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From nothingness to time, Planck's constant, and endless accretion

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Abstract

This article presents a theory of fundamental physics that quantifies a growing network of elements and links. The first two linked elements in existence arise irreversibly from nothingness. The network subsequently keeps growing by one element at a time. It has no free parameters and only one global parameter, the total number of elements N. The network statistics produce energy conservation when each link corresponds to a quantum of action. For large N, the statistics produce an internally observable three-dimensional space. Together with N, space forms a spacetime with a time dimension that follows the same statistics as each spatial dimension. Consequently, spacetime has a maximum internal speed, time reversibility, translation invariance, and it agrees with special relativity. Dense clusters of elements and links form particles, curving spacetime as in general relativity. In spacetime, particles can move and can appear as multiple instances. Observation of particles follows the rules of quantum physics, and the network agrees with the Feynman path integral.

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Contents

1 Introduction

Both cosmology and high-energy physics have been steadily working backwards towards time zero, the time when the universe presumably began in a state of high energy. But there is a lack of concordance between the major physical theories involved, general relativity for the dynamics of spacetime and quantum field theory for the dynamics within spacetime. Both theories are highly successful in their own domains, but they are in conflict where those domains overlap. A widely accepted successor to these theories has not yet been formulated.

The theory that is presented here approaches the problem from the opposite direction. It starts at time zero from a cosmological minimum and then works forward in time. It is shown here that this leads to a network with structures and dynamics that match existing physics and cosmology. Since those are huge fields of research that could not possibly be covered completely at once, the strategy here is to focus on key topics that are both crucially important and well established empirically. The theory is shown to agree on those topics. It is, thus, viable. It even has an excellent chance to be the long sought-after correct representation of reality at its most fundamental. Probed theoretical knowledge is matched well, but the theory deviates where this is expected, in areas where there is tension between existing theories or between theories and observations. Even so, many topics are not or only lightly touched, such as those concerning elementary forces and elementary particles. A major purpose of this article is, then, to kick-start this branch of research rather than to claim that it is close to completion. Although not using discrete spacetime or sets, the overall approach has parallels with causal set theory ([1], [2]). Relations with this and other theories are discussed in Section 15.

Section 2 below takes the bold step to derive the theory from the ultimate cosmological minimum, pure nothingness. The argument is exact and compelling, but it is based on physical reasoning rather than on mathematics. Readers who are primarily interested in the foundational and conceptual aspects of the theory might read that section and then go straight to the Discussion (Section 15). Readers who are primarily interested in the mathematical development of the theory might skip to Section 3 on first reading. All they need to know is the main result of Section 2, which can be summarized in a few lines. The very first structure consists of two elements connected by an undirected link. An element consists fully of the property 'being connected' and a link consists fully of the property 'connecting.' The initial structure subsequently keeps growing by new elements becoming attached, one by one. Each new element attaches with one or more new links to the then existing elements. This happens arbitrarily, regarding both to how many elements it links and to which ones in particular. Elements and links, once formed, never disappear, and links never switch between elements. The network has no free parameters and only one global parameter, N, the increasing total number of elements. That is just about all there is to the theory, and it is the starting point of the quantitative analysis in Section 3 and subsequent sections. The results support the fundamental conjecture that statistical patterns in the growing network of elements and links form not only spacetime itself, but also everything within spacetime.

The plan of the article is as follows. In Section 3 the basic statistics of the network are derived. When N becomes sufficiently large, the network forms, purely because of its statistics, a growing three-dimensional space (Section 4). Whereas N is an irreversible network time (related monotonically to the age of the universe), the locally observed time follows the same statistics as each spatial dimension. This produces a speed maximum in spacetime (Section 5), locally observed time reversibility and translation invariance, and the transformations of special relativity (Section 7). The energy and momentum transformations follow from the assumption that each link in the network corresponds in spacetime to a quantum of action. Combined with the network statistics, this gives conservation of energy in the network (Section 6). The network growth produces massive particles as clusters of elements and links (Section 6), which are shown to curve spacetime in the same way as specified by general relativity (Section 8). This enables a calculation of the present value of N from the main physical constants, including the Hubble constant (Section 8). How entropy increases in the network is derived in Section 9. Observation of particles is shown to follow the rules of quantum physics (Section 10). Combining paths in the network is consistent with a Feynman path integral (Section 11). Photons correspond to a specific subset of new elements (Section 12). Symmetries of particles (Section 13) and the high-energy creation and destruction of particles (Section 14) are briefly discussed. The article is concluded with a general Discussion (Section 15).

2 A structure emerging from nothingness

An explanation of the origin of all that exists should be based on as few assumptions as possible, preferably none. Any specific assumption would be an unexplained fact and, thus, would make the explanation incomplete. Arguably, the only starting point that leaves nothing to be specified is pure nothingness. Without anything to be specified, there are no unexplained facts. But could anything emerge from nothingness? The purpose of this section is to propose and explain a way by which this can happen. The explanation is preceded by a discussion of nothingness and by a discussion of what kind of explanation would be acceptable.

Nothingness is understood here in the strictest sense possible. There is really nothing at all. No energy, no matter, no fields, no vacuum, no space, no time, no causality, no causation, no probabilities, no probability amplitudes, nothing abstract, no sets, no relations, no structure, no state, no principles, no symmetries, no rules, no laws of physics, no dispositions, and no information. These items and anything else must be excluded, first of all, because their presence would conflict with 'really nothing at all.' But many of them must be excluded for an additional reason, a reason that is also used in the argument further below. It is, then, useful to discuss this in a bit more detail here, focussing on the last three items above. The additional reason for excluding them is that they depend implicitly on a power to point. The human mind clearly has this capacity to be directed towards something or to let one thing point to or refer to another thing, a capacity that is essential for language and mathematics. A power to point is presumably also present, in weaker forms, in other biological species than humans. It is highly likely an invention of biological evolution, plausibly originating from an evolvable form of estimation ([3], [4]). That the power to point is strictly a biological capacity, irrespective of its detailed explanation, underlies the current analysis. Then it makes no sense to let this power participate in nature at a time when there was no life yet. The fact that the last three items in the list above depend on pointing can be understood from the following considerations. First, regarding laws of physics: the constituents of such laws are meaningful only when they point to specific parts of nature and to how these parts are related. In other words, laws could only work, by themselves, if they somehow had an internal power to point. Since such a power is a biological capacity, laws cannot point by themselves and, thus, cannot produce, control or prescribe nature's regularities. Note that this is not an issue when laws are used, as in a scientific theory, to predict and describe rather than to prescribe. Then the human power to point takes the role of letting the constituents of the law refer to the right parts of nature and to the right types of relation or operation. Second, regarding dispositions: a disposition (in the sense of a propensity or potency) can be useful as well for predicting and describing. But any innate disposition, potential, or power of something to produce change depends on an implicit reference to possible happenings in the future. Neither nothingness nor anything fundamental can possess such a capacity to point across time. And, finally, regarding information: this depends on comparing and counting states, which requires pointing to such states. Again, information is useful for describing, but at a fundamental level it could not drive nature.

Suppose that we start with this rigorously minimal form of nothingness and that we are interested in how it might change. What kind of explanation of a change would then be acceptable? Two things are clearly not acceptable. First, there could be no laws that drive change. There are no laws to start with, and if changes somehow give rise to an apparent pattern or regularity, then that would not indicate a prescribing law. At most, it could be formulated as a describing law, depending on the human power to point. Second, a change to nothingness could not originate from something external, since there is nothing external. Yet, a change that occurs without depending on a law or on something external might be acceptable as part of an explanation. If, because of such a change, nothingness gets one or more specific properties, it ceases to be non-existing and starts to be an entity (the term 'entity' is used here for anything that exists). For example, one might suppose that a primordial entity with at least one physical property, such as energy, could spring into existence spontaneously. A problem here is that nearly all choices for physical properties require novel, specific information. Physical properties usually have to be specified with respect to dimension and quantity. If the created property would be of a fixed and determined nature, its occurrence would presuppose a law of physics, which is not acceptable. Alternatively, one might propose that dimension and quantity are not fixed and determined, but occur randomly or arbitrarily. But a random occurrence requires a range of options with associated probabilities, which both depend on a power to point. Even an arbitrary occurrence requires at least a range of options, which would require pointing in this particular

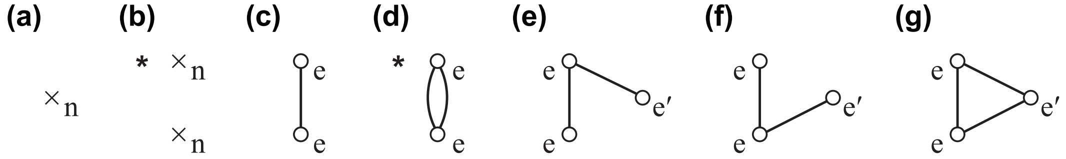

Figure 1: **a** The cross symbolizes nothingness, denoted by n. **b** Two copies of nothingness. The asterisk indicates that this is an impossible state. **c** Two linked copies of nothingness converted to entities e (called 'elements') because of the link (an entity symbolized by the line segment). **d** Two elements connected by two links. The asterisk indicates that this is an impossible state. **e-g** All possible structures with three elements (two existing e and a new e′).

case, because nothingness does not contain any such range by itself. More generally, any change to nothingness that appears to be imported from the outside, with imposed external information, must be ruled out if we want to avoid invoking, tacitly, a power to point.

From the discussion above it is clear that any change to nothingness has to proceed from nothingness itself. But there is not much to work with, because there is only nothingness, and nothingness itself is non-existent. Naively, one might propose that a possible change might be when a second copy of nothingness arises. Having two is different from having one, and such a change would not require anything external. But there are two major problems with this proposal. The first problem is that nothingness is not the kind of thing that could be counted. It is not even a thing. Saying that there is one nothingness seems equally ill-defined as saying that there is zero nothingness or saying that there are two or more copies of nothingness. Counting nothingness does not make sense. Even if we ignore this problem for the moment, there is a second problem. That problem is related to the fact that nothingness means not only that there is nothing, but also that there is nothing beyond nothingness. There is not even a beyond. Then where would a second copy of nothingness go? The two copies of nothingness would need to be separated in some way in order to be countable. But there is no space to separate the copies spatially, no time to separate them temporally, and indeed nothing at all to keep them separate. Therefore, if there would be two copies of nothingness, they could not be separate but would, by necessity, be connected. But if a copy of nothingness is connected, it has a primordial property (the property 'being connected'). Then, it has stopped being pure nothingness and has become something that exists, an entity. Importantly, being connected is here a property that requires no external information. It is a purely internal property. What we have here appears, thus, to be a possible state: two entities connected by a third entity, a link. The two entities are each nothingness brought to existence because they have a link, and the link exists because it connects two entities. Figure 1 illustrates this idea. In Fig. 1a the cross marked by n stands for nothingness. This is intended as purely descriptive and n does not stand for something that actually exists, since nothingness is not an entity. Still, Fig. 1a depicts a possible state, the state of pure nothingness (we will use the terms ‘state’ and ‘structure’ in a few cases as just a figure of speech: nothingness is not literally a state). Figure 1b shows two copies of n, again purely descriptively. The asterisk indicates that this is an impossible state, for the two reasons given above (n is not a countable entity and cannot be separated anyway). Figure 1c shows the state with two linked entities, with the link (that is, the linking entity) symbolized by a line segment and the linked entities by open circles marked by e. Whereas e was n before, it is now an entity (because of the link) and will be called ‘element’ below. This state is possible. The elements and the link are three basic entities that form the basic structure of Fig. 1c. All entities consist fully of a single property: either ‘being connected’ or ‘connecting.’ For convenience, the further explanation below will use *a*-*g* for referring to the states depicted in Figs. 1a-g.

There is no time yet and not even an ordering. The states *a* and *c* are each possible, but we have not yet discussed whether they can transition into each other. Since there is no time or any other dimension, we even cannot assume that they lie along a shared dimension. For the moment, they are just distinct, possible states. So let us discuss whether transitions are possible. Going from *a* to *c* seems to require stepping through an intermediate, impossible state, *b*: first get a second copy of nothingness, then let it connect to the first copy, and as a result arrive at *c*. However, such a way of thinking tacitly assumes time and a dynamic. But there is no time, and any transition can be discrete at most. Then skipping *b* and going straight from *a* to *c* appears possible: there is no reason to rule it out and there is nothing around that could block it. The transition to *c* and the creation of link and elements cannot be driven by a law or by anything else, but must occur spontaneously. Importantly, the transition from *a* to *c* is a purely internal affair. No external information is used, no laws, and no pointing of any kind. Moreover, it would be wrong to state that if *c* arises from *a*, that *a* was not really nothingness because apparently it had a property, namely the potential or disposition to produce *c*. That argument is false because, as discussed above, dispositions depend on the tacit assumption that there is a power to point (in this case, a power of nothingness to point to a potential state *c*). Since nothingness cannot have a biological capacity such as the power to point, it cannot have a dispositional property.

Having established that a transition from *a* to *c* is possible, we can now analyze whether a transition from *c* to *a* is possible as well. That would involve a transition of a structure with three entities (the two elements and the link) towards nothingness. Thus, all entities must disappear. But the link cannot disappear as long as the elements exist: if only the link would disappear, the state *b* would arise, which is not possible. Then, alternatively, perhaps an element could disappear? An element could only disappear if it would return to nothingness. But it is already nothingness, with a link. If it would return to nothingness, it would become nothingness with the link still existing, and thus would remain an element. This means that an element cannot disappear as long as the link exists. One might think that the elements of c could disappear together with the link, in one transition. But how can the elements and the link know that they are supposed to disappear together, in a single transition? Recall that there is no dynamics, nor time. Then synchronized change across more than one entity—here the synchronized disappearance of elements and link—would require a kind of law-like coordination that depends on a power to point and that is not possible without such a power. Since there is no power to point, the transition from c to a is blocked. Note that the transition from a to c does not depend on a coordinating power to point, because the entities do not yet exist before the transition. Thus, there is nothing that could be coordinated and synchronized. If the entities could exist independently of one another, it would indeed be impossible for them to appear at the same transition, because there would be no way to coordinate that. But there is no such independence. For existing, the entities necessarily appear together, without requiring any coordination. In conclusion, the transition from nothingness to the basic structure of c not only creates something that exists, but also does so irreversibly. This establishes a basic order: a to c, but not c to a.

Even if the transition from a to c is possible, one may wonder if it is necessary, that is, if it has to occur. If not, it might just not happen, and the state of nothingness would then never change. A related question is how easy or likely the transition from a to c is. However, there is no plausible way to define, let alone determine, a probability for this transition. Fortunately, the fact that the transition from c to a is blocked helps here. No matter how hard or improbable it is to go from a to c, if the transition occurs, then there is no way back. One can think here, by analogy, of a chemical reaction where two different forms X and Y of a substance can be converted into each other. Now suppose that the conversion rate from Y to X is zero, that is, the transition from Y to X is fully blocked. Then no matter how small the conversion rate from X to Y is, it will always result in all X becoming converted to Y. It may just take a very long time. Because in our case time does not yet exist, even an infinitesimal chance that the transition from a to c occurs must result in c. In other words, unless the transition from a to c is completely impossible, c is bound to result. Since we have already established that the transition from a to c is possible, it follows that this transition has to occur. The transition does not take a short time or a long time, because there is no time yet. It is just a discrete step that is certain.

The structure c consists of two elements and a link. The link ensures the existence of the elements and the elements ensure the existence of the link. However, we have tacitly assumed here that there is only a single link, that is, that the connection consists of a single entity. Could there be more than one link between the elements, such as two links (d) or even more? The elements of d would then still exist because they are connected, and the links of d would still exist because they connect elements. Yet, states such as d are actually impossible (as indicated by the asterisk) for the following reasons. If the connection could consist of more than one link, then the number of links would be open, since any number would be consistent with the existence of the two elements. The number of links could then either be fixed, arbitrary or random. Each of these possibilities fails. A fixed number of links, say always 2 or always 21, presupposes a prescriptive law with externally imposed information (the specific number), which is not acceptable. An arbitrary number of links (arbitrary in the sense of random without being controlled by probabilities) would not require a law if there would be a definite range of options. But a definite range is lacking here, for there would be no upper limit on the number of links. An unbounded number of links would make no sense physically, whereas assuming an upper limit would presuppose a law. Finally, a random number of links would require a range of options and associated probabilities, which would both require externally imposed information and must, therefore, be ruled out. The only possibility is a connection consisting of a single link. Although having exactly one link may be viewed as a (special) case of probability distribution (with only one possible outcome), it is the only case that does not require counting, pointing, or any external information. It is internally generated in the transition from a to c. Because having only one link is necessary, one might argue that it is a law in disguise. But it would only be a descriptive law for a fact that arises in another way than by being prescribed by a law.

Once we have structure c, a third element can become linked. This only requires a new copy of nothingness to become attached to the existing structure. As before, the new copy and the existing structure cannot be separate, because there are no means to have them separated. By necessity, any new copy of nothingness attaches as a new element to the existing structure. The attachment cannot modify the existing structure, because such a change would require synchronized coordination between the structure and the new element. Therefore, any link of the new element must attach to an existing element. This would not change the existing element, since elements cannot count and exist in the same way irrespective of the number of links that they have (if at least one). In contrast, we can rule out that a new link attaches to the existing link of c, because that would change the nature of that link and thus would require coordination between the new element and the existing structure. There are now several possibilities for how the new element can link to elements of the existing structure. All existing elements are equivalent from the point of view of the new element, as there is no way by which the new element can know how the existing elements are embedded in the existing structure and, thus, no way to tell them apart. Therefore, the new element must attach to the existing elements arbitrarily, regarding both to how many elements it links and to which ones in particular. From the point of view of the new element it is indistinguishable to how many existing elements it attaches (because it cannot count), as long as it is at least to one (because otherwise the new element would not exist). The new element e$'$ may then attach exclusively to one of the elements (e), exclusively to the other element (f), or to both elements (g). More than one link between the new element and any of the existing elements must be ruled out for the same reasons as *d* must be ruled out (see above). Thus, *e*-*g* are the only possible states. Possibilities *e* and *f* add two new entities (an element and a link), whereas possibility *g* adds three new entities (an element and two links). Neither of these possibilities require synchronized coordination, since the new entities do not yet exist before the transition. They can appear in the same transition because they depend on one another for existing. One might think that when there are two links, these links are independent of each other, since taking one away would not affect the other. However, that is not correct: the links are not independent, for they exist under the condition that there is at least one link. In conclusion, all three possibilities are free of implicit pointing and all are realizable. Which possibility is actually realized can only be arbitrary, driven by neither law nor probabilities.

Again, the creation of a new element is irreversible. The new element cannot disappear as long as any of its links exist, and if all links would be gone an impossible state would result (a copy of nothingness separate from the existing structure). As before, a synchronized disappearance of the new element and all of its links cannot occur without tacitly invoking a power to point. Therefore, the new element is there to stay. Since there is no time and the attachment of a new element is possible and irreversible, it has to occur. One may now wonder whether going from *a* (pure nothingness) to a structure such as in *e* (with a total of three elements) requires going via the intermediate structure *c* with two elements. In other words, is it possible to go directly from *a* to *e*? At first sight, one might think this is possible. All that is needed is two new copies of nothingness attaching as two elements to the nothingness of *a*, all occurring in the same transition. However, the two new elements would exist independently of each other: each element would exist irrespective of whether the other element would exist or not. Having them both appear in a single transition is then impossible: there is no way by which that could be synchronized. A direct transition from *a* to *f* is then also impossible, as *f* is equivalent to *e*. A direct transition from *a* to *g* would require e′ to become linked not only to the nothingness of *a* but also to an element e that does not yet exist and whose existence would be independent of e′. That is not possible either, again because it would require a coordination that is not available (e′ cannot know when e appears, whereas e can appear irrespective of the status of e′). The attachment of a single new element to *c* is the only possibility to get to one of the states *e*-*g*.

The structure will keep growing by the attachment of further new elements. That the structure can grow by only one element at a time is generally true. This can be understood by showing that the alternative does not work. Let us assume, then, that we have already a structure S consisting of at least two elements and that a group of two or more new elements is added, in a single transition (Fig. 2 shows an example). One might think that such a group could form an independent structure S′ that would remain separate from S (Fig. 2a). But that is not possible, since there are still no means to keep S and S′ separated (the asterisk in the figure indicates that the state

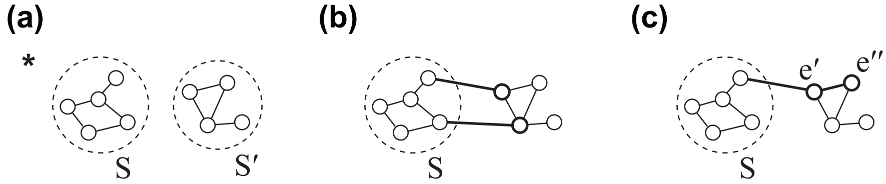

Figure 2: **a** A new structure S′ cannot arise and exist as separate from an existing structure S for lack of means to separate them. The asterisk indicates that this is an impossible state. **b** When at least two elements of a group of new elements link directly to S, then they exist independently of each other and could not have appeared in the same transition. **c** When only one new element e′ links directly to S and another new element e″ links indirectly to S via e′, then these elements could not have appeared in the same transition (because e′ exists independently of e″, whereas e″ depends on e′ existing).

is impossible). Of course, one might still conceive of S and S′ as separate, but that would depend on pointing and could not be realized. Therefore, at least one of the new elements must attach directly to S, and all new elements must be connected to S either directly or indirectly (via other new elements). Now there are two cases. The first case is when at least two of the new elements link directly to S (Fig. 2b shows an example). Each of these two new elements would then exist irrespective of whether the other one would exist or not. Then it would be impossible for them to appear in the same transition, because there would be no way to coordinate that. Although the state of Fig. 2b is possible, it can only be realized by a series of transitions. The second case is when there is only one element, say e′, that links directly to S (Fig. 2c shows an example). Then there must be at least one other new element, say e″, that links directly to e′ in order to be connected to S, as in a cascade. But the existence of e″ makes a single transition impossible, because e″ would have to link in that transition to an element e′ that does not yet exist and whose existence would be independent of e″. Again, this would require a coordination that is not there (e″ cannot know when e′ appears, whereas e′ can appear irrespective of e″). We conclude that our original assumption (adding a least two elements to S in a single transition) does not lead to possible transitions. Therefore, elements must become added one by one, without exception. Each addition is certain, for the same reason as given above for *c* following *a* (the addition is possible and irreversible, with time absent). Structures grow from nothingness to two elements, then three, then four, then five, and so on. Since each transition is irreversible, the resulting structures are ordered. One can think of them as lying along a single dimension that has some of the characteristics of time. The attachment of each new element then resembles the tick of a clock. Other than with ordinary clocks, though, time does not exist between the ticks.

The precise form of the series of structures (with the exception of c) depends on how each element was added exactly, that is, to which existing element or elements it became attached. These attachments must always be arbitrary with respect to how many and which of the existing elements become linked, for the same reasons as discussed above for which states can follow c. Once a possible state has been realized (such as e), it stays like that no matter how many elements are subsequently added to the structure. An existing element can get new links with newly forming elements, but not with already existing elements. Existing elements cannot form new links with one another for the following reasons. First of all, forming such a link would not depend on the creation of a new element. The formation would, therefore, not be irreversible and would lack the associated necessity. One might propose that links could form spontaneously and randomly, but that would require a law that specifies the probability that a link forms between subsequent ticks of structure growth. Since such a law would depend on external information, it must be ruled out. Thus, an existing structure cannot acquire new internal links. Moreover, it also cannot lose existing links, for several reasons. First, existing links cannot disappear because that would either be reversible or lead to an impossible state (with an element or group of elements becoming separated from the main structure, whereas there is still nothing that can do the separation). A specific link could not know whether disappearing would be reversible or not, since it cannot know whether it is the only link keeping the connected element in existence. Second, a random disappearance of links would require a probability that would depend on external information. And finally, we note that existing links must be persistent, as all links are equal (they are fully specified by the primordial property of connecting two elements) and the link of c was shown to be persistent. Then all links must be persistent, for a specific link cannot know whether it is the link of c or any other link. Knowing the position of a link in the structure would require a capacity to point.

In conclusion, what we have established here is the necessary creation of something from nothingness, irreversibility (creation but no destruction), an ordered, time-like sequence of states, and an ever expanding world—a growing structure of linked elements—that does not depend on a power to point. The creation and sequence of states give rise to regularities that are internally produced but that can nevertheless be described conveniently by laws. Indeed, it is possible to derive equations for the statistics of the sequence of states (Section 3 and beyond). They follow from the arbitrary attachment of new elements, which is not an assumption but rather the only possibility that avoids externally imposed information. The resulting equations have interesting properties and consequences, but they are purely descriptive. Thus, we also see the emergence of laws of physics, albeit purely descriptive ones, as expected.

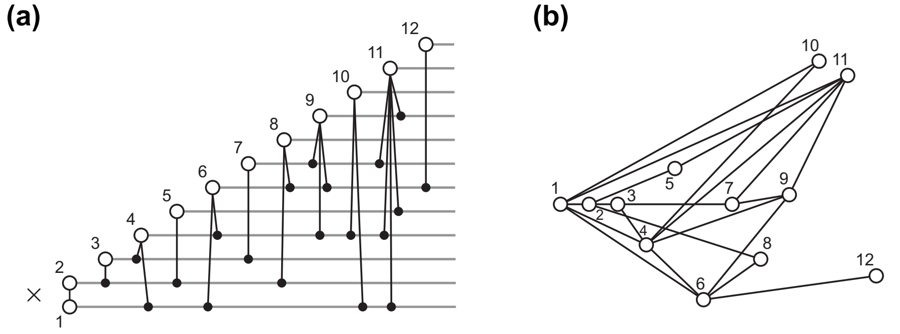

Figure 3: **a** Example of network growth, represented as a stack diagram. Open circles stand for subsequently produced elements, which remain present permanently (grey horizontal lines). A new element becomes linked to one or more existing elements (links symbolized by downward lines, drawn slightly skewed for clarity). Links do not change, once formed. The cross denotes nothingness, as in Fig. 1a. The numbers indicate the rank order of the elements (arbitrary for the first two elements). **b** Alternative representation of the same network. There is no special structure when the number of elements is small.

3 The growing network

The structure described in the previous section resembles a network. It consists of elements connected by links. Henceforth, the total structure will be called 'element network' (or 'network' for short). We can then use the term 'structure' for any substructure that arises within the overall network.

When a new element attaches to the existing network, it forms links with one or more existing elements. As these elements exist before the new one, we will call them the parents of the element. The new element is then a child of each of these existing elements. Other than in biology, though, it is not a parent but the child that produces both childhood and parenthood. As argued in the previous section, the only way by which a new element can be formed is by attaching arbitrarily to the existing elements. Thus, when element $N+1$ forms in a situation where there are already N elements in existence, this new element might be linked to any number of elements $k \in [1, 2, \ldots, N]$, where k is the number of parents of element $N+1$. Figure 3a shows a stack diagram of an example of network growth. Each open circle represents a new element, with the attached grey horizontal line symbolizing its permanent existence once created. A new element links at the point of creation to one or more parents. The links are indicated by downward lines, which are drawn slightly skewed for clarity; the linked parents are indicated by solid circles. Once links exist, they never change. Figure 3b shows the same structure in another way. For small N there is no special structure, but Section 4 will show that this changes when N is large. We will now first derive several basic properties of the network that will be needed in the sections that follow.

The probability distribution of the number of parents of element $N+1$ can be derived as follows. From the point of view of a new element, any existing element is identical and thus has the same likelihood of becoming linked as any other existing element. Therefore, the likelihood of linking to k parents must by proportional to k. There are $\binom{N}{k}$ ways to choose k parents from a group of N, thus the probability P_k that a new element has k parents is

$$P_k = k\binom{N}{k}p^k(1-p)^{N-k}, \tag{1}$$

with p the probability that any particular parent is chosen. As there are N parents to choose from, $p = 1/N$. This gives

$$P_k = k\binom{N}{k}\left(\frac{1}{N}\right)^k\left(1-\frac{1}{N}\right)^{N-k}. \tag{2}$$

Note that this distribution depends only on parameters (N and k) that are properties of the network itself, and no external information is used (in agreement with Section 2). The distribution describes the probabilities of specific outcomes when a new element connects to the existing network, but connecting is in no way controlled or driven by probabilities: it is fully arbitrary given the N possible parents. The distribution P_k is normalized, which can be readily seen by starting from (1)

$$\sum_{k=1}^{N} P_k = \sum_{k=1}^{N} k\binom{N}{k}p^k\left(1-p\right)^{N-k} = Np = N\frac{1}{N} = 1, \tag{3}$$

where we used the fact that the second expression is identical to the mean of a binomial distribution, which is known to be Np. The mean number of parents per element is

$$\sum_{k=1}^{N} kP_k = \sum_{k=1}^{N} k^2\binom{N}{k}\left(\frac{1}{N}\right)^k\left(1-\frac{1}{N}\right)^{N-k} = 2-\frac{1}{N}. \tag{4}$$

This follows from the fact that the middle expression is identical to the second moment of a binomial distribution, which is known to be $pN + p^2N(N-1)$, here with $p = 1/N$. For $N \gg 1$, the mean number of parents per element is 2, not depending on N.

Below, N will be assumed to be extremely large (with $\ln N \gg 1$, and $k \ll N$). By using $\lim_{N\to\infty}\left(1+\frac{x}{N}\right)^N = e^x$, (2) can then be approximated by

$$P_k \approx k\frac{N^k}{k!}\left(\frac{1}{N}\right)^k e^{-1} = \frac{1}{e}\frac{1}{(k-1)!}, \tag{5}$$

with $P_0 = 0$. This shows that an element has a probability of $1/e$ (about 37%) to have one parent, also $1/e$ for two parents, $1/(2e)$ for three, and then quickly decreasing probabilities for more parents. The probability distribution is effectively invariant for any new element (unless it was created when N was small, where the approximation is invalid and (2) should be used). It can be recognized as a Poisson distribution $\lambda^k e^{-\lambda}/k!$ with $\lambda = 1$ and shifted one unit to the right. Then it is clear that it is normalized and that its mean is $\lambda + 1$, which equals 2, as before. Its variance is λ, which equals 1. The number of parents of an element, that is, the number of links produced when the element is formed, is an important parameter when we discuss mass, gravity, spin, quantum physics, and photons. For convenience, we therefore define the symbols ^{1}e for a one-parent-element (i.e., an element with one parent), ^{2}e for a two-parent-element, and so on. A symbol such as $^{3+}$e indicates an element with three or more parents.

The expected number of children per element can be calculated as follows. For this we need to denote each element by its rank order, that is, an element gets the number i if there are $(i-1)$ elements in existence at the moment it is created (see Fig. 3). The first two elements in existence get, in arbitrary order, the numbers 1 and 2, by definition. Suppose now that we want to calculate the expected number of children of element i when there are N elements in total. When element $(i+1)$ was created, its parents had to come from the i existing elements. For a mean of two parents, element i then had a $2/i$ probability of being a parent of element $(i+1)$ (assuming $i, j \gg 1$). In a similar way, element i had a $2/(i+1)$ probability of being a parent of element $(i+2)$. Summing all contributions from elements $(i+1)$ to N gives the expected number of children n_c of element i

$$n_c\left(i\right) = \sum_{k=i+1}^{N} \frac{2}{k-1} \approx \int_{\frac{i}{N}}^{1} \mathrm{d}x \frac{2}{x} = 2\ln\left(\frac{N}{i}\right), \tag{6}$$

with $x = (k-1)/N$. This shows that the most recently formed elements (i close to N) are unlikely to have children, whereas element 1 is expected to have $2\ln N$ children. The latter also applies, to good approximation, to other elements i close to 1 (with $\ln i \ll \ln N$). Averaging over all existing elements gives the overall mean number of children per element

$$\frac{1}{N}\sum_{i=1}^{N} n_c\left(i\right) = \frac{1}{N}\sum_{i=1}^{N} 2\ln\left(\frac{N}{i}\right) \approx -2\int_{\frac{1}{N}}^{1} \mathrm{d}x \ln x = -2\left[x\ln x - x\right]_{\frac{1}{N}}^{1} \approx 2. \tag{7}$$

This is again 2, equal to the overall mean number of parents per element.

In the next section we will need expressions for the expected number of links between a particular element i and another particular element j. Let us first assume that $i > j$ (and that $i \gg 1$). The only way by which element i can become directly linked to an already existing element j is when j is one of i's parents when i is created. Because

i has, on average, 2 parents, and all $(i-1)$ existing elements have an equal chance of being a parent of i, the expected number of links $w(i,j)$ between i and j is

$$w(i,j) = \frac{2}{(i-1)} \quad (i > j). \tag{8}$$

If, on the other hand, $i < j$, then the only way by which i can become directly linked to j is when i is one of j's parents when j is created. On average, i has two chances to be j's parent, which gives (assuming that $j \gg 1$)

$$w(i,j) = \frac{2}{(j-1)} \quad (i < j). \tag{9}$$

Because an element cannot link to itself, $w(i,j) = 0$ for $i = j$. With $i \neq j$, (8) and (9) can be summarized by

$$w(i,j) = \frac{2}{(\max(i,j) - 1)} \quad (i \neq j). \tag{10}$$

One can interpret $w(i,j)$ as the relative frequency of links between i and j that one would find when a large ensemble of networks would be generated. In M networks one would expect to find a total of $w(i,j)M$ links between elements i and j.

A variant of (8) and (9) is $w_{\mathrm{p}}(i,j)$, the expected number of links between elements i and j under the condition that j is a parent of i $(i > j)$

$$\begin{aligned} w_{\mathrm{p}}(i,j) &= \frac{2}{(i-1)} \qquad (i > j) \\ &= 0 \qquad\qquad (i \leq j) \end{aligned} \tag{11}$$

Another variant is $w_{\mathrm{c}}(i,j)$, the expected number of links between elements i and j under the condition that j is a child of i $(i < j)$

$$\begin{aligned} w_{\mathrm{c}}(i,j) &= \frac{2}{(j-1)} \qquad (i < j) \\ &= 0 \qquad\qquad (i \geq j). \end{aligned} \tag{12}$$

Below, calculations will require $w(i,j)$ to be summed over i or j. For those calculations it is convenient to approximate the summations by integrations. For example

$$\sum_{i=1}^{N} w(i,j) = \sum_{i=1}^{j-1} \frac{2}{(j-1)} + \sum_{i=j+1}^{N} \frac{2}{(i-1)} \approx \int_0^y \mathrm{d}x \frac{2}{y} + \int_y^1 \mathrm{d}x \frac{2}{x} \ , \tag{13}$$

with $x = i/N$ and $y = j/N$. Here x and y can be interpreted as normalized rank orders, corresponding to the rank orders i and j. We can now define a function $g(x,y)$ such that

$$\sum_{i=1}^{N} w(i,j) \approx \int_0^1 \mathrm{d}x\, g(x,y) \quad \text{with} \quad g(x,y) = \begin{cases} \frac{2}{x} & (x \geq y) \\ \frac{2}{y} & (y > x) \end{cases} . \tag{14}$$

Note that $g(x, y) = g(y, x)$. Averaging a function $f(i)$ is approximated in a similar way

$$\frac{1}{N}\sum_{i=1}^{N} f(i) \approx \int_0^1 \mathrm{d}x\, f(x), \tag{15}$$

with $x = i/N$ and $\mathrm{d}x = \Delta i/N = 1/N$.

4 An emerging three-dimensional space

The network introduced above is growing, that is, the total number of elements, N, increases. When one looks at the network for a given N (thus taking a snapshot, with temporarily suspended growth), the elements and links resemble a connected graph (Fig. 3). In mathematical graph theory, elements are usually called vertices (or nodes), and the links between them are usually called edges. Vertices and edges are then abstract ways to describe a connected system (which may be of any kind, such as physical, sociological, or purely symbolical). In order to emphasize that the present network literally describes a physical system with physical connections, we will keep using the terms 'elements' and 'links.' Both refer to entities, that is, elements and links exist. Links are not relations, but physical entities (one might think of them as dots of glue). One consequence of this is that self-loops would make no sense, whereas such loops are possible in graph theory.

Links are produced when a new element is added to the existing network. They are not directed and indeed lack internal structure. Hence, the network resembles an undirected graph. Any two elements are then connected by stepping through a series of links, forming a path between the two elements. Elements may be connected via more than one path. Because of the way the network is generated, all elements are necessarily connected by going through elements 1, 2, or both. But usually there will be other, shorter paths as well. For example, elements 6 and 8 in Fig. 3 have a direct connection, which is shorter than going via elements 1 and 2. We will be interested here in the shortest path between elements, defined as the minimal number of links separating them.

The network is overall sparsely connected, since on average only two links are formed when a new element is added. However, links are formed arbitrarily, which means that there will also be, purely by chance, densely connected clusters of elements. Densely connected means here that there are many different short paths between the elements that belong to such a cluster. Compound structures consisting of many clusters might arise as well. How would the network appear to such structures, when viewed from within? Elements, clusters and compounds are then regarded as internal observers O, with the term 'observing' meant here in the general physical sense of engaging in an interaction (rather than in the specific biological sense). An observer O engages in an interaction with the network when a new element e attaches to the network with at

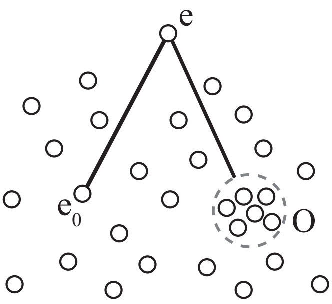

Figure 4: An observer O (an element, a cluster of elements, or a compound of clusters, symbolized by the dashed circle) interacts with the network via a newly attaching element e that happens to link to the observer and, in addition, links to an arbitrary element e_0 of the existing network. Elements are symbolized by the small circles, links other than the ones involved in the observation are not drawn for the sake of clarity.

least two links, one to O and one to any existing element e_0 of the network (Fig. 4). O thus observes the network through the latter link via the new element. Now we can ask how many direct neighbours the linked existing element e_0 has on average (i.e., the number of elements at link distance 1), how many neighbours of neighbours it has (link distance 2), and so on, for arbitrary link distance n. Using the function $w(i, j)$ defined in (8) and (9), we can calculate the expected number of neighbours W_n at link distance n as follows (under certain assumptions that are discussed further below)

$$W_n = \frac{1}{N} \sum_{i=1}^{N} \sum_{i_1=1}^{N} w(i, i_1) \sum_{i_2=1}^{N} w(i_1, i_2) \cdots \sum_{i_{n-1}=1}^{N} w(i_{n-2}, i_{n-1}) \sum_{i_n=1}^{N} w(i_{n-1}, i_n). \tag{16}$$

The first summation and division by N averages over all N existing elements i. This averaging takes account of the fact that observation of the network occurs via a new element that links arbitrarily to any element e_0 of the existing network (Fig. 4). We want to find the expected number of neighbours at link distance n for each of these i. For a particular i, the $w(i, i_1)$ in the second summation gives the expected number of links between i and i_1. The total expected number of neighbours is then obtained by summing over all possible i_1. Each of these i_1 has neighbours itself, as given by the third summation over i_2, and so forth.

The summations of (16) can be replaced with integrals by using (15) for the first summation and the function $g(x, y)$ of (14) for the remaining ones

$$W_n = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}x_1\, g(x, x_1) \int_0^1 \mathrm{d}x_2\, g(x_1, x_2) \cdots \int_0^1 \mathrm{d}x_{n-1}\, g(x_{n-2}, x_{n-1}) \int_0^1 \mathrm{d}x_n\, g(x_{n-1}, x_n). \tag{17}$$

Here $x = i/N$, $x_1 = i_1/N$, and so on (in a similar way as for (13).

Equation (17) assumes that the path $x \to x_1 \to \cdots \to x_n$ produced by iterating $g(x_i, x_j)$ is the shortest path between x and x_n. This is reasonable when $W_n \ll N$, because links are established arbitrarily and the expected number of parents and children of an arbitrary element is extremely small (2 each) compared with the total number of elements N (recall that $\ln N \gg 1$). Then the probability that a link from an element that is participating in W_n is to an element that already participated in any $W_m|_{m \leq n}$ of a particular starting element x, is extremely small as well. However, the assumption is not valid for large n. This will be quantified later, after we have analyzed (17).

Substituting (14) into (17) gives

$$\begin{aligned} W_n = &\int_0^1 \mathrm{d}x \left(\int_0^x \mathrm{d}x_1 \frac{2}{x} + \int_x^1 \mathrm{d}x_1 \frac{2}{x_1} \right) \left(\int_0^{x_1} \mathrm{d}x_2 \frac{2}{x_1} + \int_{x_1}^1 \mathrm{d}x_2 \frac{2}{x_2} \right) \cdots \\ &\left(\int_0^{x_{n-1}} \mathrm{d}x_n \frac{2}{x_{n-1}} + \int_{x_{n-1}}^1 \mathrm{d}x_n \frac{2}{x_n} \right) \\ = 2^n &\int_0^1 \mathrm{d}x \left(\int_0^x \mathrm{d}x_1 \frac{1}{x} + \int_x^1 \mathrm{d}x_1 \frac{1}{x_1} \right) \left(\int_0^{x_1} \mathrm{d}x_2 \frac{1}{x_1} + \int_{x_1}^1 \mathrm{d}x_2 \frac{1}{x_2} \right) \cdots \\ &\left(\int_0^{x_{n-1}} \mathrm{d}x_n \frac{1}{x_{n-1}} + \int_{x_{n-1}}^1 \mathrm{d}x_n \frac{1}{x_n} \right). \end{aligned} \tag{18}$$

The iterated integral has straightforward analytical solutions, but the number of terms quickly rises with increasing n. Computer algebra software (Mathematica; see appendix A for the code) shows that the integral (excluding the factor 2^n) evaluates to 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58768, and 208012 for $n = 1 \ldots 11$, respectively. According to the On-Line Encyclopedia of Integer Sequences [5], this corresponds to the Catalan numbers C_{n+1} (sequence A000108). Hence, we conjecture that

$$W_n = 2^n C_{n+1} = 2^n \frac{1}{n+2} \binom{2(n+1)}{n+1}. \tag{19}$$

Proving that this is indeed the general solution of (18) should be possible, but such a proof is still missing. Nevertheless, the number of corresponding digits in the sequence indicates that the conjecture of (19) is highly unlikely to be false.

Equation (19) can be further simplified for sufficiently large n, by using Stirling's approximation for factorials

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e} \right)^n, \tag{20}$$

which gives

$$\begin{aligned} W_n &= 2^n \frac{1}{n+2} \frac{(2\,(n+1))!}{(n+1)!\,(n+1)!} \\ &\approx 2^n \frac{1}{n+2} \frac{2^{2n+2}}{\sqrt{(\pi\,(n+1))}} = \frac{4}{(n+2)\sqrt{(\pi\,(n+1))}} \left(2^n\right)^3 = a_n \left(2^n\right)^3, \end{aligned} \tag{21}$$

where we define

$$a_n = \frac{4}{(n+2)\sqrt{(\pi\,(n+1))}} \approx \frac{4}{\sqrt{\pi} n^{3/2}}. \tag{22}$$

For sufficiently large n, (21) is dominated by the final factor. Then

$$W_n \propto (2^n)^3 . \tag{23}$$

Equation (23) shows that the expected number of neighbours at link distance n rises as the third power of a function of n. The element volume V_n, i.e., the expected total number of neighbouring elements up to a link distance n, can then be obtained by summation

$$V_n = \sum_{m=1}^{n} W_m = \sum_{m=1}^{n} a_m (2^m)^3 \approx a_n (2^n)^3 \left(1 + \frac{1}{8} + \frac{1}{64} + \cdots\right) = \frac{8}{7} a_n (2^n)^3 \propto (2^n)^3 , \tag{24}$$

assuming that n is sufficiently large such that a_m is almost a constant for values of m close to n. The element volume V_n increases in the same way as W_n, because the result is dominated by the final (most distant) term.

Equation (24) shows that V_n is proportional to the third power of a factor 2^n. We can then define a distance r_n from the observed element e_0 of the network to any element e' as $r_n = 2^n$, with n the link distance between e_0 and e'. The associated element volume V_n (defined as the expected number of elements surrounding e_0) then scales as r_n^3. Because this is a power law with an exponent of three, it corresponds to a Hausdorff (i.e., fractal) dimension of three. If V_n is the volume of a 3D sphere, then one might think that W_n would be proportional to the surface area of this sphere. However, W_n corresponds to a shell with thickness $\Delta n = 1$ and thus $\Delta r_n \propto 2^n$. Instead, for obtaining the surface area we need a shell with unit thickness $\Delta r_n = 1$. Thus, $W_n \propto (2^n)^3$ must be divided by 2^n, which shows that the surface area of the sphere scales as r_n^2, as expected.

Although this analysis suggests that the network appears as a three-dimensional space to an internal observer, this conclusion depends on the choice of $r_n = 2^n$. Another choice would lead to a different Hausdorff dimension. Furthermore, a Hausdorff dimension is not necessarily identical to a physical dimension, whereas it is the physical dimension that we are interested in. The physical dimension of a space is understood here as the number of coordinates required for indicating any position in that space. Below, we will show that the physical dimension of the network as observed from within is three as well. This is shown by induction, in a similar way as one can show that ordinary 3D Euclidean space has indeed three physical dimensions. The latter procedure is illustrated in Fig. 5a. One starts with a 3D sphere of radius r_1 around a point p_0. A sphere has a boundary (the surface) that is two-dimensional, thus one

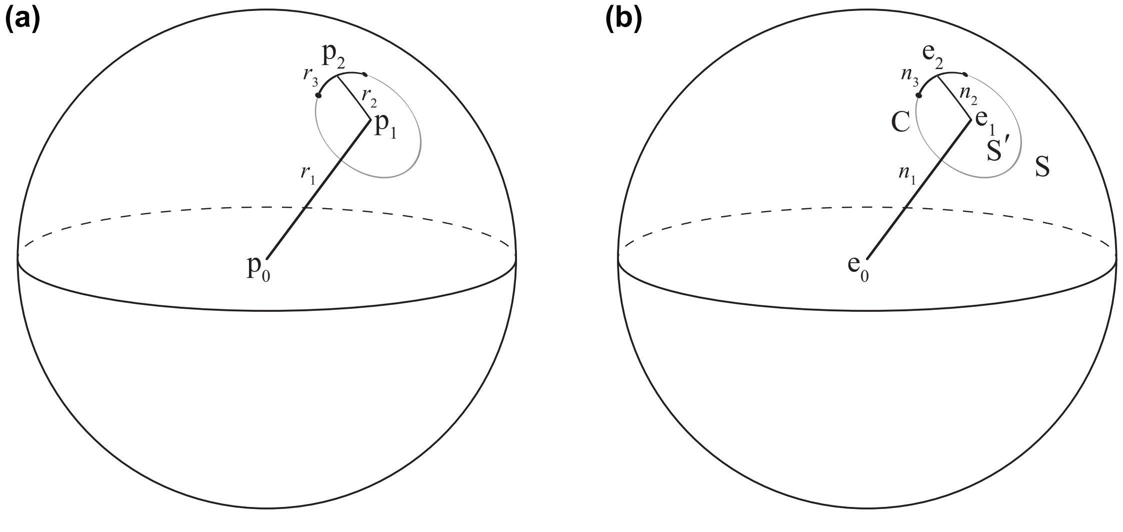

Figure 5: Physical dimension. **a** 3D Euclidean space. **b** Element network.

dimension down from the volume of the sphere. An area on the surface that contains all points lying within a distance r_2 (measured along the surface) from an arbitrary point p_1 on the surface is two-dimensional as well. But the boundary of this area (its circumference shown by the grey circle in the figure) is one-dimensional; again, this is one dimension down. Finally, the line that contains all points lying within a distance r_3 (measured along the circumference) from an arbitrary point p_2 on the circumference is one-dimensional, but the boundary of that line (the points indicated by the small black dots in the figure) is zero-dimensional. We conclude that three steps of a unitary dimensional reduction are required in order to get to zero dimensions. For induction, the argument goes in the other direction: starting from what is certainly zero dimensions, we need three steps of a unitary dimensional increase in order to get to the original sphere. This shows that the original space is indeed three-dimensional. An arbitrary point in the space can be indicated by three coordinates.

Applying this procedure to the network goes as follows (Fig. 5b). We have shown above that the volume of the elements that lie at link distance up to and including n_1 around an arbitrarily observed element e_0 is proportional to $(2^{n_1})^3$. Suppose now that an element e_1 belongs to the boundary S of such a volume and that it is observed, again from within the network through interaction via a newly attaching element (Fig. 4 with e_1 instead of e_0). Then how many neighbouring elements W'_{n_2} would it have that belong to S and that are at a link distance n_2 from e_1? A direct neighbouring element e' of e_1 must either be a child or a parent of e_1. Since the network is sparse, it is then highly likely that e' has a link distance $n_1 + 1$ or $n_1 - 1$ to e_0, respectively. Subsequently, a neighbour e'' of e' can then have the exact link distance n_1 to e_0, if either of the two following possibilities applies. The first possibility is that e'' is a parent of e' when e' is a child of an element of S. The second possibility is that e″ is a child of e′ when e′ is a parent of an element of S. In other words, we can stay as close as possible to a surface S by following a path along elements that are intermittently children and parents of one another. This is true for any surface S and any observed element e_1. If we start with a child first (for parent first see below), we get the following equation

$$W'_{n_2} = \frac{1}{N}\sum_{i=1}^{N}\sum_{i_1=1}^{N} w_{\mathrm{c}}(i,i_1)\sum_{i_2=1}^{N} w_{\mathrm{p}}(i_1,i_2)\cdots\sum_{i_{n_2-1}=1}^{N} w_{\mathrm{c}}(i_{n_2-2},i_{n_2-1})\sum_{i_{n_2}=1}^{N} w_{\mathrm{p}}(i_{n_2-1},i_{n_2}). \tag{25}$$

Here $w_{\mathrm{c}}(i,j)$ denotes the expected number of links between elements i and j under the condition that j is a child of i (12) and $w_{\mathrm{p}}(i,j)$ denotes the expected number of links between elements i and j under the condition that j is a parent of i (11). The first summation and division by N in (25) averages over all N existing elements i. This averaging takes account of the fact that observation of the network occurs via a new element that links arbitrarily to any element of the existing network. We want to find the number of neighbours that lie together with any of these i on any surface S, at link distance n_2 from i. For a particular i, the $w_{\mathrm{c}}(i,i_1)$ in the second summation gives the expected number of links between i and a child i_1. The total expected number of neighbours is then obtained by summing over all possible i_1. For a particular i_1, the $w_{\mathrm{p}}(i_1,i_2)$ in the third summation gives the expected number of links between i_1 and a parent i_2, and so forth.

Using (11), (12) and approximations similar to (13) we get

$$\begin{aligned} W'_{n_2} &= \int_0^1 \mathrm{d}x\int_x^1 \mathrm{d}x_1\frac{2}{x_1}\int_0^{x_1}\mathrm{d}x_2\frac{2}{x_1}\int_{x_2}^1\mathrm{d}x_3\frac{2}{x_3}\int_0^{x_3}\mathrm{d}x_4\frac{2}{x_3}\cdots\int_0^{x_{n_2-1}}\mathrm{d}x_{n_2}\frac{2}{x_{n_2-1}} \\ &= 2^{n_2}\int_0^1 \mathrm{d}x\int_x^1 \mathrm{d}x_1\frac{1}{x_1}\int_0^{x_1}\mathrm{d}x_2\frac{1}{x_1}\int_{x_2}^1\mathrm{d}x_3\frac{1}{x_3}\int_0^{x_3}\mathrm{d}x_4\frac{1}{x_3}\cdots\int_0^{x_{n_2-1}}\mathrm{d}x_{n_2}\frac{1}{x_{n_2-1}}. \end{aligned} \tag{26}$$

The iterated integral has again analytical solutions, with a quickly rising number of terms. Computer algebra software shows that the integral (excluding the factor 2^{n_2}) evaluates to 1, 1, 2, 2, 5, 5, 14, 14, 42, 42, 132 and 132 for $n_2 = 1\ldots12$, respectively. Focussing on even n_2, we see that this corresponds again to the Catalan numbers. Thus, we conjecture that

$$W'_{n_2} = 2^{n_2}\frac{1}{\frac{n_2}{2}+1}\binom{n_2}{\frac{n_2}{2}}. \tag{27}$$

With Stirling's approximation this leads to

$$W'_{n_2} \approx \frac{4}{(n_2+2)\sqrt{(2\pi n_2)}}\left(2^{n_2}\right)^2, \tag{28}$$

and, for sufficiently large n_2,

$$W'_{n_2} \propto (2^{n_2})^2 . \tag{29}$$

Following an argument similar to the one following (23) then gives

$$V'_{n_2} \propto (2^{n_2})^2 , \tag{30}$$

where V'_{n_2} is the expected surface area at link distance up to and including n_2 around an observed element. These equations are derived from (25), which assumes an intermittent child-parent path starting with a child. With parent first, (26) is changed to

$$W'_{n_2} = 2^{n_2} \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}x_1 \frac{1}{x} \int_{x_1}^1 \mathrm{d}x_2 \frac{1}{x_2} \int_0^{x_2} \mathrm{d}x_3 \frac{1}{x_2} \int_{x_3}^1 \mathrm{d}x_4 \frac{1}{x_4} \cdots \int_{x_{n_2-1}}^1 \mathrm{d}x_{n_2} \frac{1}{x_{n_2}}, \tag{31}$$

which evaluates (excluding the factor 2^{n_2}) to 1, 2, 2, 5, 5, 14, 14, 42, 42, 132, 132 and 429 for $n_2 = 1 \ldots 12$, respectively. This corresponds again to the Catalan numbers. Then, (29) and (30) remain valid for this case too.

Finally, we consider the collection of elements C that lie as close as possible to the boundary of a surface S′ (a circular patch of surface S), that is, at link distance n_2 from e_1 and still at link distance n_1 from e_0. Suppose that an observer within the network observes an arbitrary element e_2 that belongs to C. Then we can ask how many elements that belong to C lie at a link distance n_3 from e_2. Integrated iterations, either arbitrary or through intermittent child-parent paths, are now not possible because these were already used for the constraints on n_1 and n_2. The only possibility is a chain of direct links, either directly between e_2 and another element belonging to C (the case of $n_3 = 1$), or via one or more intermediate elements that all belong to C (the case of $n_3 > 1$). When there is a direct link between two elements, they belong to both W_{n_1} and either W_{n_1-1} or W_{n_1+1}, and also to both W'_{n_2} and either W'_{n_2-1} or W'_{n_2+1}. That is, they necessarily belong to the small number of cases that were neglected for the formulation of (17) and (25). In order to minimize further overlap between the various sets of W and between those of W', child links and parent links should be intermittent. For a link to a child we get

$$W''_1 \propto \int_0^1 \mathrm{d}x \int_x^1 \mathrm{d}x_1 \frac{2}{x_1} = 2 \tag{32}$$

and for a link to a parent

$$W''_1 \propto \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}x_1 \frac{2}{x} = 2. \tag{33}$$

Then a chain of n_3 intermittent child and parent elements gives

$$W''_{n_3} \propto 2^{n_3}, \tag{34}$$

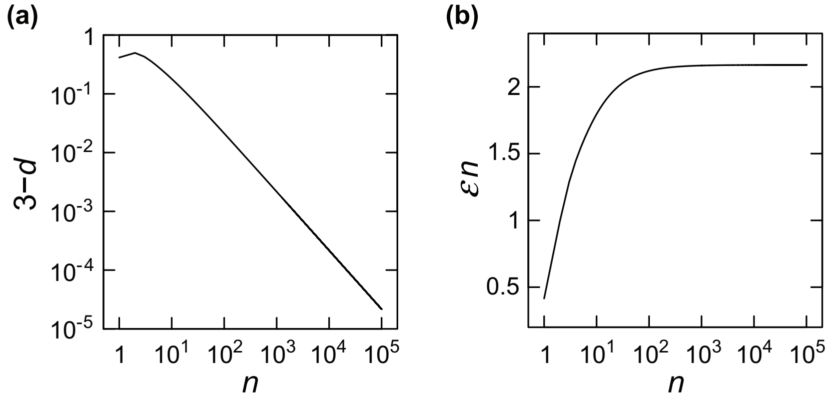

Figure 6: **a** Deviation from 3 of the slope d of $\log_2 V_n$. **b** The ratio of assumed 3D volume $\hat{V}_n$ and actual volume V_n scales as $2^{\varepsilon n} = 2^{(3-d)n}$. For sufficiently large n, εn becomes a constant. See the main text for explanation.

and, following an argument similar to the one following (23)

$$V''_{n_3} \propto 2^{n_3}, \tag{35}$$

where V''_{n_3} is the length at link distance up to and including n_3 around an element e_2.

From the analysis above we conclude that, referring to Fig. 5b, the volume of the network scales as $(2^{n_1})^3$, the boundary of the volume as $(2^{n_2})^2$, the boundary of that boundary as 2^{n_3}, and the boundary of the latter consists of two single elements, that is, it has dimension zero. Thus, three steps of a unitary dimensional reduction are required in order to get to zero dimensions. Again, the argument goes in the other direction for induction. Starting from what is certainly zero dimensions, we need three steps of a unitary dimensional increase in order to get to the original volume. This shows that the physical dimension of the network is indeed three, identical to the Hausdorff dimension, with distance given by $r_n = 2^n$. Therefore, an arbitrary point in the space can be indicated by three coordinates.

For which range of n can we expect to get a three-dimensional space? For small n, neither Stirling's approximation (20), nor the transition from (21) to (23), is valid. Thus, n should be sufficiently large. This can be quantified by computing how quickly V_n approaches $(2^n)^3$. We do this by directly computing W_n (19), see appendix B. The first summation of (24) then gives V_n. The slope of $\log_2 V_n \propto 3n$ as a function of n should ideally be 3, with Fig. 6a showing how the computed slope d differs from 3. The difference is a few tenths up to 0.5 for small n and approaches zero for large n. An error would be made when a three-dimensional space is assumed, whereas in reality the power is not 3 but d, slightly smaller than 3 (say $d = 3 - \varepsilon$, that is, $\varepsilon = 3 - d$). The error can be quantified as follows. For a range of n surrounding a given operating point n_0, the assumed volume $\hat{V}_n$ scales as 2^{3n} and the actual volume V_n as $2^{(3-\varepsilon)n}$. The ratio of assumed and actual volume $\hat{V}_n/V_n$ then scales as $2^{\varepsilon n}$. The $\log_2$ of this function, εn, is shown in Fig. 6b, with ε given by Fig. 6a. For sufficiently large n, εn becomes a constant. Then V_n and $\hat{V}_n$ could be made identical, in a large range surrounding a given operating point n_0, by introducing a suitable scaling constant in front of the definition of $\hat{V}_n$. But εn is a quickly changing function of n for small n, which makes the assumption of a three-dimensional space there too variable and inaccurate to be useful. The cross-over point between no space and space will depend on the application, but the order of magnitude is around $n = 100$.

The range of n compatible with a three-dimensional space is not only bounded from below, but also from above. When n becomes too large, (23) cannot be valid any more. A first, roughly determined bound follows from the requirement that W_n (and V_n) cannot be larger than the total number of elements N. With (21) we get

$$\log_2 (W_n) \approx 3n + \log_2 \left(\frac{4}{(n+2)\sqrt{(\pi (n+1))}} \right) \approx 3n \leq \log_2 (N), \tag{36}$$

where the second approximation is valid for large n. We find that n can be at most $\log_2 (N)/3$. A slightly more sophisticated analysis goes as follows. The W_n of (17) assumes that there is no shorter path than the one followed. For W_1 this is valid. But W_2 should be corrected for the probability that element x_2 happens to be directly connected to x, rather than only indirectly through x_1. Because element x_2 is picked at random, the probability that it is an element that was already included in W_1 equals W_1/N. We can then correct W_2 for this, as $\widehat{W}_2$

$$\widehat{W}_2 = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}x_1\, g(x, x_1) \int_0^1 \mathrm{d}x_2\, g(x_1, x_2) \left(1 - \frac{W_1}{N}\right) = \left(1 - \frac{W_1}{N}\right) W_2. \tag{37}$$

In a similar way, W_3 should be corrected for the probability that element x_3 happens to belong to either W_1 or $\widehat{W}_2$ (where potential overlap between W_1 and $\widehat{W}_2$ is neglected)

$$\begin{aligned} \widehat{W}_3 \approx & \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}x_1\, g(x, x_1) \int_0^1 \mathrm{d}x_2\, g(x_1, x_2) \left(1 - \frac{W_1}{N}\right) \\ & \int_0^1 \mathrm{d}x_3\, g(x_2, x_3) \left(1 - \frac{W_1 + \widehat{W}_2}{N}\right) \\ = & \left(1 - \frac{W_1}{N}\right) \left(1 - \frac{1}{N} \left(W_1 + \left(1 - \frac{W_1}{N}\right) W_2\right)\right) W_3. \end{aligned} \tag{38}$$

Neglecting all factors with $1/N^2$, we find

$$\widehat{W}_3 \approx \left(1 - \frac{1}{N} (2W_1 + W_2)\right) W_3. \tag{39}$$

In the general case, this procedure gives

$$\widehat{W}_n \approx \left(1 - \frac{1}{N}(W_{n-1} + 2W_{n-2} + 3W_{n-3} + \cdots)\right) W_n. \tag{40}$$

Since $W_n \propto (2^n)^3$, the first correcting factor, $W_{n-1} \approx W_n/8$, dominates. Then

$$\widehat{W}_n \approx W_n - \frac{1}{8N} W_n^2 = a_n\,(2^n)^3 - \frac{1}{8N} a_n^2 (2^n)^6, \tag{41}$$

where (21) was used. For a three-dimensional space, the correction term with exponent 6 should be negligible. Thus we demand

$$\frac{1}{8N} a_n^2 (2^n)^6 \ll a_n\,(2^n)^3\,, \tag{42}$$

or

$$(2^n)^3 \ll 8N/a_n. \tag{43}$$

With $n \gg 1$ we have $a_n \ll 1$ (see (22)). Therefore, $8/a_n \gg 1$. But because $8/a_n \lll N$, $\log_2(8/a_n)$ can be neglected compared with $\log_2(N)$. Then taking the $\log_2$ of (43) produces

$$n \lesssim \log_2(N)/3 \tag{44}$$

as a condition for getting a three-dimensional space.

Although n can be small, the equations for W_n (21) and V_n (24) show that almost all elements are separated by link distances close to the maximum possible. Then

$$n \sim \log_2(N)/3. \tag{45}$$

This means that the link distance between two arbitrary elements out of the N in existence is nearly always close to $\log_2(N)/3$.

There is no three-dimensional space when n becomes too small or too large, but this does not mean that space can be taken for granted at intermediate n. The derivation above is based on the typical case where elements are sparsely connected. But links are formed arbitrarily, which means that there can also be densely connected clusters of elements. Within a cluster, link distances δn around n are then small. As a result, (17) is not valid there, and the associated three-dimensional space does not exist. Such clusters are embedded in the space that is observed from within the network. But before discussing this in more detail, we first need to address the topics of time and dynamics.

5 Dynamics, time, and spacetime

Parts of the element network for which n is in the appropriate range will appear as a three-dimensional space to an internal observer. If the network had a fixed N, nothing would change. But N increases, hence the network is dynamical. This increase is required for observing the network from within, via newly added elements that happen to link to a specific observer in addition to linking to the network in general. In addition, new elements that do not link directly to a specific observer may affect, indirectly, how the network appears to that observer, by changing the statistics that produce space through (17). The continuous stream of new elements is then likely to change how elements and clusters of elements are observed to be related spatially. An ongoing change of spatial relations is equivalent to dynamics playing out in time. The progress of time is, thus, produced by the continual addition of new elements to the network (as was already indicated by the analysis in Section 2).

When a new element becomes attached to the network with one or more links, this cannot be undone. The transition is irreversible (Section 2). Therefore, network time has the same asymmetry. If one would retrace an existing path across links as they were produced from parents to children, one would appear to travel forward in time. If one would retrace an existing path across links from children to parents, one would appear to travel backward in time. Choosing a particular time direction, we can then perform an analysis that is analogous to the one in Section 4. Again we assume that there is an observer O that views the network from within, by getting a link with a new element that also links with an arbitrary existing element of the network (Fig. 4). We want to calculate how many time-directed neighbours an observed element has on average (i.e., the number of elements at link distance 1), how many neighbours of neighbours it has (at link distance 2), and so on, for arbitrary link distance n. Let us first look at the forward time direction, iterating from parent to child. With $w_{\mathrm{c}}(i,j)$ (12) this leads to

$$W_n^{\mathrm{F}} = 2^n \int_0^1 \mathrm{d}x \int_x^1 \mathrm{d}x_1 \frac{1}{x_1} \int_{x_1}^1 \mathrm{d}x_2 \frac{1}{x_2} \cdots \int_{x_{n-2}}^1 \mathrm{d}x_{n-1} \frac{1}{x_{n-1}} \int_{x_{n-1}}^1 \mathrm{d}x_n \frac{1}{x_n}, \tag{46}$$

where the F stands for the forward time direction. If we denote the iterated integral by X_n with $n = 0, 1, \cdots$, one can show by partial integration that $X_n = X_{n-1}$ for any $n \geq 1$. Since $X_0 = 1$, the integral equals 1 for any n. Alternatively, this can be shown by a direct integration of X_n

$$\begin{aligned} &\int_0^1 \mathrm{d}x \left(\int_x^1 \mathrm{d}x_1 \frac{1}{x_1} \cdots \int_{x_{n-1}}^1 \mathrm{d}x_n \frac{1}{x_n} \right) = \int_0^1 \mathrm{d}x \left((-1)^n \frac{1}{n!} (\ln x)^n \right) \\ &= (-1)^n \frac{1}{n!} \left[\sum_{k=0}^{n} (-1)^k \frac{n!}{(n-k)!} x (\ln x)^{(n-k)} \right]_0^1 \\ &= (-1)^n \frac{1}{n!} (-1)^n \, n! = 1. \end{aligned} \tag{47}$$

We conclude that

$$W_n^{\mathrm{F}} = 2^n. \tag{48}$$

The backward time direction requires iterating from child to parent. With $w_\mathrm{p}(i,j)$ (11) this gives

$$W_n^\mathrm{B} = 2^n \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}x_1 \frac{1}{x} \int_0^{x_1} \mathrm{d}x_2 \frac{1}{x_1} \cdots \int_0^{x_{n-2}} \mathrm{d}x_{n-1} \frac{1}{x_{n-2}} \int_0^{x_{n-1}} \mathrm{d}x_n \frac{1}{x_{n-1}}, \tag{49}$$

where B indicates the backward time direction. It is clear that the iterated integral equals 1 for any n, hence

$$W_n^\mathrm{B} = 2^n. \tag{50}$$

This is consistent with the fact that the expected number of parents is 2 for any element (4).

The total number of neighbouring elements up to a link distance n is obtained by summation

$$V_n^\mathrm{F} = \sum_{m=1}^{n} W_m^\mathrm{F} = \sum_{m=1}^{n} 2^m \approx 2^n \left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right) = 2\ 2^n \propto 2^n, \tag{51}$$

where the latter steps assume, as before, $n \gg 1$. The same result as (51) is obtained for V_n^B. We can then use $t_n = 2^n$ for quantifying distance in time, with an associated element volume (defined as the number of nearby elements) that scales as t_n^1. This is consistent with time being one-dimensional as opposed to space being three-dimensional. It should be noted that $V_n^\mathrm{F} = V_n^\mathrm{B}$ gives time reversal invariance: fundamental laws describing regularities should keep the same form irrespective of whether they are running forward in time or backward. However, this only applies to the observed time defined here. Network time (related monotonically to the age of the universe) is still directed in the forward direction, because elements are added irreversibly.

Distances in both time and space are quantified by 2^n for a link distance n. Moreover, elements can be interpreted as contributing either to time or to space. Therefore, time and space can be regarded as different aspects of a single spacetime. One consequence of this close relationship is that there is a maximum speed for causal influences within spacetime. This can be understood as follows. We consider two elements e_1 and e_2 with a connecting path that is strictly in the time direction (the left path of Fig. 7, where the dashed line symbolizes intermediate elements along the path). The n_t links are to elements with increasing rank order, i.e., they were produced at increasing values of N. Following convention, elements e_1 and e_2 will be called time-like separated, where the existence of element e_2 depends on the prior existence of element e_1, whereas the existence of element e_1 does not depend on the existence of element e_2. In contrast, space-like separated elements lack a direct time path and exist independently of one another. They are necessarily still connected by a path that includes links in the backward time direction. Time-like separated elements such as e_1 and e_2 could, in addition, have a separate space path (the shortest path along elements of any rank order), such as the right path in Fig. 7. A space path can be much shorter than the

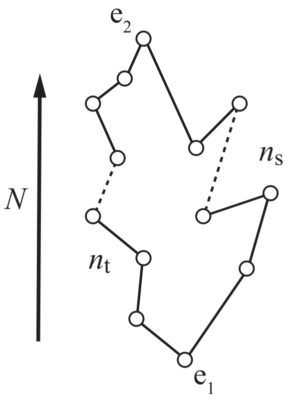

Figure 7: Maximum speed. Two time-like separated elements, e_1 and e_2, are connected by a time path (left) consisting of n_t links to elements with increasing rank order (i.e., in the direction of increasing N). The elements are also connected by a space path (right) consisting of n_s links that connect elements of any rank order. The space path can be equal or shorter than the time path ($n_s \leq n_t$), but never longer than the time path, because then the space path (defined as the shortest path between e_1 and e_2) must be the same as the time path (because any time path is also a valid space path). The dashed lines symbolize a large number of intermediate elements and links.

time path, even with n_s so small that there is no spatial separation (when n_s is below the valid range for a three-dimensional space). Long space paths are possible too, up to $n_s = n_t$. It is not possible to have n_s larger than n_t, because then the space path must be the same as the time path. This is so because any time path is also a valid space path, just a special case with strictly increasing rank orders.

We can now define a speed v as the ratio of spatial separation and temporal separation. Since there is a limit to n_s, v lies between zero and a maximum. By defining $t_n = 2^n$ (below (51)) and $r_n = 2^n$ (below (24)) we have, in effect, scaled the unit of space relative to the unit of time such that this maximum, with $n_s = n_t$, equals one ($v_{\max} = 1$). Then with $v = 2^{n_s}/2^{n_t}$ and $n_s \leq n_t$ we get

$$0 \leq v \leq 1. \tag{52}$$

The conclusion that the space paths of time-like separated elements can never be longer than their time paths depends on properties of the element network and on how space and time are observed by any internal observer. Therefore, it must be true without exception.

As derived above, internally observed distance in time can be quantified by $t_n = 2^n$. But we have also seen in Section 2 that time for the entire network appears equivalent to growing by one element at a time. Time then progresses along with N. We can define N as the network time, which must be related monotonically to the age of the universe (called cosmic time in cosmology). The linear progress of network time with N is in apparent contrast with the strongly nonlinear dependence of t_n on n. However, we will show now that the nonlinearity of $t_n = 2^n$ can work in a considerable range where it is effectively linearized, and is, thus, consistent with the progress of N. Suppose that there is a time path of length n_t between elements e_1 and e_2 such as in Fig. 7, and that e_2 is at the front of network growth, that is, e_2 has no children yet at the time when the network has N elements. We are interested in how this path is extended when time progresses, where we assume that e_1 remains fixed. Extension would certainly happen when e_2 acquires a link to a new element, that is, when n_t becomes $n_\mathrm{t} + 1$. But on average, when N changes to $N + 1$, the chance that element e_2 acquires a link is only $2/N$ (since a newly added element links on average to two elements out of N). The expected change of n_t will therefore be to $n_\mathrm{t} + 2/N$. Then the expected t_n changes to $t_n + \delta t_n$, with

$$\delta t_n = 2^{n_\mathrm{t}+\frac{2}{N}} - 2^{n_\mathrm{t}} \approx 2^{n_\mathrm{t}} \ln(2)\frac{2}{N}. \tag{53}$$

When a large series of ΔN time steps are added to N, the chance $2/N$ that e_2 acquires a link per time step slowly declines as N increases. Then the expected change of n_t is

$$\Delta n_\mathrm{t} = \sum_{i=N}^{N+\Delta N} \frac{2}{i} \approx \int_1^{1+\Delta N/N} \mathrm{d}x\, \frac{2}{x} = 2\ln\left(1 + \frac{\Delta N}{N}\right) \tag{54}$$

with $x = i/N$. We see here that n_t changes only very slowly: $\Delta n_\mathrm{t} = 1$ requires $\Delta N/N = \sqrt{e} - 1 \approx 0.65$, a considerable increase of N. The Δn_t of (54) gives an expected change Δt_n

$$\Delta t_n = 2^{n_\mathrm{t}+2\ln(1+\Delta N/N)} - 2^{n_\mathrm{t}} \approx 2^{n_\mathrm{t}} 2\ln(2)\frac{\Delta N}{N} = \Delta N \delta t_n, \tag{55}$$

with the approximation valid for $\Delta N/N$ sufficiently small. We conclude that the expected progress of observed time is related linearly to the progress of network time N over a considerable range. For a single element, the progress of observed time is only realized when it acquires a new link, that is, with very low probability. We will see below, though, that clusters of elements can consist of a huge number of elements. Then it is much more likely that a cluster acquires new links, and the progress of time for such a cluster is then realized more gradually.

In a similar way we can analyze how the position of element e_2 is expected to change when N changes to $N + 1$. Since space is three-dimensional, position and changes in position need to be treated as three-dimensional vectors. Again, when N changes to $N + 1$, the chance that element e_2 acquires a link is $2/N$. The expected change of n_s will therefore be to $n_\mathrm{s} + 2/N$. The expected change in position $\delta\mathbf{r}$ can then be in any direction, but its magnitude is given by

$$|\delta\mathbf{r}| = 2^{n_\mathrm{s}+\frac{2}{N}} - 2^{n_\mathrm{s}} \approx 2^{n_\mathrm{s}} \ln(2)\frac{2}{N}. \tag{56}$$

For a large series of ΔN time steps added to N and with $\Delta N/N$ sufficiently small, we get an expected change in position $\mathbf{\Delta r}$ with

$$|\mathbf{\Delta r}| \approx \Delta N |\delta \mathbf{r}|. \tag{57}$$

The expected change in observed position is related linearly to the progress of network time N over a considerable range. As with time, a change in position will be realized more gradually for clusters of elements than for single elements. We will discuss further below that subsequently realized changes in position need not be aligned, which makes the case of space different from the one-dimensional case of forward time.

We have seen above that both observed time and observed position are expected to be related linearly to the progress of network time over a considerable range. This means that it makes sense to define speeds as the ratio of space and time intervals Δr and Δt that are neither fixed nor infinitesimal. The approximate linearity of observed spacetime also enables viewing spacetime as approximately Minkowski spacetime, as we will see below. This is an approximation, because space exists only for the appropriate range of n and is neither smooth nor continuous, as a standard mathematical space would be.

6 Physical interpretation of links and matter

The network consists of elements and links, neither of which has a physical dimension. They can be counted subject to various conditions, their statistics can be studied, and they might be combined. Still, no combination will be other than dimensionless. Nevertheless, we have seen that they can be given a physical interpretation, including physical dimensions, when an internal observer interprets the network as forming a spacetime. When the network grows as new elements attach one by one, each new element appears to let the entire network take one step forward in time (Section 2). Moreover, an internal observer can interpret elements as being part of either space (Section 4) or time (Section 5). Thus, we have tentative physical interpretations of how elements can appear in spacetime. Links, on the other hand, have not yet been given a physical interpretation. We will use here an interpretation that remains an assumption for the moment. How likely it is that it is correct can then be judged by how well its consequences agree with the physical world as it is known empirically and theoretically. Therefore, we will in the sections below first explore the main consequences of the interpretation. In the Discussion (Section 15) we will then evaluate its plausibility.

The assumed interpretation is that in spacetime each link corresponds to a quantum of action, which will be denoted by h_ℓ (where ℓ stands for link). In the sections on wavefunctions and photons below, the Heisenberg uncertainty relation (177) as well as the Planck relation for the energy of a photon (196) show that $h_\ell = h/2$, with h the constant of Planck. Because action has the dimension of energy×time, the ratio of the total number of links in the network (total action) to the total number of elements (total network time) then has the dimension of energy. For the entire network, we know that each element has, on average, two parents (4) and two children (7). An element has then four links on average. Since links are always shared between two elements, a network of N elements is, thus, expected to contain a total of $2N$ links. The ratio of links to elements is then 2, which must be the total energy present in the network. It is the ratio of the total action added to the network since it began and the total time elapsed since it began. This remains so when the network grows, because the number of links and the number of elements grow in proportion (as each new element has two parents on average). Therefore, the total energy in the network remains constant across time, at least on average and for sufficiently large N. In other words, energy is conserved. The total energy in the network is not zero. Energy is absent when there is nothingness (Fig. 1a), starts at 0.5 (Fig. 1c) and jumps to 2/3 or 1 (Figs. 1e-g), and then quickly approaches its asymptotic value of 2 (4). We conclude that conservation of energy is a consequence when links are interpreted as quanta of action. Alternatively, one could start with the fact that the ratio of links to elements remains the same across network time, which is a (statistical) symmetry that indicates a conserved quantity. If we call this quantity energy, then each link must be a fixed quantity of energy×time.

Energy in the network is strictly positive, being the ratio of two positive numbers. However, in spacetime this becomes more complex, in particular because energy can become concentrated there in clusters of elements and links. For understanding that we will now first discuss how particulate matter can arise in the network. The emergence of a three-dimensional space within the network depends on the fact that the network is, overall, sparsely connected. But the network can also contain, for statistical reasons, densely connected clusters of elements. A densely connected cluster of elements cannot have any spatial differentiation within the cluster, because (17) is not valid there. All elements of such a cluster can then only have spatial features equal to those of the total cluster (such as spatial distances to other clusters). Moreover, Sections 2 and 5 show that time is equivalent to the growth of the network, when new elements become attached. New elements cause changes in the spatial relations between existing elements and thus produce dynamics. But within a densely connected cluster of elements no such dynamics occur, because there are no spatial relations between the elements of the cluster. Without dynamics, time cannot be differentiated within the cluster. In other words, all elements of the cluster can only have dynamical and temporal features equal to those of the total cluster (such as velocities relative to other clusters and time distances to other clusters). The term ‘particle’ is used below to denote a cluster consisting of elements that have identical features in spacetime and that together act as a unit in spacetime. Note that this use of the term excludes compound particles with a spatial extent.

Even if particles have no internal structure in spacetime, their internal structure in

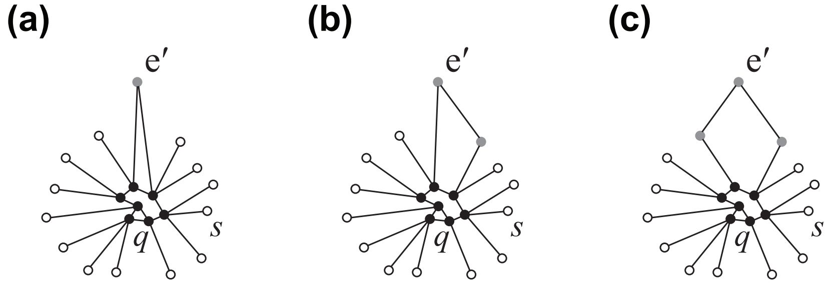

Figure 8: The core of a cluster consists of q densely connected elements (black filled circles). The core is directly connected to a mantle of s one-parent-elements ^{1}e (open circles) that belong to the cluster. They can become linked as parents to the rest of the network. **a-c** Three different ways by which a new element e′ can let the core grow; the elements that are indicated by grey filled circles then become part of the core. See the main text for further explanation.

the element network might give them properties that are observable in spacetime. A condition for the latter is that such internal structure remains stable over some time at least, that is, whilst the network is growing. What kind of stable properties clusters can have and what this implies for different types of particles is a large research topic that still needs to be explored in detail (see the discussion in Section 13). Nevertheless, it is possible to define a basic particle and derive its properties, which is explained next.

For the basic particle we assume that it is a cluster with a core that consists of $q(N)$ elements. Each of the core elements is assumed to be directly linked to at least two other elements of the core (illustrated in Fig. 8 by the black filled circles). Then the elements of the core do not comply with (17) and are not spatially separated from one another. The core is connected by $s(N)$ links to $s(N)$ one-parent-elements ^{1}e. These elements will be called the mantle of the particle, and they are taken to belong to the particle since they have no other parents. The elements of the mantle may gradually get further connections to the rest of the network. We can now analyze how such a particle will grow over time. When a new element e′ is added to the network, there are three ways by which this element could increase q. First (Fig. 8a), it might directly connect with two of its links to existing elements of the core (the set of existing core elements is denoted by $\{q\}$). Then q increases by one. The probability that this happens is q/N for each link, because the link connects arbitrarily to the N existing elements of which q belong to the core. For two links the probability is then $(q/N)^2$, assuming $q \ll N$ (and acknowledging that the two links cannot pick the same core element, see Section 2, Fig. 1d). Second (Fig. 8b), e′ might connect with one of its links to an element of $\{q\}$ and with another one of its links to an element that belongs to $\{s\}$ (the set of mantle elements). Then the latter element as well as e′ are added to

$\{q\}$, because they are then, as a result, both connected to at least two elements of the updated $\{q\}$. Thus, q increases by two. And finally (Fig. 8c), e$'$ might connect with two of its links to elements that belong to $\{s\}$. Then the latter elements as well as e$'$ are added to $\{q\}$, because all three elements are, as a result, connected to at least two elements of the updated $\{q\}$. Thus, q increases by three. The above considerations lead to

$$\frac{\mathrm{d}q}{\mathrm{d}N} = \left(\frac{q}{N}\right)^2 + 2\left(\frac{q}{N}\right)\left(\frac{s}{N}\right) + 3\left(\frac{s}{N}\right)^2, \tag{58}$$

with $\mathrm{d}N = 1$ and assuming that $q, s \ll N$. The actual dynamics are stochastic, regarding the number of links that a new element forms as well as the variables q and s, but it is assumed here that for sufficiently large q and s the dynamics can be approximated by those of means. A similar approximation can be formulated for the increase of s, with the provision that newly linked elements must be one-parent-elements ^{1}e, which make up $1/e \approx 37\%$ of all new elements (see below (5))

$$\frac{\mathrm{d}s}{\mathrm{d}N} = \frac{1}{e}\frac{q}{N}, \tag{59}$$

where each new ^{1}e is expected to increase s by q/N.

Whereas $\mathrm{d}q/\mathrm{d}N$ is proportional to $1/N^2$, $\mathrm{d}s/\mathrm{d}N$ is proportional to $1/N$. This means that for sufficiently large N the growth of q can be neglected compared with the growth of s, that is, q then becomes almost a constant compared with s. Suppose that, in effect, q stops growing relative to s when $N \sim N_q$. Integrating between N_q and N, we find for (59) the approximate solution

$$s(N) \sim \frac{q}{e}\ln(N/N_q). \tag{60}$$

This is consistent with (6), which shows that an element i is expected to have $2\ln(N/i)$ children, which would be $\sim(1/e)\ln(N/i)$ if the children were ^{1}e. Here i is the rank order of the element, that is, the network time when it was created. Hence, a collection of q such 'early' (low-rank-order, $< N_q$ with $N_q \ll N$) elements will have $\sim(q/e)\ln(N/N_q)$ children if the children are exclusively ^{1}e.

When N gets larger and larger, the growth of s can be neglected compared with the increase of N, that is, s becomes then almost constant compared with N. Another way to put this is that the derivative of $\ln N$ becomes almost zero when N becomes extremely large. Then s is, for all practical purposes, a constant as a function of time, with time related linearly to N. Since almost all elements of $\{s\}$ are formed before N becomes extremely large, they are expected to have low rank order, though not as low as the elements of $\{q\}$. Still, the rank orders i of $\{s\}$ are expected to comply with $i \ll N$ for N sufficiently large.

The s elements of the particle are connected with s links to $\{q\}$, the core of the particle. Because elements and links have different roles in the network and have different interpretations in spacetime, it is helpful to distinguish s_e, the number of elements, from s_ℓ, the number of links. For the particle, we have $s_\mathrm{e} = s_\ell$, because the mantle consists of ^{1}e and we can neglect the core. When a cluster of elements is observed as a particle in spacetime, s_ℓ or, equivalently, s_e is observed as its rest energy, as will be argued now. Because (60) shows that $s_\ell \gg q$, s_ℓ is much larger than the number of links that internally connect the q elements of the core. Therefore, the total number of links within the cluster equals s_ℓ to excellent approximation. Then s_ℓ is the total action within the cluster, because each link is assumed to be a quantum of action. A cluster acts as a unit in spacetime, as if it were a single space-forming element. However, the total action belonging to a single space-forming element is on average only 2, the mean ratio of links to elements in the network. Compared with a single space-forming element, the cluster thus has $s_\ell - 2 \approx s_\ell$ more links, that is, s_ℓ more quanta of action. Since energy is action per unit of time, the s_ℓ units of action then correspond to an energy of s_ℓ along the proper time of the particle. In other words, s_ℓ must be interpreted as the rest energy—or mass—of the particle. The action s_ℓ that is internal to a particle may be called the particle-bound action, in order to distinguish it from the action (i.e., the links) from which empty spacetime is made.

The analysis above makes some simplifying assumptions. The s links and elements are assigned to the cluster when interpreted as a spacetime unit. This is reasonable, because the elements of $\{s\}$ are only one link away from the elements of $\{q\}$, and are therefore not separated spatially. Moreover, being one-parent-elements, they are initially connected only to the cluster. When a new element connects to an element of $\{s\}$, there are two distinct possibilities. If the new element is a ^{1}e element, it would be added to $\{s\}$. But if it is a $^{2+}$e element, it would not be added to $\{s\}$. Instead, it could change the spatial embedding of the cluster or provide interactions with other clusters. Thus, ^{1}e network elements can let $\{s\}$ grow not only by directly connecting to $\{q\}$, but also by connecting to $\{s\}$, at least when their link distance to $\{q\}$ remains small enough to avoid becoming spatially separated from $\{q\}$. The formulation above is, in effect, assuming a hard cut between the particle and spacetime. In reality, the transition from no space to space must be gradual, when the approximations (20) and (23) gradually attain validity (Fig. 6b). A more sophisticated theory should take this into account and avoid a hard cut. The current analysis must, therefore, be regarded as a first-order approximation only.

7 Motion in spacetime

In the previous section we have defined a basic particle, which effectively consists of s elements and s links. Because the particle acts as a unit in spacetime, we can define time and space distances to other units in spacetime. Then we can perform an analysis that is analogous to the one following Fig. 7, if we replace the element e_2 with a particle p. Figure 9a shows a modified version of Fig. 7. The left path symbolizes a

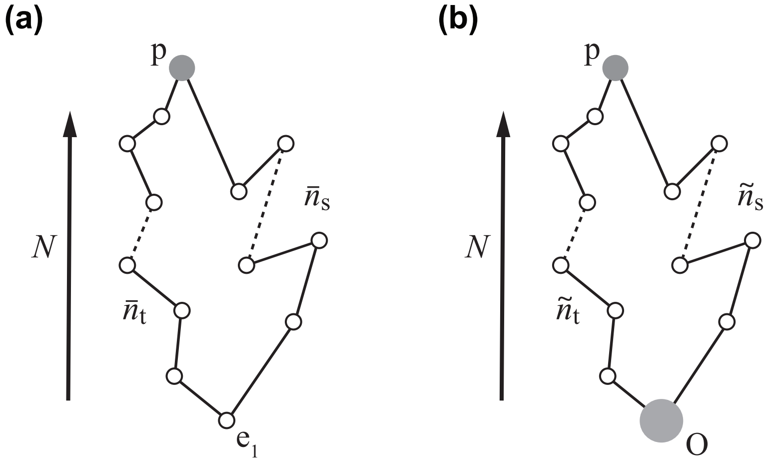

Figure 9: **a** Time-like separated element e_1 and particle p (consisting of s elements and links) are connected by s time paths (left), on average consisting of $\bar{n}_\mathrm{t}$ links along elements with increasing rank order. In addition, e_1 and p are connected by s space paths (right), on average with $\bar{n}_\mathrm{s}$ links along elements in any rank order. **b** A macroscopic observer O and a particle p are connected by s effective time paths (left) and space paths (right).

large collection of time paths between e_1 and the s elements of p. Then we can define a mean time-directed link distance $\bar{n}_\mathrm{t}$

$$\bar{n}_\mathrm{t} = \frac{1}{s}\sum_{i=1}^{s} n_\mathrm{t}(i), \tag{61}$$

where $n_\mathrm{t}(i)$ is the time-directed link distance between e_1 and element i of p. The time distance between e_1 and p is then $2^{\bar{n}_\mathrm{t}}$. Note that we must take the mean of $n_\mathrm{t}(i)$ here and not the mean of $2^{n_\mathrm{t}(i)}$, because there are no temporal and spatial distances within a particle (i.e., all of its elements are spacetime equivalents). In a similar way we can define a mean link distance $\bar{n}_\mathrm{s}$ for the space paths between e_1 and p

$$\bar{n}_\mathrm{s} = \frac{1}{s}\sum_{i=1}^{s} n_\mathrm{s}(i), \tag{62}$$

with $n_\mathrm{s}(i)$ the link distance between e_1 and element i of p. The space distance between e_1 and p is then $2^{\bar{n}_\mathrm{s}}$. Because for any i we must have $n_\mathrm{s}(i) \leq n_\mathrm{t}(i)$, we must also have

$$\bar{n}_\mathrm{s} \leq \bar{n}_\mathrm{t} \tag{63}$$

and for the speed $v = 2^{\bar{n}_\mathrm{s}}/2^{\bar{n}_\mathrm{t}}$ we therefore get

$$0 \leq v \leq 1. \tag{64}$$

There is again a maximum speed, the same as the one for a single element.

We can now analyze what happens to p when the network time increases from N to $N+1$, where we again assume that e_1 remains fixed. On average, the new network element will attach two new links arbitrarily to the N existing elements. Of those, s belong to p. Then we expect $2/N$ links being added to each of the s time paths connecting e_1 and p (and in total $2s/N$ links to p). The expected change of $\bar{n}_{\mathrm{t}}$ is

$$\delta\bar{n}_{\mathrm{t}} = \frac{1}{s}\sum_{i=1}^{s}(n_{\mathrm{t}}(i) + 2/N) - \frac{1}{s}\sum_{i=1}^{s} n_{\mathrm{t}}(i) = 2/N. \tag{65}$$

Thus, the particle is expected to move forward in time at the same rate as a single element (such as e_2 in Fig. 7). Because the analysis of (53)-(55) remains valid, we get

$$\delta t = 2^{\bar{n}_{\mathrm{t}}+\frac{2}{N}} - 2^{\bar{n}_{\mathrm{t}}} \approx 2^{\bar{n}_{\mathrm{t}}}\ln(2)\frac{2}{N}. \tag{66}$$

and

$$\Delta t = 2^{\bar{n}_{\mathrm{t}}+2\ln(1+\Delta N/N)} - 2^{\bar{n}_{\mathrm{t}}} \approx 2^{\bar{n}_{\mathrm{t}}}2\ln(2)\frac{\Delta N}{N} = \Delta N\delta t. \tag{67}$$

Again, the expected progress of observed time for the particle is related linearly to the progress of network time N over a considerable range. However, the number of realized steps forward per unit of time is expected to be s times larger for the particle than it is for a single element, with each step s times smaller. For large s, the realized progress of time is then much more gradual.

For the s space paths connecting e_1 and p we also expect $2/N$ links being added to each of these paths when the network time increases from N to $N+1$. For a particular path i we then get

$$\delta n_{\mathrm{s}}(i) = 2/N. \tag{68}$$

However, this does not lead to an equivalent of (65), because such a $\delta\bar{n}_{\mathrm{s}}$ would ignore the three dimensions of space. Links added to p might take the particle in any direction. The expected result then depends on how strongly these directions are expected to be correlated, and the actual result depends on how strongly the realized directions are actually correlated. Fully correlated directions give, per time step of the network time,

$$|\delta\mathbf{r}|_{\max} = 2^{\bar{n}_{\mathrm{s}}+\frac{2}{N}} - 2^{\bar{n}_{\mathrm{s}}} \approx 2^{\bar{n}_{\mathrm{s}}}\ln(2)\frac{2}{N}, \tag{69}$$

For fully correlated directions and a large series of ΔN time steps added to N (and with $\Delta N/N$ sufficiently small), we get an expected change in position $\mathbf{\Delta r}$ that is maximal, with

$$|\mathbf{\Delta r}|_{\max} \approx \Delta N|\delta\mathbf{r}|_{\max}. \tag{70}$$

If the directions produced by subsequent link additions are expected to be uncorrelated, the expected change in position would be zero. Similarly, uncorrelated directions produced by realized links would result in small and random actual changes in position. The net effect of this jitter would be close to zero because s is large. Therefore

$$0 \leq |\mathbf{\Delta r}| \leq |\mathbf{\Delta r}|_{\max}. \tag{71}$$

For a given level of correlation, the expected change in position would still be related linearly to the progress of the network time over a considerable range. Then spacetime is approximately linear for a particle in a similar way as discussed for a single element at the end of Section 5.

The analysis above was performed relative to a single element e_1 that was assumed to remain fixed. However, single elements may get new links at any time and are only rudimentary observers. We will therefore replace e_1 with an observer O that is assumed to be macroscopic (Fig. 9b). Macroscopic means here that the observer consists of a very large number of particles and, thus, a huge number of elements. The key point is that changes in proper time of the observer are almost perfectly smooth, because the individual steps produced by link additions are too small to be observable. Moreover, the position of the observer would become almost perfectly fixed, because the jitter produced by uncorrelated links in the space direction would cancel out almost completely. Although O has inevitably a spatial extent, it is still possible to define a mean position of O and effective link distances $\check{n}_\mathrm{s}(i)$ and $\check{n}_\mathrm{t}(i)$ from O to element i of p. This requires a mixed averaging at the particles of O over n within particles (because there is no spacetime within particles) and over 2^n between particles (because particles are separated in spacetime). The averaging over the s elements of p remains the same, with the mean of $\check{n}_\mathrm{s}(i)$ denoted by $\tilde{n}_\mathrm{s}(i)$ and the mean of $\check{n}_\mathrm{t}(i)$ by $\tilde{n}_\mathrm{t}(i)$. Then all the results of (65)-(71) remain valid if we replace $n_\mathrm{s}(i)$ with $\check{n}_\mathrm{s}(i)$, $n_\mathrm{t}(i)$ with $\check{n}_\mathrm{t}(i)$, $\bar{n}_\mathrm{s}(i)$ with $\tilde{n}_\mathrm{s}(i)$, and $\bar{n}_\mathrm{t}(i)$ with $\tilde{n}_\mathrm{t}(i)$.

Both O and p have their own observational reference frames, coordinate frames that move along with their own position. That means that O does not see itself move relative to its own reference frame. The same goes for p. Both O and p still get new links, which are all interpreted, by O and by p themselves, to produce changes in forward time. Following convention, we will call the self-observed time of O the proper time of O and the self-observed time of p the proper time of p.

We can now look in more detail at what would be observed, in spacetime, to happen to an element or particle when the network grows. For simplicity, we will assume here that elements and particles are free, that is, that they only interact with the elements that form spacetime and not with elements that belong to particles. Figure 10 shows this for a single element e. On average, one expects then $2/N$ links to become attached to e when the network time proceeds from N to $N + 1$. The expected direction in which a link is attached must be purely in the time direction of spacetime when viewed from the element itself, but when viewed from an external observer O there are several

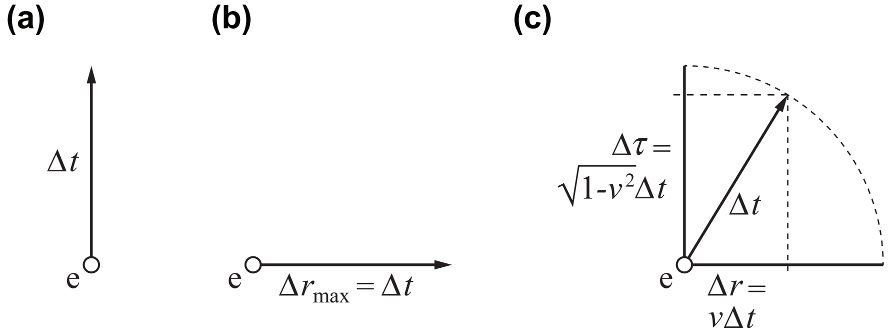

Figure 10: **a** Links added purely in the time direction let an element e move in the time direction of spacetime with time steps Δt. **b** Links added purely in the space direction let an element e move in the space direction with space steps $\Delta r_{\max} = v_{\max}\Delta t = \Delta t$. **c** With $0 \leq v \leq 1$ the link of fixed magnitude Δt points in any spacetime direction, with space component $\Delta r = v\Delta t$ and, therefore, time component $\Delta\tau = \sqrt{1 - v^2}\Delta t$.

possibilities. The direction in which the link is attached can then be purely in the time direction (Fig. 10a), purely in the space direction (Fig. 10b), or somewhere in between (Fig. 10c). Each link is assumed to correspond to a quantum of action, which means that the magnitude of the addition remains the same, irrespective of its orientation in spacetime. Therefore, the added link can be regarded here as a vector of fixed length and variable orientation. When it points purely in the forward time direction (Fig. 10a), it produces a step in the proper time of O, say Δt. When it points purely in the space direction (Fig. 10b), it produces the maximum possible step in space, $\Delta r_{\max}$. Because we know from (52) and (64) that the maximum speed $v_{\max}$ equals 1, we have $\Delta r_{\max} = v_{\max}\Delta t = \Delta t$. For smaller speeds we get

$$\Delta r = v\Delta t, \tag{72}$$

and therefore

$$\Delta\tau = \sqrt{1 - v^2}\Delta t, \tag{73}$$

as illustrated in Fig. 10c. We can use Pythagoras' theorem here, because the axis of proper time must be orthogonal to space, as there is nothing in the element network that could bias the proper time axis towards any spatial direction. Whereas the observer O views its own (proper) time to proceed with steps of Δt, O observes the proper time of e (such as would be indicated by a clock co-moving with e) to proceed with steps of $\Delta\tau = \sqrt{1 - v^2}\Delta t$. Then $\Delta\tau < \Delta t$ when $v > 0$, which means that τ progresses more slowly than t, with smaller steps. Thus, there is an observed time dilation. Following convention, we can define

$$\gamma = 1/\sqrt{1 - v^2} \tag{74}$$

and rewrite (73) as

$$\Delta t = \gamma \Delta \tau. \tag{75}$$

Then $\Delta t \geq \Delta \tau$ because $\gamma \geq 1$.

The analysis remains essentially the same when we reverse the roles of e and O. We have seen above that particles consisting of s elements progress in proper time at the same rate as single elements (65). This must be equally true for compound observers that consist of many particles. Moreover, distance in space is defined such that the distance from A to B equals the distance from B to A (as both depend in the same way on the same link distance). Then changes in distance are also equal, as well as the ratio of change in distance to change in proper time (defined by a link distance as well). This means that when O observes that the reference frame of e moves at a speed v relative to its own reference frame, that e must observe that the reference frame of O moves relative to its own reference frame at the same speed, in the opposite direction. Then (73) is again valid, and e observes the time of O to proceed more slowly than its own proper time. By a similar argument, we can also replace e with a particle p and show that O observes the proper time of p according to (73). The same goes for p observing the proper time of O.

In addition to a time dilation, there is also a length contraction: the observer O sees spatial distances along the direction of motion of e shorter than e sees them. This can be understood as follows. We assume that two elements e and e′ are separated by a distance Δl in the reference frame of e along the direction of the relative motion v of the reference frames of e and O. Suppose that e observes that e and e′ pass a given point P at times t and t', respectively. With $\Delta t_l = |t' - t|$ we must have

$$\Delta l = v \Delta t_l \tag{76}$$

in the reference frame of e. However, O sees the time of e proceed more slowly, such that it observes $\sqrt{1 - v^2}\Delta t_l$ for the difference in times when e and e′ pass point P. Because v is the same for e and O, this means that

$$\Delta r = \sqrt{1 - v^2}\Delta t_l v = \sqrt{1 - v^2}\Delta l, \tag{77}$$

with Δr the distance between e and e′ as observed by O. We conclude that $\Delta r < \Delta l$ when $v > 0$, that is, the space of e in the direction of its motion appears to be contracted as observed by O. In terms of γ (74) we can reformulate (77) as

$$\Delta l = \gamma \Delta r, \tag{78}$$

where it is understood that Δl and Δr are aligned with the direction of motion. Again, these results remain essentially the same when we reverse the roles of e and O, or replace e with a particle p.

Above we have assumed small steps in time and space, either because they are the expected effect of a small expected change in action ($2/N$ for a single element), or because the actual effect of a single link is shared with a huge number of elements (for a particle). However, changes in time and space are related linearly to the network time over a considerable range (Section 5). Then we can generalize the results to larger time and space intervals. If v varies over the time interval this has to be done by integrating, but if v remains approximately constant the result is particularly simple. From (72) and (73) we then get

$$(\Delta r)^2 + (\Delta\tau)^2 = (\Delta t)^2 \tag{79}$$

or, rearranging,

$$(\Delta\tau)^2 = (\Delta t)^2 - (\Delta r)^2. \tag{80}$$

Here $\Delta\tau$ is an interval of proper time of the observed element or particle. A given interval of proper time is equivalent to a fixed link distance in the direction of network time. Because a link distance is a property of the element network, it must be an invariant, that is, the same for any observer in spacetime. Then $(\Delta\tau)^2$ is an invariant, as well as $(\Delta t)^2 - (\Delta r)^2$.

The analysis so far provides all the conditions that are needed for deriving the Lorentz transformations. Here we will just state the result, for use in Section 10. Assuming that a coordinate frame with coordinates (t', x', y', z') is moving with a velocity v along the x-axis relative to a frame with coordinates (t, x, y, z), we get

$$t' = \gamma(t - vx) \tag{81}$$

$$x' = \gamma(x - vt) \tag{82}$$

$$y' = y \tag{83}$$

$$z' = z. \tag{84}$$

Figure 10 is equally valid for an element e as for a particle p. However, a particle has a considerable rest energy (Section 6), because it consists of a large number of links, that is, a large internal quantity of action. Because energy is action per unit of time, a particle moving through proper time represents a fixed quantity of energy in the rest frame of the particle. Action has not only the dimension of energy×time, but also of momentum×length. A particle moving with a fixed velocity through space in the rest frame of an observer should then represent a fixed quantity of momentum in that frame. This suggest that an analysis that is similar to the one of Fig. 10 can be performed for energy and momentum rather than for time and space. Figure 11 illustrates how this works. A particle p, consisting of s elements and s links, moves in its inertial frame along the direction of proper time by link additions (Fig. 11a). Then the rest energy E_0 of the particle is a constant, equal to the total action it contains (i.e., s quanta of action) per unit of its proper time, defined here as the time shift $\Delta\tau$ produced by a link addition. Time and space have the same definition in terms of link distances in the inertial frame. Therefore, when, hypothetically, a link would be added in the space direction, this would produce a shift in space (i.e., a length) that would

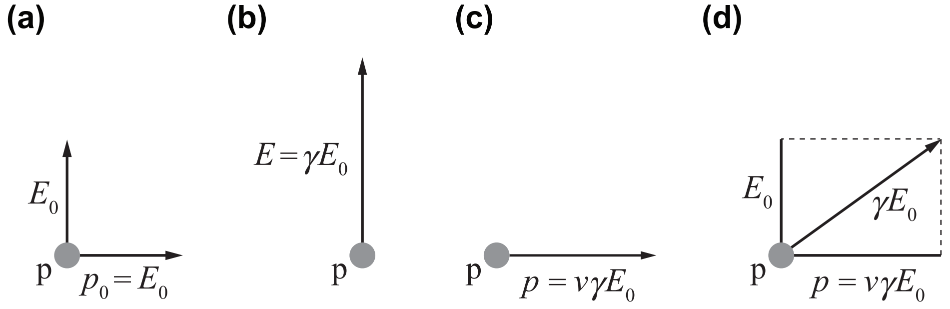

Figure 11: **a** A particle p with a rest energy E_0 makes a step of given magnitude in proper time when a link is added in its rest frame. A hypothetical link added in the space direction would produce a step of the same magnitude, with $p_0 = E_0$. **b** When an observer O views p moving with speed v, the observed energy of p would be the observed action per step of p's proper time, giving γE_0. **c** The observed momentum p equals the observed energy $v\gamma E_0$ in the direction of space. **d** Summary diagram. Depending on v, the total observed energy of magnitude γE_0 points in different spacetime directions, with space component $v\gamma E_0$ and time component E_0. See the main text for further explanation.

have the same magnitude as $\Delta\tau$. This would correspond to a momentum p_0 that is again a constant, equal to the total action s within the particle per unit of length, that is, per unit with a magnitude $\Delta\tau$. Then we must have

$$p_0 = E_0 \tag{85}$$

(Fig. 11a). Now suppose that an observer O views the particle moving at a velocity v relative to O's reference frame. Then the observer's time t runs faster than the particle's proper time τ by a factor γ (75). Thus, the action of the particle is, according to the observer, $E_0\Delta t = E_0\gamma\Delta\tau$ rather than the $E_0\Delta\tau$ in the particle's frame. The observed action per unit of the particle's proper time is then γE_0 (Fig. 11b). This is the total energy that the observer assigns to the particle. In the direction of space, action corresponds to momentum p rather than energy. According to the observer, the action is

$$p_0\Delta x = p_0 v\Delta t = p_0 v\gamma\Delta\tau = E_0 v\gamma\Delta\tau, \tag{86}$$

where we used (75) and (85). The observed action per unit of the particle's proper time is then $E_0 v\gamma$ (Fig. 11c). This is the energy $E_0 v\gamma$ that the observer assigns to the particle in the direction of space, or, equivalently, the momentum $p_0 v\gamma$ that the observer assigns to the particle. These results are summarized in the diagram of Fig. 11d. For any v, the total observed energy can be viewed as a vector of magnitude γE_0 pointing in different spacetime directions depending on v, with a space component $v\gamma E_0$ and a time component E_0. Note that this diagram is consistent with $\gamma^2 - v^2\gamma^2 = 1$ (which follows from $\gamma = 1/\sqrt{1-v^2}$). We get

$$(v\gamma E_0)^2 + (E_0)^2 = (\gamma E_0)^2 \tag{87}$$

or, rearranging to make each side of the equation invariant (since E_0 is a network property),

$$(E_0)^2 = (\gamma E_0)^2 - (v\gamma E_0)^2. \tag{88}$$

Here E_0 is the invariant rest energy (the mass m) of the particle, γE_0 its total energy (E) as observed by O, and $v\gamma E_0$ the observed relativistic momentum (p) of the particle. Then (87) can also be written as

$$E^2 = m^2 + p^2, \tag{89}$$

which is known as the energy-momentum relation of special relativity. For $v \to 0$ we get $\gamma \to 1$, which gives the classical momentum $p = mv$. For $v \to 1$ we get $\gamma \to \infty$, which makes the left-hand side of (88) vanishingly small compared with the terms on the right-hand side. Therefore, when $v = 1$, the rest energy equals zero and $E = p$.

8 Gravity and the expansion of space

Section 4 showed that W_n and V_n are proportional to $(2^n)^3$, where the derivation was based on the fact that elements are sparsely linked. While this is typically true, Section 6 showed that, in addition, there can be clusters of densely linked elements, observable as particles in spacetime. How would such network particles affect the calculation of W_n? For simplicity, we will assume below that any particle consists of s_e elements and s_ℓ links. This gives no loss of generality, because the result will be shown to depend on a sum over particles, not on differences between particles. Recall that W_n is calculated as follows (with (16) reproduced here for convenience)

$$W_n = \frac{1}{N}\sum_{i=1}^{N}\sum_{i_1=1}^{N} w(i,i_1)\sum_{i_2=1}^{N} w(i_1,i_2)\cdots\sum_{i_{n-1}=1}^{N} w(i_{n-2},i_{n-1})\sum_{i_n=1}^{N} w(i_{n-1},i_n). \tag{90}$$

Here W_n is the expected number of elements that are at link distance n from an observed element, and $w(i,j)$ is the expected number of links between the element with rank order i and the element with rank order j. Now suppose that in one of the summations of (90), say the summation over i_p, a particle p is encountered. This means that element i_p is part of the set of particle elements j_k, with $k \in [1, 2, \ldots, s_\mathrm{e}]$. Figure 12 shows the situation. The elements of p are at small link distances from the densely connected core of the particle. As a result, there is no space within the particle. Therefore, all elements j_k are, from the point of view of spacetime, acting as

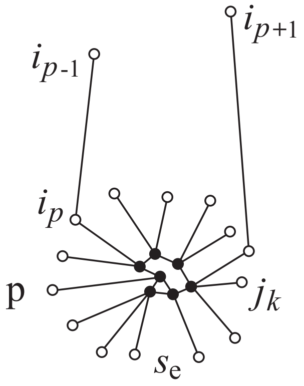

Figure 12: An element i_p is part of a particle p that consists of s_e elements and s_ℓ links. All elements j_k of p, including i_p, are elements of low rank order and consequently are almost always parents of the connecting elements i_{p-1} and i_{p+1}. The change in connectivity produced by the particle is s_e, because the connection to i_{p+1} can be routed through any element j_k, and, thus, the expected number of links between i_p and any i_{p+1} is s_e times larger than it would be for a single element i_p.

if they are, together, a single element. For the calculation of W_n, all links to p should, then, be treated as if they are connected to a single element. As already mentioned in Section 6, the rank orders of the elements of p are expected to be low (with $j_k \ll N$), because N is assumed to be sufficiently large such that the particle has effectively stopped growing (according to (60). Almost all elements of the particle's mantle were, thus, added early in time. The element i_p that is connected to i_{p-1} is then almost always a parent of i_{p-1}. Then w must be replaced with w_p (11) (where the p stands for parent, not particle). Therefore, whenever $w(i_{p-1}, i_p)$ occurs in (90) it takes the form $w(i_{p-1}, i_p) = w_\mathrm{p}(i_{p-1}, i_p) = 2/(i_{p-1} - 1)$ for any i_{p-1}. Similarly, the element i_{p+1} that is connected to any j_k is almost always a child of i_p, and whenever $w(i_p, i_{p+1})$ occurs in (90) it takes the form $w(i_p, i_{p+1}) = w_\mathrm{c}(i_p, i_{p+1}) = 2/(i_{p+1} - 1)$ (12) for any i_{p+1}. If there would be no particle and only a single element i_p of low rank order, then the relevant part of (90) would be

$$\begin{aligned} W_n &= \cdots \sum_{i_p=1}^{N} w(i_{p-1}, i_p) \sum_{i_{p+1}=1}^{N} w(i_p, i_{p+1}) \cdots \\ &= \cdots \sum_{i_p=1}^{N} \frac{2}{(i_{p-1} - 1)} \sum_{i_{p+1}=1}^{N} \frac{2}{(i_{p+1} - 1)} \cdots . \end{aligned} \tag{91}$$

When there is a particle with i_p as one of the particle's elements j_k, as in Fig. 12, then this becomes

$$\begin{aligned} W_n(\mathrm{p}) &= \cdots \sum_{i_p=1}^{N} w(i_{p-1}, i_p) \sum_{k=1}^{s_\mathrm{e}} \sum_{i_{p+1}=1}^{N} w(j_k, i_{p+1}) \cdots \\ &= \cdots \sum_{i_p=1}^{N} \frac{2}{(i_{p-1}-1)} \sum_{k=1}^{s_\mathrm{e}} \sum_{i_{p+1}=1}^{N} \frac{2}{(i_{p+1}-1)} \cdots \\ &= \cdots \sum_{i_p=1}^{N} \frac{2}{(i_{p-1}-1)}\, s_\mathrm{e} \sum_{i_{p+1}=1}^{N} \frac{2}{(i_{p+1}-1)} \cdots \\ &= s_\mathrm{e} W_n. \end{aligned} \tag{92}$$

This analysis is based on the condition that exactly one particle is encountered in each of the linked paths producing W_n. If there would be more than one particle in a linked path, the number of elements of the particles would multiply, which would produce a nonlinear term in W_n. For example, there would be a term with $s_i s_j$ when two particles i and j are on the same linked path and a term with $s_i s_j s_k$ when three particles i, j and k are on the same path. We will assume below that the number of particles is small enough such that no path has more than one particle and W_n remains in the linear regime.

The result of (92) is valid for any n, and therefore also for V_n

$$V_n(\mathrm{p}) = \sum_{i=1}^{n} W_i(\mathrm{p}) = \sum_{i=1}^{n} s_\mathrm{e} W_i = s_\mathrm{e} V_n. \tag{93}$$

This assumes again that there is exactly one particle in each of the linked paths producing V_n. Equation (93) shows that the presence of particles in a volume increases the amount of volume without affecting link distances, that is, the density of space increases. In effect, a particle acts as a path multiplier, replacing the multiplication by 2 of a single element with a multiplication by $2s_\mathrm{e}$.

The analysis so far is still confined to the element network, which is intrinsically dimensionless. If we want to infer how this would be observed in spacetime, with the right dimensions, we first need to look in more detail at how time and space are scaled in network terms. With respect to time, we have scaled that in earlier sections to $t_n = 2^n$, but the actual result was (51), which is henceforth our definition of t_n

$$V_n^\mathrm{F} = 2\ 2^n = b_\mathrm{t}\, 2^n = t_n, \tag{94}$$

with

$$b_\mathrm{t} = 2 \tag{95}$$

a scaling constant. With respect to space, the actual result was (24), which can be equated to the volume of a sphere with radius r_n

$$V_n = \frac{8}{7} a_n (2^n)^3 = \frac{4}{3} \pi \, r_n^3. \tag{96}$$

Then we find

$$r_n = \left(\frac{6a_n}{7\pi} \right)^{1/3} 2^n = b_\mathrm{r} \, 2^n, \tag{97}$$

with

$$b_\mathrm{r} = \left(\frac{6a_n}{7\pi} \right)^{1/3} \tag{98}$$

a scaling constant. The ratio of r_n and t_n shows how space changes relative to time, as a function of n

$$b = \frac{r_n}{t_n} = \frac{b_\mathrm{r}}{b_\mathrm{t}} = \frac{1}{2} \left(\frac{6a_n}{7\pi} \right)^{1/3} \approx \frac{1}{\sqrt{\pi}} \left(\frac{3}{7} \right)^{1/3} n^{-1/2} = b_1 n^{-1/2}, \tag{99}$$

where $a_n \approx \frac{4}{\sqrt{\pi}} n^{-3/2}$ (22) was used, and b_1 is defined as

$$b_1 = \frac{1}{\sqrt{\pi}} \left(\frac{3}{7} \right)^{1/3}. \tag{100}$$

From $b_\mathrm{r} = b \, b_\mathrm{t}$ we get a simple expression for b_r

$$b_\mathrm{r} = 2 b_1 n^{-1/2}. \tag{101}$$

We can now proceed with analyzing (93) further. That result is based on the assumption that each linked path of W_n contains exactly one particle. The total number of thus assumed particles in V_n can be obtained as follows. We first take a closer look at W_n, which was shown in (19) and (21) to be equal to

$$W_n = 2^n C_{n+1} = 2^n \frac{1}{n+2} \binom{2(n+1)}{n+1} \approx a_n \left(2^n \right)^3. \tag{102}$$

Thus, W_n consists in fact of two factors, a factor 2^n that is produced by n consecutive elements that each multiply the number of paths by a factor 2 (that is, the 2 that is in the definition of $w(i,j) = 2/(\max(i,j) - 1)$ (10)), and a factor $C_{n+1} \approx a_n \left(2^n \right)^2$ that is produced by the combinatorics of following different paths back and forth through the network. This factor would remain the same if elements had been different by not producing two links on average when created, but another number. For example, with $w(i,j) = 3/(\max(i,j) - 1)$ the result would have been $W_n = 3^n C_{n+1}$. By producing two links on average, each element is, in effect, a ×2 path multiplier. The assumption made above is that exactly one of the elements of each set of n elements that form a linked path is replaced with a particle, that is, by a $\times 2s_{\mathrm{e}}$ path multiplier. This is assumed for each of the C_{n+1} combinatoric paths through the network. Therefore, the total number of particles assumed is C_{n+1}. Because this analysis is valid for each shell W_i that contributes to V_n, the total number of particles u_n in V_n that were implicitly assumed for (93) follows from summation

$$\begin{aligned} u_n &= \sum_{i=1}^{n} C_{i+1} = \sum_{i=1}^{n} \frac{W_i}{2^i} = \frac{W_n}{2^n} + \frac{W_n/8}{2^{n-1}} + \frac{W_n/64}{2^{n-2}} + \cdots \\ &= \frac{4}{3}\frac{W_n}{2^n} = \frac{4}{3}\frac{7}{8}\frac{V_n}{2^n} = \frac{7}{6}b_{\mathrm{r}}\frac{V_n}{b_{\mathrm{r}}2^n} = \frac{7}{6}b_{\mathrm{r}}\frac{V_n}{r_n}, \end{aligned} \tag{103}$$

where we used (24) and (97). The number of particles u_n is $\propto (2^n)^2$, which is huge. Let us assume that the probability of encountering a particle in a path is α. In principle, α depends on the path length, n. However, because nearly all paths have length n or close to n when $n \gg 1$, a negligible error is made when α is assumed to be a constant. Moreover, we assume that $\alpha \ll 1$, small enough such that the probability of encountering two or more particles in a path is negligible (thus avoiding the nonlinearity mentioned below (92)). More precisely, we require that the contribution of two particles in a path of V_n is negligible compared with the contribution of one particle in a path, that is, $\alpha^2 s_{\mathrm{e}}^2 \ll \alpha s_{\mathrm{e}}$, or

$$\alpha s_{\mathrm{e}} \ll 1. \tag{104}$$

Then the probability of encountering no particle in a path is $1-\alpha$. Of the u_n paths, $(1-\alpha)u_n$ paths contain no particle and αu_n paths contain one particle. The number of particles K in the volume V_n is then αu_n and

$$\alpha = \frac{K}{u_n}. \tag{105}$$

Because a fraction $1-\alpha$ of the paths of V_n contains no particle and a fraction α contains one particle, we must have

$$V_n(K) = (1-\alpha)V_n + \alpha V_n(\mathrm{p}) = (1-\alpha)V_n + \alpha s_{\mathrm{e}} V_n, \tag{106}$$

using (93). Here $V_n(K)$ denotes the volume when it contains K particles, and V_n, as before, the volume when it contains no particles. With (105) and (103) this becomes

$$\begin{aligned} V_n(K) &= (1+(s_{\mathrm{e}}-1)\alpha)V_n \approx (1+s_{\mathrm{e}}\alpha)V_n = (1+\frac{Ks_{\mathrm{e}}}{u_n})V_n \\ &= (1+\frac{Ks_{\mathrm{e}}}{(7/6)b_{\mathrm{r}}V_n/r_n})V_n = (1+\frac{6}{7}\frac{1}{b_{\mathrm{r}}}\frac{Ks_{\mathrm{e}}}{V_n}r_n)V_n. \end{aligned} \tag{107}$$

Here Ks_e is the total number of particle-bound elements in the volume V_n. Therefore, the ratio of Ks_e and V_n is the density $\tilde{\rho}_\mathrm{e}$ of particle-bound elements

$$\tilde{\rho}_\mathrm{e} = \frac{Ks_\mathrm{e}}{V_n}. \tag{108}$$

The tilde indicates that $\tilde{\rho}_\mathrm{e}$ is a network variable, in order not to confuse it with a spacetime variable. In other variables, such as r_n, this is indicated by the index n. We can still use V_n rather than $V_n(K)$ in this equation, because the change of V_n in (107) is tiny, with $s_\mathrm{e}\alpha \ll 1$ (104). Now (107) becomes

$$V_n(\tilde{\rho}_\mathrm{e}) = (1 + \frac{6}{7}\frac{1}{b_\mathrm{r}}\tilde{\rho}_\mathrm{e} r_n)V_n. \tag{109}$$

Each particle has not only s_e elements, but also $s_\ell = s_\mathrm{e}$ links. Each link is assumed to be a quantum of action in spacetime, thus s_ℓ is the action bound to each particle. We have shown in Section 6 that the particle-bound action corresponds to a fixed rest energy of a particle when moving along the proper time of the particle. In other words, s_ℓ is proportional to the mass of the particle. Therefore, (109) shows a relationship between the effective volume of space and the amount of particle-bound action—or mass—contained in space. In order to investigate this further, we replace Ks_e with Ks_ℓ, $\tilde{\rho}_\mathrm{e}$ with the link density $\tilde{\rho}_\ell$, and rewrite (109) as

$$V_n(\tilde{\rho}_\ell) = (1 + \frac{6}{7}\frac{1}{b_\mathrm{r}}\tilde{\rho}_\ell r_n)V_n. \tag{110}$$

Whilst Ks_e is numerically identical to Ks_ℓ, they have different physical interpretations. Whereas Ks_e is a path multiplier that increases the volume V_n, Ks_ℓ is the total particle-bound action contained in V_n. Thus, (110) tells us how the effective volume of space depends on the presence of particle-bound action.

The volume V_n is scaled by a dimensionless factor that is slightly larger than one. This scaling must be the same irrespective of whether the scaled volume is expressed as a network volume V_n or as a spacetime volume V. We will therefore focus on

$$\tilde{\rho}_\ell r_n, \tag{111}$$

which can be expressed as

$$[\text{number of particle-bound links in a volume } V_n] \times \frac{r_n}{V_n}. \tag{112}$$

In spacetime, r_n and $V_n \propto r_n^3$ must be expressed in terms of r, and, thus, we need to find the proper spacetime scaling for $1/r_n^2$. This can be done by first finding the proper spacetime scaling for t_n by using the Hubble constant H_0, as will be explained next.

The Hubble constant is the factor by which the mean recession velocities of cosmic objects, such as galaxies, grow as a function of distance. It signifies the cosmic expansion of space. At the time of writing, there is some uncertainty about the correct value of H_0, where measurements with different methods fall in the range 66–76 in the units shown below (see figure 1 in ([6])). For the present purpose, we will use the typical value that is based on direct observations

$$H_0 \approx 73 \text{ km/s/Mpc} \approx 2.37 \cdot 10^{-18} \text{ s}^{-1}. \tag{113}$$

Here Mpc stands for the astronomical distance measure megaparsec, with 1 Mpc $\approx 3.086 \cdot 10^{16}$ m. The subscript 0 indicates that it is the value at the current cosmic time. Below we will follow that convention by writing n_0 for the current operating point of n in order to distinguish it both from values of n at other epochs of cosmic time and from intervals of n around n_0. Notational clutter will be reduced by abbreviating the latter by η, that is

$$\eta = \delta n. \tag{114}$$

The subscript η will also be used for t_n, r_n and V_n, to indicate that these are to be interpreted as resulting from intervals around n_0. Because these variables are exponential in n, the equations for intervals keep the same form (apart from factors $\ln 2$). For example, (112) becomes

$$[\text{number of particle-bound links in a volume } V_\eta] \times \frac{r_\eta}{V_\eta}. \tag{115}$$

From the units of H_0, (m/m) per second, it is clear that it can be interpreted as the relative expansion of space per unit of time. The relative expansion of r_η in the current theory can be obtained from (97)

$$\frac{\delta r_\eta}{r_\eta} = \frac{b_\mathrm{r} \ln(2)\, 2^{n_0}\eta}{r_\eta} = \ln(2)\,\eta, \tag{116}$$

where we neglect the derivative of b_r that follows from (101) $\mathrm{d}b_\mathrm{r}/\mathrm{d}n \propto b_\mathrm{r}/n \ll b_\mathrm{r}$, because $n_0 \gg 1$. We conclude that $H_0/\ln 2$ equals the change η, that is, the change in link distance, per second

$$\eta/\mathrm{s} = H_0/\ln 2. \tag{117}$$

A change η in link distance produces a change not only in r_η, but also in t_η according to (94)

$$\delta t_\eta = b_\mathrm{t} \ln(2)\, 2^{n_0}\eta. \tag{118}$$

Because η per second equals $H_0/\ln 2$, the Hubble constant provides a way to convert a time interval δt in SI units to a time interval δt_η in network units. However, the conversion includes an unknown factor 2^{n_0}, which is huge but constant. Fortunately,

$$\begin{tikzcd}
t_\eta \arrow[r, "\times\frac{1}{b}"] & r_\eta \\
t \arrow[u, "\times\frac{H_0}{\ln 2}"'] & r \arrow[l, "\times\frac{1}{c}"']
\end{tikzcd}$$

Figure 13: Factors for converting the spacetime variables r and t and the network variables r_η and t_η into one another. Here c is the spacetime speed of light, H_0 the Hubble constant, and b the network scaling of space and time (99).

this factor drops out when the conversion to δr_η and δr is included, as will become clear below.

The factor $b_\mathrm{t} \ln(2)\, 2^{n_0}$ in (118) is fixed and can be defined as the unit $[t_{n_0}]$ that converts an interval η into an interval δt_η

$$\delta t_\eta = [t_{n_0}]\eta, \tag{119}$$

with

$$[t_{n_0}] = b_\mathrm{t} \ln(2)\, 2^{n_0}. \tag{120}$$

Similarly, the unit for r_η is

$$[r_{n_0}] = b_\mathrm{r} \ln(2)\, 2^{n_0}. \tag{121}$$

However, we have established in Section 5 that the maximum speed v_max in network terms equals one (see Fig. 7 and explanation). Therefore, we will work with units that are equal for space and time, and do that consistently both in spacetime (with $ct = r$) and in the network. Then the network unit $[r_{n_0}]$ must be multiplied by $b_\mathrm{t}/b_\mathrm{r} = 1/b$ in order to make it equal to the unit $[t_{n_0}]$. This means that if we want to convert t_η to r_η in normalized units, we have to multiply by $1/b$. Note that this is different from what one might think, at first sight, namely that we have to multiply by b because of (99). However, the b there does not indicate a scaling of units, but a scaling of spacetime itself. Hence the inversion that we get here when we normalize to the maximum speed. All network units and b are still dimensionless.

Figure 13 summarizes the steps we have taken above. Together this produces

$$r_\eta = \frac{1}{b} t_\eta = \frac{1}{b} \frac{H_0}{\ln 2} t = \frac{H_0}{bc \ln 2} r, \tag{122}$$

with c the speed of light in spacetime. The r_η/V_η in (115) now becomes

$$\frac{r_\eta}{V_\eta} = \left(\frac{bc \ln 2}{H_0} \right)^2 \frac{r}{V} \ . \tag{123}$$

The [number of particle-bound links in a volume V_η] in (115) becomes [number of particle-bound links in a volume V]. Because each link is assumed to be a quantum of action, h_ℓ, with dimension energy×time, (111) becomes, combined with (123)

$$\tilde{\rho}_\ell r_\eta = \frac{E\,t}{h_\ell}\left(\frac{bc\ln 2}{H_0}\right)^2 \frac{r}{V} = \frac{\rho_{\mathrm{E}} t}{h_\ell}\left(\frac{bc\ln 2}{H_0}\right)^2 r, \tag{124}$$

where $\rho_{\mathrm{E}} = E/V$ is the energy density in spacetime. With $t = r/c$, $b = b_1 n_0^{-1/2}$ (99), and $b_{\mathrm{r}} = 2b_1 n_0^{-1/2}$ (101), we can write (110) as an expression in spacetime

$$\begin{aligned} V(\rho_{\mathrm{E}}) &= (1 + \frac{(6/7)(1/(2b_1 n_0^{-1/2}))b_1^2 n_0^{-1} c\,(\ln 2)^2}{h_\ell H_0^2}\rho_{\mathrm{E}} r^2)V \\ &= (1 + \frac{(3/7)b_1 n_0^{-1/2} c\,(\ln 2)^2}{h_\ell H_0^2}\rho_{\mathrm{E}} r^2)V. \end{aligned} \tag{125}$$

The dimension of the scaling part of this expression is, with E representing the dimension of energy,

$$\dim\left(\frac{(3/7)b_1 n_0^{-1/2} c\,(\ln 2)^2}{h_\ell H_0^2}\rho_{\mathrm{E}} r^2\right) = \frac{\mathsf{L}\,\mathsf{T}^{-1}}{\mathsf{E}\,\mathsf{T}\,\mathsf{T}^{-2}}\,\mathsf{E}\,\mathsf{L}^{-3}\,\mathsf{L}^2 = \mathsf{1}. \tag{126}$$

The scaling is dimensionless, as it should be.

We have shown before (see below (108)) that the term added to 1 in the factor that scales V, is tiny. Thus, $\sim r^2$ in (125) represents a tiny correction $\sim r^5$ on the dominant r^3 of V. It can be interpreted as a curvature of space, which will be explained in the paragraphs below. Before doing that, it is useful to remark that (125) does not mean that space in regions without particles remains flat. Because ρ_{E} is defined for regions of any size, varying this region shows that the influence of particles must decline gradually. There can be no discontinuities, nor singularities, because the network would not support those. Very close to a particle or a cluster of particles, space would not blow up, but rather cease to exist (gradually, as when moving to the left in Fig. 6b).

Hartle ([7], pp. 20 and 147-148) shows that the volume V of a sphere of radius r embedded in a curved three-dimensional space with radius of curvature a is given by

$$V = 4\pi a^3\left\{\frac{1}{2}\sin^{-1}\left(\frac{r}{a}\right) - \frac{r}{2a}\left[1 - \left(\frac{r}{a}\right)^2\right]^{1/2}\right\}. \tag{127}$$

For small r/a this can be approximated by

$$V \approx 4\pi a^3\left(\frac{1}{3}\left(\frac{r}{a}\right)^3 + \frac{1}{10}\left(\frac{r}{a}\right)^5\right) = \frac{4}{3}\pi r^3\left(1 + \frac{3}{10}\left(\frac{r}{a}\right)^2\right). \tag{128}$$

In general relativity, the local curvature of space is not specified by a local radius of curvature, but by the scalar curvature (or Ricci scalar) R. The scalar curvature R is related to the sectional curvature K of a manifold as $R = n(n-1)K$ in n dimensions, and the sectional curvature K equals $1/a^2$ for a local radius of curvature a. Then a and R are related as $1/a^2 = R/6$ for a three-dimensional space. Whereas a has the dimension L, R has the dimension L^{-2}. Now (128) can be written as

$$V \approx \frac{4}{3}\pi r^3 \left(1 + \frac{R}{20} r^2\right). \tag{129}$$

This has the same form as (125) and leads to the following expression for R

$$R = \frac{(60/7) b_1 n_0^{-1/2} c\,(\ln 2)^2}{h_\ell H_0^2} \rho_{\mathrm{E}}. \tag{130}$$

Hence, the curvature of space, R, and the energy density within space, ρ_{E}, are related linearly. This relationship is consistent with one that can be derived from the Einstein field equations, as will be briefly explained now. We will ignore the cosmological constant here. The Einstein equations are then given by (e.g., [8], p.184)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}, \tag{131}$$

where $R_{\mu\nu}$ is the Ricci tensor, R the Ricci scalar, $g_{\mu\nu}$ the metric tensor, $T_{\mu\nu}$ the stress-energy tensor, and $\kappa = 8\pi G/c^4$ the Einstein gravitational constant, which depends on the speed of light c and on G, the Newtonian constant of gravitation. The analysis above assumes implicitly what in cosmology is called 'dust,' a collection of particles or objects that are not interacting other than through gravity, and that are moving slowly ($\ll c$) in the observer's local inertial frame (see [9], section 4.2). The stress-energy tensor has then only a single non-zero component, namely $T_{00} = \rho_{\mathrm{E}}$. Taking the trace of (131) gives

$$R_{\mu\nu} g^{\mu\nu} - \frac{1}{2} R g_{\mu\nu} g^{\mu\nu} = R - \frac{1}{2} R\ 4 = -R = \kappa T_{\mu\nu} g^{\mu\nu} = -\kappa \rho_{\mathrm{E}}, \tag{132}$$

or

$$R = \kappa \rho_{\mathrm{E}}. \tag{133}$$

Some of the signs in (131) and (132) depend on how one combines the sign conventions used for three different definitions (see the Table of Sign Conventions at the beginning of [10]). This issue need not concern us here, because any specific combination of sign conventions that one chooses can only lead to the same sign for the physical result (133). Positive energy density ρ_{E} comes with positive curvature R.

The analysis above shows that particles affect the curvature of space in a way that is consistent with general relativity. Particles do not affect time directly, because time is equivalent to paths consisting exclusively of links traversed in the forward time direction. The mechanism of Fig. 12 then would not work, because it requires that links are traversed in both directions, since particles consist mostly of low-rank-order elements. However, particles affect time indirectly, because the spacetime and energy-momentum relations of Section 7 still apply, at least locally. The curvature of space shown above is, then, a curvature of spacetime in the general case. The consequences of this can be worked out quantitatively, but some effects are immediately clear qualitatively. For example, when a section of space becomes more dense because of the presence of particles, the maximum speed of light means that crossing such a section would take more time. Time in such a section would progress more slowly when measured by an external observer.

How particles move through curved space, in the sense of classical physics, is not addressed here explicitly. However, the classical limit of quantum physics, as it follows from the Feynman path integral, is equally valid for any spacetime produced by the network, including ones with gravity (see the end of Section 11). This fully defines classical trajectories through curved space.

An expression for n_0 follows from (130) and (133)

$$n_0 = \left(\frac{(60/7) b_1 c \, (\ln 2)^2}{\kappa h_\ell H_0^2} \right)^2 . \tag{134}$$

Here b_1 is given by (100), $c = 2.9979 \cdot 10^8 \,\mathrm{m/s}$, $\kappa = 2.0767 \cdot 10^{-43} \,\mathrm{s^2/(m\,kg)}$ when $G = 6.6743 \cdot 10^{-11} \,\mathrm{m^3/(kg\,s^2)}$, $h_\ell = h/2$ with $h = 6.6261 \cdot 10^{-34} \,\mathrm{kg\,m^2/s}$, and H_0 is given by (113). Then this produces

$$n_0 = 1.8 \cdot 10^{240}, \tag{135}$$

a huge number. But it is tiny compared with the total number of elements N_0

$$N_0 \approx V_{n_0} = \frac{8}{7} a_{n_0} \left(2^{n_0}\right)^3 = \frac{8}{7} \frac{4}{\sqrt{\pi} n_0^{3/2}} \left(2^{n_0}\right)^3 \sim 10^{1.7 \cdot 10^{240}} . \tag{136}$$

These estimates show that approximations in earlier sections that were based on $n \gg 1$, $N \gg 1$ and $\ln N \gg 1$ are well motivated, at least in the current cosmic epoch. Because both n and N decrease when going back in time, the scaling factor b between r_n and t_n would increase and is expected to affect the numerical value of some parameters, such as κ and G. However, this would affect only the very early universe.

The number of elements one could observe when looking back i links from n_0 grows as 2^i, and the number of existing elements going forward j links from the first two elements grows as 2^j. Because the number of observable elements cannot become larger than the number of existing elements, the maximum number of observable elements occurs at $i \approx j \approx n_0/2$. Therefore, the number of elements one could observe right now is about $N_0^{1/2} \sim 10^{10^{240}}$, and the total universe is about $N_0^{1/2} \sim 10^{10^{240}}$ larger than the observable universe. Nearly all of the current universe is causally disconnected from the here and now, and will remain so.

The derivation above is based on slowly moving 'dust.' When the particles are moving at relativistic speeds, together or in more complex ways, the spacetime and energy-momentum relations of Section 7 can be used to analyze the consequences (along the lines of Chapter 4 of [9]). It requires methods from statistical physics, in particular how the energy would be expected to be distributed amongst the particles of an ensemble, at a given temperature. Moreover, interactions between particles would affect the energy density. These topics will not be elaborated upon here.

The present theory does not produce a fixed expansion rate of spacetime. When a new element becomes linked to the existing network, its effect depends on where exactly its links attach. If each link goes to an element of the spatial part of the network, then the new element will contribute to expansion. If at least one of its links goes to a particle and another one to space, it will contribute to modifying the spatial embedding of that particle, that is, it will take part in the movement of the particle. And if two or more of the links of a new element go to different particles, it will mediate an interaction between these particles, that is, a force. This means that when the density of particles in the network is larger, a smaller fraction of the new elements will contribute purely to space, and the expansion of space will be slower. Because the density of particles is a dynamic variable, the expansion of space is dynamic as well.

The theory proposed here produces gravity in the same way as general relativity, by producing curvature of spacetime. Still, the theories are not identical. In particular, the present theory produces a spacetime that is inevitably expanding, which is just a consequence of the growth of the network and the progress of time. Moreover, space is inevitably flat for large N, because eventually the presence of matter will become so much diluted that empty, flat space dominates. In contrast, general relativity allows for different outcomes with respect to expansion and flatness. This does not necessarily mean that it is inconsistent with the present theory, just that it has more degrees of freedom than are required for describing the reality that is conjectured here.

Cosmological measurements indicate that spacetime is indeed flat of nearly flat. The flatness is incorporated in the standard ΛCDM cosmological model by setting a constraint on the energy density in the universe. It is required to be equal to the critical density ρ_{crit} that produces flatness according to the standard FLRW metric. In normalized densities (with $\Omega_i = \rho_i/\rho_{\text{crit}}$), the density for matter (baryonic+dark) is estimated to be only $\Omega_m \approx 0.3$ at the present time, and the density of radiation is estimated to be negligible. The above constraint then makes it necessary to introduce another energy density of unknown origin, with $\Omega_{\text{DE}} = 1 - \Omega_m \approx 0.7$. Here DE stands for dark energy. In the ΛCDM model it is described by the cosmological constant Λ that can be added to the Einstein field equations. The cosmological constant would be a homogeneous energy source that produces negative pressure, with $\Omega_\Lambda \approx 0.7$. It can drive the expansion of space and keep it going at a constant or increasing rate, which would otherwise be difficult to understand in general relativity because gravity would oppose expansion. The theory presented here does not need and should not have an additional parameter that produces flatness and unabated expansion. However, it may still have properties that can be interpreted by general relativity as such a component. Below it will be argued that space in the presence of matter indeed has a homogenous energy density that does not gravitate in the normal manner, but makes space less rather than more dense.

As discussed in Section 6, the ratio of links to elements in the network is and remains 2, which is equivalent to conservation of energy. But in particles this ratio equals 1, because the number of particle-bound elements, s_{e}, is equal to the number of particle-bound links, s_ℓ. The reason is that the particle consists of ^{1}e elements (apart from a core of negligible size). This means that there is a shortage of ^{1}e elements in the space part of the network, and that the ratio of links to elements there must be slightly larger than 2. How much larger follows from the following simple considerations. Suppose that the mean link density of matter is $\tilde{\rho}_\ell$ in a network of size N. The number of particle-bound links is then $N\tilde{\rho}_\ell$ and the number of particle-bound elements also $N\tilde{\rho}_\ell$. Overall, a network of N elements must have $2N$ links. Thus, the number of links in the space part of the network is $2N - N\tilde{\rho}_\ell$ and the number of elements there $N - N\tilde{\rho}_\ell$. The ratio of links to elements there is then not 2, but

$$\frac{2N - N\tilde{\rho}_\ell}{N - N\tilde{\rho}_\ell} \approx 2 + \tilde{\rho}_\ell, \tag{137}$$

with $\tilde{\rho}_\ell \ll 1$. Therefore, the excess link density in the space part of the network equals the mean link density of matter. Link density must be proportional to energy density in spacetime (see above). The link density of empty space, 2, would not be observable from within space, because it is a constituent of space itself. Moreover, empty space does not gravitate. A deviation from 2 might nevertheless be observable. If it is identified as dark energy or Λ, then one would expect $\Omega_{\mathrm{DE}} = \Omega_m = 0.5$, or just $\Omega_{\mathrm{DE}} = \Omega_m$ if ρ_{crit} is not assumed to be a tipping point. This is different from current estimates of $\Omega_{\mathrm{DE}} \approx 0.7$ and $\Omega_m \approx 0.3$, but at least it is of the same order of magnitude. Establishing whether there is a real discrepancy, and if so, whether it can be resolved, would require a more detailed study than can be done here. In any case, ρ_m decreases gradually as matter is diluted by the expansion of space, and according to the present analysis ρ_{DE} should then decrease in proportion.

One might wonder how the excess link density of space discussed above affects the density of space, that is, whether it has effects on gravity. The mechanism of Fig. 12 does not apply, because the relaying elements are not of low rank order and the condition that there is at most one particle per path is not fulfilled. If formulated in those terms, each element here is a $\times(2 + \tilde{\rho}_\ell)$ path multiplier. In contrast to the mechanism of Fig. 12, it affects not only space but also time directly. Replacing the factor 2 in the definition of $w(i,j)$ by $(2+\tilde{\rho}_\ell)$ affects the definition of time as

$$t'_n = b_\mathrm{t}\,(2+\tilde{\rho}_\ell)^n \tag{138}$$

and of space as

$$\begin{aligned} V'_n &= \frac{8}{7}W'_n = \frac{8}{7}(2+\tilde{\rho}_\ell)^n C_{n+1} = \frac{8}{7}(2+\tilde{\rho}_\ell)^n \frac{1}{n+2}\binom{2(n+1)}{n+1} \\ &= \frac{8}{7}a_n(2+\tilde{\rho}_\ell)^n\,(2^n)^2\,, \end{aligned} \tag{139}$$

where (19) and (24) were used. Note that the combinatorial factor C_{n+1} is not affected. Then using the time t'_n as reference (as we should, because $v_\mathrm{max}=1$ remains true), we can rewrite V'_n as

$$\begin{aligned} V'_n &= \frac{8}{7}a_n(2+\tilde{\rho}_\ell)^n\,(2^n)^2 \approx \frac{8}{7}a_n\left((2+\tilde{\rho}_\ell)^n\right)^3\left((1-\tilde{\rho}_\ell/2)^n\right)^2 \\ &\approx \frac{8}{7}a_n\left((2+\tilde{\rho}_\ell)^n\right)^3(1-\tilde{\rho}_\ell n)\,. \end{aligned} \tag{140}$$

The factor $(1-\tilde{\rho}_\ell n)$ shows that space contains fewer elements than one would expect based on t'_n. In other words, the energy density $\tilde{\rho}_\ell$ makes space less rather than more dense, an effect opposite to that of regular gravity. It is a global effect, though, that is small locally.

9 Entropy and the arrow of time

Since the network is stochastic, it is interesting to investigate its entropy and how that changes over time. This cannot be translated directly into an entropy defined in spacetime, because entropy depends on the units considered. Would we consider molecules, elementary particles, or elements? Even when considering, say, molecules, one has to decide on coarse graining, that is, on the scale of the analysis. Here we will just focus on the network and its elements, because that is rather straightforward. Nevertheless, the network entropy and spacetime entropy are, of course, not unrelated. A major observable coupling is constituted by photons, which are argued in Section 12 to consist of a subset of new ^{2}e elements. These are amongst the most common new elements (5).

We will use here the Gibbs entropy S, equal to Shannon's entropy used in information theory, with any scaling constant set to one (e.g., the Boltzmann constant $k_\mathrm{B}=1$)

$$S = -\sum_i p_i \ln p_i. \tag{141}$$

Here i denotes the possible states and p_i the probability of state i. For small N some of the states of the network that can arise when a new element is added, are degenerate. For example, the states depicted in Figs. 1e and f are identical. However, when N becomes large, the number of degenerate states will quickly become small compared with the total number of possible states. Hence, by focussing on large N, we can neglect this issue. From equation (2) we know that the probability of getting a new state with k links connecting with the new element is

$$P_k = k\binom{N}{k}\left(\frac{1}{N}\right)^k\left(1-\frac{1}{N}\right)^{N-k}. \tag{142}$$

This is actually a group of $\binom{N}{k}$ individual states, each with a probability

$$\tilde{P}_k = k\left(\frac{1}{N}\right)^k\left(1-\frac{1}{N}\right)^{N-k}, \tag{143}$$

with

$$\binom{N}{k}\tilde{P}_k = P_k. \tag{144}$$

These states are the ones that can arise when a new element is added to a given network that has already N elements. The entropy $S(N)$ is

$$S(N) = -\sum_{k=1}^{N}\binom{N}{k}\tilde{P}_k\ln\tilde{P}_k. \tag{145}$$

For large N and $k \ll N$ we have

$$\tilde{P}_k \approx k\left(\frac{1}{N}\right)^k e^{-1} \tag{146}$$

and

$$\ln\tilde{P}_k \approx \ln k - k\ln N - 1 \approx -k\ln N. \tag{147}$$

Then

$$\begin{aligned} S(N) &= -\sum_{k=1}^{N}\binom{N}{k}\tilde{P}_k\ln\tilde{P}_k \approx \ln(N)\sum_{k=1}^{N}k\binom{N}{k}\tilde{P}_k \\ &= \ln(N)\sum_{k=1}^{N}kP_k = 2\ln N. \end{aligned} \tag{148}$$

where (144) was used, and the final step follows from the fact that the sum is the mean of P_k, which equals 2 (see below (5)). We see that the entropy equals the sum of the entropies of each of the two links that a new element has on average, where each link can choose from N existing elements (which gives N possible states, each with a probability of $1/N$).

The $S(N)$ of (148) is the entropy added to the total entropy of the network when a new element becomes attached. Entropies from subsequent additions of elements can be added, because the additions are independent from the state of the existing network. We use here the fact that the entropy of a joint probability distribution of independent variables is the sum of the entropies of the marginal distributions. The total entropy $S_\mathrm{c}(N)$ (where c stands for cumulative) of the network is then

$$S_\mathrm{c}(N) = \sum_{i=1}^{N} S(i) = \sum_{i=1}^{N} 2\ln i = 2\ln N! \approx 2N\ln N. \tag{149}$$

The final step follows from Stirling's approximation expressed as $\ln n! = n\ln n - n + \mathcal{O}(\ln n)$.

The very first structure in existence (Fig. 1c), when $N = 2$, represents a single state, with $p_1 = 1$. Thus, $S(2) = S_\mathrm{c}(2) = 0$. From there, the entropy steadily increases with N, both the entropy $S(N)$ added at each step in network time and the total entropy $S_\mathrm{c}(N)$. This agrees with the notion that the entropy of the universe always increases and never decreases. This is often used as an explanation for why the arrow of time points forward, whereas this direction of time is not stated explicitly by the fundamental laws of physics as formulated in spacetime (which is indeed time-symmetrical because of the equality of (48) and (50)). However, that explanation for the arrow of time is false according to the present theory. Time runs forward, fundamentally, because the network time depends on the irreversible addition of new elements (Section 2). And the entropy grows, fundamentally, because the addition of new links is arbitrary. In addition, observed entropy grows in spacetime when one applies coarse graining when observing. Coarse graining means losing information about the state of a system, for statistical reasons. Losing information means increasing the observed entropy. But this is not a fundamental issue. The fundamental increase of entropy would happen even if one could know the current state of the network perfectly. Still, it is clear that the latter is not possible, since there is no way by which one could observe or infer the network in detail.

10 Wavefunctions and the Born rule

In this section and the next one we will take a closer look at how well the present theory agrees with quantum physics. Quantum physics is incorporated from the very start, since the network is connected by and grows by discrete links that are quanta of action in spacetime. The theory is literally a theory of quantum accretion. Yet, at first sight it is not clear how the network could lead to a description that is based on travelling and interfering waves. Where are those waves and what would produce them? There seems to be nothing that could produce a harmonic oscillator, with a restoring force proportional to a displacement. However, there are in fact two distinct mechanisms in the growing network that can be interpreted as waves. One of these is discussed in the section on photons (Section 12) and the other one is discussed here.

Since quantum physics has a long and ongoing history of interpretational uncertainty, some cautionary remarks may be appropriate. The statistical approach used here might seem classical at first glance, but it is in fact nothing of the kind. In particular, there is no continuous and smooth classical spacetime. Network time is fundamental yet discrete, and observed time is stochastic. Space is stochastic too, but not fundamental. The network is non-deterministic at its core, and has no deterministic 'hidden variables.' The theory is neither local nor nonlocal, because within the network there is no space and the concept of locality is inapplicable, fundamentally. Although the formulation below does not use an operator algebra, Section 11 shows that the network is consistent with Feynman's path integral formulation of quantum physics; this formulation is known to be equivalent to other standard formulations.

The key observation we will use here is that, even without oscillations, the network does contain frequencies in the form of the number of links that attach to particles per unit of time. We have seen in Section 6 (at (67) and below) that a particle with s elements will receive s times more links per unit of time than a single space-forming element. Yet, on average its progress in time is the same as that of single elements, since each of its time steps is s times smaller. Because the particle consists almost completely of ${}^{1}\mathrm{e}$ elements, it also has s links and its mass m is proportional to s. The proper time τ of a particle p with mass m thus progresses with time steps

$$\delta\tau \propto \frac{1}{m}. \tag{150}$$

Then $1/\delta\tau$ can be interpreted as an intrinsic frequency f_{p} of the particle, with f_{p} the number of steps per unit of time. For convenience, we will scale the units in the following analysis such that $\delta\tau = 1/m$ and

$$f_{\mathrm{p}} = m. \tag{151}$$

This scaling corresponds to setting $h_\ell = 1$, because m has the dimension of energy (with $c = 1$ and $v_{\max} = 1$), and f_{p} is the number of links, that is, the number of h_ℓ (with dimension energy×time) per unit of time.

We will now first consider how a particle would appear in the time domain, and only after that extend that to spacetime. We first note that the frequency f_{p} that is defined above is not the frequency of a smooth harmonic oscillation. This becomes clear when we compare the progress of the particle's proper time τ with the proper time t_{o} of a comoving macroscopic object (such as a clock that records time or is being read out by an observer). The macroscopic object is assumed to have a mass $M \gg m$, such that

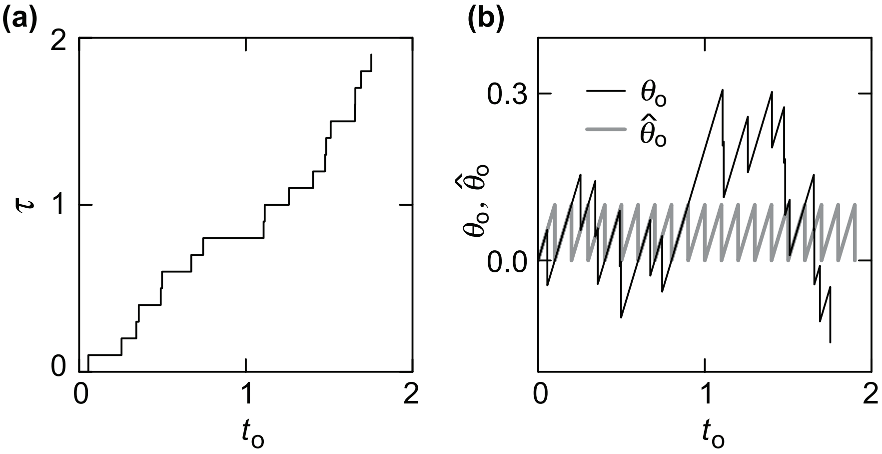

Figure 14: **a** Microscopic proper time τ of a particle p with mass m as a function of the (smooth and regular) proper time t_o of a macroscopic observer o. Addition of, on average, m links per unit of time (with $m = 10$ in this example) produces steps of $\delta\tau = 1/m$ according to a Poisson point process. **b** The difference $\theta_\mathrm{o} = t_\mathrm{o} - \tau(t_\mathrm{o})$ is an irregular saw-tooth oscillation. It can be estimated by a regular one, $\widehat{\theta}_\mathrm{o}$, at the same rate. Here $\widehat{\theta}_\mathrm{o}$ is defined as the expected elapsed time since the latest expected step in τ. A new observation will reset the phase of the estimate $\widehat{\theta}_\mathrm{o}$ to the actual phase of θ_o, which is set to zero (i.e., τ is set to t_o) at the moment of observation (when a new ^{2}e element links to both o and p). An observation is assumed at time $t_\mathrm{o} = 0$.

t_o is, in effect, progressing smoothly, with high-frequency time steps that are too small to be observable. In contrast, with sufficiently small m, τ depends measurably on the random occurrences of new links. The arrival of new links must occur according to a Poisson point process, because the occurrences are uncorrelated and the process is fully defined by its rate, f_p. Specifically, the probability that a link to a new element is acquired by one of the particle's elements is proportional to m (because a new link is equally likely to attach to any of the N existing elements). Whenever a new link attaches to the particle, a single step $\delta\tau$ is made relative to the macroscopic t_o, with no progress in time between steps. Figure 14a shows an example of τ versus t_o (for $m = 10$), and Fig. 14b (black line) shows the function $\theta_\mathrm{o} = t_\mathrm{o} - \tau(t_\mathrm{o})$, indicating how t_o and τ differ. As can be seen, θ_o fluctuates as an irregular saw-tooth oscillation.

The moments when τ makes a step are usually not directly observable by a comoving clock. Newly added links to the particle will nearly always be to new elements that do not, in addition, link to the clock. This is necessarily true for new ^{1}e elements (about 37% of all new elements, see (5)), which are not observable, since they only link to the particle. In contrast, ^{2}e elements (also about 37%) that link to the particle might also link to the clock, but most would link to elements that are distinct from the clock (usually remote in physical space, including at space-like distances). The remaining $^{3+}$e elements would, even if linked to both particle and clock, usually have links elsewhere and, therefore, would provide ambiguous information at best. Thus, only a tiny fraction of the steps made by τ could be observed directly, when a ^{2}e element links to both particle and clock. When such an observation occurs, the step in τ coincides with the time t_{o}. The state of the particle in terms of the macroscopic proper time is then fully known, with $\tau = t_{\mathrm{o}}$ and $\theta_{\mathrm{o}} = 0$.

Subsequently, t_{o} will run forward in a well-defined way, but how θ_{o} (and thus τ) progresses can only be known in probabilistic terms. The clock cannot know exactly when τ makes new steps, since they are random. Only when a new observation is made, the corresponding step is observed. Observation is here a random process that may take a long time (in terms of number of steps) to be realized, depending on the masses of particle and clock. Between observations, a scientifically minded observer might want to make an estimate of the state of the particle, which in this case is fully given by the value of its proper time, τ. This is not a common type of measurement, since measurements usually concern properties in spacetime; moreover, quantum theory assumes that the particle's proper time is equally smooth as the observer's time. Still, it is instructive to see how this estimate would work in this particular case. A useful estimate of the state of the particle would be $\widehat{\theta}_{\mathrm{o}}$, defined as the expected elapsed time since the latest expected step in τ. Although steps in τ follow a Poisson process, and thus are equally likely to occur at any point in time, there is still the constraint that the expected rate of the process is f_{p} events per unit of time. We assume here that the value of f_{p} is known, because the mass m of the particle is known or has been estimated beforehand. With an expected rate of f_{p}, θ_{o} makes, on average, a downward step after a time lapse of $1/f_{\mathrm{p}}$. This means that $\widehat{\theta}_{\mathrm{o}}$ must make such steps every $1/f_{\mathrm{p}}$ units of time. Figure 14b (grey line) illustrates $\widehat{\theta}_{\mathrm{o}}$ as a function of t_{o}. It is not expected to be identical to the actual θ_{o} (as the example of the black line), but it is the best that can be done given the sole constraint that is produced by f_{p}. It is the θ_{o} that is expected on average (this follows from the property of a Poisson process that intervals are independent, with an exponential distribution with a fixed mean of $1/f_{\mathrm{p}}$). Between observations, we can then define $\widehat{\theta}_{\mathrm{o}}$ or a function of $\widehat{\theta}_{\mathrm{o}}$ as the expected state of the particle. Once a new observation is made, the phase of $\widehat{\theta}_{\mathrm{o}}$ is reset to the phase of θ_{o}, which itself is set to zero (i.e., with $\tau(t_{\mathrm{o}}) = t_{\mathrm{o}}$) by the act of observation, irrespective of its phase just before the observation. For example, this happens at time $t_{\mathrm{o}} = 0$ in Figure 14b, where an observation is assumed. We note that an observation produces an actual change of the particle, because it acquires a link that changes its proper time. In addition, an observation changes the observer's ongoing estimate of the state of the particle.

The argument above assumes that the frequency f_{p} associated with the particle's mass m is exactly proportional to m. That would be true if each time step N to $N+1$, that is, each new element, would always produce exactly two new links. Then the chance of linking to the particle would indeed be exactly proportional to m. However, two new links are only the mean of a probability distribution P_k (5) that applies to any number of links $k \in [1, 2, \ldots, N]$ according to $P_k = 1/(e(k-1)!)$. Because P_k is a shifted Poisson distribution with a rate $\lambda = 1$ and shifted one unit to the right relative to a standard Poisson distribution, its mean $\mu_P = 2$ and its variance $\sigma_P^2 = 1$ (i.e., $\frac{1}{2}$ of the mean μ_P). Then the curve of Fig. 14a should not be the time integral of a Poisson point process with fixed Poisson rate, but the time integral of a Poisson process with variable Poisson rate. For each step of τ, that is, for each interval preceding the step, the Poisson rate is then a stochastic variable defined by P_k. For a single step one expects, thus, a frequency f_p with a mean $\mu_{f_\mathrm{p}} = m$ and a variance $\sigma_{f_\mathrm{p}}^2 = m/2$ (as for P_k, the variance is $\frac{1}{2}$ of the mean, irrespective of m). For n steps $\delta\tau$, taking the mean of n independent realizations of P_k reduces the variance of f_p by a factor n. Moreover, for sufficiently large n the resulting probability distribution of f_p will become Gaussian (through the averaging of n independent, identically distributed random variables). Then the probability distribution of f_p is a normal (Gaussian) distribution $\mathcal{N}$ with mean m and variance $m/(2n)$ (with the condition that $f_\mathrm{p} > 0$, because the composing distributions P_k are only non-zero for $k \geq 1$)

$$f_\mathrm{p} \sim \mathcal{N}(m, \frac{m}{2n}). \tag{152}$$

The number of steps n in a time interval Δt will, on average, equal $\Delta t/\delta\tau$, with $\delta\tau = 1/m$ as before, hence

$$f_\mathrm{p} \sim \mathcal{N}(m, \frac{m}{2m\Delta t}) = \mathcal{N}(m, \frac{1}{2\Delta t}). \tag{153}$$

We see here that the variance of f_p becomes small for large Δt, whereas it goes to the large value $m/2$ when Δt approaches $\delta\tau$ from above. In the latter case, the distribution is not Gaussian, but the discrete and skewed distribution P_k (5), which excludes $k = 0$. Therefore, with an interval $\Delta t = \delta\tau$ where, on average, one step in τ is expected, the possible values of f_p equal the integer multiples of $m/2$, that is, $f_\mathrm{p} = k(m/2)$ with $k \in [1, 2, \ldots, N]$, occurring with probability P_k.

Variations of f_p are observed as equivalent to variations in the mass m (i.e., the rest energy E_0) of the particle. When the particle receives more links per unit of time, the total action that attaches per unit of time is larger, and therefore the observed energy is too. But it is only a temporary increase, because the average rate remains the same. Increases of observed energy at one moment in time must be compensated by decreases of observed energy at another moment. Now (153) gives

$$\sigma_{f_\mathrm{p}} = \sigma_{E_0} = \frac{1}{2\Delta t}, \tag{154}$$

or

$$\sigma_{E_0}\Delta t = \frac{1}{2}. \tag{155}$$

This resembles an uncertainty principle, were it not that time t is not an observable in quantum physics, hence time and energy do not produce a standard uncertainty relation. Nevertheless, the relation shows that the shorter the interval Δt is, the larger the uncertainty in observed energy. This is consistent with quantum physics.

Because the mass of a particle appears to vary randomly over time, we will denote it by a random variable μ, with mean $\langle\mu\rangle = m$ and a probability density function (pdf) $\eta(\mu)$, where $\eta(\mu)$ corresponds to (153) with $f_\mathrm{p} = \mu$.

$$\eta(\mu) = \mathcal{N}(m, \frac{1}{2\Delta t}). \tag{156}$$

Here μ is the observed mass and m is the actual, or proper mass. The latter is an invariant network property, because it depends only on s, the number of elements and links of the particle. The observed mass μ is always larger than zero, but for convenience, and without loss of generality, we will define $\eta(\mu)$ over the whole real axis of μ and just set $\eta(\mu) = 0$ for $\mu \leq 0$. The width of $\eta(\mu)$ depends on the interval Δt according to (156), but we will assume that Δt and $\eta(\mu)$ remain fixed for the following discussion. For each value of μ, a different function $\widehat{\theta}_\mathrm{o}(\mu)$ applies, because the corresponding frequency μ of added links per unit of time is different. Then these different $\widehat{\theta}_\mathrm{o}(\mu)$ have to be added somehow, weighed by $\eta(\mu)$. But some caution is needed here, because $\widehat{\theta}_\mathrm{o}(\mu)$ is a case of circular data, that is, data lying on a circle as opposed to linear data lying on an infinite or semi-infinite axis. This is clear from the definition of $\widehat{\theta}_\mathrm{o}(\mu)$ as

$$\widehat{\theta}_\mathrm{o}(\mu) = \text{the expected elapsed time since the latest expected step in } \tau, \tag{157}$$

which will over a time interval $1/\mu$ increase to a maximum of $1/m$ and then abruptly return to zero and continue increasing from there. The grey line in Fig. 14b shows this for $\mu = m$, where each saw-tooth has a slope 1. For $\mu > m$, the graph would consist of a higher frequency saw-tooth with a larger slope μ/m, which would still reach a maximum $\delta\tau = 1/m$ (Fig. 15). The latter follows from the fact that the time steps made by the particle only depend on its proper mass m, that is, on s, which is the fixed number of its elements and links. Similarly, for $\mu < m$ the saw-tooth would still have the same amplitude $1/m$, but its frequency would be lower than m. The circularity of $\widehat{\theta}_\mathrm{o}(\mu)$ is also clear from the following equivalent way to represent it (assuming an observation at $t_\mathrm{o} = 0$, as in Fig. 14)

$$\widehat{\theta}_\mathrm{o}(\mu, t_\mathrm{o}) = \frac{1}{2\pi m}\mathrm{Arg}(e^{i2\pi\mu t_\mathrm{o}}), \tag{158}$$

where Arg is the principal value of the argument of a complex number, with the value defined here to lie in the interval $[0, 2\pi)$. Thus, $\widehat{\theta}_\mathrm{o}(\mu, t_\mathrm{o})$ behaves as the phase of a steadily rotating vector in the complex plane. Averaging circular data cannot be done

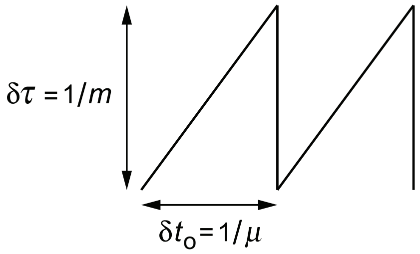

Figure 15: A particle with a proper mass m and an observed mass μ produces an expected saw-tooth oscillation (such as the grey line in Fig. 14b) with a saw-tooth that consists of time steps $\delta t_\mathrm{o} = 1/\mu$ of the observer's time and time steps $\delta\tau = 1/m$ of the particle's proper time, here with $\mu > m$. The slope of the saw-tooth equals $\delta\tau/\delta t_\mathrm{o} = \mu/m$, hence the observed mass (or rest energy) is $\mu = m\delta\tau/\delta t_\mathrm{o}$.

in the standard way for linear data. For example, linearly averaging $\widehat{\theta}_\mathrm{o} = \tau - \epsilon$ and $\widehat{\theta}_\mathrm{o} = \epsilon$ (with $|\epsilon| < \tau/2$) would produce $\widehat{\theta}_\mathrm{o} = \tau/2$, whereas it should be $\widehat{\theta}_\mathrm{o} = 0$. Similarly, the naive way to get the expected value of $\widehat{\theta}_\mathrm{o}$ by linearly averaging (158) over μ would be incorrect

$$\left\langle \widehat{\theta}_\mathrm{o}(t_\mathrm{o}) \right\rangle \neq \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, \frac{1}{2\pi m} \mathrm{Arg}(e^{i2\pi\mu t_\mathrm{o}}) \tag{159}$$

The correct way to average circular data is by adding them as two-dimensional vectors, or, equivalently, as complex numbers ([11] Ch. 1.3). Then we get

$$\left\langle \widehat{\theta}_\mathrm{o}(t_\mathrm{o}) \right\rangle = \frac{1}{2\pi m} \mathrm{Arg} \left(\int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi\mu t_\mathrm{o}} \right). \tag{160}$$

We can now define a wavefunction $\psi(t_\mathrm{o})$ as

$$\psi(t_\mathrm{o}) = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi\mu t_\mathrm{o}}, \tag{161}$$

and rewrite (160) as

$$\psi(t_\mathrm{o}) = |\psi(t_\mathrm{o})| e^{i2\pi m \left\langle \widehat{\theta}_\mathrm{o}(t_\mathrm{o}) \right\rangle}. \tag{162}$$

Following [11], we will write the norm of $\psi(t_\mathrm{o})$ as R

$$R = \left| \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi\mu t_\mathrm{o}} \right|. \tag{163}$$

Here R provides a measure of how concentrated the circular data is [11]. When there is zero spread, that is, when $\eta(\mu)$ is a Dirac delta function $\delta(\mu - m)$, $R = 1$. When the spread is maximal, that is, when $\eta(\mu)$ spreads the data homogeneously over the unit circle, $R = 0$. We conclude that $\psi(t_\mathrm{o})$ as defined in (161) is an excellent estimator of the state of the particle. Its phase is a simple function of $\left\langle \widehat{\theta}_\mathrm{o}(t_\mathrm{o}) \right\rangle$ and its amplitude provides the relevant information on $\eta(\mu)$.

The particle can thus be characterized by a state $\psi(t_\mathrm{o})$ that consists of the summed contributions of each possible μ of the particle, weighted by how likely such a μ is present (161). When an observation is initiated and actually realized at time t'_o, the phase is reset to zero for each μ, producing

$$\psi\left(t'_\mathrm{o}\right) = \int_{-\infty}^{\infty} \mathrm{d}\mu \, \eta(\mu) \, e^{i2\pi 0} = 1, \tag{164}$$

where the latter follows from the fact that $\eta(\mu)$ is a normalized pdf.

Before an observation is made, the probability that the particle is observed at time t_o depends on how the contributions from the various μ add up in (161). The probability should scale as a function of $|\psi\left(t_\mathrm{o}\right)|$, because we have seen above that this is a measure of how concentrated the data is, that is, how small the spread. We get $|\psi\left(t_\mathrm{o}\right)| = 1$ when all phases are equal (that is, when all rotating vectors align), whereas we expect $|\psi\left(t_\mathrm{o}\right)| \approx 0$ when phases are evenly random. Intermediate levels of alignment must be related monotonically to $|\psi\left(t_\mathrm{o}\right)|$. Thus, we can focus on candidate pdfs that have the form $\propto |\psi\left(t_\mathrm{o}\right)|^\beta$, with β a constant to be determined. Equation (161) shows that $\psi\left(t_\mathrm{o}\right)$ is the Fourier transform of $\eta(\mu)$, which means that $\eta(\mu)$ is the inverse Fourier transform of $\psi\left(t_\mathrm{o}\right)$

$$\eta(\mu) = \int_{-\infty}^{\infty} \mathrm{d}t_\mathrm{o} \psi\left(t_\mathrm{o}\right) e^{-i2\pi\mu t_\mathrm{o}}. \tag{165}$$

We use here the convention for defining Fourier transforms that is common in the fields of signal processing and Fourier optics. Then Parseval's theorem for Fourier transform pairs states

$$\int_{-\infty}^{\infty} \mathrm{d}t_\mathrm{o} \left|\psi\left(t_\mathrm{o}\right)\right|^2 = \int_{-\infty}^{\infty} \mathrm{d}\mu \left|\eta(\mu)\right|^2. \tag{166}$$

The right-hand-side of this equation is a pure property of the particle as defined in the network, fully independent of any considerations of observation procedures and spacetime. It is an invariant. This means that the left-hand-side of the equation is an invariant as well. With a proper normalization, $\left|\psi\left(t_\mathrm{o}\right)\right|^2$ can then be interpreted as the pdf for observing the particle at time t_o. We conclude that $\beta = 2$ is the right choice for the candidate pdf, guaranteeing conservation of probabilities. Since $\left|\psi\left(t_\mathrm{o}\right)\right|^2$ is a probability density function, $\psi\left(t_\mathrm{o}\right)$ can be called a probability amplitude. It is, in general, a complex number, capable of producing interference, whereas its squared norm has the properties of a classical probability.

For extending these results to space, we will ignore the jitter for the moment, and focus first on the frequency $f_\mathrm{p} = m$ that corresponds to the mean time between steps. How would this oscillation appear to an observer O_v who views the particle as moving with a velocity v along the x-axis? We will follow here an argument by de Broglie [12]. To another observer O_0 who would be comoving with the particle, the oscillation of the particle would be represented by a periodic function with an argument proportional to $f_{\mathrm{p}} t_{\mathrm{o}}$. To observer O_v, who sees the particle moving at velocity v (and who uses a coordinate frame with coordinates t and x), this function has the argument

$$f_{\mathrm{p}} t_{\mathrm{o}} = f_{\mathrm{p}} \gamma (t - v x). \tag{167}$$

Here we have used the Lorentz transformation of (81), and also the fact that f_{p} is an invariant of spacetime because it is a network property. A periodic function with the right-hand side of (167) as argument corresponds to a wave of frequency $f_{\mathrm{p}}\gamma$ travelling at a velocity of $1/v$. But $1/|v|$ is here larger than one, thus the wave cannot correspond to a physical movement (in the sense of carrying energy or of transferring a signal). This is consistent with the fact that $1/v$ is merely the phase velocity v_{ph} of the wave. For physical movement we need the group velocity v_{g} of a superposition of waves, such as of a wave front or some other wave disturbance. Indeed, we have seen above that, as observed, the particle does not have a single frequency $f_{\mathrm{p}} = m$, but rather a band $\eta(\mu)$ of likely frequencies $f_{\mathrm{p}} = \mu$ around m. In general, the phase and group velocities of waves are given by $v_{\mathrm{ph}} = \omega / k$ and $v_{\mathrm{g}} = \mathrm{d}\omega / \mathrm{d}k$, respectively (with $\omega = 2\pi f_{\mathrm{p}}$ angular frequency, $k = 2\pi/\lambda$ wavenumber, and λ wavelength). Then

$$v_{\mathrm{g}} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\frac{\mathrm{d}\omega}{\mathrm{d}v}}{\frac{\mathrm{d}k}{\mathrm{d}v}} = \frac{\frac{\mathrm{d}\omega}{\mathrm{d}v}}{\frac{\mathrm{d}(\omega/v_{ph})}{\mathrm{d}v}} = \frac{\frac{\mathrm{d}\left(2\pi f_{\mathrm{p}}(1-v^2)^{-1/2}\right)}{\mathrm{d}v}}{\frac{\mathrm{d}\left(2\pi f_{\mathrm{p}} v (1-v^2)^{-1/2}\right)}{\mathrm{d}v}} = v. \tag{168}$$

We see that the group velocity does not depend on frequency, and is therefore the same over the entire frequency band of the particle. Thus, the intrinsic frequency band of a particle moving with a velocity v relative to an observer O_v corresponds, to that observer, to a superposition of waves that moves with a group velocity v, along with the particle. In other words, the particle displays the properties of a travelling wave packet. These waves are commonly referred to as matter waves. They originate here from the Lorentz transformation combined with the fact that a particle displays a band of frequencies (ultimately because of P_k, (5)).

The frequency f_{p} is proportional to the mass (or rest energy E_0) as $f_{\mathrm{p}} = m = E_0$. Therefore, f_{p} is the rest frequency of the particle. Equation (167) shows that the frequency of an observed moving particle can be formulated in a similar way,

$$f'_{\mathrm{p}} = \gamma f_{\mathrm{p}} = \gamma m = E, \tag{169}$$

with E the energy of the observed moving particle (89). The phase velocity v_{ph} and f'_{p} together define the de Broglie wavelength of the particle

$$\lambda_{\mathrm{B}} = \frac{v_{\mathrm{ph}}}{f'_{\mathrm{p}}} = \frac{1}{v f'_{\mathrm{p}}} = \frac{1}{v \gamma m} = \frac{1}{p}, \tag{170}$$

where p is the relativistic momentum of the particle (defined below 88).

As discussed above, the particle does not produce a single frequency, but rather a distribution $\eta(\mu)$ of frequencies, where the actual number of links added per unit of time, μ, fluctuates around the mean, m. Each frequency μ then produces a wave in the manner of (167). The state of the particle $\psi(t_{\mathrm{o}})$ as observed by a comoving O_0

$$\psi(t_{\mathrm{o}}) = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi\mu t_{\mathrm{o}}}, \tag{171}$$

is therefore observed by O_v as

$$\psi(x,t) = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi\mu\gamma(t-vx)} = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi(Et-px)}, \tag{172}$$

where we defined the energy $E = \gamma\mu$ and momentum $p = \gamma\mu v$ in accordance with the definitions given for m, see (88) and (89). We used here the fact that $\eta(\mu)$ is a network property and is the same for any observer in spacetime. The positive sign at t in (172) means that many of the signs in the equations derived below are switched compared with standard conventions. However, that does not change the physics and could be changed if one so wishes.

We will first study the wave packet at a fixed time $t = 0$. From (172) we get

$$\psi(x) = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{-i2\pi\mu\gamma vx} = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{-i2\pi px}, \tag{173}$$

where we defined $\psi(x) = \psi(x,0)$. Because $p = \gamma\mu v$ with v and γ given, the pdf $\eta(\mu)$ can be transformed to a pdf as a function of p, say $\varphi(p)$. Conservation of probabilities then gives $\eta(\mu)\mathrm{d}\mu = \varphi(p)\mathrm{d}p$. Therefore

$$\psi(x) = \int_{-\infty}^{\infty} \mathrm{d}p\, \varphi(p)\, e^{-i2\pi px}, \tag{174}$$

and consequently

$$\varphi(p) = \int_{-\infty}^{\infty} \mathrm{d}x\, \psi(x)\, e^{i2\pi px}. \tag{175}$$

We have shown above that, for a sufficiently large time interval Δt, the pdf $\eta(\mu)$ is a Gaussian (153). Then $\varphi(p)$ is a scaled Gaussian and $\psi(x)$ is a Gaussian too (being the Fourier transform of a Gaussian). Gaussian Fourier transform pairs have standard deviations that are related as

$$\sigma_x \sigma_p = \frac{1}{2\pi}. \tag{176}$$

With $p = \gamma\mu v$ and with γv dimensionless, we need to multiply p in the same way as μ in order to express it in SI units, that is, we need to multiply by h_ℓ. If we use $h_\ell = h/2$ we obtain the standard Heisenberg uncertainty principle for position and momentum

$$\sigma_x \sigma_p = \frac{1}{2\pi}\frac{h}{2} = \frac{1}{2}\hbar, \tag{177}$$

with $\hbar = h/(2\pi)$ the reduced Planck constant.

Finally, we will look at what happens when an observation is made. If the particle is observed at time $t = 0$ at a particular position x, then this corresponds to a particular value t_o of the particle's macroscopically observed proper time. The event of observation means that one of the elements in the observing (macroscopic) object obtains a new link from a new element, which, in addition, links to the observed particle. Then the exact time of that particular step in the proper time τ of the particle is known to be t_o, and the phase for each frequency μ of the particle is reset to zero. Thus at an observation, (173) gives

$$\psi(x) = \int_{-\infty}^{\infty} \mathrm{d}\mu \, \eta(\mu) \, e^{-i2\pi 0} = 1, \tag{178}$$

again because $\eta(\mu)$ is a normalized pdf. Between observations, we use the middle expression of (173)

$$\psi(x) = \int_{-\infty}^{\infty} \mathrm{d}\mu \, \eta(\mu) \, e^{-i2\pi\mu\gamma v x} \tag{179}$$

and scale x to $\xi = \gamma v x$ (with v and γ given), writing $\psi(x) = \psi_\xi(\xi)$ to obtain

$$\psi_\xi(\xi) = \int_{-\infty}^{\infty} \mathrm{d}\mu \, \eta(\mu) \, e^{-i2\pi\mu\xi}. \tag{180}$$

Since $\psi_\xi(\xi)$ and $\eta(\mu)$ are Fourier transform pairs, Parseval's theorem now gives

$$\int_{-\infty}^{\infty} \mathrm{d}\xi \, |\psi_\xi(\xi)|^2 = \int_{-\infty}^{\infty} \mathrm{d}\mu \, |\eta(\mu)|^2. \tag{181}$$

As before, the right-hand-side of this equation is a network property and therefore an invariant in spacetime. Following a similar argument as given above for the time domain, $|\psi_\xi(\xi)|^2$ can be interpreted, with a proper normalization, as the pdf for observing the particle at position ξ. Scaling ξ and $|\psi_\xi(\xi)|^2$ back to x and $|\psi(x)|^2$, and using $|\psi_\xi(\xi)|^2 \, \mathrm{d}\xi = |\psi(x)|^2 \, \mathrm{d}x$ because of conservation of probabilities, we find

$$\int_{-\infty}^{\infty} \mathrm{d}x \, |\psi(x)|^2 = \int_{-\infty}^{\infty} \mathrm{d}\mu \, |\eta(\mu)|^2. \tag{182}$$

The conclusion is that, with a proper normalization, $|\psi(x)|^2$ can be interpreted as the pdf for observing the particle at position x. This is the Born rule.

If the analysis above had been performed for a fixed time $t = t_1$ rather than for $t = 0$, that would have produced only a fixed phase shift in $\psi(x)$. Then $|\psi(x, t_1)|^2$ would still be the pdf for observing the particle at position x. Similarly, an analysis performed for a fixed position $x = x_1$ would have produced $|\psi(x_1, t)|^2$ as the pdf for observing the particle at time t. Because x and t are independent coordinates, the general pdf for observing the particle at coordinates (x,t) must be proportional to $|\psi(x,t)|^2$, and that is equally true for a 3D-position $\mathbf{x}$ with a pdf proportional to $|\psi(\mathbf{x},t)|^2$.

We will end this section with a few remarks about quantum measurement. An observation produces an actual change of the particle, because it acquires a link that changes its proper time. In addition, the observation changes the state of the particle as estimated by the observer. How this actual change of state and the change of estimated state affect the states of other particles (either actual or estimated states), or the states estimated by other observers, depends on the details of how such particles and observers are correlated with one another. In the next section we will show that the theory is consistent with the path integral formulation of quantum physics, and, thus, with established quantum phenomena in general, including non-classical correlations.

A limitation of quantum theory is that it assumes that the observer's own time is smooth and regular compared with the time of the observed object (as in Fig. 14). This means that the observer's mass must be much larger than the object's mass. When the masses are of the same order of magnitude or the object's mass is larger than the observer's, the irregularity of the observer's own time would destroy most or all of the information a measurement could provide about the object's time. Consequently, wavefunctions will lose most of their predictive power when applied to macroscopic objects. It would not be difficult to quantify this using information theory, but that will not be done here.

11 The Feynman path integral

Deriving a Feynman path integral from a wavefunction can be done by a time-slicing method ([13] and, for example, [14]). Rather than doing that here, we will take a more straightforward approach by deriving the path integral directly from the network. Such an approach is promising, since the network contains paths that consist of action in the form of chained links. Hence, the network is already close to a path integral, that is, a physically realized one.

For simplicity, we will focus here on scalar positions x. Suppose that we want to know how the wavefunction of a particle at a particular position and time (x',t') is related to its wavefunction at another position and time (x'',t''). These positions must be time-like separated, with proper times τ' and τ'' of the particle, where we assume $\tau'' > \tau'$. A co-moving macroscopic observer O_0 records proper times of t'_o at the start and t''_o at the end of this particular segment of the particle's time line. Then from (171) we get

$$\psi(t'_\mathrm{o}) = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi\mu t'_\mathrm{o}} \tag{183}$$

and

$$\psi(t''_{\mathrm{o}}) = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi\mu t''_{\mathrm{o}}}. \tag{184}$$

We can now define a transition amplitude $K(t'_{\mathrm{o}}; t''_{\mathrm{o}})$ for obtaining $\psi(t''_{\mathrm{o}})$ from $\psi(t'_{\mathrm{o}})$. However, there are two major problems when we want to use (183) and (184) to infer $K(t'_{\mathrm{o}}; t''_{\mathrm{o}})$. The first problem is that μ is a stochastic variable that is independently realized in the measurements of (183) and (184). Then we cannot divide $e^{i2\pi\mu t'_{\mathrm{o}}}$ out in order to get the transition amplitude. This problem can be resolved when we assume that $(t''_{\mathrm{o}} - t'_{\mathrm{o}}) \gg 1/m$, that is, that the period of the frequency m is negligible compared with the time between measurements. This condition is easily satisfied, since $1/m$ for a typical particle, say an electron, is about 10^{-20} s. Because $\eta(\mu)$ is a Gaussian distribution (156), adjacent frequencies $1/\mu$ will then in general produce strongly varying phases over a distance $(t''_{\mathrm{o}} - t'_{\mathrm{o}})$ and interfere destructively, on average. Only at a stationary point, that is, at the peak of the Gaussian, adjacent frequencies can interfere constructively. In other words, only the frequency m and a tiny neighbourhood around it will survive. Thus, we must replace $\eta(\mu)$ in (183) and (184) with a Dirac delta function $\delta(\mu - m)$. Then the probability transition amplitude only concerns m, in excellent approximation. We may now be tempted to formulate

$$K(t'_{\mathrm{o}}; t''_{\mathrm{o}}) \overset{?}{=} e^{i2\pi m(t''_{\mathrm{o}} - t'_{\mathrm{o}})}. \tag{185}$$

However, this does not yet take care of the second of the problems mentioned above. This is the problem that $(t''_{\mathrm{o}} - t'_{\mathrm{o}})$ is a single macroscopic (i.e., classical) time trajectory. In reality, the network provides many ways by which the particle at time τ' can be connected to the particle at time τ'' through a pure time path in the network (such as the left path in Fig. 7), even many paths that have $\tau'' - \tau' \approx t''_{\mathrm{o}} - t'_{\mathrm{o}}$. The reason for the latter is that a particle consists of a huge number of elements. Each of these elements shares its position and time with all the others (see (61) in Section 7). Each time path between the particle as observed at t'_{o} and observed at t''_{o} must then take the form of (185), but with the action (in units of h_ℓ) taken along that path rather than along the classical time trajectory (which contains the classical action $m(t''_{\mathrm{o}} - t'_{\mathrm{o}})$). Then we obtain

$$K(t'_{\mathrm{o}}; t''_{\mathrm{o}}) = \sum_{\mathcal{P}} e^{i2\pi S_{\mathcal{P}}} = \int_{t'_{\mathrm{o}}}^{t''_{\mathrm{o}}} \mathcal{D}t\, e^{i2\pi S_{\mathcal{P}}}, \tag{186}$$

where the path integral $\int \mathcal{D}t$ symbolizes summing over all time paths $\mathcal{P}$ that connect the particle at t'_{o} with the particle at t''_{o}, with $S_{\mathcal{P}}$ denoting the total action (in units of h_ℓ, that is, being the number of links) along each of these paths.

Suppose that observer O_0 is replaced with an observer O_v, who uses a coordinate frame with coordinates x and t, and who sees the particle moving with a speed v from

(x',t') to (x'',t'') (thus $v = |x''-x'|)/|t''-t'|$). Then we need (172) rather than (171)

$$\psi(x',t') = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi(Et'-px')} \tag{187}$$

and

$$\psi(x'',t'') = \int_{-\infty}^{\infty} \mathrm{d}\mu\, \eta(\mu)\, e^{i2\pi(Et''-px'')}, \tag{188}$$

with $E = \gamma\mu$ and $p = \gamma\mu v$ as before. Again, we must replace $\eta(\mu)$ with $\delta(\mu - m)$ for the same reason as above. Then $(\gamma m t'' - \gamma m v x'')$ is the classical action (apart from the sign convention, see below (172)). Furthermore, we must replace the classical trajectory with all network paths $\mathcal{P}$ that connect (x',t') and (x'',t''). This finally leads to

$$K(x',t';x'',t'') = \sum_{\mathcal{P}} e^{i2\pi S_{\mathcal{P}}} = \int_{(x',t')}^{(x'',t'')} \mathcal{D}t\, e^{i2\pi S_{\mathcal{P}}} \tag{189}$$

and

$$\psi(x'',t'') = \int_{(x',t')}^{(x'',t'')} \mathcal{D}x\, e^{i2\pi S_{\mathcal{P}}} \psi(x',t'). \tag{190}$$

This equation shows that knowledge of the wavefunction at a particular point (x',t'), combined with knowledge of how the action changes across spacetime, allows one to calculate the wavefunction at any other point (x'',t''). The expression is identical to the Feynman path integral. The path integral approach to quantum mechanics is known to be fully equivalent to the operator-based formulation of quantum mechanics [13]. All the results of non-relativistic quantum mechanics follow from (190), including the Schrödinger equation, interference, and non-classical correlations.

The action $S_{\mathcal{P}}$ is in units of the action per link, that is, it equals the number of links (not an integer for particles, see (61) and (62)). The precise value of h_ℓ is irrelevant, because m, as defined below (151), scales automatically with h_ℓ. The reason is that both m and $S_{\mathcal{P}}$ refer to a number of links. This means that the path integral does not provide a way to confirm the correct scaling of h_ℓ in terms of h.

The transition to classical physics, where particles follow definite trajectories, is readily understood from the path integral [15]. The action per link is tiny compared with any classical action. Then $S_{\mathcal{P}}$ is huge at classical scales, which means that the factors $e^{i2\pi S_{\mathcal{P}}}$ vary strongly and are likely to average out for most paths. Only for paths where $S_{\mathcal{P}}$ is at a stationary point (that is, at an extremum or an inflection point), the factors $e^{i2\pi S_{\mathcal{P}}}$ can interfere constructively. Such paths then form the classical trajectory of a particle. This depends only on network paths, and is, thus, equally applicable to flat spacetime as to spacetime that is distorted by gravity (Section 8).

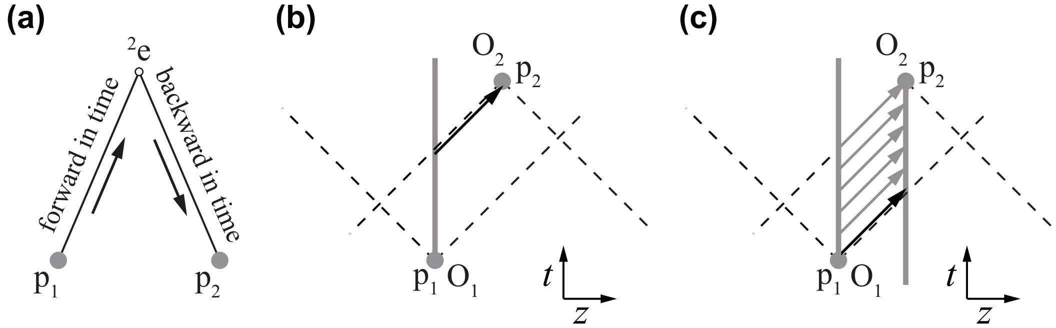

Figure 16: **a** A ^{2}e element connecting two particles p_1 and p_2 appears to make a zigzag in time as viewed from each particle (illustrated for p_1). **b** A photon emitted by p_1 and absorbed by p_2, as recorded by a macroscopic observer O_2. The photon connects an element of p_2 with an element of p_1 that has a rank order at the intersection of p_1's timeline (vertical grey line) and the past light cone of p_2 (dashed line). The photon appears to travel along a trajectory indicated by the black arrow. **c** A macroscopic observer O_1 conjectures that a photon is emitted by p_1 and travels along the black arrow to be absorbed by p_2. The photon connects an element of p_1 with an element of p_2 that has a rank order at the intersection of p_2's timeline (right vertical grey line) and the future light cone of p_1 (dashed line). It is still recorded by O_2 at O_2's own position in spacetime, since all elements of p_2 always share one another's position in spacetime. Neither O_1 nor O_2 can know which trajectory was taken by the photon (black arrows or any intermediate one, such as the grey arrows), since the photon did not actually take a specific trajectory.

12 Photons

As mentioned in the section on wavefunctions for particles, there is a second mechanism in the network that can produce an interpretation of travelling waves. This is illustrated in Fig. 16a. When a ^{2}e element connects to two particles p_1 and p_2, it represents a zigzag in time from the point of view of each particle. The arrows illustrate this for particle p_1: there is step forward in time when following the link from p_1 to ^{2}e, and then a step backward in time when following the link from ^{2}e to p_2. Particle p_2 sees it as just the other way around. The zigzag in time is the equivalent of a single cycle of an oscillation. This will be worked out below in quantitative detail. But before doing that, we will first argue that all photons are new ^{2}e elements (but not the other way around).

There are several reasons why photons must correspond to ^{2}e elements at their time of birth. A first reason is that photons are massless particles, which means that they should not have many links. New elements with a small number of parents would qualify for that, with ^{2}e elements blending in perfectly as they have the same number of links as space has on average. A second reason follows from the rotational symmetry of new elements, that is, how much rotation is needed in three dimensional space in order to return to the starting orientation. A new ^{1}e element is attached to space with only one link, and can be rotated by any angle in any direction without being changed. Hence, it has spin 0. A new ^{2}e element is attached to space with two links, which we assume to be to different positions. Then it can be rotated around a single axis connecting these two positions, and will return to its original orientation after a rotation by 2π. Hence it has spin 1. New $^{3+}$e elements, as well as massive particles, are attached to space with at least three links, assumed to be to at least three different positions. Then two full rotations, in total by 4π, are needed in order to return to the original orientation (this can be illustrated by the Dirac belt trick and follows from the properties of the 3D rotation group). Hence, they have spin $\frac{1}{2}$. Since photons have spin 1 (or rather the corresponding helicity), this points again to ^{2}e elements (without excluding that other types of spin 1 particles can be ^{2}e elements as well). And finally, only new ^{2}e elements can dynamically connect two particles across spacetime unambiguously, as expected from photons.

However, not all new ^{2}e elements can be photons. Many ^{2}e will just link to two parts of spacetime itself, thus contributing to spacetime and its expansion. Other ^{2}e will attach to a particle with one link and to spacetime with the other link, thus contributing to the particle's position and velocity in spacetime. Still other ^{2}e may link to a particle with one link and to parts of spacetime beyond the light cone of the particle, even to the vast parts of the universe that lie beyond the observable universe. And finally, it is not clear at this point in time what corresponds to the electric charge of a particle, that is, what makes a particle 'dark' or not, or what makes a particle have an electric charge rather than another type of charge. In the following we will assume a ^{2}e connecting two electrically charged particles p_1 and p_2, which are separated along a z-axis (following convention we will use z here, such that x and y are available for transverse directions). The particles lie inside each other's past and future light cones, respectively (dashed lines in Fig. 16b,c).

Each particle is assumed to have been formed early in time (as in Section 6), and continues to receive links as part of the process of going forward in time (as in Fig. 14a). Therefore, each particle is directly connected to elements with rank orders spread out throughout network time. These elements are symbolized by the vertical grey lines in the figure. We assume that the photon is emitted by p_1 and absorbed by p_2. When the corresponding ^{2}e attaches to both p_1 and p_2, this happens at exactly the same moment, according to the photon. To the photon, being emitted and being absorbed is the same event. This is consistent with special relativity in spacetime, since travelling at the speed of light brings the flow of time to a halt. The network view agrees on this, because the ^{2}e element does not receive new links while connecting p_1 and p_2, hence the ^{2}e does not travel forward in time. But that is not how p_1 and p_2 see things. We will now first discuss the view of p_2 (using Fig. 16b) and only after that the different view of p_1 (using Fig. 16c).

In Fig. 16b, p_2 receives a photon from p_1. Although the particle p_2 itself cannot know that the photon came from p_1, that could be inferred by a local macroscopic observer O_2 (who may have an optical imaging device and the appropriate background knowledge). Because O_2 interprets the incoming ^{2}e as a photon, there must be a light-like trajectory in spacetime between an element of p_1 and an element of p_2. If there were no such trajectory, the ^{2}e would not be interpreted as a photon, but presumably as noise. Since the photon is taken to follow a light-like trajectory, which has the same link distance in the network for time paths and space paths (see Fig. 7), the inferred point of origin is the intersection of p_1's timeline (vertical grey line in the figure) with p_2's light cone (dashed line). As a result, the trajectory of the photon, as inferred by O_2, is represented by the black arrow.

The particle p_1 (or rather a local macroscopic observer O_1) has a different view. In Fig. 16c a photon is emitted by p_1. It is absorbed by p_2, according to the photon itself (which simultaneously forms links with both p_1 and p_2). But O_1 cannot know that. Photons emitted by p_1 might end up anywhere. Even if a photon is directed towards p_2 (say with the help of a laser), it might be absorbed on its way to p_2. Or the ^{2}e that is conjectured by O_1 to be on its way might connect to a part of space outside its light cone. O_1 could infer only with hindsight that a photon detected at p_2 probably came from p_1, with the inferred point of destiny the intersection of p_1's light cone (dashed line) with p_2's timeline (vertical grey line). The black arrow then represents the trajectory of the photon, according to O_1. Observer O_2 will still detect the photon at O_2's own position in time, since all elements of p_2 share a single position in space and time (irrespective of their rank order). If p_2 is observed to move at a particular time, because of the impact of a photon, the movement could have been mediated through any of its elements. This means that trajectories intermediate between the black arrows of Fig. 16b and c are equally valid (the grey arrows show examples). Neither O_1 nor O_2 can know the actual trajectory, fundamentally because there is no actual trajectory.

A potential worry may be that the instant connection between p_1 and p_2 might conflict with special relativity by allowing faster than light transmission of information across space. However, that is not the case. Viewed in isolation, the connection to p_2 consists solely of a new link of unknown origin. It could have been a link from a ^{1}e or $^{3+}$e element, and it could have come from a part of the network that is space-like connected. There is no specific information other than the local event of receiving a link. Only when an observer correlates that event with previous events and with knowledge of the detector (such as the properties of its optics and sensor), it can be interpreted as carrying specific information. For the same reason an observed ^{2}e connecting with an element in the unobservable parts of the universe can never provide information about those parts, because there is no way by which a suitable correlation could be inferred.

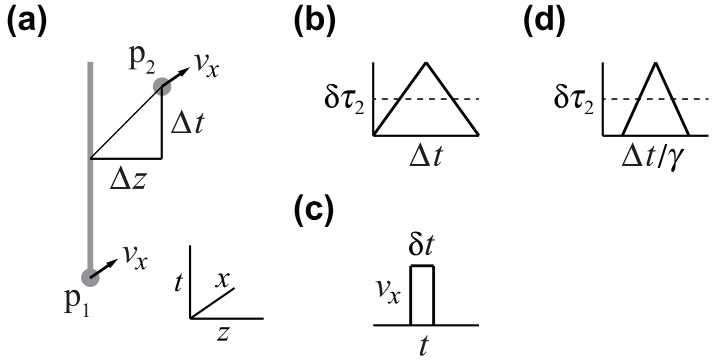

Figure 17: **a** Two electrically charged particles p_1 and p_2 at an observed distance Δz from each other exchange a photon (i.e., a ^{2}e). They have a velocity v_x in the x-direction. **b** The observed time course of the ^{2}e when $v_x = 0$. The zigzag is equivalent to a single cycle of a triangular oscillation about its central value (dashed line). **c** A brief movement in the x-direction. **d** The observed time course of the ^{2}e is compressed because of time dilation produced by the movement of **c**.

In the following we will focus on the situation of Fig. 16b, where a photon originating at p_1 is detected at p_2. What would be the properties of this photon? Figure 17a is a modified version of Fig. 16b. In addition to the time- and z-axis, there is a perpendicular x-axis. Along this axis, both particles can move briefly with the same velocity v_x. Let us first analyze the case of $v_x = 0$. Then, according to O_2, the time course of the ^{2}e is the one shown in Fig. 17b. The link between p_2 and ^{2}e corresponds to a time step that is equal to a step of $\delta\tau_2 = 1/m$ in the forward direction of the proper time τ_2 of p_2, with m the mass of p_2. The reverse link is an identical step backwards in time, as far as O_2 can know. O_2 has already inferred that the photon came from p_1 at a distance $\Delta z = \Delta t$ (with speed $v_{\max} = 1$ and $c = 1$). Then the single cycle of the oscillation took a time Δt (Fig. 17b). This corresponds to a frequency band $\eta(\nu_0)$ with a mean frequency $\bar{\nu}_0 = 1/\Delta t$, and a travelling wave packet ϕ consisting of a superposition of periodic functions with argument $\nu_0(t - z)$, expressed in O_2's coordinates (z, t)

$$\phi(z,t) = \int_{-\infty}^{\infty} \mathrm{d}\nu_0 \, \eta(\nu_0) \, e^{i2\pi\nu_0(t-z)}. \tag{191}$$

Since Δt will in general be long, the mean frequency $\bar{\nu}_0$ is low. Now suppose that the particles make, synchronously with the photon, a brief step of duration δt in the x-direction, during which their velocity is v_x (Fig. 17c). This setup is kept deliberately as simple as possible, because the purpose here is to explain the principle, rather than to make a detailed model. Because v_x is perpendicular to the z-direction, the particle's observed z-properties (such as distance and speed) are not affected. But observed time is dilated according to (75)

$$\Delta\tau = \Delta t/\gamma, \tag{192}$$

with

$$\gamma = \frac{1}{\sqrt{1 - v_x^2}}. \tag{193}$$

As a result, O_2 observes the photon's time course not as in Fig. 17b, but as in Fig. 17d. Then the observed mean frequency of the photon is $\bar{\nu} = 1/\Delta\tau = \gamma/\Delta t = \gamma\bar{\nu}_0$, and the travelling wave packet consists of frequencies $\nu = \gamma\nu_0$

$$\phi(z,t) = \int_{-\infty}^{\infty} \mathrm{d}\nu\, \eta(\nu)\, e^{i2\pi\nu(t-z)}. \tag{194}$$

According to O_2, the photon consists of two links in series that span a mean time interval $\Delta\tau$. Since the two links are together equal to an action of $2h_\ell$, the energy $E_{\bar{\nu}}$ of the photon must be

$$E_{\bar{\nu}} = \frac{2h_\ell}{\Delta\tau} = 2h_\ell\bar{\nu}. \tag{195}$$

We obtain the Planck relation for the energy of a photon of observed frequency $\bar{\nu}$ if we use $h_\ell = h/2$, with h the constant of Planck,

$$E_{\bar{\nu}} = h\bar{\nu}. \tag{196}$$

The photon has an energy $E_{\bar{\nu}}$ according to O_2, and travels with the speed of light from p_1 to p_2. Hence, it must have a momentum given by $p = E_{\bar{\nu}}$ in accordance with special relativity (see below (89)). As a result, photons transfer not only energy, but also momentum, and thereby act as force carriers.

According to the observer, the frequency is $\bar{\nu}$, which corresponds to a period $\Delta\tau = h/E_{\bar{\nu}}$. Then the δt of Fig. 17c must be similar, that is

$$\delta t \sim h/E_{\bar{\nu}}, \tag{197}$$

or

$$E_{\bar{\nu}}\,\delta t \sim h. \tag{198}$$

This is consistent with the form of uncertainty relation derived in the section on wavefunctions, (155).

The analysis above is formulated for a unimodal velocity pulse in the x-direction, but it could have equally well been formulated for a pulse in the y-direction or a combination of x and y. A bimodal pulse (back and forth) in either direction, or a combination of x and y out of phase would lead to something resembling linear and circular polarizations. However, these are just qualitative remarks, and a proper analysis must await a good understanding of what properties of a particle correspond to electric charge (presumably, which are the statistical symmetries of its core, see Section 13).

Although the macroscopic observer O_2 infers only a single shortest trajectory between p_1 and p_2, there are in reality many paths in the network that connect p_1 and p_2. This situation is the analogue of the path integral for particles with mass (Section 11). Again, one must sum over all possible paths with the proper phase depending on the length of each path. Only paths that have nearly the same phase as other paths will interfere constructively.

An experimenter can manipulate the available paths by placing appropriate barriers in the vicinity of an experimental setup. Thus, it is possible to reduce the number of photons that reach an experiment and that potentially disturb it. However, there is no way to reduce the number of ^{2}e elements attaching to the elements that constitute the setup. Within the network, each existing element still has the same chance as any other existing element to get a new link when a new element becomes added to the network. Nevertheless, shielding a setup can reduce the fraction of ^{2}e elements that are interpreted as photons (by having an interpretation in spacetime). The other, larger fraction of ^{2}e elements that is interpreted as noise is then increased. In other words, shielding shifts the balance of how ^{2}e elements are interpreted.

The view of photons explained here can help to resolve the conceptual paradoxes that they can produce. For example, in a delayed-choice experiment [16] one manipulates one of the pathways in a double-slit interference experiment after a photon has allegedly passed the slits, but before its interference is recorded. The apparent paradox is that the result is the same as when the manipulation was done before the photon arrived at the slits. The paradox is produced by the false assumption that the photon was actually travelling before the recording. That is only a macroscopic interpretation, since in reality the photon is emitted by its source at the very moment it is recorded, and the available network paths at that moment are the relevant ones for the result. Those paths are the same irrespective of when the manipulation took place, as long as it happened before the moment of recording.

This explanation remains essentially the same for a delayed-choice experiment performed on matter, such as electrons, rather than on photons. Although massive particles move more slowly than the speed of light, they do not actually travel through space. Within the network, particles stay where they are. Motion is just a macroscopic interpretation of changes in the network, in particular changes that are interpreted as changes in space (a statistical pattern). The resulting shifts in the spatial relations between particles are then interpreted as physical motion. The result of double-slit interference on electrons only depends on the available network paths at the moment of recording (Section 11). It does not matter at which moment before the moment of recording one has manipulated the double slit. However, things will become different when one would attempt to perform the experiment on larger and larger objects, up to macroscopic ones. Then the assumptions on which quantum theory is based become gradually invalid (see the final paragraph of Section 10).

13 Symmetries of particles

This section is mostly programmatic. It contains few results, but discusses promising ways to make further progress. A key issue is which specific properties particles have. Mass has been discussed in Sections 6 and 8. It is almost completely produced by ^{1}e elements, which are spin-0 elements (Section 12) and amongst the most common new elements. The ^{1}e elements together form a mantle that is connected to the densely connected core of the particle (Fig. 8). It was shown in the section on wavefunctions (Section 10) that the attachment of new links to the elements of a particle produces a frequency f. For example, the electron, with a mass of about $m_\mathrm{e} = 9.1 \cdot 10^{-31}\,\mathrm{kg}$, produces a frequency

$$f_\mathrm{e} = m_\mathrm{e} c^2 / h = 1.24 \cdot 10^{20}\,\mathrm{Hz}. \tag{199}$$

Here h rather than h_ℓ must be used, because in network terms the mass of a particle and the progress in time are both measured by a number of links. Then the standard SI units are appropriate in spacetime. The frequency f_e is the number of links attaching to the electron per second. This must be equal to the number of elements of the electron, N_e, times the change in link distance per second, $\eta/\mathrm{s} = H_0/\ln 2$ (117). The latter was determined from the Hubble constant H_0 (113) in the section on gravity (Section 8). Therefore

$$N_\mathrm{e} = \frac{f_\mathrm{e} \ln 2}{H_0} = 3.6 \cdot 10^{37}. \tag{200}$$

The ratio of the number q of core elements to the number s of mantle elements is not yet known, neither for the electron nor for any other particle. It must be small, because q is expected to grow as $\sim 1/N^2$ (58), whereas s is expected to grow as $\sim 1/N$ (59). Even if the ratio q/s is tiny, say 10^{-20}, the hugeness of N_e means that q will still be large. The core is then likely to be amenable to statistical analysis.

As a side remark, we note that $N_\mathrm{e} = 3.6 \cdot 10^{37} \approx 2^{125}$. With $n_\mathrm{e} \sim \log_2(N_\mathrm{e})/3$ (the analogue of (45)) we find $n_\mathrm{e} \sim 42$. Then Fig. 6b shows that the mantle of the electron is indeed too small to have a 3D spatial extent (even when disregarding that it is made of ^{1}e elements). Currently known elementary particles p have masses falling roughly in a range of $10^6 \approx 2^{20}$ smaller or larger than the electron, suggesting $n_\mathrm{p} \in [\sim 35 \ldots \sim 49]$.

For the wavefunction of particles, the path integral formulation of (190) assumes that each path begins and ends at a network particle, and that a particle acts as if it is a regular element of spacetime. The latter is true in the sense that all elements of a particle have an indefinite, though common and shared, position in spacetime. However, the particle may have an internal structure that can affect the outcome of the path integral. This can be understood by revisiting the analysis in the beginning of Section 7, in particular (61) and (62). The link length of a path between particles

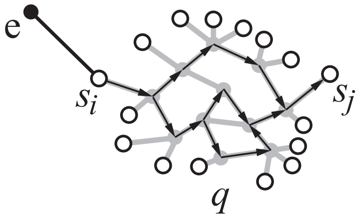

Figure 18: Two elements s_i and s_j belong to the mantle of s ^{1}e elements of a particle (open circles). They are connected through the q elements (grey filled circles) of the densely connected core of the particle. The arrows show three of the possible paths connecting s_i and s_j. An external element e connects through s_i and the core to the other mantle elements, such as s_j. From the point of view of e, the path lengths towards all elements of the particle must be averaged, with the appropriate phases. The symmetries of the core then determine how this adds up.

requires averaging over the lengths to each element within a particle. In principle, the averaging goes over both the q elements of the core and the s elements of the mantle, but it can be approximated by averaging only over s, since $s \gg q$. The averaging includes paths that reach a particular element of the mantle, say s_j, by going through another element of the mantle, say s_i, via the intermediate core. Then the number of those paths results from multiplying by the number of possible paths through the core. Figure 18 illustrates this. The lengths of these paths depend on the properties of the core. The path length shows up as periodic per length of one link in the path integral, as $\exp(i2\pi S_{\mathcal{P}})$. Therefore, the effect of the core depends on the resulting distribution of path lengths (modulo 1 link). If this distribution is flat, the paths through the core interfere destructively, and the particle cannot be observed directly (it is 'dark'). Indirectly, though, its effects can be observed because it will still produce gravitation. On the other hand, if the distribution has a clear peak, many of the paths interfere constructively, enabling observation.

The above argument assumes that the core has a spherical symmetry, that is, that it does not matter which pair of mantle elements is involved. But the core may have other symmetries than a spherical one. This could produce observable effects if mantle elements are correlated both with this internal symmetry and with external spacetime properties, such as orientation in space. Moreover, core symmetries of two different particles might have synergistic effects when a new ^{2}e element connects these particles.

There are reasons to suspect that the core of some particles may indeed acquire internal symmetries. The dominant mode of growth was shown in Fig. 8c. This mode outgrows the other two modes (58), because it depends on s^2 rather than on qs and q^2, with $s \gg q$. The dominant mode of growth forms ring-shaped structures of various sizes. Thus, it produces a structure with internal correlations, though not as regular as in a crystal. One can speculate that techniques borrowed from the study of quasicrystals [17], or similar, may be of use here.

14 Particle creation and destruction

The theory developed so far concerns a fixed number of particles, at least a fixed number of massive particles. Photons, as a subset of new ^{2}e elements, will appear to be created and destroyed all the time. However, massive particles correspond to densely connected clusters of elements and links (Section 6). They consist of a core and a mantle that grow in proportion to $\sim 1/N^2$ and $\sim 1/N$, respectively. Then they must have formed during the earlier stages of the universe, and new network particles are not expected to be formed during the later stages, when N is huge. Neither are they expected to be destroyed. At first sight, this seems to conflict with observations, which show that the number of particles is variable, in particular in the presence of sufficiently high energy, such as mediated by high-energy photons or at high-speed collisions of particles. The purpose of this section is to show that observing variable numbers of particles is consistent with the present theory. Ultimately, the theory will have to make a quantitative connection with quantum field theory, but that lies in the future. Here, the principle of how massive particles can be observed as varying in numbers will be explained.

We will use here the results from the beginning of Section 10 on observing a particle p with proper mass m (that is, with m elements and links). Because of the variability of the proper time of p, the observed mass (i.e., the observed rest energy) varies randomly over time, according to a random variable μ with pdf $\eta(\mu)$ (156)

$$\eta(\mu) = \mathcal{N}(m, \frac{1}{2\Delta t}). \tag{201}$$

This is a Gaussian with mean $\langle\mu\rangle = m$ and variance $1/(2\Delta t)$. Here Δt is the time interval, which is usually large, resulting in a small variance of μ. However, when one observes at high energies, Δt becomes small. Then $\eta(\mu)$ gradually ceases to be Gaussian, but reverts to the discreteness of the distribution P_k (5) of which it is composed (see the explanation above and below (152) and (153)). For single steps in the proper time of the particle, when $\Delta t \approx \delta\tau = 1/m$, we find discrete values for μ

$$\mu = k\frac{m}{2} \;\text{ with }\; k \in [1, 2, \ldots, N], \tag{202}$$

occurring with probability P_k (see below (153), with $f_{\mathrm{p}} = \mu$). Then $\eta(\mu)$ can be written as

$$\eta(\mu) = \sum_{k=1}^{N} P_k\, \delta(\mu - k\frac{m}{2}), \tag{203}$$

with δ the Dirac delta function. The mean is still m, $\langle\mu\rangle = m$, and the possible values of the observed mass are $m/2$, m, $3m/2$, $2m$, $5m/2$, and so on. These are the values of the rest energy of the particle, not values of the total energy as increased by relativistic speeds. Then how would the observer interpret these observations? A value of m is clear, it would be just a single particle. A value of $2m$ will be interpreted as two particles, that is, two observed copies of the original particle p. Although this originates from a single cluster in the network, this cluster lacks a particular position in spacetime. Then there is no reason why it could not produce two different observed trajectories in spacetime of two different observed instances of the particle. The same is true for an observed mass $n \cdot m$, corresponding to n observed particles. Nevertheless, this would be only a temporary observed state, since the mean of $\eta(\mu)$ is still m. Therefore, on average only a single particle can be observed per network particle.

The uneven values of the observed rest energy are more complicated. In particular $m/2$ cannot correspond to an observation of the particle p, because it is too small. Still, it represents an energy, but an energy in the absence of an observed particle, though still in the presence of the corresponding network particle. We might tentatively call it the zero-point energy, m_{ZP}, of the network particle. This energy level must be there, because otherwise the mean of $\eta(\mu)$ could never be m. The other uneven values of the observed mass might be interpreted as a number of particles plus a zero-point energy. For example, $5m/2$ might be viewed as two particles plus m_{ZP}.

Although this explanation is tentative and far from complete, it shows the way by which the number of particles can vary, at least the observed number of particles. It also indicates what the origin may be of some of the most persistent issues of quantum field theory (QFT). The zero-point energy can be present at most once per existing network particle. That is far less than the standard assumption of QFT that it is present at each point in spacetime (as a consequence of having a quantum harmonic oscillator at each point in spacetime). Moreover, the number of spacetime points is limited by the fact that space is not smooth and continuous, and that there is no space within particles and at small link distances around them. Together these deviations from standard assumptions should go a long way towards explaining QFT's unwanted infinities and the much too large estimate of the vacuum energy of the quantum fields.

15 Discussion

By growing, the network produces a variety of patterns. Although these patterns consist of dimensionless elements and links, they are assumed to get a physical interpretation when observed from within the network. The results in this article are based, first, on the emergence of a (3+1)-dimensional spacetime (Sections 4 and 5), and, second, on the assumption made in Section 6 that each link is interpreted as a quantum of action in spacetime. The plan there was to first explore how well the consequences of this interpretation agree with the physical world as it is known empirically and theoretically, and then to evaluate the plausibility of the interpretation in the Discussion. We have seen in the preceding sections that there is considerable agreement with established physics and cosmology: conservation of energy, time reversal invariance, the transformations of special relativity, massive particles, curvature of spacetime, expansion of space, a generally increasing entropy (though for a different reason than usually assumed), quantum physics describing particle observation, uncertainty relations, the Born rule, the Feynman path integral, photons and the Planck relation for their energy, multiple instances of particles at high energies, and a zero-point energy. Since the network produces a vast *terra incognita*, and the author is a generalist rather than a specialist in any of the topics investigated, it is possible, or perhaps likely, that the analysis contains some errors. But even when taking that possibility into account, it appears that the proposed physical interpretation of links is plausible to such an extent that we can safely consider it to be correct. Each link corresponds to $h/2$, that is, half the constant of Planck. On average, each new element is connected by two new links, together producing one h. But there is considerable variation in a new element's realized number of links, ranging from one up to the number of existing elements (but the latter with a negligible probability when N is large (2)).

The theory has an interesting relative in the causal set theory (CST) of quantum gravity ([1], [2], [18], [19]). In CST, spacetime consists of discrete spacetime events (there called elements, too), which together form a partially ordered set. In [2] (p.024002-2) a sequential growth dynamics for spacetime is described as a process of "cosmological accretion" by which "an element of the set comes into being as the 'offspring' of a definite set of existing elements." The process of growth is thought of as "constituting time." It is clear that this vision is nearly identical to the basics of the theory derived in Section 2. Yet, a major difference with CST is that the elements in the present theory are not bits of spacetime, but rather bits of network time (at least as how they can be interpreted when newly formed). In contrast, space and spacetime are (statistically) emergent. A second major difference with CST is that the network here has neither causality (the principle that causes precede effects) nor causation. New elements and links result from irreversibility, as in a ratchet, not from a cause-and-effect relationship. Causation is, ultimately, a dynamical relation, which the network cannot have, first, because it has no internal time and dynamics, and, second, because relations are merely descriptive tools that depend on pointing. Nevertheless, the network contains an ordering that resembles causality. A third major difference with CST is that not only the elements exist in the current theory, but also the entities that connect the elements, the links. Both are physical; links are not relations, but physical entities. This points to a general problem with using sets and order theory for fundamental physics. A set consisting of two or more members could not exist physically, because it lacks the physical means to keep its members together. For example, Fig. 2a could be viewed as a two-member set, but it cannot be realized physically. Sets are only held together by the physicists' minds who define them, by pointing (see Section 2). Even when used purely as descriptive devices, connected graphs are in a better position than sets.

Connected graphs have indeed been used widely for theories of fundamental physics. Examples are quantum graphity ([20]) and loop quantum gravity ([21]). These and similar approaches differ from the one presented here by using the graph's components (vertices, edges, or combinations) to directly represent quantum states. Then their primitives are too complex to have a fundamental existence: quantum states depend on pointing (according to the arguments in Section 2) and cannot be fully fundamental. In causal dynamical triangulations ([22]) small (3+1)-dimensional building blocks ($\sim$ triangles) are combined using quantum rules (which again produces an issue with fundamentality). In simulations it can produce a (3+1)-dimensional spacetime at large scales ([23]). It is not immediately clear whether this is related, perhaps indirectly, to the results presented here. Finally, graphs form the basis of the Wolfram model ([24]). The links there are not entities, but directed relations that even can form self-loops. The underlying assumption of this and quite a few other approaches is that reality is produced computationally, by sets of rules or laws. Consequently, such approaches tend to lean too much on mathematics (which consists completely of implicit pointing) and too little on physics (which is about reality devoid of pointing). Fundamentally, there can be no rules at all (Section 2).

A potentially related theory that should be mentioned here is that of Oppenheim [25] [26]. It couples classical gravity to quantum field theory, and is fundamentally stochastic. It is clear that it cannot describe the emergence and initial stages of the current network, in particular because spacetime, classical or not, does not yet exist there. Nevertheless, it would be interesting to see if hybrid classical-quantum gravity could act as an intermediate statistical description halfway between these initial stages and the stages that are well described by quantum field theory and general relativity.

The theory that is presented here starts from pure nothingness. The notion that nothingness is a natural starting point for fundamental physics is not new (reviewed in [27]). It is sometimes argued that nothingness might produce something through a quantum vacuum fluctuation, but that does not involve pure nothingness and would depend on pointing. There would still be an assumed vacuum and its laws. As to the question why there is something rather than nothing, [27] distinguishes five options: creation by an external entity, the universe being embedded in something larger, the universe having an underlying principle, nothingness being an incoherent concept, and the existence of the universe being an unexplainable, brute fact. From the perspective of Section 2, not one of these options works. The first three conflict with 'really nothing at all,' whereas the last two concern potential issues of pointing, and can, therefore, have no consequences for the existence of the universe. However, we can now add a sixth option, the one that is shown here to be viable: nothingness leads to something as a consequence of irreversibility with time absent (Section 2).

The first things that arise from nothingness are network time (stepped forward by each new element) and quanta of action (each link). Only much later space emerges as a statistical consequence of network growth (Section 4). Observable spacetime is formed by space combined with a statistically defined time (Section 5). The latter time is related nonlinearly to network time, but is in practice linearized over a considerable range (Section 5). Questions about the nature of space and time, including whether one is more fundamental than the other, have a long and tortuous history in physics. No attempt will be made here to refer to, let alone review, the vast literature on this topic. The present theory comes to a firm conclusion: both the space and the time of spacetime are emergent, but there is indeed a fundamental network time. The latter runs forward by itself and does not depend on observation or entropy.

Since its inception, quantum physics has provoked lively debates about its interpretation, that is, what it refers to in physical reality (its ontology, what exists 'out there'). Some questions refuse to go away, such as what constitutes a measurement, must it involve a human observer, how to differentiate observer from observed, whether a wavefunction is a fundamental entity or not, does observation make it collapse as in a physical process, and how to interpret phenomena that seem to be nonlocal or even anticausal. The theory developed here can resolve these issues, since it is crystal clear how and from what the network is being built (Section 2) and what constitutes a measurement (Section 10, in particular the last two paragraphs). Reviewing the existing interpretations from the perspective of the current theory would require interpreting these interpretations (which may have variants and may not be completely clear on all points), which will not be done here.

Nevertheless, one of the debated points mentioned above, about human observers, requires some further comments here. It is a recurrent theme in the debates, sometimes even involving consciousness and free will. First of all, it should be clear that the network will continue to grow autonomously, irrespective of whether there are biological observers or not. The major statistical patterns that form a (3+1)-dimensional spacetime and matter will occur inevitably, as a result of overwhelming statistics. The same must be true in general for the formation of other dynamical structures, such as galaxies, stars, planets, and nearly all processes studied by physics and chemistry. None of that depends on observers. The statistical patterns are there, even if there is no one observing them. It is not known how often life will form, because its formation and evolution depend on chance and the right circumstances to a much larger extent than is the case for most physical and chemical systems. Evolution of the Darwinian kind is particularly interesting, because it can lead to a power to point and to other powers that are not seen elsewhere in nature. A plausible theory (full disclosure: the author's and not mainstream, yet) as to how such powers can arise is that organisms have evolved mechanisms that produce an implicit estimate of their own evolutionary fitness (roughly, the chance to survive and reproduce). Quantitative simulations ([3]) show that such an estimate can improve the evolutionary outcome in variable environments. The required mechanism is that organisms decrease the variability of their behaviour when the estimated fitness is high ('never change a winning team'), and increase it when it is low ('desperate times call for desperate measures'). Two things are needed here. The first thing required is estimation, which entails a power to point, since any estimate points to what is estimated. And the second thing required is a source of fundamental physical randomness (as opposed to the apparent randomness that is produced by incomplete knowledge). In ([3]) it required indirect arguments to justify the assumption of such randomness, since both classical and quantum physics claim determinism (with quantum physics only in a formal way, through the unitary evolution of the state vector; determinism is then interpreted as 'fully governed by laws' with conserved information and conserved entropy). In contrast, the current theory has, fundamentally, no determinism at all. Instead, determinism is the statistical (though highly accurate) consequence of patterns that arise in randomness. Hence, living organisms would have no trouble to tap into the fundamental randomness of reality and to utilize it. Then the power to point can emerge in a weak sense (that is, as a novel and surprising property, yet explainable) from the network randomness combined with the evolved material system that is the organism. In contrast, the power to point appears to emerge in a strong sense (that is, as a novel and surprising property, and fundamentally unexplainable) relative to only the material system. In short, the power to point is here a biologically generated and sustained statistical pattern in the network, on a par with spacetime and matter.

Estimating fitness is a form of pointing. One can conjecture that from this generic form more specific forms of pointing can evolve, as well as derived powers. Agency (the power to act) can be viewed as behaviour combined with pointing, in particular pointing to a deliberate goal. Consciousness can be viewed as the power to communicate pointing (see [28] for a semi-quantitative study, and Ch. 12 in [29] for a qualitative general essay). Since these powers depend on the network statistics just as spacetime and matter do, they are as real as the latter, albeit not classifiable and observable in terms of spacetime and matter. In particular, they cannot be produced by computation. It is clear that the power to point, agency, and consciousness play crucial roles in the practice of doing physics. Any experiment requires agency and consciousness, which combined produce a form of free will. Any physical theory requires pointing and, when formulated and eventually communicated, consciousness. In that sense quantum physics does not differ from classical physics or any other science. It is just more confusing because it clearly displays the consequences of the discreteness and irregularity of time, of the statistical nature of spacetime and matter, and of the fact that observation requires interaction.

The numbers of elements and links in the network are huge. Even the tiny electron has about 10^{37} elements and links (200). As a result, much of current spacetime physics is deterministic for almost all practical purposes. Exceptions can be found at early cosmic times and presumably in sub-particle structures. There are everyday exceptions as well. One such exception is when a biological power such as agency has effects on matter (to be understood as a statistical pattern affecting a statistical pattern). Another everyday exception is the arrow of time, which in classical and quantum physics can be reversed without changing the fundamental dynamical laws of physics. This is also true, in good approximation, within the observed spacetime in the network, but not for the fundamental network time. This time progresses irreversibly (Section 2) and the arbitrary addition of quanta of action leads to an ever increasing entropy (Section 9). The increase in entropy is not attributable to coarse graining, but is fundamental. Hence, contrary to commonly held views, the arrow of time is not produced by an increase of entropy, even though the two are correlated.

Let us conclude on a general note. Normally, when one uses or produces a model or theory for a particular empirical purpose, the foremost question is 'could it be right?' Consequently, the model can be evaluated by fitting it to experimental data, discrepancies can be investigated, the fitted parameter values can be judged by their consistency with existing knowledge, and the performances of alternative models or theories can be compared. None of that normality seems to apply to the theory that has been developed here. Now, the foremost question is rather a rhetorical 'could it not be right?' The theory has exceptional properties. It does not require initial conditions or initial structures, and it has no free parameters. There are no numbers or structures that could be adjusted. If the argument in Section 2 is correct and has no loopholes (or at least has no loopholes that cannot be closed), the network is the necessary outcome of starting from nothingness. Then the theory is not one of many, but one of one. Arguably, it is the simplest theory possible. Slightly simplifying the first step, the recipe would read 'start with one element and then keep adding single elements that stick arbitrarily to the existing elements.' Even simpler than that, the elements pop into existence by themselves and the mix is self rising. As shown in this article, this simple mechanism produces considerable agreement with existing physics and cosmology, across the board. If all of that would be produced by a structureless zero-parameter theory as a result of happenstance, it would truly be a coincidence of cosmic proportion. Uncommonly, the theory inverts the normal direction of scientific discovery. Although science sometimes works with overarching concepts (such as symmetries and action in physics, evolution in biology, and information in neuroscience), the normal direction is to use empirical findings to discover and test models and underlying theories. Science has long ago learned to trust empiricism more than rationalism. Here, however, one seems forced to view any tension with well-established theories or with empirical data as an indication that the interpretation of the present theory is at fault rather than the theory itself. The challenge is, thus, to develop effective ways to understand the statistics of the network, which in its raw form is not very informative. Some of that has been done here, but considerable effort is still required to establish a seamless connection with the effective field theories of high-energy physics and with theories about the earliest phases and overall dynamics of the cosmos.

A Mathematica code for computing W_n

The code below is for computing (18) (excluding the factor 2^n) for $n = 4$; add or delete terms for other n.

```
g(x_,y_):=Piecewise[{{1/x,x>=y},{1/y,y>x}}]
Wn=Integrate[g[x0,x1]g[x1,x2]g[x2,x3]g[x3,x4],
   {x0,0,1},{x1,0,1},{x2,0,1},{x3,0,1},{x4,0,1}]
```

B Numerical computation of W_n

W_n can be computed directly from (19), but for large n it is more convenient to approximate the part with the factorials by an accurate approximation formula for the Catalan numbers ([30]), with $M = 4n + 7$

$$\frac{1}{n+2}\binom{2(n+1)}{n+1} \approx \frac{4^{n-1}}{M\sqrt{M\pi}}\left(128 + \frac{160}{M^2} + \frac{84}{M^4} + \frac{715}{M^6} + \frac{10180}{M^8}\right). \tag{204}$$

Then W_n is 2^n times this result, and

$$W_n = a'_n \left(2^n\right)^3, \tag{205}$$

with

$$a'_n = \frac{1}{4M\sqrt{M\pi}}\left(128 + \frac{160}{M^2} + \frac{84}{M^4} + \frac{715}{M^6} + \frac{10180}{M^8}\right). \tag{206}$$

These equations provide, for small and intermediate n, an approximation that is more accurate than (21) and (22). For large n, the leading term gives $a'_n \approx 4/(\sqrt{\pi}n^{3/2})$, consistent with (22).

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