

Real-Alignment Interpretation: Contextual Phase Synchronization as a Mechanism for Quantum Measurement

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Abstract

We develop a minimal phase-dynamical extension of the Real-Alignment Interpretation for quantum measurement, in which a system state $f(t)$ interacts with a contextual state $g(t)$ through weak coupling and near-resonant dynamics. Starting from a two-mode Schrödinger description, we obtain an effective Adler-type equation for the relative phase $\psi = \arg f - \arg g$. The resulting dynamics exhibits two qualitatively distinct regimes: a locked regime, in which the phase approaches a stable fixed point and the system–context pair becomes real-aligned, and a drifting regime, in which the phase explores the circle and alignment events occur intermittently. The locked regime yields a characteristic event time, while the drifting regime gives rise to inter-click statistics and effective probabilistic behavior under coarse-graining. We show that the dynamics can be represented as overdamped motion in a tilted washboard potential and that Born-type probabilities are recovered under ergodic phase sampling and detector-symmetric or calibrated channel response. In this picture, measurement is modeled as a synchronization process rather than an instantaneous collapse. The framework provides an analytically tractable mechanism by which contextual coupling can generate discrete outcomes, event timing, emergent probabilities, and possible phase-sensitive deviations from ideal Born statistics.

Keywords: Quantum measurement, Phase synchronization, Adler equation, Contextual dynamics, Quantum foundations, Emergent probability

1 Introduction

Quantum mechanics (QM) provides an exceptionally accurate framework for predicting experimental outcomes, yet its conceptual foundations remain unsettled. In particular, the measurement problem, namely how definite outcomes arise from the linear superposition principle, remains unresolved [8, 10]. The standard formalism associates observables with operators and outcomes with eigenvalues but does not supply a dynamical mechanism for the selection of a single outcome in an individual measurement. Instead, the projection

postulate introduces a discontinuous collapse of the wavefunction, external to unitary Schrödinger evolution and lacking a clear physical basis.

Several approaches attempt to address this tension. Decoherence theory explains the suppression of interference through entanglement with environmental degrees of freedom and accounts for the emergence of classical pointer states [4, 9]. However, it does not explain outcome selection in single experimental runs. Hidden variable theories, most notably Bohmian mechanics, restore determinism through particle trajectories guided by the wavefunction [2, 11], but the Born rule is imposed via a quantum equilibrium hypothesis rather than derived dynamically. Objective collapse models modify the Schrödinger equation by introducing stochastic nonlinear terms that induce localization [5, 12, 13], yet the origin of the stochastic field remains phenomenological.

A structurally distinct perspective arises from contextuality. The Kochen-Specker theorem demonstrates that measurement outcomes cannot be assigned independently of the measurement context [6]. This excludes noncontextual hidden variable models and establishes contextual dependence as a fundamental feature of quantum theory. Nevertheless, contextuality is typically formulated as an algebraic constraint on value assignments rather than as a dynamical physical process. Consequently, standard formulations do not provide a mechanism by which the measurement context actively participates in outcome formation.

In parallel, synchronization phenomena in nonlinear dynamics provide a well established mechanism for the emergence of stable collective behavior in coupled systems. Weakly interacting oscillators can exhibit phase locking despite detuning and noise, a behavior captured by the Adler equation and its generalizations [1, 3, 7]. The transition between phase locking and phase drift is governed by a competition between coupling and detuning and is associated with qualitative changes in system dynamics. Such systems demonstrate how discrete and robust states can emerge from continuous interactions.

These observations motivate the exploration of a framework in which contextuality is promoted to a dynamical variable and measurement outcomes arise through synchronization between system and context. The Real-Alignment Interpretation (RAI) introduces a contextual state and postulates that a measurement event occurs when the system and context satisfy a real-valued alignment condition [14]. While this condition identifies the occurrence of definite outcomes, its original formulation does not specify the underlying dynamics. In this work, we develop a minimal dynamical model for system–context interaction. Under weak coupling and near-resonance conditions, we show that the relative phase obeys an Adler-type equation with stochastic forcing. Within this framework:

- measurement corresponds to a phase-locking transition,
- quantum probabilities arise from ergodic phase dynamics,
- the classical limit emerges as a strong locking regime, and
- contextuality becomes a dynamical degree of freedom.

The resulting formulation preserves the Hilbert space structure of QM while providing a continuous dynamical account of measurement. It establishes a direct connection between quantum foundations and synchronization theory and offers a physically transparent mechanism for the emergence of discrete outcomes and probabilistic statistics. The paper is organized as follows. Section 2 derives the contextual phase dynamics and analyzes the locked and drifting regimes. Section 3 discusses the implications for measurement,

probability, contextuality, and the classical limit. Section 4 provides a broader discussion of the model and its limitations. Section 5 concludes with future directions. Technical derivations of the Adler reduction, locking time, inter-click time, and stationary phase distribution are collected in Appendices A–D.

2 Contextual Phase Dynamics

In this section we formulate the reduced phase dynamics governing the interaction between a system state and a contextual state within the Real-Alignment framework. The central aim is to show how the real-alignment condition can arise dynamically, rather than being imposed only as a static constraint. We begin with a minimal two-mode Schrödinger model in which the system and its measurement context are represented by complex amplitudes $f(t)$ and $g(t)$, coupled through a weak Hermitian interaction. Under weak-coupling and near-resonance assumptions, the fast carrier oscillations can be separated from the slow evolution of the relative phase. This reduction leads to an Adler-type equation for the system–context phase difference, from which the locked and drifting regimes, effective potential, event times, and probabilistic behavior can be analyzed.

2.1 System–context phase dynamics

We consider a minimal two-mode system–context model,

$$\Psi(t) = \begin{pmatrix} f(t) \\ g(t) \end{pmatrix}, \quad f(t), g(t) \in \mathbb{C},$$

evolving according to

$$i\hbar \frac{d}{dt} \begin{pmatrix} f \\ g \end{pmatrix} = \hbar \begin{pmatrix} \omega_f & k \\ k & \omega_g \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix}.$$

Equivalently,

$$i\dot{f} = \omega_f f + kg, \quad i\dot{g} = \omega_g g + kf.$$

Here ω_f and ω_g are the intrinsic system and contextual frequencies, while $k \in \mathbb{R}$ is a weak coupling parameter satisfying $|k| \ll \omega_f, \omega_g$. This form makes explicit that the system and context are not evolving independently, but are coupled through off-diagonal interaction terms. To extract the physically relevant degrees of freedom, we express both amplitudes in polar form,

$$f = r_f e^{i\theta_f}, \quad g = r_g e^{i\theta_g},$$

and introduce the relative phase

$$\psi = \theta_f - \theta_g.$$

This phase difference encodes the instantaneous alignment between the system and its context. In the present two-mode phase reduction, the real-alignment condition

$$\text{Im}\langle f, g \rangle = \text{Im}(f^* g) = 0$$

is satisfied when the relative phase obeys

$$\psi = 0 \pmod{\pi},$$

corresponding to phase alignment or anti-alignment between the system and its context.

We focus on the near-resonant regime,

$$|\omega_f - \omega_g| \ll \omega_f, \omega_g,$$

where ω_f and ω_g denote the intrinsic frequencies of the system and contextual modes. In this regime, the dynamics naturally separates into fast carrier oscillations and a slow evolution of the relative phase. This separation of timescales is the key that allows a reduced description. Applying standard averaging methods, or equivalently phase-reduction techniques familiar from nonlinear dynamics, and retaining the leading harmonic of the effective phase coupling, one obtains an Adler-type equation for the slow evolution of ψ ,

$$\dot{\psi} = \Delta - K \sin \psi + \eta(t),$$

where $\Delta = \omega_g - \omega_f$ is the detuning between the intrinsic frequencies, and K is an effective coupling strength whose precise form depends on the microscopic interaction and the coarse-graining procedure. The term $\eta(t)$ represents coarse-grained stochastic fluctuations arising from unresolved contextual degrees of freedom, environmental noise, or microscopic complexity within the measurement apparatus. This equation is of Adler type and constitutes the minimal dynamical model for synchronization between the system and its context. It shows explicitly that real alignment is not a discrete postulate, but the dynamical fixed-point structure of a phase evolution equation. The derivation of the Adler-type equation from the underlying two-mode Schrödinger dynamics is given in Appendix A.

2.2 Locked and drifting regimes and effective potential

The Adler-type phase equation exhibits two qualitatively distinct dynamical regimes, governed by the competition between detuning Δ and coupling strength K . These regimes provide a direct dynamical interpretation of measurement and probability within the Real-Alignment framework.

When

$$|\Delta| < K,$$

the system admits a stable fixed point ψ^* satisfying

$$\Delta - K \sin \psi^* = 0, \quad \sin \psi^* = \frac{\Delta}{K}.$$

In this locked regime, the relative phase converges exponentially toward ψ^* , and the system and context become phase-aligned. This synchronized configuration is robust under small perturbations and acts as an attractor of the dynamics. Within the RAI framework, such phase locking can be interpreted as corresponding to the emergence of a definite measurement outcome, reflecting a sustained alignment between the system and its context. In this picture, different measurement outcomes correspond to distinct alignment channels, with the realized outcome determined by the stable phase-locked solution selected by the dynamics.

In contrast, when

$$|\Delta| > K,$$

no stable fixed point exists. The relative phase evolves continuously, and the system fails to maintain synchronization with the context. Instead, $\psi(t)$ drifts across the full

circle, exploring its phase space over time. In this drifting regime, alignment occurs only intermittently, and any assignment of outcomes must be statistical, giving rise naturally to probabilistic behavior.

These regimes also admit a complementary and geometrically intuitive description in terms of an effective potential. The deterministic part of the Adler equation,

$$\dot{\psi} = \Delta - K \sin \psi,$$

can be written in gradient form as

$$\dot{\psi} = -\frac{dV}{d\psi},$$

with the effective potential

$$V(\psi) = -K \cos \psi - \Delta\psi.$$

Here $V(\psi)$ is understood on the lifted phase variable

$$\psi \in \mathbb{R},$$

the universal cover of the phase circle, while the physical phase itself is taken modulo 2π . This is why the detuning term $-\Delta\psi$ produces a tilted washboard potential rather than a strictly periodic potential.

Indeed,

$$-\frac{dV}{d\psi} = \Delta - K \sin \psi,$$

which recovers the deterministic Adler equation. In this representation, the phase dynamics corresponds to overdamped motion in a tilted washboard potential: the cosine term promotes phase alignment, while the linear term introduces a detuning-dependent tilt. The resulting deformation of the potential landscape and the disappearance of stable minima at the critical threshold $|\Delta| = K$ are illustrated in Fig. 1. In the locked regime, the potential contains local minima corresponding to stable synchronized states, and the phase relaxes into one of these minima, producing a definite and repeatable outcome. As $|\Delta|$ increases, the tilt progressively eliminates these minima, leading to unbounded phase evolution in the drifting regime.

In the presence of stochastic fluctuations, the phase undergoes noise-driven motion in this potential landscape, leading to intermittent excursions between regions of alignment. In this way, measurement is associated with the existence of stable attractors in phase space, whereas probabilistic behavior emerges when these attractors are absent and the phase explores its domain ergodically. Taken together, these results show that alignment, discreteness, and probabilistic behavior emerge from a single underlying phase-dynamical mechanism.

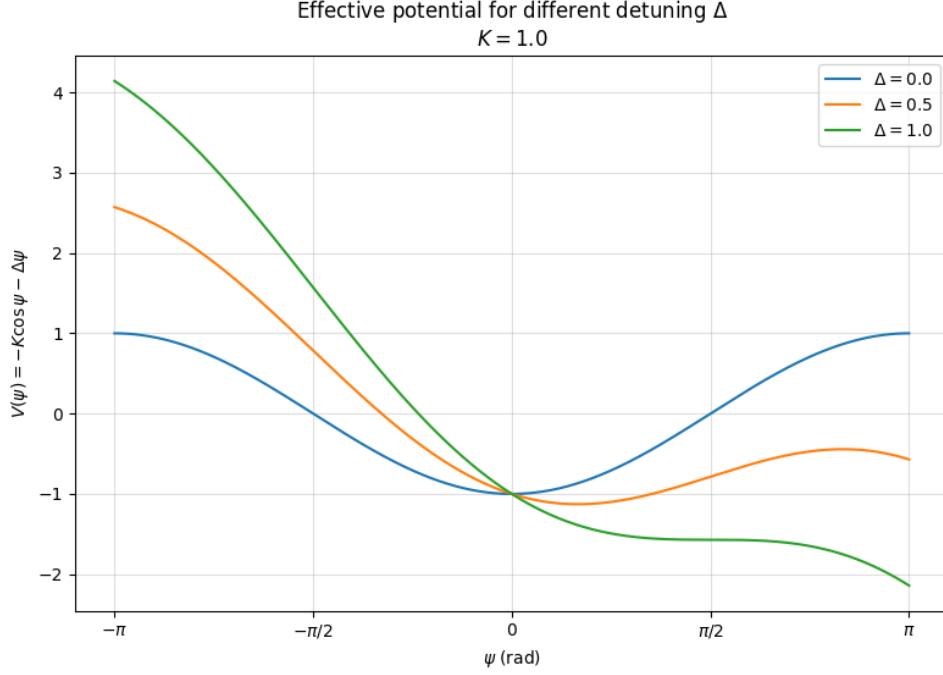


Figure 1: Effective potential $V(\psi) = -K \cos \psi - \Delta \psi$ for different values of detuning Δ , with $K = 1$ used for illustration. For $\Delta = 0$, the potential is symmetric with a minimum at $\psi = 0$, corresponding to exact phase alignment. For $0 < |\Delta| < K$, the potential is tilted and the minimum shifts to $\psi^* = \arcsin(\Delta/K)$, indicating a phase-locked state. At the critical value $|\Delta| = K$, the stable minimum and unstable maximum merge in a saddle-node bifurcation and disappear, signaling the transition to the drifting regime.

2.3 Characteristic event and inter-click times

An important quantity in the locked regime is the timescale over which phase alignment is established. This timescale determines how rapidly a definite measurement outcome emerges within the Real-Alignment framework. It therefore identifies the measurement timescale as an emergent dynamical quantity, rather than an externally imposed parameter.

In the regime $|\Delta| < K$, the Adler equation admits a stable fixed point ψ^* satisfying

$$\Delta - K \sin \psi^* = 0.$$

To quantify the approach to this state, we consider small deviations $\delta(t)$ around the fixed point,

$$\psi(t) = \psi^* + \delta(t), \quad |\delta| \ll 1.$$

Linearizing the phase dynamics yields

$$\dot{\delta} = -K \cos \psi^* \delta,$$

which describes exponential relaxation toward the locked configuration. Using, on the stable branch,

$$\cos \psi^* = \sqrt{1 - \left(\frac{\Delta}{K}\right)^2},$$

we obtain

$$\dot{\delta} = -\sqrt{K^2 - \Delta^2} \delta.$$

Thus, the deviation decays as

$$\delta(t) = \delta_0 e^{-t/\tau_{\text{lock}}},$$

with characteristic locking time

$$\tau_{\text{lock}} = \frac{1}{\sqrt{K^2 - \Delta^2}}.$$

This timescale sets the duration over which the system and context dynamically align. In the RAI interpretation, τ_{lock} can be understood as the characteristic time required for the emergence of a definite measurement event. As the system approaches the critical threshold $|\Delta| \rightarrow K$, the locking time diverges, reflecting the loss of stable phase alignment and the onset of the drifting regime. The derivation of the locking time and its divergence near the critical threshold is given in Appendix B.

In addition to the locking time, one may define the time interval between successive registered alignment events. Let t_n denote the time of the n -th real-alignment event, defined by the crossing condition

$$\psi(t_n) = 0 \pmod{\pi},$$

or, more generally, by entry into a small alignment window around the real-alignment condition. Since real alignment occurs at both $\psi = 0$ and $\psi = \pi$ modulo 2π , we introduce a small window width $\epsilon > 0$ and define

$$W_\epsilon = \{\psi : |\psi| < \epsilon\} \cup \{\psi : |\psi - \pi| < \epsilon\} \pmod{2\pi}.$$

The inter-click time is then

$$\tau_n = t_{n+1} - t_n.$$

In the deterministic drifting regime, where noise is negligible and $|\Delta| > K$, the phase obeys

$$\dot{\psi} = \Delta - K \sin \psi.$$

The time required for the phase to complete one full cycle is

$$T_{\text{drift}} = \int_0^{2\pi} \frac{d\psi}{\Delta - K \sin \psi}.$$

For $|\Delta| > K$, this evaluates to the positive winding period

$$T_{\text{drift}} = \frac{2\pi}{\sqrt{\Delta^2 - K^2}}.$$

For $\Delta < -K$, the phase winds in the opposite direction; the physical period is understood as the absolute winding time, giving the same positive expression.

Since real alignment occurs whenever $\psi = 0 \pmod{\pi}$, there are two alignment crossings per full 2π cycle. Because the phase velocity is generally not symmetric over the two half-cycles, the two individual crossing intervals need not be identical. The cycle-averaged deterministic inter-click time is therefore

$$\bar{\tau}_{\text{click}} = \frac{T_{\text{drift}}}{2} = \frac{\pi}{\sqrt{\Delta^2 - K^2}}.$$

The evaluation of the drift period and the cycle-averaged inter-click time is given in Appendix C.

In the presence of stochastic fluctuations, the inter-click time becomes a random variable. The phase evolves according to

$$\dot{\psi} = \Delta - K \sin \psi + \eta(t), \quad \langle \eta(t) \rangle = 0, \quad \langle \eta(t)\eta(t') \rangle = 2D\delta(t - t').$$

The waiting-time distribution may then be defined as

$$p(\tau) = \langle \delta(\tau - (t_{n+1} - t_n)) \rangle,$$

where the average is taken over noise realizations and initial conditions. Here the δ -symbol in $p(\tau)$ denotes the Dirac delta distribution, not the small deviation $\delta(t)$ used above. Equivalently, if $S(t)$ denotes the survival probability that no alignment event has occurred up to time t , then

$$p(\tau) = -\frac{dS(\tau)}{d\tau}.$$

In regimes where alignment events are rare and approximately independent, the waiting-time distribution approaches an exponential form,

$$p(\tau) = r e^{-r\tau},$$

where r is the effective click rate. The mean inter-click time is then

$$\langle \tau \rangle = \frac{1}{r}.$$

Thus, the model predicts distinct characteristic event-time statistics in different regimes: exponential relaxation toward a locked event in the synchronized regime, regular cycle-averaged inter-click intervals in deterministic drifting motion, and exponential-like waiting times in noise-dominated ergodic regimes.

2.4 Interpretation

The central insight of this framework is that the relative phase ψ is not merely an auxiliary variable, but the primary dynamical quantity governing the relation between system and context. The weak interaction induces a tendency toward phase synchronization, while detuning and stochastic fluctuations oppose it, producing a competition between phase locking and phase drift. The observable behavior of the system is determined by the outcome of this competition.

Within this picture, measurement can be interpreted as a dynamical phase-locking process. A definite outcome corresponds to the stabilization of the relative phase at a stable fixed point ψ^* , where the system and context evolve with a fixed phase relation. This phase-locked configuration is dynamically selected and stable under perturbations, providing a mechanism for the persistence and repeatability of measurement outcomes. In contrast, when no stable fixed point exists, or when stochastic fluctuations dominate, the phase undergoes continuous or noise-driven motion. In this regime, no single aligned configuration is selected, and outcomes must be described statistically.

This interpretation connects quantum measurement directly to synchronization phenomena in nonlinear dynamics. The locking condition

$$|\Delta| < K$$

marks the existence of a stable synchronized state, while the critical threshold

$$|\Delta| = K$$

marks the loss of this state through the disappearance of the stable phase-locked solution. Discrete outcomes therefore emerge when the phase stabilizes in an alignment channel, rather than being imposed as instantaneous collapse events.

The emergence of probability follows from the stochastic phase dynamics. In the presence of noise, the relative phase undergoes diffusion across its domain, and alignment events occur intermittently. Over long times, this leads to effective sampling of phase space, with outcome statistics determined by the stationary or current-carrying distribution of the underlying stochastic dynamics. In this sense, randomness is treated as effective: it arises from phase diffusion, incomplete synchronization, and unresolved contextual degrees of freedom within the reduced model.

More generally, the existence of stable fixed points partitions the phase space into basins of attraction, each corresponding to a distinct alignment channel. The realized outcome is determined by which phase-locked state is selected by the dynamics, while fluctuations and initial conditions influence which basin is reached in a given realization. The inter-click times between successive alignment events therefore carry physical information about the regime of the system: regular cycle-averaged intervals indicate deterministic drift, exponential-like waiting times indicate noise-dominated ergodic behavior, and rapid relaxation indicates stable locking.

Contextuality appears naturally as a dynamical feature rather than an abstract constraint. The contextual state $g(t)$ enters explicitly through the coupling, detuning, noise strength, and alignment channels that shape the phase dynamics. Different measurement setups therefore correspond to different dynamical environments, leading to different alignment conditions and outcome statistics. In this sense, the context is not passive; it actively participates in the selection and stabilization of outcomes.

From this perspective, the wavefunction observed in measurement can be understood as an effective description emerging during phase alignment. When the phase locks, the system and context combine into a phase-coherent composite configuration, providing the operational object described by the usual wavefunction formalism. This does not require abandoning Hilbert-space quantum mechanics; rather, it suggests that the operational wavefunction is recovered in the regime where system–context alignment is dynamically stable.

The quantum-to-classical transition also acquires a natural dynamical interpretation. In the strong-locking regime,

$$K \gg |\Delta|, \quad D \ll K,$$

the phase rapidly converges to a stable fixed point and fluctuations are too weak to destroy the locked state. Outcomes then become effectively deterministic. In contrast, weak coupling or strong noise leads to drifting phases, intermittent alignment, and probabilistic behavior. The classical limit therefore corresponds to robust synchronization across many contextual degrees of freedom.

Taken together, these elements suggest a unified picture in which measurement, discreteness, probability, contextuality, event timing, and classicality all emerge from a single underlying phase-dynamical mechanism. The Real-Alignment framework thus recasts the measurement problem as a problem of synchronization: the emergence, persistence, or loss of phase locking determines whether the system exhibits definite, drifting, or probabilistic behavior.

3 Implications for Quantum Theory

The contextual phase dynamics developed above provides an effective dynamical layer for interpreting measurement, probability, contextuality, and the emergence of classical behavior. The purpose of this section is not to replace the Hilbert-space formalism of quantum mechanics, but to clarify how system–context alignment can generate definite events and statistical outcome frequencies within a reduced phase model. We first interpret measurement as synchronization, then analyze the stochastic phase distribution, channel probabilities, weak-locking limits, effective system–context states, and possible deviations from ideal Born statistics. A full microscopic derivation of detector-specific channel couplings is left for future work.

Throughout this section, the effective phase dynamics is taken to be

$$\dot{\psi} = \Delta - K \sin \psi + \eta(t), \quad \langle \eta(t) \rangle = 0, \quad \langle \eta(t)\eta(t') \rangle = 2D \delta(t - t'),$$

where D is the effective phase-diffusion strength and $\delta(t - t')$ denotes the Dirac delta distribution. The deterministic drift is

$$F(\psi) = \Delta - K \sin \psi.$$

3.1 Measurement as synchronization

Within the Real-Alignment framework, measurement is interpreted as a dynamical synchronization process between the system state $f(t)$ and the contextual state $g(t)$. In the present two-mode phase reduction, the real-alignment condition

$$\text{Im}\langle f, g \rangle = 0$$

is satisfied when the relative phase obeys

$$\psi = 0 \pmod{\pi},$$

so that measurement events correspond to alignment or anti-alignment between system and context.

In the deterministic limit $D = 0$, phase locking occurs when

$$|\Delta| < K.$$

The stable fixed point ψ^* satisfies

$$\Delta - K \sin \psi^* = 0, \quad \sin \psi^* = \frac{\Delta}{K}.$$

Thus the onset of measurement is associated with the existence of a stable synchronized configuration. In the exact-resonance case $\Delta = 0$, this fixed point coincides with exact phase alignment, $\psi^* = 0$. For nonzero detuning, the locked state is shifted but remains dynamically synchronized, representing stable alignment within the corresponding effective alignment channel.

In this regime, the relative phase relaxes toward ψ^* , and the locked configuration provides a dynamically stable condition for a definite outcome. This interpretation links quantum measurement to synchronization phenomena familiar from nonlinear dynamics. Discrete outcomes are not inserted as instantaneous collapses, but arise when the system–context phase relation becomes locked in one of the available alignment channels.

3.2 Stochastic phase dynamics and stationary measure

To describe probabilistic behavior in the drifting and noise-dominated regimes, the deterministic phase equation must be supplemented by stochastic fluctuations. These fluctuations represent unresolved contextual degrees of freedom, environmental noise, or microscopic detector complexity. The resulting stochastic phase dynamics determines the long-time distribution of the relative phase and, consequently, the statistics of alignment events.

Let $P(\psi, t)$ denote the probability density of the relative phase ψ at time t . The stochastic phase equation induces the Fokker–Planck equation

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial \psi} [(\Delta - K \sin \psi)P] + D \frac{\partial^2 P}{\partial \psi^2}.$$

Equivalently,

$$\frac{\partial P}{\partial t} = -\frac{\partial J}{\partial \psi},$$

where the probability current is

$$J(\psi, t) = (\Delta - K \sin \psi)P - D \frac{\partial P}{\partial \psi}.$$

In a stationary state, J is constant. If $\Delta = 0$, detailed balance holds and the stationary current vanishes. In this equilibrium case,

$$P_{\text{st}}(\psi) = \frac{1}{Z} \exp\left(\frac{K \cos \psi}{D}\right), \quad Z = \int_0^{2\pi} \exp\left(\frac{K \cos \psi}{D}\right) d\psi.$$

This distribution is periodic and normalizable on the phase circle.

For $\Delta \neq 0$, the system is a tilted diffusion on the circle and generally has a nonzero stationary current. In that case, the stationary distribution is not simply proportional to

$$\exp\left[\frac{K \cos \psi + \Delta \psi}{D}\right],$$

since this expression is not periodic. A periodic steady-state solution can be written in the standard current-carrying form

$$P_{\text{st}}(\psi) = \frac{1}{\mathcal{N}} \exp\left[-\frac{V(\psi)}{D}\right] \int_{\psi}^{\psi+2\pi} \exp\left[\frac{V(u)}{D}\right] du,$$

where

$$V(\psi) = -\Delta \psi - K \cos \psi, \quad -\frac{dV}{d\psi} = \Delta - K \sin \psi,$$

and \mathcal{N} is fixed by

$$\int_0^{2\pi} P_{\text{st}}(\psi) d\psi = 1.$$

This expression is periodic and accounts for the nonzero probability current induced by detuning. In the zero-detuning limit, it is consistent with the detailed-balance distribution above after normalization.

The long-time statistics of alignment events are therefore determined by the stationary measure of this stochastic phase process. In regimes where the phase explores its domain ergodically, time averages of event frequencies coincide with stationary ensemble averages. The derivation of the corresponding Fokker–Planck equation and the stationary phase distributions is given in Appendix D.

3.3 Dynamical origin of channel probabilities

The phase dynamics described above determines when alignment events occur, but outcome probabilities require specifying how different detector channels respond to those events. To connect phase dynamics with outcome probabilities, consider a set of measurement channels indexed by i . Each channel is associated with a detector mode a_i , occupation

$$n_i = \langle a_i^\dagger a_i \rangle,$$

and absorption rate Γ_i . Let ρ denote the reduced density operator of the system and H_S its system Hamiltonian. For a Markovian detector, the reduced dynamics of a single channel may be modeled by the Lindblad term

$$\dot{\rho} = -\frac{i}{\hbar}[H_S, \rho] + \Gamma_i \mathcal{D}[a_i]\rho,$$

where

$$\mathcal{D}[a_i]\rho = a_i \rho a_i^\dagger - \frac{1}{2}\{a_i^\dagger a_i, \rho\}.$$

In the corresponding quantum-jump description, the instantaneous click rate in channel i is

$$R_i(t) = \Gamma_i \text{Tr}(a_i^\dagger a_i \rho(t)) = \Gamma_i n_i(t).$$

When the contextual phase dynamics is coarse-grained over many microscopic absorption attempts, we denote by \tilde{K}_i the effective alignment strength of channel i , taken to scale with its click rate:

$$\tilde{K}_i \propto R_i \propto \Gamma_i n_i.$$

Let c_i denote the complex amplitude associated with channel i . In the coherent-state or single-mode amplitude approximation, the occupation satisfies

$$n_i \simeq |c_i|^2,$$

and therefore

$$\tilde{K}_i \propto \Gamma_i |c_i|^2.$$

The probability of a registered event in channel i , conditioned on the occurrence of some event, is then

$$P_i = \frac{\tilde{K}_i}{\sum_j \tilde{K}_j}.$$

Using $\tilde{K}_i \propto \Gamma_i |c_i|^2$, this gives the detector-weighted channel probability

$$P_i = \frac{\Gamma_i |c_i|^2}{\sum_j \Gamma_j |c_j|^2}.$$

For detector-symmetric or properly calibrated channels,

$$\Gamma_i = \Gamma \quad \text{for all } i,$$

this reduces to

$$P_i = \frac{|c_i|^2}{\sum_j |c_j|^2}.$$

For a normalized state, $\sum_j |c_j|^2 = 1$, and therefore

$$P_i = |c_i|^2.$$

Thus, the Born rule is recovered under explicit assumptions: ergodic phase sampling, common alignment criteria, and detector-symmetric or properly calibrated channel response. This phase-dynamical model therefore supplies an explicit mechanism for the ergodic dominance-region weighting used in the geometric formulation of RAI.

If the detector rates are not equal, the channel probabilities remain detector-weighted:

$$P_i = \frac{\Gamma_i |c_i|^2}{\sum_j \Gamma_j |c_j|^2},$$

which represents a detector-biased distribution rather than the ideal Born rule. This distinction is important: ideal Born statistics require either symmetric detector coupling or calibration that removes channel-dependent efficiencies.

3.4 Weak-locking expansion and alignment windows

The preceding channel argument can also be expressed in terms of alignment windows in phase space. Let W_i denote the alignment region associated with channel i , and let $P_{\text{st}}^{(i)}(\psi)$ denote the stationary phase distribution associated with the effective dynamics of that channel, including its channel-dependent coupling \tilde{K}_i . The event probability is

$$P_i = \frac{\int_{W_i} P_{\text{st}}^{(i)}(\psi) d\psi}{\sum_j \int_{W_j} P_{\text{st}}^{(j)}(\psi) d\psi}.$$

In the weak-locking regime $\tilde{K}_i/D \ll 1$, and for equal window widths and equal background rates, the leading channel-dependent contribution is proportional to \tilde{K}_i . Therefore,

$$P_i \simeq \frac{\tilde{K}_i}{\sum_j \tilde{K}_j}.$$

Together with $\tilde{K}_i \propto \Gamma_i |c_i|^2$, this gives the normalized channel probability obtained above.

This form avoids treating the uniform background phase density as a channel probability. Only the channel-dependent alignment contribution is counted as a registered event. In this way, null intervals or non-detection periods are naturally separated from physical channel clicks.

3.5 Effective system–context state

During phase locking, the system and context may be represented by an effective coherent composite state

$$\Psi_{\text{eff}} = f + \alpha g,$$

where α is a slowly varying coefficient determined by the coupling and relative phase. For simplicity, α may be taken real, or its phase may be absorbed into the definition of g . When $\psi \approx 0$, the two components share a common phase direction and one obtains

$$\Psi_{\text{eff}} \simeq (r_f + \alpha r_g) e^{i\theta_f}.$$

This construction should be understood as an effective description rather than an independent postulate. It expresses the fact that, in a locked regime, the system and context behave as a single phase-coherent composite. The standard wavefunction description is therefore recovered operationally in the regime where system–context alignment is stable.

A full derivation of Schrödinger evolution for Ψ_{eff} requires specifying the microscopic dynamics of f , g , and α . In the present framework, Ψ_{eff} is best regarded as the coarse-grained state observed once alignment has been established.

3.6 Classical limit as strong locking

The quantum-to-classical transition acquires a dynamical interpretation in the strong-locking regime. When

$$K \gg |\Delta|, \quad D \ll K,$$

the stationary distribution becomes sharply concentrated near the stable fixed point ψ^* . The phase relaxation time

$$\tau_{\text{lock}} = \frac{1}{\sqrt{K^2 - \Delta^2}}$$

is short, and the system rapidly settles into a stable aligned configuration.

In this regime, outcomes are effectively deterministic because phase diffusion is too weak to destroy the locked state. Macroscopic classicality can therefore be interpreted as the collective stabilization of phase alignment across many degrees of freedom. Conversely, weak coupling or large noise produces drifting phases, intermittent alignment, and probabilistic behavior.

3.7 Ergodicity, partial locking, and deviations from Born statistics

The recovery of Born statistics depends on ergodic sampling of the relevant alignment regions and on detector-symmetric channel response. If either condition fails, deviations may occur.

Nonergodic contextual dynamics can bias the time spent in different alignment regions, so that observed frequencies need not agree with the stationary-window prediction, where $P_{\text{st}}(\psi)$ denotes the stationary phase distribution for the relevant effective dynamics:

$$P_i^{\text{obs}} \neq \frac{\int_{W_i} P_{\text{st}}(\psi) d\psi}{\sum_j \int_{W_j} P_{\text{st}}(\psi) d\psi}.$$

Similarly, unequal detector couplings give

$$P_i = \frac{\Gamma_i |c_i|^2}{\sum_j \Gamma_j |c_j|^2},$$

rather than the ideal Born expression. Partial locking may also produce temporal correlations between successive events, which would appear as deviations in waiting-time distributions or autocorrelation functions.

These effects provide potential empirical tests of the framework. Controlled variation of K , Δ , and D should change the transition between locked, drifting, and partially locked regimes.

3.8 Quantum randomness as emergent

In this model, randomness is an emergent consequence of deterministic drift combined with environmental phase diffusion. The deterministic term $-K \sin \psi$ drives the phase toward alignment, while the stochastic term $\eta(t)$ induces diffusion across the phase circle and between alignment regions.

Individual outcomes may be unpredictable because microscopic contextual fluctuations are unresolved, but their statistics are governed by the stationary and ergodic properties of the phase dynamics. Randomness is therefore treated as effective within the present reduced model.

This interpretation is analogous in spirit to statistical mechanics: the underlying dynamics may be deterministic or only weakly stochastic, but limited access to microscopic contextual degrees of freedom produces effectively probabilistic event statistics. Thus, the apparent randomness of measurement outcomes reflects the coarse-grained behavior of the system–context phase dynamics rather than an independent postulate.

3.9 Contextuality as a dynamical variable

In RAI, contextuality is represented by the explicit dynamical state $g(t)$, rather than by a static logical constraint alone. The contextual state determines the detuning, coupling, noise strength, and alignment channels that enter the effective phase equation. Thus different measurement arrangements correspond to different dynamical environments and may lead to different alignment statistics.

This provides a concrete dynamical meaning to contextuality: outcomes depend not only on the system state, but also on the evolving measurement context. The same system state may therefore lead to different alignment behavior when coupled to different contextual states. Contextuality is not merely a restriction on value assignments; it becomes a time-dependent physical relation between system and apparatus.

For multipartite systems, a rigorous account would require extending the present two-mode model to coupled networks of contextual phases. Such an extension may describe correlations through shared or interacting contextual degrees of freedom, but this remains a subject for future work.

3.10 Collapse and decoherence as dynamical regimes

Within the present framework, apparent collapse and decoherence can be interpreted as different regimes of the same underlying phase dynamics. Collapse-like behavior corresponds to entry into a locked region, where the phase remains near a stable fixed point and a definite outcome is recorded. Decoherence-like behavior corresponds to diffusion or dephasing among contextual alignment regions under stochastic forcing.

This statement is interpretive rather than a replacement for standard decoherence theory. A full derivation of environmental decoherence would require tracing over many contextual degrees of freedom. The present model provides a reduced phase-level description of how such loss of coherence may appear in the alignment dynamics.

The value of this viewpoint is that it places outcome formation and loss of phase coherence within a common dynamical language. Both are controlled by the same variables K , Δ , and D : strong coupling favors stable locking, while noise and detuning favor diffusion, dephasing, and loss of persistent alignment.

3.11 Experimental consequences

At the level of the reduced phase-dynamical model, several testable signatures are suggested:

- threshold behavior as the ratio $K/|\Delta|$ crosses the locking condition;
- changes in waiting-time distributions as the noise strength D is varied;
- temporal correlations between events in partially locked regimes;
- detector-dependent deviations of the form

$$P_i = \frac{\Gamma_i |c_i|^2}{\sum_j \Gamma_j |c_j|^2}$$

when channel efficiencies are not calibrated;

- possible departures from Born statistics when contextual phase exploration is nonergodic.

Such effects may be probed in controlled quantum platforms where coupling, detuning, and noise can be engineered, including superconducting qubits, trapped ions, photonic systems, and optomechanical resonators.

A particularly direct test would be to measure how event rates and waiting-time distributions change as the effective detuning or noise strength is varied. The model predicts a qualitative change between locked, drifting, and noise-dominated regimes, so deviations from standard uncorrelated click statistics could provide evidence for underlying contextual phase dynamics.

3.12 Structural implications

The framework suggests a two-level structure for quantum theory:

1. *Kinematic layer*: the Hilbert-space structure, superposition principle, and linear operators remain intact.
2. *Dynamical layer*: the contextual phase dynamics determines when events occur and how outcome frequencies are generated.

In this view, standard quantum probabilities are recovered when the dynamical layer satisfies the appropriate ergodic and detector-symmetry conditions. Deviations from these conditions provide possible empirical windows into the underlying contextual dynamics.

This separation clarifies the role of the proposed model. It does not replace the usual Hilbert-space formalism, but supplements it with a dynamical account of event formation. The kinematic layer specifies the possible outcomes and amplitudes, while the dynamical layer determines how real-alignment events occur in time.

4 Discussion

The reduced phase model developed in this work provides a compact dynamical description of measurement within the Real-Alignment Interpretation. Its central advantage is that it makes the system–context relation explicit. The system is not treated as an isolated object whose measurement outcome is imposed externally; rather, it is described as phase-coupled to a contextual degree of freedom, with the relative phase ψ determining whether real alignment can occur.

The Adler-type equation

$$\dot{\psi} = \Delta - K \sin \psi + \eta(t)$$

captures the essential competition between detuning, coupling, and stochastic contextual fluctuations. When the coupling dominates the detuning, $|\Delta| < K$, the dynamics admits a stable locked solution. In this regime, the system and context relax toward a fixed phase relation, and the measurement process can be interpreted as the emergence of a stable real-aligned configuration. The associated locking time,

$$\tau_{\text{lock}} = \frac{1}{\sqrt{K^2 - \Delta^2}},$$

provides an intrinsic dynamical timescale for outcome formation. Its divergence as $|\Delta| \rightarrow K$ reflects the loss of stable alignment at the transition to the drifting regime.

The drifting regime, $|\Delta| > K$, is equally important. In this case no stable fixed point exists, and the phase evolves around the circle. Real-alignment events then occur intermittently, and the relevant observable quantity becomes the inter-click time between successive alignment events. In the deterministic drifting regime, the cycle-averaged inter-click time is

$$\bar{\tau}_{\text{click}} = \frac{\pi}{\sqrt{\Delta^2 - K^2}},$$

while in the presence of stochastic fluctuations the waiting-time distribution becomes a statistical object. This provides a concrete route by which probabilistic behavior can emerge from the same underlying phase dynamics.

The effective-potential formulation further clarifies the structure of the model. With the convention

$$\dot{\psi} = \Delta - K \sin \psi,$$

the corresponding potential is

$$V(\psi) = -K \cos \psi - \Delta \psi,$$

so that

$$-\frac{dV}{d\psi} = \Delta - K \sin \psi.$$

The dynamics is therefore equivalent to overdamped motion in a tilted washboard potential. In the locked regime, stable minima correspond to synchronized configurations. At the critical point $|\Delta| = K$, the stable minimum and unstable maximum merge and disappear, giving rise to phase drift. This potential picture provides a useful geometric interpretation of the transition between definite and probabilistic behavior.

A key implication of the model is that probabilities are not introduced as primitive objects at the level of individual events. Instead, they arise from stochastic phase dynamics,

ergodic sampling, and detector-channel response. Under the assumptions of ergodic phase exploration, common alignment criteria, and detector-symmetric or calibrated channels, the channel probability reduces to the Born form,

$$P_i = |c_i|^2.$$

If these assumptions are relaxed, the model allows detector-weighted probabilities of the form

$$P_i = \frac{\Gamma_i |c_i|^2}{\sum_j \Gamma_j |c_j|^2},$$

where Γ_i represents the response strength of the i -th detector channel. This distinction is important because it separates ideal Born statistics from detector-biased or nonergodic regimes and suggests possible empirical tests.

The model is intentionally minimal. It does not claim to describe the full microscopic complexity of realistic measurement apparatuses, which involve many environmental degrees of freedom, internal detector dynamics, and possible memory effects. Its value lies in isolating the basic mechanism: weak coupling, detuning, synchronization, stochastic phase diffusion, and real-alignment events. This makes the framework analytically transparent while still allowing experimentally meaningful quantities such as locking times, inter-click times, waiting-time distributions, and deviations from Born statistics to be identified.

Several extensions are natural. First, multi-channel measurement contexts should be modeled explicitly, with each outcome channel associated with its own effective coupling, detuning, and noise strength. Second, open-system derivations should be developed to connect the phenomenological noise term $\eta(t)$ and diffusion constant D to microscopic bath parameters. Third, detector-specific models are needed to relate the effective coupling K and channel response Γ_i to physical apparatus properties. Finally, networks of coupled contextual phases may provide a route toward describing multipartite correlations and the emergence of macroscopic classicality through collective synchronization. To make these implications explicit, Table 1 summarizes the main conceptual consequences of the phase-dynamical formulation.

Table 1: Conceptual comparison between standard quantum mechanics and the Real-Alignment Interpretation with contextual phase dynamics.

Aspect	Standard QM	RAI with contextual dynamics
Born rule	Postulated	Emergent under ergodic and detector-symmetric conditions
Measurement	Collapse postulate	Phase-locking event
Randomness	Fundamental or postulated	Effective, from phase diffusion and unresolved context
Quantum-classical divide	Axiomatic or decoherence-based	Strong-locking limit
Contextuality	Logical/operational constraint	Dynamical system-context relation
Collapse/decoherence	Separate descriptions	Interpretable as regimes of phase dynamics

Thus, the present model should be viewed as a first dynamical layer beneath the Real-Alignment Interpretation. It preserves the Hilbert-space structure of quantum mechanics while adding an explicit phase-dynamical mechanism for when events occur, how outcomes stabilize, and how statistical behavior emerges.

5 Conclusion

We have developed a minimal phase-dynamical model for quantum measurement within the Real-Alignment Interpretation. Starting from the system–context relation, the measurement process is described by an Adler-type equation for the relative phase,

$$\dot{\psi} = \Delta - K \sin \psi + \eta(t),$$

which captures the competition between detuning, coupling, and stochastic contextual fluctuations.

The model identifies two central dynamical regimes. In the locked regime, $|\Delta| < K$, the relative phase relaxes toward a stable fixed point, producing persistent system–context alignment and providing a dynamical mechanism for definite outcomes. In the drifting regime, $|\Delta| > K$, stable alignment is lost and real-alignment events occur intermittently, giving rise to waiting-time statistics and effective probabilistic behavior.

The effective potential

$$V(\psi) = -K \cos \psi - \Delta \psi$$

shows geometrically that the transition between these regimes corresponds to the disappearance of stable minima in a tilted washboard potential. This connects definite outcomes with stable phase locking and probabilistic behavior with drift, diffusion, and incomplete synchronization.

The framework also clarifies how contextuality enters dynamically. The contextual state $g(t)$ actively shapes the coupling, detuning, noise, and alignment channels that determine outcome statistics. Under ergodic sampling and detector-symmetric or calibrated response, the channel probabilities reduce to the Born rule; when these conditions fail, the model allows possible deviations through nonergodic phase exploration, partial locking, or detector-dependent response.

Thus, within the present reduced model, measurement, discreteness, probability, contextuality, event timing, and the classical limit can be understood as consequences of a single phase-dynamical mechanism. The Real-Alignment Interpretation therefore recasts the measurement problem as a synchronization problem: definite outcomes correspond to stable phase locking, while probabilistic behavior arises from drift, diffusion, and unresolved contextual dynamics.

Future work should derive the effective parameters K , Δ , and D from microscopic open-system models, extend the theory to multi-channel and multipartite contexts, and compare predicted locking thresholds and waiting-time distributions with controlled quantum experiments.

A Derivation of the Adler-Type Phase Equation

This appendix derives the reduced contextual phase equation used in the main text. The goal is to show how an Adler-type equation for the relative phase can arise from a weakly coupled two-mode Schrödinger model after phase reduction and coarse-graining.

A.1 Two-mode Schrödinger model

We consider two coupled complex amplitudes $f(t)$ and $g(t)$, interpreted as the system and contextual modes. The state vector is

$$\Psi(t) = \begin{pmatrix} f(t) \\ g(t) \end{pmatrix}, \quad f(t), g(t) \in \mathbb{C}.$$

We take the Hamiltonian to be

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}},$$

with

$$\hat{H}_0 = \hbar \begin{pmatrix} \omega_f & 0 \\ 0 & \omega_g \end{pmatrix}, \quad \hat{H}_{\text{int}} = \hbar k \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Here ω_f and ω_g are the free frequencies, and k is a real coupling constant with units of inverse time. The Schrödinger equation

$$i\hbar \frac{d}{dt} \Psi(t) = \hat{H} \Psi(t)$$

gives the component equations

$$i\dot{f} = \omega_f f + kg, \tag{1}$$

$$i\dot{g} = \omega_g g + kf. \tag{2}$$

We write both amplitudes in polar form,

$$f = r_f e^{i\theta_f}, \quad g = r_g e^{i\theta_g},$$

with $r_f, r_g \geq 0$, and define the relative phase

$$\psi = \theta_f - \theta_g.$$

A.2 Interaction expectation value

The interaction expectation value is

$$\langle \hat{H}_{\text{int}} \rangle = \Psi^\dagger \hat{H}_{\text{int}} \Psi.$$

Since

$$\hat{H}_{\text{int}} \Psi = \hbar k \begin{pmatrix} g \\ f \end{pmatrix},$$

we obtain

$$\langle \hat{H}_{\text{int}} \rangle = \hbar k (f^* g + g^* f).$$

Using the polar forms,

$$f^* g + g^* f = r_f r_g (e^{i(\theta_g - \theta_f)} + e^{-i(\theta_g - \theta_f)}) = 2r_f r_g \cos(\theta_g - \theta_f).$$

Since $\theta_g - \theta_f = -\psi$, this becomes

$$\langle \hat{H}_{\text{int}} \rangle = 2\hbar k r_f r_g \cos \psi.$$

The numerical factor 2 arises directly from $f^*g + g^*f$. It may be absorbed into the definition of an effective coupling constant, depending on convention.

A.3 Exact amplitude and phase equations

Differentiate

$$f = r_f e^{i\theta_f}, \quad g = r_g e^{i\theta_g},$$

to obtain

$$\dot{f} = (\dot{r}_f + i r_f \dot{\theta}_f) e^{i\theta_f}, \quad \dot{g} = (\dot{r}_g + i r_g \dot{\theta}_g) e^{i\theta_g}.$$

For the f -equation,

$$i(\dot{r}_f + i r_f \dot{\theta}_f) e^{i\theta_f} = \omega_f r_f e^{i\theta_f} + k r_g e^{i\theta_g}.$$

Dividing by $e^{i\theta_f}$ gives

$$i\dot{r}_f - r_f \dot{\theta}_f = \omega_f r_f + k r_g e^{i(\theta_g - \theta_f)}.$$

Since $\theta_g - \theta_f = -\psi$,

$$e^{i(\theta_g - \theta_f)} = \cos \psi - i \sin \psi.$$

Equating real and imaginary parts yields

$$\dot{r}_f = -k r_g \sin \psi, \tag{3}$$

$$\dot{\theta}_f = -\omega_f - k \frac{r_g}{r_f} \cos \psi. \tag{4}$$

Similarly, the g -equation gives

$$i(\dot{r}_g + i r_g \dot{\theta}_g) e^{i\theta_g} = \omega_g r_g e^{i\theta_g} + k r_f e^{i\theta_f}.$$

Dividing by $e^{i\theta_g}$ and using

$$e^{i(\theta_f - \theta_g)} = e^{i\psi} = \cos \psi + i \sin \psi,$$

we obtain

$$\dot{r}_g = k r_f \sin \psi, \tag{5}$$

$$\dot{\theta}_g = -\omega_g - k \frac{r_f}{r_g} \cos \psi. \tag{6}$$

Thus the exact amplitude equations are

$$\dot{r}_f = -k r_g \sin \psi, \tag{7}$$

$$\dot{r}_g = k r_f \sin \psi, \tag{8}$$

and the exact phase equations are

$$\dot{\theta}_f = -\omega_f - k \frac{r_g}{r_f} \cos \psi, \tag{9}$$

$$\dot{\theta}_g = -\omega_g - k \frac{r_f}{r_g} \cos \psi. \tag{10}$$

A.4 Population imbalance and exact relative-phase equation

Define the amplitude imbalance

$$N = r_f^2 - r_g^2.$$

Then

$$\dot{N} = 2r_f\dot{r}_f - 2r_g\dot{r}_g.$$

Using the amplitude equations,

$$\dot{N} = 2r_f(-kr_g \sin \psi) - 2r_g(kr_f \sin \psi),$$

so that

$$\dot{N} = -4kr_f r_g \sin \psi.$$

The relative phase satisfies

$$\dot{\psi} = \dot{\theta}_f - \dot{\theta}_g.$$

Using the phase equations,

$$\dot{\psi} = -\omega_f + \omega_g - k \left(\frac{r_g}{r_f} - \frac{r_f}{r_g} \right) \cos \psi.$$

Let

$$\Delta\omega = \omega_f - \omega_g.$$

Then

$$\dot{\psi} = -\Delta\omega - k \left(\frac{r_g}{r_f} - \frac{r_f}{r_g} \right) \cos \psi.$$

Since

$$\frac{r_g}{r_f} - \frac{r_f}{r_g} = \frac{r_g^2 - r_f^2}{r_f r_g} = -\frac{N}{r_f r_g},$$

we obtain the exact relative-phase equation

$$\dot{\psi} = -\Delta\omega + k \frac{N}{r_f r_g} \cos \psi.$$

A.5 Adiabatic elimination and phase reduction

To obtain a closed phase equation, we eliminate the amplitude imbalance N . Physically, N may relax because of weak dissipation, unresolved contextual degrees of freedom, detector coupling, or environmental damping. We model this by adding a phenomenological relaxation rate $\Gamma > 0$:

$$\dot{N} = -\Gamma N - 4kr_f r_g \sin \psi.$$

In the adiabatic regime, N relaxes faster than the slow relative phase. Setting $\dot{N} \simeq 0$, we obtain

$$N \simeq -\frac{4k}{\Gamma} r_f r_g \sin \psi.$$

Substituting this into the exact phase equation gives

$$\dot{\psi} \simeq -\Delta\omega + k \frac{1}{r_f r_g} \left(-\frac{4k}{\Gamma} r_f r_g \sin \psi \right) \cos \psi.$$

Therefore,

$$\dot{\psi} \simeq -\Delta\omega - \frac{4k^2}{\Gamma} \sin \psi \cos \psi.$$

Let

$$C = \frac{4k^2}{\Gamma}.$$

Then

$$\dot{\psi} = -\Delta\omega - C \sin \psi \cos \psi.$$

Using

$$\sin \psi \cos \psi = \frac{1}{2} \sin(2\psi),$$

the explicit adiabatic elimination generates a second-harmonic phase coupling. At the level of reduced phase dynamics, however, the effective phase force may generally be represented by a periodic odd coupling function,

$$F(\psi) = \sum_{m \geq 1} K_m \sin(m\psi).$$

Retaining only the dominant first harmonic after coarse-graining yields the Adler-type normal form. With the sign convention

$$\Delta = -\Delta\omega, \quad K > 0,$$

we obtain

$$\dot{\psi} = \Delta - K \sin \psi.$$

Including unresolved contextual fluctuations gives

$$\dot{\psi} = \Delta - K \sin \psi + \eta(t).$$

The effective coupling K is therefore determined by the microscopic coupling and the relaxation/coarse-graining process. In the simplest adiabatic reduction, the effective coupling scale is set by

$$K_{\text{eff}} \sim \frac{4k^2}{\Gamma},$$

up to numerical factors and harmonic-renormalization effects introduced by the coarse-graining.

A.6 Regime of validity

The Adler-type reduction rests on the following assumptions:

1. *Weak coupling:* $k \ll \omega_f, \omega_g$, so that the fast carrier oscillations remain well defined.
2. *Near resonance:* $|\omega_f - \omega_g| \ll \omega_f, \omega_g$, so that the relative phase evolves slowly.
3. *Nonzero mode amplitudes:* $r_f r_g \neq 0$, so that the relative phase and the ratios appearing in the phase equations are well defined.
4. *Fast amplitude relaxation:* the imbalance N relaxes on a timescale shorter than the phase drift.

5. *Leading-harmonic approximation*: the slow phase force is dominated by its first harmonic.

Under these assumptions, the system–context phase dynamics is consistently reduced to

$$\dot{\psi} = \Delta - K \sin \psi + \eta(t).$$

B Locking Time and Critical Slowing Down

In the deterministic locked regime,

$$|\Delta| < K,$$

the Adler equation

$$\dot{\psi} = \Delta - K \sin \psi$$

has a stable fixed point ψ^* satisfying

$$0 = \Delta - K \sin \psi^*.$$

Hence

$$\sin \psi^* = \frac{\Delta}{K}.$$

To derive the locking time, perturb the phase around the fixed point:

$$\psi(t) = \psi^* + \delta(t), \quad |\delta| \ll 1.$$

Substituting into the Adler equation gives

$$\dot{\delta} = \Delta - K \sin(\psi^* + \delta).$$

Expanding to first order,

$$\sin(\psi^* + \delta) \simeq \sin \psi^* + \delta \cos \psi^*.$$

Therefore,

$$\dot{\delta} = \Delta - K \sin \psi^* - K \cos \psi^* \delta.$$

Since

$$\Delta - K \sin \psi^* = 0,$$

we obtain

$$\dot{\delta} = -K \cos \psi^* \delta.$$

Using, on the stable branch,

$$\cos \psi^* = \sqrt{1 - \sin^2 \psi^*} = \sqrt{1 - \left(\frac{\Delta}{K}\right)^2},$$

we find

$$K \cos \psi^* = \sqrt{K^2 - \Delta^2}.$$

Thus

$$\dot{\delta} = -\sqrt{K^2 - \Delta^2} \delta.$$

The solution is

$$\delta(t) = \delta_0 \exp[-t/\tau_{\text{lock}}],$$

with

$$\tau_{\text{lock}} = \frac{1}{\sqrt{K^2 - \Delta^2}}.$$

As $|\Delta| \rightarrow K$, the denominator tends to zero and the locking time diverges:

$$\tau_{\text{lock}} \rightarrow \infty.$$

This divergence is the critical slowing down associated with the loss of the stable locked state at the transition to the drifting regime.

C Drifting Period and Inter-Click Time

In the deterministic drifting regime,

$$|\Delta| > K,$$

the Adler equation

$$\dot{\psi} = \Delta - K \sin \psi$$

has no stable fixed point. The phase therefore advances around the circle. The time required for one full 2π cycle is

$$T_{\text{drift}} = \int_0^{2\pi} \frac{d\psi}{\Delta - K \sin \psi}.$$

For $\Delta > K$, the standard integral

$$\int_0^{2\pi} \frac{d\psi}{\Delta - K \sin \psi} = \frac{2\pi}{\sqrt{\Delta^2 - K^2}}$$

gives the positive time for one full winding in the increasing- ψ direction. For $\Delta < -K$, the phase winds in the opposite direction; the physical period is the absolute winding time and is again

$$T_{\text{drift}} = \frac{2\pi}{\sqrt{\Delta^2 - K^2}}.$$

Thus the formula holds as a positive period for all $|\Delta| > K$.

Real alignment occurs at

$$\psi = 0 \pmod{\pi},$$

so that there are two alignment crossings per full 2π cycle. However, because the phase velocity

$$\dot{\psi} = \Delta - K \sin \psi$$

is generally not symmetric over the two half-cycles, the two individual crossing intervals need not be identical. Therefore, the relevant deterministic inter-click time used in the main text is the cycle-averaged inter-click time,

$$\bar{\tau}_{\text{click}} = \frac{T_{\text{drift}}}{2} = \frac{\pi}{\sqrt{\Delta^2 - K^2}}.$$

This expression is valid only in the deterministic drifting regime $|\Delta| > K$. In the locked regime, the appropriate timescale is instead the locking time

$$\tau_{\text{lock}} = \frac{1}{\sqrt{K^2 - \Delta^2}}.$$

D Stationary Phase Distribution

We consider the stochastic Adler equation

$$\dot{\psi} = \Delta - K \sin \psi + \eta(t),$$

with

$$\langle \eta(t) \rangle = 0, \quad \langle \eta(t) \eta(t') \rangle = 2D \delta(t - t').$$

Equivalently, in Itô form,

$$d\psi = (\Delta - K \sin \psi) dt + \sqrt{2D} dW_t.$$

The corresponding Fokker–Planck equation for the phase density $P(\psi, t)$ is

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial \psi} [(\Delta - K \sin \psi)P] + D \frac{\partial^2 P}{\partial \psi^2}.$$

This can be written as a continuity equation,

$$\frac{\partial P}{\partial t} = -\frac{\partial J}{\partial \psi},$$

where the probability current is

$$J(\psi, t) = (\Delta - K \sin \psi)P - D \frac{\partial P}{\partial \psi}.$$

D.1 Zero-detuning case

When $\Delta = 0$, detailed balance holds and the stationary current vanishes:

$$J = 0.$$

Thus,

$$(-K \sin \psi)P_{\text{st}} - D \frac{dP_{\text{st}}}{d\psi} = 0.$$

This gives

$$\frac{1}{P_{\text{st}}} \frac{dP_{\text{st}}}{d\psi} = -\frac{K}{D} \sin \psi.$$

Integrating,

$$\ln P_{\text{st}} = \frac{K}{D} \cos \psi + \text{const.}$$

Hence

$$P_{\text{st}}(\psi) = \frac{1}{Z} \exp\left(\frac{K \cos \psi}{D}\right),$$

with

$$Z = \int_0^{2\pi} \exp\left(\frac{K \cos \psi}{D}\right) d\psi.$$

D.2 Nonzero detuning and current-carrying steady state

For $\Delta \neq 0$, the phase dynamics corresponds to diffusion in a tilted washboard potential. With the convention

$$\dot{\psi} = \Delta - K \sin \psi,$$

the effective potential is

$$V(\psi) = -K \cos \psi - \Delta \psi,$$

so that

$$-\frac{dV}{d\psi} = \Delta - K \sin \psi.$$

Because $V(\psi)$ contains the linear term $-\Delta\psi$, it is not periodic when $\Delta \neq 0$. Therefore the naive zero-current expression

$$P(\psi) \propto \exp \left[-\frac{V(\psi)}{D} \right] = \exp \left(\frac{K \cos \psi + \Delta \psi}{D} \right)$$

is not a periodic probability density on the phase circle. Instead, the stationary state generally has a nonzero constant current.

A periodic current-carrying stationary solution may be written as

$$P_{\text{st}}(\psi) = \frac{1}{\mathcal{N}} \exp \left[-\frac{V(\psi)}{D} \right] \int_{\psi}^{\psi+2\pi} \exp \left[\frac{V(u)}{D} \right] du,$$

where \mathcal{N} is fixed by normalization,

$$\int_0^{2\pi} P_{\text{st}}(\psi) d\psi = 1.$$

This expression is periodic on the circle and accounts for the nonzero probability current induced by detuning.

Thus, for $\Delta = 0$, the phase distribution reduces to an equilibrium-like detailed-balance form, while for $\Delta \neq 0$, the appropriate stationary state is a current-carrying distribution. This distinction is important for interpreting long-time alignment statistics in the drifting and weak-locking regimes.

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