Smugglers of reference: Expressivist-inferentialism and the inevitability of model denotation

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Khalifa, Millson and Risjord (2022) have proposed a "thoroughgoing" inferentialist account of scientific representation. They claim that their account overcomes what they call the "smuggling" objection, namely, the fact that inferentialist accounts always require implicitly to rely on some more fundamental relation between the model and the target, such as similarity, isomorphism, or denotation. These authors attempt to avoid the smuggling objection by exporting Brandom's (1994) expressivist theory of reference in the philosophy of language to the case of scientific representation. In this paper, I set aside similarity and isomorphism, and focus on denotation. I take the so-called DEKI account of scientific representation (Frigg and Nguyen 2020), which assumes denotation as a crucial element of representation, as a point of reference to critically analyse Khalifa, Millson and Risjord's proposal. I argue that the authors do not successfully address the smuggling objection, and that the account is also open to further objections. I conclude that denotation is a required element for a working account of scientific representation, and that thoroughgoing inferentialism remains unsatisfactory.)

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1. Introduction

The great majority of philosophers agree that the relation holding between a model and its target is one of representation[1] but disagree on the nature of this relation. While some philosophers focus on similarity (Giere 2004; Weisberg 2013), or on mathematical morphism (Da Costa & French 1990, 2000; French & Ladyman 1999; van Fraassen 2008), many focus on referential relations like denotation and exemplification (Goodman 1976; Elgin 2010; Frigg & Nguyen 2020). A third great family of accounts is usually called *inferentialism*. All inferentialists tend to take the surrogative reasoning (that is, the inferences from a model to its target) that scientists perform via models not as a possible result or consequence of representation, but as a fundamental bedrock of the concept of representation itself.[2]

Khalifa, Millson and Risjord (KMR) argue that one of the main objections against inferentialism is what they call the "smuggling objection" (2022, p. 266). The objection is that, despite all their promises of deflationism, inferentialism eventually must rely, at least implicitly, on some "substantive" relationships – like similarity, isomorphism, or denotation. If the objection succeeds, KMR hold, the original, intuitive appeal of inferentialism and its deflationary approach is lost. As a response, KMR have proposed "expressivist-inferentialism", or "thoroughgoing inferentialism" (*ibid.*, p. 264), with which they try to demonstrate that a good inferentialist account can work without any appeal to denotation.[3] By tackling the smuggling

[1] Isaac (2013) and Knuuttila (2024) are dissenting voices on this. For a reply cf. my article (Sartori 2026).

[2] Inferentialism is diverse and ranges from deflationary accounts (Suárez 2004, 2024), to more inflated ones (Contessa 2007, Díez 2020).

[3] KMR also want to exclude any appeal to similarity or isomorphism; I focus on denotation only.

objection and proposing a denotation-free account of model representation, KMR want to show the preferability of their version of inferentialism with respect to other accounts of scientific representation. Among these accounts, KMR (*ibid.*, pp. 265-6, 277, 282) explicitly include the so-called DEKI account of scientific representation, developed in (Frigg & Nguyen 2020), which takes denotation as a fundamental element of representation, together with the other three components that together form the "DEKI" acronym: exemplification, keying-up, and imputation.

The aim of this paper is to assess whether KMR's account succeeds in responding to the smuggling objection and demonstrating its preferability to the DEKI account. My answer will be negative: denotation still plays a crucial role in KMR's account, and if this role were denied, the account would not be able to make sense of the epistemology of scientific models. In addition, KMR's account does not seem to fare better than the DEKI account in numerous other regards. So, the latter view remains preferable to KMR's thoroughgoing inferentialism.

The paper is structured as follows. In section 2, I introduce the DEKI account of scientific representation. As KMR's inferentialism emerges from a critique of the role of denotation in accounts like DEKI, it is pivotal to understand the account KMR are competing with. In section 3, I provide the fundamental elements of KMR's inferentialism. Then, in section 4, I argue that KMR's account does not overcome the smuggling objection. Section 5 provides two further objections against KMR's account. In section 6, I suggest that KMR's suspicions against denotation may just be unmotivated. The overall conclusion of my analysis is that DEKI remains a preferable alternative to KMR's inferentialism.[4]

2. DEKI with An Example: the Lotka-Volterra Model

As an example to illustrate the DEKI account,[5] let us consider the Lotka-Volterra model of a prey-predator population system, which is also the main case study used by KMR in their paper. The Lotka-Volterra model is a mathematical model constituted by two differential equations:

$$\frac{dV}{dt} = rV - (aV)P \qquad ; \qquad \frac{dP}{dt} = b(aV)P - mP$$

If V and P are the number of prey and predators, respectively, the first equation says that the rate of change of V is equal to a certain increase of V by the growth rate r, minus the so-called functional response $(aV)P$ – i.e., predators eat the prey proportionally to the sizes of the two populations. In the second equation, m is the rate of natural death among the predators,

[4] My investigation focuses on KMR's proposal only and is not a criticism of inferentialism overall. Further criticisms are offered in (Frigg & Nguyen 2020) and (Nguyen & Frigg 2022).

[5] My illustration of the DEKI account will be extremely concise. More details can be found in (Frigg & Nguyen 2020) and (Nguyen & Frigg 2022).

and *b(aV)P* expresses that the predator population grows by eating the prey. Let us call the uninterpreted mathematics the *carrier* of the representation, and the *model system M* is the carrier endowed with an interpretation *I*. *I* is a function associating each mark with its physical-biological meaning.

Even when interpreted, a model system is just a system, with its properties and internal dynamics. It is not a representation of anything, yet. The Lotka-Volterra model, even when endowed with the physical-biological interpretation I have just provided, fails to describe any real population. This is because, as Volterra himself notes, the model is highly idealised. He points out that "[T]he size of the model populations are measured by real [i.e., non-integer] values; births take place continuously [and] each species is taken to be homogeneous [...] by ignoring variations of age and size" (Volterra 1928, p. 6). In addition, the model says that an individual prey dies *only* if it meets a predator, which is completely unrealistic. In general, there is no real population that complies with the model's description: real populations consist of individuals that die of hunger and disease, are different in age, size and many other properties, and whose births take place irregularly. So, even if we call *V* the "prey" and *P* the "predators", we are not talking about any real prey or predator yet. We need something further in order to explain how this model *epistemically represents* a real target system. The DEKI account provides an answer to this question by defining "epistemic representation" as follows:

> **DEKI epistemic representation$_{\textbf{(def)}}$:** A model system *M* – understood as an object *X* endowed with an interpretation *I* – epistemically represents a target system *T iff*:
>
> (i) *M denotes T*,
>
> (ii) *M exemplifies* properties $P_1, \ldots, P_n$,
>
> (iii) $P_1, \ldots, P_n$ are associated with a second set of properties $Q_1, \ldots, Q_n$ via a *key*,
>
> (iv) $Q_1, \ldots, Q_n$ are *imputed* to *T*.

Here, *denotation* is one of the four fundamental ingredients of epistemic representation. It expresses Goodman's basic intuition that for a model to represent a target system, the former "must be a symbol for it, stand for it, refer to it" (Goodman 1976, p. 5). If this referential relation from the model to the target is not established, there would be no reason to start talking about representation at all.

In philosophy of language, denotation is the relation holding between a name and its bearer. In DEKI, this is extended: denotation is the relation from a symbol to the referent of that symbol. The Lotka-Volterra model, understood as a system of equations endowed with a physical-biological interpretation, denotes the fish populations in the Adriatic Sea after WWI that Volterra was trying to study when he created his model. Minimal as it may be, denotation is crucial to establish an initial referential relation between an idealised description and a real target system. While the model as a whole denotes the target as a whole, it will usually be the case that parts of the model will denote *parts* of the target – e.g. the model prey (i.e., the mathematical variable interpreted as an idealised prey population) denotes the prey fish population in the Adriatic Sea. Clearly, DEKI holds that denotation alone is not enough. When we represent phenomena in science, we do not limit ourselves to establishing a referential relation – like when we assign sites to a table with name tags. Model representation is aimed at obtaining some epistemic gain, and the other three elements of DEKI are meant to explain how.

In technical terms, an object X *exemplifies* a certain property A iff X instantiates A and also refers to A. Samples are paradigmatic examples of exemplification: they possess a certain property and, given a certain context, they refer to it. As Elgin (1983, 1996, 2010) has emphasised at length, "reference to a property" here means that that property is made epistemically accessible and highlighted. To illustrate this with our example, Volterra's model exhibits the so-called Volterra Property (Weisberg & Reisman 2008, p. 113) – namely, that when a biocide occurs in both populations, the relative amount of the prey population increases (Volterra 1926, p. 558). This explains an odd phenomenon of post-war abundance of predatory species. When fishing boats and fisheries were in full activity before the war, the relative amount of the prey was higher than the predators. But when the war started and fishing in the Adriatic Sea became prohibited or more arduous, the relative size of the predacious species grew at the expense of the prey.

The ultimate goal of our modelling is to *impute* some of the model's exemplified properties to the denoted target system in the real world. Imputation is the fourth element of DEKI and should be understood as mere property attribution, with no guarantee of correctness. This is crucial, because we want cases of misrepresentation to still count as instances of representation, and not to be ruled out as cases of non-representation. A geocentric model of the solar system still represents the solar system, if admittedly in an inaccurate way.

However, models rarely exemplify properties that are *directly* imputed to the target system. Even when one focuses on *relevant* properties only, it is seldom the case that these properties are simply imputed to the target system as they are. In the Lotka-Volterra model, the Volterra Property directly follows from the model assumptions: anytime there is a biocide, the change in relative size *does* in fact occur in the model. However, as Nguyen correctly points out, the Volterra Property "needn't be exported directly to the target system" but rather, it is better to think of the model as making the claim that "[f]ishing [...] increases the prey-to-predator ratio's *susceptibility* to rise" (2020, p. 1029). In other words, a *factual* statement in the model is translated into a *susceptibility* statement when it is applied to the target system. This is an example of what Frigg and Nguyen (2021) call a *key*, which is defined as a function associating relevant properties exemplified by the model to properties eventually imputed to the denoted target system. Other examples of keys are: scale factors; functions translating limit values in mechanic models into more realistic values (Nguyen & Frigg 2020); "approximation" keys;

geometrical projections in maps (Nguyen & Frigg 2022) and pictures (Sartori 2025).[6] The presence of a key does not automatically make our imputations correct. The key is simply part of how a system has to be interpreted in order to be used as a representation. So, we can still get things wrong and misrepresent targets even when the key is in place.

In this regard, Frigg and Nguyen (2022) have introduced a useful distinction between two types of correctness of our representational inferences. *Derivational* correctness concerns whether we are *reading* the representation correctly. In the case of the Lotka-Volterra model, we identify the bio-physical interpretation of the mathematical symbols, we mathematically derive the Volterra Property, and then impute it to the target system via a susceptibility key. The justification for this derivational correctness is internal to the single representational framework – namely, the Lotka-Volterra model itself. On the other hand, *factual correctness* concerns whether the target possesses those properties that we are imputing to it or not. Frigg and Nguyen insist that the justification for factual correctness is unavoidably *extrinsic* to the model itself. Modelling is an ampliative form of reasoning, and a model alone cannot provide the (entire) justification about the correctness of our results about the target. What models offer are testable hypotheses, the factual correctness of which can be justified only by "looking" outside the model – e.g., by checking the target system directly if possible or appealing to further empirical and theoretical reasons. With the Lotka-Volterra model, scientists modified the original assumptions and tested the robustness of the original model's properties, the Volterra Property included (Weisberg & Reisman 2008).

Now that I have introduced DEKI as a comparison example, let us move to KMR's proposal.

3. Expressivist Inferentialism

3.1. Inferentialism without denotation

The first and main motivation for KMR's "thoroughgoing" inferentialist account of scientific representation is to overcome the smuggling objection (2022, p. 266). The objection is that, despite all their promises of deflationism, inferentialist accounts eventually must rely, at least implicitly, on some other presumed representation relationship – such as similarity, isomorphism, or denotation. KMR's proposal, if successful, would then fare well "without appeal to substantive model-target representation relationships"(*ibid.*, p. 264). KMR attempt to produce such an account via an explicit importation of the expressivist strategy employed by Brandom's (1994) expressivist approach in the philosophy of language – about which I will discuss below.

As KMR remind us, other authors attempted to use Brandom's expressivism for the same

[6] See also my previous work (Sartori 2023a; 2023b) for keys involved in representations with model organisms and scientific thought experiments, respectively.

purposes (de Donato Rodriguez & Zamora Bonilla 2009) but, KMR hold, their account does not survive the smuggling objection. This is because Brandom's expressivism is a theory of linguistic meaning and reference, not of scientific representation, so it cannot be applied directly to the case of models (KMR 2022, p. 266). In contrast, KMR claim that their expressivist-inferentialist account is immune to the smuggling objection, thus presenting their view as virtually the best inferential account of representation currently available in the literature.

Before moving to the details of KMR's proposal, it is important to give further specifics on their doubts about denotation. First, KMR "use 'reference' as the relation between words and objects, and hold 'denotat[ion]' for the relation between model elements and target objects" (*ibid.*, p. 267). This terminology clashes with mine: I use "reference" as an umbrella term including many different relations between objects and symbols – among which denotation and, as we have seen above, exemplification. I understand denotation as the specific referential relation from a name to its bearer, or more generally from an interpreted symbol (or stand-in) to the system for which that symbol stands. For clarity's sake, I keep my use consistent for what follows and use "linguistic denotation" and "model denotation" to convey KMR's terms and avoid confusion.

KMR characterise their opponents' views about representation and denotation as follows: "[f]or representationalists, surrogative inference can only be justified if the representation relation [like similarity, isomorphism, or denotation] holds between model and target" (*ibid.*, p. 265). This way to describe the other accounts of representation, and DEKI specifically, may sound confusing, at least as concerns denotation, so let us unpack this issue.

Here, it is not clear whether KMR's concept of justification concerns only the derivational correctness of our inferences or also their factual correctness – a distinction that I have drawn at the end of section 2. Below, I will further show that while KMR's position is ambivalent on this issue, they most likely mean factual correctness. However one wants to define justification, DEKI holds that denotation is a necessary (but insufficient) condition for the justification of our inferences. For what concerns derivational correctness, we cannot even start imputing properties, and thus making inferences, from a model about a target, if there are no forms of denotational relation between (parts of) the model and (parts of) the target system. If we are talking about factual correctness, then as we have seen, its justification follows from factors extrinsic to the specific representation in question (checking the target system directly, converging results from other models, and so on). Nevertheless, even to be factually correct, our inferences from the model to the target system must be interpreted as following from the model assumptions *and* being about the designated target system. So, in this indirect way, denotation is a necessary semantic condition for any subsequent epistemic use of the model, including the justification for factual correctness. The intuition is that if we don't specify that the representation is a representation *of* something in the first place, there seems to be little hope to even start making an inference, even though the main source of justification for the factual correctness of our inferences is to be found outside and beyond the model itself.

All this to say that, if KMR are concerned simply with the idea that denotation does not provide *sufficient* justification for the derivational or factual correctness of our inferences, there is simply no disagreement here: in DEKI, denotation does not provide this sort of complete justification for our inferences, nor is it supposed to. To make sense of KMR's criticism, and the rest of the arguments they provide (illustrated below), however, we must understand their thesis to be stronger. According to KMR, we should give an account of representation without *any* appeal to, or requirement of, denotative relations, explicit or implicit they may be. For this is required in order to avoid the smuggling objection.

3.2. Justification and epistemic entitlements

Let's start by introducing KMR's definition of scientific representation (KMR 2022, p. 265):

> **KMR-representation$_{\mathbf{(def)}}$:** A model *M* is a scientific representation of a target system *T* iff *M* has scientifically justified surrogative consequences that are answers to questions about *T*.

The account is clearly inferential as it defines the ability to make inferences from models to target as the constitutive element of scientific representation.[7] Specifically, KMR's definition hinges on the concept of "justified surrogative consequences" drawn from a model about a target system (*ibid.*,). Given that the inferences flow from one system to another, the concept of "surrogative" does not seem to play any role in the definition. This is also because, if it did, then it would seem to already implicitly presume some form of relation between the model and the target, namely, a referential relation like denotation. So, let us focus on the concept of justified inferences.

When it comes to what counts as justification, KMR list five epistemic "entitlements" (*ibid.*, p. 269). These entitlements, once all "secured", would constitute the "justification structure" of our inferences (*ibid.*, p. 274). KMR call such a structure the "inferential pedigree" (*ibid.*, p. 274) of the model with respect to its target. Therefore, the securing of the five epistemic entitlements define the overall representational status of the model.[8] KMR's five epistemic entitlements are (i) characterisation, (ii) derivation, (iii) measurement, (iv) no-defeaters, and (v) relevance. Let us now look at these entitlements one by one, with some comparisons with the DEKI account as a touchstone.

[7] "Epistemic" and "scientific representation" are synonyms here, as the question of a demarcation criterion for specifically scientific representations is not relevant here.

[8] Unfortunately, KMR do not give further specifics about the inferential pedigree.

Characterisation is a physical (or biological, psychological, economic...), plausible interpretation of the model's mathematical formalism or of its material, non-interpreted components. I take KMR's characterisation to be equivalent to the *I*-function we have seen in DEKI, which associates elements and features of the uninterpreted carrier to an interpretation and thus to the model system *M*.

The *derivation* entitlement is secured for an inference when that inference actually follows from the model assumptions. This entitlement roughly corresponds to what I have called derivational correctness of our inferences from a representation. However, KMR do not make reference to a key, leaving this inferential "step" implicit in the process.

As the name suggests, *measurement* indicates some form of measurement aimed at justifying a certain inference for a certain target. KMR are extremely liberal on their concept of measurement, leaving unspecified any potential criteria of adequacy to demarcate measuring actions more precisely. Even with all its flexibility, this entitlement seems problematic, because there are clearly models that were built and even used without any direct measurement. Interestingly, this also applies exactly to the Lotka-Volterra model. There, Volterra made casual observations about fisheries (and maybe one wants to call this measurement), but those observations were exactly what the model had to explain. So, it would be odd to say that the model was justified by such measurements. Also, the original model itself never gave rise to any measurements, nor was it ever tested against data.[9]

KMR are also very flexible in their characterisation of the *no-defeaters* entitlement, which would be secured if there are no "facts that either refute the conclusion or undercut the premise's support of the conclusion" (*ibid.*, p. 269). This is a very general justificatory strategy: we go on with our inferences until we have reasons to doubt. One can see, though, why in many other accounts of representation, DEKI included, there is no such requirement: in modelling, we may often have *some* grounds to doubt our assumptions, particularly when they involve distortions. In the end, there may well be, and often there are, some defeaters, weak as they may be, which oppose our models' assumptions. Thus, the no-defeaters entitlement may end up being too strong and rule out many potentially useful hypotheses connecting a model to a target. To address this issue, it may be sufficient to weaken the requirement as follows: if there *were* in fact objections, they should not be *too* strong, depending on the context.

Finally, *relevance* is the most flexible and general of the entitlements: the model's inferences are answers to questions we are actually interested in about a given target system. Defined like this, relevance seems a requirement about any epistemic practice: if we are making inferences of any worth, we will have some questions in mind we want to answer. Of course, these questions may change with time depending on the practices we perform: if a certain material

[9] This is why many accounts of representation, DEKI included, do not mention measurement.

model used as a surrogative system does not allow to answer a certain class of questions but it is useful to solve other interrogatives, the relevance entitlement may shift, and its securing compromised or strengthened accordingly.

A few further remarks on the general characterisation of KMR's entitlements are in order. First, KMR insist that in order to constitute the structure of justification for our inferences, these entitlements must have been "secured" (*ibid.*, p. 274). This seems to suggest, in the most charitable reading, that what matters is that, for each entitlement, we have to reach a reasonable, sufficient level of success, however we intend to define it: no defeaters (or at least not too many or too strong); enough relevance; correct derivation from the model's assumptions; some successful measurement involved; and a plausible characterisation of the formal or concrete elements of the model. Furthermore, KMR hold that (at least) two of their entitlements – no-defeaters, and derivation – are *epistemic* entitlements, and not *semantic* ones: they do not concern the meaning of a proposition, but rather epistemic "statuses achieved by a scientific community" (*ibid.*, p. 269). It is not clear whether KMR meant that all five entitlements are epistemic in this sense, even though they later insist that their analysis is not, primarily, semantic but epistemological (*ibid.*, p. 279) – I will come back to this in section 4.3. Finally, KMR allow a high level of flexibility on how a certain entitlement can be established within an epistemic community: they mention that the consequence of a model, for example, "might be established in a scientific paper, subjected to peer review, and perhaps explored in a wider debate" (*ibid.*, p. 269). Arguably, the epistemological nature of KMR's epistemic entitlements is compatible with quite a few different interpretations. I suggest that a plausible and charitable way to interpret KMR's epistemic entitlements is to understand them as weak, defeasible externalist epistemic warrants,[10] even though nothing in the arguments that follow strictly depends on this characterisation.

3.3. The smuggling objection

By looking at some of KMR's entitlements, it is difficult not to suspect that denotation is already implicitly at work, thus substantiating the threat of the smuggling objection. Let us spell out how the objection would go, and then explain how KMR propose to deal with it.

First of all, consider the characterisation entitlement, where a certain variable in the mathematical equations of the model has to be endowed with a bio-physical interpretation. However, characterisation relates uninterpreted elements of the model to their theoretical interpretation, but not to a specific target system. KMR themselves make this point clear when they say that "[a]ssociating a linguistic descriptor like "rabbit population" with the variable x does not establish a relation of scientific representation or denotation between the model element, x, and a set

[10] I thank an anonymous reviewer for this suggestion.

of rabbits in Canada or anywhere else" (2022, p. 275). To clarify where the problem lies, let us take again the Lotka-Volterra model. KMR tell us that, by their characterisation entitlement, we interpret, say, the letter V as a prey population. However, this model-prey, interpreted as we are supposed to (via characterisation in KMR's proposal, or the I-function in DEKI), does not describe any actual prey yet. For the prey in the Lotka-Volterra model is an abstract (or fictional) idealised object: it is not constituted by discrete entities but it is continuous, it grows exponentially unless a predator eats them, and it exhibits many other features that make the model system relevantly different from any real ecological system. So, even admitting characterisation, it is not clear that the model's results are in any way relevant for the target system. The same can be said about relevance: how do we know that the model's results, i.e. what is true in the model, are relevant in any sense to answer our questions about the target? If there is no reference (and thus, denotation) to our target system, then there is no way to even think of moving from the model to the target as "relevant".

This is, I hold, the version of the smuggling objection tailored for KMR's view: denotation seems at least implicitly assumed in the very concept of relevance, and it is required for characterisation to connect a theoretical interpretation of a model system to an actual target.

As one can see, the problem is distinct from, and logically prior to, the question about the justification of the idealisations in the model. If that were the problem, KMR would say that securing the entitlements is our way to deal with our idealisations: by appealing to no-defeaters, relevance, measurement and so on, one can show that the idealisations in the model are either irrelevant or actually useful to answer our questions about the target. But this is not the problem at stake here. Even before justifying the idealisations, we must recognise the properties of the model as *idealised* in the first place. However, an idealised property is never intrinsically so: it is idealised *relative to* something else. In the case of models, an idealisation is relative to a target. So, we first of all need a way to identify the target and put it in relation with the model, in order to then talk about any form of idealisation and its potential justification. In other words, we need denotation.

Consider that my argument about characterisation and relevance has a cascade effect on other entitlements. Without denotation, and thus without a clear sense of relevance, we cannot say how (or even whether) the process of measurement would be performed, or which objections would really count as defeaters. Even derivation may be affected, as it can be seen with a quick comparison with the DEKI account. There, the key allows to "adjust" the model's exemplified properties to the ones actually imputed to the target. The key thus makes explicit the further inferential step scientists have to take when they move away from the idealised setting of the model to a specific target system. The key, however, does not provide the identification of the target, but presupposes it. The identification of the target is instead provided by the relation of denotation, which requires us to (even vaguely) identify both *relata*. As KMR do not want to appeal to denotation, the separation between what is true about the model and what the model says about the target becomes fuzzy: it is not clear exactly what really "follows" from the model *about the target*, specifically. Overall, this is the smuggling objection: without an implicit assumption about denotation, KMR's inferential pedigree seems to irremediably collapse.

3.4. The expressivist way

KMR's strategy to reply to the smuggling objection explicitly appeals to Brandom's (1994) expressivist theory of truth and reference and exports the same intuitions to the case of scientific representation. The expressivist strategy in the philosophy of language proceeds in two steps. First, it holds that the content of a proposition can be explained by its inferential role, which "is captured by the (semantic) commitments and entitlements undertaken by affirming the proposition" (KMR 2022, p. 277). Second, it gives an expressivist account of semantic vocabulary, including "truth" and "(linguistic) denotation", so that any referential concept is explained via previous uses and inferences. The first part of the argument, concerning the expressivist treatment of propositional content, is not relevant for our purposes. The second, concerning expressivism about semantic operators, is instead useful because it provides the template for KMR's later export of the strategy to the context of modelling. I will thus focus on this part of Brandom's argument alone. In addition, as Brandom's original theory would require a monograph on its own, I will focus on KMR's own illustration of his argument – and I will speak of KMR-Brandom, to highlight this fact.

Before proceeding, it is important to notice that Brandom's original inferentialism does not *identify* the semantics of operators like truth and reference with their inferential role, nor does it *reduce* the former to the latter. The semantics of language is still conceptually distinct from its use, inferences included. As clarified by Murzi and Steinberger (2017) in their introduction to this topic, Brandom's point is meta-semantic and concerns the *explanatory* direction of our theory of language: while the traditional theories of language explain the (inferential) use of terms and propositions on the basis of their semantic content and referential concepts, the inferential theory of language reverses the explanation order, and takes inferences and use to explain truth, meaning, and reference. So, truth and meaning do *not* disappear, and remain conceptually distinct from the inferences that would explain them. However, the challenge for KMR seems relevantly different in nature. They want to show how one can *reduce* representation to inferences, without implicitly assuming model denotation.[11] Otherwise, it would not be clear

[11] If, like Rosen (2010), we thought of reduction in terms of grounding, my distinction here between explanation and reduction may become more vague. However, this is a controversial issue to say the least, and the burden of proof would still fall on KMR's shoulders.

how they would really avoid the smuggling objection, which states that denotation is implicitly required by any inferential account of representation. So, analogous as they may be, KMR's enterprise seems quite different in nature than Brandom's original argument.

Let us now look at the expressivist treatment of operators like truth and reference. KMR-Brandom suggests that, when we say that a proposition p is true, we are not expressing a (new) relation between p and the world: we are simply stating exactly the same propositional content originally expressed by p. The truth operator is just expressing an *endorsement* of that propositional content on the part of someone – the speaker or the person that we are talking about (KMR 2022, p. 278). KMR emphasise that endorsement here "is an activity, not a further proposition" (*ibid.*, p. 278). This should shed light on the general strategy: given that also the propositional content of p can be expressed via previous and present use (inferences included), it seems that we do not need any appeal to correspondence to reality or problematic referential assumptions. The reference, implicit as it may be, is instead to previous uses, inferences and propositions. Things get more complicated when we move from truth to linguistic denotation. Let me quote KMR at length:

> Not all semantic vocabulary can be treated as [truth is], since not all will have sentences as their anaphoric antecedents. "... refers ....," for example, has indefinite descriptions and deictic expressions as antecedents. In general, Brandom's strategy for semantic vocabulary is to treat such items as "proform" operators that anaphorically inherit content from antecedent expressions and endorse it in some way. (*ibid.*, p. 278)

Let us assume, for the sake of KMR's argument, that KMR-Brandom's expressivism proves itself to work well with linguistic denotation (that is, the relation from words to objects). Then, we would have an *explanation*, in purely inferential-expressivist terms, of how some terms acquire their referents. However, linguistic denotation is admittedly *still there*. Brandom's thesis is, again, meta-semantic, and concerns the explanatory order between use and reference, not the replacement of the latter with the former. Indeed, KMR insist that they do not think that scientific representation can be entirely reduced to a question of linguistic reference,[12] and they themselves specify that Brandom's strategy " will not apply directly to scientific representation" (*ibid.*, p. 278). However, they suggest that one can export the expressivist strategy to the case of representation, because "'M represents T' and 'm denotes o' are very near cousins"[13] and the "*epistemic* function [of representation and model denotation] in scientific discourse is analogous to that of [truth and linguistic reference] operators" (*ibid.*, p. 278). In the next section, I discuss KMR's export of the expressivist strategy to the case of scientific representation.

[12] The only ones to dissent are Callender and Cohen (2006). For a rebuttal, see Frigg and Nguyen (2020, pp. 23-30); see also Ruyant (2021).

[13] KMR's remark on denotation and representation being "cousins" sounds puzzling, as most literature on representation argues against such an analogy (Frigg & Nguyen 2020, pp. 23-30).

3.5. Expressivist representation, expressivist denotation

KMR attempt to apply an analogous expressivist strategy to both the concepts of "scientific representation" (2022, p. 279) and "model denotation" (*ibid.*, p. 282). Let's look at both expressivist treatments one by one. To argue that representation can be cashed out in expressivist terms, KMR's first assume, as a matter of general, "uncontroversial" (*ibid.*, p. 279) consensus, that representation implies the justification of model inferences:

> *All* parties to the debate over scientific representation agree that when [a model] *M* represents [a target system] *T*, some surrogative inferences from what *M* says to conclusions about *T* are justified. (*ibid.*, p. 279, my emphasis)

As a second premise, KMR remind us that the "surrogative inferences endorsed are exactly those justified by the inferential pedigree" (*ibid.*, p. 279). In this sense, one only needs to appeal to the epistemic entitlements mentioned above. KMR can then claim as follows:

> "[r]epresents" is thus analogous to "true" [...]: it expresses and endorses an epistemic entitlement that is based on an independent body of epistemic entitlements. And in so doing, it introduces no new epistemic entitlements or semantic content (*ibid.*, p. 279).

In this way, the entire concept of representation is expressed via the epistemic entitlements and the inferential pedigree endorsed by scientists, with no appeal to denotation or reference. Then, KMR say that they clearly need to "explain how '*M* represents *T*' is related to a particular inferential pedigree such that treating *M* as a representation can endorse the entitlements it provides" (*ibid.*, p. 279). They "cheerfully shoulder this explanatory burden" (*ibid.*, p. 279) by appealing to the entitlements, as well as to how the model is presented, constructed, and used:

> The entitlements of the inferential pedigree are embedded in the scientific process of building, operationalizing, and working with a particular model, a process wherein the target is described. It is in virtue of that process that the surrogative conclusions drawn from *M* are about *T*, rather than another target. (*ibid.*, p. 279)

KMR then conclude that "'*M* represents *T*' [...] expresses that some surrogative inferences from *M* to conclusions about *T* are justified" (*ibid.*, p. 280).[14] So, all the work would actually be done by the entitlements, with no need to appeal to referential relations.

Finally, KMR claim that their expressivist strategy can also be applied to model-target denotation specifically (*ibid.*, p. 282). They argue that denotative statements can be easily reformulated in terms of inferential patterns. For example, the statements "[i]n the model *M* the element *m* denotes the element *o* in the target system *T*" can be expressed as "[i]nfer that *o*

[14] KMR explicitly require only an existential claim that *some* inferences are justified, with no specification about on which specific inference (2022, p. 280). This sounds deeply puzzling, as it is not clear how one can endorse the justification for unspecified inferences.

exists when *m* occurs in *M*" (*ibid.*, p. 283). The same would hold for inferences about relations in the models, and other more complex inferences. For instance, if *m* has a property *P1* in the model, then the part of the target *o* denoted by *m* has a property *P2* defined via the derivation entitlement; and so on. In order to explain how we establish such inferential patterns, KMR appeal once again to their five epistemic entitlements: any "specific instance of these inferences will be justified (or unjustified) by their inferential pedigree" (*ibid.*, p. 283).

4. The Inevitability of Denotation

Let me provide here a schematic summary of KMR's argument:

- P1. It is uncontroversial that "*M* represents *T*" implies an endorsement of the justification of a set of model inferences from *M* to *T*.
- P2. The model-target relation is already embedded in the use of the model and thus in the epistemic entitlements constituting the justification mentioned in (P1).
- C. We can give an inferentialist account of representation without any reference to denotation.

This section critically assesses KMR's premises. Section 4.1 challenges KMR's assumption (P1) that all parties agree that representation implies the endorsement that some inferences are justified. In section 4.2, I argue that even if premise (P1) were vindicated, the smuggling objection persists due to either the scientists' implicit acknowledgment or KMR's epistemic entitlements. My objection here applies to both KMR's expressivist treatments of representation and model denotation. Section 4.3 considers a possible rebuttal by KMR, consisting in a shift from semantics to epistemology, and shows that such a line of defence is ineffective.

4.1. The justification of model inferences

KMR take for granted that it is uncontroversial for all parties that representation implies an endorsement that certain inferences from the model to the target are justified. Let me call such a thesis the Endorsement Thesis (ET) for short. Admittedly, it is unclear whether ET concerns the justification for the *derivational* correctness or the *factual* correctness of our inferences, which I distinguished at the end of section 2. However, there are good reasons that KMR are talking about the latter.

To begin with, their epistemic entitlement "derivation" is basically equivalent to my definition of internal correctness. Then, it would be odd if KMR said that the justification for internal correctness, and thus their derivation entitlement, was obtained by *all* the entitlements, derivation included. Some sort of problematic circularity would be involved. And there seems to be no reason or plausible way available to KMR to distinguish their derivation entitlement from my internal correctness. Furthermore, there is even stronger evidence that KMR's Endorsement Thesis has to do with the justification for *factual* correctness. First, when KMR define inferential pedigree, they specify that it corresponds to the "structure of justification" of our inferences, and this justification is provided by the five entitlements when they "have been *secured*" (2022, p. 274; my emphasis). While they offer no clear definition of how one can "secure" an entitlement, it seems plausible that "securing an inference" is not equivalent to merely considering that inference as the correct reading of a model's implications. "Securing" here means providing reasons for the factual correctness of the inference in question. Second, KMR themselves talk about ET in terms of truth:

> To claim that *M* represents *T*, then, is to endorse just that set of surrogative inferences where propositions derived from *M* are inferred to be *true of T* (*ibid.*, p. 280, my emphasis).

So, KMR *do* in fact say that the endorsement of a model's inferences about its target implies an endorsement of the justification for their factual correctness. If my analysis is correct, KMR's Endorsement Thesis is incorrect in at least two ways: it is semantically implausible and empirically inaccurate. *Semantically*, it is simply not true that when we understand a model as a representation, then we are already endorsing that the inferences we are drawing within the model are also correct about the target. When I look at the Ptolemaic model of the Solar System, I understand the former as a representation of the latter, but I don't automatically *endorse* that what the model tells me about its target is justified. So, interpretation of a model as a representation of a target system does not imply an endorsement of the justification of the factual correctness of those imputations.

Moreover, KMR's assumption is arguably *empirically* inadequate too. Scientists use models to produce new testable hypotheses, but it is a stretch to say that deeming a model a representation of a phenomenon implies an endorsement, on the part of the scientists, of the justification for the factual correctness of those hypotheses. As a paradigmatic example, let us consider the case of the steady-state model of cosmology (Bondi & Gold 1948; Hoyle 1948), which tried to explain the expansion of the universe without assuming an initial Big Bang. After being intensively studied and developed, this model was eventually rejected, mostly because of observations that contradicted the predictions of the model – especially, the model explicitly excluded the existence of the so-called cosmic microwave radiation (CMR), an observation predicted by the Big Bang Theory that was in fact confirmed in 1964 (Penzias & Wilson 1965).

Now, scientists developing and using the steady-state model simply did not possess the required justification for the model's implications. For example, they did not know whether CMR would have been observed or not. Still, they used and improved the model anyway for almost twenty years, to provide testable hypotheses to be then empirically tested. KMR could object that when scientists studied and developed the steady-state model, they *did* endorse the justification of their inferences about the expansion of the universe; they simply stopped when those inferences were proven wrong. However, first, scientists – as well as laypeople – did not stop considering the steady-state model a model of cosmology once it was rejected because of the CMR observations. Second, in fact, some still thought of that model as a *good* model, even long after CMR was detected – as famous physicist Weinberg's 1972 writing testifies:

> The steady state model is so attractive that many of its adherents still [...] hope that the evidence against it will eventually disappear as observations improve. (Weinberg 1972, p. 463-464)

So, not only is it arguable that an endorsement of the justification for the factual correctness of the steady-state model's predictions was not required to develop, study, and use the model. But even when the model was in fact questioned on the basis of strong empirical evidence, people still hoped that the model could be vindicated again empirically in the future. All in all, this case directly contradicts KMR's first assumption: the model was meant to provide testable hypotheses that could be vindicated and justified or not. There is no reason to imply any endorsement, on the part of scientists, of the justification for the correctness of those hypotheses.

The example of the steady-state model is by no means an exception. Once we look at cases beyond those involving extremely reliable models, which are very rare, the conjectural nature of models' predictions becomes strikingly evident. For instance, model organisms are used to formulate hypotheses about biological mechanisms occurring in humans well before one can elaborate a full-fledged justification for the correctness of these hypotheses; models of the climate provide predictions that exhibit a high level of uncertainty; and economic models offer appealing conjectures on how to intervene in monetary trends that however may remain purely theoretical possibilities. Overall, justification is something we aim at *acquiring*, not something we *endorse* from the start. So, what KMR suggest simply diverges from what seems to happen in actual scientific practice.

As a consequence of both its semantic and empirical inadequacy, ET is also rejected by most accounts of scientific representation, making this thesis deeply *controversial*. The DEKI account, for one, does not agree with such a statement: to represent something does not imply an endorsement that the factual correctness of our model inferences is justified about the target. All that is required in DEKI is that we impute properties to the target system. But then, one can perfectly acknowledge that such imputations are tentative, conjectural, and possibly even false hypotheses.[15] The justification for the factual correctness of those hypotheses, if ever acquired, should be better thought of as the desired *end* of our scientific enquiries, and not something the endorsement of which is directly implied by our use of the model as a representation. DEKI is no exception in this regard: other accounts of representation, like the representation-as accounts

[15] This is also in continuity with the work by Popper (1963/2014) and Lakatos (1976; 1978) about the conjectural nature of our scientific hypotheses *tout court*.

(Goodman 1976, Elgin 2010), but also those based on similarity and isomorphism, and other accounts (Hughes 1997; Toon 2012) do *not* take justification as a constitutive part or as an inevitable implication of representation at all. Probably, not even deflationary inferentialists like Suárez would accept KMR's premise. So, KMR's first assumption about endorsement of justification of our inferences turns out to be both semantically and empirically incorrect, as well as highly controversial in the debate. In this sense, we can legitimately conclude that truth and representation are *not* relevantly analogous for an expressivist treatment. While truth can be recast in terms of an endorsement of a proposition, representation doesn't express an endorsement of the justification of our inferences.

4.2. The smuggling objection strikes back

Let us suppose that KMR's first premise was vindicated in some way. So, let's analyse KMR's second assumption, namely, that the very construction, definition and use of the model, and the securing of their epistemic entitlements, already expresses the relation between the model and the target via a specific inferential pedigree with no requirement of denotation.

First, the very fact that a specific target system T is constantly mentioned in the construction and use of a model (a fact that KMR grant) certainly constitutes strong evidence that some form of denotation is involved. Only because the denotative relation remains implicit does not make it any less important. Second, besides these implicit appeals to an already designated target, KMR's account can count here only on the epistemic entitlements, which are the only constitutive, definitional elements of their expressivist representational relationship – the inferential pedigree being defined as the sum of the entitlements when secured. But, as we have seen in section 3.3, the epistemic entitlements *do* require some denotation in order to even start working. Indeed, this was the very reason to start the entire expressivist enterprise in the first place.

Similar thoughts to what I have said so far about KMR's expressivist account of representation can be naturally applied to their expressivist treatment of model denotation. KMR give this treatment by translating denotative relations into endorsements about justification of their epistemic entitlements. But this strategy cannot work, exactly for the same reasons it didn't work for representation. First, analogously to what I said for KMR's premise (1), it is not true that when I take an element of the model to stand-in for an element of the target I am implicitly endorsing the justification of relevant entitlements. It may be purely tentative and hypothetical. Second, the entitlements themselves presuppose that some concept of denotation is in place.

KMR do not provide any further argument to defend their first premise, or to "explain" how their second premise would escape the smuggling objection. But, as my analysis illustrates, the first premise is incorrect or at least highly controversial, and the second premise does not grant a solution to the smuggling objection.

4.3. Semantics vs. epistemology?

At this point, KMR could complain that I am confusing a semantic level of analysis with an epistemological one. In fact, before they illustrate their expressivist treatment of representation, KMR remind us of the important difference between Brandom's programme in philosophy of language and their own endeavour concerning scientific representation:

> First [...] Brandom's project aims at understanding the *semantic* function of '...is true,' while we are interested in understanding the capacity of models to represent their targets (and not the semantic function of the sentence '*M* represents *T*'). Second, the endorsement of surrogative inference in *M* represents *T* is the endorsement of an entitlement to draw surrogative conclusions, not the endorsement of a propositional content. (KMR 2022, p. 279, my emphasis)

This passage is arguably puzzling, because the exact goals of KMR's analysis start to become quite unclear. In the rest of their paper, KMR clearly speak as if fully committed to a *semantic* analysis of representation. First, they have given a definition of representation at the beginning of their article. Second, they have set as their main aim a purely inferentialist account of representation, understood in the same sense as other accounts, like DEKI (but also Suárez' and the similarity- or isomorphism-based views) that *do* deal with the semantics of representation. Third, they identify as their opponents exactly those accounts, and argue that thoroughgoing inferentialism is preferable to them. Fourth, in one of the extracts quoted above, they explicitly draw an identity between the proposition "*M* represents *T*" and an endorsement of some epistemic entitlements about the inferences we draw from the model to the target (*ibid.*, p. 280). In contrast, in the last quoted passage, KMR clearly claim that they are *not* interested in the semantics of representation. The two sets of claims are simply in overt conflict with each other. Despite their ambivalence, KMR seem to be in fact interested in the semantics of representation:

> "*M* represents *T*" inherits the entitlements that justify the inference of model derivations (*M* says that *P*) to a proposition true or false of a target (*C*). It thereby *expresses* entitlement to a set of surrogative inferences (*ibid.*, my emphasis).

Here, KMR explicitly say that statements about representation "express" the endorsement of epistemic entitlements. So, the relation is not (only) of epistemic dependence, but a matter of semantics as well. And so, all my previous arguments still apply. In any case, even if my reading here was wrong, it is clear that shifting the discussion from semantics to epistemology would not save KMR's inferentialist proposal. First, if KMR's account were not concerned with the semantics of representation, their level of analysis would simply be different than the one of all the other accounts of representation, DEKI included. So, it would not be clear any more whether KMR's comparison of their own account with those that include denotation is in fact meaningful – before even asking whether it is effective – and the dispute would simply dissolve. Second, it would remain unclear how KMR can show that denotation is not involved in their epistemology of models without *also* talking about the semantics of the inferential activities involved in modelling practices. Such an analysis would simply leave the semantic aspects of those practices implicit, thus confirming that the smuggling objection is still in place.

5. Two Further Problems for Thoroughgoing Inferentialism

In the previous section, I have shown that KMR do not manage to tackle the smuggling objection. In this section, I want to raise two further issues for their view. One is that the account fails to distinguish model inferences from any other sort of inference, while the second is that their account implies that each model has indefinitely many target systems.

The first issue concerns KMR's epistemic entitlements, which are the only constitutive element of representation in KMR's inferentialism. Arguably, these entitlements seem to apply easily to many types of inferences that do not seem representational at all. As an extreme example, consider a deductive syllogism, like "All humans are mortal. Socrates is a human. Therefore Socrates is mortal". Now, this syllogism satisfies all KMR's five entitlements. For the syllogism is relevant for a certain question about Socrates (is Socrates mortal?); it definitely includes a derivation (a deduction); some of its premises will be obtained from some form of measurement (that Socrates is a man, that humans seem to be mortal), at least in the general terms KMR use to define "measurement"; it works if no defeaters of its premises emerge; and it involves a characterisation of linguistic signs as real objects (the words "Socrates" and "humans" are interpreted as referring to real human beings). However, it is counterintuitive to affirm that the syllogism is a scientific representation of Socrates, at least in the same sense as, scientific models are representations of their designated target systems. But this is exactly what KMR's thoroughgoing inferentialism seems to entail.

KMR do not notice this problem, and thus do not offer any possible way to escape it. If their strategy resembled their expressivism about representation, they could appeal to implicit endorsements of the scientists. However, this answer would be unsatisfactory, leaving an actual illustration of the peculiarities of surrogative reasoning still to be provided. The other option I can envisage is to bite the bullet and admit that model-based inferences are in fact not distinguishable from other types of inferences. However, it would be then mysterious why one should even try to develop an account of surrogative reasoning in the first place. To provide a comparison example: DEKI gives a more precise characterisation of representational inferences, defining them via denotation, interpretation, exemplification and keying-up. The syllogism itself (i) does not denote Socrates, nor (ii) does it exemplify properties that are imputed to Socrates. The key would not help here, because it is not clear what properties of the syllogism, even linguistically interpreted, would have to be translated and how they would be translated.

Let us move to the second issue: according to KMR's view, each model turns out to have indefinitely many target systems. Here, I remind the reader that by KMR's definition (3.2), for a representational relationship to obtain, the only condition is that the model offers justified inferences about the target system. To make my argument clear, I will make use of an analogy. Let us consider an everyday tool: a hammer. Now, the hammer is a system, with its shape, material and other properties. This system clearly provides the basis to draw many justified inferences. I can make justified inferences about the hand that could wield it, and in general the limbs, articulations, and cognitive system an organism needs in order to use it. The hammer also allows us to learn something about what materials are commonly used to build it, available to the manufacturers, and involved in the production process. Furthermore, if I study the hammer's use, I can infer many properties of the very nails and other objects that an agent can hit with it. So, our hammer can justifiably answer a lot of questions about numerous other things: its users, the way in which it is produced and used, the practical contexts in which it is used, and even the history and culture of the society that created it.

If this is true about a hammer, it is definitely true about model systems as well. Similarly to the hammer, a mathematical model like the Lotka-Volterra model allows justified inferences not only about its designated target: the model also offers justified answers to questions about our mathematical symbolism and how it is used; about us as cognitive agents able to perform certain types of mathematical reasoning; about other theorems and theories assumed in the background, and so on. But then, according to KMR's definition, the model represents, *by definition*, not only its target system, but all these other things. In the case of the Lotka-Volterra model, we would have the implication that the model does not only represent the fish population in the Adriatic Sea, but also it would represent the users of the model (first of all, Volterra himself!), the practices in which it is employed, and it would even represent mathematics itself. So, if one remains liberal about which questions one can answer on the basis of the model, as KMR seem to do, and no further criterion of identification of the target is provided, one ends up trivialising the concept of representation, because a model would end up representing far too many things.

While DEKI solves this problem trivially with denotation, which identifies the designated target system, KMR's account does not have the required resources to tackle it. It cannot appeal to the entitlements: most of them are so general that will be relevant for the justification of any inference we are drawing. The justification of the inferences, say, from the model to its user will also have to be assessed by appeal to relevance, correct derivation, no defeaters, and arguably some measurements. Even characterisation: to understand the model as a model, it needs to be an interpreted object (and not, say, a random string of marks of a page). This second issue is particularly pressing, as it contradicts KMR's suggestion that the best quality of inferentialism is that it provides "a superior picture of how models are used in scientific practice" (2022, p. 266). In fact, that any model turns out to have indefinitely many target flies in the face of an intuitive understanding of scientific practice, as both scientists and philosophers clearly acknowledge that *there is* a designated system, a specific set of phenomena, that the model is supposed to stand for and represent, distinct from other things about which we could draw inferences from the model.

Again, KMR do not notice the problem, and thus do not deal with it in their paper. And, the prospect of trivialising the concepts of representation seems serious enough to rule out the possibility of simply biting the bullet. KMR may want to argue that the distinction between the actual target system and the rest of systems about which we can draw inferences from the model is, again, just implicitly known by the practitioners and the relevant epistemic community. However, this is a definitive concession to the advocates of denotation. For denotation, in DEKI and other denotation-based accounts, is defined in very minimal terms and can be established in numerous different ways. In particular, it does not need to be explicitly acknowledged by practitioners, no more than when people use proper names in everyday language do not explicitly acknowledge a relation of denotation.

6. A Final Note on Denotation and Inferentialism

From the very beginning, KMR labelled denotation as a substantive relation. However, it is not at all clear what "substantive" really means here. In some anti-representationalist circles, this term identifies mind-independent relations, like similarity (see e.g., Knuuttila 2011). However, this would not work for denotation, which is clearly mind-dependent. In general, it is not clear how denotation would be more "substantive" than the inferences with which KMR would like to replace it. KMR do not offer further explanation, but it seems to suggest some strong and potentially controversial assumptions about denotation. This brings me to legitimately cast some doubts on the very motivations for KMR's iconoclastic attitude against denotation. KMR say that while some inferentialist accounts (Hughes 1997; Contessa 2007) and the DEKI account take denotation as a requirement for representation, "such views put the cart before the horse: denotation is consequent on surrogative inference, not the other way around" (2022, p. 282). They further explain their view as follows:

> [I]t is a mistake [...] to explain surrogative inference in terms of a prior notion of denotation. Doing so misses the function of denotation. To say that the model denotes some object is to say that the model licenses inferences about that object (*ibid.*, p. 284).

First, as I have argued above, it is controversial, to say the least, that representation implies licensing our inferences – and the same applies a fortiori to denotation. Second, KMR's claims indicate some confusion on the kind of explanation we are interested in. In DEKI, inferences *logically* depend on denotation, and representation is semantically explained by appeal to denotation, in combination with the rest of the constituents of the DEKI account. The epistemic function of models thus is also explained by reference to denotation and the rest of the elements of the account. Yet, there is no claim in DEKI that denotation could not *causally* depend on the amount of success of our imputations – and thus, on licensed inferences – as KMR seem to suggest. The sort of dependences that KMR and DEKI are interested in, then, are simply different, and thus compatible with each other. And, I am happy to grant that how a certain representation turns out to denote a certain target *is* often the result of the historical evolution of our imputation attempts and their success: they effectively exemplify certain properties that we manage to successfully map onto our target systems of interest via a suitable key.[16] But nothing in DEKI denies this fact: the four elements of the account are not to be read as temporally ordered, and all four contribute to representation and affect each other constantly.

While the two orders of explanations in DEKI and KMR's inferentialism are orthogonal, I take it that DEKI's focus on the semantic priority is more beneficial. First, denotation does have a philosophical priority, because it is a semantic requirement for any eventual inferential success: there is no way to make sense of a surrogative inference if we are not already interpreting the model system as a surrogate system that refers to the target system. Also, denotation does not *always* or necessarily depend on the past success or licensing of our inferences, as it can simply be obtained by fiat. Finally, the role granted to denotation in DEKI does not rule out the historical dependence emphasised by the inferential accounts. In contrast, KMR's excessive focus on the inferential aspect, and thus on the "historical" sense of explanation risks making them oblivious of the important referential relations implicitly at play in scientific representation.

7. Conclusion

In this paper, I argued that Khalifa, Misson and Risjord's (2022) inferential account does not escape the smuggling objection. Also, I showed that their view (i) includes controversial assumptions about the justification of model inferences, (ii) is unable to distinguish model inferences from other types of inferences, and (iii) has the problematic implication that each model has indefinitely many targets. Finally, I cast doubts on KMR's very motivations to get rid of denotation. All in all, it seems that the costs of embarking on an expressivist-inferentialist endeavour simply outweigh the admittedly scarce benefits of such an enterprise.

[16] Goodman's idea of "entrenchment" (1983) applies the same strategy: certain predicates are chosen over others because they have worked better in the past. This did not stop Goodman from using denotation for his own theory of representation (1976).

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