Classical Heraclitus Spacetimes and the Equivalence of Local and Global Structure*

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Abstract

In any spacetime theory, a model is Heraclitus if no distinct points share the same local structure. Heraclitus models are known to exist in general relativity. Here, we present three examples of Heraclitus models within the classical spacetime context: (i) a geometrized classical spacetime (with curved derivative operator), (ii) a classical cosmological model whose underlying spacetime is Galilean (with flat derivative operator), and (iii) a classical cosmological model whose underlying spacetime is Leibnizian (with no derivative operator). The third example is of special interest since it shows a sense in which non-rigid Leibnizian spacetime can be "rigidified" by adding matter. This means Leibnizian spacetime+matter can be more deterministic than Leibnizian spacetime itself. We close with a general theorem which holds in any spacetime theory: Heraclitus models have the same local structure if and only if they have the same global structure.

1 Introduction

Consider a collection of spacetime models of the form $(M, O_1, ..., O_n)$ where M is a smooth manifold and $O_1, ..., O_n$ are geometric objects on M.[1] Let $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ be any models in the collection and let $U \subseteq M$ and $U' \subseteq M'$ be any open sets. A diffeomorphism $f : U \to U'$ is an *isomorphism* from $(U, O_1, ..., O_n)$ to $(U', O'_1, ..., O'_n)$ if $f^*(O'_i) = O_i$ on U for $i = 1, ..., n$. Let us now consider a general notion of a "Heraclitus" spacetime (Manchak and Barrett 2023).

Definition. A spacetime model $(M, O_1, ..., O_n)$ is *Heraclitus* if, for any distinct points $p, q \in M$ and any open neighborhoods U and V respectively, there is no isomorphism $f : U \to V$ such that $f(p) = q$.

If a spacetime model $(M, O_1, ..., O_n)$ is Heraclitus, then any distinct points $p, q \in M$ fail to share the same local structure. In such a model, one could

*Thanks to Hans Halvorson, David Malament, Oliver Pooley, James Read, and Jim Weatherall for helpful comments and discussions.

[1]To be precise, let us restrict attention to smooth tensor fields and derivative operators.

say that it is impossible to step twice in the same river. We know there exist Heraclitus spacetimes within the context of general relativity where models are of the form (M, g_{ab}) with g_{ab} a Lorentzian metric on a connected, Hausdorff manifold M (Manchak and Barrett 2023). Indeed, it seems that Heraclitus models are generic in this context although this is difficult to capture precisely.[2] What about other spacetime theories?

Here, we present three examples of Heraclitus models within the classical spacetime context: (i) a geometrized classical spacetime (with curved derivative operator), (ii) a classical cosmological model whose underlying spacetime is Galilean (with flat derivative operator), and (iii) a classical cosmological model whose underlying spacetime is Leibnizian (with no derivative operator). The third example is of special interest. Because it has no derivative operator, Leibnizian spacetime fails to be rigid like Galilean spacetime. But if one adds a sufficiently asymmetric matter field to the spacetime, the resulting Leibnizian cosmological model becomes rigid. This means that Leibnizian spacetime+matter can be more deterministic than Leibnizian spacetime itself. We close with a general theorem which holds in any spacetime theory whatsoever: Heraclitus models $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ have the same local structure if and only if they have the same global structure.

2 Examples

We start with the definition of a classical spacetime.[3] We follow the presentation of Malament (2012) except that we do not restrict attention to the four-dimensional case. This will allow us to present simple two-dimensional Heraclitus examples. A spacetime model $(M, t_{ab}, h^{ab}, \nabla)$ is *classical* if (i) M is a connected, Hausdorff manifold of dimension $n \geq 2$, (ii) t_{ab} is a symmetric tensor of signature $(1, 0, ..., 0)$, (iii) h^{ab} is a symmetric tensor of signature $(0, 1, ..., 1)$ which satisfies the orthogonality condition $h^{ab}t_{bc} = \mathbf{0}$, (iv) ∇ is a derivative operator that satisfies the compatibility condition $\nabla_a t_{bc} = \nabla_a h^{bc} = \mathbf{0}$.

We next construct a simple "Galilean" classical spacetime. Let $M = \mathbb{R}^2$ in (t, x) coordinates. Let $t_{ab} = (d_a t)(d_b t)$ and $h^{ab} = (\partial/\partial x)^a(\partial/\partial x)^b$. Let ∇ be the flat derivative operator associated with the (t, x) coordinates. One can easily verify that the spacetime model $(M, t_{ab}, h^{ab}, \nabla)$ counts as classical. It is two-dimensional *Galilean spacetime*. On this spacetime, let $\phi, \rho : M \to \mathbb{R}$ be a pair of smooth scalar fields satisfying Poisson's equation:

$$h^{ab}\nabla_a\nabla_b\phi = 4\pi\rho$$

The scalar ρ represents the Newtonian mass-density function. The scalar ϕ represents a gravitational potential where the force on a particle with mass m

[2]See Sunada (1985, Theorem 1) for a proof that Heraclitus models are generic among the collection of compact Riemannian manifolds.

[3]For foundational texts concerning classical spacetimes, see Earman (1989) and Malament (2012).

is given by the (spacelike) vector $-mh^{ab}\nabla_a\phi$. So a particle with tangent vector ξ^a subject to no forces except gravity must satisfy the equation of motion $\xi^a\nabla_a\xi^b = -h^{ab}\nabla_a\phi$. One can now use ϕ to define a second derivative operator $\tilde{\nabla}$ on M such that a curve with tangent vector ξ^a is a geodesic with respect to $\tilde{\nabla}$ if and only if it satisfies the equation of motion just given with respect to ∇.[4] This is the "geometrization lemma" of Trautman (1965) where $\tilde{\nabla}$ is given by the derivative operator related to ∇ by the connection field $C^a_{bc} = -t_{bc}h^{ad}\nabla_d\phi$. One finds that $(M, t_{ab}, h^{ab}, \tilde{\nabla})$ is a classical spacetime and that the Ricci tensor associated with $\tilde{\nabla}$ comes out as $R_{ab} = 4\pi\rho t_{ab}$. We now use this geometrization lemma to show that a simple choice of ϕ and ρ results in $(M, t_{ab}, h^{ab}, \tilde{\nabla})$ being Heraclitus.

Example 1. Let $(M, t_{ab}, h^{ab}, \nabla)$ be the two-dimensional Galilean spacetime defined above. Let $\phi, \rho : M \to \mathbb{R}$ be the smooth scalar fields defined as follows:

$$\phi(t, x) = 4\pi(x^2e^t/2 + e^x)$$
$$\rho(t, x) = e^t + e^x$$

It is easily verified that Poisson's equation $h^{ab}\nabla_a\nabla_b\phi = 4\pi\rho$ is satisfied. So from the geometrization lemma, we know there is a classical spacetime $(M, t_{ab}, h^{ab}, \tilde{\nabla})$ with associated Ricci tensor $R_{ab} = 4\pi\rho t_{ab}$. Let $p = (t_p, x_p)$ and $q = (t_q, x_q)$ be any points in M and let U and V be any neighborhoods of p and q respectively. Suppose $f : U \to V$ is an isomorphism such that $f(p) = q$. The classical spacetime $(M, t_{ab}, h^{ab}, \tilde{\nabla})$ is Heraclitus if we can show that $p = q$.

Since f is an isomorphism, it must preserve ρ. This follows because f preserves both $R_{ab} = 4\pi\rho t_{ab}$ and t_{ab}. Let $\sigma : M \to \mathbb{R}$ be the scalar field given by $\sigma = h^{ab}(\tilde{\nabla}_a\rho)(\tilde{\nabla}_b\rho)$. This field represents the magnitude of the derivative of ρ according to h^{ab}. After a brief calculation, one finds:

$$\sigma(t, x) = e^{2x}$$

Since f is an isomorphism, it must preserve σ given that it preserves h^{ab}, $\tilde{\nabla}$, and ρ. Because $\sigma(p) = \sigma(q)$, we have $e^{2x_p} = e^{2x_q}$. It follows that $x_p = x_q$. Because $\rho(p) = \rho(q)$, we have $e^{t_p} + e^{x_p} = e^{t_q} + e^{x_q}$. Since $x_p = x_q$, it follows that $t_p = t_q$. So $p = q$. So the classical spacetime $(M, t_{ab}, h^{ab}, \tilde{\nabla})$ is Heraclitus. □

Example 1 can be converted into a type of Heraclitus classical "cosmological model" whose underlying spacetime is Galilean. Consider the following definition (cf. Malament 2012, p. 291): a structure $(M, t_{ab}, h^{ab}, \nabla, \rho)$ is a *classical cosmological model* if $(M, t_{ab}, h^{ab}, \nabla)$ is a classical spacetime and ρ is a smooth scalar field on M. As before, ρ represents the Newtonian mass-density function.

[4]See Malament 2012, Proposition 4.2.1 and note that the proof goes through for all dimensions $n \geq 2$.

Example 2. Let $(M, t_{ab}, h^{ab}, \nabla)$ be the two-dimensional Galilean spacetime defined above in (t, x) coordinates. Let $\rho(t, x) = e^t + e^x$. Define $\sigma : M \to \mathbb{R}$ to be the scalar field given by $\sigma = h^{ab}(\nabla_a \rho)(\nabla_b \rho)$. We find $\sigma(t, x) = e^{2x}$ as before. Since both ρ and σ must be preserved under any isomorphism, the classical cosmological model $(M, t_{ab}, h^{ab}, \nabla, \rho)$ is Heraclitus. $\square$

Example 2 is a Heraclitus classical cosmological model $(M, t_{ab}, h^{ab}, \nabla, \rho)$ whose underlying spacetime $(M, t_{ab}, h^{ab}, \nabla)$ is Galilean. Surprisingly, this cosmological model remains Heraclitus even after one throws away the derivative operator ∇. In other words, one has a type of Heraclitus cosmological model $(M, t_{ab}, h^{ab}, \rho)$ whose underlying spacetime (M, t_{ab}, h^{ab}) is "Leibnizian" (see Earman 1989).

Example 3. Let $(M, t_{ab}, h^{ab}, \nabla, \rho)$ be the classical cosmological model defined above. Now consider the structure $(M, t_{ab}, h^{ab}, \rho)$. Define $\sigma : M \to \mathbb{R}$ to be the scalar field given by $\sigma = h^{ab}(d_a \rho)(d_b \rho)$ where d is the exterior derivative operator on M. We find that $\sigma(t, x) = e^{2x}$ as before. Let p and q be any points in M with neighborhoods U and V respectively. Suppose $f : U \to V$ is an isomorphism such that $f(p) = q$. Since f is an isomorphism, it must preserve ρ. But this means that f must also preserve $d_a \rho$. This follows since d is the exterior derivative operator and thus $f^*(d_a \rho) = d_a(f^* \rho)$ (see Hawking and Ellis 1973, p. 26). Since f preserves h^{ab} and $d_a \rho$, it must also preserve σ. Since both ρ and σ must be preserved by f, we find that $p = q$ as before. So the "Leibnizian cosmological model" $(M, t_{ab}, h^{ab}, \rho)$ is Heraclitus. $\square$

Example 3 is striking given that the model $(M, t_{ab}, h^{ab}, \rho)$ has no derivative operator. Consider the following definition (Geroch 1969): a spacetime model $(M, O_1, ..., O_n)$ is *rigid* if, for any open set $U \subseteq M$ and any isomorphism $f : M \to M$, if f acts as the identity on U, then f is the identity map on all of M.[5] Any model that is Heraclitus must also be rigid.[6] Because the two-dimensional Leibnizian spacetime (M, t_{ab}, h^{ab}) fails to have a derivative operator, it fails to be rigid like Galilean spacetime. But now we see that if one adds a sufficiently asymmetric matter field ρ to the spacetime, one can use the exterior derivative operator on M to show that the resulting Leibnizian cosmological model $(M, t_{ab}, h^{ab}, \rho)$ is rigid since it is Heraclitus. In other words, matter can "rigidify" a Leibnizian spacetime.

It follows that Leibnizian spacetime+matter can be more deterministic than Leibnizian spacetime itself. Let us make this precise. A collection of spacetime models is *de re deterministic* if, for any $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ in the collection and any open sets $U \subseteq M$ and $U' \subseteq M'$, if there is an isomorphism

[5] For recent work concerning rigidity, see Halvorson and Manchak (2025), Dewar (2025), Manchak et al. (2025), Grimmer and Read (2026), and Read and Manchak (2026).

[6] Suppose a model $(M, O_1, ..., O_n)$ fails to be rigid. There must be an isomorphism $f : M \to M$ which is not the identity map. So there must be distinct points $p, q \in M$ such that $f(p) = q$. Since M counts as an open neighborhood for both p and q, we see that $(M, O_1, ..., O_n)$ is not Heraclitus.

$f : U \rightarrow U'$, then there is an isomorphism $g : M \rightarrow M'$ such that $g_{|U} = f$. A collection of spacetime models is *de re* deterministic* if, for any $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ in the collection and any open sets $U \subseteq M$ and $U' \subseteq M'$, if there is an isomorphism $f : U \rightarrow U'$, then there is a unique isomorphism $g : M \rightarrow M'$ such that $g_{|U} = f$. The only difference between the definitions is the uniqueness clause at the end. Of course de re* determinism implies de re determinism. One can show that de re* determinism is equivalent to the conjunction of de re determinism and the condition that every model in the collection is rigid (see Dewar 2025 and Read and Manchak 2026). The singleton collection of Leibnizian spacetime $\{(M, t_{ab}, h^{ab})\}$ satisfies de re determinism but not de re* determinism (Manchak et al. 2025, Halvorson et al. 2026). But since the Leibnizian cosmological model $(M, t_{ab}, h^{ab}, \rho)$ defined above is rigid, we find that the singleton collection $\{(M, t_{ab}, h^{ab}, \rho)\}$ is de re* deterministic.[7]

The astute reader will have noticed that t_{ab} plays no role in the argument that Example 3 is Heraclitus.[8] It follows that even the pared down model (M, h^{ab}, ρ) is Heraclitus. In fact, one can go a step further by replacing h^{ab} with the vector field $\xi^a = (\partial/\partial x)^a$. The structure (M, ξ^a, ρ) remains Heraclitus since one can reconstruct σ by considering the scalar field $(\xi^a d_a \rho)^2 = e^{2x}$ where d_a is the exterior derivative operator. Can we throw away the vector field ξ^a altogether? No: the model (M, ρ) fails to be Heraclitus since there is a global reflection isomorphism $f : M \rightarrow M$ defined by $f(t, x) = (x, t)$.

Stepping back, one has a general question: what are the "minimal" structures that are sufficiently rich to ensure the Heraclitus property? Of course, a manifold M all by itself can never be Heraclitus since there is a diffeomorphism taking any point into any other. In one dimension, there do exist Heraclitus structures (M, α) where α is a scalar field. Just let $M = \mathbb{R}$ and let $\alpha(x) = e^x$. But in dimensions two or more, a manifold with scalar field is too meagre to be Heraclitus. What about a manifold with derivative operator? As far as we can tell, the question is open.

3 Local=Global Structure

Consider again a collection of spacetime models of the form $(M, O_1, ..., O_n)$. We say $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ are *locally isomorphic* if, for each point $p \in M$, there is a neighborhood U of p and an open set U' in M' such that there is an isomorphism $f : U \rightarrow U'$, and, correspondingly, with the roles of $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ interchanged. Naturally, models $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ are (globally) *isomorphic* if there is an isomorphism $f : M \rightarrow M'$.

We now show that any pair of Heraclitus models are locally isomorphic if and only if they are isomorphic. This has already been shown within the con-

[7] For more on Leibnizian spacetime and definitions of determinism, see Earman (1977), Stein (1977), and Weatherall (2020).

[8] The astute reader is David Malament. The remainder of this section is largely due to him.

text of general relativity (Manchak and Barrett 2023). Here, we generalize the result to apply to all spacetime theories. We begin with a foundational "gluing lemma" concerning manifolds (O'Neill 1983, p. 5).

Lemma. Let M and M' be manifolds. For each index $\alpha \in A$, let U_α be an open set on M and let $f_\alpha : U_\alpha \to M'$ be a smooth map. If, for all $\alpha, \beta \in A$, $f_\alpha = f_\beta$ on $U_\alpha \cap U_\beta$, then the unique map $f : \bigcup U_\alpha \to M'$ defined such that $f_{|U_\alpha} = f_\alpha$ for all $\alpha \in A$ must be smooth (see Figure 1).

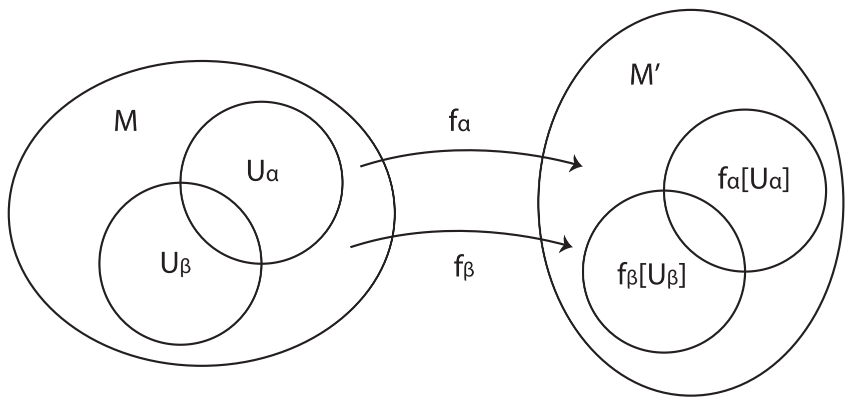

Figure 1: If the smooth maps $f_\alpha : U_\alpha \to M'$ and $f_\beta : U_\beta \to M'$ agree on the their overlapping domain $U_\alpha \cap U_\beta$, they can be combined to create a smooth map $f : U_\alpha \cup U_\beta \to M'$ such that $f_{|U_\alpha} = f_\alpha$ and $f_{|U_\beta} = f_\beta$.

The gluing lemma tells us that if all local smooth maps $f_\alpha : U_\alpha \to M'$ and $f_\beta : U_\beta \to M'$ agree on their overlapping domain $U_\alpha \cap U_\beta$, then all such maps can be combined to create a single global smooth map $f : \bigcup U_\alpha \to M'$. We now show that if $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ are locally isomorphic Heraclitus models, the gluing lemma can be invoked (twice) to combine the local isomorphisms between the models into a single global isomorphism.

Theorem. Heraclitus models $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ are locally isomorphic if and only if they are isomorphic.

Proof. One direction is trivial. Let $(M, O_1, ..., O_n)$ and $(M', O'_1, ..., O'_n)$ be locally isomorphic Heraclitus models. Since they are locally isomorphic, for each point $p \in M$, we can fix an associated open neighborhood $U_p \subseteq M$ and an isomorphism $f_p : U_p \to U'_p$ where U'_p is an open set in M'. Let p, q be any points in M and let r be any point in $U_p \cap U_q$. Let $U' = f_p[U_p \cap U_q]$ and $V' = f_q[U_p \cap U_q]$. Since f_p and f_q are isomorphisms, we know $f_q \circ f_p^{-1} : U' \to V'$ is an isomorphism that takes $f_p(r)$ to $f_q(r)$. Since $(M', O'_1, ..., O'_n)$ is Heraclitus, we have $f_p(r) = f_q(r)$ and thus $f_p = f_q$ on $U_p \cap U_q$. Since $\bigcup U_p = M$, it follows from the gluing lemma that the map $f : M \to M'$ defined such that $f_{|U_p} = f_p$ for all $p \in M$ must be smooth.

The next step is to show that $f : M \to M'$ is a bijection. Let p, q be any points in M and suppose that $f(p) = f(q)$. So $f_p(p) = f_q(q)$ where $f_p : U_p \to U'_p$ and $f_q : U_q \to U'_q$ are the isomorphisms associated with p and q. Let $U = f_p^{-1}[U'_p \cap U'_q]$ and $V = f_q^{-1}[U'_p \cap U'_q]$. Since f_p and f_q are isomorphisms, we know $f_q^{-1} \circ f_p : U \to V$ is an isomorphism which maps p to q. Since $(M, O_1, ..., O_n)$ is Heraclitus, it follows that $p = q$ and thus f is injective. Now let p' be any point in M'. Because the spacetimes are locally isomorphic, there is an isomorphism $h : N' \to N$ where N' is an open neighborhood of p' and N is an open set in M. Let $p \in M$ be the point $h(p')$ and consider its associated isomorphism $f_p : U_p \to U'_p$. Let $U' = h^{-1}[U_p \cap N]$ and $V' = f_p[U_p \cap N]$. Since h and f_p and are isomorphisms, we know that $f_p \circ h : U' \to V'$ is an isomorphism which maps p' to $f_p(p)$. Since $(M', O'_1, ..., O'_n)$ is Heraclitus, it follows that $p' = f_p(p)$. Because $f_p(p) \in f[M]$, we know $p' \in f[M]$ and thus f is surjective. So f is a bijection.

Now we show that f^{-1} is smooth. For each $p \in M$, we can consider the inverse of its associated isomorphism: $f_p^{-1} : U'_p \to U_p$. Let p, q be any points in M. Suppose there is a point $r' \in U'_p \cap U'_q$. The map f is defined such that $f_{|U_p} = f_p$ for all $p \in M$. So f must send the point $f_p^{-1}(r') \in U_p$ to the point $r' \in U'_p$. Similarly, f must send the point $f_q^{-1}(r') \in U_q$ to the point $r' \in U'_q$. Since f is injective, we know $f_p^{-1}(r') = f_q^{-1}(r')$. So $f_p^{-1} = f_q^{-1}$ on the region $U'_p \cap U'_q$ for any $p, q \in M$. Since f is surjective, it follows that $\bigcup U'_p = M'$. So f^{-1} is the unique map from $\bigcup U'_p = M'$ to M defined such that $f^{-1}_{|U'_p} = f_p^{-1}$ for all $p \in M$. By the gluing lemma, f^{-1} must be smooth.

Since $f : M \to M'$ is a smooth bijection with a smooth inverse, it is a diffeomorphism. The final step is to verify that it is an isomorphism. Consider any point $p \in M$ and its associated isomorphism $f_p : U_p \to U'_p$. We know that $f_p^*(O'_i) = O_i$ on U_p for all $i = 1, ..., n$. Since $f_{|U_p} = f_p$, we know that $f^*(O'_i) = O_i$ on U_p for all $i = 1, ..., n$. Since p was chosen arbitrarily, it follows that f is an isomorphism. □

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