

# Perspectivist Account of Truth-Theoretic Semantics in Quantum Mechanics

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## Abstract

According to various no-go results in the foundations of quantum mechanics, for any system associated to a Hilbert space of dimension higher than two, it is not possible to assign definite truth values to all propositions pertaining to the system without generating a Kochen-Specker contradiction. In this respect, the Bub-Clifton uniqueness theorem is utilized for arguing that truth-value definiteness is consistently restored with respect to a determinate sublattice of propositions defined by the state of the quantum system concerned and a particular observable to be measured. On this basis, a perspectivist/contextual account of truth valuation in the quantum domain is produced that satisfies Tarski's criterion of material adequacy for a theory of truth. In light of the latter, perspectivist truth conforms to perspective or context-bound correspondence of a *de re* nature, designating locally an objectively existing state of affairs. Such an account derives by virtue of the microphysical nature of physical reality in displaying a context-dependence of facts; thus, it essentially opposes a non-perspectival, metaphysically fixed point of reference, or a panoptical standpoint from which to state all facts of nature.

**Keywords** Quantum Mechanics · Perspectivism · Notion of Perspective · Contextuality · Bub-Clifton Uniqueness Theorem · Quantum Semantics · Perspectivist/Contextual Account of Truth

## 1 What is a Perspective on Physical Reality?

Contemporary scientific perspectivism has emerged in the twenty-first century in the context of general philosophy of science as an appealing position in overcoming traditional metaphysical dichotomies. Thus, it occupies in the philosophical spectrum a middle ground between the extremes of the context-free universals of metaphysical realism and the inherent relativism entailed by constructivist accounts of science. The prime tenet of scientific perspectivism derives from the fact that any knowledge and comprehension of something, either through a process of perception and classification or of normative structuring or purposeful acting, necessitates the endorsement of a conceptual scheme and the selection of specific perspectives. In this respect, there is no access to an absolute “God’s eye view” of the world cut off from perspective.

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Currently, scientific perspectivism is often subsumed under the thesis that knowledge of nature is possible only within the boundaries of historically well-defined scientific perspectives composed of data analysis, theoretical models and principles relative to the perspective adopted, so that one may refer, for instance, to the Newtonian perspective, the Maxwellian perspective, and so on (e.g., Giere 2006). Alternatively, in a weaker sense, scientific perspectivism is viewed as a means for assessing or evaluating, at the same time, rival modelling practices or incompatible research programmes which may give rise to perspectival knowledge (e.g., Rueger 2016). In both cases—characterising, correspondingly, the diachronic and synchronic version of perspectivism—the usual sense attributed to the notion of a “perspective” refers to “the actual—historically and culturally situated—scientific practice of a real scientific community at a given historical time” (Massimi 2022, 5). Thus, the notion of a perspective is broadly taken to include, firstly, the body of knowledge claims advanced by the scientific community at a given time, secondly, the community’s resources (theoretical, experimental, technological) available at the time for reliably generating such claims, and, thirdly, second-order assertions concerning the justification of the knowledge claims so advanced (Massimi 2022, 6). Admittedly, defined that way, a perspective echoes Kuhn’s historicism, specifically, his concept of a “disciplinary matrix” (see also Giere 2013; Chang 2022).

In view of the fact that the customary position in the general philosophy of science strictly associates a scientific perspective to a given epistemic community, we introduce alternatively a concrete spatiotemporal concept of “perspective” that aims to serve as the basis of scientific inquiry in the natural sciences. The proposed notion of “perspective” is theorised, in relation to its ordinary meaning, and is conceived as the primary vehicle of tracing and investigating the world, as the principal structural unit of probing the natural world. On this account, a perspective is characterised *endo-theoretically*, namely, within a specific discipline, by a set of variables that are used to describe systems or to partition objects into parts, which together give a systematic account of a domain of phenomena. A perspective, therefore, in its functionality to act as a probe of an object, should be structurally adaptable to the species of the object of inquiry. The condition of structural adaptability of a potential perspective is satisfied if and only if it encodes a structurally invariant context of resolution, grouping together all information that can be delineated from the investigated object in terms of the probe’s resolving power (Karakostas & Zafiris 2022). It is worth noting that the proposed conception of perspective is of an interactive nature and, therefore, should also be differentiated from the common pictorial understanding of a perspective as a visual metaphor involving a viewing projection, which, by itself, depicts the process of knowing as a passive activity.

The indicated conception of “perspective”, broadly defined at this stage, is rigorously formulated in the following section in the case of Hilbert space quantum mechanics, satisfying the desiderata put forward. Such a kind of perspective, functioning as a probe of the natural world, provides observational accessibility on the investigated system, resolves a targeted object of inquiry, acts locally as a probing frame for the individuation of events, and thus it serves as a “window” on physical reality in the most direct conceivable sense. The significance of this point is particularly pertinent to quantum theory because of the existence of incompatible physical quantities, represented by corresponding non-commuting self-adjoint operators, pertaining to any non-trivial quantum system. Experimental or measurement contexts of such quantities cannot be held simultaneously in quantum mechanics. Thus, each mode of observation of incompatible quantum mechanical

quantities—for instance, the quantities of position and momentum in a given direction or the spin components in different spatial directions—gives rise in an ineliminable way to a particular kind of representation, encoding, or description of the system. Consequently, it is legitimate to say that acquiring experimental/empirical knowledge of the quantum world is essentially of a perspectival nature in the sense that claims about what is observed cannot be detached, in all circumstances, from the context of observation.

It should be pointed out that the aim of the paper is not to provide a comparative assessment among perspectival aspects that have been associated with various diverse interpretations of the quantum formalism. The latter constitutes a distinct project that falls entirely outside the scope of the present paper. As stated in the abstract, the paper’s exclusive aspiration is the investigation of quantum mechanical semantics in view of the Bub-Clifton uniqueness theorem. It is explicitly shown towards this direction that the Bub-Clifton result is a ‘go’ theorem allowing the specification of the maximal sublattices of propositions that can be taken as simultaneously determinate in the global non-Boolean structure of Hilbert space quantum mechanics, thus giving rise in a consistent manner to a perspectivist/contextual account of truth valuation that is in harmony with the proposed endo-theoretic perspectivist approach. Based on fundamental structural features of quantum theory, most notably the endemic feature of quantum contextuality, we provide in Section 2 succinct argumentation in revealing the theory’s affinity to perspectivist reasoning.

## 2 The Perspectivist/Contextual Character of Quantum Mechanics

In the standard Hilbert space formulation of quantum theory, each physical system is associated with a separable complex Hilbert space  $\mathcal{H}$ , the unit vectors of which correspond to possible states of the system. Every physical quantity or observable  $A$  pertaining to the system is represented by a self-adjoint operator  $\hat{A}$  acting on  $\mathcal{H}$ , the spectrum of which is the set of possible values of  $A$ . In marked contrast to classical mechanics, quantum mechanical observables are, in general, non-commuting under the pointwise multiplication of operators. From a physical point of view, the non-commutativity condition means that incompatible operators are not co-measurable and thus their corresponding potential eigenvalues cannot qualify as measurement values prior to and independently of the actual measurement process.

Within this framework, quantum events or elementary propositions—that is, true/false questions concerning values of observables pertaining to a quantum system—are represented by orthogonal projection operators  $\{\hat{P}_i\}$  on the system’s Hilbert space  $\mathcal{H}$  or, equivalently, by the closed linear subspace  $\mathcal{H}_{\hat{P}_i}$  of  $\mathcal{H}$  upon which the projection operator  $\hat{P}_i$  projects. The isomorphism (one-to-one mapping) between the set of all closed linear subspaces of  $\mathcal{H}$  and the set of all projection operators, denoted by  $\mathcal{L}_{\mathcal{H}}$ , allows us to translate the lattice structure of the subspaces of Hilbert space into the algebra of projections with the appropriate lattice theoretic characterizations. Then, a quantum algebra of events is identified with the algebraic structure of all projection operators on Hilbert space, ordered by inclusion and carrying an orthocomplementation operation, thus forming a complete, atomic, orthomodular lattice. In effect, a non-classical, non-distributive, and therefore non-Boolean logical structure is induced which has its origin in quantum theory.

Specifically, a quantum event algebra  $L$  is defined as an orthomodular  $\sigma$ -orthoposet (Dalla Chiara et al. 2004), that is, as a partially ordered set of quantum events  $l \in L$ , endowed with a maximal element 1 and with an operation of orthocomplementation  $[-]^* : L \rightarrow L$ , which satisfy, for all  $l \in L$ , the following conditions: [a]  $l \leq 1$ , [b]  $l^{**} = l$ , [c]  $l \vee l^* = 1$ , [d]  $l \leq l' \Rightarrow l'^* \leq l^*$ , [e]  $l \perp l' \Rightarrow l \vee l' \in L$ , [f] for  $l, l' \in L$ ,  $l \leq l'$  implies that  $l$  and  $l'$  are compatible, where  $0 := 1^*$ ,  $l \perp l' := l \leq l'^*$ , and the operations of meet  $\wedge$  and join  $\vee$  represent, respectively, the intersection and linear span of closed subspaces. The  $\sigma$ -completeness condition, meaning that the join of countable families of pairwise orthogonal events exists, is required in order to have a well-defined theory of quantum observables over  $L$ .

It is important to note that any arbitrary pair of events  $l$  and  $l'$ , belonging to a quantum event algebra  $L$  are compatible, if the sublattice generated by  $\{l, l^*, l', l'^*\}$  is a Boolean algebra, namely, if it is a Boolean sublattice of  $L$ . This already indicates that Boolean event algebras are structurally adaptable to a quantum event algebra and, furthermore, may serve as the building blocks of quantum structures. As argued in the sequel, this is a pivotal feature characterizing the contextual/perspectivist nature of quantum theory.

The phenomenon of contextuality is most effectively revealed by Kochen-Specker's celebrated theorem and its recent ramifications (e.g., Abramsky & Brandenburger 2011; Karakostas & Zafiris 2017). According to the Kochen and Specker result, for any quantum system associated to a Hilbert space of dimension greater than two, there does not exist a two-valued homomorphism, or, equivalently, a truth-functional assignment  $h : \mathcal{L}_{\mathcal{H}} \rightarrow \{0, 1\}$  on the set of projection operators,  $\mathcal{L}_{\mathcal{H}}$ , interpretable as quantum mechanical propositions, preserving the lattice operations and the orthocomplement, even if these lattice operations are carried out among commuting elements only. The essence of the theorem, when interpreted semantically, asserts the impossibility of assigning definite truth values to all propositions pertaining to a physical system at any one time, for any of its quantum states, without generating a contradiction. In view of Gleason's (1957) fundamental theorem, Kochen-Specker's theorem reaffirms the fact that on a Hilbert space of dimension greater than two, no two-valued probability measure exists globally and thus no representation of probability measures as a convex sum of  $\{0, 1\}$ -valuations is possible.

From a physical point of view, the Kochen-Specker result shows that in a system represented by a Hilbert space of dimension  $d > 2$ , there exist projection operators  $\{\hat{P}_i\}$  such that it is not always possible to assign truth values 0 and 1 to all corresponding propositions pertaining to the system, so that the following conditions are fulfilled:

- (i) For any complete orthogonal  $i$ -tuple of projection operators,  $\{\hat{P}_i\}$ , the assignment satisfies  $\sum_i h(\hat{P}_i) = h(\hat{\mathbb{I}}) = 1$ , that is, one projection operator is mapped onto 1 ("true") and the remaining  $i - 1$  projection operators are mapped onto 0 ("false") (completeness of the basis condition).
- (ii) If a projection operator,  $\hat{P}_k$ , belongs to multiple complete orthogonal bases, then, it is consistently assigned the same value in all bases (non-contextuality condition).

The initial proof of Kochen and Specker establishes that no such assignment of  $\{0, 1\}$ -valuation is possible for a special case restricted to a finite sublattice of projection operators on a three-dimensional Hilbert space, associated to a spin-1 quantum system, in a way that preserves the non-contextuality condition. In their original proof, Kochen and Specker used a three-dimensional real Hilbert space and a set of 117 rays (rank-1 projectors) for which

the preceding two conditions are shown to be contradictory. Hence, the established contradiction entails the existence of a contradiction in higher dimensional complex Hilbert spaces. Recent developments have remarkably reduced the size and complexity of proofs of the Kochen-Specker theorem (e.g., Lisoněk et al. 2014) and transformed such proofs into experimentally testable inequalities, providing also loophole-free tests of quantum contextuality (Wang et al. 2022), thus establishing the contextual character of the theory as a *structural* feature of the quantum mechanical formalism itself. Furthermore, quantum contextuality has been established at the inception of the twenty-first century as a fundamental resource in quantum information processing, ranging from measurement-based quantum computation to communication complexity scenarios in an endeavor to significantly surpass classical computing (e.g., Howard et al. 2014).

A failure of the non-contextuality condition means that the value assigned to a quantum mechanical observable  $A$  depends on the context in which it is considered. An equivalent way of expressing the above is to say that the value of  $A$  depends on what other compatible observables are assigned values at the same time; that is, it depends on a choice that concerns operators that commute with  $\hat{A}$ . Consequently, the value of the observable  $A$  cannot be regarded as pre-fixed, as being independent of the measurement context actually chosen, as specified by the set of mutually compatible observables one may consider it with. This dependence captures the endemic feature of quantum contextuality. As opposed to the classical case, in quantum mechanics there is not a global context of measurement, a single privileged perspective, relative to which all conceivable observables of the system are co-measurable. One and the same quantum system does exhibit several possible contextual manifestations in the sense that it can be assigned several definite incommensurable properties with respect to distinct incompatible quantum observables corresponding to different aspects of reality which, in principle, cannot be considered simultaneously.

Henceforth, the notion of context giving rise to a perspective that is applied on a quantum system is formalized by a maximal set of mutually compatible observables, or, equivalently, by a complete Boolean algebra of commuting projection operators generated by this set. It should be underlined that such a complete Boolean algebra of projection operators represents the maximal information that is encodable into a quantum system at any given time by a state preparation procedure (e.g., Svozil 2009). It also bears the status of a local structural invariant characterizing a whole commutative algebra of observables that can be simultaneously spectrally resolved and hence be co-measurable. Note that mutually commuting operators decompose spectrally into identical pairwise orthogonal sets of eigenvectors, forming an orthonormal basis, which correspond to pairwise orthogonal projectors adding up to unity.

Since in the lattice of quantum events there exist incompatible physical quantities non-commuting with any considered commutative algebra of observables, there exists a multiplicity of possible Boolean algebras of orthogonal projections furnishing an invariant of this kind only at the local level of discourse. Therefore, although a quantum event structure is globally non-Boolean, it can be qualified spectrally, and hence be accessed experimentally, only in terms of Boolean event structures operating locally as structural invariants of co-measurable families of physical observables. Consequently, a complete Boolean algebra of projection operators in the lattice of quantum events picked by an observable to be measured instantiates locally a physical context, which serves as a Boolean probing frame or perspective relative to which results of measurement are being coordinatized. The requirement of such a consideration is also clearly expressed by Janas et al. (2022, 214): “The

kinematics of quantum mechanics presents us with a fundamentally non-Boolean probabilistic global structure. Upon this structure, in a given measurement context, we impose a particular Boolean frame. We do this to express our experience of the results of measurements conducted in that context—an experience of events that either do or do not occur, and that together fit into a consistent picture, in that measurement context, of the phenomenon being described”.

Historically, although Bohr did not refer to Boolean algebras or to the associated Boolean frames, it was nonetheless his primary insight that the unequivocal physical interpretation of events in the quantum domain necessitates the reference to an experimental arrangement under specified conditions, that is, it requires the consideration of a Boolean frame. In Bohr’s (1958, 392-393) words:

In the treatment of atomic problems, actual calculations are most conveniently carried out with the help of a Schrödinger state function, from which the statistical laws governing observations obtainable under specified conditions can be deduced by definite mathematical operations. It must be recognized, however, that [...] the unambiguous physical interpretation of [events] in the last resort requires a reference to a complete experimental arrangement [i.e., a Boolean frame]. Disregard of this point has sometimes led to confusion, and in particular the use of phrases like ‘disturbance of phenomena by observation’ or ‘creation of physical attributes of objects by measurements’ is hardly compatible with common language and practical definition.

Any empirical investigation of the quantum world requires the specification of a Boolean probing frame or perspective determining the kind of the interaction with the system under study. In quantum mechanics the relation between the global theoretical structure and its various empirical sub-structures is indeed such that, depending on the type of experimental context a quantum system is brought to interact, different manifested aspects of the system are disclosed, impossible to be combined into a single picture as in classical physics, although only one type of system is concerned. It has been recently shown, by applying category-theoretic reasoning to quantum mechanics, that the global structure of a quantum algebra of events can be represented in terms of structured interconnected families of Boolean probing frames, realized as locally variable perspectives on a quantum system, being capable of carrying jointly all the information encoded in the former (Karakostas & Zafiris 2022). Accordingly, the non-directly accessible quantum event structure is uniquely constituted (up to equivalence) by a multiplicity of intertwined local perspectives directed towards it and covering the system of inquiry entirely under their joint action.

### 3 Truth-Value Assignment in Quantum Mechanics

The logic of a physical theory reflects the structure of the elementary propositions referring to the ascription of properties to a physical system in the domain of the relevant theory. As indicated in Section 2, on the standard codification of quantum theory, the elementary propositions pertaining to a system form a non-Boolean lattice,  $\mathcal{L}_{\mathcal{H}}$ , isomorphic to the lattice of closed linear subspaces or corresponding projection operators of a Hilbert space. Thus, a proposition pertaining to a quantum system is represented by a projection operator  $\hat{P}$  on the system’s Hilbert space  $\mathcal{H}$ , or, equivalently, it is represented by the linear subspace  $\mathcal{H}_{\hat{P}}$  of  $\mathcal{H}$  upon which the projection operator  $\hat{P}$  projects. Since each projection operator

$\hat{P}$  on  $\mathcal{H}$  acquires two eigenvalues 1 and 0, where the value 1 can be read as “true” and 0 as “false”, the proposition asserting that “the value of the physical quantity  $A$  of system  $S$  lies in a certain range of values  $\Delta$ ”, or, equivalently, that “system  $S$  in state  $|\psi\rangle$  acquires the property  $P(A)$ ”, is said to be true if and only if the corresponding projection operator  $\hat{P}_A$  obtains the value 1, that is, if and only if  $\hat{P}_A|\psi\rangle = |\psi\rangle$ . Accordingly, the state  $|\psi\rangle$  of the system lies in the associated subspace  $\mathcal{H}_{\hat{P}_A}$  which is the range of the operator  $\hat{P}_A$ , i.e.,  $|\psi\rangle \in \mathcal{H}_{\hat{P}_A}$ . In such a circumstance, the property  $P(A)$  pertains to the quantum system  $S$ . Otherwise, if  $\hat{P}_A|\psi\rangle = 0$  and, hence,  $|\psi\rangle$  belongs to a subspace completely orthogonal to  $\mathcal{H}_{\hat{P}_A}$  ( $|\psi\rangle \in \perp\mathcal{H}_{\hat{P}_A}$ ), the counter property  $\neg P(A)$  pertains to  $S$ , and the proposition is said to be false.

Due to the essentially probabilistic structure of quantum theory, for a given quantum system, the totality of propositions represented by projection operators or Hilbert space subspaces are not partitioned into two mutually exclusive and collectively exhaustive sets representing either true or false propositions. As already pointed out, only propositions represented by subspaces that contain the system’s state are assigned the value “true” (propositions assigned probability 1 by  $|\psi\rangle$ ), and only propositions represented by spaces orthogonal to the state are assigned the value “false” (propositions assigned probability 0 by  $|\psi\rangle$ ) (Dirac 1958, 46-47; von Neumann 1955, 213-217). Hence, propositions represented by subspaces that are at some non-zero or non-orthogonal angle to the unit vector  $|\psi\rangle$ —or, more appropriately, to the ray representing the quantum state—are not assigned any truth value in  $|\psi\rangle$ . These propositions are neither true nor false; they are assigned by  $|\psi\rangle$  a probability value which is neither 0 nor 1, but between 0 and 1, and the law of bivalence is violated. Thus, they are undecidable or indeterminate for the system in state  $|\psi\rangle$  and the corresponding properties are taken as indefinite. In fact, such value indefiniteness resulting to semantic indeterminacy is a global property of quantum logical structures, since according to Kochen-Specker’s theorem, for any quantum system of dimension higher than two, there is simply no quantum state which maps the totality of quantum mechanical propositions into the two-valued Boolean algebra  $\{0, 1\}$  of truth values. Therefore, the following question naturally arises: what are the maximal determinate sublattices of  $\mathcal{L}_{\mathcal{H}}$  or the maximal structures of propositions in quantum mechanics that can be taken as simultaneously determinate, that is, as being assigned determinate (but perhaps) unknown truth values in an overall consistent manner?

## 4 Maximal Determinate Sublattices of Quantum Mechanical Propositions

In this respect, we employ the Bub-Clifton so-called “uniqueness theorem” (Bub 2009; Bub 1997; Bub & Clifton 1996). Consider a quantum mechanical system  $S$  represented by a ray or one-dimensional projection operator  $D = |\psi\rangle\langle\psi|$  spanned by the unit vector  $|\psi\rangle$  on an  $n$ -dimensional Hilbert space  $\mathcal{H}$ . Let  $A$  be an arbitrary observable of  $S$  with  $m \leq n$  distinct eigenspaces  $A_i$ , where the rays  $D_{A_i} = (D \vee A_i^\perp) \wedge A_i$ ,  $i = 1, \dots, k \leq m$ , denote the non-zero projections of the state  $D$  onto these eigenspaces. Then, according to the Bub-Clifton theorem, the unique maximal sublattice of the lattice of projection operators (or corresponding Hilbert space subspaces),  $\mathcal{L}_{\mathcal{H}}$ , representing the propositions that can be

determinately true or false of the system  $S$ , is given by,

$$\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\}) = \{P \in \mathcal{L}_{\mathcal{H}} : D_{A_i} \leq P \text{ or } D_{A_i} \leq P^\perp, \forall_i, i = 1, \dots, k\},$$

where the relation ‘ $\leq$ ’ indicates subspace inclusion and  $P^\perp$  denotes the subspace orthogonal to  $P$ .

The sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\}) \subset \mathcal{L}_{\mathcal{H}}$  is generated by: (i) the rays  $\{D_{A_i}\}$ , the non-zero projections of  $D$  onto the  $k$  eigenspaces of  $A$ , and (ii) all the rays in the subspace  $(D_{A_1} \vee D_{A_2} \vee \dots \vee D_{A_k})^\perp = (\vee D_{A_i})^\perp$  orthogonal to the subspace spanned by the  $\{D_{A_i}\}$ ,  $i = 1, \dots, k$ . Since the rays  $D_{A_i}$  are orthogonal, they are compatible and generate a Boolean sublattice of  $\mathcal{L}_{\mathcal{H}}$ . Thus,

$$(D_{A_1} \vee D_{A_2} \vee \dots \vee D_{A_k})^\perp = (D_{A_1})^\perp \wedge (D_{A_2})^\perp \wedge \dots \wedge (D_{A_k})^\perp.$$

In effect, the sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  is partitioned into  $k$ -orthogonal subspaces corresponding to a partition of the spectrum of observable  $A$  into  $k$  distinct eigenspaces. Thus,

$$\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\}) = \mathcal{L}_{\mathcal{H}}(D_{A_1}) \cap \mathcal{L}_{\mathcal{H}}(D_{A_2}) \cap \dots \cap \mathcal{L}_{\mathcal{H}}(D_{A_k}),$$

because each  $\mathcal{L}_{\mathcal{H}}(D_{A_i})$ ,  $i = 1, \dots, k$ , is generated by the ray  $D_{A_i}$  and all the rays in the subspaces  $(D_{A_i})^\perp$  orthogonal to  $D_{A_i}$ . The set of maximal (non-degenerate) observables associated with  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  includes any maximal observable with  $k$  eigenvectors along the directions  $D_{A_i}$ ,  $i = 1, \dots, k$ . The set of non-maximal observables includes any non-maximal observable that is a function of one of these maximal observables. Consequently, all the observables whose eigenspaces are spanned by rays in  $\mathcal{L}_{\mathcal{H}}(D_{A_i})$  are determinate, given the system’s state  $D$  and  $A$ .

The identification of such maximal determinate sets of observables provides a consistent assignment of truth values to the associated propositions in  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  of  $\mathcal{L}_{\mathcal{H}}$ , not to all propositions in  $\mathcal{L}_{\mathcal{H}}$ . The sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  represents the maximal subsets of propositions pertaining to a quantum system that can be assigned simultaneously determinate truth values, where a truth-value assignment is defined by a Boolean or two-valued homomorphism,  $h : \mathcal{L}_{\mathcal{H}}(\{D_{A_i}\}) \rightarrow \{0, 1\}$ . If the Hilbert space  $\mathcal{H}$  is of dimension  $d > 2$ , there are exactly  $k$  two-valued homomorphisms on  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$ , where the  $i$ th homomorphism assigns to the corresponding proposition  $D_{A_i}$  the value 1 (“true”) and the remaining propositions in  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$ ,  $i = 1, \dots, k$ , the value 0 (“false”). The determinate sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  is maximal, in the sense that, if an extra element is added to it, lattice closure generates the lattice  $\mathcal{L}_{\mathcal{H}}$  of all subspaces of  $\mathcal{H}$ , and no global two-valued homomorphism on  $\mathcal{L}_{\mathcal{H}}$  exists (Bub 2009).

In fact, the Bub-Clifton determinate sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  constitutes a modification of the ordinary textbook codification of quantum mechanics. On this standard position, an observable has a determinate value if and only if the state  $D$  of the system is an eigenstate of the observable. Equivalently, the propositions that are determinately true or false of a system are the propositions represented by subspaces that either include the ray denoting the state  $D$  of the system (and hence are assigned probability 1 by  $D$ ), or are orthogonal to  $D$  (and hence are assigned probability 0 by  $D$ ). Thus, the standard determinate sublattice can be formulated as:

$$\mathcal{L}_{\mathcal{H}}(D) = \{P \in \mathcal{L}_{\mathcal{H}} : D \leq P \text{ or } D \leq P^\perp\}.$$

It is simply generated by the state  $D$  and all the rays in the subspace orthogonal to  $D$ . If the Hilbert space  $\mathcal{H}$  is of dimension  $d > 2$ , there is one and only two-valued homomorphism on  $\mathcal{L}_{\mathcal{H}}(D)$ : the homomorphism induced by mapping the state  $D$  onto 1 and every other ray orthogonal to  $D$  onto 0. Apparently, the sublattice  $\mathcal{L}_{\mathcal{H}}(D)$  forms a subset of Bub-Clifton’s proposal  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$ . The latter will only agree with  $\mathcal{L}_{\mathcal{H}}(D)$  if  $D$  is an eigenstate of  $A$ , for then the set  $\{D_{A_i}\}$  consists of only  $D$  itself. In general, the sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  contains all the propositions in  $\mathcal{L}_{\mathcal{H}}(D)$  that it makes sense to talk about consistently with  $A$ -propositions, namely propositions that are strictly correlated to the spectral projections of a suitable observable  $A$ . From the viewpoint of the uniqueness theorem, the sublattice  $\mathcal{L}_{\mathcal{H}}(D)$  is obtained by taking  $A$  as the unit observable  $I$ . However, as Bub & Clifton (1996) rightly observe, there is nothing in the mathematical structure of Hilbert space quantum mechanics that necessitates the selection of the preferred determinate observable  $A$  as the unit observable  $I$ , whilst, in addition, this choice leads to von Neumann’s account of the quantum measurement process resulting in a sequential regress of observing observers.

Then, the following question arises: what specifies the choice of a particular preferred observable  $A$  as determinate if  $A \neq I$ ? The Bub-Clifton proposal allows, in effect, different choices for  $A$  corresponding to various different ‘no collapse’ interpretations of quantum mechanics, for example Bohm’s hidden variable theory, if the preferred observable  $A$  is fixed as position in configuration space, modal interpretations that exploit the bi-orthogonal decomposition theorem (e.g., Dieks 2022), or environmental decoherence approaches where such a preferred observable is “selected mainly by the system-environment interaction Hamiltonians” (Zurek 1993, 290).

In our view, if one wishes to stay within the framework of Hilbert space quantum mechanics and refrains from introducing additional structural elements, the most natural and immediate choice of a suitable preferred observable, especially, for confronting the problem of truth-value assignments, results in the determinateness of the observable to be measured. This is physically motivated by the fact that in the quantum domain one cannot assign, in a consistent manner, definite sharp values to all quantum mechanical observables pertaining to a microphysical system, independently of the measurement context actually specified. In terms of the structural component of quantum theory, this is due to functional relationship constraints—even among commuting self-adjoint operators only—that govern the algebra of quantum mechanical observables, as revealed by the Kochen-Specker theorem and the relevant analysis in Section 2. On this account, it is not possible, in principle, to assign to a quantum system definite non-contextual properties corresponding to all possible measurements. This means that it is not possible to assign consistently a definite unique truth value to every single elementary proposition, represented by a projection operator, independently of which subset of mutually commuting projection operators one may consider it to be a member.<sup>2</sup>

Then, let  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  be the sublattice of  $\mathcal{L}_{\mathcal{H}}$  consisting of all compatible observables with  $A$ , thus forming a complete Boolean algebra of commuting projection operators defined by the spectral decomposition of  $A$ , the projection operators representing  $A$ -propositions. One may wonder, therefore, what is the exact relationship between the determinate sublattices  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  and  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$ . The Bub-Clifton sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  differs from

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<sup>2</sup>In fact, any attempt of simultaneously ascribing context-independent, sharp values to all observables of a quantum system forces the quantum statistical distribution of value assignment into the pattern of a classical distribution, thus leading directly to contradictions of the Greenberger-Horne-Zeilinger type (Greenberger 2009).

$\mathcal{L}_{\mathcal{H}}(\{D/A\})$  only in including additional elements in  $(D_{A_1} \vee D_{A_2} \vee \dots \vee D_{A_k})^\perp$ , representing rays spanned by linear superpositions of vectors from different subspaces  $A_i$  and, thus, being incompatible with some of the spectral projectors  $\hat{A}_i$ .<sup>3</sup> Since these rays lie exclusively in the subspace  $(D_{A_1} \vee D_{A_2} \vee \dots \vee D_{A_k})^\perp$ , they are assigned probability zero by the state  $D$  and by every probability measure on  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$ . Hence, the non-Boolean sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  can be obtained from  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  by adding a maximal set of ‘null’ rays in this sense. By setting aside these impossible or, in the physicist’s language, ‘meaningless’ propositions associated with non-Boolean features, the sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  is reduced to  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$ . The latter is generated by lattice closure from the rays  $\{D_{A_i}\}$ ,  $i = 1, \dots, k$ , and all the rays in the  $k$  subspaces  $A_i - D_{A_i}$ , which are orthogonal to the state  $D$ . Equivalently, the sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  is generated from the projection operators (or propositions) in the spectral decomposition of the observable  $A$  and the elements compatible with  $A$  by lattice closure. It is for this reason that the informational content of  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  represents the maximal information encodable into a quantum system at any particular time  $t$  by a state preparation procedure, as indicated in Section 2, whereas the non-Boolean sublattice  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  cannot formally serve this purpose. Likewise,  $\mathcal{L}_{\mathcal{H}}(\{D_{A_i}\})$  falls short of encapsulating the concept of a valid experimental or measurement context, since the latter is mathematically formalized by a single maximal (non-degenerate) observable  $A$ , such that all commuting, co-measurable observables are functions thereof (e.g., Svozil 2009; Abbott et al. 2014; Karakostas & Zafiris 2022). Nonetheless, the determinate sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  may be viewed as a particular implementation of the uniqueness theorem in case that the set of definite-valued observables for a given quantum state  $D = |\psi\rangle\langle\psi|$  and a preferred non-degenerate observable  $A$  is restricted to a maximal set of commuting projection operators with values correlated to the values of  $A$  in  $D$ .

From the standpoint of the proposed perspectivist approach as analysed in Sections 1 and 2, it is precisely the selection of a particular observable  $A$  to be measured that provides empirical access to the non-Boolean quantum world through a Boolean  $A$ -perspective, defined by the determinate sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  of  $\mathcal{L}_{\mathcal{H}}$ . The selection of a particular observable to be measured necessitates also the selection of an appropriate experimental or measurement context with respect to which the measuring conditions remain intact. Formally, a measurement context  $\mathcal{C}_A(D)$  can be defined by a pair  $(D, A)$ , where, as previously,  $D = |\psi\rangle\langle\psi|$  is an idempotent projection operator denoting the general pure state of quantum system  $S$  and  $\hat{A} = \sum_i a_i \hat{P}_i$  is a self-adjoint operator denoting the measured observable. Of course,  $\mathcal{C}_A(D)$  is naturally extended to all commuting, compatible observables which, at least in principle, are co-measurable alongside of  $A$ . Then, in accordance with the Bub-Clifton theorem, given the state  $D$  of  $S$ ,  $D$  restricted to the set of all propositions concerning  $A$  is necessarily expressed as a weighted mixture  $D_A = \sum_{i=1}^k |c_i|^2 |a_i\rangle\langle a_i|$  of determinate truth-value assignments, where each  $|a_i\rangle$  is an eigenvector of  $A$  and  $|c_i| = |\langle\psi, a_i\rangle|$ ,  $i = 1, \dots, k$ . Since  $D_A$  is defined with respect to the selected context  $\mathcal{C}_A(D)$ ,  $D_A$  may be called a representative perspectivist or contextual state.

In justifying from a physical point of view the aforementioned term, it is worthy to note that the state  $D_A$ —which results as a catalogue of well-defined properties or equivalently determinate truth-value assignments selected by a two-valued homomorphism on  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$ —may naturally be viewed as constituting a state preparation of system  $S$  in the context of the preferred observable  $A$  to be measured. Thus, the state  $D_A$  should not

<sup>3</sup>For notational simplicity,  $A_i$  represents here the subspace  $\mathcal{H}_{\hat{A}_i}$  that corresponds to the projection operator  $\hat{A}_i$ ,  $i = 1, \dots, k$ .

be regarded as the final post-measurement state, reached after an  $A$ -measurement has been carried out on the system concerned. The contextual state represents here an alternative description of the general state  $D$  of  $S$  by taking specifically into account the selection of a particular observable, and hence of a suitable experimental context, on which the state of the system under measurement can be conditioned. In other words, it provides a redescription of the investigated system which is necessitated by taking specifically into account the context of the selected observable. For, it is important to realise that this kind of redescription is intimately related to the fact that both states  $D$  and  $D_A$  represent the same system  $S$ , albeit in different ways. Whereas  $D$  refers to a general initial state of  $S$  independently of the specification of any particular observable and, hence, regardless of the determination of any measurement context, the state  $D_A$  constitutes a conditionalization state preparation of  $S$  with respect to the observable to be measured, while dropping all ‘unrelated’ reference to observables that are incompatible with such a preparation procedure.<sup>4</sup> Consequently,  $D_A$  is a mixed state over a set of basis states that are eigenstates of the measured observable  $A$ , and it reproduces the probability distribution that  $D$  assigns to the values of  $A$ . Observe that, according to the Hilbert space formalism of quantum mechanics, for any observable  $A$ ,  $TrDA = TrD_AA$ . The latter equality designates the fact that the initial general state  $D$  does not have observational significance over the perspectivist or contextual state as long as predictions of measurement outcomes are restricted to the observables that are related to the selected context  $\mathcal{C}_A(D)$ . Thus, with respect to the representative contextual state  $D_A$  the following conditions are satisfied:

- (i) Each  $|a_i\rangle$  is an eigenvector of  $A$ . Thus, each quantum mechanical proposition  $D_{A_i} \equiv P_{|a_i\rangle} = |a_i\rangle\langle a_i|$ ,  $i = 1, \dots, k$ , assigns in relation to  $\mathcal{C}_A(D)$  a determinate value to  $A$ , the eigenvalue  $a_i$  satisfying  $\hat{A}|a_i\rangle = a_i|a_i\rangle$ .
- (ii) Any eigenvectors  $|a_i\rangle$ ,  $|a_j\rangle$ ,  $i \neq j$ , of  $A$  are orthogonal. Thus, the various possible propositions  $\{P_{|a_i\rangle}\}$ ,  $i = 1, \dots, k$ , are mutually exclusive. Consequently, the different orthogonal eigenstates  $\{|a_i\rangle\}$ ,  $i = 1, \dots, k$ , correspond to different values of the measured observable  $A$  or to distinct settings of the apparatus situated in the context  $\mathcal{C}_A(D)$ .
- (iii) Each  $|a_i\rangle$  is non-orthogonal to  $D = |\psi\rangle\langle\psi|$ . Thus, each proposition  $P_{|a_i\rangle}$  whose truth value is not predicted with certainty is possible with respect to  $\mathcal{C}_A(D)$ .

It is evident, therefore, that the perspectivist or contextual state  $D_A$  represents the set of all probabilities of events corresponding to quantum mechanical propositions  $\{P_{|a_i\rangle}\}$  that are associated with the measurement context  $\mathcal{C}_A(D)$ . With respect to the latter, the propositions  $\{P_{|a_i\rangle}\}$  correspond in a one-to-one manner with disjoint subsets of the spectrum of the observable  $A$  and hence generate a sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  of simultaneously determinate propositions, against a non-Boolean background of indefinite possibilities. Thus, the

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<sup>4</sup>In Dirac’s (1958, 11-12) words: “A state of a system may be defined as an undisturbed motion that is restricted by as many conditions or data as are theoretically possible without mutual interference or contradiction. In practice the conditions could be imposed by a suitable preparation of the system, consisting perhaps in passing it through various kinds of sorting apparatus, such as slits and polarimeters, the system being left undisturbed after the preparation”. Hence, the consideration of the state  $D_A$  falls entirely within Dirac’s prescription. It defines the maximal information that may be encodable into a quantum system by state preparation and being in principle extractable by experimental means. In particular, it specifies what is actually the case of system  $S$  at time  $t$  with respect to a maximal set of co-preparable (or co-measurable) observables with  $A$ .

$\{P_{|a_i}\}$  propositions are assigned determinate truth values, in the standard Kolmogorov sense, by the state  $D_A$ .

It is instructive to note at this point that producing a preparatory Boolean environment  $\mathcal{C}_A(D)$  for a system  $S$  in state  $D$  to interact with a measuring arrangement does not determine which event will take place, but it does determine the *kind* of event that will take place. It forces the outcome, whatever it is, to belong to a certain definite Boolean sublattice of events  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  of  $\mathcal{L}_{\mathcal{H}}$  for which the standard measurement conditions remain constant. Such a set of standard conditions for a definite kind of measurement constitutes a set of necessary and sufficient constraints for the occurrence of an event of the selected kind. As already explicitly stated in Section 2, we view each preparatory Boolean environment of measurement as a context that offers a perspective on a quantum system. A perspective or context is defined by a set of compatible physical observables or, more precisely, by a complete Boolean algebra of commuting projection operators generated by such a set. Physical observables in any such algebra can be given consistently definite values. Equivalently, the determinateness of observables allows for as many bivalent truth-value assignments to corresponding propositions. In this respect, each context functions locally as a Boolean probing frame for the individuation of events, thus providing a ‘window on reality’ (Zafiris & Karakostas 2013; Karakostas & Zafiris 2022). It is probably one of the deepest insights of modern quantum theory that whereas the totality of all experimental/empirical events can only be represented in a globally non-Boolean structure, the acquisition of every single event depends on a locally Boolean context.

## 5 Perspectivist/Contextual Account of Truth

The logic of a physical theory, in addition to its syntactic aspect—dealing with the logico-mathematical operations and relations among the elementary propositions pertaining to a system (Sections 3 and 4)—incorporates the task of providing a semantics, focusing particularly on the endeavor of establishing a criterion of truth or, in a strict sense, defining an account of truth. In relation to the latter, the correspondence theory of truth has frequently been regarded within physical science as the most eminent. Indeed, elementary propositions (also called experimental propositions) of the logical structures of fundamental theories of physics concern the values of physical quantities pertaining to a system and refer to the world in the most direct feasible manner. They can only be ascertained as true or false by corresponding, or not corresponding, to “facts” or, more generally, to actual “states of affairs” or, in contemporary physical terms, to “events” conceived in the physicist’s wording as genuine measurement outcomes, as concrete elements of empirical reality. In this respect, elementary propositions are posits in the process of probing the natural world.

Although the correspondence theory of truth admits various different formulations, the core of any correspondence theory is the idea that a proposition is true if and only if it corresponds to or matches reality (e.g., Burgess et al. 2011). A noticeable criticism exerted to the traditional conception of correspondence truth is that the correspondence relation between propositional terms and corresponding facts (or states of affairs) is neither specified nor explained; it is simply presupposed. For, ‘corresponds to the facts’ functions merely as a synonym, as an alternative extended way of saying ‘is true’ (e.g., Lewis 2001; Engel 2002). Using a popular example towards this direction, the proposition “snow is white” is true, simply because it corresponds to the fact that snow is white. Thus, the

fact that makes a proposition true is a restatement of the proposition itself. Facts are merely re-expressions of the propositions they make true. A full-fledged correspondence theory, however, must articulate an explanation of the correspondence relation that is more complex, and thus not amenable to the immediate restatement reply. In addition, there must provide a genuine account of facts as special kinds of entities that can be candidates for the relationship of truth-making. Certainly, the connection between propositions (or, in general, truth-bearers) and the world need not be of the simplistic relation of mirror image that is attributed to traditional correspondence truth (e.g., Karakostas 2012; Sher 2023). For a proposition to be true, it should not necessarily be tied to the world in an unmediated, context-independent manner, as assumed by the traditional alethic scheme.

Specifically, when examining the functioning of correspondence truth within the propositional structures of fundamental theories of physics, it should be clear that it requires an understanding not just of the logical form of correspondence per se, but of the specific field of knowledge in which the correspondence relation is realized. It requires, for instance, a systematic analysis of the nature of properties and relations that may be instantiated by objects within the domain of the relevant theory. Then, truth in terms of correspondence may be appropriately understood as property instantiation, in the following sense: if  $P$  is a true proposition, then  $P$  attributes a specific property to an object of the relevant domain.

In the quantum domain, truth-makers of quantum mechanical propositions, namely facts or actual states of affairs, are not pre-determined, pre-fixed; they are not ‘out there’ wholly unrestrictedly. As established by virtue of both Kochen-Specker’s and Bub-Clifton’s theorems, they are context-dependent in an essential sense. In view of the preceding considerations, therefore, we propose a perspectivist/contextual account of truth that is compatible with the propositional structure of quantum theory by conforming to the following instance of the correspondence scheme:

*Perspectivist/Contextual Correspondence* [PCC]: The proposition that  $P$ -in- $\mathcal{C}$  is true if and only if there is a state of affairs  $X$  such that (1)  $P$  expresses  $X$  in  $\mathcal{C}$  and (2)  $X$  obtains,

where  $\mathcal{C}$  denotes, in general, the context of discourse and, specifically, in relation to the aforementioned quantum mechanical considerations, the experimental context  $\mathcal{C}_A(D)$ , linked to the proposition  $P$  that is associated with a particular determinate sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  of  $\mathcal{L}_{\mathcal{H}}$ , defining a local perspective on the quantum system, and  $A$  indicates the physical quantity under investigation. As will be argued in the sequel, in the light of the alethic scheme [PCC], perspectivist truth is perspective or context-bound correspondence of a *de re* nature. Given the determination of a perspective, or the specification of a local context of reference, the attribution of truth values to propositions designates an objectively existing state of affairs that does not depend upon the epistemic system of any knower’s cognizance. Agents sharing the same perspective reproduce empirical facts of phenomena intersubjectively.

Let us initially note that the proposed perspectivist/contextual account of truth satisfies Tarski’s (1935/1956, 188) criterion of material adequacy, known as “convention T” or “schema T”, for a theory of truth:

(T) The proposition that “ $P$ ” is true if, and only if,  $P$

where the symbol “ $P$ ” in (T) represents the name of the proposition which  $P$  stands for. To this purpose, let us consider a particular proposition  $P$ : “system  $S$  has the property  $P(A)$ ”.

Assume context-dependence with regard to  $P(A)$ , that is, the latter property of  $S$  holds within context  $\mathcal{C}$ . Then, proposition  $P$  is concisely stated as: “system  $S$  has the property  $P(A)$ -in- $\mathcal{C}$ ”. Suppose now that this proposition is true. If so, the following instance of the T-schema must be true a priori:

The proposition that “system  $S$  has the property  $P(A)$ -in- $\mathcal{C}$ ” is true if, and only if, system  $S$  has the property  $P(A)$ -in- $\mathcal{C}$ .

If, however, the property  $P(A)$  of  $S$  is context-dependent upon  $\mathcal{C}$ , then the proposition that system  $S$  has the property  $P(A)$  must also be context-dependent upon  $\mathcal{C}$ . Thus, in conformity with the propositional status entering into the scheme [PCC], the preceding instance of the T-schema can be written equivalently in succinct form as:

The proposition that “ $P$ -in- $\mathcal{C}$ ” is true if, and only if,  $P$ -in- $\mathcal{C}$ .

Evidently, the logical operation of the bi-conditional in the preceding T-sentence is governed again by the T-schema, or the truism, that the content of a proposition determines the necessary and sufficient conditions under which it is true. Thus for any given true proposition which is context-dependent upon  $\mathcal{C}$ , the fact that makes it true is the context-dependent fact (or state of affairs) upon  $\mathcal{C}$  that the proposition expresses. Truth contextuality follows naturally from the contextuality of makers of propositional truths. If the latter are context-dependent, then whatever truths may be expressed about them must also be contextual.

The proposed account of truth, as encapsulated by the scheme of perspectivist/contextual correspondence [PCC], incorporates explicitly a context-dependence texture of the ‘world-word’ relation, if the world in its microphysical dimension is to be correctly describable. The truth-making relationship is now established, not in terms of a raw unconceptualized reality, as envisaged by the traditional scheme of correspondence truth, but between a well-defined portion of reality as carved out by the experimental context and the propositional content that refers to the selected context. Such interdependence of propositional content and referential context is not by virtue of some kind of philosophical predilection, but by virtue of the microphysical nature of physical reality displaying a context-dependence of facts. For, in view of Kochen-Specker’s theorem, there simply does not exist, within a quantum mechanical discourse, a global consistent binary assignment of determinately true or determinately false propositions independently of the appeal to a context; propositional content ought to be linked to a context.

It is instructive to point out in this respect that empirical access to the non-Boolean quantum world can only be gained by adopting a particular Boolean perspective, which is defined by a determinate sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  of compatible propositions or, in a more concrete sense, by the specification of an experimental context  $\mathcal{C}_A(D)$  that, in effect, selects a particular observable  $A$  as determinate. Within the context  $\mathcal{C}_A(D)$ , the  $A$ -properties we attribute to the system under investigation have determinate values. Nonetheless, the values of incompatible observables, associated with mutually exclusive experimental arrangements, are indeterminate. Hence, at any temporal moment, there is no a universal context that may allow either an independent variation of the properties of a quantum system or a unique description of the system in terms of determinate properties. This feature yields furthermore an explicit algebraic interpretation of the complementarity concept in quantum theory, in so far as quantum mechanical properties obtain effectively determinate values—equivalently, the associated propositions acquire determinate truth-value assignments—within a local Boolean substructure  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  of  $\mathcal{L}_{\mathcal{H}}$ , whereas the underlying source of

quantum mechanical ‘oddity’ is located in the fact that incompatible observables cannot be simultaneously realized or embedded into one inclusive Boolean logical structure.<sup>5</sup>

It is therefore clear that within the quantum mechanical framework the alethic scheme [PCC] functions under the globally non-Boolean constraints that quantum theory imposes upon any locally defined Boolean frame and its associated measurement context. In effect, the non-Boolean structure of quantum theory can be conceived of as a family of Boolean algebras or Boolean perspectives—each one corresponding to a set of commuting observables pertaining to a given system—that are intertwined in such a way that the whole family cannot be embedded into a single global Boolean algebra (Karakostas & Zafiris 2025). The intertwining prohibits the possibility of considering observable values in all these Boolean algebras of the collection as representing properties possessed by the system prior to their determination through measurement processes. Given that our physical description of the world is represented at the level of events by elementary propositions, a physical system is described at any particular time by a maximal set of true propositions. Precisely this functioning is accomplished by the scheme [PCC], assigning consistently values (i.e., properties) to a complete set of quantum observables that is compatible with the observable under measurement.

It is worthy to underline in this connection that the proposed account of truth of perspectivist/contextual correspondence [PCC] essentially denies that there can be a God’s eye view or an absolute Archimedean standpoint from which to logically evaluate the totality of facts of nature. Hence, the reference to a Boolean preparatory experimental context  $\mathcal{C}_A(D)$  should not be affiliated with practices of instrumentalism, operationalism and the like; it does not aim to reduce theoretical terms to products of operational procedures. It provides, in fact, the appropriate conditions under which it is possible for a quantum mechanical proposition to receive consistently a truth value. In other words, the specification of the context is part and parcel of the truth-conditions that should obtain for a proposition in order the latter to be invested with a determinate (albeit unknown) truth value. Otherwise, the proposition is, in general, semantically undecidable. In the quantum description, therefore, the introduction of the experimental context selects at any particular time  $t$  a specific sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  of compatible propositions in the global non-Boolean lattice  $\mathcal{L}_{\mathcal{H}}$  of a quantum system as co-definite; that is, each proposition in  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$  is assigned at time  $t$  a definite truth value, “true” or “false”. Equivalently, each corresponding property of the system either obtains or does not obtain. Thus, in our approach, the specification of a particular determinate sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$ , or a concrete experimental context, provides in a consistent manner the necessary conditions whereby bivalent assignment of truth values to quantum mechanical propositions is in principle applicable. This marks the fundamental difference between conditions for well-defined attribution of truth values to propositions and mere operational conditions.

This element also signifies the transition from the uncritical qualification of truth values to propositions by acknowledging them as being true or false simpliciter, as in traditional correspondence theory of truth, to the demarcation of the limits of possible experience or to the establishment of pre-conditions which make possible the attribution of truth values to

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<sup>5</sup>A category-theoretic framework of quantum event structures that allows a generalized interpretation of the complementarity principle has been given recently by Zafiris & Karakostas (2019). In view of the latter, complementarity is not only understood as a relation between mutually exclusive experimental arrangements, as envisaged by the original conception, but it is primarily comprehended as a reciprocal relation concerning information transfer between two hierarchically different structural kinds of event structure—the Boolean and quantum kinds of structure—that can be brought into local or partial structural congruence.

propositions. From the standpoint of our interpretative framework, the proposed account of truth [PCC] is associated with a perspectivist method of inquiry that seeks to investigate—from within the sphere of our worldly conditions, scientific standards and practices—the presuppositions and limits of experience and knowledge, thus opposing a non-perspectival, metaphysically fixed point of reference. In the quantum description, therefore, the consideration of the sublattice  $\mathcal{L}_{\mathcal{H}}(\{D/A\})$ , defining a Boolean perspective on the quantum system under investigation, forms indeed a pre-condition of quantum physical experience, which is necessary if quantum mechanics is to grasp empirical reality at all. Any microphysical event that ‘occurs’ is raised at the empirical level only in conjunction with the specification of an experimental context that conforms to a set of observables co-measurable by that context. The introduction of the experimental context furnishes thus the status of a presupposition of any empirical access to the quantum level of reality, of any possible empirical inquiry on the microscopic scale, and hence of any possible cognizance of microphysical objects as scientific objects of experience. In this respect, the specification of the context constitutes a methodological act preceding any empirical truth in the quantum domain and making it possible.

We note in this respect that the proposed account of truth of perspectivist/contextual correspondence [PCC] is not a relative notion itself; the propositions to which it applies are relative. They are relative to a specific Boolean sublattice of compatible propositions which are determinately true or false of a system at any given time. For, as already argued, a quantum mechanical proposition is not true or false simpliciter, but acquires a determinate truth value with respect to a well-defined context of discourse as specified by the state of the quantum system concerned and a particular observable to be measured. Thus, the conditions under which a proposition is true are jointly determined by the context in which the proposition is expressed and the actual microphysical state of affairs as projected into the specified context. What makes a proposition true, therefore, is not that is relative to the context—as an alethic relativist must hold (see, for instance, MacFarlane 2005)—but whether the conditions in question obtain. The obtainment of the conditions implies that it is possible for us to make, in an overall consistent manner, meaningful statements that the properties attributed to quantum objects are part of physical reality. In our approach, therefore, a proposition is true because it designates an objectively existing state of affairs, albeit of a perspectivist/contextual nature. In relation to the latter, we stress the fact that, in contrast to a panoptical “view from nowhere” of the classical paradigm, the general onto-epistemological implication of quantum theory acknowledges in an essential way a perspectivist/contextual character of knowledge. The classical doctrine that one can reasonably talk about ‘all entities’, as if the attribution of properties to an ‘entity’ had a unique, fixed meaning, independently of the appeal to a particular context of reference, is inadequate in the microphysical level of discourse. Nonetheless, the traditional conception of correspondence truth, involving a direct context-independent relation between singular terms of propositions and definite autonomous facts of an external reality, may be viewed as a limit case of the more generic alethic scheme of perspectivist/contextual correspondence [PCC], when the latter is applied in straightforward unproblematic circumstances where the non-explicit specification of a context of discourse poses no further consequences. This is immediately revealed if as context  $\mathcal{C}$  in [PCC] is taken the whole of reality and thus any particular reference to the conditions of a local context is silently removed.

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