Pointless Noncommutative Spacetime

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Abstract

Noncommutative geometry is often taken to imply a radical departure from relativistic spacetime at the Planck scale. The structure it describes, *noncommutaive spacetime* (NCST), lacks spacetime points (*pointlessness*), thereby challenging its status as a genuine physical spatiotemporal structure, as opposed to a merely mathematical construct. This paper challenges that conclusion. First, I argue that claims of pointlessness presuppose an ambiguous criterion for distinguishing physical from merely mathematical structure: proposed specifications either generate conflicting interpretations or collapse into triviality in the noncommutative setting. Second, I contend that even granting pointlessness, non-spatiotemporality does not follow. The inference relies on an implicit *constitution thesis* (that spacetime is constituted by its points) which proves either insufficiently precise or too weak to sustain the conclusion. By analysing four functional roles of spacetime points in classical theories, I demonstrate that none requires their retention in NCST, thereby undermining claims that spacetime disappears in the noncommutative regime.

Keywords: Noncommutative geometry, Noncommutative Spacetime, Pointlessness, Physicality, Criteria of Physicality

1 Introduction

Over the past few decades, so-called noncommutative approaches to quantum gravity (QG) have received steadily increasing interest. These postulate that, at or beyond the Planck scale, spacetime exhibits non-classical features, adequately described through a suitable *noncommutative geometry* (NCG). In this regime, the fundamental structure is commonly designated as *noncommutative spacetime* (NCST).[1]

Noncommutative approaches depart from standard relativistic frameworks in two principal aspects. First, noncommutativity induces characteristic deformations of the relativistic formalism, introducing a novel dependence on a noncommutative parameter. Second, a significant number of key mathematical and physical structures, ordinarily employed in the description of relativistic spacetime, lose their applicability and become formally ill-defined within the noncommutative setting. Consequently, these approaches face the challenge of showing that NCST can still serve as a genuinely spatiotemporal structure, notwithstanding its departures from the models of special relativity (SR) and general relativity (GR).

Seminal contributions to NCST approaches typically concur that, at quantum gravitational scales, this novel geometric structure preserves its spatiotemporal content despite the deformation. For example, (Snyder 1947, 40) characterises his NCST model as "quantum spacetime." Similarly, (Doplicher et al. 1995, *passim*) maintain that novel uncertainty relations, induced by the noncommutative parameter, motivate a new mathematical model of "Quantum Spacetime." On this understanding, 'NCST' is not merely a convenient label for a purely mathematical construct devoid of physical content, but rather a genuinely physical structure to be investigated through the formal apparatus of NCG.

Nonetheless, interpreting NCST as genuinely spatiotemporal stands in opposition with extant philosophical analyses. For example, (Huggett and Wüthrich 2025, 20) argue that "non-commutative geometry is non-spatial, in the sense that it is 'pointless', so it must be understood as a purely algebraic theory." Huggett et al. (2021) are even more forthright: within NCG, "spatial structure is derived" (4725) and "fields-first interpretations are the only game in town" (4697). Such claims suggest that, despite initial appearances, noncommutative theories actually entail the disappearance of spacetime in the quantum gravitational regime. In this respect, these theories share significant foundational and epistemological challenges with other QG approaches that likewise posit non-spatiotemporal degrees of freedom as fundamental to the relevant domain.[2]

More precisely, these arguments maintain that NCG is incompatible with a standard conception of spacetime as constituted by spatiotemporal points. In the noncommutative setting, on this view, points become ill-defined due to the specific construction of NCST itself (see, e.g., Lizzi (2009)). This feature, often described

[1] NCG exhibits wide-range applications within theoretical physics. Key contributions fall within two major approaches: the spectral triple approach and the quantum group approach. For the former, see, e.g., Connes and Marcolli (2007), as well as van Suijlekom (2025), focusing on applications to particle physics, and Marcolli (2018) for applications to cosmology. For the latter, see, e.g., Majid (1995); Szabo (2003), as well as references therein.

[2] The problem of the disappearance of spacetime has been extensively discussed in the philosophy of physics over the past two decades: see, e.g., Callender and Huggett (2004); Carlip (2014); (Crowther 2016, 13–15). Importantly, this has often been linked to the related issue of spacetime emergence: e.g., Oriti (2014); Wüthrich (2018); Huggett and Wüthrich (2025). In this paper, my primary aim is to defend NCST approaches as genuinely spatiotemporal. Accordingly, I contend that, in this context, a successful defense of their spatiotemporality renders unnecessary the development of specific solutions to the problem of spacetime disappearance, including that of emergence. If anything were to emerge in NCG, it would primarily serve to illuminate how relativistic structures are recovered in an appropriate limit, without signalling the original non-spatiotemporality of the underlying NCST structure.

as *pointlessness* or *point-freedom*, is taken to underwrite the inference to non-spatiotemporality. Intuitively, if NCST were composed of points understood as potential locations of physical events, then pointlessness would threaten its very ontological basis. On this interpretation, NCST could not meaningfully qualify as spatiotemporal at all.[3]

The central aim of this paper is to examine in detail how pointlessness bears on the spatiotemporality of noncommutative theories. I contend that this depends crucially on the criteria through which a structure is deemed "physical" rather than merely mathematical. Such criteria function as metatheoretic principles, specifying the conditions under which mathematical objects acquire physical content. I argue that pointlessness does not, by itself, undermine the foundations of NCST theories: disputes about pointlessness ultimately indicate disagreement over the appropriate criteria of physicality. Accordingly, any proponent of the disappearance of spacetime in NCG must make explicit the criteria employed. Different criteria, however, yield different assessments of pointlessness in the noncommutative regime, thereby precluding any sweeping conclusion.

Furthermore, I contend that, even if pointlessness were conceded, it would not automatically entail the disappearance of spacetime in noncommutative approaches. I therefore reject the claim that NCG is necessarily non-spatiotemporal: a noncommutative theory can coherently and meaningfully describe a spatiotemporal structure, despite the absence of physically significant points. Put differently, noncommutativity (and, more specifically, pointlessness) is not inherently incompatible with spatiotemporality.

To substantiate this position, in Section 2 I introduce the pointlessness problem in NCG and clarify the distinction between its mathematical and physical implications. In Section 3, I focus on physical (as opposed to merely mathematical) pointlessness and examine three candidate criteria of physicality: analogy with quantum mechanics (QM), tempered operationalism, and pragmatic significance. I illustrate that these criteria support divergent conclusions regarding physical pointlessness. Nevertheless, an opponent of spatiotemporality can still endorse a specific criterion. Accordingly, in Section 4, I examine the various roles that points may play within spatiotemporal theories. These may be leveraged by the adversary against spatiotemporality in NCG. Although noncommutativity does indeed challenge some of these roles in particular contexts (expecially in light of physical pointlessness), it leaves others intact. Moreover, the eliminated roles are arguably unnecessary for the definition of spacetime. Consequently, any argument for non-spatiotemporality building on these roles fails to reach its intended conclusion.

Similarly, in Section 5, I analyse the purportedly constitutive role of points in relation to spatiotemporal structure, as expressed in the so-called *constitution thesis*, which is pivotal to arguments for the disappearance of spacetime in NCG. In Section 6, I argue that this thesis is, at best, controversial and, at worst, conceptually unclear. On this basis, I defend NCST theories against the charge that they entail the disappearance of spacetime.

[3]This conclusion follows only if one adopts a localisation-based account of spacetime points. For a detailed discussion, see Section 4.1.

2 Pointlessness in NCG: Mathematical and Physical

The term *pointlessness*, or *point-freedom*, designates the absence of points within the ontology of a certain theory. On the one hand, points may be excluded on the grounds that they are ill-defined structures (*mathematical pointlessness*). That is, they cannot be defined or derived from the primitive elements of the theory, without the introduction of auxiliary assumptions or the risk of internal inconsistency. On the other hand, points may be mathematically definable, yet fail to possess any physical content (*physical pointlessness*). In this latter sense, point-based notation and concepts may still be employed instrumentally, for the purposes of calculation or heuristic guidance; however, they lack any role in the physical interpretation of the theory or explanatory relevance with respect to its intended domain of phenomena. In short, points do not qualify as physical entities, but rather as surplus structure or "ficticious entities."

Of course, pointlessness is not peculiar to noncommutative theories. For instance, in QM, points are excluded from phase space by the uncertainty relations: at best, localisation of states in phase space is achieved through coherent states, which, by definition, minimise these relations rather than eliminate them. Likewise, pointlessness has figured prominently in a number of philosophical discussions on GR, particularly in the substantivalism vs relationism debate. Algebraic formulations of general relativistic models (such as those developed by Geroch (1972)) dispense entirely with explicit reference to spacetime points, thereby avoiding ontological commitment to them. Similarly, manifold anti-realism maintains that matter fields, rather than manifold points, should be taken as fundamental. On this view, the additional advantage of pointlessness is the potential to avoid the hole problem within an algebraic framework (Earman (1989)), although this possibility has been challenged (Rynasiewicz (1992)).

Noncommutative approaches to QG exhibit pointlessness due to the introduction of a noncommutative parameter. Points become ill-defined for two distinct reasons: first, their mathematical characterisation is highly nontrivial; second, even if mathematically well-defined, they would allegedly fail to carry genuine physical content.[4] In this context, one encounters the so-called *pointlessness problem*: the apparent impossibility of reconstructing the notion of a mathematical or spacetime point within the theory. For example, Huggett et al. (2021) contend that noncommutativity is incompatible with the standard notion of sharp localisation of events within arbitrarily small spacetime regions. Consequently, the mathematical and physical status of such small regions becomes obscure in the noncommutative setting. This, in turn, motivates privileging an algebraic over a geometric perspective: "fields-first interpretations are the only game in town" (4697). On this view, pointlessness motivates "a fundamental metaphysics that eschews the concept of arbitrary localisability" (*ivi*). Moreover, NCG necessitates that physical spacetime be recovered from an underlying noncommutative algebra, thereby circumventing issues related to the disappearance of spacetime.

To elaborate further, within the specific framework of NCG, pointlessness appears to threaten the foundations of candidate theories of NCST in two respects.

[4]Of course, related considerations already appear in the broader philosophical literature on space and time: as mentioned, coordinate-based reasoning deflates the ontological and physical significance of spacetime points in GR in light of diffeomorphism invariance (see, e.g., Wallace (2019)). In NCG, however, the difficulty is more radical: the absence of points arises independently of the role of particular symmetry principles.

First, it calls into question the formal coherence of the underlying mathematical constructions. More precisely, the elimination of points renders a number of key point-based structures ill-defined. Differentiable manifolds, for instance, cannot be straightforwardly defined: noncommutative structures lack the formal resources to distinguish points via their separability properties, owing to the failure of the Hausdorff condition.[5] Consequently, mathematical theories that essentially depend on point-based structures are excluded from the noncommutative framework, as they can no longer deliver coherent representations of the intended domain of application. In their place, genuinely point-free mathematical frameworks must be identified to assume the explanatory and structural roles previously fulfilled by point-based geometric formalisms.

Second, pointlessness may challenge certain well-established interpretative postulates in the philosophical literature. (Teller 1995, 95), for example, explicitly characterises field configurations as assignments of specific values to determinable quantities at each spacetime point. On this view, field quantities are *defined* only if they have definite value *at each spacetime point*. Similarly, spatiotemporal localisation of systems often entails that the smallest region they occupy can be specified: possibly, under appropriate conditions, this may be a point. Accordingly, physical pointlessness demands a revision of some of these interpretative principles in order to preserve the physical content of the relevant theories.

Despite these challenges, upon closer examination, mathematical pointlessness does not undermine the formal consistency of noncommutative theories (Maresca (2026)). Specifically, it is possible to identify novel, arguably more sophisticated structures that replace point-based ones in line with the noncommutative structure. The latter typically include lattice-based structures, privileging specific definitions of points among inequivalent ones. For example, noncommutative extensions of the traditional Gelfand duality indicate *quantales* as viable geometric models for noncommutative approaches (Borceux and van Den Bossche (1989)). Informally, a quantale is a lattice-theoretic structure inducing an appropriate, point-free topology on an underlying set. On this view, NCST is derived as the *quantum spectrum* of its corresponding noncommutative (C^*-)algebra, thereby reconstructing the necessary topological content and separability properties.

Still, the problem raised by physical pointlessness remains open. Importantly, the central aim of this paper is to examine in detail this issue and how it bears on the spatiotemporality of noncommutative theories. For this reason, in the rest of this paper I will assume that mathematical pointlessness can be circumvented and its associated issues resolved. Instead, I will thoroughly investigate the consequences of physical pointlessness, with emphasis on its relation with spatiotemporality.

[5] Arguably, the weaker separability axiom satisfied by these geometric structures, namely T_0-separability, is insufficient to recover the ordinary notion of a point as characterised by Hausdorff-separation.

3 Physical Pointlessness and Physicality

The notion of mathematical pointlessness can give rise to serious concerns, as it can easily translate into *physical* pointlessness. Still, it is important to note that the foundational issues raised by the latter are only partially dependent on their mathematical counterparts. Mathematical pointlessness, of course, entails physical pointlessness: if the noncommutative algebra lacks points, then no point can be spatiotemporal. However, it does not follow that physical pointlessness is entirely reducible to mathematical concerns.

As discussed, the absence of mathematical points in a noncommutative setting can be challenged. Consequently, critics of NCG in the context of QG must supply independent arguments for physical pointlessness. This entails two tasks: (i) identifying noncommutative theories that include mathematical points (whether commutative or noncommutative), and (ii) demonstrating that these mathematical points cannot be promoted to physical entities. In other words, the critic must prove that NCG cannot be extended into a physical geometry adequate for the quantum gravitational regime, unless it entails the disappearance of spacetime.

Therefore, any meaningful assessment of physical pointlessness must address step (ii) of the adversary's strategy. Physicists and philosophers alike typically rely on intuitive understandings of what counts as "physical," but such intuitions often diverge in more complex cases. I argue that the problem of physical pointlessness in NCST theories gives rise to precisely this kind of disagreement. Specifically, I contend that varying definitions of the term "physical" can lead to different structures, within a NCST theory, being identified as physically significant: some including points, others without them. Each definition corresponds to a particular criterion that provides a principled basis for distinguishing between physical and unphysical structures. I refer to these as *criteria of physicality*.

More precisely, *physicality* refers to the capacity of a theory's mathematical structures to bear physical content. On this account, an interpretation of a theory assigns physical meaning only to those elements deemed "physical." Criteria of physicality thus act as metatheoretical principles that determine which mathematical structures are eligible to receive physical interpretation. Satisfying appropriate criteria of physicality is a necessary precondition for assigning a physical interpretation: elements of the formalism that fail to meet these criteria are regarded as *unphysical*. These criteria are neither dictated by the theory itself nor reducible to mere conventions. Instead, they typically reflect implicit assumptions and expectations shared by physicists working within a particular theoretical framework. Moreover, these criteria are often contingent on the investigative context in which the theory is applied.

In light of this, an advocate of physical pointlessness must identify relevant criteria of physicality (that is, specify what she means by "physical") and examine whether those criteria allow mathematical points to be interpreted as spacetime points. The claim of physical pointlessness requires demonstrating the following: no criterion of physicality permits the definition of spacetime points in a NCST theory. I argue that current formulations of this argument (most notably in Huggett et al. (2021)) suffer from a lack of clarity due to an unspecified (or *underspecified*) set of criteria of physicality (see Section 3.2). Without such specification, the argument remains too vague to be convincing.

To substantiate this claim, I examine three exemplary criteria of physicality: analogy with QM (Section 3.1), tempered operationalism (Section 3.2), and pragmatic significance (Section 3.3). I assess their respective capacities to address the problem of physical pointlessness in the context of NCG. I argue that the first criterion provides essential heuristic guidance for the development of NCST theories; however, it fails as a robust criterion of physicality due to substantial differences between QM and NCST. By contrast, the operationalist criterion may support the advocate of physical pointlessness under specific conditions, despite its notable limitations. Finally, the criterion of pragmatic significance does raise a problem of physical pointlessness, but this is not unexpected: the same issue arises even in classical relativistic theories.

3.1 The Analogy with QM

From a historical perspective, the analogy between QM and NCST has served as fruitful heuristic in the development of noncommutative frameworks. QM has provided suitable techniques and methods for analysing NCST algebras, deforming the relevant symmetry algebras, and exploring the formalism of spectral triples.[6] This analogy is already evident in the earliest contributions to NCG in the context of QG (Snyder (1947); Flint (1948); see also Maresca (2025)). Specifically, it clarifies certain ideas and expectations that guided the formulation of NCST theories, particularly regarding the status of spacetime points.

QM offers a precise analysis of the concept of a point. In classical mechanics, each state is described by a point in phase space. Quantisation implies that the state of any point-like quantum system is instead represented as a normalised distribution; these are constrained by uncertainty relations that prevent sharp localisation when the relevant operators do not commute. As a result, states in QM cannot, in general, be sharply localised. The states of an algebra generated by noncommuting operators can, at best, be coherent states that minimise the uncertainty relations. Yet even these coherent states possess a minimal variance and therefore do not correspond to points in phase space: that is, they are not dispersionless states. The optimal localisation of states is thus limited by irreducible uncertainty.

A similar situation arises in NCG. Classically, physical events can be sharply localised and described by states acted upon by coordinate operators, which indicate their precise location. In NCST, however, sharp localisation is typically precluded: nontrivial commutation relations between coordinate operators give rise to generalised uncertainty relations that constrain the admissible states. In some cases, the NCST algebra may lack any dispersionless states at all: observers (if definable) cannot localise physical events at a distance (through see Lizzi et al. (2019) for further specification). At best, the algebra may admit optimal localisation states, i.e., coherent states that minimise these generalised uncertainty relations. Like in QM, these states are not dispersionless and exhibit residual variance.

[6] See Connes (1995) for a thorough examination.

An advocate of physical pointlessness could attempt to elevate this heuristic analogy to a fully developed criterion of physicality. In other words, she may propose that structures in NCST acquire physical content insofar as they correspond, by analogy, to physical structures in QM.[7] This leads to the formulation of the following criterion:

(QM) Consider a noncommutative theory T. A mathematical structure in T can receive physical content only if it corresponds, by analogy, to a physical structure in QM.

This criterion implies that the ascription of physicality in NCST theories is mediated by the analogy with QM: the analogy preserves the physical significance of the quantum structures. Importantly, the criterion does not amount to a direct translation of physical interpretation from QM to NCST. Rather, it identifies which structures in the NCST theory are suitable *candidates* for physical interpretation, based on their analogical relation to physical structures in QM: for example, dispersionless states.

An implicit assumption behind (QM) is that QM itself provides a clear and unambiguous distinction between physical and unphysical structures, based on a settled definition of physicality in the quantum domain. It is agreed that possible outcomes of measurement procedures are physical. Still, the ascription of physicality to superposition states is controversial: are all possible outcomes described by the superposition physical, even before a measurement? This assumption is debated. Consequently, the definition of physicality in QM remains a subject of ongoing philosophical debate: different interpretations of quantum theory offer divergent views about which elements of the formalism are genuinely physical.

Moreover, even if such a definition were available, one may ask why it could not be directly applied to NCST theories. (QM) admits this possibility, but warns of a potential error: the domain-specific differences between QM and NCST may render the direct application of the quantum mechanical definition inappropriate. In this respect, the criterion functions as a mediating principle: it adapts the quantum mechanical definition of physicality to the NCST setting but also avoids problematic misapplications.

At this point, the advocate of physical pointlessness may argue that (QM) justifies her claim. Even if the NCST theory under consideration includes mathematical points, these would not generally correspond to dispersionless states and, by analogy, would fail to qualify as physical entities. However, I contend that this line of reasoning is weak. As noted, the quantum mechanical definition of physicality is itself unclear. Moreover, the analogy between QM and NCST breaks down in crucial respects.

First, in QM, position uncertainty can be minimised at the expense of increased momentum uncertainty. This trade-off is permitted because certain operators (such as position or momentum) can be preferentially treated within the formalism. One could argue that physicality should remain independent of any such choice: all observables that can be selected through privileging ought to reflect genuine physical properties of the quantum system. In other words, the reasoning goes, *any* operator corresponding to a measurable quantity should be considered physical, *precisely because* it could, in

[7] This kind of interpretation process is analysed, for example, in De Haro and de Regt (2018).

principle, be singled out by an arbitrary choice of measurement process and produce a definite outcome.

By contrast, NCST theories typically do not permit for such privileging. Elevating one coordinate operator over another would require a principled justification; however, in a spatiotemporal context, where all coordinates should be treated symmetrically, no such justification exists. Consequently, arbitrariness in measurement settings may ordinarily justify treating quantum mechanical observables as equally physical; however, in NCST, this argument is no longer available. Privileging one coordinate operator over another requires a principled justification that the theory does not provide: there is no trivial sense in which positions along each possible coordinate axis can be considered physical.

Second, QM permits the transformation of states between different bases via the Fourier transform, provided the states exhibit suitable regularity and are complex-valued distributions. This transformation typically relies on the canonical conjugacy between certain operators (e.g., position and momentum) and supports the idea that physicality should be symmetric between conjugate pairs: if a state is physical when expressed in one operator basis, one naturally expects it to remain physical when transformed into the basis of its conjugate operators. In this sense, the physicality of a quantum state is basis-independent.

By contrast, in NCST, such transformations are not always available. Not all states admit a Fourier transform, and even when basis changes are possible, they often require more elaborate integral transforms. Furthermore, the coordinate operators in NCST may lack a well-defined notion of canonical conjugacy altogether. With a given NCST algebra, there is often no meaningful sense in which one coordinate operator stands as the conjugate of another. As a result, the theory lacks the structural foundations necessary to support (let alone guarantee) the basis-independence of this notion of physicality.

In essence, the concept of physicality in NCST theories is structurally different from that in QM. What qualifies as a physical structure in NCST may differ substantially from a physical quantum state, due to significantly different features.

These considerations suggest that (QM) is inadequate for identifying physical structures in NCST theories. More precisely, it fails to provide a reliable demarcation between those structures that can receive physical content and those that cannot. For instance, in QM, the ability to localise states in position space (at the cost of increased momentum uncertainty) permits physical interpretations that are not available in NCST. Similarly, the physical significance of generalised uncertainty relations in NCST is obscured by the absence of a clear interpretation of the coordinate operators.[8] If their relation is unclear, then what physical role do they play? And how does this ambiguity affect the interpretation of states? These questions challenge the legitimacy of using (QM) as a premise for denying the physicality of mathematical points in NCST theories.

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[8] To briefly illustrate this point, compare, for example, the operationalist interpretation of Flint (1948) with the statistical interpretation sketched in Ballesteros et al. (2021).

3.2 Pointlessness in Light of Operationalism

An alternative criterion of physicality is provided by certain forms of operationalism.[9] Operationalism has been a key methodological assumption in the development of NCST theories from the 1930s to the early 1990s. The viability of operationalist criteria has since been significantly challenged by a series of thought experiments, most notably by Bronstein (2012); Mead (1964); Doplicher et al. (1995). Moreover, operationalism typically involves measurement procedures based on clocks and rulers; these, in turn, require a sufficiently developed geometric theory: the ability to extract such a theory via noncommutative algebraic-geometric dualities becomes a necessary condition for any operational approach to spatiotemporality. One can reasonably expect that different versions of operationalism will yield divergent views on which mathematical structures are to be regarded as physical.

In the context of NCST theories, Huggett et al. (2021) discuss a weak version of operationalism, which they term *tempered operationalism*. This serves as criterion of physicality in the following formulation:

(TO) Structures in a NCST theory are physical only if it is possible to describe a hypothetical operation for measuring the associated magnitudes.

Prima facie, (TO) offers a plausible criterion for distinguishing between physical and unphysical structures: the former include all mathematical structures representing potentially measurable quantities. Structures that fail to satisfy (TO) should be treated as mere mathematical abstractions. In this respect, (TO) is arguably more specific than (QM). Indeed, it does not rely on analogy with another theory, but states a precise self-contained condition, namely, the possibility of designing and describing a measurement procedure for the relevant magnitude. Accordingly, (TO) can only be violated by structures for which *no* conceivable measurement procedure can be articulated.

Operationalism has, of course, been largely criticised since Bridgman's early work.[10] Tempered operationalism, however, is a weaker variant thereof, and so it avoids many of these criticisms.[11] An advocate of physical pointlessness may thus invoke (TO) to eliminate spatiotemporal points from the ontology of NCST theories.

To illustrate, consider an operationalist interpretation of a NCST theory. Suppose that spacetime coordinate operators represent procedures to localise events in NCST, relative to a specific reference frame. According to (TO), the NCST algebra possesses physical meaning only if we can specify a corresponding measurement protocol to sharply localise a designated target event. However, in most nontrivial cases, noncommutative effects preclude sharp localisation (though the specifics vary from model to

[9]Note that, historically, the distinction between these two criteria has often been blurred. See Maresca 2025 for an overview of their relationship.

[10]Chang (2019) offers a thorough discussion of the main criticisms.

[11]Tempered operationalism departs from traditional operationalism in a key aspect: the existence of an operation is regarded as a necessary, but not sufficient, condition for the ascription of physical meaning (compare this with Bridgman (1927)). As a result, the physical content of a theory's mathematical structure need not be entirely reducible to its associated operations. Tempered operationalism does not provide a strict semantic criterion, but rather reflects a broadly shared attitude among physicists, one that arguably aligns more closely with Bridgman's original intent (see Chang (2019)). Furthermore, the measurement procedures associated with operationalised concepts need not be described within the same theory under investigation; that is, the theory need not be *Einstein-Feigl complete* (see Carrier (1990)).

model). As a result, mathematical points (interpreted as potential locations for events in NCST) fail to satisfy (TO) and cannot be considered physical objects. In this sense, the advocate of physical pointlessness concludes, NCST contains no physical points.

Huggett et al. (2021) support this argument with a thought experiment. They consider a target particle and a measurement device, each described by a Gaussian distribution with spread α, peaked at positions x_1 and x_2, respectively. Classically, in the limit $\alpha \to 0$, the convolution of these distributions peaks at x_2: the location of the target is thus operationally identified as the point x_2. However, in NCST, the two Gaussians are composed via a star product, which introduces noncommutative features governed by a deformation parameter. The resulting distribution is interpreted as a probability amplitude for finding the target at x_2; it does not peak sharply at that point due to an irreducible variance α. In this case, increasing measurement precision does not improve localisation beyond the fundamental noncommutative scale. Consequently, the authors argue, there is no operational method for identifying a point in NCST with the location of a physical system.

According to Huggett, Lizzi, and Menon, this illustrates "the impossibility of operationalising position measurements below the noncommutativity scale" (4719), and hence, that NCG "does not have the resources to make meaningful claims about localisability beyond a certain magnitude" (4696). Thus, points in NCST are rendered *meaningless* in two distinct senses: they are undefinable from a mathematical perspective (though see Section 2 and references therein for counterarguments), and they are non-operationalisable, in that they violate (TO). Accordingly, in the star-product formalism, "the manifold is exactly that, a component of the representation [i.e., the formalism], with excess representational structure for the true, essentially algebraic, fundamental objects" (4720). Huggett, Lizzi, and Menon therefore conclude that NCST theories entail the disappearance of spacetime: "noncommutative spacetime" is a mere label for a purely algebraic, non-spatiotemporal structure.

However, upon closer scrutiny, this thought experiment only supports a far weaker conclusion than the outright violation of (TO). Indeed, sharp localisability is prohibited through a specific localisation scheme, one in which locations are modelled by Gaussian distributions. This is a direct localisation procedure which, arguably, would be unrealistic in most physical settings. Its failure to localise events in NCST is thus not surprising, but expected. Moreover, the failure of one specific procedure is not sufficient to demonstrate a violation of (TO). In fact, (TO) requires that *no* conceivable operational procedure could ever localise the target within NCST. While Huggett, Lizzi, and Menon provide a valuable and insightful result, their analysis actually highlights the *difficulty* of establishing physical pointlessness on the basis of (TO). Their desired conclusion, that NCST has no points, would require the analysis of a broader range of localisation strategies.

Furthermore, in their paper the authors argue that the alleged failure of (TO) implies that manifolds are no longer well-defined structures within a noncommutative setting. While the justification for this inference is debatable (see below), the elimination of manifolds from the physical ontology of NCST theories can be supported by other arguments, such as the breakdown of classical algebraic-geometric dualities. On this view, only field algebras are capable of receiving physical content in NCST: operationally, they represent measurable quantities and thus satisfy (TO). By contrast, as previously discussed, the manifold becomes surplus representational structure with respect to the fundamental objects, namely, the noncommutative fields. Manifolds are indeed composed of points that violate (TO), and so cannot be regarded as physical structures, but merely as mathematical devices for performing calculations.

This conclusion, however, is problematic. In order to carry out calculations, indeed, the elements of a noncommutative field algebra are usually represented in their integral form. In the classical setting, integration variables can denote points in an underlying space. It is necessary for this space to be mathematically well-defined, if the integral itself is also well-defined. For example, fields can be defined as distributions acting on the regions of an underlying Minkowski spacetime, a well-defined structure in a classical setting. In a noncommutative setting, however, the integration variables can no longer be identified with points. This raises a foundational question: how are fields to be explicitly defined in the absence of spacetime points?

3.3 Pragmatic Significance

The preceding criteria apply to noncommutative theories by stipulating from the outset which structures should be considered physical. Disagreement over the correct criterion of physicality can therefore lead to conflicting conclusions. For example, Huggett et al. (2021) put forth the idea of using coherent states as substitutes of standard mathematical points for the definition of field quantities. These states arguably satisfy (TO), and thus may serve as genuine physical entities for recovering the eliminated point-dependent structures. However, the introduction of coherent states as physical entities stands in contrast with the (QM) criterion, depending on which exact criteria of physicality are adopted in QM. Specifically, if one admits that preparation procedures produce genuinely physical states, such as coherent states, then, by (QM), these states can also be regarded as physical in the noncommutative setting. By contrast, if the only physical states are produced through projective measurements, coherent states remain devoid of physical content.

Additionally, Huggett, Lizzi, and Menon admit coherent states as physical entities in the noncommutative setting solely in virtue of their behaviour in the commutative limit. Yet (QM), as previously formulated, makes no such appeal to limits.[12]

Despite such conflicting conclusions, physicality is often taken as a necessary condition for the existence of theoretical structures: intuitively, such structures are not mere mathematical constructions, but acquire physical content, allow the theory to make contact with experiments, and enhance our understanding of nature. However,

[12] Huggett et al. (2021) introduce smearing as an "interpretational postulate" (4725), intended to bridge the noncommutative composition of fields at high energy with the "manifest image" (4697) of classical spacetime reconstructed from a commutative field algebra. Smearing is thus presented as "a way to operationally use an object, a point, as an approximate avatar to connect with classicality" (4723). Observable fields are approximated by noncommutative smeared fields defined over regions larger than the fundamental length. Accordingly, only variations of field quantities above this scale possess physical significance; finer structure is regarded as "representational baggage, required by the formalism that we have adopted to formulate NCFT [sc. noncommutative field theory], and connect it with experience" (4724). Consequently, smearing is intended to motivate the use of coherent states as integration variables in NCST theories. Moreover, this use of smearing as part of a reduction strategy is methodologically unusual and raises questions about its role in the overall construction.

claims as "Spacetime points exist as physical entities" are often as uninformative as the conflicting criteria of physicality that underlie them.

Curiel (2018) criticises this mode of discourse about physicality. He emphasises the deep connection between physical structures and the application of physical theories to practical contexts. In actual scientific practice, he argues, spacetime structures are defined by how they are employed in modelling real-world target systems. On this view, the distinction between physical and unphysical structures is a pragmatic issue: it can only be determined by examining how theories are applied in specific modelling contexts. More precisely, any criterion of physicality must account for a particular investigative context or "framework." According to Curiel, such a framework consists of two components: (i) a family of physical structures, and (ii) a set of experimental practices that enable investigation of the kinds of systems modelled by those structures.

In light of this, Curiel explicitly formulates his criterion of physicality as follows:

(PS) "[A]n entity, purportedly represented by a theoretical structure, has physicality if one has reasons to take that structure seriously in a physical sense, namely, if one can show that it plays an ineliminable or at least fruitful and important role in the way that theory and experiment make contact with each other" (472).

Accordingly, the question of the physicality of spacetime points[13] can only be settled within specific investigative contexts. These contexts provide information about which structures are physically significant for defining physical events and phenomena. Crucially, Curiel argues that spacetime points fail to satisfy (PS) in any application of GR to practical contexts: they do not contribute to the stress-energy tensor, are unnecessary for formulating an initial value problem, do not couple directly to physical fields, and so forth. Therefore, spacetime points are not physical in any straightforward or unproblematic sense: they do not "exist" as physical entities under (PS).

It is important to note that Curiel's discussion is confined to the classical regime. The advocate of physical pointlessness might argue, a fortiori, that spacetime points will likewise fail to satisfy (PS) in the noncommutative regime. On this view, physical pointlessness follows from (PS), but is no more troubling in the noncommutative context than it already was in GR. In particular, the conclusion does not follow from noncommutativity *per se*: spacetime points "never existed" to begin with.

By contrast, one might respond by identifying investigative contexts in which spacetime points do in fact fulfil physically significant functions. While the energy scales of NCST theories prevent direct empirical testing, they do not preclude meaningful phenomenological considerations. Moreover, spacetime points can serve a plethora of functions within a physical theory, some of which may turn out to be crucial for the definition of physical events. Consequently, a thorough examination of the roles that spacetime points may play within NCST theories is essential. Such an examination must take into account the range of potential applications of NCST theories across diverse investigative physical contexts.

[13] Or, as Curiel discusses it in the paper, whether spacetime points exist as physical entities.

4 The Role of Spacetime Points

In the preceding section, I have argued that claims of physical pointlessness are significantly weakened by the lack of a well-specified criterion of physicality. The available criteria diverge both in their implications and in the difficulties they encounter. Consequently, disagreement regarding the precise criteria yields different assessments of physical pointlessness in noncommutative approaches. Among the three considered, (PS) appears to be the most promising. As discussed, this criterion entails physical pointlessness in the noncommutative setting, and thus seems (at least initially) to advance the programme of spacetime disappearance.[14]

However, the pragmatic significance criterion understands such pointlessness as a mere consequence of a much broader issue. In other words, physical pointlessness is not confined to noncommutative approaches, but also extends to classical relativistic theories. Consequently, (PO) risks trivialising the problem of pointlessness. If so, noncommutativity does not entail physical pointlessness; rather, it exacerbates this pre-existing features.

Likewise, criterion (TO) can be rehabilitated, provided that its main issue (namely, the definability of fields in the absence of spacetime points) is satisfactorily addressed. To this end, Huggett et al. (2021) introduce a distinction between two types of structures. On their view, *phenomenal points* are geometric elements belonging to the commutative regime: they trivially satisfy (TO). These points may be used in the integral representation of field quantities without issue; they represent possible locations of physical events at low-energy scales. Coherent states serve as suitable phenomenal points, insofar as they approximate well-localised states in the commutative limit.

By contrast, *mathematical points* are ideal elements required for calculations. They lack physical content and must disappear in the commutative limit of the theory.

Importantly, this distinction arguably weakens the authors' proposal, yet might still provide a defence of (TO). Its viability depends on a thorough clarification of the roles fulfilled by these two types of points within a spaiotemporal theory, and of the issues arising from their absence in a non-spatiotemporal one. More precisely, mathematical points, being unphysical under (TO), cannot ground spatiotemporality. By contrast, phenomenal points might plausibly do so, but only if they fulfil relevant functions relative to spacetime. Huggett, Lizzi, and Menon must therefore clarify which roles are essential to spatiotemporality and determine whether phenomenal points actually realise them.[15]

[14]One might object that the pragmatic significance criterion remains too vague to bear substantive weight. In particular, the criterion does not provide a precise catalogue of the functions that points should satisfy in order to be physically significant. A partial list of these key roles is discussed in the following sections. Nonetheless, I emphasise that, on Curiel's view, this would probably constitute a very weak criticism, one that he would be ready to accept. Specifically, Curiel could reply that this complaint misconstrues the methodological intent of the proposal. The pragmatist undertones of his paper suggest that such a complete list of functions cannot be definitely compiled; rather, any adequate specification of those functions must be articulated relative to specific investigative contexts or "frameworks." It is thus reasonable for different contexts to indicate different families of functions. General, unqualified claims about physicality would therefore be methodologically suspect. I am sympathetic to this response. A (partial) articulation of the key functions that physical points are expected to fulfil in a spatiotemporal theory is developed in Sections 4 and 5.

[15]A detailed assessment of the viability and limitations of the phenomenal/mathematical distinction is provided in Section 4.2.

An advocate of spacetime disappearance in NCG may still adopt one of the foregoing criteria, despite their respective shortcomings, and maintain that the resulting physical pointlessness signals the failure of NCST to qualify as genuinely spatiotemporal. Underdetermination in the selection of an appropriate criterion of physicality does not, by itself, dissolve the issue; at most, it renders the pointlessness problem insufficiently articulated. Instead, to block the inference from physical pointlessness to spacetime disappearance, one must demonstrate that the presence or absence of physical points is, in a precise sense, irrelevant to the foundations of a spatiotemporal theory. In what follows, I accept this challenge and address it directly.

Notably, the structure of a spatiotemporal theory is generally extremely complex. Textbook presentations often simplify its construction by presenting key results in a linear progression. However, a more accurate picture reveals that each structure serves precise formal and interpretational roles within the theory. For example, the elimination of the conformal structure in GR precludes the definition of a Weyl structure on a manifold, and thus the specification of a Lorentzian metric.

In this context, a thorough reconstruction of the roles fulfilled by a given structure within the theory sheds light on the potential consequences of its removal. These roles must be delineated with respect to three contexts.

First, a structure can serve specific roles within *mathematical geometry*. This delineates and provide a mathematical model of spacetime.

Second, the same structure can play different roles within a *physical geometry*. This is a description of spacetime as a physical structure, according to our best scientific theories or, more specifically, the theory under consideration. Physical geometry represents spacetime as object of physical investigation within specific research contexts.

Finally, a structure can contribute to the definition of a *phenomenological geometry*. This provides a model that accounts for all empirically accessible information concerning spatiotemporal phenomena within a designated regime. Such geometry is constrained by extra-theoretic bounds (including the choice of setup, the sensitivity of instruments, and approximation methods) which can influence the potential functions of the intended structure.

Building on this classification, points can fulfil four principal functions within a spatiotemporal theory: the localisation of events (Section 4.1); the integral formulation of fields (Section 4.2); the evaluation of physical quantities (Section 4.3); and the construction of spacetime (Section 5 below).

4.1 Localisation

Spacetime points designate the sites at which physical events can potentially occur. On this view, they are necessary for events to be localised in spacetime with arbitrary precision. Operational accounts require a detailed description of this localisation process (e.g., via a scattering procedure between a probe and a designated target) and an assessment of whether it can be executed within a specific domain. Accordingly, spacetime points can serve a localisation function relative to the definition of physical geometry.

This function raises immediate concerns regarding the identification of individual spacetime points through localisation. Each localisation procedure is conditional on a reference frame and a coordinate system. According to the standard view, this implies that a spacetime point is defined by its coordinates with respect to this frame and coordinate system. This, however, conflicts with a corollary of diffeomorphism invariance: coordinates are not invariant under diffeomorphisms and therefore should not be considered physically meaningful.

Nevertheless, the identification of spacetime points with their coordinates is not necessarily implied by the localisation function. Here, localisation simply means that points serve as *support* for events: events do not identify the spacetime points they occupy; rather, they *presuppose* them. In other words, a spatiotemporal theory must include a structure capable of supporting events. Classically, this structure is the set of spacetime points.

As previously discussed, NCG raises serious concerns about the localisation of events. Huggett et al. (2021), for instance, explicitly claim that points cannot serve this function in NCST. I agree that *sharp* localisability of events (within arbitrarily small regions, as Huggett, Lizzi, and Menon have in mind) is generally precluded in NCST, albeit with a few caveats (see Lizzi et al. (2019)). This limitation arises from the noncommutative parameter, which imposes a minimum uncertainty and bounds the size of possible localisation regions.

However, localisability does not necessarily require that these regions be "arbitrarily small." It is expected that appropriate notions of *event* and *localisation accuracy* depend on the theory in question. Classical constructions admit point-like events and sharp localisation, because they are compatible with specific idealisation schemes. However, these features do not generalise across all spatiotemporal theories and regimes, nor do they define the notion of localisation itself. Regions of nonzero minimal size can still fulfil the localisation function and support (possibly nonlocal) events within an appropriate domain.

One could also argue that equating localisation with sharp localisation was implausible from the start. In phenomenological geometry, for instance, no single point corresponds to the location of a system: the resolution of a localisation procedure can only be optimised up to a certain bound. Similarly, idealisations do not yield pointwise localisation, unless such localisation is already grounded in a classical framework, which clearly does not apply in the noncommutative case. Consequently, I take sharp localisation to be an unnecessary function of spacetime points. These need only possess the properties required to support the notion of an event within the relevant domain.

4.2 Expression of field quantities

Points also play a mathematical role in the explicit formulation of field-theoretic quantities. To write such quantities in their integral form, points must be well-defined, serving as integration variables. (Huggett et al. 2021, 4722) propose this function as a reason to distinguish between phenomenal and mathematical points: the latter are introduced to ensure well-defined integrals, even though they are, strictly speaking, unphysical in a noncommutative context (see also Section 3.2).

It is important to note that fields can be defined independently of their integral representation, e.g., as elements of an algebra. However, this algebraic approach may be impractical from a physical perspective. First, it is insufficient to constrain the propagation of information between interacting fields.[16] Second, it does not support explicit calculations: such calculations require fields to be represented in integral form (e.g., as tempered distributions), and may involve additional mathematical structures to aid both their use and interpretation.

In this regard, mathematical pointlessness poses no serious problem for the integral formulation of fields. On the one hand, appropriate structures can be identified to replace point-based notation in computations. On the other, one may simply accept the use of classical points as primitive notational devices (or "fictitious elements"), without committing to their ontological status or definability.

4.3 Evaluation

In physical geometry, spacetime points have often been regarded as necessary elements for supporting the quantities posited by physical theories. This view is clearly articulated, for example, by (Teller 1995, 95–96):

> A *Field Configuration for a determinable* (*or collection of determinables*) is a specific assignment in which each space-time point gets assigned a value of the determinable (or a value of each determinable in the collection). [...] A *Constitutive Determinable* is a determinable the values of which are properties of individual space-time points. A *Field Determinable* is a determinable the values of which are the full field configurations. (Emphasis in the original)

In QFT, Teller raises concerns about the notion of a determinable,[17] but notably, he does not question the dependence of fields on spacetime points. On the contrary, he correlates the definability of fields with the existence of an underlying point-based spacetime structure.

Teller's criterion reflects a widely held interpretation of QFT. On this view, the elimination of physical points, as posited in NCG, would undermine the definability of field theories altogether.

However, this criterion faces several challenges. Most significantly, fields do not require points as their fundamental support. Instead, field theories are more robustly characterised by anchoring fields to spacetime *regions*.[18] Adopting regions (rather than points) as domains over which fields are defined does not inherently lead to nonlocality, provided that key principles (such as microcausality) are preserved.

Moreover, the definability of fields does not necessarily presuppose any underlying spacetime structure. For example, in loop quantum gravity, fields are associated with

[16] Note that algebraic QFT postulates the existence of spacetime regions *alongside* field algebras. The reason is to introduce a suitable notion of locality implemented via the microcausality axiom. See (Haag 1996, 105).

[17] According to him, quantum fields are not determinables that assign specific operators to each spacetime point, but rather specify expectation values for possible field experiments performed at different locations (Teller 1995, 103). Crucially, Teller still commits to the introduction of spacetime points as support for field quantities. I will not discuss further the advantages and limitations of this definition.

[18] This is stated explicitly in algebraic QFT, or implied by the smearing condition in Wightman's axiomatisation.

spin networks and graph-based structures; in NCG, they are defined over noncommutative algebras, which may bear, in the general case, no direct relation to a NCST model. Accordingly, physical pointlessness does not preclude the definability of fields. Fields can be understood as well-defined elements of appropriate algebras; moreover, they can be evaluated by identifying suitable structural frameworks, none of which need involve commutative points, or even explicitly spatiotemporal entities.

This observation has important implications for phenomenological contexts. Fields can be meaningfully defined even when the notion of a spacetime point cannot be experimentally realised (except via approximations). Furthermore, one might argue that field quantities should not be defined on spacetime at all, but rather on momentum space: in fact, phenomenological models and experimental setups are primarily energy-dependent (see, e.g., Amelino-Camelia et al. (2011)). On this view, the disappearance of spacetime poses no obstacle for phenomenological geometry. Nonetheless, it should be acknowledged that this last point remains the subject of ongoing debate.

5 The Constitution Thesis

The preceding remarks indicate that classical spacetime points are either unnecessary to fulfil key functions (contrary to the standard view), or replaceable with alternative structures, such as noncommutative points (see Borceux and van Den Bossche (1989)). Nevertheless, the proponent of spacetime disappearance in NCG may point to further cases in which commutative points appear to be indispensable. In particular, she might appeal to the following claim:

(CT) Spacetime is constituted by spacetime points.

I refer to (CT) as the *constitution thesis*. Notably, it licences the following inference:

(CT') If a structure is spatiotemporal, then it is constituted by spacetime points.

Given (CT'), physical pointlessness would entail (by *modus tollendo tollens*) that no structure in the noncommutative setting can be spatiotemporal. In this way, the constitution function traditionally assigned to spacetime points within classical spatiotemporal theories underwrites an argument for the disappearance of spacetime in NCG. However, as currently formulated, (CT) is vague: it does not specify what is meant by "constituted".[19]

In this section, I argue that (CT) is not a necessarily true statement. Consequently, it cannot be employed as a reliable premise against the spatiotemporality of NCST theories. To demonstrate this, in Section 5.1 I investigate the relevant notion of spatiotemporality underlying both (CT) and (CT'): what does it mean for a structure to count as *spatiotemporal*? I argue that there is no single, agreed answer. Rather, spatiotemporality is a *multifaceted* concept.[20]

[19] As discussed below, this vagueness is indeed conceived as advantage for the advocate of (CT), as it allows versatility to respond to several counterpoints.

[20] See, e.g., Baker (2019) on his cluster view. I also take this claim to align with Stein (1977), who argues against a categorical (i.e., theory-independent and a priori) concept of metrical structure in the classical and relativistic settings. In particular, the idea of spacetime as a multifaceted concept resonates with the pragmatist attitude expressed in the following quote: "Tell me exactly what *you mean* by 'spacetime.' [...] 'I mean,' says X, 'the mere four-dimensional differentiable manifold, independently of any further structure'; 'I,' says Y, 'mean the smooth 4-manifold, with the distinction of directions at each point into *spacelike* and

Next, in Section 5.2, I consider possible specifications of the constitution relation in (CT), in light of the differing characterisations of spatiotemporality. Finally, in Section 5.3, I explore a potential constructivist refinement of (CT), and identify the conceptual limitations it faces.

5.1 Spatiotemporality

Physicists generally agree on what spacetime is, at least in broad terms. Intuitively, it is the domain in which all physical events may occur; its geometry is described by our best physical theories. More specifically, spacetime is composed of a collection of interrelated structures; depending on the specific theory, these may include: set-theoretic, topological, differential, affine, projective, metrical, conformal structure, among many others. Spatiotemporal theories, such as GR, describe the subtle interplay between these structures.

Furthermore, spacetime is expected to fulfil a number of theoretical roles (see Section 4), including: supporting physical events; supporting field quantities (but see Section 4.3); and possibly imposing symmetries (via Earman's principle: see (Earman 1989, 46), though this principle remains controversial). In addition, spacetime is characterised by several features that are typically assumed by, and encoded in, spatiotemporal theories. These include: enabling the sharp localisation of events; possessing a well-defined causal structure; having four dimensions; admitting a Lorentzian signature; and satisfying smoothness and Hausdorff conditions. A definition of "spatiotemporal structure" thus involves specifying a subset of these structures, roles, and features.

However, physicists and philosophers often disagree on which specific aspects, among those listed here, are essential to characterise a structure as spatiotemporal. Accordingly, "spatiotemporality" is not a well-defined concept, unless a detailed list of such relevant structures, roles, and features is clearly articulated and agreed upon. This thesis is defended, for example, by Jaksland and Salimkhani (2023): the disappearance and emergence of spacetime are not well-posed problems unless one specifies what qualifies as a spatiotemporal property, and what is said to disappear in the quantum gravitational regime. Similarly, Baker (2019) proposes that "spacetime" should be treated as a cluster concept, in order to capture the various roles played by spacetime structures (or their functional substitutes) in different theories.

On this view, which I endorse, there is no unique answer to the question "What makes a structure spatiotemporal?" Rather, such an answer is context-dependent and ultimately relative to the physical scenario in which the question is formulated. However, this observation brings us no closer to a clarification of the constitution thesis (CT). I suggest that in order to clarify (CT) in the context of the disappearance of spacetime in NCG, one must address a more specific question: What makes a *mathematical* structure spatiotemporal? In other words, under what conditions can a purely mathematical geometry be interpreted as a physical geometry?

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causal (timelike or null) directions.' Ah, yes, I say, for X and Y the metric is extrinsic to space-time; whereas for Y, but not for X, the conformal structure is intrinsic. I not only have no criterion for adjudicating among these views; I have no *desire* to adjudicate, *see no point* in adjudicating, among them" ((Stein 1977, 394–395); emphasis in the original).

This is a question of interpretation and its preconditions. It requires identifying the mathematical structures that are sufficient to bear spatiotemporal properties and to fulfil the corresponding theoretical roles. On this basis, the advocate of spacetime disappearance might then assert the following: if such-and-such mathematical structures are necessary for spatiotemporality, and a NCST theory fails to recover them, then that theory entails the disappearance of spacetime.

Philosophers of physics often endorse what (Janssen 2009, 27) refers to as the "'angle-brackets-M-O-sub-i' religion."[21] According to this view, all spacetime models take the form of an ordered tuple $\langle M, O_1, \ldots, O_n \rangle$, where M is a smooth four-dimensional manifold, and $O_1, \ldots, O_n$ are geometric objects that satisfy the dynamical equations of the theory.[22] These tuples are taken to adequately represent the spacetime structure, independently of the specific matter fields defined on them. This representation is sometimes enriched by the *container hypothesis*: spacetime serves as a background structure that is ontologically independent of its content.[23] However, this assumption is not required in order to define spatiotemporality (Knox 2013, 346, fn. 2).

This formal view has been challenged on several grounds. First, the status of the tensor fields in the tuple is ambiguous: whether they are essential to defining spatiotemporality (alongside the bare manifold) depends crucially on one's interpretation of the hole argument.[24] Second, this framework assumes a sharp distinction between spacetime and matter content, a distinction that has been recently questioned (Martens (2019)). Third, spacetime can be described in a number of ways other than as a manifold (bare or enriched with tensor fields). These alternatives include, for example: sections of $SO(1,3)$-principal fiber bundles; manifolds equipped with spin structures and additional topological structures; manifolds characterised by Synge's world function; solutions to the Palatini action; and various other formal representations.

While this formal characterisation has the merit of bringing spatiotemporal theories closer to the tools of model theory, it also risks neglecting the pragmatic considerations that often guide the classification of a structure as spatiotemporal. In particular, different structures may serve as spacetime in different investigative contexts.

To illustrate, localisability is an indispensable feature of spacetime within certain phenomenological contexts. In such cases, the spacetime structure must permit the definition of area and volume, and allow the introduction of local reference frames and

[21] This position is typically associated, for example, with the works of Friedman (1983) and Earman (1989).

[22] To be precise, $\langle M, O_1, \ldots, O_n \rangle$ describes a *dynamically possible model* of the theory: see (Friedman 1983, 48 ff.).

[23] The container hypothesis derives from a specific interpretation of the Scholium to the Definitions in Newton's *Principia*, I. For further details, see Rynasiewicz (995a,b) and (Pooley 2013, §§2; 4). For a criticism of the container hypothesis, see, e.g., (Brown 2005, 67), who quotes (Earman 1989, 155).

[24] This problem has been extensively discussed by contemporary philosophy of physics, starting from the seminal paper by Earman and Norton (1987). For more recent contributions, see, e.g., Belot (1996); Gaul and Rovelli (2000); Landsman (2023); Bradley and Weatherall (2022). Interestingly, Curiel (2018) argues that the dispute cannot be settled on purely mathematical grounds, but ultimately depends on the investigative context. I take his remark to imply that a proper characterisation of spatiotemporality in terms of the 'angle-brackets-M-O-sub-i' religion would not be unique: pragmatic considerations are essential for the identification of spatiotemporal structures.

coordinate systems. However, these features may prove insufficient in contexts where, for instance, one must define a stress-energy tensor or a chiral field.

In conclusion, spatiotemporality is not a univocal notion; it is inherently *multi-faceted*. Its definition depends on the specific investigative context at hand, and so, too, does its characterisation in terms of the mathematical structures deemed necessary to support it. Accordingly, any claim concerning the spatiotemporality of a given theory must be accompanied by an explicit specification of both the relevant context and the mathematical structures that are taken to embody the spatiotemporal content of that theory.

5.2 Building spacetime from points

For the sake of the argument, suppose that the advocate of the constitution thesis has provided a suitable definition and characterisation of spatiotemporality. She must then specify the relationship between spacetime and spacetime points. Constitution is an asymmetric relation, in which the constituted entity is less fundamental than the constituent. This relation can be formulated in terms of various conceptions of fundamentality: that is, specifications of the fundamentality relation in terms of other partial ordering relations, including, for example, mereological composition, formal priority, or reduction. Notably, (CT) aims to define this constitution relation for each spatiotemporal theory in isolation: that is, constitution is here understood as an intra-theoretic relation.

In its weakest form, constitution requires that the constituted entity be derivable from its constituents. On this interpretation, (CT) asserts that spacetime is derived from spacetime points. This view is supported by the conception of spacetime as a compound of multiple structures, coalescing appropriately, which are imposed upon a set of primitive entities, namely, spacetime points. The derivation is assumed to be *linear*: primitive elements are first identified and defined; then, other structures are successively derived from these primitives and previously derived entities.

Reconstructing the constitution thesis thus raises the challenge of specifying the derivation relation linking spacetime points to spacetime. In particular, the advocate of (CT) must show that spacetime points are sufficient to reconstruct all the structures required by the relevant notion of spatiotemporality. This problem has been explored in the context of axiomatic reconstructions of spacetime geometry, i.e., attempts to organise and understand spacetime theories under a linearity assumption (Carrier (1990)). These reconstructions typically begin by identifying a complete list of primitive entities, chosen on extra-theoretic grounds; then, they show how the rest of the theory can be recovered through stepwise derivations from the primitives. The aim is to enhance theoretical clarity: each structure is either primitive or derived through an intelligible and transparent procedure.[25]

These reconstructions motivate a *deductive approach* to the constitution thesis. On this view, one starts with a set of points as primitives and shows how spatiotemporal structures can be deduced from them. To facilitate this, points are supplemented

[25]See (Adlam et al. 2025, 118–121) for a discussion of this strategy and its Carnapian influences. Notably, the authors distinguish between constructivist axiomatisations, which merely assume linearity, from proper constructivism, which also requires the primitives to be experimentally accessible. See Section 5.3.

with logical relations and background mathematical frameworks, such as set theory, first- and second-order logic, specific ordering relations, or pre-geometric axioms. In this framework, spacetime is reconstructed by imposing a suitable topology on the set of points (Benda (2008)), constructing an intended model through representation theorems (Andréka et al. (2010); Cocco and Babic (2021)), or identifying appropriate relations between points (Madarász et al. (2004); Mould (1959); Mundy (1986); Robb (1936)).[26]

From this, three specifications of the constitution thesis can be distinguished:

1. *Mereological constitution:* Spacetime is mereologically constituted by spacetime points.[27] On this view, the elimination of points due to physical pointlessness entails the disappearance of spacetime itself.
2. *Supervenience:* Spacetime supervenes on spacetime points, that is, it depends on them, but also may involve irreducible geometric or physical properties. On this view, certain point-independent structures may survive physical pointlessness, but are insufficient to count as genuinely spatiotemporal.
3. *Relational derivation:* Spacetime is derived by imposing non-mereological relations on spacetime points. For example, Robb (1936) and Mundy (1986) reconstruct geometry from kinematic displacements, while Mould (1959) appeals to dynamical shifts. This view allows for a radical relationalism (akin to eliminativism): spacetime can be defined without reference to points, which are reduced to mere placeholders for relational structures. While this avoids the outright disappearance of spacetime, it arguably dilutes (CT) to the point of irrelevance.

The deductive approach to (CT) has been criticised for being limited to mathematical geometry. That is, the physical content of spacetime arguably cannot be reduced to a system of logical or formal relations among ideal primitives. One possible response is to "add physical content by hand:" assume spatiotemporal structures as primitive, and derive spacetime points via restriction to local structures.

However, this raises two problems. First, global features do not trivially determine local (i.e., point-based) features. Second, this global-first perspective is incompatible with (CT). Specifically, if spacetime points are eliminated due to physical pointlessness, then the existence of primitive spatiotemporal structures is insufficient to counter arguments for the disappearance of spacetime based on (CT). A different approach to the relation between spacetime and points is therefore required.

5.3 The Constitutive Role of Points in NCG

Spacetime constructivism has made significant contributions to the reconstruction of spacetime geometry, particularly in operationally motivated frameworks. These approaches reconstruct physical geometry from a set of statements about immediately observable facts, progressing via increasingly abstract conceptualisations (Carrier

[26]Pre-geometric approaches face a challenge, in that they must assume some spatiotemporal structures as primitives alongside points from the outset. See, e.g., Bunge (1967) and Covarrubias (1993).

[27]See Hellman and Shapiro (2018), especially chs. 2, 5, 6. For such mereological approaches, see also Baron and Le Bihan (2022b); Pohlmann (2024). Mereological projects to the constitution of spacetime have recently been challenged by QG theories exhibiting disappearance of spacetime: see Baron and Le Bihan (2022a) for a discussion.

(1990)). The requirement that primitives be empirically testable, together with the linearity of derivation, enhances the logical clarity of the theory: it clarifies both the definitions and inferential processes by which the theory is built.

Constructive approaches typically introduce additional primitives that are neither spacetime nor spacetime points. For example, Reichenbach (1969) famously based his reconstruction on light rays, while Ehlers et al. (1972) also include primitive notions of event and particle.[28] On this view, both spacetime and spacetime points are derived structures, acquiring their physical content from more fundamental primitives.[29]

Two remarks are crucial here. First, as is well known, constructive approaches cannot proceed without presupposing certain spatiotemporal features or properties from the outset. This does not mean assuming the full structure of spacetime, but at least a minimal subset of its properties to supplement the primitive elements. Iterative reconstructions attempt to soften this assumption: primitives are progressively refined to coalesce with derived structures at each step. In this way, additional properties are not simply postulated, but emerge as necessitated by the construction itself. However, this methods risk circularity, whose acceptability in specific contexts is open to debate (see (Adlam et al. 2025, §2.3.2)).

Second, spacetime constructivism still permits a hierarchical relation between points and spacetime: the former can be more fundamental in the order of derivation. That is, spacetime points may be derived from the primitives prior to the full derivation of spacetime, and can serve as intermediate steps in that process. Consequently, (CT) can remain compatible with constructivist approaches, and may still support arguments for the disappearance of spacetime in cases of physical pointlessness. In particular, this implies that spacetime points cannot be derived from the primitives by physically salient processes, such as intersections of light rays. A fortiori, spacetime itself cannot be fully derived by physically salient procedures: it would depend on the missing points, among other things.

This modified version of (CT) avoids the issue of physical content by presupposing it, and only allows for content-preserving derivations. In contrast, the argument for the disappearance of spacetime does not entail the elimination of all spatiotemporal structures; rather, it only eliminates those structures that depend on spacetime points for their well-definiteness. For instance, structures introduced alongside the primitives are unaffected by pointlessness. Moreover, some of the eliminated structures may still be recovered without the intermediate step of deriving spacetime points, thereby circumventing the issue entirely.

6 Noncommutative Spacetime

As discussed, the constitution thesis must be refined in order to serve as a viable premise for the disappearance of spacetime. The relevant definition of spatiotemporality remains both vague and unspecified, due to its inherently multifaceted nature.

[28] Alternative sets of primitives can be introduced, depending on whether the reconstruction relies on a well-delineated observation theory or not. See, e.g., Castagnino (1968); Hayashi and Shirafuji (1977); Hehl and Obukhov (2006); Perlick (1987); Schelb (1996). For a discussion, see (Adlam et al. 2025, §2.3.1).

[29] Compare with the discussion on physical salience in Huggett and Wüthrich (2013). Here, the primitives are directly testable. On the authors' view, physical salience propagates from them to the derived structures: it is bottom-up salience without scale-transition.

Moreover, it is unclear what exactly should count as composition. The advocate of the disappearance of spacetime maintains that whatever conceptual difficulties (CT) might raise in the classical contexts, these are significantly amplified in the framework of NCG. Indeed, in classical spacetime theories, the shortcomings of (CT) may be dismissed as merely conceptual, with no impact on calculations or predictive success. This is no longer the case in the noncommutative, quantum gravitational regime.

Recall the structure of the argument available to the advocate of spacetime disappearance. NCST theories exhibit physical pointlessness. By (CT), spacetime is constituted by spacetime points. Therefore, no spatiotemporal structure can be sustained within NCG: the phrase "noncommutative spacetime" becomes a label for an object that is not spatiotemporal in any significant sense. Of course, if physical pointlessness is undermined, the argument collapses trivially: (CT) becomes insufficient to support the conclusion (see Section 3).

However, for the sake of the argument, suppose that the NCST theory under consideration does exhibit physical pointlessness. Does this, by itself, entail the disappearance of spacetime?

The answer hinges on how (CT) is refined. One possibility is to adopt a deductive approach, leading to the following formulation:

(CT1) Spacetime is derived from (i.e., constituted by) spacetime points upon specification of a derivation relation.

This formulation is problematic in two respects. First, as discussed above, it is too vague: it fails to specify what is meant by "spacetime," and thereby fails to identify which structures would be eliminated were the argument to succeed. Second, even with a clearer specification, the derivation relation is insufficient to guarantee that the derived structures possess spatiotemporal content. As it stands, (CT1) is untenable.

These issues can be mitigated by shifting to a constructive approach to (CT). On this view, the key premise can be restated as:

(CT2) Spacetime is physically constructed from the non-spatiotemporal primitives of the theory.

If spacetime points do not figure in the construction of spacetime, then (CT2) cannot ground the argument for the disappearance of spacetime on the basis of physical pointlessness. By contrast, if spacetime points do appear as intermediate constructions in the derivation of spatiotemporal structure, then constructivism deflates the main objections raised against (CT1). In this case, spatiotemporality is characterised by the construction process itself, and the derivational role of points must be explicitly illustrated. Furthermore, the use of directly testable entities as primitives endows the construction with physical salience: this ensures that the resulting structures possess genuine physical content.

Nevertheless, even under this refined formulation, physical pointlessness does not necessarily imply the disappearance of spacetime. There may exist alternative methods for constructing NCST that entirely avoid spacetime points. In other words, while the disappearance of spacetime is certainly a possibility, it can only be demonstrated by ruling out all conceivable alternative constructions of NCST. Thus, the advocate of spacetime disappearance must embrace the burden of proof: it is incumbent upon her to show that no such construction is viable.

7 Conclusion

Noncommutativity challenges the ordinary status of points in spatiotemporal theories. Not only does this raise a mathematical issue: points cannot be defined within a consistent noncommutative theory. It also challenges a number of standard interpretational postulates. Indeed, points are demoted from their potential status as physical entities. As discussed, ascriptions of physical significance to spacetime points raise controversy already in relativistic theories. In this regard, the common expectation is for these worries to be exacerbated in the noncommutative setting. Specifically, if spacetime points are absent from the ontology of a candidate QG theory, then that theory cannot include any spatiotemporal structure within the quantum gravitational domain: it exhibits disappearance of spacetime.

In this paper, I have challenged this view on two accounts. First, I have argued that the elimination of physical points from noncommutative theories, termed *physical pointlessness*, actually stems from specific assumptions about which mathematical structures should be considered physical and which ones should not. These assumptions are articulated through different criteria of physicality. However, physicists and philosophers alike often disagree on which criterion, if any, can successfully demonstrate that NCST theories entail the disappearance of spacetime. More specifically, I have examined three such criteria (analogy with QM, tempered operationalism, and pragmatic significance) and shown that they each face significant challenges. The plurality of available criteria suggests that noncommutativity does not necessarily entail physical pointlessness. Rather, the latter can only follow if the selected criterion of physicality can be supported by additional arguments, despite its expected shortcomings.

Still, the advocate of spacetime disappearance in the noncommutative setting might contend that such a criterion can be identified and properly defended. Consequently, she can leverage this result to argue that NCST is a mere placeholder for a mathematical structure devoid of spatiotemporal content. This instance of disappearance of spacetime in the noncommutative setting demands careful scrutiny. In particular, the implication from physical pointlessness to spacetime disappearance relies on a family of specific functions that points are supposed to fulfil in spatiotemporal theories, but fail to realise in the noncommutative setting.

Importantly, the adversary's argument rests on the extension of commutative results to a novel, unexplored, high-energy regime, where it is conjectured that the underlying geometry becomes noncommutative. Specifically, the argument for the disappearance of spacetime necessitates three elements. First, it requires an account of the relationship between spacetime points (or the disappearance thereof) and spacetime. Second, it requires the proponent of the argument to clarify the features of a properly physical structure, in the sense of offering precise criteria of *physicality* and *spatiotemporality* to characterise the geometry as mathematical, physical, or phenomenological. Finally, it requires a clear interpretation of the physical effects of noncommutativity.

In this paper, I have challenged the argument for spacetime disappearance and argued that pointlessness, if proved, can only provide limited justification for the disappearance of spacetime at high energies under restrictive conditions. Indeed, without the indicated specifications, the argument against NCST deflates to a mere suggestion, based on an analogy with alternative theories of QG.

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