A New Origin for Entropy: Jauch's Conservation Law and the Geometry of Thermodynamics

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Abstract

A central question for the foundations of thermodynamics is which conceptual structures underpin the existence of entropy and temperature. Jauch claimed that entropy and temperature could be derived from a novel conservation law. This paper reconstructs the physics and mathematics of Jauch's claim and finds that his original proof is not valid. Remarkably, his theorem is still true. The alternative proof provided here uses geometric ideas from modern gauge theory, revealing a deep geometric structure in the heart of thermodynamics. This result also helps to settle an old debate by showing that entropy and temperature can be defined in a new, conceptually clear way without appeal to an irreversible assumption like the second law.

1. Introduction

There is an old debate about whether thermodynamic entropy and temperature exist because of an assumption of irreversibility like the second law, or because of facts about adiabatic accessibility as defined by Carathéodory (1909), Ehrenfest-Afanassjewa (1925), or Lieb and Yngvason (1999), or because of some other reason entirely. This is a pressing question for the philosophical foundations of thermodynamics because of the central role that entropy and temperature play in both its conceptual structure and its application across the sciences. Its answer also has an important theoretical component, in part due to the difficulty of defining thermodynamic entropy and temperature using operational measurements.[1] Albert (2000) follows most textbooks in adopting the second law approach, while other philosophers defend the generality and precision of the other approaches (cf. Uffink and Valente 2021; Lavis and Frigg 2025). The debate also has implications for whether entropy is intrinsically linked to a thermodynamic

[1]This is especially true for entropy, but also for temperature: one should not be too cavalier about defining temperature as 'just what is measured by thermometers' before reviewing the extraordinary conceptual challenge of defining thermometers, as breathtakingly chronicled by Chang (2004).

arrow of time.[2] However, few have considered the implications of an alternative approach published in this journal by Josef-Maria Jauch (1972), who claimed that thermodynamic entropy and temperature can be derived from a conservation law.[3]

Jauch's conservation law states roughly that if an adiabatic process restores all the work configuration variables to their initial values, then that process cannot perform work. Jauch gave this principle some physical motivation and a precise mathematical expression, pointing out that it does not contain any irreversible ingredient like the second law. He also expressed the hope that it would provide a novel foundation for thermodynamics that frees us once and for all from the "wrong impression that the existence of entropy and temperature are characteristic consequences of irreversibility" or the second law (Jauch 1972, p.328). Jauch's new foundation may not be widely known, but it has still been mentioned and interpreted by many authors.[4] A development of his idea appears in Part I of a treatise published just after his untimely death (Jauch 1975). No second part was ever published.

My purpose in this paper is to reconstruct the physics and mathematics of Jauch's proposal, in order to correct and complete it. I first develop the physical and mathematical basis for Jauch's conservation law, arguing that it is indeed physically plausible and, unlike the second law, contains no irreversible ingredient. Unfortunately, a correction is also needed: Jauch's derivation of entropy and temperature from this conservation law is not valid, and his argument strategy cannot be straightforwardly repaired. Nevertheless, I will show that Jauch's theorem is actually true. The alternative proof that I will provide begins by showing how standard thermodynamics can be studied using tools from Yang-Mills gauge theory, and then uses those tools to reformulate Jauch's theorem as the requirement of a flat connection for a line bundle describing thermodynamics. This reformulation renders the proof into a straightforward consequence of the Frobenius theorem, but which makes use of a different argument from Carathéodory's theorem. The result is a promising new foundation for thermodynamics, and in particular for the origin or thermodynamic entropy and temperature, which helps to clarify its geometric structure along the way.

[2]Common wisdom that temporal direction arises from entropy increase or the second law has been challenged by Price (1996), Uffink (2001), Brown and Uffink (2001), Henderson (2014), Marsland III et al. (2015), and Roberts (2022, Chapter 6).

[3]Jauch appears to have stumbled onto this idea in a coauthored paper on Szilard's Paradox where they Jauch and Báron (1972, p.223) write in a footnote: "It seems not to be generally known that the existence of the integrating factor, hence the existence of the function entropy, is a consequence of the first principle of thermodynamics for conservative systems under reversible quasistatic variations. This was discovered by T. Ehrenfest. A new proof of this statement will be given in a subsequent publication."

[4]Cf. Emch (1984); Kuzemsky (2020); Jauch (1975); Walter (1978); Uffink (2001, 2007); Roberts (2022).

I begin in Section 2 by explaining the physics and mathematics of Jauch's conservation law, and then turn to his derivation in Section 3 to show where it falters. In Section 4 I introduce the geometric structure needed to reformulate Jauch's conservation law, and then use it to provide a corrected proof. Section 5 then briefly discusses the relationship between this theorem and the work by Ehrenfest-Afanassjewa (1925), before concluding in Section 6.

2. A Conservation Law

Jauch's aim is to derive thermodynamic entropy and temperature in a way that avoids the difficulties of the standard 'Carnot cycle' derivation. His work is thus in the tradition of Carathéodory, who had a similar aim.[5] However, instead of beginning with Carathéodory's principle, Jauch begins with a conservation law. This conservation law is Jauch's main idea, and indeed is retained entirely in the corrected proof that I will present below. I thus begin by explaining it in some detail: by first giving it some physical motivation, and then providing a formal mathematical statement.

2.1. **Physical motivation.** Imagine an engine with two cylinders, and thus with work described by two volume variables V_1, V_2. As the engine cycles through its various volume states, it traces out a closed loop in the space of work variables called a *work cycle* (Figure 1).

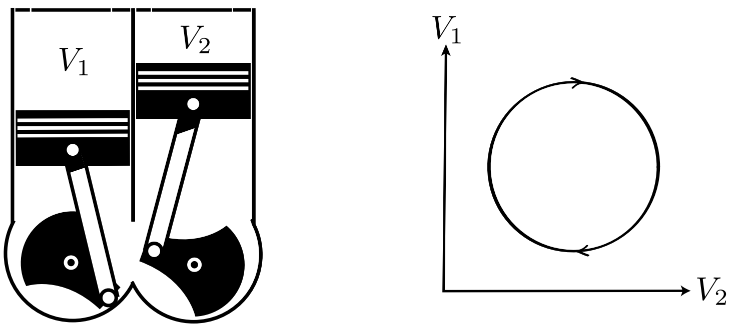

Figure 1: A cyclic engine with two volume variables.

During such a cycle, the engine's energy U does not necessarily return to its initial value. Instead, it may exchange energy with its environment, either by perform-

[5] Carathéodory wrote that "in order to be able from the outset to treat systems with arbitrarily many degrees of freedom, instead of the Carnot cycle that is almost always used, but is intuitive and easy to control only for systems with two degrees of freedom, one must make use of a theorem from the theory of Pfaffian differential equations" (Carathéodory 1909, p.357). For more concerns about the Carnot cycle approach see Uffink (2001, §6.2).

ing work or by exchanging heat. So, when the energy of the engine is included in the configuration space, its trajectory forms a downward spiral. The fact that the engine performs a work cycle is reflected in the fact that this spiral projects down onto a closed curve in the plane of work variables (Figure 2).

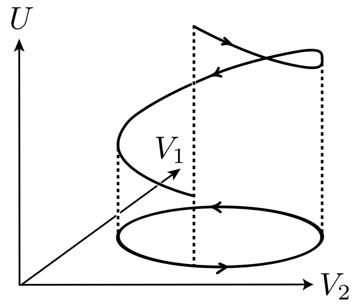

Figure 2: During an engine's cycle, energy does not return to its initial value even when volume does.

This downward spiral can be motivated by Kelvin's Principle, which I will state as follows (cf. Buchdahl 1966, p.89):

a cyclic process cannot absorb heat and convert it entirely into work.

Applied to our engine, this means that the process of absorbing heat to perform work must be limited in a way that leads our engine to lose heat to its environment. The result is an overall decrease in energy, which means the curve in Figure 2 can spiral down, but it cannot spiral up. Thus, Kelvin's Principle famously captures an 'irreversible' aspect of realistic engines, that they can traverse this curve in one direction but not the other.

Jauch's conservation law concerns the special case of a process with no heat exchange whatsoever, which is to say the process is *adiabatic*. In that case, the spirit of Kelvin's Principle would be to say: without heat exchange, there can be no work. Work energy must come from somewhere, and so if it does not come from heat exchange, then there cannot be any work. This is what I will call *Jauch's conservation law:*

a cyclic process without heat exchange cannot perform work.

This assumption is implicit throughout Carnot's famous *Reflexions on the Motive Power of Fire*: if a heat-engine receives no heat, then it cannot produce motive power or work.[6]

[6] One explicit such statement by Carnot is: "The result of these first operations has been the production of a certain quantity of motive power and the removal of caloric from the body A to the body B. The result of the inverse operations is the consumption of the motive power produced and the return of the caloric from the body B to the body A; so that these two series of operations annul each other, after a fashion, one neutralizing the other" (Carnot 1824, pp.66-67).

A consequence of it is that, if a cyclic engine is adiabatic, then it cannot result in any change in total energy. In terms of our diagram, this says that if an adiabatic curve produces a closed loop in the space of work variables, then the curve must be closed.

Thus, although Jauch's conservation law may be viewed as inspired by Kelvin's Principle, it does not introduce any irreversible ingredient into thermodynamics. For this reason, Jauch refers to it as "the static form of what is usually called the second principle of thermodynamics", noting that it "refers only to adiabatic processes, hence it is weaker than the second [Kelvin's] principle" (Jauch 1975, pp.122-3).

2.2. **Formal statement of the conservation law.** To state Jauch's conservation law mathematically, let M be a smooth connected manifold of dimension n with local coordinates $(U, V_1, \ldots, V_{n-1})$, where U represents energy and the V_i represent work configuration variables. The full phase space of thermodynamics is usually taken to be a larger $(2n+1)$-dimensional manifold with both intensive and extensive variables; however, it is generally equipped with n equations of state which, together with the first law, constrain physical behaviour to an n-dimensional manifold like this one.

For some set of smooth real-valued functions $P_1, \ldots, P_{n-1}$ on M, let *work* be defined[7] by $\omega := -\sum_{i=1}^{n-1} P_i dV_i$ and let *heat* be $\xi = dU + \omega$. A curve $\gamma : [0,1] \to M$ will be called an *adiabat* if there is no heat along it: $\xi(\bar{\gamma}) = 0$, where $\bar{\gamma}$ is the tangent vector field to γ. A curve will be called a *work cycle* just in case it is closed in the work variables, in that for $i = 1, \ldots, n-1$

$$V_i(\gamma(0)) = V_i(\gamma(1)). \qquad (1)$$

A work cycle is not necessarily a closed curve in M, which would require in addition that $U(\gamma(0)) = U(\gamma(1))$. Now Jauch's conservation law, that a cyclic process without heat exchange cannot perform work, can be stated:

(C_1) in a neighbourhood of every point, if γ is an adiabat and a work cycle, then $\int_\gamma \omega = 0$.

Jauch's own notion is slightly awkward here: Jauch refers to what I have called a work cycle as a closed curve, despite the fact that it is not closed in M in the ordinary topological sense. He later adjusts the terminology to refer to a work cycle as 'closed' (with scarequotes) (Jauch 1975, p.122). I will continue to use the term 'work cycle' in

[7] I adopt the sign convention $\omega = -PdV$ represents 'inward-directed' work done on the system, while $-\omega = PdV$ represents the work performed by the system on its environment.

what follows to avoid confusion, but Jauch's terminology is needed to understand his own statement of his conservation law in the paper, where he also writes δA for the work one-form that I have called ω:

> "The fundamental hypothesis that characterizes conservative thermal systems is that for every closed [in the sense of being a work cycle] adiabatic c, $\int_c \delta A = 0$" (Jauch 1972, p.329).

3. Jauch's Proof

Jauch's proof is an attempt to derive entropy and temperature, so let me briefly recall why this is a central problem in the foundations of thermodynamics. When work has the form PdV, heat can always be written as TdS: this follows immediately from the existence and uniqueness theorems for ordinary differential equations.[8] However, this is not generally true when work includes more than one volume variable like $\omega = -P_1 dV_1 - P_2 dV_2$. Then there need not exist any smooth functions T and S such that $\xi = TdS$. So, a further postulate is needed to describe the general circumstances under which this is the case. Jauch, Carathéodory, and many others are concerned with the question of which physical postulate is appropriate for that purpose, and thus for the conclusion that entropy and temperature exist.

Jauch's claim is that his conservation law, which he calls *hypothesis (1)*, is the appropriate physical postulate. I have adopted a more standard notation than him, so I will briefly summarise the differences before quoting him. Jauch denotes the configuration space manifold by $\mathcal{X}$ instead of M, and takes its dimension to be $n+1$ instead of n. His local coordinates are $(x_0, x_1, \ldots, x_n)$ instead of $(U, V_1, \ldots, V_{n-1})$, with energy denoted by x_0 instead of our U. Finally, Jauch writes heat as δQ instead of ξ and work as δA instead of ω. Jauch then claims:

> "**Theorem.** Under hypothesis (1), the differential form $\delta Q = dx_0 + \delta A$ admits an integrating factor T in a suitable neighbourhood of any point in $\mathcal{X}$ so that $\delta Q = TdS$ where S is some function" (Jauch 1972, p.329).

[8] *Proof:* An adiabat satisfying $0 = \xi = dU - PdV$ is equivalent to a solution to $P = dU/dV$. By the local existence and uniqueness theorems for ordinary differential equations, this equation has a solution in a neighbourhood of every point. Let S be any function that is constant along the resulting local congruence of adiabats. Then $dS = 0$ if and only if $\xi = 0$, and so since a non-vanishing one-form is determined up to a scale factor by its kernel, there is a smooth T such that $\xi = TdS$ in that neighbourhood. In contrast, with more volume terms like $\xi = dU - P_1 dV_1 - P_2 dV_2$, adiabats are characterised by a Pfaffian equation for which no comparable result is available.

In our notation, this says: *given* (C_1), *every point admits a neighbourhood in which* $\xi = dU + \omega$ *is equal to* TdS *for some smooth functions* T *and* S.

Jauch's argument proceeds as follows. Fixing a point $a \in M$, Jauch defines a function $\mathcal{U}$ by,

$$\mathcal{U}(x) := \int_a^x \omega + \text{constant} \tag{2}$$

where for each x the integral is taken along any adiabat γ from a to x (Jauch 1972, Equation 2). If there is more than one such adiabat, Jauch points out that they will nevertheless produce the same value for $\mathcal{U}(x)$. This is where Jauch makes use of his conservation law (C_1): since an adiabatic work cycle c satisfies $\int_c \omega = 0$, it follows that for the adiabat γ in Equation (2), "the value of the integral is independent of path (as long as it is an adiabat)" (Jauch 1972, p.330). This is the main step in the proof. The function $\mathcal{U}$ is then used as the basis for a coordinate transformation on the neighbourhood, which is then in turn used to construct entropy and temperature functions.

Unfortunately, the main step is not valid. Although Jauch addresses the possible non-uniqueness of adiabatic paths in defining $\mathcal{U}$, he does not address their possible non-existence: there might be points x and a that are not adiabatically connected. Without this assurance, the function is not defined on the entire neighbourhood, as would be required to use it as Jauch does to define a new coordinate in a coordinate transformation.

The problem can be made stark by asking, what is the domain of $\mathcal{U}$? It is not clear from Jauch's writing. However, three lines above the definition of $\mathcal{U}(x)$, he defines x to be an arbitrary point in the neighbourhood. This is problematic: it implicitly assumes each point x is connected to a by an adiabat, whereas Carathéodory's principle says that there are points in every neighbourhood that are *not* adiabatically connected to a. The problem is that Carathéodory's principle is known to be equivalent[9] to Jauch's conclusion, that there exist smooth functions T, S in a local neighbourhood of every point such that $\xi = TdS$. So, Jauch's argument appears to introduce an implicit assumption that is incompatible with his conclusion.

Even if Jauch were to accept that $\mathcal{U}$ is not defined on the entire neighbourhood, it then cannot be straightforwardly used to define a coordinate function on that neighbourhood, and so the rest of the argument falls apart. As a potential fix, one

[9]See Carathéodory (1909), as well as Bernstein (1960), Boyling (1968), or Bryant et al. (1991, Theorem 3.5) for a rigorous proof.

might hope to define a foliation of the neighbourhood into adiabatically connected hypersurfaces $r \mapsto \Sigma_r$. The hope would be to define a one-parameter *set* of functions $\mathcal{U}_r(x) = \int_a^x \omega$, each defined for all $x \in \Sigma_r$, and then to use this one-parameter set to define $\mathcal{U}$. However, Jauch has given no indication of how such a foliation could be constructed, or why each surface is guaranteed to be adiabatically connected. Such statements rather appear to require the very integrability condition that Jauch is trying to prove.

In my view, there is no straightforward repair to this proof strategy. Remarkably, Jauch's theorem does still turn out to be true. But, the proof that I will give of this fact makes use of an entirely different strategy, which is the subject of the next section.

4. Corrected Proof

This section proceeds in three stages. First, I reformulate thermodynamics on a line bundle, in order to generalise Jauch's separation of volume and energy variables in thermodynamics. Second, in that language, I give two equivalent reformulations of Jauch's conservation law that help to clarify its mathematical significance. Finally, I show that these reformulations provide an elegant proof of Jauch's theorem that avoids the problems with his original argument.

4.1. **Line bundle framework.** The configuration space of our engine in Figure 2 involves two manifolds: the 2-dimensional manifold in which a loop is traced in volume space, and the 3-dimensional manifold that includes energy, and in which the complete trajectory of the engine is a spiral. Every system in thermodynamics has these two general components: an $(n-1)$-dimensional manifold N, whose points are the observable configurations characterising work, and an n-dimensional manifold M, with an added dimension describing total energy (internal energy). These two manifolds are related by a smooth projection,

$$\pi : M \to N. \tag{3}$$

Thus, mathematically, we begin by casting a model of thermodynamics as a fibre bundle with one-dimensional fibres, also called a *line bundle*. We will only consider fibre bundles that admit a local trivialisation.[10]

[10] A fibre bundle $\pi : M \to N$ is called *locally trivial* if for each point $q \in N$ there is a neighbourhood B and a bundle isomorphism $\phi : M\big|_B \to B \times F$ such that each fibre $\pi^{-1}(p)$ is mapped to $\{p\} \times F$. The pair (B, ϕ) is then called a *local trivialisation* (cf. Baez and Muniain 1994, Chapter 2).

Jauch's definitions of work, energy, and heat can now be given more general geometric definitions:

- *Work is any one-form that annihilates vertical vectors,* $\pi_*(X) = 0 \Rightarrow \omega(X) = 0$. This reflects the fact that work incorporates only observable degrees of freedom. It also implies the standard coordinate expression of work: in local adapted coordinates $(U, V_1, \ldots, V_{n-1})$ for M, it follows[11] that $\omega = -\sum_{i=1}^{n-1} P_i dV_i$ where each $P_1, \ldots, P_{n-1}$ is a smooth function on M.

- *Energy is a vertical coordinate function* $U : M \to \mathbb{R}$ *in that if* $\pi_*(X) = 0$ *and* $X \neq 0$*, then* $dU(X) \neq 0$. In adapted local coordinates $(U, V_1, \ldots, V_{n-1})$ for M this is just what guarantees that energy is a coordinate variable. It says that it is always possible to have changes in energy without any changes in work.

- *Heat is defined by* $\xi := dU + \omega$. This means that heat is unobservable energy, in the sense that when one subtracts the change in observable energy $\sum_{i=1}^{n-1} P_i dV_i$ from the change in total energy dU, the result is heat, $dU - \sum_{i=1}^{n-1} P_i dV_i = dU + \omega = \xi$.

With these definitions, we will say as before that a curve γ is an *adiabat* if $\xi(\bar{\gamma}) = 0$, and is a *work cycle* if $\pi[\gamma]$ is closed in N. The latter is a more general, coordinate-free way to express the statement that the curve γ starts and ends at the same point in the work configuration variables.

4.2. **Reformulation of the conservation law.** The line bundle framework allows Jauch's conservation law to be given a more informative expression. I will develop this in two steps. The first is to formulate it as a relationship between two kinds of closed loops. The second is to state it in the language of holonomies and curvature.

Let $\pi : M \to N$ be a line bundle, with work, energy, and heat defined as above. If γ is an adiabat, then $\xi(\bar{\gamma}) = 0$, which means that

$$\int_\gamma \omega = \int_\gamma (\xi - dU) = -\int_\gamma dU. \tag{4}$$

Thus, $U(\gamma(0)) = U(\gamma(1))$. When γ is also a work cycle, this means that γ is closed. So, Jauch's conservation law (C_1) is equivalent to,

(C_2) There is a neighbourhood of every point in which if $\gamma : [0, 1] \to M$ is an adiabat and $\pi[\gamma]$ is closed, then γ is closed too.

[11] A one-form that annihilates all vertical vectors is called *semi-basic* and has the property that $\omega = -\sum_{i=1}^{n-1} P_i dV_i$ (Montgomery 2002, Def. 7.4.2).

This loop-based expression is highly suggestive of the holonomy formulation of Yang-Mills gauge theory.[12] Readers familiar with that theory may indeed notice that our spiral in Figure 2 looks suspiciously like a holonomy. Given a fibre bundle with an affine connection, a holonomy measures the extent to which a closed loop in the base space lifts to an open curve in the total space, like the spiral in Figure 2. The failure to lift to a closed curve indicates the presence of curvature, as when one parallel transports a vector around a closed loop on the surface of a sphere: due to the curvature of the sphere, the closed loop in the base space lifts to an open curve in the tangent bundle.

To apply this thinking to thermodynamics, we would need a precise notion of a 'lift'. In Yang-Mills theory it is defined using an (often affine) connection. However, a lift can also be defined using a more general Ehresmann connection, which is available in thermodynamics. Following Hermann (1975), an *Ehresmann connection* is any sub-bundle $H \subseteq TM$ that splits the tangent bundle into horizontal and vertical components at every point,

$$T_pM = H_p \oplus V_p. \tag{5}$$

The vertical subspace V_p here consists of those vectors X such that $\pi_*(X) = 0$.

When the heat one-form ξ in thermodynamics is non-vanishing, it defines an Ehresmann connection given by the 'adiabatic' distribution of vectors X satisfying $\xi(X) = 0$, which we denote,

$$H = \ker \xi. \tag{6}$$

This H is an Ehresmann connection essentially because vertical vectors are never adiabatic. If they were, then a non-zero vertical vector would satisfy both $\omega(X) = 0$ and $\xi(X) = 0$, and hence that $dU(X) = 0$, which is impossible because U is a vertical fibre coordinate. So, a vertical process is never adiabatic, and $H_p \cap V_p = \{0\}$. But $\dim H_p = n - 1$ (as is the case for the kernel of every non-vanishing one-form) and $\dim V_p = 1$, and so $T_pM = H_p \oplus V_p$ as claimed. When $H = \ker \xi$ is the Ehresmann connection defining a set of adiabats in this way, I will call it an *adiabatic connection*.

For our purposes, the key feature of an Ehresmann connection is that it defines a lift: for each initial point $p \in M$ and for each curve $\gamma_N : [0, 1] \to N$ beginning at $\pi(p)$, the *lift* of γ_N is the curve $\gamma : [0, 1] \to M$ such that $\pi(\gamma(t)) = \gamma_N(t)$ for all $t \in [0, 1]$ and $\bar{\gamma} \subset H$. For each point in the fibre over $\pi(p)$, this lift is unique whenever it exists

[12] The philosophical significance of the holonomy formulation has been the subject of much discussion; see Belot (1998), Healey (2007), Myrvold (2011), Rosenstock and Weatherall (2016), and Jacobs (2023).

(Hermann 1975, Theorem 2.2, pp.69-70). I will refer to the lift defined by an adiabatic connection as the *adiabatic lift*.

In summary, *a thermodynamic system defines a line bundle* $\pi : M \to N$ *and a one-form* ξ *for which* $H = \ker \xi$ *is an Ehresmann connection defining the adiabats, and which uniquely defines an adiabatic lift from* N *to* M. In this language, Jauch's conservation law can be expressed as:

(C_3) There is a neighbourhood of every point in which the adiabatic lift of every closed loop in N is a closed loop in M.

In other words, (C_3) says that the adiabatic connection only has trivial holonomies. In gauge theory, a connection with trivial holonomy is called *flat*. Thus, Jauch's conservation hypothesis in this language is the assumption that *the adiabatic connection is flat*.

The discussion above suggests thermodynamics admits a formulation as a full-blown gauge theory. That is indeed the case (Roberts 2026). However, it goes beyond the purpose of this note to develop that theory here, where our aim is only to correct and complete the proof of Jauch's theorem.

4.3. **Proof of the theorem.** Given a one-form $\xi = dU + \omega$ on a manifold M, Jauch's theorem states that his conservation law implies the existence of local functions T, S such that $\xi = TdS$. We have now seen that these conditions generalise to a line bundle equipped with an adiabatic (Ehresmann) connection, where his conservation law is equivalent to (C_3), which expresses that the adiabatic connection has trivial holonomy. This allows for a reformulation of Jauch's theorem as a consequence of the Frobenius theorem, from which Jauch's original theorem follows as an immediate corollary:

Theorem 1. *Let* $\pi : M \to N$ *be a line bundle, and let* $H = \ker \xi$ *be an Ehresmann connection for some non-vanishing one-form* ξ*. If, over some contractible neighbourhood of* N*, the lift of every closed loop is a closed loop in* M*, then there exist smooth functions* T, S *on* M *such that* $\xi = TdS$ *in that neighbourhood.*

Proof. The hypothesis implies that the adiabatic connection H has trivial holonomy, which is to say that it is flat, and hence involutive (Hermann 1975, Theorem 2.1, p.87). But, by the Frobenius theorem (Bryant et al. 1991, Theorem 1.1), $H = \ker \xi$ is involutive if and only if it is integrable, in which case, in a neighbourhood of every point, there are smooth functions T, S such that $\xi = TdS$. □

Jauch's original theorem can now be derived by retracing the discussion above: given local coordinates $(U, V_1, \ldots, V_{n-1})$ for M and one-forms $\omega = -\sum_{i=1}^{n-1} P_i dV_i$ and $\xi = dU + \omega$, we have seen that the projection onto the coordinates $(V_1, \ldots, V_{n-1})$ defines a line bundle $\pi : M \to N$. Moreover, the subbundle $H = \ker \xi$ is an Ehresmann connection when ξ is non-vanishing. Jauch's conservation law (C_1) now says that an adiabatic work cycle produces no work. This implies (C_2), that if an adiabat γ projects onto a closed loop $\pi[\gamma]$ in N then γ is closed in M. And, that implies (C_3), that the adiabatic lift of a closed curve in N is closed in M. By Theorem 1, it follows that $\xi = TdS$ for some local functions T and S, and so Jauch's theorem is proved.

This discussion shows that, like Carathéodory's theorem,[13] Jauch's theorem is fundamentally local. Indeed, the conclusion need not hold globally on non-contractible manifolds, analogous to the circumstances that give rise to a geometric phase in Yang-Mills gauge theories. This is not a failure of the framework, but a new physical prediction for thermodynamics, which enjoys some empirical support in the phenomenon of anomalous topological heat transfer (Roberts 2026).

5. Connection to Tatiana Afanassjewa

Before concluding, let me briefly discuss the relationship between Jauch's foundation for thermodynamics and the one proposed by Tatiana Ehrenfest-Afanassjewa (1925, p.223). In an early remark about his result, Jauch said that it was "discovered by T. Ehrenfest" (Jauch and Báron 1972), referring to a pair of papers she published in 1925 and 1926. I have not been able to find any evidence of Jauch's theorem in these papers, or elsewhere in Afanassjewa's work. Instead, she consistently relies on Carathéodory's theorem, which is a different statement that is proved in a different way.

However, Jauch does appear to have been inspired by her thesis that "the irreversibility of non-static processes has no relevance for the existence of entropy" (Ehrenfest-Afanassjewa 1925, p.178), which she defended throughout her career. Jauch also thought that the lack of a physical basis for her axioms kept this observation from being widely accepted.[14] I would like to point out that Jauch's conservation hypothesis

[13] The local nature of Caratéodory's theorem was pointed out by Bernstein (1960) and its implications discussed by Boyling (1968) and Lavis and Frigg (2025, §10.3.2).

[14] Jauch wrote, "she pointed out that the existence of an integrating factor for the differential 1-form representing the reversible heat transmitted to a thermal system has nothing to do with the irreversibility of all spontaneous processes occurring in time. ... A conceivable reason for the lack of interest in T. Ehrenfest's important remark may be that her fundamental hypothesis, which replaces the second principle of thermodynamics, has no concrete physical interpretation" (Jauch 1972, p.328).

in fact follows from Afanassjewa's 'fundamental hypothesis', which she calls *Axiom A* (the Entropy Axiom). She stated her axiom as follows:

> "If the integral $\int_1^2 \delta Q$ is non-zero along a quasi-static path that connects states 1 and 2 of a thermally homogeneous system, then the system cannot be brought from one of these states into the other along an adiabatic quasi-static path." (Ehrenfest-Afanassjewa 1925, p.172)

As Jauch points out, the physical motivation for this principle could be further developed. But it serves her purposes because it both avoids the assumption of irreversibility and also implies Carathéodory's principle.[15] Thus she concludes that heat can be written $\xi = TdS$ by Carathéodory's theorem.

Stripping away some unnecessary details, Afanassjewa's axiom essentially states, *if* $\int_\gamma \xi \neq 0$ *along a curve that connects* p *and* q*, then* p *and* q *cannot be connected by an adiabat*, or in equivalent contrapositive form,

(A) If p and q are adiabatically connected, then $\int_\gamma \xi = 0$ on every curve γ from p to q.

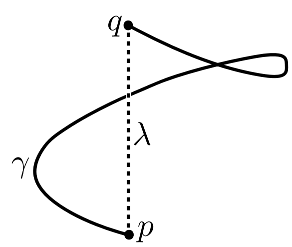

Figure 3: An adiabatic work cycle γ defining a vertical curve λ.

This axiom implies Jauch's conservation law in the form (C_2): for suppose there is an adiabatic work cycle γ from p to q. Since a work cycle satisfies $\pi(p) = \pi(q)$, there must be a vertical curve λ from p to q, as in Figure 3, which by (A) satisfies $\int_\lambda \xi = 0$. But, since λ is vertical, it must satisfy $\int_\lambda \omega = 0$ as well, which means that $\int_\lambda dU = 0$, and hence $U(p) = U(q)$. Thus, $p = q$, and the original adiabatic work cycle γ must be closed.

Since the converse does not hold, Jauch's conservation law is strictly weaker than Afanassjewa's axiom. This also means that our result is strictly stronger than hers, in that Theorem 1 reaches the same conclusion by assuming less.

[15] For evidence that Carathéodory's principle already avoids any irreversible ingredient see Marsland III et al. (2015).

6. Conclusion

Jauch's conservation law does indeed provide a new foundation for thermodynamics, which has many virtues: its physical implications are clear and plausible, it avoids appeal to 'adiabatic accessibility', and it does not depend on any irreversible ingredient like the second law. The essential structures it requires are just the distinction between work and total (internal) energy, and the undirected notion of an adiabat. In this sense, Jauch's approach appears to have advantages over many other approaches to deriving entropy and temperature.

Although Jauch's own line of argument unfortunately did not succeed in establishing this, a new and more revealing expression of his conservation law does enable a successful proof. Most remarkably, this new proof involves viewing thermodynamics as a gauge theory, and in particular as a line bundle with an Ehresmann connection interpreted as defining the adiabats. In that framework, Jauch's conservation law becomes the statement that there are only trivial holonomies, which is to say that the adiabatic connection is flat. This immediately implies the integrability condition needed to ensure that heat can be written as TdS. Our framework thus converts Jauch's theorem into an elegant part of the geometric structure of thermodynamics. That geometric structure itself appears to be an interesting new area of potential research.

References

Albert, D. Z. (2000). *Time and chance*, Cambridge, MA: Harvard University Press.

Baez, J. and Muniain, J. P. (1994). *Gauge fields, knots and gravity*, Series on Knots and Everything Vol. 4, London: World Scientific Publishing.

Belot, G. (1998). Understanding electromagnetism, *The British Journal for the Philosophy of Science* **49**(4): 531–555.

Bernstein, B. (1960). Proof of Carathéodory's local theorem and its global application to thermostatics, *Journal of Mathematical Physics* **1**(3): 222–224.

Boyling, J. (1968). Carathéodory's principle and the existence of global integrating factors, *Communications in Mathematical Physics* **10**(1): 52–68.

Brown, H. R. and Uffink, J. (2001). The Origins of Time-Asymmetry in Thermodynamics: The Minus First Law, *Studies in History and Philosophy of Modern Physics* **32**(4): 525–538. http://philsci-archive.pitt.edu/217/.

Bryant, R. L., Chern, S., Gardner, R., Goldschmidt, H. and Griffiths, P. (1991). *Exterior Differential Systems*, New York: Springer-Verlag New York Inc.

Buchdahl, H. A. (1966). *The Concepts of Classical Thermodynamics*, Cambridge: Cambridge University Press.

Carathéodory, C. (1909). Untersuchungen über die Grundlagen der Thermodynamik, *Mathematische Annalen* **67**: 355–386.

Carnot, S. (1824). *Reflections on the Motive Power of Heat*, London: Macmillan & Co. 1890 English Translation by R.H. Thurston, https://www.google.co.uk/books/edition/Reflections_on_the_motive_power_of_heat/_tHxDntE2GQC.

Chang, H. (2004). *Inventing Temperature: Measurement and Scientific Progress*, New York: Oxford University Press.

Ehrenfest-Afanassjewa, T. (1925). Zur Axiomatisierung des zweiten Hauptsatzes der Thermodynamik, *Zeitschrift für Physik* **33**: 933–945. Translated into English by Marina Baldissera Pacchetti as, 'On the Axiomatization of the Second Law of Thermodynamics', in Uffink et al. (2021) *The Legacy of Tatiana Afanassjewa*, Springer Nature Switzerland AC, pp.170-183.

Emch, G. G. (1984). *Mathematical and conceptual foundations of physics*, Amsterdam:Elsevier Science Publishers B.V.

Healey, R. (2007). *Gauging What's Real: The Conceptual Foundations of Contemporary Gauge Theories*, New York: Oxford University Press.

Henderson, L. (2014). Can the second law be compatible with time reversal invariant dynamics?, *Studies in History and Philosophy of Modern Physics* **47**: 90–98. http://philsci-archive.pitt.edu/10698/.

Hermann, R. (1975). *Gauge fields and Cartan-Ehresmann Connections, Part A*, Brookline, MA: Math Sci Press.

Jacobs, C. (2023). The metaphysics of fibre bundles, *Studies in History and Philosophy of Science* **97**: 34–43. http://philsci-archive.pitt.edu/21468/.

Jauch, J. M. (1972). On a new foundation of equilibrium thermodynamics, *Foundations of Physics* **2**(4): 327–332.

Jauch, J. M. (1975). Analytical Thermodynamics, Part 1: Thermostatics—General Theory, *Foundations of Physics* **5**(1): 111–132.

Jauch, J. M. and Báron, J. (1972). Entropy, information and Szilard's paradox, *Helvetica physica acta* **45**(2): 220–232.

Kuzemsky, A. (2020). In search of time lost: Asymmetry of time and irreversibility in natural processes, *Foundations of Science* **25**(3): 597–645.

Lavis, D. A. and Frigg, R. (2025). *The fundamentals of thermodynamics*, Cham, Switzerland: Spring Nature Switzerland AG.

Lieb, E. H. and Yngvason, J. (1999). The physics and mathematics of the second law of thermodynamics, *Physics Reports* **310**(1): 1–96. https://arxiv.org/abs/cond-mat/9708200.

Marsland III, R., Brown, H. R. and Valente, G. (2015). Time and irreversibility in axiomatic thermodynamics, *American Journal of Physics* **83**(7): 628–634.

Montgomery, R. (2002). *A Tour of Subriemannian Geometries, Theor Geodesics and Applications*, Providence, RI: The American Mathematical Society.

Myrvold, W. C. (2011). Nonseparability, classical, and quantum, *The British Journal for the Philosophy of Science* **62**: 417–432. https://philsci-archive.pitt.edu/5335/.

Price, H. (1996). *Time's Arrow and Archimedes' Point: New Directions for the Physics of Time*, New York: Oxford University Press.

Roberts, B. W. (2022). *Reversing the Arrow of Time*, Cambridge: Cambridge University Press. Open Access: https://doi.org/10.1017/9781009122139.

Roberts, B. W. (2026). Heat as gauge. Unpublished Manuscript: https://arxiv.org/abs/2503.08753.

Rosenstock, S. and Weatherall, J. O. (2016). A categorical equivalence between generalized holonomy maps on a connected manifold and principal connections on bundles over that manifold, *Journal of Mathematical Physics* **57**(10): 102902. http://philsci-archive.pitt.edu/11904/.

Uffink, J. (2001). Bluff your way in the second law of thermodynamics, *Studies In History and Philosophy of Modern Physics* **32**(3): 305–394. http://philsci-archive.pitt.edu/313/.

Uffink, J. (2007). Compendium of the Foundations of Classical Statistical Physics, *in* J. Butterfield and J. Earman (eds), *Philosophy of Physics Part B*, Elsevier, pp. 923–1074. http://philsci-archive.pitt.edu/2691/.

Uffink, J. and Valente, G. (2021). Afanassjewa and the foundations of thermodynamics, *in* J. Uffink, G. Valente, C. Werndl and L. Zuchowski (eds), *The Legacy of Tatjana Afanassjewa*, Cham, Switzerland: Spring Nature Switzerland AG, chapter 3, pp. 55–82.

Walter, J. (1978). On the definition of the absolute temperature—a reconciliation of the classical method with that of Carathéodory, *Proceedings of the Royal Society of Edinburgh* **82A**: 87–84.