and

particularism. However, ontological dependence can also be conceived as a symmetric relation, as in coherentism.

Another assumption generally associated with the notion of ontological dependence is metaphysical foundationalism or well-foundedness (WF). Asymmetric dependence, by its irreflexive and transitive character, induces a partial order among particular objects. If dependence is well-founded, there exists a domain of basic objects that are considered absolutely fundamental, where chains of dependence end. (WF) can be formalized as follows (see Calosi 2014):

$$\forall x(Bx \vee \exists y(By \wedge Dxy)) \quad (12)$$

Here,  $B$  is the property of being a basic object, and  $D$  is a binary predicate corresponding to ontological dependence. Equation (12) states that for all objects  $x$ , either  $x$  is basic, or there exists an object  $y$  that is basic such that  $x$  ontologically depends on  $y$ . These basic objects can be located at any level of a mereological hierarchy. If they are at the lowest level, (WF) aligns with a form of atomistic particularism. If the basic object is at the highest level, the single whole, then (WF) aligns with a form of monistic holism. Recall that ontological dependence can also have relata belonging to different ontological categories. Therefore, it is possible for chains of dependence to terminate in a basic structure, as in structuralism. If (WF) is rejected, the chains of dependence may extend indefinitely (infinetism). If (WF) is rejected and reciprocal or symmetric dependence is admitted, then chains of dependence can loop back on themselves, as in the case of coherentism.

Although some metaphysicians consider the notion of ontological dependence to be primitive and therefore unanalyzable—merely a binary predicate  $D$  (see Schaffer, 2009)—there are several proposals for analyzing it. First, the modal-existential analysis of ontological dependence is mentioned (see Tahko & Lowe, 2020):

$$D^{\text{mod}}_{x_1 x_2} \equiv \Box(E_{x_1} \rightarrow E_{x_2}) \quad (13)$$

Equation (13) tells us that  $x_1$  depends on  $x_2$  iff, necessarily, if  $x_1$  exists, then  $x_2$  exists. According to Fine, in his paper on ontological dependence (1995a), the modal-existential analysis is too weak to account for the

notion of ontological dependence and is consequently vulnerable to counterexamples, such as Socrates' dependence on his singleton. Fine then suggests that ontological dependence should be analyzed in more restrictive terms and that the notion should be tied to the essence of the dependent object. Then he proposes the following analysis:

$$D^{\text{ess}}x_1x_2 \equiv \Box_{x_1}(Ex_1 \rightarrow Ex_2) \quad (14)$$

In equation (14), unlike equation (13), the indexed modal operator  $(\Box_{x_1})$  has been introduced, which belongs to Fine's logic of essence (1995b), to indicate that necessity here is in virtue of the essence of  $x_1$ . Equation (14) represents Fine's essential-existential analysis of the notion of ontological dependence. Notice the resemblance between this analysis of dependence and the analysis of monadic essential property in equation (6). This formal similarity suggests that, under this analysis of dependence, the existence of the independent object ( $x_2$ ) is part of the essence of the dependent object ( $x_1$ ).

Finally, what may be called the "identity-based analysis", originally proposed by Lowe (1998) and further developed in Tahko and Lowe (2020), deserves mention. The idea behind this proposal is to ground the notion of ontological dependence not in the existence of the involved entities but in their respective identities. The identity involved here is not numerical but qualitative. The qualitative identity of the dependent object is taken to be essentially related to the qualitative identity of an independent object, in the sense that a one-way relation to that object figures among the properties that are constitutive of the dependent object's identity. Since this relation is identity-constituting, it is deemed essential to the dependent object. Given that the dependent object's identity is thus necessarily tied to an external and independent entity, this asymmetric identity-constitutive relation is taken, within this framework, to suffice for ontological dependence. Calosi (2020) suggests two ways to formalize this analysis.

$$D^{\text{id-1}}x_1x_2 \equiv \Box_{x_1}(Ex_1 \rightarrow Ex_2 \wedge \exists R(Rx_1x_2)) \quad (15)$$

$$D^{\text{id-2}}x_1x_2 \equiv \Box_{x_1}(Ex_1 \rightarrow \exists R(Rx_1x_2)) \quad (16)$$

In equation (15),  $x_1$  depends on  $x_2$  if, in virtue of the essence of  $x_1$ , the existence of  $x_1$  implies the existence of  $x_2$  and the existence of a relation between  $x_1$  and  $x_2$ . In equation (16), in virtue of the essence of  $x_1$ , the existence of  $x_1$  implies only the existence of the relation between  $x_1$  and  $x_2$  (no longer the existence of  $x_2$ ). Notice the similarity between the analysis of dependence in equation (16) and the analysis of the dyadic essential predicate in equation (8). This formal similarity suggests that, under this analysis of dependence, an object is dependent because it is internally related to another. Hence, ontological dependence and asymmetric internal relation could be considered equivalent, at least according to the interpretation of internal relations made in this article. Interestingly, Calosi (2020) proposes (in a footnote) a simpler, third formulation of this analysis, in which the existential predicate is not directly involved. Namely:

$$D^{\text{id-3}}_{x_1x_2} \equiv_{x_1} \exists R(Rx_1x_2) \quad (17)$$

Although equivalent to  $D^{\text{id-2}}$  (see equation 8),  $D^{\text{id-3}}$  makes the equivalence between ontological dependence and asymmetric internal relation even clearer. Please note that, for simplicity, we have considered only rigid dependence. Nonetheless, generic counterparts of dependence under the various analyses considered can be easily construed by mimicking the formal procedures employed in Subsection (3.1) for internal relations. Before proceeding, some partial conclusions derived from the work conducted in this section are highlighted:

- (1) An understanding of internal relations as essential relations was achieved, and formal means were found to express them.
- (2) It was shown that the fundamental or derivative nature of internal relations depends on the adopted metaphysical framework. If particularist metaphysics is rejected, internal relations can be considered fundamental.
- (3) Various analyses of the notion of ontological dependence were made available. In particular, it was found that the analysis based on identity may have close connections to the notion of internality.

#### **4. The Metaphysical Nature of Entanglement**

Having examined the metaphysics of relations and ontological dependence, we are now in a position to address the central question of this paper: What is the metaphysical nature of quantum entanglement? It will be shown that this question does not have a unique or absolute answer, but rather one that is relative to the metaphysical framework one chooses to adopt. First, we discuss the inadequacy of treating entanglement as an external relation. Then, we defend an understanding of entanglement as an internal relation. Finally, we explore how this view aligns with structuralism, monism, and coherentism.

##### **4.1. Entanglement as an External Relation**

In this subsection, we examine the challenges of treating entanglement as an external relation within a particularist metaphysical framework. "Particularism" (a term likely coined by Teller, 1986), "particularist metaphysics", or "object-oriented metaphysics" (a term coined by structuralists) is a framework or family of frameworks that prioritize particulars over relations or structures. A prominent example is Lewis' Humean supervenience (1986). This view posits a plurality of basic particulars, with relations among them regarded as derivative, supervening on their intrinsic properties. This contrasts with structuralist or holistic frameworks, which treat relations or wholes as fundamental. In particularist metaphysics, fundamental essential features are regarded as exclusively monadic, while internal relations are considered merely derivative, as they are assumed to necessarily supervene on these features. Within this framework, only external relations are admitted in the basic ontology. Indeed, if internal relations were fundamental, modal facts would no longer be supervenient, and basic objects would become mutually dependent. Thus, positing fundamental internal relations contradicts the conjunction of Hume's dictum (that basic objects are modally free) and particularism (that there is a plurality of basic objects). As a result, unless we are willing to abandon Hume's dictum, if fundamental internal relations are accepted, the metaphysical picture provided by particularism ultimately collapses into holism.

Though dominant for most of the last century, the metaphysical nature of entanglement poses a serious challenge to particularist metaphysics. Due to its non-supervenience, entanglement in particularism cannot be accommodated but as an external relation incorporated into the basic ontology, alongside space-time

relations. This move appears as a conservative way to deal with the metaphysical nature of entanglement without abandoning particularist assumptions. Some proponents of Humean supervenience consider this solution (see, for example, Darby, 2012). Under this approach, entanglement could take the place of the world-making relation (Jaksland, 2021). However, by treating entanglement as an external relation, particularist metaphysics faces significant difficulties, most of them previously recognized in Calosi and Morganti (2016). We outline them below:

- First, external relations do not seem capable of imposing modal constraints on their relata, of the kind required by the entanglement relation. In fact, space-time relations typically admitted at the supervenience basis generally do not impose modal constraints on their relata. For example, it is accepted that the spatial position of one basic object is independent of the position of another basic object. If entanglement is accepted as an external relation that imposes modal constraints, then basic or primitive modality is being assumed, which contradicts Hume's dictum.
- Second, the proposed solution introduces a new element into the basic ontology with no additional benefit, which may be seen as an ad hoc and not very parsimonious move.
- Third, as shown in Section (2), the entanglement relation is not transitive and therefore cannot be decomposed into a series of dyadic relations. It must be conceived as a relation involving all its relata at once, in a global manner. In contrast, in particularist metaphysical frameworks such as Humean supervenience, everything basic is local and particular. For instance, space-time relations can be decomposed into multiple dyadic relations, ensuring locality. Thus, introducing a global item into the basic ontology could make the particularist metaphysical picture appear incoherent: global matters of particular fact replace the local ones, so the world cannot be built piece by piece—a clear departure from Lewis's mosaic metaphor (see Darby, 2012).

In summary, the attempt to accommodate entanglement as an external relation within a particularist or object-oriented metaphysics seems unpromising.

## 4.2. Entanglement as an Internal Relation

Having outlined several reasons that make it problematic to regard quantum entanglement as an external relation, we now present an argument in favor of understanding it as an internal relation. The fact that entanglement is a non-supervenient relation should not be taken as an indication that it is necessarily an external relation. Accepting entanglement, due to its non-supervenience, as an external relation is forced by the prior adoption of a reductionist view of internal relations within the framework of a particularist, object-oriented metaphysics. If we step out of that framework and distinguish between internal and external relations not based on their derivative or fundamental status but based on their essential or non-essential character (or by their modal strength, if the notion of essence seems too metaphysically loaded, see Subsection 3.1), then it becomes possible to conceive of entanglement as a non-supervenient internal relation. This would require introducing an important additional assumption—the thesis of the entangled universe, in either its weaker or stronger variant—which will be discussed in the next subsection.

In contrast to external relations, internal relations, due to their essential character, can naturally give rise to the modal connections typical of entanglement. This is perhaps the most compelling reason to endorse the internal relation view of entanglement. Let us return to our previous analysis of different types of internal relations in Subsection (3.1). Given that essential propositions are stronger than modal ones, the analysis of, for instance, symmetric rigid internal relations provided in equation (10) implies:

$$R^{\text{int.rig.sym}} x_1 x_2 \rightarrow \Box (E x_1 \wedge E x_2 \rightarrow \exists R^{\text{sym}} (R x_1 x_2)) \quad (18)$$

Before moving on, note that here we focus on symmetric internal relations, given the symmetry of the entanglement relation discussed in Section (2). Additionally, leaving aside subtle questions concerning the identity conditions of quantum systems that we cannot address here, we consider, for simplicity, rigid internal relations—although the same reasoning would apply if the reference of the quantum systems involved were generic. In that case, the relevant equation would be (11), not (10). That said, given the equivalence proposed in equation (3),  $R x_1 x_2$  can be replaced by  $(U x_1 \leftrightarrow D x_2) \wedge (D x_1 \leftrightarrow U x_2)$  in the consequent of equation (18).

Thus, for the case of entanglement resulting from the singlet state (the particular case addressed in equation (3)), we obtain:

$$R^{\text{int.rig.sym}}_{x_1x_2} \rightarrow \Box (Ex_1 \wedge Ex_2 \rightarrow \exists R^{\text{sym}} ((Ux_1 \leftrightarrow Dx_2) \wedge (Dx_1 \leftrightarrow Ux_2))) \quad (19)$$

Note that, according to equation (19), if entanglement resulting from the singlet state is considered an internal relation  $R^{\text{int.rig.sym}}_{x_1x_2}$ , then it follows that, necessarily, the existence of systems  $x_1$  and  $x_2$  entails the existence of a relation  $R^{\text{sym}}$  such that the correlations  $(Ux_1 \leftrightarrow Dx_2)$  and  $(Dx_1 \leftrightarrow Ux_2)$  associated with the singlet state obtain. Please note that correlations due to entanglement can be stronger or weaker in a statistical sense. The modal strength of the entanglement relation should correspond to the physical degree of entanglement. In particular, in cases of maximal entanglement, the modal strength of the relation should be such that, once one outcome is fixed, the other is necessarily determined, excluding any alternative possibilities. Such modal strength is properly captured by the necessity operator applied to the consequent in equation (19).

Nonetheless, this should not be taken as an indication that entanglement is necessarily maximal on the internal relation view. Had we a more nuanced modal logic to capture different degrees of modal strength, different degrees of entanglement could be represented within this formalism.

The above scheme could be generalized to entanglement relations other than the one resulting from the singlet state, and to cases where the reference of the quantum systems involved is generic. This suggests that entanglement is best conceived as a type of internal relation—symmetric, either rigid or generic—as this naturally supports the modal connections associated with it. While entanglement remains non-supervenient in this approach, its essential nature as an internal relation ensures that the correlations associated with it possess the required modal strength. This avoids the issues that arise when entanglement is treated as an external relation.

A further clarification is in order concerning the precise notion of essence at stake and its associated modal entailments. As mentioned, in the modal account of essence—as discussed for instance in Fine (1994)—a property is taken to be essential to an entity if it follows necessarily from the existence of that entity. In

possible-worlds terms, a property is essential iff the entity possesses it in every possible world in which it exists. As discussed in Subsection (3.1), Fine argued that this definition is too weak, and proposed instead that an essential property must follow from the existence of the entity in virtue of its essence, whatever that is ultimately taken to be. Nonetheless, even in Fine's framework, essence retains strong modal entailments.

Against this background, treating entanglement as an essential feature may seem problematic. Entangled states are itself contingent: quantum systems are not necessarily entangled, and separable states are perfectly admissible within standard quantum mechanics. This may suggest that any claim about the essentiality of entanglement is ill-posed from the outset: how could entanglement be an essential property of a quantum system if it is possible for that system to be in a separable state?

The answer lies in the fact that the notion of essence at issue here is not intended to be absolute, in the sense of requiring that the entity possess the relevant property in every possible world in which it exists. As is common in the metaphysics of science, our interest is not in unrestricted modal space, but in domains of possible worlds constrained by scientific theories. The notion of essence employed here is therefore a relativized one: a property is essential to an entity iff that entity possesses the property in all possible worlds within a specified domain (for a defense of relativized essentialism and its associated modality, see Hirèche 2022).

In the present case, the relevant domain is restricted to worlds in which the universal quantum state is entangled. Within such a domain, entanglement can be construed as an essential relation in the following sense: given an entangled universal quantum state with unitary dynamics, every subsystem of the universe is at least minimally entangled with some other subsystem in every world belonging to that domain. Thus, the essentiality claim concerns entanglement relative to a globally entangled universe, rather than entanglement simpliciter. It is this relativized notion of essence that is operative here. We further discuss this crucial physical hypothesis in the next subsection.

#### 4.2.1. Physical and Mereological Assumptions

Before advancing with the detailed explanation of how entanglement as an internal relation fits within structuralism, monism, and coherentism, it is necessary to discuss certain assumptions required. As made explicit in equations (18) and (19) above, one necessary condition for entanglement to be considered an internal relation is that it occurs necessarily among the quantum systems involved, whether their reference is rigid or generic. For this reason, if one is willing to construe entanglement as an internal relation, it must first be assumed that entanglement is a widespread phenomenon affecting all quantum systems in the universe. This relates to the physical hypothesis of an "entangled universe" (EU). Calosi (2014) states this thesis as follows:

(EU) There is a quantum state of the universe and it is an entangled state.

Although controversial, this assumption has physical motivation. In its favor, there is the fact that the dynamics of quantum mechanics (the Schrödinger equation) preserves entanglement. That is, an entangled quantum state cannot spontaneously evolve into a separable state. Moreover, in general, interactions result in entangled states. This allows for the assumption that the state of the universe is entangled or will eventually reach that condition. Non-collapse frameworks—Everettian, certain modal, and decoherence-based interpretations—are naturally compatible with or supportive of global entanglement, as the universal wave function evolves unitarily and typically becomes increasingly entangled through system-environment interactions. However, some influential interpretations (including the so-called orthodox interpretation) postulate the occurrence of "collapses" under certain circumstances, meaning non-unitary jumps from superpositions to eigenstates, causing quantum systems to disentangle. Thus, it must be acknowledged that (EU) is sensitive to the interpretation of quantum mechanics one chooses to adopt.

Strictly speaking, for entanglement to be considered a rigid internal relation, not only is (EU) necessary but also a stronger form of that assumption. In fact, states such as the one proposed by Bigaj (2012) for multipartite systems (see Section 2), though globally entangled, allow for particular subsets of subsystems to be unentangled (see the example in equation 4). If that is the case, entanglement can only be construed as a

generic internal relation, where the references of the relata are not necessarily fixed. If one is willing to construe entanglement as a rigid internal relation, a stronger version of (EU) is required. In terms of Calosi (2014):

(EU\*) There is a quantum state of the universe, and it is a true n-multipartite entangled state.

This principle postulates not only that the state of the universe is entangled but also that each concrete object is entangled with all others. The thesis is stronger and, therefore, more controversial than (EU). However, entanglement is known for being preserved between very distant objects. This, along with certain commonly accepted cosmological scenarios where the universe begins in a simple quantum state (see Hartle 2003), probably an entangled state, as well as with no-collapse interpretations of quantum mechanics, makes this assumption reasonable.

Before moving on, let us discuss two quantum mechanical principles that impose constraints on a hypothetical quantum state of the universe: subadditivity and monogamy. Quantum subadditivity is an entropy inequality which states that the uncertainty contained in a composite system is never greater than the sum of the uncertainty contained in its subsystems. Correlations between subsystems, rather than adding to the overall uncertainty, actually serve to reduce it. More simply, knowing how systems are correlated provides a more complete picture than considering them in isolation. The principle of monogamy establishes that a quantum system's capacity for entanglement is a limited resource. If a system  $x_1$  is maximally entangled with a system  $x_2$ , they cannot share any entanglement whatsoever with a third system  $x_3$ . Maximal entanglement, in this sense, is exclusive; it cannot be freely or universally shared among multiple partners.

These principles constrain the possible forms a globally entangled universe could take. Subadditivity does not forbid global entanglement but places an upper bound on it, ruling out global states in which the uncertainty of the whole exceeds the sum of the uncertainties of its parts. The monogamy principle, in turn, rules out a universe in which pairs of subsystems are maximally entangled, since maximal entanglement between two subsystems entails their complete disentanglement from the rest of the universe. Subadditivity and monogamy are not obstacles for (EU) and (EU\*), but structuring principles. In particular, since (EU)

allows subsystems to be pairwise unentangled, it leaves more room for strong entanglement among other pairs. By contrast, (EU\*) requires that every pair of subsystems instantiate at least a minimal degree of entanglement, thereby constraining the extent to which strong pairwise correlations can be realized.

Moreover, both (EU) and (EU\*) are compatible with the local preparation of approximately separable states or approximately maximally entangled pairs. This is because the constraints imposed by monogamy and subadditivity govern the global distribution of entanglement rather than its local manipulation. Local operations allow one to temporarily isolate and control a small set of subsystems, effectively decoupling them from the rest of the universe to create strong, localized correlations, without violating the global constraints.

Importantly, the argument for entanglement as an internal relation does not require (EU\*), nor does it require the universe to be in a maximally entangled state. It suffices that some degree of global entanglement is conserved. Please note that the entangled universe thesis—though controversial—is not an ad hoc move introduced here to defend internality, but a position already explored in both theoretical physics and metaphysics of science that plays an important heuristic role in both domains. My aim is not to provide a full defense of (EU)—let alone of (EU\*)—since that would exceed the scope of this paper, but rather to show that, if either hypothesis is adopted, conceiving entanglement as an internal relation becomes a serious metaphysical option.

Finally, it is important to note that the internal relation view does not require global entanglement itself to be metaphysically necessary. It suffices that the universe happens to be contingently entangled. Given this assumption, a non-collapse interpretation of quantum mechanics guarantees—via unitary Schrödinger evolution—that entanglement will be preserved over time. As a result, necessity arises only relatively: at the level of subsystems within an entangled universe, not at the level of the universe as a whole. Global entanglement need not be a physical, let alone a metaphysical, necessity. Modal entailments associated with the essentiality of entanglement thus emerge conditionally, once global entanglement is in place. As stressed above, the notion of essence at stake is not absolute but relative, with respect to a domain of possible worlds constrained by a given scientific hypothesis.

We now briefly present two additional mereological principles required by one of the metaphysical frameworks that will be discussed: monism. Please note that these are not requirements of the internal relation view of entanglement itself, but only of monism. The first is the principle of unrestricted composition:

(UC) For every non-empty set of objects, there exists an object that is the mereological sum of those objects.

In a monistic metaphysical framework, (UC) ensures that there exists an object that is the mereological sum of all concrete particulars, which we call the universe. The second principle required by monism is the tiling constraint (TC) (Calosi, 2014):

(TC) The universe is a mereological sum of mereologically disjoint basic entities.

The adoption of (TC) is necessary for the option between monism and particularism to be decidable. In fact, from (TC), it follows that if the universe is basic, then nothing else can be. There is no need to go further into the meaning of these principles here. Interested readers may delve deeper into the literature on monism (Schaffer, 2010a, 2010b; Calosi, 2014).

#### 4.2.2. Entanglement in Structuralism

With these physical and mereological assumptions in place, we can now explore how entanglement, understood as an internal relation, fits within structuralism. Before proceeding, a brief note: in the following discussions, we consider only the rigid symmetric internal relation view of entanglement (not the generic one), without assuming commitment to (EU\*), but solely for the sake of formal simplicity. Structuralism is a metaphysical framework that has been gaining recognition in the area of the metaphysics of science for several years, becoming an important rival to particularist, object-oriented, metaphysics. Structuralism holds that concrete particular objects asymmetrically depend on a structure considered fundamental. This structure is precisely composed of a set of relations that mediate between those objects. In this metaphysical framework, relations are conceived as more fundamental than their relata, making the objects mere nodes or points of intersection within the structure. Entanglement, as a non-supervenient relation, can naturally occupy

the role of the fundamental structure within this metaphysical framework. Of course, commitment to at least (EU) is required. Calosi and Morganti (2021) propose a formalization of structuralism in which they employ the essential-existential analysis of the notion of ontological dependence ( $D^{\text{ess}}$ , see equation 14). For simplicity, suppose the universe is bipartite, containing only two objects,  $x_1$  and  $x_2$ . Structuralism then proposes that:

$$\Box_{x_1 x_2} (Ex_1 \wedge Ex_2 \rightarrow \exists R^{\text{irr, sym}} (Rx_1 x_2)) \quad (20)$$

where  $R$  takes the place of the fundamental structure. In this formulation,  $D^{\text{ess}}$  is a relation where the dependent terms are the objects  $x_1$  and  $x_2$ , and the structure  $R$  is the independent term.

With a formulation of structuralism at our disposal, it becomes clear that entanglement is advantageously understood as an internal relation within this framework. This is because the internality of the structure allows us to infer an asymmetric dependence relation between the objects and the structure, as required by this form of structuralism. Consider that, from our analysis of the rigid symmetric internal relation (equation 10), it follows:

$$R^{\text{int, rig, sym}} x_1 x_2 \rightarrow \Box_{x_1 x_2} (Ex_1 \wedge Ex_2 \rightarrow \exists R^{\text{sym}} (Rx_1 x_2)) \quad (21)$$

Note that the consequent in equation (21) approaches the formal expression of structuralism proposed in equation (20). In equation (20) it is only added that the structure is an irreflexive relation, a condition that is satisfied if we accept that the structure is identified with the entanglement relation, which is considered irreflexive (see Section 2). This indicates that, starting from entanglement as an internal relation, an argument in favor of this form of structuralism can be constructed. McKenzie (2014) arrives at the same conclusion, presenting a similar argument, in which the starting point is the existence of relations that weakly discern indistinguishable fermions and result from the application of the symmetrization postulate (Muller and Saunders, 2008). Nonetheless, if one intends to provide a complete defense of this form of structuralism, it would be necessary to discuss the ontological status that should be assigned to the properties of quantum systems that do not depend on the state and thus could not be entangled, such as mass, charge, etc. It is

unlikely, at least in non-relativistic quantum mechanics, that such magnitudes can be reduced in terms of relations. Additionally, an argument should be introduced to establish that the dependence between objects and structure is not reciprocal. In other words, it would be necessary to exclude the moderate variant of structuralism. This is not the appropriate place to delve further into these points.

Now, an objection raised by Calosi and Morganti (2021) is briefly addressed. They argue that taking entanglement as the fundamental structure is not enough to account for a rigid dependence between the objects and the structure, but only for a generic collective dependence. The authors argue that it is not possible to precisely select which of the many entanglement relations that can occur between a number of quantum systems supports the rigid dependence of the objects on the structure. The authors assume that entanglement can be properly expressed in terms of a plurality of entanglement relations, each relative to a particular observable. However, as discussed in Section (2), Cinti et al. (2022) demonstrated that entanglement is a single relation that cannot be relativized to observables. This allows for the establishment of rigid dependence of objects on the structure, thus dispelling the skeptical doubt raised by the aforementioned authors.

#### 4.2.3. Entanglement in Monism

This subsection explores how entanglement, as an internal relation, can be accommodated within a monistic framework, particularly in priority monism. While structuralism prioritizes relations, monism emphasizes the primacy of the whole over its parts. We first mention a weaker thesis, metaphysical holism (see Calosi, 2014). This doctrine holds that if two (or more) objects exhibit modal connections, they cannot be basic but must be dependent parts of a common whole. According to Calosi and Morganti (2021), holism can be formalized as follows:

$$\Box_{x_1, x_2} (Ex_1 \wedge Ex_2 \rightarrow \exists u (x_1 \ll u \wedge x_2 \ll u)) \quad (22)$$

Here,  $\ll$  represents the mereological relation of proper parthood (i.e., if  $x \ll u$ , then  $x$  is a proper part of  $u$ ). In this formulation, the notion of ontological dependence analyzed in essential-existential terms ( $D^{\text{ess}}$ ) is

again employed, so that  $x_1$  and  $x_2$  depend on  $u$ . Note that the fundamental whole to which a number of concrete particulars depend can be their mereological sum or a larger whole of which the particulars at issue are proper parts. It should be emphasized that the argument in favor of holism crucially depends on Hume's dictum regarding the independence of basic objects, a principle shared by proponents of particularist metaphysical frameworks. Recall that, according to this principle, basic objects must be independent, meaning there cannot be modal connections between them. Therefore, if modal connections are found between objects, they cannot be basic but must be parts of a fundamental common whole. In terms of Ismael and Schaffer (2020), modally connected objects have a common ground. Actually, the argument of these authors is somewhat subtler: where there are modal connections, while it is not possible to establish a common cause, there is a common ground.

If those modal connections are only local, then we remain in a form of relatively innocuous holism. However, there is currently a stronger form of holism in the metaphysics of science that is gaining some recognition in recent years. This is priority monism, defended by Schaffer (2010a, 2010b). According to priority monism, ontological chains of dependence are well-founded and culminate in the universe, the only basic entity. In contrast to radical monistic theses that deny the existence of a plurality of concrete particulars, priority monism represents a moderate form of monism that accepts the existence of a plurality of concrete particulars, though it denies their basic status. Priority monism, as a strong form of holism, requires that all objects in the universe be modally connected. At this point, one could appeal to entanglement or some other type of physical relation to ensure the required modal connections. In one of Schaffer's works, a basic universe, integrated as a whole, is obtained from the idea that "all things are internally related" (2010b). Of course, if entanglement takes the place of the internal relation that modally connects all things, then commitment to at least (EU) is required. Additionally, by its very nature, priority monism requires that there be a mereological sum of all objects in the universe, i.e., (UC) is required. Furthermore, to ensure that the universe is the only basic object, (TC) is required.

Suppose once again, for simplicity, that the universe has only two parts,  $x_1$  and  $x_2$ . In continuity with equation (22), the inference of priority monism from the internal relation that modally connects all objects could be formalized as follows:

$$R^{\text{int.rig.sym}} x_1 x_2 \rightarrow \Box_{x_1, x_2} (E x_1 \wedge E x_2 \rightarrow \exists u (u = x_1 + x_2)) \quad (23)$$

Here,  $+$  represents the mereological sum, and  $u$  refers to the universe as a whole.  $R^{\text{int.sym}}$  is the internal relation from which Schaffer infers priority monism, a role that could be fulfilled by entanglement under (EU). Schaffer himself does not provide an analysis of this kind, as he treats the notion of ontological dependence as a primitive, unanalyzable notion (Schaffer, 2009). There are no available attempts in the literature to analyze priority monism in terms of an analysis of ontological dependence. Therefore, establishing the validity of equation (23) is a pending task.

Before concluding this subsection, two clarifications are made. First, for Schaffer's argument to work, the internal relation in question does not need to be considered internal in the strongest sense being adopted here, i.e., as an essential polyadic feature. It only needs to be the case that the internal relation in question imposes modal constraints on its relata. Of course, the argument proceeds smoothly if internal relations are understood in terms of essence, as modal claims follow from essential ones (see Subsection 3.1). Second, monistic entanglement can be somehow considered a derivative relation, in contrast to structuralist entanglement. This is because the entanglement relation between parts can be considered necessitated by a certain monadic property of the whole. In the example of the two entangled fermions provided in Section (2), it was mentioned that a property associated with a certain value (0) of total spin is assigned to the composite system. In monism, it can be considered that the entanglement relation resulting from the singlet state depends on (0) total spin.

#### 4.2.4. Entanglement in Coherentism

Calosi and Morganti (2021) are recent proponents of coherentism. They reject hierarchical metaphysical frameworks in which chains of ontological dependence run in a one-way direction from objects to the fundamental whole or from them to a fundamental structure. Their main reason for this rejection is that both

structuralism and monism involve a series of controversial assumptions, such as (EU) and (WF). Monism, moreover, requires the mereological principles (UC) and (TC) (see Subsection 4.2.1). The authors only accept reciprocal or symmetric relations of ontological dependence, resulting in a non-hierarchical metaphysical framework, known as coherentism.

This subsection examines how entanglement, as an internal relation, can be naturally accommodated within coherentism by establishing symmetric dependence relations between objects. Unlike structuralism and monism, which posit hierarchical dependence distinct from entanglement, coherentism can directly accept entanglement as the symmetric dependence relation assumed in this framework. This non-hierarchical approach aligns naturally with entanglement's symmetric and non-transitive character. This may make the integration of entanglement and coherentism appear more parsimonious than in structuralism and monism.

Let us spell this out. In structuralism and monism, entanglement is not itself considered an ontological dependence relation, but rather a relation from which asymmetric dependence relations follow. These asymmetric dependence relations take the set of the relata of the entanglement relation as one of their terms. The other term may be the whole (in monism) or the entanglement relation itself (in structuralism). Coherentism is simpler in that it avoids this complexity and can consider entanglement as a relation that is equivalent to reciprocal dependence, or at least one from which reciprocal dependence follows for the same relata. The formalization of coherentism that Calosi and Morganti (2021) propose is:

$$\Box_{x_1}(Ex_1 \rightarrow Ex_2) \wedge \Box_{x_2}(Ex_2 \rightarrow Ex_1) \quad (24)$$

The equation (24) means that for  $x_1$  it is essential that  $x_2$  exists and vice versa. Again, the notion of dependence being used here is the essential-existential ( $D^{\text{ess}}$ ). In equation (24), two asymmetric reciprocal relations are present, whose conjunction amounts to a symmetric dependence relation.

We now show how reciprocal dependence follows from entanglement when considered an internal relation, thereby finding a natural place within coherentism. Calosi and Morganti (2021) themselves propose that

entanglement can be accepted as an essential relation (for us, internal) that endorses coherentism. In their equation (25), the authors formalize the relation between two entangled objects as follows:

$$\Box_{x_1x_2} Rx_1x_2 \quad (25)$$

Note that it is precisely the intended meaning of the symmetric internal relation  $R^{\text{int.rig.sym}}x_1x_2$  according to the perspective assumed in this article, that is, a relation that holds necessarily in virtue of both relata's essences (see equation 10). The fact that reciprocal dependence follows from the internal relation between two objects can be expressed in the following terms:

$$R^{\text{int.rig.sym}}x_1x_2 \rightarrow Dx_1x_2 \wedge Dx_2x_1 \quad (26)$$

Equation (26) means that if  $x_1$  and  $x_2$  are symmetrically and internally related, then reciprocal dependence relations exist between them. If we accept that entanglement is an internal relation and if equation (26) is valid, then entanglement can be considered compatible with coherentism. Next, we evaluate under what conditions the validity of equation (26) can be established. Recall that in Subsection (3.2), a close similarity was found between the notion of internality and ontological dependence analyzed in terms of identity ( $D^{\text{id}}$ ). In particular, the second and third formulations of that analysis ( $D^{\text{id-2}}$  and  $D^{\text{id-3}}$ ) seem to be equivalent to an asymmetric internal relation (see equations 16 and 17). Based on equation (17) and from

$$R^{\text{int.rig.sym}}x_1x_2 \equiv \Box_{x_1x_2} Rx_1x_2 \equiv \Box_{x_1} Rx_1x_2 \wedge \Box_{x_2} Rx_2x_1 \quad (27)$$

it is possible to establish that a symmetric internal relation (such as the one instantiated by the relation of entanglement) is equivalent to two reciprocal asymmetric ontological dependence relations. Namely:

$$R^{\text{int.rig.sym}}x_1x_2 \equiv D^{\text{id-3}}x_1x_2 \wedge D^{\text{id-3}}x_2x_1 \quad (28)$$

This simply means that if  $x_1$  and  $x_2$  are internally and symmetrically related, then their identities are reciprocally dependent. This suggests that this third formulation of the identity-based dependence analysis is preferable for use within coherentism. It is clear that equation (28) implies equation (26), which was the

expression intended to be proven valid. Therefore, it can be concluded that the internality of entanglement is fully compatible with the framework provided by coherentism. Moreover, coherentism appears to be the metaphysical framework that benefits most from considering entanglement as an internal relation, as it provides the most parsimonious fit.

### **Final Remarks**

In this paper, we have argued that quantum entanglement should be understood as a non-supervenient internal relation. We have considered that internal relations are essential polyadic features that are allowed to be fundamental, resolving the tension between the non-supervenience of entanglement and its typical modal strength. This perspective offers several advantages over the external relation view of entanglement within particularist metaphysical frameworks. In particular, it naturally sustains the modal constraints associated with entanglement—something that particularism cannot achieve without introducing several issues, as discussed in Subsection (4.1). Additionally, it provides a more flexible integration of entanglement within a variety of metaphysical frameworks. Adapting to these frameworks, entanglement can be considered fundamental, derivative, or even a relation of symmetric dependence. Furthermore, treating entanglement as an internal relation, as defended in this article, allows for the formulation of arguments in support of structuralism, monism, or coherentism.

These contributions advance the existing literature by offering a novel perspective on internal relations and the metaphysics of entanglement. While previous work has treated entanglement as an external relation or focused on its non-supervenience, our approach reinterprets it in terms of essence, providing strong motivations for integrating entanglement into various metaphysical frameworks beyond particularism.

A final remark is in order. As discussed in Subsection (4.2.4), coherentism appears to be the metaphysical framework that accommodates entanglement in the most parsimonious way. However, coherentism can also be made compatible with holism or even priority monism, provided one accepts the additional assumptions required by this framework. It may even be worth exploring the possibility of treating coherentism as a form of moderate structuralism, where structure and related objects hold ontological parity. As Calosi and

Morganti (2021) suggest, such an approach could allow coherentism to reconcile key insights from both structuralism and monism, bringing together the best of both worlds.

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