

Semantic Completion: A View From The Boundary

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Abstract

The concept of a boundary condition was developed to encode a scientific model's behaviour at spatial boundaries, and has since become an expansive and indispensable part of scientific practice. Unfortunately, its philosophical nature remains obscure. Boundary conditions fit poorly into familiar philosophical categories such as the contingent-nomological distinction, and philosophical accounts are often too narrow or too broad. In this paper, we argue that boundary conditions have a single overarching character: to constrain a model's behaviour by appeal to degrees of freedom not included in that model. This generalises the notion of a boundary to allow not just boundaries in space but in representation more generally, including scale. We refer to this practice as *Semantic Completion*, arguing that it better captures the rich variety of boundary conditions in science while providing a constructive framework for reconciling some old philosophical disagreements.

1. Introduction

Boundary conditions play a pervasive role in physics, and yet philosophical discussions of their scientific function remain fragmentary. On the one hand, boundary conditions express physical facts, like the fact that there is a dissipative force between a pipe and the water flowing through it. On the other hand, philosophers have traditionally partitioned physical facts into categories that are not obviously suitable for boundary conditions: those that are true in virtue of a natural law, called *nomological*

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facts, and those that are not, called *contingent facts*. To take an example that has been widely discussed by philosophers,¹ it is a nomological fact that no 60kg cube of uranium-235 exists (since this would exceed its critical mass), while it is a contingent fact that no 60kg cube of gold exists. But a more complete discussion would also ask: what kind of facts are the boundary conditions for these cubes? The ambient air pressure may seem contingent, and yet if it were sufficiently high then it would change each cube's critical mass, with the latter often viewed as nomological.

Traditional philosophical accounts have made little progress on this puzzle. One philosophical tradition, which arises out of Hadamard's classification of partial differential equations, treats boundary conditions as just an essential component for ensuring that an initial value problem is well-posed—or, as philosophers put it, for ensuring that a set of equations evolve deterministically. On this view, boundary conditions play the same explanatory role as initial conditions, the latter of which are widely viewed as contingent. Unfortunately, that assimilation obscures the distinctive function of boundary conditions in physics (Wilson 1990; Morrison 1999; Bursten 2021), and more generally the role that context plays in defining a physical system (Wilson 2006). Thus, more recent philosophers have eschewed the nomological/contingent distinction and argued that boundary conditions consist in a plurality of disparate modelling practices (Sykora 2019; Bursten 2021).

We argue for an alternative and unifying account. One can recognise that boundary conditions do not fall neatly into the nomological-contingent division without requiring a disunified plurality of what boundary conditions are. Our thesis is that boundary conditions play one overarching role in physics, which we call *semantic completion*: to constrain the behaviour of a model on the basis of how it interacts with or couples to another system whose degrees of freedom are external to it. In Section 2, we formulate two existing philosophical positions on the boundary condition debate, which we call the initial value problem approach and the pluralist approach. In Section 3 we articulate our alternative account, and in Section 4 present two examples of boundary conditions that illustrate the generality of our approach. We conclude in Section 5 with a prospectus outlining some broader implications for an array of debates in the philosophy of science, including inter-scale modelling, theory change, scientific representation, and the unity of science.

¹The gold cubes are due to Reichenbach (1947, p.368), although the uranium-235 example may be due to Tooley (see Van Fraassen 1987, p.246).

2. Two Views of Boundary Conditions

2.1. A brief history of boundaries. The technique of constraining a physical problem by subjecting it to external boundary conditions appeared surprisingly late in the history of physics. It is largely absent in the work of Leibniz, Newton, or any of their predecessors. The 1696 brachistochrone problem of Bernoulli, which asked what curve provides the fastest descent for a body sliding along it under the force of gravity, was formulated using boundary conditions for the string. However, the first solutions appear to have used geometric techniques rather than variational ones, and so boundary conditions were more a part of the formulation of the problem than its solution (de Icaza Herrera 1994). Similarly, early 18th century analyses of a vibrating violin string by Taylor and Bernoulli made little substantial use of the boundaries of the string in their calculations.²

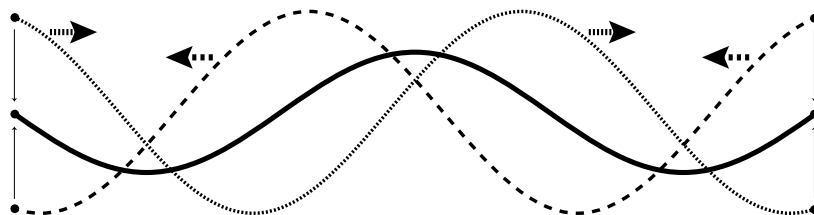


Figure 1: Two oppositely-directed waves constrained to sum to zero at the boundary.

A more substantial use of boundary conditions appeared in the great creative leap of [D'Alembert \(1747\)](#), who modelled a vibrating string as a superposition of two waves moving in opposite directions, and making early use of partial differential equations. He then used a boundary condition to restrict which pairs of waves are possible: namely, those summing to zero at the endpoints ([Figure 1](#)). The result is the familiar periodic behaviour of a standing wave.

An explosion of similar uses of this technique soon followed, including boundary conditions in:

- the derivation of the Euler-Lagrange equations through action extremisation;
- the Lavoisier-Laplace ice calorimeter with a fixed-temperature boundary;

²A history of the vibrating string analysis can be found in [Kline \(1972, §21.4 and §22.2\)](#).

- the introduction of no-slip conditions into fluid flow equations;
- the boundary-flux integral theorems by Gauss, Green, and Stokes;
- Riemann’s solution to the Dirichlet problem;
- Hadamard’s initial value problem analysis of partial differential equations;

and many, many more. Today, boundary conditions are an essential part of the study and application of partial differential equations, especially in physics.

Thus, boundary conditions arose through a continuous historical development spanning several centuries, often overlapping the development of partial differential equations, and leading to a productive modern technique. This raises the philosophical question of what the nature of that technique amounts to. For example, are all these uses of boundary conditions unified by any mathematical, physical, or explanatory character? We will begin by identifying two existing responses to this question, before turning to our own.

2.2. The IVP Account. An *initial value problem* (IVP) or *Cauchy problem* is the problem of determining when a partial differential equation admits a solution that is unique and depends continuously on initial data.³ If it does, then we say that the problem is *well-posed*. This is obviously a central problem for both physics and philosophy. For example, when initial values represent present facts and solutions represent past and future facts, then the IVP is equivalent to the problem of determinism (Earman 1986, §II.11).

For some equations (like the wave equation), initial conditions exist that are sufficient to uniquely determine a solution. However, for others (like the Poisson equation), initial conditions are not enough: a unique solution is at best guaranteed in the presence of further assumptions, which are then called ‘boundary conditions’. From this perspective, initial and boundary conditions play essentially the same explanatory role, as when Arfken points out in a book on mathematical methods that for the wave equation, “[t]he term boundary conditions includes as a special case the concept of initial conditions” in which only initial position and velocity are given (Arfken 1985, p.503). We refer to this as the *IVP Account of boundary conditions*: boundary condi-

³See Fattorini (1984, §1.2). Note that we use ‘initial data’ and ‘initial conditions’ interchangeably. Continuous dependence will not concern us here; see Gyenis (2014) for a philosophical discussion of its significance.

tions are any assumptions besides initial data that are required to render a differential equation well-posed.

Boundary conditions in this sense are exceedingly common. Many partial differential equations (PDEs) used in physics can be classified into one of three types identified by [Hadamard \(1923\)](#),⁴ two out of three of which require boundary conditions for well-posedness:

- *Hyperbolic* (Example: Wave equation). The initial value problem is well-posed given appropriate initial conditions.
- *Parabolic* (Example: Heat equation). The initial value problem is not well-posed on the basis of initial data alone: *boundary conditions are often required*.
- *Elliptic* (Example: Poisson equation). The initial value problem is not well-posed on the basis of initial data alone: *boundary conditions are always required*.

Moreover, even hyperbolic differential equations may require boundary conditions when the initial data is limited. John Earman gives a pithy statement of this fact in his *Primer on Determinism*, although he does not endorse it as a general account of boundary conditions:

“Trying to determine the weather in Minneapolis tomorrow from even the most precise meteorological data today in Minneapolis is a thankless task since tomorrow’s weather can be influenced by what is now happening in North Dakota and Wisconsin.... Two remedies may be contemplated. This first is to erect imaginary boundary walls (W_1 and W_2 ...) to record the incoming influences as they penetrate the boundaries of the system. This gives rise to a non-Laplacian initial-boundary value problem: given the appropriate initial data on S and the appropriate boundary data on $W_1 \cup W_2$, determine the state in the interior region R . For field theories where there is action by contact such initial-boundary value problems are often well posed” ([Earman 1986](#), p.33)

⁴A linear second-order PDE of the form $Af_{xx} + Bf_{xy} + Cf_{yy} + Df_x + Ef_y + F = 0$ is called *hyperbolic*, *elliptic*, or *parabolic* if the discriminant $B^2 - 4AC$ is (respectively) positive, negative, or zero. This classification generalises to linear Pfaffian systems ([Bryant et al. 1991](#), §V.2), Banach spaces ([Fattorini 1984](#)), and even non-linear PDEs like the Einstein equations through linearisation techniques. Regularity assumptions like Lipschitz continuity are still assumed, without which further failures of well-posedness like Norton’s dome can arise ([Norton 2008](#); [Malament 2008](#)). See [Callender \(2017\)](#) and [James \(2022\)](#) for a philosophical discussion of hyperbolic PDEs.

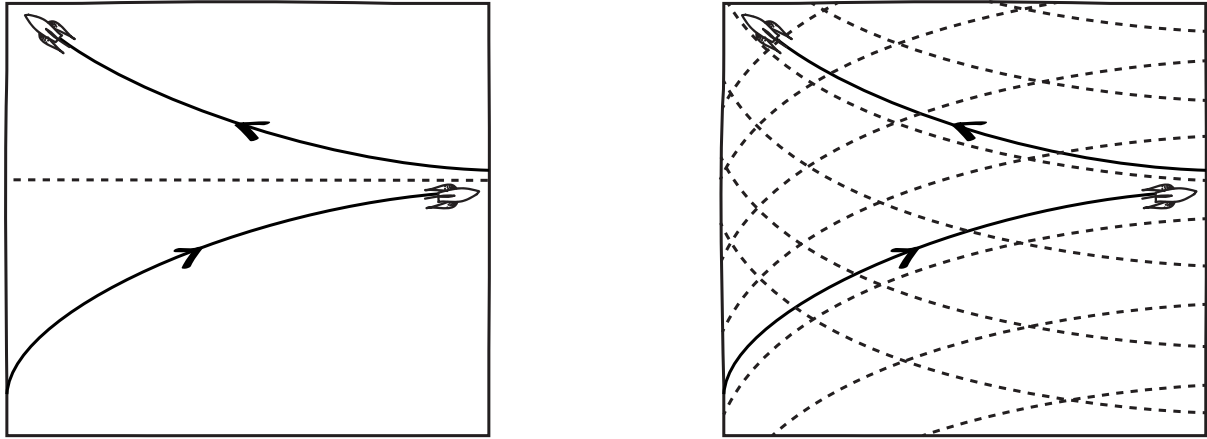


Figure 2: Space escaper and space invader trajectories in Newtonian spacetime (left) and anti-de Sitter spacetime (right).

The example of the wind around Minneapolis might lead one to think that boundary conditions are set on the boundary of a finite spatial region, in order to control external influences that enter the region. However, as Earman goes on to point out, boundary conditions can also be applied to initial value problems associated with all of spacetime, as is the case in most cosmological models. Their role is particularly visible in the well-known problem of ‘space invaders’ (Geroch 1977; Earman 1986). In Newtonian gravitation, the lack of an upper bound for velocity allows for scenarios in which a system of gravitating masses are accelerated towards an asymptote, at which moment the masses will have escaped all of space in finite time (Xia 1989). Conversely, such masses may ‘invade’ at arbitrarily high velocity from an asymptote into a region that was a vacuum for all previous times. A similar problem appears in relativity theory in the universal covering of the anti-de Sitter solution (Hawking and Ellis 1973, p.133). Both are illustrated in Figure 2.

To make such an IVP well-posed, it is common practice to set boundary conditions on the formal boundary of spacetime, which is infinitely far away in space. This is common in the analysis of anti-de Sitter spacetime, but also in more prosaic spacetimes like those that are asymptotically flat. We will return to this practice in Section 3.4.

2.3. Difficulties for the IVP Account. The IVP account faces the challenge of treating boundary conditions as unjustified and *ad hoc* mathematical assumptions, drummed up to ensure well-posedness whether or not we have any reason to believe it. In most philosophical discussions, including that of Earman (1986), the particular case of determinism is not a priori knowledge, but rather a question to be studied using

the structure of our best scientific theories. From this perspective, the IVP Account of boundary conditions is dubious and unmotivated.

The IVP Account has also come under fire for failing to do justice to the many roles that boundary conditions play in science. We might agree that initial and boundary conditions do sometimes play the same role in securing the determinism in an IVP. However, fusing initial and boundary conditions together in this way dramatically limits the generality of the account. [Wilson \(1990\)](#) characterises this fusion as ‘initialandboundaryconditions’, treated as one word:

“One of the quarrels that I... have with the stock philosophy of science picture of ‘initialandboundaryconditions’ stems from the presumption that boundary conditions play an essentially inert role in science—it is the *laws* that serve as the real engine driving a D-N explanation forward, etc.” ([Wilson 1990](#), p.566)

Wilson points out that on the famous ‘Deductive-Nomological’ (D-N) account of [Hempel and Oppenheim \(1948\)](#), according to which a scientific explanation of P is a deduction of P from a nomological sentence (the ‘law’) and a contingent sentence (the ‘initial conditions’), the role of boundary conditions is particularly unclear. For, as we have seen above, boundary conditions do not fall neatly into either side of the contingent-nomological distinction. Concern about a ‘Hempelian’ approach that forces boundary conditions into the initial conditions category has been echoed by [Bursten \(2021\)](#). However, the problem is not just the D-N account: even amongst the great number of alternative accounts of scientific explanation (cf. [Woodward 2014](#)), we are not aware of any unified general account of the role of boundary conditions.

This suggests the IVP Account is too narrow. But there is also a sense in which it is too broad: if boundary conditions are just formal conditions that assure well-posedness, then what do we make of the common practice of placing such conditions on quantities that have no representatives in reality, such as gauge fields? Despite being formal mathematical ‘boundary conditions’, they may not be boundary conditions in any substantial physical sense. For example, the Coulomb gauge in $U(1)$ electromagnetism requires solving an elliptic PDE that is not well-posed without boundary conditions.⁵ But, those conditions are placed on pure gauge degrees of freedom, and

⁵The Coulomb gauge selects a local representative A of the gauge potential such that $\nabla \cdot A = 0$. Since each gauge transformation $g_\phi \in U(1)$ transforms $A \mapsto A_\phi = A + \nabla\phi$, it follows that $\nabla \cdot A_\phi = \nabla^2\phi$, which is an elliptic PDE (the Poisson equation). To ensure that it is well-posed, boundary conditions must be placed to ensure a solution ϕ exists and is unique. A common sufficient condition for uniqueness is

so setting them has no effect on any measurable quantity like the electromagnetic field tensor. A good account of boundary conditions should distinguish between purely formal boundary conditions and those of a more physically substantial kind. The IVP Account on its own does not.

2.4. The Pluralist Account. The *Pluralist Account* states that boundary conditions play a multiplicity of different explanatory roles, which might have no unifying character and are not necessarily anything like the role that initial values play in science. [Wilson \(1990\)](#) indicates the direction of travel towards this view when he points out the substantially different roles that boundary conditions play for a vibrating string, an elastic body, a heated object, a cube of ice in water, a flow of water around an obstruction, and a fluid flowing through a container. The IVP account does not adequately capture any of these examples. Although Wilson himself does not endorse the use of the word ‘pluralism’, [Bursten \(2021\)](#) does. Her thesis is that boundary conditions simply fulfil a multiplicity of different explanatory roles, referring to her view as a kind of “compatible pluralism” about boundary conditions ([Bursten 2021](#), p.254). She identifies that there are at least four such roles:

1. *Scope-Setting.* Some boundary conditions determine the scope under which a model can be applied, such as the Prandtl boundary-layer model of fluid flow ([Morrison 1999](#)).
2. *Stable Descriptions.* Other boundary conditions provide “stable descriptions of law-like behaviour” (p.248), such as the theory of standing waves made possible by boundary conditions fixing the endpoints of the wave.
3. *Novel phenomena.* Some boundary conditions, such as the degree of slippage between the boundary of a fluid and its container, describe “novel phenomena” (p.249) that themselves become the object of study.
4. *Coordination.* Boundary conditions sometimes help one consistently coordinate two incompatible descriptions, such as a boundary-layer model’s separation of the viscous region from the ideal fluid region (p.250).

$\phi(x) \rightarrow 0$ as $x \rightarrow \infty$; see [Griffiths \(1999, §10.1.3\)](#). More generally, the relationship between gauge freedom and constraints on admissible data can be understood geometrically using a variational principle: consistency of the boundary variation implies initial value constraints that restrict the independently specifiable degrees of freedom, allowing one to identify dynamical redundancy in the model. See [Gryb and Thébault \(2024, §7–8\)](#), who extend [Woodhouse \(1992, §2\)](#).

Bursten contrasts her view with two theses that she calls ‘Hempelian’, that (i) initial and boundary conditions play the same explanatory role, and (ii) that natural laws play a more significant explanatory role. Both theses would seem to be endorsed by the IVP Account, and neither one seems plausible. She also claims that a “particularly telling instance of the persistence of the Hempelian theses” (Bursten 2021, p.237) appears in Butterfield’s discussion of explanation in cosmology. Butterfield (2014, §3.2.1) presents and discusses the well-known methodological principle that, if initial conditions in cosmology are merely contingent “happenstance”, then a satisfying explanation in cosmology should explain not just one initial condition but also those that are typical. Butterfield then remarks in a footnote that “one can, and some authors do, state the same idea for final and-or boundary conditions” (Butterfield 2014, p.64, footnote 14). Bursten writes about this passage that “the details of his account are not important here. What is worth noting, instead, is the appearance of both Hempelian theses in one short passage and its footnote” (Bursten 2021, p.237).

What Bursten calls ‘Hempelian’ theses are indeed problematic views of boundary conditions, and we agree that her four categories make progress in clarifying the roles that boundary conditions play in scientific explanations. However, we submit that she has misread Butterfield: in fact, neither ‘Hempelian’ thesis appears in his short passage or footnote. Butterfield (2014) does discuss the effort to explain initial and boundary conditions in cosmology, but neither thesis (i) or (ii) appears in that discussion. On the contrary, his footnote takes boundary conditions to constitute a case that is distinct from initial conditions, against (i). And, his Sections 3 and 4 argue that typicality explanations of initial or boundary conditions often include lawlike dynamical processes⁶, against (ii). Thus, a consequence of his thinking is that *if* a given boundary condition is merely contingent happenstance, whether it be for Minneapolis or for AdS spacetime, then a more satisfying explanation would include typicality considerations. In our view, this is correct for both initial conditions and boundary conditions.

We take Bursten’s central (and correct) point to be that initial and boundary conditions do not always serve the same role in a scientific explanation. But, once this is recognised, we take it that a more expansive pluralism could absorb typicality discussions of cosmological boundary conditions, and indeed even the IVP Account,

⁶For example, Butterfield (2014, p.66) points out that to solve the ‘flatness problem’, viz. that the universe’s mass density parameter Ω is very close to one, inflationary cosmology proceeds by showing that for generic/typical initial conditions, the dynamical process of inflation implies that “ Ω will be driven very close to 1 by the end of the inflation period” through the dynamical process of inflation.

as part of the plurality in certain circumstances. Indeed, [Wilson \(1989, p.510\)](#) refers to conditions associated with the constraint of a problem's solution space as "side conditions" so as not to assume they are necessarily honest boundary conditions, but he does not forbid it either. Even the occasional contingent boundary condition seems compatible with the broader insights of Wilson and Bursten, that boundary conditions play many substantial explanatory roles in physics.

2.5. Difficulties for the Pluralist Account. The advantage of pluralism, that it allows many things to count as boundary conditions, also presents a difficulty. The danger is that, if one were to call everything a boundary condition, the concept would lose its meaning and we would have accounted for nothing. For example, the four explanatory roles that [Bursten \(2021\)](#) describes—scope-setting, stable descriptions, describing new phenomena, and coordination—are so general that they are in danger of encompassing nearly all of physics. Even specifying that boundary conditions should be described at the boundary of some initial value problem's state space, the concern is that one could simply adjoin any arbitrary physical model to the boundary of that state space and it would count as a boundary condition on this view.

At the same time, there is a sense in which the Pluralist Account might be too restrictive, in that most of the examples discussed by [Wilson \(1990\)](#) and [Bursten \(2021\)](#) focus on boundary conditions that separate continuum systems in space. This leaves open the puzzling explanatory role that boundary conditions often play at the boundary of infinite spatial regions, as well as with boundaries that are not associated with space at all but rather with other degrees of freedom that appear on different scales. As pluralists themselves have argued, an adequate account of boundary conditions should do justice to the full richness of their role in physics.

Despite the many virtues of pluralism, we would not want to miss the explanatory opportunity that a unifying account can provide. We are reminded of the proliferation of electric and magnetic laws before Maxwell's synthesis, and the many laws of thermodynamics before Planck and the Ehrenfests. Of course, the balance of one's judgement may push towards pluralism about boundary conditions. However, we would like to point out that an alternative, unified account of boundary conditions still exists.

3. The Semantic Completion Account

3.1. **Adequacy Criteria.** One is free to define words however one likes, through what Grünbaum (1962, p.420) calls ‘Trivial Semantic Conventionalism’. However, the term ‘boundary condition’ is used by most scientists and mathematicians to refer to a definite concept in physics. In our view, that concept can be stated with as much clarity as other central concepts in physical modelling, like that of a ‘spacetime metric’ or a ‘central potential’.

However, before we give our account of boundary conditions, it is worth saying briefly what kind of problem we are trying to solve. We take it that every adequate account of boundary conditions must at least satisfy the following:

1. *Boundary conditions restrict the possible behaviours of a system, in the sense that arose amongst eighteenth century physicists and continues to this day.*
2. *Not every restriction is a boundary condition.* If boundary conditions were to include every restriction on the possible behaviour of a system, they would be too general to be of conceptual interest.
3. *They can be infinitely far away.* One must capture the common physical practice of setting boundary conditions that are in some sense set at the edge of spacetime, whether or not that spacetime is finite.
4. *They can be off the boundary of solution space.* One must capture the physical practice of setting boundary values that are not in any straightforward sense located on the boundary of space or a solution space.
5. *They should be distinct from purely formal boundary conditions, for example in models that include descriptive redundancy like those of gauge physics.*

Satisfying all these conditions requires an account of boundary conditions that is narrower than an unconstrained pluralist account, but broader than the IVP account.

3.2. **Boundary conditions as semantic completion.** Let us begin by saying something about what ‘boundary’ means, if it is not necessarily the boundary of a spatial region. We take ‘boundary’ to refer to the boundary of the set of degrees of freedom represented explicitly in the model, and thus a boundary for that model’s capacity to represent the world using its available degrees of freedom. To take two canonical examples in the theory of boundary value problems:

- *Boundary of a wave.* D'Alembert's wave model has the capacity to specify the displacement, velocity, curvature, and restoring forces at each point along a violin string, but not the degrees of freedom that give rise to the constraining forces on the boundary, such as those of the tuning peg and tailpiece of a violin string, or those confining an electron to remain near the atomic nucleus.
- *Boundary of a fluid.* A Navier-Stokes model of fluid flow specifies fields representing the density, velocity, pressure, and viscosity of a fluid in a pipe, but not the degrees of freedom associated with the container or its interaction with the molecules of the fluid.

If we only consider the built-in degrees of freedom, there is a sense in which each description is 'incomplete': namely, it does not describe how the system interacts with and couples to some external system. A fluid model provides an incomplete description of a fluid-container interaction, and a wave model is an incomplete description of a wave-constraint interaction.

What then is a boundary condition? On our view, it is any restriction on the behaviour of a model whose purpose is to *complete* the model with the relevant implications of those missing interactions. Here is the statement of the view.

The Semantic Completion Account. A *boundary condition* of a model M is any restriction on the possible behaviour of M that follows from facts about how M interacts with (or couples to) degrees of freedom that are not in the model M .

Boundary conditions on this account have two key features: 1) they are *modal*, in that they restrict the possible behaviour of a model (typically by restricting the possible solutions of a PDE), and they are *semantic*, in that they incorporate semantic content arising from physical facts that are not expressible using the model's ordinary resources for representing degrees of freedom.

Here is how those two dual features of boundary conditions arise in the two examples above:

- *Boundary conditions for a wave* include setting the position of the wave to be fixed at the boundary of some open interval. This constrains the possible waveforms of the model, while also encoding the consequences of forces and degrees of freedom that arise from the coupling of the wave to an external constraint.

- *Boundary conditions for a fluid* include the no-slip condition that the fluid velocity vanishes at the boundary. This restricts the possible solutions to the Navier-Stokes equations, and arises from viscous forces determined by the interaction of the fluid molecules with those of the external container, which are not included in the phase space of the fluid model itself.⁷

Thus, in models in which boundary conditions are relevant, a term like ‘water’ has incomplete meaning until boundary conditions are imposed.

This incompleteness is similar to the view of meaning expressed more generally by semantic externalists in the philosophy of language and philosophy of mind. According to semantic externalists, the meaning of such a word is not complete on the basis of one’s mental state alone. For example, Putnam famously placed meaning not only in one’s head, but in “the contribution of society and the contribution of the real world” that lies external to one’s head (Putnam 1973, p.711). In particular, the notorious ‘twin earth’ example aimed to illustrate how two people with exactly the same mental states can refer to different substances when they utter the word ‘water’, H_2O and XYZ. Without entering into the debate over Putnam’s example, we would like to point out that in a similar way, two fluid flow systems in physics may share all the same properties at every point, and indeed even share all the same boundary values, but still refer to different things due to the presence of different boundary conditions giving rise to that boundary behaviour.⁸ Boundary conditions serve to complete the model by restoring facts about degrees of freedom that are missing from the model, in much the same way that ‘the world’ is supposed to restore facts about the molecular structure in Putnam’s example.

3.3. Adequacy and the Nomological-Contingent Distinction. The Semantic Completion Account manifestly satisfies the adequacy conditions introduced in Section 3.1: it is by definition a restriction on the possible behaviour of a system, but not just any restriction. It must be motivated by facts that lie outside the descriptive resources of the given model, for example because they involve an external theory or alternative model. This account does not require that two objects be in physical contact with each other, or that ‘boundary’ be interpreted as the formal boundary of a manifold. Historically

⁷This boundary condition arose in the Prandtl modification of Euler’s equation discussed by Wilson (1990), Morrison (1999) and Bursten (2021), to include a viscous force that is negligible except near the boundary, allowing solutions in which velocity vanishes there.

⁸A concrete example of this for fluid dynamics will be given in Section 3.5.

and practically they often arise in this way, but we will see examples below in which they do not.

What of the nomological-contingent distinction? The Semantic Completion account allows boundary conditions to arise from both contingent and nomological facts, so long as those facts arise from outside the model's available degrees of freedom. For example, boundary conditions may be the result of nomological facts like the electromagnetic and mechanical laws underpinning the viscosity of water in a pipe, or of contingent facts like the temperature of an object's boundary. The distinction between a constraint's source arising from 'inside' or 'outside' a model's set of available degrees of freedom is thus best viewed as orthogonal to the nomological-contingent distinction.

Our account endorses certain aspects of both the IVP and Pluralist accounts, while proposing friendly amendments to other aspects of them. They are as follows.

3.4. Relationship to the IVP account. Boundary conditions in our sense are often, but not always, enough to make an IVP well-posed. In some such cases initial and boundary conditions may even play the same functional role in an explanation, *pace* [Wilson \(1990\)](#) and [Bursten \(2021\)](#). The difference is that on the Semantic Completion account, boundary conditions are defined by the presence of a coupling to degrees of freedom that are not in the model, which may or may not be enough to make an IVP well-posed.

This difference is particularly visible in the Coulomb gauge example introduced in Section 2.3. What distinguishes purely formal examples of boundary conditions in gauge physics from physically substantial ones? According to the Semantic Completion account, boundary conditions are only physically substantial if they encode how a system couples to an external one. In the Coulomb gauge, those boundary conditions are not arbitrary if one takes them to encode facts about how a local system couples to its environment. For example, boundary conditions can encode how a local subsystem 'smoothly connects' to a nearby subsystem, in the fact that the gauge fields for both must smoothly align at their boundary interface. A detailed general account of how gauge fields can encode subsystem coupling of this kind has been given by [Gomes \(2021, §3\)](#).⁹

⁹Similar views specifically about gauge as encoding coupling can be found in [Rovelli \(2014\)](#) and [Gomes et al. \(2021\)](#). This account also appears broadly aligned with the subsystem recursivity of [Wallace \(2022\)](#). A further interesting connection in the context of dualities is to the notions of 'extendible interpretations' discussed in [De Haro and Butterfield \(2025\)](#), see in particular pp. 80-81, and pp. 291-92.

The latter example may seem strange: how can the boundary of a spatially infinite region encode the behaviour of an external environment? The answer is, it is common physics practice to represent a finite region using an infinite amount of space. For example, [Penrose \(1982, p.632\)](#), [Gomes \(2024, p.23\)](#), [Gomes and Rovelli \(2024, p.14\)](#), and [Shi \(2025, p.2\)](#) emphasise that asymptotic flatness is a boundary condition that often encodes dynamical isolation from an environment: such isolation is unavoidable in relativity if one wishes to define a concept like ‘energy’ that depends on this. Even if a spacetime may be inextendible, and thus cannot represent any more spacetime points, asymptotic flatness is still a boundary condition that can encode its dynamical isolation from an environment:

“Asymptotically flat spacetimes are interesting, not because they are thought to be realistic models for the entire universe, but because they describe the gravitational fields of isolated systems, and because it is only with asymptotic flatness that general relativity begins to relate in a clear way to many of the important aspects of the rest of physics, such as energy, momentum, radiation, etc.” ([Penrose 1982, p.632](#))

More generally, as [Shi \(2025, p.2\)](#) has eloquently pointed out, in a relativistic spacetime “the boundary conditions contain the trace of ignored material sources.” To see one striking example of this that is relevant to the modern study of Hawking radiation, let us return to anti-de Sitter (AdS) spacetime.

A spacetime with a well-posed initial value problem is one that has a Cauchy surface, which determines all future and past facts on the basis of facts about an instant.¹⁰ AdS spacetime does not. However, there is a causal structure-preserving map (called a *conformal map*) from AdS into the Einstein Static Universe (Figure 3), and the latter does have a Cauchy surface.¹¹ So, in a purely mathematical sense, the data on this larger Cauchy surface are enough to ensure determinism. The initial conditions can be identified with the portion of the surface lying inside the original AdS spacetime, while boundary conditions are those that lie outside it. As far as the initial value problem is concerned, both play the same functional role. However,

¹⁰A *Cauchy surface* is a maximal spatial surface that is intersected by every inextendible timelike curve exactly once; a spacetime has a Cauchy surface if and only if it is globally hyperbolic ([Geroch 1970](#)). These are exactly the spacetimes for which general relativity has a well-posed initial value problem in the sense of [Choquet-Bruhat and Geroch \(1969\)](#).

¹¹This technique was proposed by [Penrose \(1964\)](#) and developed for AdS spacetime by [Avis et al. \(1978\)](#) and later [Ishibashi and Wald \(2004\)](#).

only boundary conditions play the role of capturing physical facts arising from strictly outside the original model, and in our view this is their defining feature.

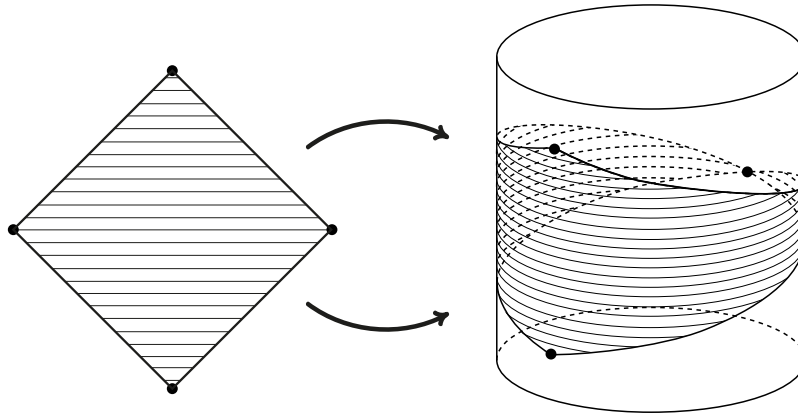


Figure 3: Conformal embedding of anti-de Sitter spacetime (left) into the Einstein static universe (right).

The exact physical significance of a boundary condition depends on what the model represents, and AdS spacetime can be applied in a variety of different ways. One of the simplest occurs when the Hawking radiation emitted by certain extremal black holes escapes to a region that is asymptotically AdS. As a result, as with space invaders, this radiation ‘reaches the boundary’ in finite time. Setting a boundary condition that reflects that radiation back towards the black hole, one arrives at a cavity-radiation model of a black hole in thermal equilibrium, thereby avoiding the well-known instabilities of ordinary asymptotically flat black holes.¹² In contrast, by setting a boundary condition in which radiation disappears in finite time without returning, one can naturally model AdS spacetime as an open system.

So, we endorse the use of boundary conditions in conjunction with initial value problems, in line with the IVP account. However, this does not make boundary conditions into *ad hoc* mathematical mechanisms for restoring determinism. The mathematical notion of a boundary value that secures a well-posed IVP does not by itself provide us with any physical reason to choose one boundary value over another. In contrast, our physical definition of a boundary condition does: one should adopt the boundary value that accurately describes how the system couples to external degrees of freedom, whether or not it is enough to make an IVP well-posed. We suspect this might already be the default perspective amongst physicists studying AdS spacetime,

¹²See [Gibbons \(2000\)](#) for a classic overview.

but if it is not then we think it should be:

“From the mathematical perspective, the boundary condition choice is arbitrary. We need physical input to fix it. Not always but quite often, this leads to a unique choice.” (Dias and Santos 2013, p.2)

3.5. Relationship to the Pluralist account. We agree with the insight of Wilson (1990) and Bursten (2021) that boundary conditions play a great variety of different explanatory roles. What the Semantic Completion account adds is a story about what unifies this great variety: they represent the variety of ways in which a model can couple to degrees of freedom not in its original description. Our friendly addendum is thus that the plurality of boundary conditions are not irreducibly distinct, but rather unified by their role in the semantic completion of an otherwise incomplete model.

This is particularly visible from the fact that the very same boundary *values* can sometimes arise from different boundary *conditions*. For example, in the viscous flow model discussed by Wilson (1990) and Bursten (2021), the no-slip boundary value is achieved by setting the velocity of the fluid at the boundary to be $v = 0$. This choice is generally motivated by appeal to the way the fluid couples to its container, which is at rest. However, a variety of different proposals were given during the historical development of modern fluid mechanics as to what that coupling actually is:

- In the solidified-layer model of Navier (1821, p.253), one assumes a thin layer of the fluid has “solidified” along the wall and couples mechanically to the moving fluid in a way that requires $v = 0$.
- In the rarefaction model of Maxwell (1878) for low-pressure fluids, the coupling is thermodynamical, in that the slip-velocity is taken to be inversely proportional to pressure. The slip-velocity thus approaches $v = 0$ in the large-pressure limit.
- In his viscosity model, Prandtl (1905, p.486) takes the coupling to involve a fluid that “sticks” (*hafte*) to the container walls in a way that generates viscosity, and thus slip velocity approaches $v = 0$ where viscosity becomes appreciable.

Despite the fact that all of these models arrive at the same boundary value $v = 0$ for the velocity of fluid flow at the edge of its container, they still arise from very different boundary *conditions*, in the sense of different proposals for how the fluid couples to degrees of freedom that are absent in the original fluid flow model. On

our account, each approach provides a different semantic completion of the original incomplete model.

4. Coupling as a Boundary Condition

4.1. Beyond boundaries in space. One of the most radical features of the Semantic Completion account is that it dramatically generalises the notion of a ‘boundary’, from its historical origin in spatial boundaries and endpoints to the more general boundaries of a model’s set of degrees of freedom. In this section we begin by clarifying just how general the Semantic Completion account of boundary conditions is. We will then argue that this level of generality is necessary to capture modern physical practice.

Most historically-motivated accounts of boundary conditions characterise them as boundaries of some region in space or solution space, such as Wilson:

“Boundary conditions, roughly characterized, represent claims about how a certain portion of the universe interacts with its surroundings along their mutual boundary” (Wilson 1990, p.566).

Similarly, here is Bursten:

“In physics, the term ‘boundary condition’ is used to denote a particular sort of component of some mathematical models. In particular, boundary conditions are specified sets of values that a differential equation must take at the boundary region of the problem’s solution space” (Bursten 2021, p.238).

These perspectives track common textbook presentations in continuum physics, and are compatible with boundary conditions for vibrating strings, fluid flow in a container, and even AdS spacetime, which all occur at the ‘edge’ of a finite or infinite spacetime region.

However, on the Semantic Completion account, being a boundary of space or solution space is neither necessary nor sufficient for being a boundary condition. It rather requires a coupling to degrees of freedom that do not appear in the original model, whether or not they are in a different region of space. This allows our account to capture the common practice of using boundary conditions to represent the coupling of an effective field to some omitted degrees of freedom.

A simple example of what we have in mind is the damped harmonic oscillator. When it is described as a one-particle system, omitting the many degrees of freedom of the environment, we represent the overall influence of the environment by introducing a damping term into the dynamics.¹³ Although this is not normally considered a boundary condition, we propose to view it as a boundary not of space but of scale. More generally, let us take an *effective field* to be any physical model that omits a large number of degrees of freedom from its target of representation. When the overall behaviour of the omitted degrees of freedom is encoded as an interaction term in the dynamics, then that interaction term is a boundary condition in our sense. But, instead of a boundary in space, it is a boundary of scale.

The damped harmonic oscillator is not usually the kind of system one associates with boundary conditions. However, we argue that there is a good reason to view it that way: the more familiar cases of boundary conditions for strings, fluids, and potentials all share the property of having their behaviour restricted by omitted degrees of freedom, which happen to lie on the boundary of a finite spatial region. However, for many systems of physical interest, such as AdS spacetime, there may be no finite spatial boundary at all. Thus, the appropriately general way to characterise the practice is by dropping the spatial boundary component, which results in the Semantic Completion account. Note in particular that we are *not* saying that every interaction term in an equation of motion must be a kind of boundary condition. Some boundary conditions are indeed encoded in interaction terms, such as the viscosity term in the Navier-Stokes model discussed above, but it is the fact that they help to capture the influence of omitted degrees of freedom that makes them boundary conditions.

To illustrate, we will consider two examples of effective field theories in quantum mechanics that first omit degrees of freedom, and then reintroduce them into the dynamics as boundary conditions: the Gross-Pitaevskii equation for Bose-Einstein condensates, and the three-particle Efimov effect. Looking at these examples in detail helps to illustrate how this practice really does consist in setting boundary conditions, in the same tradition begun by D'Alembert, Euler, Lagrange, and others.

4.2. Boundary conditions for Bose-Einstein condensates. Our first example is the macroscopic description of a Bose-Einstein condensate, through what is known as

¹³See [Roberts \(2022, §7.2\)](#) for a discussion of this example in the context of incomplete representations and irreversibility, and [Ladyman and Thebault \(2026\)](#) and [Thébault \(2026\)](#) in the context of autonomy and the modelling of open systems via contact Hamiltonian mechanics. The latter account builds upon important work on the philosophy of open quantum systems by [Cuffaro and Hartmann \(2024\)](#).

the Gross–Pitaevskii equation. Bose-Einstein condensates (BECs) are novel phases of matter arising when the same state is occupied by many particles.¹⁴ These many-particle states can be described using a bosonic Fock space description of quantum field theory, typically with 10^4 to 10^7 particles, giving rise to quantum effects that are essentially macroscopic. This includes novel quantum behaviour that is observable at the scale of ordinary objects, like the strange frictionless flow of superfluids and superconductors.

In practice, scientists often model BECs using an effective model called the Gross–Pitaevskii equation, which describes the collective behaviour of a gas using an effective field instead of the full quantum field theory of its underlying parts. To help appreciate the ingenious use of boundary conditions that arise in this model, we will briefly sketch how the Gross-Pitaevskii equation arises.

The underlying quantum field is given by a standard Fock space representation of creation and annihilation operators for bosonic particles in quantum field theory. However, when a very large portion of those particles occupy the same state, the average behaviour of the system as a whole can be accurately approximated by a single field $\psi(x)$ called the *condensate order parameter* or *effective field*.¹⁵ It has the formal appearance of a wavefunction, and indeed exists in a space of square-integrable functions just like an ordinary quantum wavefunction. However, unlike in ordinary quantum mechanics, $|\psi(x)|^2$ does not represent the probability of an occurrence, but rather the density of particles occupying a given state. This mean-field style modelling technique was pioneered by [Bogoliubov \(1947\)](#) and is now called the *Bogoliubov approximation*. The Gross-Pitaevskii equation is just the evolution equation that one derives for ψ when applying this approximation to a dilute gas of trapped bosonic particles.¹⁶ Formally,

¹⁴(cf. [Blank et al. 2008](#), pp. 419-420). Multi-particle occupation is not possible for fermions, which are subject to Pauli’s principle. However, BECs can still be realised using neutral atoms with integral spin that obey Bose statistics. First studied theoretically by [Bose \(1924\)](#) and [Einstein \(1925\)](#), BECs were demonstrated experimentally by two groups in 1995 using a now widely-repeated experimental protocol where a magneto-optical trap, together with laser and evaporative cooling, are used to contain and cool a small sample of neutral atoms ([Anderson et al. 1995](#); [Davis et al. 1995](#); [Pethick and Smith 2002](#)).

¹⁵Formally, one begins with a symmetric Fock space $\mathcal{F}_s(\mathcal{H})$ for a one-particle Hilbert space \mathcal{H} , with creation and annihilation operators $a^*(x)$ and $a(x)$ together with some Hamiltonian. Writing the one-body density matrix as $\gamma(x, y) := \langle \Phi, a^*(x)a(y)\Phi \rangle$, we say that a Bose–Einstein condensate occurs when γ is dominated by a single function ψ , called the *condensate order parameter* or *effective field*, in that $\gamma(x, y) \approx \psi^*(x)\psi(y)$, and with particle density given by $\rho(x) = \langle \Phi, a^*(x)a(x)\Phi \rangle \approx |\psi(x)|^2$ ([Pethick and Smith 2002](#); [Dalfovo et al. 1999](#)).

¹⁶Up to a choice of units the Gross-Pitaevskii equation is $i(d/dt)\psi(t) = (-\nabla^2 + V + a_s|\psi(t)|^2)\psi(t)$. It is a non-linear Schrödinger equation with an external potential V modelling the magneto-optical trap, together with a non-linear potential $a_s|\psi(t)|^2$, which uses the scattering length a_s to model the short-range low-energy scattering of the particles in place of a detailed model of their interaction (cf. [Pethick and Smith 2002](#), §5.2.1 and §6.1).

it is just the Schrödinger equation with two interaction terms: an external potential V describing how the magneto-optical trap contains the gas, and a non-linear interaction term $a_s|\psi(t)|^2$ that arises from the short-distance scattering behaviour, where a_s is a constant called the *scattering length*. The resulting model is usually interpreted as an effective or ‘classical’ field equation, because it replaces the dynamics of a multiparticle system with the dynamics of its statistical average.

The Gross-Pitaevskii equation is of special interest to us because it requires one to expand the notion of boundaries beyond boundaries in space, to include the coupling of an effective field to some omitted degrees of freedom. In particular, on the Semantic Completion Account the non-linear interaction term satisfies the dual roles of a boundary condition:

1. the effective field ψ omits degrees of freedom; and
2. the overall effects of those omitted degrees of freedom are re-introduced by the non-linear interaction term $a_s|\psi(t)|^2$.

The state space of the effective field ψ models the average behaviour of the gas as a whole, and so it excludes the degrees of freedom for describing multi-particle behaviour like scattering. Thus, those particle degrees of freedom are external to the effective field model. However, the external particle interactions still constrain the effective field’s behaviour: the cumulative effect of the particle scattering is an effective macroscopic pressure that influences the average behaviour of the gas.¹⁷ This pressure appears in the model through the non-linear interaction term $a_s|\psi(t)|^2$, which constrains the behaviour of ψ through the Gross-Pitaevskii equation.

The result is a prime example of a boundary condition in the sense of semantic completion: the behaviour of the model is restricted on the basis of physical facts about how that model couples to external degrees of freedom. Remarkably, this boundary condition does not refer to a boundary between two systems in space. Instead, it is a boundary between the macroscopic degrees of freedom that the model represents and the microscopic degrees of freedom that it does not.

This observation may shed light on a recent debate in the philosophy of science. In the case of superconductivity, a special case of BECs, there is a similar macroscopic

¹⁷This ‘pressure’ is more than an analogy: writing $\psi = \sqrt{\rho}e^{iS}$, Madelung (1927) famously pointed out that the pure real part of Schrödinger evolution (after taking the gradient) is the Euler equation $m(\partial_t v + (v \cdot \nabla)v) = -\nabla V - \nabla Q$, with a ‘quantum’ potential $Q := -\nabla^2 \sqrt{\rho}/2m\sqrt{\rho}$ in place of the ordinary pressure term. Remarkably, for the Gross-Pitaevskii Hamiltonian, the non-linear term $a_s|\psi(t)|^2$ becomes an ordinary pressure term in this fluid dynamical analogy, so that one derives *exactly* the Euler equation for a compressible fluid (see Pethick and Smith 2002, p.169).

wave equation that gives rise to the London equations, which have been the subject of much debate amongst philosophers.¹⁸ On the one hand, Cartwright and her collaborators refer to these models as ‘phenomenological’ arguing that they enjoy a certain amount of independence from the underlying theory:

“What is needed is the recognition of the independence from theory, in methods and aims, of the scientific activity we have come to call phenomenological model building. Anything less than that will fail to make sense of the justification offered by the London brothers for their proposal of a new model for superconductivity” (Cartwright et al. 1995, p.148).

French and Ladyman (1997) resist this independence claim, arguing that there is structural correspondence between the effective model and the underlying theory:

“Once attention is paid to these correspondences it can be seen that the construction of the London-London model did not proceed ‘phenomenologically’, in the sense of being independent of theory” (French and Ladyman 1997, p.382).

We believe the Semantic Completion account provides an intermediate view. Namely, the degrees of freedom of the underlying quantum field theory really are external to the effective field model. However, the effective model is not independent of the underlying theory—not even in methods and aims—because it encodes the influence of those degrees of freedom in the form of boundary conditions.

It is not difficult to see that the structure of this example is ripe for generalisation. Indeed, arguably all effective constants that we find in the modelling of many-body systems via classical or quantum collective state dynamics will be boundary conditions in our sense. Examples include the effective mass term used in the modelling of electron dynamics in crystalline solids, the dielectric constant used in macroscopic electrodynamics, and any non-fundamental equation-of-state or transport parameter, such as viscosity or conductivity, used in collective macroscopic models of systems with microscopic degrees of freedom.

4.3. Boundary conditions for the Efimov effect. Our second example is the Efimov effect, which describes an emergent long-range interaction between three non-relativistic

¹⁸See Cartwright et al. (1995); French and Ladyman (1997); Suárez (1999); Cartwright (1999); Howard (2007); Suárez and Cartwright (2008); Bueno et al. (2012).

particles that was discovered by [Efimov \(1970\)](#) and recently became a hot topic in experimental physics.¹⁹ As with BECs, it is an example of boundary conditions that arise not in space, but in the way an effective model interacts with degrees of freedom that lie outside that model’s scope. But, whereas BECs involve the coupling of a macroscopic model to microscopic degrees of freedom through an effective interaction term in the model, in the Efimov model boundary conditions encode the effects of short-range physics via a parameterised family of conditions on the wavefunction.

The simplest case of the Efimov effect arises for the wavefunction of a triple of identical bosonic particles. Some everyday boundary conditions encode the fact that each *pair* of particles is attractive when close, but these are not the ones of interest to us. In sketch, under these circumstances one can define a wavefunction that is ‘effective’ in the sense of only depending on just two real parameters, the radius and angle of the three-particle trio, without any of the other details of their individual degrees of freedom. This effective wavefunction obeys an interaction equation,²⁰ which unexpectedly includes a long-range $1/R^2$ force of attraction. That long-range force is called the *Efimov effect*. Since it is symmetric and destroyed if any one of the three particles are removed, it is sometimes described as a quantum realisation of Borromean rings (Figure 4). Remarkably, the effective $1/R^2$ interaction term has no characteristic length scale, which after setting the short-distance boundary condition leads to the discrete scale-invariance or *universality* of the effect.²¹

The boundary condition that we are interested in here is what makes the energy observable for this effective interaction well-defined. The energy derived in the Efimov effect is not of the standard form that is usually required of observables in quantum

¹⁹[Efimov \(2009, p.534\)](#) described the evolution of his effect from “questionable to pathological to exotic to being a hot topic of today’s ultracold physics” where they now have a wide variety of experimental realisations ([Gopalakrishnan 2006](#); [Kraemer et al. 2006](#); [Naidon and Endo 2017](#)).

²⁰Consider a three-particle wavefunction space $L^2(\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3)$ and a translation and particle permutation invariant Hamiltonian that satisfies the Bethe-Peierls boundary condition; thus, the scattering length is large compared to the characteristic range over which any two bosons interact. A particular energy eigenstate ψ can be selected and shown to imply an ‘effective energy’ eigenvalue equation,

$$-\frac{1}{2m_0} \left(\frac{d^2}{dR^2} + \frac{1}{R} \frac{d}{dR} - \frac{s^2}{R^2} \right) f(R) = \lambda f(R), \quad (1)$$

where R is the radius of the three-particle configuration and $f(R)$ is a wavefunction on $L^2(0, \infty)$. Moreover, $s^2 < 0$, so the ‘effective force’ s^2/R^2 is attractive, thus arising for the three-body description but not for any of the individual pairs. See [Efimov \(1973\)](#) or [Naidon and Endo \(2017\)](#) for details.

²¹See [Naidon and Endo \(2017\)](#). This is universality in the strong sense of invariance under type-level variation ([Gryb et al. 2021](#); [Palacios 2022](#)) since the quantum states of the system are fully characterised by the $1/R^2$ model, independently of whether the effect involves nucleons, atoms, or other resonantly interacting particles. Even more remarkably, the structure of the model can be applied in the context of unimodular type approaches to quantum cosmology ([Gryb and Thébault 2019](#)).

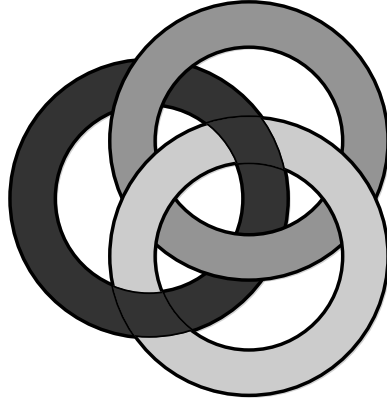


Figure 4: Borromean rings: three rings linked such that removing any one unlinks the remaining two.

mechanics, in that it is not a self-adjoint operator.²² However, the effective Hamiltonian always admits self-adjoint *extensions*.²³ Each such extension encodes a statement about the behaviour of the wavefunction at $R = 0$, when all three particles are very close together. The degrees of freedom required to exactly characterise that short-range interaction are not included in the effective description, which only has radius and angle degrees of freedom. Nevertheless, their *effect* is still encoded in the choice of a self-adjoint operator to extend the Efimov energy observable. Each self-adjoint extension is thus a boundary condition on our account, in that it arises from facts about an external coupling.

Similar boundary conditions are used to describe a wide variety of couplings to external degrees of freedom in physics, including the coupling discussed above of AdS spacetime to an external system, as it is carried out by [Ishibashi and Wald \(2004\)](#). They also appear in various approaches to renormalisation in quantum field theory, where boundary conditions are encoded within dimensionful parameters, restricting the behaviour of a model on the basis of physical couplings that have their origins in physics at shorter or larger scales than can be represented using the degrees of freedom of a given model.²⁴ In all such cases, the model does not include the degrees of freedom necessary to characterise some interaction, and so it is characterised through

²²Though see [Roberts \(2018\)](#) for a sense in which the usual requirement can be relaxed.

²³This was essentially shown in an early work by [Minlos and Faddeev \(1961\)](#); see also [Michelangeli \(2021\)](#). An introduction to Naimark's theory of self-adjoint extensions can be found in [Blank et al. \(2008, §4.7\)](#) or [Reed and Simon \(1980\)](#).

²⁴Formally, this is unsurprising since the $1/R^2$ model is the example of a singular potential and the extension procedure can be shown to be asymptotically equivalent to a class of renormalisation based approaches to the same problem ([Gopalakrishnan 2006](#)). Thus, we would encourage resisting the suggestion that self-adjoint extension parameters are merely technical artefacts of unphysical idealisation or, in line with the IVP view, merely formal tools used to restore determinism (cf. [Earman 2009](#)).

a self-adjoint extension instead.

5. Prospectus

The examples we have discussed only scratch the surface of the rich variety of physical problems that boundary conditions are used to solve. Much remains to be done to produce a more complete account. We also suggested in the introduction that focusing on the role of boundary conditions, understood in terms of the Semantic Completion Account, provides a new and insightful perspective on an array of debates in the philosophy of science such as inter-scale modelling, theory change, scientific representation, and the unity of science. We have seen two examples already, in the debate about the nature of boundary conditions and the debate about reduction and superconductivity. In this final section we will offer some further commentary to justify our bold introductory claim. We start with the most straightforward case, given the discussion of the previous section: inter-scale modelling.

Evidently, if boundary conditions include constants or parameters that encode short-distance behaviour of a physical system which we can effectively model in some low-energy limit then there is a clear sense in which one part of their functional roles is to mediate relationships between microscopic and macroscopic scales by allowing a macro scale theory to encode relevant physical facts from degrees of freedom at the micro scale without requiring the representation of these degrees of freedom. Scientific modelling often, however, requires us to model the interface between two scales using mediating inter-scale models. Such ‘models in the middle’ have been discussed not only by Cartwright and her collaborators, but also by [Batterman \(2000\)](#) in the context of hydrodynamics and [Winsberg \(2001\)](#) in the context of computer simulation of crack dynamics in condensed matter systems. In such contexts, the Semantic Completion Account implies that the intermediate modelling framework will include boundary conditions encoding physical facts from *both* the shorter and longer scales. We suggest that the account should prove particularly fruitful for the analysis of scientific practice in such cases.

This brings us to the second, and seemingly more surprising, topic on which the Semantic Completion Account may offer new insights: theory change. In an oddly neglected passage of a highly influential article, [Post \(1971, p. 236\)](#) noted that under theory change, a law in the old theory may turn out to be a consequence of a boundary condition in the new. In other words: if a new theory does not have the expressive

capacity to capture the successful models of the old laws, then the old laws may sometimes be captured using boundary conditions. We can thus, in some cases, find boundary conditions as encoding physical facts from old theories. Obvious examples include the way the flat curvature of Minkowski spacetime in special relativity provides a boundary condition for asymptotically flat spacetimes in general relativity, and the way Newton's law of gravity provides a boundary condition for the Schwarzschild solution in the weak gravitational limit. What is even more startling, and unanticipated by Post, is that we might also think of boundary conditions as encoding facts from *future* theories. Indeed, this is one way of thinking about the role of the Bekenstein entropy in black hole thermodynamics: as encoding constraints that a future theory of quantum gravity will ultimately explain.

Ultimately, we hope the reader will be satisfied that our aim in proposing the semantic completion account of boundary conditions should not be understood in terms of a dialectical push-back on the existing, highly valuable, work on boundary conditions within the philosophical literature. Rather our goal is to push-forward; not only can the semantic completion account allow us to preserve the virtues of IVP and pluralist accounts, but, in adopting the view from the boundary that we recommend, various new constructive approaches to reconciling old disagreements are made visible.

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