Classical logic and quantum properties

Patrick Fraser, Michael E. Miller, and Jessica Wilson

We articulate a collection of desiderata for an account of the dynamical quantities of a physical theory, and we present a theory that meets these desiderata in the case of quantum mechanics. Our theory retains a distinction between the values of dynamical quantities and the truth values of sentences asserting that a system has a particular value of a particular quantity. This allows our theory to incorporate the phenomenon of quantum indeterminacy as a pattern in the properties instantiated by a system in a particular quantum state, while also retaining the semantics of classical logic. We contrast our theory with quantum logic, which flattens the distinction between quantity values and truth values, and we address a series of objections that have been raised to previous attempts to reconcile quantum indeterminacy with classical logic.

1. Introduction. Dynamical physical theories represent how the properties of physical systems change over time. Properties are represented on the state space of the theory, and fixing a state in the state space determines which value of the properties the physical system instantiates at a time. The properties in question are quantities as the possible values of the properties are represented in a space of numerical values. Dynamical evolution changes the state and, as a result, changes the values of the dynamical physical quantities. These general observations suggest the following desiderata for an account of the dynamical quantities of a given physical theory: such an account should specify what quantities there are according to the theory, what the possible values of each quantity are, which of those quantities are instantiated by particular physical systems, and the conditions under which a system has a particular value of a particular quantity.

Dynamical physical quantities and their values are naturally characterized in terms of determinable and determinate properties. Determinables and their determinates stand in a characteristic specification relation. A determinate property is a specific way a determinable might be instantiated. Physical quantities can be thought of as determinables and their values can be thought of as their determinates. The values of quantities are specific ways that those quantities can be instantiated. A state in a state space specifies which determinate value of each determinable quantity is instantiated at a given instant in time.

Providing an account of the dynamical quantities of *classical* mechanics that meets the desiderata is straightforward. The determinable properties are represented by smooth functions from phase spaces to the real numbers, and the determinates are represented by the image values of those functions. A system instantiates a particular quantity if the state of that system is represented by a point in the phase space on which that quantity is defined. When we specify which point is the actual state of the system at a time, we uniquely fix the image value of each of the smooth functions on its phase space and hence we uniquely fix which determinate value of each determinable property is instantiated when the system is in that state. This treatment of the dynamical quantities of classical mechanics in terms of determinables and determinates is so straightforward that one might wonder why it would be the subject of philosophical discussion at all: it seems to simply be a restatement of the standard way of thinking about classical mechanical quantities.

Providing an account of the dynamical quantities of *quantum* mechanics which meets the same desiderata is less straightforward, and the subject of controversy. This is so notwithstanding that each of the components in the classical case has an analog in the quantum case. Determinable properties are represented by Hilbert space operators, the determinates of those determinables are represented by the real numbers in the spectra of those observables, a system instantiates a determinable just in case that determinable is represented on the Hilbert space on which the system's state is defined, and the eigenstate-eigenvalue link specifies the conditions under which a system in a given state instantiates a particular determinate of each of the determinables. These are kinematic features of the state space of the theory and so should be shared by any interpretation which retains that structure. The controversy is thus distinct from the controversy over which interpretation of quantum mechanics we should adopt.[1] Rather, the controversy reflects unclarity over how to reconcile an account of quantum properties with the phenomenon of quantum indeterminacy. [2]

The phenomenon of indeterminacy is highly contested even before we concern ourselves with its quantum incarnation. There are accounts which treat indeterminacy as epistemic, semantic, and metaphysical in character, and unclarity about the nature of indeterminacy in general has filtered into debates about quantum indeterminacy in particular. Results such as the Kochen-Specker theorem indicate that the indeterminacy at issue in quantum mechanics is a feature of the world itself, not our representation or knowledge of the world. This suggests quantum indeterminacy is metaphysical and not semantic or epistemic, but for much of the history of the development of quantum mechanics, there were no well-articulated theories of metaphysical indeterminacy. During this period discussions of quantum indeterminacy floated free of more general philosophical approaches to indeterminacy.

This situation has changed and there are now two approaches to metaphysical indeterminacy which are viable candidates for characterizing quantum indeterminacy. On a 'meta-level' approach, metaphysical indeterminacy is a matter of reality being unsettled with respect to which fully determinate state of affairs actually obtains.[3] There have been attempts to analyze quantum indeterminacy as an instance of this phenomenon,[4] and several objections to doing so.[5] We agree with these objections: they show that meta-level approaches cannot characterize quantum indeterminacy.

On an 'object-level' approach, metaphysical indeterminacy is a failure of individual states of affairs to be fully determinate. There are several ways to say what it might take for a state of affairs to fail to be fully determinate.[6] One prominent view characterizes a failure of fully determinacy in terms of the patterns of instantiation of determinable and determinate

[1] Hidden variable theories, such as Bohmian mechanics, involve modifications to the kinematic structure of the state space and will thus give rise to changes to the theory of quantities. Such theories can also be treated using the methods we introduce in this article with appropriate modification, though we will not discuss this application of our view here.

[2] The controversy has become the subject of a substantial literature: (Skow, 2010; Darby, 2010; Bokulich, 2014; Wolff, 2015; Glick, 2017; Torza, 2017; Calosi and Wilson, 2019; Calosi and Mariani, 2020; Torza, 2021; Corti, 2021; Darby and Pickup, 2021; Mariani, 2021; Calosi, 2021; Calosi and Mariani, 2021; Calosi and Wilson, 2021; Fletcher and Taylor, 2021a,b; Schroeren, 2021; Torza, 2022; Lewis, 2022; Glick, 2022; Darby and Pickup, 2022; Calosi and Wilson, 2022; Calosi, 2022; Menon, 2024; Mariani, 2024; Fletcher and Taylor, 2024a,b; Calosi and Toader, 2025; Fraser and Miller, 2025; Cinti et al., 2025; Nørgaard, 2025).

[3] (Akiba, 2004; Barnes, 2010; Barnes and Williams, 2011; Rosen and Smith, 2004)

[4] (Darby, 2010; Darby and Pickup, 2021, 2022)

[5] It has been noted, for instance, that quantum states are fully specified, and hence do not seem to encode unsettled goings-on (Schroeren, 2021; Menon, 2024). Various other objections are given in (Skow, 2010; Bokulich, 2014; Calosi and Wilson, 2019; Corti, 2021).

[6] (Burgess, 1990; Greenough, 2008; Goswick, 2021)

properties.[7] On such a determinable-based account, a state of affairs exhibits indeterminacy if, in that state of affairs, a determinable is instantiated but no unique determinate of that determinable is instantiated. This account of metaphysical indeterminacy has been put forward as a candidate analysis for quantum indeterminacy as well[8] and it has also been the subject of criticism.[9]

One of the central points of contention over object-level analyses of quantum indeterminacy concerns how to give semantic content to propositions which attribute properties to quantum systems. Some advocates of object-level analyses have argued that their accounts of quantum properties are compatible with a classical semantics. This claim has been challenged by several authors who argue that a proper treatment of quantum indeterminacy requires embracing the alternative semantic assumptions of the quantum logic program.[10]

At bottom, the dispute here concerns how we should think about the relationship between values of quantities on the one hand, and truth values of propositions asserting that a system has particular values of those quantities on the other. The quantum logic program flattens the distinction between these two kinds of values and, as a consequence, undermines efforts to reconcile determinable-based indeterminacy with classical logic. There is no need to flatten this distinction, however, and in fact, doing so requires one to embrace a series of nonstandard meta-semantic assumptions. If one is careful to retain the distinction between the values of quantities and the truth value of propositions, the path to reconciling a determinable-based account of quantum indeterminacy with classical logic becomes clear.

Below we develop an account of quantum properties in terms of determinables and determinates. We identify how determinable and determinate structure arises naturally in the state space of quantum mechanics, and we show that this leads to determinable-based indeterminacy. The account retains the distinction between the values of quantitative properties and the truth values of propositions about those properties, and thus we arrive at an account of quantum properties which naturally captures quantum indeterminacy in terms of the values of quantities, but where the resulting logic of propositions is just classical logic.

More specifically, we proceed as follows. In Section 2, we introduce our preferred theory of the dynamical quantities of quantum mechanics, and we discuss how this theory incorporates the phenomenon of metaphysical indeterminacy. In Section 3, we introduce the rudiments of a semantic theory that expresses which propositions about quantum properties are true according to this theory, and contrast it with the semantic theory adopted in the quantum logic program. In Section 4, we discuss the meta-semantic differences between our semantic theory and the semantic theory of quantum logic. In Section 5, we conclude.

2. Quantum Determinables and Their Determinates. In this section, we present an account of the dynamical quantities of quantum mechanics which meets the desiderata specified in Section 1, and we show how determinable-based metaphysical indeterminacy arises as a consequence of this proposal.[11]

[7] (Wilson, 2012, 2013, 2017)

[8] (Bokulich, 2014; Calosi and Wilson, 2019; Calosi and Mariani, 2020; Calosi, 2021; Calosi and Mariani, 2021; Calosi and Wilson, 2021, 2022; Calosi, 2022; Schroeren, 2021)

[9] (Glick, 2017, 2022; Torza, 2017, 2021; Fletcher and Taylor, 2021a,b, 2024a)

[10] (Torza, 2017, 2021; Fletcher and Taylor, 2021a,b, 2024a)

[11] Various realist interpretations of quantum theory may reach different conclusions about the status of quantum metaphysical indeterminacy. Here, we restrict attention to Hilbert space formulations in which quantities are represented by self-adjoitn operators. These considerations straightforwardly generalize to contexts in which quantities are instead represented by elements of a C^*-algebra.

2.1. Our proposal. The self-adjoint operators defined on a particular Hilbert space represent the dynamical physical quantities of systems whose states are represented on that space.[12] For instance, the identity operator on the spin-1/2 Hilbert space represents the property of being a spin-1/2 system, the $\hat{\sigma}_z$ operator represents the property of spin in the z-direction, and the projection operator $\hat{\mathbb{P}}_{\uparrow_z} = |\uparrow_z\rangle\langle\uparrow_z|$ represents the property of being spin-up in the z-direction. Different Hilbert spaces have different collections of self-adjoint operators defined on them. For example, the self-adjoint operators on the Hilbert space $L^2(\mathbb{R})$ include the $\hat{X}$-operator, representing the property of position in one dimension, and projections onto regions, $\hat{\mathbb{P}}_\Delta$, representing having a position in the region Δ. These self-adjoint operators are the mathematical representations of dynamical quantities on Hilbert space.

The different ways in which the quantities represented by self-adjoint operators may be instantiated are labeled by their eigenvalues, or more generally, their spectral values. For instance, the eigenvalues of $\hat{\sigma}_z$, $\hbar/2$ and $-\hbar/2$, represent the ways of having spin in the z-direction—namely, 'up' and 'down.' The spectral values of $\hat{X}$, all the numbers in $\mathbb{R}$, represent ways of being exactly located at different positions in space with respect to some coordinate system. Similarly, projection operators are self-adjoint operators with spectral values 1 and 0, and so they are dynamical quantities with two distinct determinates. How to interpret the spectral values of projection operators is a point of controversy, and lies at the heart of some of the issues we discuss below. The spectral values of self-adjoint operators are the mathematical representations of the determinates associated with dynamical quantities on Hilbert space. Taken together, these observations specify what quantities there are according to quantum theory, and what the possible values of each quantity are:

> **Determinable Specification**: $\hat{O}$ represents a quantum determinable if and only if $\hat{O}$ is a self-adjoint operator defined on a Hilbert space $\mathcal{H}$.

> **Determinate Specification**: The determinates of the quantum determinable represented by $\hat{O}$ are represented by the numbers in the spectrum of $\hat{O}$, $\mathrm{sp}(\hat{O})$.

These principles express how determinable and determinate properties are represented on the state space of quantum mechanics. On our proposal, determinable properties and determinate properties are represented by different kinds of mathematical objects, with the former being represented by self-adjoint operators, and the latter being represented by real numbers. That determinables and determinates are represented by strictly distinct classes of mathematical objects reflects that, in the context of quantum mechanics, these are strictly distinct classes of properties. In quantum mechanics, determinates are never determinables, nor vice versa. This type separation will be important in what follows, and marks a departure from applications of determinable properties in other contexts.

We now need to express the conditions under which a particular physical system in a particular quantum state instantiates these determinable and determinate properties. We begin with the determinables. When we specify the Hilbert space on which the state of a

[12] The debates we are engaged with all restrict attention to determinable quantum properties represented by self-adjoint operators, and often only those defined on finite-dimensional Hilbert spaces. This requires generalization along a number of axes. First, many realistic quantum systems are represented using infinite-dimensional Hilbert spaces, and many of their important properties have continuous spectra. Second, one might think that normal operators, or 'unsharp' POVM observables also represent determinable quantum properties (Roberts, 2017, 2018). Our account accommodates the infinite-dimensional cases. It may be used to accommodate properties more general than those usually considered in this literature, but we do not elaborate on this here.

quantum mechanical system is represented, we fix which dynamical quantities the system has. Distinct systems whose states are represented on the same Hilbert space have the same dynamical quantities, and systems whose states are represented on distinct Hilbert spaces have distinct collections of dynamical quantities. For example, electrons, muons, and tauons all have spin states represented on the spin-1/2 Hilbert space, and thus have all of the same spin-related dynamical quantities. On the other hand, photons, W bosons, Z bosons, and gluons all have spin states represented on the spin-1 Hilbert space, and thus have all of the same spin-related dynamical quantities, different from the spin-related dynamical quantities of the spin-1/2 particles. This suggests that a system instantiates all and only the self-adjoint operators on the Hilbert space in which its state is represented:

> **Determinable Instantiation**: Let $\mathcal{S}$ be a physical system whose quantum state is defined on the Hilbert space $\mathcal{H}$ and let $\hat{O}$ be a self-adjoint operator defined on the Hilbert space $\mathcal{H}'$. Then $\mathcal{S}$ instantiates a determinable represented by $\hat{O}$ if and only if $\mathcal{H} = \mathcal{H}'$.

The eigenstate-eigenvalue link (EEL), once correctly articulated in terms of determinables and determinates, specifies the conditions under which a system in a particular quantum state instantiates a particular determinate of a given determinable.[13]

> **Determinate Instantiation (EEL)**: A physical system $\mathcal{S}$ in quantum state $|\psi(t)\rangle \in \mathcal{H}$ at time t instantiates the determinate represented by $\lambda \in \mathrm{sp}(\hat{O})$ of the determinable represented by $\hat{O}$ at t if and only if $|\psi(t)\rangle \in \mathrm{dom}(\hat{O})$ and $(\hat{O} - \lambda\hat{I})|\psi(t)\rangle = 0$.[14]

By presenting the account in terms of the spectra of self-adjoint operators, we have ensured that it is well-defined for dynamical quantities defined on finite and infinite dimensional Hilbert spaces, and for quantities with discrete and continuous spectra.

2.2. Quantum metaphysical indeterminacy. With an account of quantum properties in hand, we turn to the task of investigating the relationship between this account of properties and the phenomenon of quantum indeterminacy. Our approach to this question does not proceed by assuming a particular analysis of the phenomenon of indeterminacy, and revising the account of quantum properties so that it exhibits indeterminacy of that form. Rather, we simply look at the theory of quantum properties just articulated, and check which form of indeterminacy it exhibits, if it exhibits indeterminacy at all. We argue that this account of quantum properties leads to determinable-based metaphysical indeterminacy as a consequence.

Recall that determinable-based metaphysical indeterminacy obtains in a given state of affairs just in case, in that state of affairs, a given determinable is instantiated but no unique determinate of that determinable is instantiated. As previously, the failure of uniqueness might stem from the instantiation of more than one determinate, in which case the resulting indeterminacy is said to be glutty, or it might stem from the failure of any determinate of the determinable to be instantiated, in which case the indeterminacy is said to be gappy. It is straightforward to see that according to the account of quantum properties we have

[13] The first characterization of EEL in terms of determinables and determinates is due to Lewis (2016). As Lewis observes, this is a choice point where different interpretations of quantum mechanics disagree.

[14] In finite dimensions this reduces to the more familiar condition that $|\psi(t)\rangle$ is an eigenstate of $\hat{O}$ with eigenvalue λ.

provided, at least some quantum states of affairs will exhibit this form of metaphysical indeterminacy. Consider, for example, a spin-1/2 particle in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)$. Since this state is defined on the same Hilbert space as the $\hat{\sigma}_z$ operator, it instantiates the 'spin in the z-direction' determinable. However, in this state, the particle is not in an eigenstate of the $\hat{\sigma}_z$ operator, and thus fails to instantiate any of its determinates. Hence, such a state of affairs exhibits gappy metaphysical indeterminacy.

This turns out to be a very general feature of the account of quantities we have articulated: *every* quantum state of affairs exhibits metaphysical indeterminacy for almost all instantiated determinables. To see that every quantum state of affairs exhibits such indeterminacy, it suffices to note that for every quantum state $|\psi\rangle$, there exists some other quantum state $|\phi\rangle$ which is orthogonal to $|\psi\rangle$. Given this fact, the quantum state of affairs represented by the state $|\psi\rangle$ will not be an eigenstate of the determinable property given by the determinable $\hat{O} = \frac{1}{2}(|\psi\rangle + |\phi\rangle)(\langle\psi| + \langle\phi|)$. This shows that every quantum state of affairs is metaphysically indeterminate for some of its determinables. Assuming that we are valuing determinates using EEL, quantum indeterminacy is, moreover, always gappy. If a system in $|\psi\rangle$ instantiates two $\hat{O}$-determinates λ and τ, then $\lambda|\psi\rangle = \hat{O}|\psi\rangle = \tau|\psi\rangle$ by EEL. Since $|\psi\rangle \neq 0$, it follows that $\lambda = \tau$, and so at most one determinate of a given determinable may be instantiated.[15] But gappy quantum indeterminacy is not just present in all quantum states of affairs; it is ubiquitous within each one. Among the set of quantum determinables instantiated by a particular quantum system, the subset for which that system also instantiates a determinate is measure zero.[16] Hence, determinacy is exceedingly rare, and indeterminacy is typical.

So, on the account we have provided, quantum metaphysical indeterminacy in its gappy form is ubiquitous. These results follow directly from the account of properties we have adopted: the pattern of property instantiation generated by **Determinable Instantiation** and **Determinate Instantiation** is always one in which there is determinable-based metaphysical indeterminacy.[17] It is important to note that in arriving at these results, we have not appealed to any conceptual resources that are semantic in character. Our claims have concerned only quantity valuations, not the content of, or truth conditions for, propositions. In the next section, we argue that despite exhibiting metaphysical indeterminacy, our account of quantum properties is compatible with the semantic assumptions normally adopted in the context of classical logic.

[15] There are two distinct revisions to the determinate valuation scheme we have advanced here which would give rise to glutty indeterminacy instead. First, glutty indeterminacy can arise if one adopts a valuation scheme which is more permissive than EEL. Some have been motivated to consider such valuation schemes in response to considerations concerning spontaneous collapse theories (Albert and Loewer, 1996; Clifton and Monton, 1999; Lewis, 2016; Myrvold, 2018), while others have been pulled in this direction for metaphysical reasons (Calosi and Wilson, 2022). Second, glutty indeterminacy can arise if the determinate valuation scheme is specified relative to some index. For example, one might think that in Everettian quantum mechanics, valuation is relativized to a branch; see (Lewis, 2016; Calosi and Wilson, 2021).

[16] In finite dimensions, self-adjoint linear operators themselves form a vector space equipped with a Lebesgue measure. The points in this space of operators for which the eigenstate relation is satisfied for a given vector lie in a lower-dimensional subsurface and are thus measure zero. In the infinite-dimensional case, the only uniform measure over the resulting space of self-adjoint operators values every measurable set measure zero.

[17] Glick (2017; 2022) has argued that one should to modify **Determinable Instantiation** in order to eliminate this determinable-based metaphysical indeterminacy. This seems to get things the wrong way around. Whether one's account of quantum properties exhibits metaphysical indeterminacy should be a consequence of the instantiation conditions one adopts.

3. Semantic Theories for Quantum Propositions. We now seek a semantic theory that allows us to specify the propositional content of quantum theory. Such a theory should identify what the propositions expressed by quantum theory are, how those propositions may be logically combined with one another, and what their truth conditions are. Many discussions of quantum indeterminacy have not explicitly focused on these semantic issues. One notable exception is the approach that accounts for the phenomenon of quantum indeterminacy using the semantic theory associated with the quantum logic program. After presenting our semantic theory, we will contrast it with this approach.

3.1. Our semantic theory. We proceed by first articulating the syntax and grammar of a language whose sentences are intended to express the propositional content of quantum theory. We then give these sentences interpretations that imbue them with meaning by providing their truth conditions. The propositions expressed by the sentences at issue are understood to be propositions about the instantiated values of dynamical physical quantities, and their truth conditions should be articulated in such a way that the truth or falsity of a proposition is determined by the state state of the system as represented on Hilbert space.

In this section, we restrict attention to developing a semantics for a propositional language, as this is the setting in which the framework of quantum logic is typically presented. Ultimately, however, we are concerned with how the semantic content of quantum theory interfaces with a satisfactory account of quantum properties. Hence in Section 4 we provide the required generalization to a more expressive predicative language which includes labels for the properties that appear in quantum theory. There, we discuss the conspicuous absence of predication in the framework of quantum logic, and identify the collection of meta-semantic assumptions adopted in that framework which explain its absence.

Our propositional language $\mathcal{L}$ is a set of sentences constructed from a set of atomic sentence letters SL and a set of sentential connectives $\{\neg, \wedge, \vee\}$. $\mathcal{L}$ is defined inductively:

Base Case: If $p \in SL$, then $p \in \mathcal{L}$.

Inductive Case: If $p \in \mathcal{L}$, then $\neg p \in \mathcal{L}$.

Inductive Case: If $p, q \in \mathcal{L}$, then $p \wedge q \in \mathcal{L}$.

Inductive Case: If $p, q \in \mathcal{L}$, then $p \vee q \in \mathcal{L}$.

Closure: There are no other sentences in $\mathcal{L}$.

Without a supplemental semantics, $\mathcal{L}$-sentences are merely strings of symbols: they have no interpretation—no meaning—and so cannot be thought of as being true or false. Nor is there any consequence relation between sentences in virtue of which one entails another. Once $\mathcal{L}$-sentences are given an explicit truth-conditional interpretation, they may be thought of as expressing propositions. Our task now is to characterize such interpretations.

Sentences in $\mathcal{L}$ are intended to express propositions about the determinable and determinate properties—dynamical physical quantities and their values—instantiated by quantum systems. On our account, this will be achieved by taking the atomic sentence letters in SL to express instances of determinable and determinate property attribution. Correspondingly, we would like SL to include such sentences as 'the electron has spin in the x-direction', 'the electron is spin-up in the z-direction', 'the photon has energy', 'the energy of the photon is $\hbar\omega/2$', and so forth. Of course, which of these atomic sentence letters turn out to be true will depend on which Hilbert space is used to interpret $\mathcal{L}$, and what quantum state a system is taken to occupy. We would also like $\mathcal{L}$ to include more grammatically complex sentences generated from these atomic sentence letters by the standard sentential connectives, such as 'it is not the case that the electron is spin-up in the x-direction', 'either the electron is spin-up in the z-direction or the electron is spin-down in the z-direction', 'the photon has energy and it is not the case that the photon has spin in the x-direction', and so forth. We can articulate a semantic theory that achieves these aims by using the account of quantity valuation developed above to specify truth conditions for the atomic sentence letters, and then use the standard classical inductive truth conditions to give meaning to grammatically complex sentences.

For simplicity, we will interpret $\mathcal{L}$ as expressing propositions about a single system $\mathcal{S}$ whose state at a time t is represented by a unit vector $|\psi(t)\rangle \in \mathcal{H}$ for a particular Hilbert space $\mathcal{H}$. Interpretations of $\mathcal{L}$ will then be maps which assign Hilbert space subsets $H_p^C \subseteq \mathcal{H}$ to every sentence $p \in \mathcal{L}$. These subsets will ultimately be understood as containing all and only the states in which 'p' is true under the classical semantics we are articulating. Under such an interpretation, an $\mathcal{L}$-sentence 'p' is true in the instantaneous state of affairs represented by $|\psi(t)\rangle$ just in case $|\psi(t)\rangle \in H_p^C$.

We want the atomic sentence letters in SL to be interpreted as sentences which attribute dynamical physical quantities and their values to quantum systems. $\mathcal{L}$ is a propositional language, so its sentences are not sufficiently fine-grained to be identified with instances of predication. However, if the interpretation of $\mathcal{L}$ is to be compatible with this aim, the subsets of $\mathcal{H}$ serving as the interpretations of atomic sentence letters must be compatible with our account of quantity valuation. It follows from **Determinable Instantiation** and **Determinate Instantiation** that the interpretations of atomic sentence letters must be closed linear subspaces of $\mathcal{H}$. This is because **Determinable Instantiation** entails that the set of states in which a determinable property is instantiated comprises the entire set of normalized vectors in a particular Hilbert space: for a given determinable, either every state in $\mathcal{H}$ instantiates the determinable or no state does. Similarly, **Determinate Instantiation** entails that the set of states which instantiate a determinate value of a given determinable quantity comprise the entire set of normalized vectors in a closed linear subspace; namely, the eigenspace associated with the spectral value representing the determinate in question. Thus, on our semantic theory, an interpretation of the atomic sentence letters $SL \subseteq \mathcal{L}$ will be any map which obeys the following condition:

Base Case: If $p \in SL$, then H_p^C is a closed linear subspace of $\mathcal{H}$.

This base case makes the truth conditions for atomic sentence letters sensitive to the geometry of $\mathcal{H}$. Once we have specified the truth conditions for the atomic sentence letters, the remaining sentences are grammatical complexes of sentences whose truth conditions have already been specified. While Hilbert spaces have rich mathematical structure, it is not mandatory that the truth conditions for these grammatically complex sentences preserve all of the Hilbert space structure. After all, the role of a Hilbert space in quantum theory is to specify what the dynamical physical quantities are, and how the values of such quantities are to be attributed to the relevant physical systems. This job is done before the job of specifying the meanings of grammatically complex sentences begins.

With this in mind, we will still use a Hilbert space as the underlying meaning structure in which to interpret the rest of $\mathcal{L}$, but we will now forget about most of its structure, viewing it only as a set. This is possible because Hilbert spaces are ultimately sets on which standard set-theoretic operations can be performed. This choice marks a critical point of departure from the semantic theory of quantum logic. On our semantic theory, given an interpretation of the atomic sentence letters of $\mathcal{L}$, we may uniquely inductively extend this interpretation to an interpretation of all of $\mathcal{L}$ as follows:

Inductive Case: If $p \in \mathcal{L}$, then $H^C_{\neg p} = \mathcal{H} \setminus H^C_p$.

Inductive Case: If $p, q \in \mathcal{L}$, then $H^C_{p \wedge q} = H^C_p \cap H^C_q$.

Inductive Case: If $p, q \in \mathcal{L}$, then $H^C_{p \vee q} = H^C_p \cup H^C_q$.

These conditions define the sets H^C_p which serve as the interpretations of $\mathcal{L}$-sentences. A sentence 'p' will be true in the instantaneous state of affairs represented by the state $|\psi(t)\rangle \in \mathcal{H}$, which we will write as $|\psi(t)\rangle \vDash_C p$, just in case $|\psi(t)\rangle \in H^C_p$. When '$p$' is true in every such state of affairs regardless of the interpretation—when 'p' is a tautology—we will simply write $\vDash_C p$. As we show below, the set of tautologies associated with this semantics includes all axioms of classical logic.

Consider a system of the 'spin-1/2' physical kind represented on the Hilbert space $\mathbb{C}^2$, such as an electron's spin degrees of freedom. The set of states in which it is true that 'either the electron is spin-up in the x-direction or the electron is spin-down in the z-direction' is the set-theoretic union of the $\hbar/2$-eigenspace of $\hat{\sigma}_x$ and the $-\hbar/2$-eigenspace of $\hat{\sigma}_z$. Hence, this sentence is true only for states in the set $\mathrm{span}\{|\uparrow_x\rangle\} \cup \mathrm{span}\{|\downarrow_z\rangle\}$.[18] Superpositions of these states are not states for which this sentence is true. Similarly, the states for which 'it is not the case that the electron is spin-up in the x-direction' is true are all the states in the set-theoretic complement of $\{|\uparrow_x\rangle\}$—that is, states in $\mathcal{H} \setminus \mathrm{span}\{|\uparrow_x\rangle\}$—including the state $|\downarrow_x\rangle$ and all of the non-trivial superpositions of spin-up and spin-down in the x-direction.

3.2. The semantics of quantum logic. The semantic theory just given provides one way of characterizing the propositional content of quantum theory. It does so by giving a semantics for a language about quantum systems that uses Hilbert spaces as the underlying meaning structures. To our knowledge, the quantum logic program provides the only other explicitly worked out account of the propositional content of quantum mechanics. This approach takes the same language and provides a distinct interpretation of the same underlying meaning structures. Our aim at this stage is simply to identify where exactly quantum logic comes apart from the theory we have articulated.

It begins with the same language, $\mathcal{L}$, constructed from the same atomic sentence letters and the same connective symbols.[19] Interpretations of $\mathcal{L}$ are once again maps from sentences $p \in \mathcal{L}$ to Hilbert space subsets $H^Q_p \subseteq \mathcal{H}$, and in fact, the base case of quantum logical interpretations is the same as in our theory:

Base Case: If $p \in SL$, then H^Q_p is a closed linear subspace of $\mathcal{H}$.

Thus, $H^C_p = H^Q_p$ for all $p \in SL$. This, however, is where the similarities end. Whereas in our theory we use Hilbert space structure to articulate truth conditions for the atomic sentences and then adopt the standard classical semantics for the connectives, in quantum logic the semantics for the connectives is defined in a manner that also tracks Hilbert space structure. In particular, in quantum logic the inductive extension of the interpretation of $\mathcal{L}$ is carried out as follows:

[18] $\mathrm{span}\{H\}$ denotes the set of all linear combinations of elements of H.

[19] The labels for the connective symbols are sometimes selected differently because they are eventually given a distinct interpretation; however, the associated grammatical rules for forming sentences from the connective symbols remain fixed.

Inductive Case: If $p \in \mathcal{L}$, $H^Q_{\neg p} = H^{Q\perp}_p$.

Inductive Case: If $p, q \in \mathcal{L}$, $H^Q_{p\wedge q} = H^Q_p \cap H^Q_q$.

Inductive Case: If $p, q \in \mathcal{L}$, $H^Q_{p\vee q} = \text{span}\{H^Q_p \cup H^Q_q\}$.

$H^\perp$ denotes the orthocomplement of H, that is, the set of all elements of $\mathcal{H}$ which are each orthogonal to every element of H. The sentence 'p' is true in the instantaneous state of affairs represented by $|\psi(t)\rangle$, which we will write as $|\psi(t)\rangle \vDash_Q p$, if and only if $|\psi(t)\rangle \in H^Q_p$. When '$p$' is true in every such state under every interpretation—when 'p' is a tautology—we shall simply write $\vDash_Q p$. It will turn out that the tautologies with respect to $\vDash_Q$ fail to include all tautologies of $\vDash_C$, and so are not the tautologies of classical logic. This is because the above inductive extension of the interpretation of $\mathcal{L}$ defines alternative interpretations of $\mathcal{L}$-sentences which do not, in general, agree with our classical semantics: for some $p \in \mathcal{L}$, $H^Q_p \neq H^C_p$.

The differences between our semantic theory and the semantic theory of quantum logic can ultimately be traced back to the following deeply entrenched assumption:[20]

> **Central Tenet of Quantum Logic:** Let $\mathcal{H}$ be the Hilbert space over which $\mathcal{L}$ is interpreted. For every $p \in \mathcal{L}$, there exists some projection operator $\hat{\mathbb{P}}$ on $\mathcal{H}$ such that 'p' is true in the instantaneous state of affairs represented by the quantum state $|\psi(t)\rangle \in \mathcal{H}$ if and only if $\hat{\mathbb{P}}|\psi(t)\rangle = |\psi(t)\rangle$.

In short, this is the stipulation that to every sentence there corresponds some Hilbert space projection which mediates its truth conditions. It is a requirement on the mathematical form of the truth conditions for *all* the sentences in $\mathcal{L}$, not just the atomic sentence letters in SL as is the case on our proposal.

Before discussing the motivation for this assumption and critically evaluating its metaphysical consequences, we first demonstrate that *if* one adopts the central tenet, then one is forced to adopt a semantics for the connectives distinct from the classical semantics adopted in our theory. Suppose for reductio both that the inductive truth conditions for sentential connectives are the classical ones we propose, and that the central tenet of quantum logic obtains. Let 'p' be the sentence 'the electron is spin-up in the x-direction' and let the kind of system in question be an electron. Then there are some states in $\mathcal{H}$ in which 'p' is true, such as $|{\uparrow_x}\rangle$, and others in which it is not true, such as $|{\downarrow_x}\rangle$. Since $H^C_{\neg p} = \mathcal{H} \setminus H^C_p$, any quantum state of the form $|\phi\rangle = \alpha\,|{\uparrow_x}\rangle + \beta\,|{\downarrow_x}\rangle$ with $\alpha, \beta \neq 0$ will be an element of $H^C_{\neg p}$. From the central tenet of quantum logic, there is then some projection operator $\hat{\mathbb{P}}$ such that both $\hat{\mathbb{P}}\,|{\downarrow_x}\rangle = |{\downarrow_x}\rangle$ and $\hat{\mathbb{P}}|\phi\rangle = |\phi\rangle$. By the linearity of $\hat{\mathbb{P}}$:

$$\hat{\mathbb{P}}\,|{\uparrow_x}\rangle = \hat{\mathbb{P}}\left(\frac{1}{\alpha}|\phi\rangle - \frac{\beta}{\alpha}\,|{\downarrow_x}\rangle\right) = \left(\frac{1}{\alpha}|\phi\rangle - \frac{\beta}{\alpha}\,|{\downarrow_x}\rangle\right) = |{\uparrow_x}\rangle\,.$$

Thus, $|{\uparrow_x}\rangle \in \mathcal{H}\setminus H^C_p$ from the central tenet of quantum logic, while $|{\uparrow_x}\rangle \in H^C_p$ by hypothesis: a contradiction. A similar contradiction may be derived for the classical inductive truth conditions for disjunction. A classical semantic theory is therefore unavailable to anyone endorsing the central tenet of quantum logic.

[20] This assumption is embraced in von Neumann's classic textbook (von Neumann, 1955, pp. 160-161), it serves as the guiding assumption in the landmark paper of Birkhoff and von Neumann (1936), and it subsequently reappears at every major stage of development in quantum logic. For a small selection of examples, see (Mackey 1963, pp. 64–76; Jauch and Piron 1963, p. 829; Piron 1964, p. 443; Kochen and Specker 1968, p. 65; Bub 1974, p. 55).

3.3. Logical comparison. Both our classical semantic theory and the quantum-logical semantic theory characterize the propositional content of quantum theory. They use the same language and they both give truth conditions for that language in terms of the same underlying meaning structures. They differ only with respect to the form of their truth conditions. Moreover, the quantum-logical theory is the minimally invasive modification of our classical semantic theory which satisfies the central tenet: the truth conditions for atomic sentence letters and conjunctions are identical, and there is a sense in which the truth conditions for negation and disjunction for the quantum-logical semantic theory are, respectively, the broadest restriction and the narrowest relaxation of the classical truth conditions compatible with the central tenet.

Despite the differences in their truth conditions, both theories satisfy the law of non-contradiction and the law of excluded middle. With respect to both $\vDash_C$ and $\vDash_Q$, for all $p \in \mathcal{L}$ we have:

$$\begin{array}{rll} \textit{Non-Contradiction:} & \vDash_C \neg(p \wedge \neg p) & \vDash_Q \neg(p \wedge \neg p) \\ \textit{Excluded Middle:} & \vDash_C p \vee \neg p & \vDash_Q p \vee \neg p \end{array}$$

So, on both theories, 'it is not the case both that the photon has energy $\hbar\omega/2$ and the photon does not have energy $\hbar\omega/2$' will be true no matter what state the photon is in, and 'either the electron is spin-up in the x-direction or the electron is not spin-up in the x-direction' will be true no matter what state the electron is in.

The differences in their truth conditions does, however, generate at least two significant differences between these theories. The first significant difference is that their associated sets of tautologies are different. In particular, our proposed classical semantics validates the law of distributivity:

$$\text{For all } p, q, r \in \mathcal{L}, \quad |\psi(t)\rangle \vDash_C p \wedge (q \vee r) \text{ if and only if } |\psi(t)\rangle \vDash_C (p \wedge q) \vee (p \wedge r).$$

However, on the quantum-logical semantics, it can happen for some $p, q, r \in \mathcal{L}$ and some states $|\psi(t)\rangle$ that $|\psi(t)\rangle \vDash_Q p \wedge (q \vee r)$ and $|\psi(t)\rangle \nvDash_Q (p \wedge q) \vee (p \wedge r)$. The second significant difference regards the status of bivalence. A semantics for a language is bivalent just in case for each model, every sentence in the language is either true or false in that model. Thus, bivalence is a prohibition on truth value gaps. Whereas classical semantics is the hallmark of a bivalent semantic theory, quantum-logical semantics is commonly thought to violate bivalence.

To evaluate whether a particular semantics is bivalent, one must consider how both truth and falsity are defined. Up to this point we have only defined truth in the two semantic theories under consideration, and so there is not yet a clear verdict about the status of bivalence in either case. There are two ways one might define falsity: (i) 'p' is false when 'p' fails to be true; or (ii) 'p' is false when '$\neg p$' is true. Roughly, (i) says that being false coincides with being untrue, while (ii) says that negation acts to invert the truth value of a sentence. In order to say whether either of the above semantic theories is bivalent, a choice between (i) and (ii) must be made.

For our classical semantics, the choice does not matter because (i) and (ii) are equivalent, and bivalence holds. Every sentence is rendered true or false by every quantum state no matter whether one defines negation according to (i) or (ii). In the quantum-logical semantics, however, the choice between (i) and (ii) becomes substantive. In particular, for all

$|\psi(t)\rangle$, there are some $p \in \mathcal{L}$ such that $|\psi(t)\rangle \nvDash_Q p$ and $|\psi(t)\rangle \nvDash_Q \neg p$. Several results suggest that if negation is to fill its intended role, (ii) must be adopted, and so quantum logic must violate bivalence;[21] but some have argued that bivalence may be restored to quantum logic by adopting a conception of negation that accords with (i) instead.[22]

There has been considerable debate about the status of the quantum-logical semantic program. Following Quine's suggestion[23] that logic may be revisable on empirical grounds, Putnam[24] argues that the empirical considerations compelling us to supplant classical mechanics with quantum mechanics likewise compel us to supplant classical logic with quantum logic. Putnam, moreover, suggests that adopting quantum logic potentially resolves many of the interpretive challenges of quantum theory.[25] Many, including eventually Putnam himself, have found this suggestion unconvincing.[26] We do not aim to revisit these debates, as they seem be predicated on what we can now see to be the mistaken assumption that quantum logic provides the only available semantic theory for quantum mechanics. In any case, the availability of our theory demonstrates that quantum mechanics does not require any departure from a classical semantic theory.

4. Meta-Semantic Theories for Quantum Properties. We have just shown how the same meaning structures may be used to give two distinct accounts of the semantic content of quantum theory. While we allowed that the atomic sentence letters in the language $\mathcal{L}$ might express instances of predication, strictly speaking each of the proposals discussed in the last section were cast in terms of a propositional rather than a predicative language. As predicates are the syntactic objects which represent properties, and we are articulating a theory of the semantics of quantum property attribution, an adequate account of the semantic content of quantum theory should be cast in a predicative language in which the predicates explicitly label the properties.

In Section 4.1, we present a predicative generalization of the classical propositional theory developed in Section 3.1. In Section 4.2, we discuss the status of properties in quantum logic. Interestingly, there is no predicative generalization of the propositional theory discussed in Section 3.2: predicates are conspicuously absent in quantum logic. We identify two meta-semantic commitments of the quantum logic program which explain why quantum logic has no use for syntactic objects labeling properties. Though we do offer some reasons for thinking that these meta-semantic assumptions are problematic, our central aim here is not to argue against them. Rather, our central aim is to show that these assumptions are not obligatory, and that each is rejected in our classical semantic theory. In Section **??**, we show that several objections to previous efforts to reconcile determinables-based quantum indeterminacy with classical logic turn on these meta-semantic assumptions—assumptions which only advocates of quantum logic endorse.

4.1. The classical logic of quantum properties. Our aim now is to extend our semantic theory to a finer-grained predicative language which is sufficiently structured to be sensitive to the underlying theory of quantum properties. The logic of this semantic theory turns out

[21] (Jauch and Piron, 1963; Kochen and Specker, 1968; Bacciagaluppi, 2009)
[22] (Demopoulos, 1976; Hellman, 1980; Calosi and Toader, 2025)
[23] (Quine, 1951)
[24] (Putnam, 1969)
[25] Contemporary efforts to characterize the phenomenon of quantum indeterminacy using the resources of quantum logic can be thought of as contributions to this tradition.
[26] (Gardner, 1971; Dummett, 1978; Hellman, 1980; Putnam, 1981; Maudlin, 2005; Rumfitt, 2015; Kripke, 2023)

to be classical. Hence, contrary to claims made otherwise,[27] classical logic and determinable-based metaphysical indeterminacy are entirely compatible with one another.

The language we consider, $\mathcal{L}_{\text{Pred}}$, is an unquantified predicative language equipped with a single name $\mathcal{S}$ and the familiar logical connectives $\{\neg, \wedge, \vee, \rightarrow\}$. The predicates in this language will all be unary predicates. Concretely, $\mathcal{L}_{\text{Pred}}$ is the set of sentences grammatically defined in the following way:

Terms: There is a single name '$\mathcal{S}$'.

Determinable Predicates: For each self-adjoint operator $\hat{O}$ defined on any Hilbert space, there is a unary predicate denoted $F_{\hat{O}}$.

Determinate Predicates: For each spectral value λ of each self-adjoint operator $\hat{O}$ defined on any Hilbert space, there is a unary predicate denoted $F^{\lambda}_{\hat{O}}$.

Predicate Closure: There are no other predicates.

Base Case: For each predicate F, $F(\mathcal{S}) \in \mathcal{L}_{\text{Pred}}$.

Inductive Case: For all $\phi \in \mathcal{L}_{\text{Pred}}$, $\neg\phi \in \mathcal{L}_{\text{Pred}}$.

Inductive Case: For all $\phi, \chi \in \mathcal{L}_{\text{Pred}}$, $\phi \wedge \chi \in \mathcal{L}_{\text{Pred}}$.

Inductive Case: For all $\phi, \chi \in \mathcal{L}_{\text{Pred}}$, $\phi \vee \chi, \in \mathcal{L}_{\text{Pred}}$.

Inductive Case: For all $\phi, \chi \in \mathcal{L}_{\text{Pred}}$, $\phi \rightarrow \chi \in \mathcal{L}_{\text{Pred}}$.

Sentence Closure: There are no other sentences in $\mathcal{L}_{\text{Pred}}$.

The set of structures in which $\mathcal{L}_{\text{Pred}}$ is interpreted is given by the set of all pure quantum states defined on arbitrary Hilbert spaces. That is, for any Hilbert space $\mathcal{H}$, every unit vector $|\psi\rangle \in \mathcal{H}$ is a structure. Hence, individual structures will be ordered pairs of the form $\langle|\psi\rangle, \mathcal{H}\rangle$ where $|\psi\rangle \in \mathcal{H}$. To interpret $\mathcal{L}_{\text{Pred}}$ in some structure $\langle|\psi\rangle, \mathcal{H}\rangle$, we define an interpretation map $\mathcal{I}$ as follows:

Names: $\mathcal{I}(\mathcal{S}) = |\psi\rangle$.

Determinable Extensions: For all $|\alpha\rangle \in \mathcal{H}$, $|\alpha\rangle \in \mathcal{I}(F_{\hat{O}})$ iff $\hat{O}$ is defined on $\mathcal{H}$.

Determinate Extensions: For all $|\alpha\rangle \in \mathcal{H}$, $|\alpha\rangle \in \mathcal{I}(F^{\lambda}_{\hat{O}})$ iff $\hat{O}$ is defined on $\mathcal{H}$, $|\alpha\rangle \in \text{dom}(\hat{O})$, and $(\hat{O} - \lambda\hat{I})\,|\alpha\rangle = 0$

This interpretation ensures that the theory of determinable and determinate instantiation proposed above is satisfied: whenever our theory asserts that a quantum property is instantiated by a system, this interpretation makes true the sentence that asserts that the property is instantiated by the system. Given such an interpretation, we may now state the conditions under which an arbitrary $\mathcal{L}_{\text{Pred}}$-sentence is true in a structure $\langle|\psi\rangle, \mathcal{H}\rangle$. Let $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \phi$ mean that ϕ is true in $\langle|\psi\rangle, \mathcal{H}\rangle$, and stipulate that ϕ is false in $\langle|\psi\rangle, \mathcal{H}\rangle$ otherwise. For every predicate F (which may label either a determinable or a determinate), and for all sentences $\phi, \chi \in \mathcal{L}_{\text{Pred}}$, we adopt the following truth conditions:

[27](Torza, 2017, 2021; Fletcher and Taylor, 2021a,b, 2024a)

Base Case: $\langle|\psi\rangle, \mathcal{H}\rangle \vDash F(\mathcal{S})$ iff $\mathcal{I}(\mathcal{S}) \in \mathcal{I}(F)$.

Inductive Case ($\neg$): $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \neg\phi$ iff $\langle|\psi\rangle, \mathcal{H}\rangle \nvDash \phi$.

Inductive Case ($\vee$): $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \phi \vee \chi$ iff either $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \phi$ or $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \chi$.

Inductive Case ($\wedge$): $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \phi \wedge \chi$ iff both $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \phi$ and $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \chi$.

Inductive Case ($\rightarrow$): $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \phi \rightarrow \chi$ iff either $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \neg\phi$ or $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \chi$.

On this semantics, the tautologies of $\mathcal{L}_{\text{Pred}}$ include all of the theorems of classical logic.

Theorem (Soundness)**.** *If ϕ is a theorem of classical logic, then $\langle|\psi\rangle, \mathcal{H}\rangle \vDash \phi$ for all $\langle|\psi\rangle, \mathcal{H}\rangle$.*

Proof. This follows by simple computation from the stated truth conditions applied to any Hilbert-style axiomatization of classical logic. □

In particular, our semantic theory validates the law of excluded middle, the law of non-contradiction, and the law of distributivity. It can also be shown that this semantics is truth functional and bivalent. These results capture the sense in which the logic of our semantics is just classical logic.[28]

We should clarify that while the semantics thus articulated admits of a classical logic, our point, from a broader perspective, is not that the correct logic of quantum mechanics is or must be classical. Rather, our point is that nothing about the quantity valuation scheme of quantum theory compels a revisionary logic with a non-classical semantic theory. In providing a classical semantics for quantum theory, we are simply showing that quantum theory itself, even understood as involving genuine metaphysical indeterminacy, does not pose a problem for logical orthodoxy.

4.2. Two meta-semantic commitments of quantum logic. The availability of a classical semantics for quantum property ascription raises the question: why has the quantum-logical semantic theory been the primary focus of efforts to articulate the semantic content of quantum theory? In Section 3, we demonstrated that a departure from classical semantics is made necessary by adherence to the central tenet of quantum logic. Hence to understand the motivation for the quantum-logical semantics, we need to understand the motivation for adherence to the central tenet.

The standard justification for adopting the central tenet can be traced back to the beginning of quantum logic, in the work of Birkhoff and von Neumann, and reflects commitment to the verificationist theory of meaning in vogue at that time.[29] According to verificationism, the meaning of an empirical proposition is given by the conditions under which that

[28] One might think that, because our account is bivalent, it must violate the Kochen-Specker theorem (Kochen and Specker, 1968), which establishes that there can be no value-definite, non-contextual account of quantum properties. Our account, however, is straightforwardly value-indefinite. Bivalence is only in tension with the Kochen-Specker theorem if one flattens the distinction between truth values of propositions and numerical values of dynamical physical quantities. If one maintains this distinction, however, bivalence is perfectly consistent with the Kochen-Specker theorem, as our account demonstrates. Our account, moreover, serves as a constructive demonstration that determinable-based quantum metaphysical indeterminacy is not incompatible with classical logic, despite various arguments to the contrary (Torza, 2017; Fletcher and Taylor, 2021a, 2024b).

[29] The central tenet is articulated in both (von Neumann, 1955), and (Birkhoff and von Neumann, 1936). In a letter to Birkhoff during the preparation of the latter, von Neumann registers that issues related to the motivation for the central tenet are "dangerous ground", and worries that these are "too philosophical" to discuss in the paper (von Neumann, 2005, p. 49).

proposition may be experimentally verified.[30] This principle is embraced in quantum logic by requiring that the meaning of every proposition is given by an experimental question admitting of 'yes' and 'no' as its possible answers, where the verification of a proposition proceeds via an experiment which answers the associated question. These verifications are then represented by measurements of two-valued dynamical physical quantities whose possible values are 1 and 0, with these values corresponding to the 'yes' and 'no' answers to the experimental question. Every two-valued dynamical physical quantity whose possible values are 1 and 0 is represented by a projection operator. The demand to characterize the meanings of propositions in terms of verifications leads one to pair every proposition with a projection operator in such a way that the proposition is true if and only if the system instantiates the 1 determinate of that quantity. The central tenet thus gives meanings to quantum propositions in verificationistically acceptable terms.

While the verificationism underlying the motivation for the central tenet has since been widely abandoned, the central tenet itself has lived on in the quantum logic program. As we have just shown, the central tenet amounts to the requirement that the truth conditions for every sentence in the language are coordinated with the instantiation conditions of a property represented by a projection operator. Because this coordination applies to all sentences, it must be applicable not just to atomic sentences, but also to grammatically complex sentences—a crucial point, to which we will shortly return.

Upholding the central tenet requires two meta-semantic commitments. To identify these commitments, it is helpful to first break the tenet into two independent principles which jointly entail the tenet. If $\mathcal{S}$ denotes the system with respect to which $\mathcal{L}$ is interpreted, and $|\psi\rangle$ denotes the state of $\mathcal{S}$, these principles are:

> **Sentence–Property Correspondence (SPC):** For every $p \in \mathcal{L}$, there exists some property P such that 'p' is true iff $\mathcal{S}$ has P.
>
> **Property–Projection Correspondence (PPC):** For every property P, there exists some projection $\hat{\mathbb{P}}$ on $\mathcal{H}$ such that $\mathcal{S}$ has P iff $\hat{\mathbb{P}}|\psi\rangle = |\psi\rangle$.

SPC coordinates the truth conditions of $\mathcal{L}$-sentences with the instantiation conditions of properties, and **PPC** coordinates the instantiation conditions of properties with the geometry of Hilbert space characterized in terms of its projective structure. By simultaneously adopting **SPC** and **PPC**, the need for the instantiation conditions of properties drops out, and we are left with the central tenet, which coordinates the truth conditions of all $\mathcal{L}$-sentences directly with the closed linear subspaces of Hilbert space—that is, with the ranges of its projections.

We are now in position to identify the two meta-semantic commitments required to uphold the central tenet. The first concerns the status of predicates. Since the role of the instantiation conditions of properties is rendered redundant by the conjunction of **SPC** and **PPC**, the central tenet removes the need for syntactic objects that explicitly label the properties. Put another way, quantum logic obviates the need for a predicative language by making predication and propositional assertion equivalent. These observations explain why predication is absent from quantum logic. This meta-semantic commitment also flattens the distinction between the truth values of propositions and the numerical values of dynamical quantities: assigning the determinate value 1 to a quantity is equivalent to assigning the semantic value 'true' to the corresponding proposition, and assigning 0 is equivalent to assigning 'false'.

[30] There were many subtly different versions of the verificationist thesis. For discussion, see Uebel (2019).

The second meta-semantic commitment required to uphold the central tenet concerns the truth conditions for grammatically complex sentences. As we have noted, quantum logic values every sentence, including the grammatically complex sentences, using Hilbert space structure. The demand for this stems not from **SPC**,[31] but rather from **PPC**. In particular, **PPC** requires that all properties of quantum systems, including those that **SPC** pairs with grammatically complex sentences, be represented by self-adjoint operators or their spectral values. This amounts to the demand that the properties that **SPC** pairs with grammatically complex sentences are themselves dynamical physical quantities or their associated determinate values, and that the truth conditions of such grammatically complex sentences be given in terms of Hilbert space structure rather than the semantics of the connectives used to form the sentence.

It seems to have gone unnoticed just how revisionary this approach to articulating the semantic content of a state space theory turns out to be. To appreciate this revisionary aspect, recall the standard property ascription scheme for classical mechanics that we introduced in Section 1, according to which dynamical quantities are represented by smooth functions on phase spaces, and their determinates are represented by their associated image values. We can read off the semantic content of atomic property-attributing sentences from the standard valuation scheme: a sentence attributing a dynamical quantity or a numerical value of such a quantity to a system is true just in case that system instantiates that quantity or that value, respectively. The second meta-semantic commitment operative in the quantum logic program, if adapted to this classical-mechanical context, would demand that grammatically complex sentences be paired with dynamical quantities represented as smooth functions on phase space, or their associated numerical values. To consistently implement this commitment, as in the case of quantum logic, one would similarly be forced to revise the semantics for at least some of the classical connectives.[32] If, on the other hand, we only value the atomic sentences using the kinematic structure of the theory, and we value the grammatically complex sentences using the standard classical semantics for the connectives used to form the sentences, no such revision is required.

In our classical theory, we reject both of the meta-semantic commitments required to uphold the central tenet, as well as the central tenet itself. Our theory does not render property attribution by way of predication and propositional assertion equivalent, and we have explicitly shown how to incorporate predication in the language. Moreover, the distinction between numerical values of quantities and truth values of propositions has been

[31] **SPC** is a version of a familiar principle from typed languages ('λ-abstraction'), according to which any sentence may be converted into a predicate whose instantiation is logically equivalent to the original sentence.

[32] Consider a classical point particle moving in one dimension. One property such a particle may instantiate is that of being located at coordinate x_0. This is the determinate value represented numerically by x_0 of the dynamical quantity 'position', represented by the smooth phase space function $X(x,p) = x$. Another property such a particle may instantiate is that of not being located at coordinate x_0, which the particle instantiates if and only if the sentence 'the particle is not located at coordinate x_0' is true. If one upholds the classical truth conditions for negation, this sentence is true if and only if the state of the system is (x,p) with $x \neq x_0$. If one upholds the second meta-semantic commitment operative in the quantum logic program in the context of classical mechanics, one further requires that the property 'not being located at coordinate x_0' corresponds to a determinate value of some classical dynamical quantity represented by a smooth phase space function. So, in order to uphold both the classical truth conditions for negation and the second meta-sematic commitment under consideration in the setting of classical mechanics, one requires that there exists a smooth phase space function F, representing a classical dynamical quantity, with some determinate value z such that $F(x,p) = z$ if and only if $x \neq x_0$. Any such function, however, must be discontinuous at all points in phase space of the form (x_0, p); it cannot be smooth. Hence, there does not exist any such phase space quantity.

retained in our theory. We value atomic sentence letters using Hilbert space structure, but then we use the standard classical semantics for the connectives to value grammatically complex sentences. The existence of this classical alternative demonstrates that the central tenet and its attendant meta-semantic requirements are not mandated by the structure of quantum theory or the phenomenon of quantum indeterminacy. They are meta-semantic choices that stand in need of independent motivation.

One is free to adopt whichever meta-semantic commitments one likes, including those associated with the quantum logic program, provided one is prepared to accept the consequences of those commitments. The meta-semantic commitments associated with the quantum logic program, however, have consequences that strike us as undesirable. The first meta-semantic commitment renders predication dispensable, playing no independently significant role in how the semantic content of quantum theory is expressed. This is at odds with the fact that the primary aim of a semantic theory for any physical theory is to give meanings to sentences involving the attribution of physical properties to the systems described by that theory. Obviating the need for predication undercuts this aim.

The second meta-semantic commitment flattens the distinction between kinematics and semantics by taking all sentences, including grammatically complex ones, to have the same meanings as sentences which attribute dynamical quantities or determinate values to physical systems. This is at odds with how the kinematics of a physical theory and the semantics of a language are ordinarily conceived. Kinematics is largely concerned with characterizing the dynamical quantities and determinate values posited by physical theories in terms of the state space structure of that theory, and is not responsible for characterizing, for instance, the inductive truth conditions of negation, conjunction, or disjunction. Semantics, conversely, is concerned with characterizing the meanings of sentences in languages, and is not responsible for characterizing which dynamical quantities and determinate values a physical theory posits. These undesirable consequences of the meta-semantic commitments of quantum logic are not, however, obligatory, as our classical theory demonstrates.

5. Conclusion. We have articulated an account of the dynamical quantities of quantum theory in terms of determinables and determinates which naturally codifies the kinematics of the theory, and demonstrated that the account exhibits determinable-based quantum metaphysical indeterminacy. We have further demonstrated that this account may be expressed by a predicative language which may be fitted with a bivalent, truth-functional semantics that validates classical logic. Much of the confusion surrounding the possibility of such a view has stemmed from meta-semantic commitments internal to the quantum logic program. Nothing about quantum theory, nor the phenomenon of quantum indeterminacy, however, demands that we adopt those commitments.

References

Akiba, Ken (2004). Vagueness in the World. *Noûs 38*(3), 407–429.

Albert, David Z. and Barry Loewer (1996). Tails of Schrödinger's Cat. In Rob Clifton (Ed.), *Perspectives on Quantum Reality: Non-Relativistic, Relativistic, and Field-Theoretic*, pp. 81–92. Dordrecht: Springer Netherlands.

Bacciagaluppi, Guido (2009). Is logic empirical? In Kurt Engesser, Dov M. Gabbay, and Daniel Lehmann (Eds.), *Handbook of Quantum Logic and Quantum Structures: Quantum Logic*. Elsevier.

Barnes, Elizabeth (2010). Ontic Vagueness: A Guide for the Perplexed. *Noûs 44*(4), 601–627.

Barnes, Elizabeth and J. Robert G. Williams (2011). A Theory of Metaphysical Indeterminacy. In Karen Bennett and Dean W. Zimmerman (Eds.), *Oxford Studies in Metaphysics volume 6*, pp. 103–148. Oxford University Press.

Birkhoff, Garrett and John von Neumann (1936). The Logic of Quantum Mechanics. *Journal of Symbolic Logic 2*(1), 44–45. Publisher: Association for Symbolic Logic.

Bokulich, Alisa (2014). Metaphysical Indeterminacy, Properties, and Quantum Theory. *Res Philosophica 91*(3), 449–475. Publisher: Philosophy Department, St. Louis University.

Bub, J. (1974). *The Interpretation of Quantum Mechanics*. The Western Ontario Series in Philosophy of Science. Springer Netherlands.

Burgess, J. A. (1990). Vague Objects and Indefinite Identity. *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition 59*(3), 263–287.

Calosi, Claudio (2021). Determinables, location, and indeterminacy. *Synthese 198*(5), 4191–4204.

Calosi, Claudio (2022). Quantum modal indeterminacy. *Studies in History and Philosophy of Science 95*, 177–184.

Calosi, Claudio and Cristian Mariani (2020). Quantum relational indeterminacy. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics 71*, 158–169.

Calosi, Claudio and Cristian Mariani (2021). Quantum Indeterminacy. *Philosophy Compass 16*(4), e12731.

Calosi, Claudio and Iulian D. Toader (2025). Bivalent Quantum Indeterminacy. arXiv:2501.15226 [physics].

Calosi, Claudio and Jessica M. Wilson (2019). Quantum Metaphysical Indeterminacy. *Philosophical Studies 176*(10), 2599–2627. Publisher: Springer Verlag.

Calosi, Claudio and Jessica M. Wilson (2021). Quantum indeterminacy and the double-slit experiment. *Philosophical Studies 178*(10), 3291–3317.

Calosi, Claudio and Jessica M. Wilson (2022). Metaphysical Indeterminacy in the Multiverse. In Valia Allori (Ed.), *Quantum Mechanics and Fundamentality : Naturalizing Quantum Theory between Scientific Realism and Ontological Indeterminacy*, pp. 375–395. Cham: Springer International Publishing.

Cinti, Enrico , Cristian Mariani, and Marco Sanchioni (2025). The Unbearable Indefiniteness of Spacetime. *Foundations of Physics 55*(1), 14.

Clifton, Rob and Bradley Monton (1999). Losing Your Marbles in Wavefunction Collapse Theories. *The British Journal for the Philosophy of Science 50*(4), 697–717.

Corti, Alberto (2021). Yet Again, Quantum Indeterminacy is Not Worldly Indecision. *Synthese 199*(3-4), 5623–5643. Publisher: Springer Verlag.

Darby, George (2010). Quantum Mechanics and Metaphysical Indeterminacy. *Australasian Journal of Philosophy 88*(2), 227–245. Publisher: Taylor & Francis.

Darby, George and Martin Pickup (2021). Modelling Deep Indeterminacy. *Synthese 198*, 1685–1710. Publisher: Springer Verlag.

Darby, George and Martin Pickup (2022). Defending the Situations-Based Approach to Deep Worldly Indeterminacy. In Valia Allori (Ed.), *Quantum Mechanics and Fundamentality : Naturalizing Quantum Theory between Scientific Realism and Ontological Indeterminacy*, pp. 365–373. Cham: Springer International Publishing.

Demopoulos, William (1976). The Possibility Structure of Physical Systems. In William L. Harper and Clifford Alan Hooker (Eds.), *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science: Volume III Foundations and Philosophy of Statistical Theories in the Physical Sciences*, pp. 55–80. Dordrecht: Springer Netherlands.

Dummett, Michael (1978). Is Logic Empirical? In *Truth and Other Enigmas*, pp. 269–289. Harvard University Press.

Fletcher, Samuel C. and David E. Taylor (2021a). Quantum Indeterminacy and the Eigenstate-Eigenvalue Link. *Synthese 199*(3-4), 1–32. Publisher: Springer (Springer Science+Business Media B.V.).

Fletcher, Samuel C. and David E. Taylor (2021b). Two Quantum Logics of Indeterminacy. *Synthese 199*(5-6), 13247–13281. Publisher: Springer Verlag.

Fletcher, Samuel C. and David E. Taylor (2024a). Axiomatization of an Orthologic of Indeterminacy. *Journal of Philosophical Logic*.

Fletcher, Samuel C. and David E. Taylor (2024b). The representation and determinable structure of quantum properties. *Synthese 204*(2), 44.

Fraser, Patrick and Michael E. Miller (2025). From classical to quantum indeterminacy, and back. *Philosophy of Science*. forthcoming.

Gardner, Michael R. (1971). Is Quantum Logic Really Logic? *Philosophy of Science 38*(4), 508–529.

Glick, David (2017). Against Quantum Indeterminacy. *Thought: A Journal of Philosophy 6*(3), 204–213.

Glick, David (2022). Quantum Mechanics Without Indeterminacy. In Valia Allori (Ed.), *Quantum Mechanics and Fundamentality : Naturalizing Quantum Theory between Scientific Realism and Ontological Indeterminacy*, pp. 319–335. Cham: Springer International Publishing.

Goswick, Dana (2021). Odd Objects: LEM Violations and Indeterminacy. *Erkenntnis 86*(6), 1615–1633.

Greenough, Patrick (2008). Indeterminate Truth. *Midwest Studies In Philosophy 32*(1), 213–241.

Hellman, Geoffrey (1980). Quantum Logic and Meaning. *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association 1980*, 493–511.

Jauch, J.M. and C. Piron (1963). Can hidden variables be excluded in quantum mechanics? *Helvetica Physica Acta 36*(VII), 827–837. Publisher: Birkhäuser.

Kochen, Simon and E.P. Specker (1968). The Problem of Hidden Variables in Quantum Mechanics. *Indiana University Mathematics Journal 17*(1), 59–87.

Kripke, Saul A. (2023). The Question of Logic. *Mind 133*(529), 1–36.

Lewis, Peter J. (2016). *Quantum Ontology: A Guide to the Metaphysics of Quantum Mechanics*. Oxford University Press USA.

Lewis, Peter J. (2022). Explicating Quantum Indeterminacy. In Valia Allori (Ed.), *Quantum Mechanics and Fundamentality : Naturalizing Quantum Theory between Scientific Realism and Ontological Indeterminacy*, pp. 351–363. Cham: Springer International Publishing.

Mackey, George W. (1963). *The Mathematical Foundations of Quantum Mechanics*. New York: W. A. Benjamin.

Mariani, Cristian (2021). Emergent Quantum Indeterminacy. *Ratio 34*(3), 183–192.

Mariani, Cristian (2024). The Determinacy Problem in Quantum Mechanics. *Foundations of Physics 54*(6), 73.

Maudlin, Tim (2005). The Tale of Quantum Logic. In Yemima Ben-Menahem (Ed.), *Hilary Putnam*, Contemporary Philosophy in Focus, pp. 156–187. Cambridge University Press.

Menon, Tushar (2024). On algebraic naturalism and metaphysical indeterminacy in quantum mechanics. *Studies in History and Philosophy of Science 105*, 1–16.

Myrvold, Wayne C. (2018). Ontology for Collapse Theories. In Shan Gao (Ed.), *Collapse of the Wave Function: Models, Ontology, Origin, and Implications*, pp. 97–123. Cambridge: Cambridge University Press.

Nørgaard, Maria (2025). Quantum indeterminacy: a matter of degree? *European Journal for Philosophy of Science 15*(1), 13.

Piron, Constantin (1964). Axiomatique quantique. *Helvetica Physica Acta 37*(IV-V), 439.

Putnam, Hilary (1969). Is logic empirical? In *Boston studies in the philosophy of science*, pp. 216–241. Springer.

Putnam, Hilary (1981). Quantum mechanics and the observer. *Erkenntnis 16*(2), 193–219.

Quine, W. V. O. (1951). Two Dogmas of Empiricism. *The Philosophical Review 60*(1), 20–43.

Roberts, Bryan W. (2017). Unreal Observables. *Philosophy of Science 84*(5), 1265–1274.

Roberts, Bryan W. (2018). Observables, disassembled. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics 63*, 150–162.

Rosen, Gideon and Nicholas J. J. Smith (2004). Worldly Indeterminacy: A Rough Guide. *Australasian Journal of Philosophy 82*(1), 185–198.

Rumfitt, Ian (2015). *The Boundary Stones of Thought: An Essay in the Philosophy of Logic*. New York: Oxford University Press.

Schroeren, David (2021). Quantum metaphysical indeterminacy and the ontological foundations of orthodoxy. *Studies in History and Philosophy of Science Part A 90*, 235–246.

Skow, Bradford (2010). Deep Metaphysical Indeterminacy. *Philosophical Quarterly 60*(241), 851–858.

Torza, Alessandro (2017). Quantum Metaphysical Indeterminacy and Worldly Incompleteness. *Synthese* (10), 1–14. Publisher: Springer Verlag.

Torza, Alessandro (2021). Quantum Metametaphysics. *Synthese 199*(3-4), 1–25. Publisher: Springer Verlag.

Torza, Alessandro (2022). Derivative Metaphysical Indeterminacy and Quantum Physics. In Valia Allori (Ed.), *Quantum Mechanics and Fundamentality : Naturalizing Quantum Theory between Scientific Realism and Ontological Indeterminacy*, pp. 337–350. Cham: Springer International Publishing.

Uebel, Thomas (2019). Verificationism and (Some of) its Discontents. *Journal for the History of Analytical Philosophy 7*(4). Number: 4.

von Neumann, John (2005). *John von Neumann: Selected Letters*. American Mathematical Society.

von Neumann, John (2018 [1955]). *Mathematical Foundations of Quantum Mechanics: New Edition*. Princeton University Press.

Wilson, Jessica M. (2012). Fundamental Determinables. *Philosophers' Imprint 12*. Publisher: Mpublishing.

Wilson, Jessica M. (2013). A Determinable-Based Account of Metaphysical Indeterminacy. *Inquiry: An Interdisciplinary Journal of Philosophy 56*(4), 359–385. Publisher: Routledge.

Wilson, Jessica M. (2017). Are There Indeterminate States of Affairs? Yes. In Elizabeth Barnes (Ed.), *Current Controversies in Metaphysics*, pp. 105–125. Taylor & Francis.

Wolff, Johanna (2015). Spin as a Determinable. *Topoi 34*(2), 379–386. Publisher: Springer Netherlands.