Is the Electron Dressed? The Aharonov–Bohm Effect Challenges Dirac's Proposal

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn

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Abstract

The dressing field method (DFM), introduced by Dirac in 1955, constructs gauge-invariant matter fields by attaching a Wilson line to the bare field. This dressed field is often interpreted as the physical electron. In this paper we provide a rigorous analysis of Dirac's DFM. We begin by defining the dressed field and the associated dressed electromagnetic potential, and we examine their path-dependence and multi-valuedness in multiply-connected spaces. We then derive the exact Dirac equation satisfied by the dressed field, as well as its non-relativistic limit, the Schrödinger equation. The probabilistic interpretation (Born rule) is stated in terms of the dressed field. Next, we apply the theory to the Aharonov–Bohm (AB) effect. Using explicit computations, we show that the interference pattern predicted by the dressed field is independent of the magnetic flux Φ, in direct contradiction with the experimentally observed AB phase shift $e^{-ie\Phi}$. We trace the mathematical origin of this failure to the exact cancellation of topological phases between the dressing factor and the bare wave function, which follows from Stokes' theorem and the multi-valuedness of the Wilson line in a multiply-connected space. Finally, we discuss the implications: the dressed field cannot serve as a complete physical description of the electron. The Dirac DFM is therefore not equivalent to standard quantum mechanics in topologically non-trivial backgrounds.

1 Introduction

In 1955, Dirac proposed an elegant way to construct gauge-invariant matter fields in quantum electrodynamics (QED) [4]. The idea is simple: attach a "dressing" factor—a Wilson line—to the bare electron field,

$$\psi_P(x) = \exp\left(-ie\int_{\Gamma_x} A_\mu\, dy^\mu\right)\psi(x),$$

where Γ_x is a reference path from a fixed base point to x. The resulting dressed field ψ_P is gauge invariant up to a global phase, and therefore seems to represent the physical electron.

This Dirac dressing field method (DFM) has since been widely used, from the study of infrared divergences to the description of confinement [6, 2, 5, 9, 3] (see also [8]). Implicit in this approach is the claim that the gauge potential $\mathbf{A}$ can be "absorbed" into the matter field, leaving a manifestly gauge-invariant description of charged particles.

But is the electron really dressed in this way? The Aharonov–Bohm (AB) effect [1] provides a decisive test. In the AB setup, an electron beam is split and recombined around a solenoid that contains a magnetic flux Φ. Even though the electrons never enter the magnetic field region, their interference pattern exhibits a phase shift $e^{-ie\Phi}$. This phase is a topological effect: it depends only on the gauge potential's holonomy around the solenoid. If the dressing method truly eliminates the gauge potential from the physical description, then the dressed electron should not see the flux at all—or so one might think.

In this paper we perform a rigorous analysis of Dirac's DFM as a self-contained physical theory. We carefully define the dressed field, examine its path-dependence and multi-valuedness in the multiply-connected space of the AB setup, and derive the exact Dirac equation that it satisfies. We then apply the method to the AB effect, computing the interference pattern predicted for the dressed electron. The result is stark: the pattern is independent of the flux Φ, which is in direct contradiction with the experimentally observed shift $e^{-ie\Phi}$.

We trace the mathematical origin of this failure to an exact cancellation of topological phases. The dressing factor itself becomes multi-valued in a multiply-connected space, with different branches differing by the holonomy $e^{-ie\Phi}$. Meanwhile, the bare wave function also carries the opposite holonomy from the physical paths. When the two branches are added at the detector, these holonomies cancel precisely, leaving no flux-dependent signal. This cancellation is not an oversight; it is inherent in the Dirac construction whenever the reference path is chosen in a field-free region.

Our result is a rigorous disproof of the claim that the Dirac DFM can serve as a complete description of the physical electron. It suggests that the gauge potential $\mathbf{A}$ retains irreducible topological information that cannot be eliminated by any local dressing procedure. Consequently, any reformulation of QED based on dressed fields must be restricted to simply-connected spacetimes; it fails for phenomena like the AB effect that depend on the non-trivial holonomy of the gauge connection.

The paper is organized as follows. Section 2 defines the Dirac dressing field and its basic properties, including its multi-valuedness in multiply-connected spaces. Section 3 derives the exact dynamics of the dressed field, culminating in a Dirac (and Schrödinger) equation that, in the AB setup, reduces to free propagation. Section 4 computes the interference pattern for the dressed electron in the AB effect, showing it is flux-independent. Section 5 discusses the implications for the DFM. Section 6 concludes.

2 Dirac Dressing Field: Definitions and Properties

2.1 The Bare Theory

We work in natural units $\hbar = c = 1$. The metric is $(+, -, -, -)$. The bare Dirac field $\psi(x)$ satisfies the minimally coupled Dirac equation

$$(i\gamma^\mu D_\mu - m)\psi = 0, \qquad D_\mu = \partial_\mu - ieA_\mu, \tag{1}$$

where γ^μ are the Dirac matrices, $A_\mu = (\phi, -\mathbf{A})$. The electromagnetic field strength is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, with components $F_{0i} = E_i$, $F_{ij} = \epsilon_{ijk} B_k$.

We will also need the non-relativistic limit. The corresponding Schrödinger equation for a spinless particle is

$$i\partial_t \psi = \frac{1}{2m}\big(-i\nabla + e\mathbf{A}\big)^2 \psi - e\phi\,\psi. \tag{2}$$

2.2 Definition of the Dressed Field

Choose a fixed reference point x_0 (e.g., at spatial infinity on the left). For each point x, fix a smooth path Γ_x from x_0 to x. The family of paths is assumed to vary smoothly with the endpoint x. Define the dressing factor

$$h(x) = \exp\!\left(-ie \int_{\Gamma_x} A_\mu(y)\, dy^\mu\right) = \exp\!\left(ie \int_{\Gamma_x} \mathbf{A}(y)\cdot d\mathbf{y} - ie \int_{\Gamma_x} \phi(y)\, dt\right). \tag{3}$$

In the static magnetic case ($\phi = 0$), this simplifies to $h(x) = \exp\!\Big(ie \int_{\Gamma_x} \mathbf{A}\cdot d\mathbf{y}\Big)$. The dressed field is

$$\psi_P(x) = h(x)\,\psi(x). \tag{4}$$

2.3 Gauge Transformation Properties

Under a $U(1)$ gauge transformation,

$$\psi(x) \to e^{-ie\chi(x)}\psi(x), \qquad A_\mu(x) \to A_\mu(x) - \partial_\mu\chi(x). \tag{5}$$

The line integral transforms as

$$\int_{\Gamma_x} A_\mu(y) dy^\mu \to \int_{\Gamma_x} A_\mu(y) dy^\mu - \chi(x) + \chi(x_0). \tag{6}$$

Hence

$$h(x) \to \exp\!\left(-ie \int_{\Gamma_x} A_\mu dy^\mu + ie\chi(x) - ie\chi(x_0)\right) = h(x)\, e^{ie\chi(x)} e^{-ie\chi(x_0)}. \tag{7}$$

Then

$$\psi_P(x) = h(x)\psi(x) \to h(x) e^{ie\chi(x)} e^{-ie\chi(x_0)} \cdot e^{-ie\chi(x)}\psi(x) = e^{-ie\chi(x_0)} h(x)\psi(x) = e^{-ie\chi(x_0)}\psi_P(x). \tag{8}$$

Thus ψ_P changes only by a global constant phase $e^{-ie\chi(x_0)}$, which is physically irrelevant. Therefore ψ_P is a gauge-invariant object in the quantum mechanical sense.

2.4 Path-Dependence and Multi-Valuedness in Multiply-Connected Spaces

The AB effect takes place in the space $\mathcal{M} = \mathbb{R}^2 \setminus \{0\}$ (the plane with the solenoid removed). This space is multiply-connected: there exist closed loops that cannot be continuously contracted to a point. Consider a point x on the right side of the solenoid. From the reference point x_0 (say, at infinity on the left) to x, there are two homotopically distinct families of paths: those that pass above the solenoid (call them Γ_+) and those that pass below (Γ_-). Their line integrals differ by the magnetic flux:

$$\int_{\Gamma_-} \mathbf{A} \cdot d\mathbf{y} = \int_{\Gamma_+} \mathbf{A} \cdot d\mathbf{y} + \Phi, \tag{9}$$

where $\Phi = \oint \mathbf{A} \cdot d\mathbf{l}$ is the total flux. Consequently,

$$h_-(x) = \exp\left(ie \int_{\Gamma_-} \mathbf{A}\right) = \exp\left(ie \int_{\Gamma_+} \mathbf{A} + ie\Phi\right) = h_+(x)\, e^{ie\Phi}. \tag{10}$$

Thus the dressing factor $h(x)$ is multi-valued: it takes different values at the same spatial point x depending on the homotopy class of the chosen reference path. To define a single-valued function, one must fix a branch. This introduces a branch cut (e.g., along the negative x-axis). However, in an interference experiment, the two wave packets naturally correspond to the two different branches: the upper packet uses Γ_+, the lower packet uses Γ_-. This is essential for the calculation.

2.5 Dressed Electromagnetic Potential

In modern DFM, one also introduces a dressed gauge potential via the standard DFM transformation:

$$A^P_\mu(x) = h^{-1}(x) A_\mu(x) h(x) + \frac{i}{e} h^{-1}(x) \partial_\mu h(x). \tag{11}$$

For the Abelian case, with $h(x) = e^{-ie\Theta(x)}$ and $\Theta(x) = \int_{\Gamma_x} A_\mu(y) dy^\mu$, we have $h^{-1} = e^{ie\Theta}$. Then

$$A^P_\mu = A_\mu + \frac{i}{e} e^{ie\Theta} \partial_\mu (e^{-ie\Theta}) = A_\mu + \frac{i}{e} e^{ie\Theta} (-ie\partial_\mu \Theta) e^{-ie\Theta} = A_\mu - \partial_\mu \Theta. \tag{12}$$

Using the formula derived in the Appendix:

$$\partial_\mu \Theta = A_\mu(x) + \int_{\Gamma_x} F_{\mu\nu}(y)\, dy^\nu, \tag{13}$$

we obtain

$$A^P_\mu = A_\mu - \left(A_\mu + \int_{\Gamma_x} F_{\mu\nu}(y)\, dy^\nu\right) = -\int_{\Gamma_x} F_{\mu\nu}(y)\, dy^\nu. \tag{14}$$

Thus the dressed electromagnetic potential is directly given by the non-local line integral of the field strength along the reference path Γ_x. This expression is manifestly gauge invariant because $F_{\mu\nu}$ is gauge invariant.

In the AB setup, the reference paths Γ_+ and Γ_- are chosen to lie entirely in the field-free region (outside the solenoid). Along these paths, $F_{\mu\nu} = 0$, so the integral vanishes:

$$A^P_\mu = 0 \qquad \text{for both } \Gamma_+ \text{ and } \Gamma_-.$$

Hence the dressed potential is identically zero and carries no topological information about the enclosed flux. The entire topological phase (the holonomy) resides in the dressing factor $h(x)$ itself, as shown in Eq. (10).

3 Dynamics of the Dressed Field

3.1 Derivation of the Dirac Equation

We derive the equation of motion for the dressed field from the bare theory.

From $\psi_P = h\psi$ we have

$$\partial_\mu \psi_P = (\partial_\mu h)\psi + h\,\partial_\mu \psi. \tag{15}$$

Solving for $h\partial_\mu\psi$ and substituting $\psi = h^{-1}\psi_P$ gives

$$h\partial_\mu\psi = \partial_\mu\psi_P - (\partial_\mu h)h^{-1}\psi_P. \tag{16}$$

Then compute $hD_\mu\psi$ with $D_\mu = \partial_\mu - ieA_\mu$:

$$hD_\mu\psi = h\partial_\mu\psi - ie\,hA_\mu\psi. \tag{17}$$

Insert (16) and $\psi = h^{-1}\psi_P$:

$$hD_\mu\psi = \Big[\partial_\mu - (\partial_\mu h)h^{-1} - ie\,hA_\mu h^{-1}\Big]\psi_P. \tag{18}$$

Using the definition of the dressed potential (11), this becomes the exact identity

$$hD_\mu\psi = (\partial_\mu - ieA^P_\mu)\psi_P. \tag{19}$$

Finally, start from the bare Dirac equation

$$(i\gamma^\mu D_\mu - m)\psi = 0, \tag{20}$$

and multiply by h:

$$h(i\gamma^\mu D_\mu\psi - m\psi) = 0. \tag{21}$$

Using $\psi_P = h\psi$ and the identity above, we obtain

$$i\gamma^\mu(\partial_\mu - ieA^P_\mu)\psi_P - m\psi_P = 0. \tag{22}$$

Thus the dressed field ψ_P satisfies a Dirac equation in which the original gauge potential A_μ is replaced by the gauge-invariant, non-local field A^P_μ. In the AB setup, one can choose reference paths Γ_+ and Γ_- for the upper and lower arms that lie entirely in the field-free region outside the solenoid. Along both paths $F_{\mu\nu} = 0$, so $A^P_\mu = 0$. Consequently, each dressed component $\psi_{P,\pm}$ satisfies the free Dirac equation, even during interference at the detector.

3.2 Non-Relativistic Limit

The non-relativistic limit of (22) for a spinless particle gives the Schrödinger equation

$$i\partial_t\psi_P = \frac{1}{2m}\big(-i\nabla - e\mathbf{A}^P\big)^2\psi_P - e\phi^P\psi_P. \tag{23}$$

In the AB setup with the reference path avoiding the solenoid, $\mathbf{A}^P = 0$ and $\phi^P = 0$, so we obtain the free Schrödinger equation.

3.3 Probabilistic Interpretation (Born Rule)

In standard quantum mechanics, the probability density for a particle at position $\mathbf{x}$ at time t is $\rho(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2$. Since $|\psi_P|^2 = |h\psi|^2 = |h|^2|\psi|^2 = |\psi|^2$ (because $|h| = 1$), the dressed field yields the same probability density:

$$\rho_P(\mathbf{x}, t) = |\psi_P(\mathbf{x}, t)|^2 = |\psi(\mathbf{x}, t)|^2. \tag{24}$$

Therefore, the Born rule is preserved when using the dressed field. For interference experiments, the total wave function at the detector is the sum of the contributions from the two paths: $\Psi_P(D) = \psi_{P,1}(D) + \psi_{P,2}(D)$. The probability is $|\Psi_P(D)|^2$. This is the quantity we will compute.

4 The Aharonov–Bohm Test

4.1 Setup and Geometry

Consider the standard magnetic AB setup. An infinite solenoid of negligible radius is placed along the z-axis, carrying a magnetic flux Φ. The electron source S is located at a point on the left ($x = -L, y = 0$), and the detector D at a point on the right ($x = +L, y = 0$). Two classical paths from S to D are considered: γ_1 passes above the solenoid (upper half-plane), γ_2 passes below (lower half-plane). The magnetic field is zero everywhere along these paths, but the vector potential $\mathbf{A}$ is non-zero. In the Coulomb gauge, $\mathbf{A}(r, \theta) = \frac{\Phi}{2\pi r}\hat{\theta}$.

The space $\mathcal{M} = \mathbb{R}^2 \setminus \{0\}$ is multiply-connected. We fix an arbitrary reference point x_0 (e.g., at infinity on the left). Choose a fixed reference path Γ_S from x_0 to the source S. This path may lie, for instance, in the upper half-plane; the exact choice is irrelevant because it will be common to both branches.

4.2 Dressed Fields and Cancellation of the Vector Potential

For the upper branch, we define the reference path from x_0 to a point x on γ_1 as the concatenation

$$\Gamma_x^{(1)} = \Gamma_S \circ \gamma_1|_{S\to x},$$

where $\gamma_1|_{S\to x}$ is the segment of the physical path from S to x. For the lower branch we similarly use

$$\Gamma_x^{(2)} = \Gamma_S \circ \gamma_2|_{S\to x}.$$

Because the upper and lower half-planes are each simply connected (they do not contain the solenoid), any other choice of reference path within the same half-plane would give the same line integral of $\mathbf{A}$ (by Stokes' theorem, since the difference would enclose zero flux). Therefore the above choice is completely general.

The dressing factors at a point x on the respective branches are

$$h_1(x) = \exp\left(ie\int_{\Gamma_x^{(1)}} \mathbf{A}\cdot d\mathbf{l}\right) = \exp\left(ie\int_{\Gamma_S} \mathbf{A}\cdot d\mathbf{l}\right)\,\exp\left(ie\int_{\gamma_1|_{S\to x}} \mathbf{A}\cdot d\mathbf{l}\right), \tag{25}$$

$$h_2(x) = \exp\left(ie \int_{\Gamma_x^{(2)}} \mathbf{A} \cdot d\mathbf{l} \right) = \exp\left(ie \int_{\Gamma_S} \mathbf{A} \cdot d\mathbf{l} \right) \exp\left(ie \int_{\gamma_2|_{S \to x}} \mathbf{A} \cdot d\mathbf{l} \right). \qquad (26)$$

The bare wave functions at the same point x (before reaching the detector) are

$$\psi_1(x) = \psi_0 e^{iS_0(x)} \exp\left(-ie \int_{\gamma_1|_{S \to x}} \mathbf{A} \cdot d\mathbf{l} \right), \qquad \psi_2(x) = \psi_0 e^{iS_0(x)} \exp\left(-ie \int_{\gamma_2|_{S \to x}} \mathbf{A} \cdot d\mathbf{l} \right), \qquad (27)$$

where $S_0(x)$ is the free action from S to x (equal for both paths for simplicity). Hence the dressed fields at any point along the branches are

$$\begin{aligned} \psi_{P,1}(x) &= h_1(x)\, \psi_1(x) \\ &= \exp\left(ie \int_{\Gamma_S} \mathbf{A} \cdot d\mathbf{l} \right) \exp\left(ie \int_{\gamma_1|_{S \to x}} \mathbf{A} \cdot d\mathbf{l} \right) \psi_0 e^{iS_0(x)} \exp\left(-ie \int_{\gamma_1|_{S \to x}} \mathbf{A} \cdot d\mathbf{l} \right) \\ &= \exp\left(ie \int_{\Gamma_S} \mathbf{A} \cdot d\mathbf{l} \right) \psi_0 e^{iS_0(x)}, \end{aligned} \qquad (28)$$

$$\begin{aligned} \psi_{P,2}(x) &= h_2(x)\, \psi_2(x) \\ &= \exp\left(ie \int_{\Gamma_S} \mathbf{A} \cdot d\mathbf{l} \right) \exp\left(ie \int_{\gamma_2|_{S \to x}} \mathbf{A} \cdot d\mathbf{l} \right) \psi_0 e^{iS_0(x)} \exp\left(-ie \int_{\gamma_2|_{S \to x}} \mathbf{A} \cdot d\mathbf{l} \right) \\ &= \exp\left(ie \int_{\Gamma_S} \mathbf{A} \cdot d\mathbf{l} \right) \psi_0 e^{iS_0(x)}. \end{aligned} \qquad (29)$$

Thus, along each branch the $\mathbf{A}$-dependent exponentials cancel exactly pointwise. In particular, at the detector D we have

$$\psi_{P,1}(D) = \psi_{P,2}(D) = \exp\left(ie \int_{\Gamma_S} \mathbf{A} \cdot d\mathbf{l} \right) \psi_0 e^{iS_0(D)}. \qquad (30)$$

4.3 Predicted Interference Pattern

The total dressed wave function at the detector (up to an overall normalization) is

$$\Psi_P(D) = \psi_{P,1}(D) + \psi_{P,2}(D) = 2\, \exp\left(ie \int_{\Gamma_S} \mathbf{A} \cdot d\mathbf{l} \right) \psi_0 e^{iS_0(D)}. \qquad (31)$$

The probability density (interference pattern) is

$$\rho_P(D) = |\Psi_P(D)|^2 = 4|\psi_0|^2, \qquad (32)$$

which is independent of the magnetic flux Φ. The constant factor $\exp\left(ie \int_{\Gamma_S} \mathbf{A}\right)$ cancels in the absolute value. Thus DFM predicts no flux-dependent phase shift, in direct contradiction with the experimentally observed AB shift $e^{-ie\Phi}$.

Note that in the AB setup, the dressed field satisfies the free Dirac equation (or free Schrödinger equation) for each branch, as shown in Sec. 3. Since both branches originate from the same initial dressed field at the source (the reference path is unique there) and evolve freely, they remain identical throughout propagation. Consequently, at the detector we have $\psi_{P,1}(D) = \psi_{P,2}(D)$, and the interference pattern is independent of the flux Φ, in agreement with the explicit calculation above.

4.4 Extension to Quantum Dressing Operators

The above analysis treated the dressing factor as a classical c-number phase. A natural objection is that in QED the gauge field is operator-valued, and the dressing should be defined as a Weyl operator

$$\hat{W}(\Gamma_x) = \exp\left(-ie \int_{\Gamma_x} \hat{A}_\mu(y)\, dy^\mu\right),$$

which creates a coherent photon cloud [9]. However, in the AB experiment the magnetic flux Φ is produced by a macroscopic solenoid, and the electromagnetic field is prepared in a coherent state $|\Phi\rangle$ with large mean flux. For any closed loop γ enclosing the solenoid, the Wilson loop operator $\hat{W}(\gamma)$ satisfies

$$\langle\Phi|\hat{W}(\gamma)|\Phi\rangle = e^{-ie\Phi} + \mathcal{O}(\hbar), \tag{33}$$

where the $\mathcal{O}(\hbar)$ term is negligible in the semiclassical (large-flux) limit. In this limit we may treat $\hat{W}(\gamma)$ as the c-number phase $e^{-ie\Phi}$ when acting on $|\Phi\rangle$. Therefore, the quantum dressing factor effectively reduces to its classical counterpart, and the previous cancellation proof applies. In other words, DFM does not restore the AB phase either in the full quantum theory.

5 Implications

The calculation in Section 4 shows that the Dirac DFM predicts a flux-independent interference pattern for the AB effect, in direct contradiction with experiment. This failure has several important implications for DFM and its claimed equivalence to standard QED.

5.1 The Electron is not Dressed (in Dirac's Sense)

The title of this paper asks whether the electron is dressed. Dirac's DFM gives a specific prescription: the physical electron is the gauge-invariant combination $\psi_P = h\psi$. Our calculation in Section 4 shows that this dressed field predicts no AB phase shift – the interference pattern is independent of the magnetic flux Φ, in contradiction with experiment. Therefore, if "dressed" means "described by Dirac's dressed field", then the electron is not dressed.

Why does the method fail? In a multiply-connected space, the dressing factor h carries a holonomy $e^{ie\Phi}$ while the bare wave function carries the opposite holonomy $e^{-ie\Phi}$. Their product cancels the flux dependence exactly. The very mechanism that makes ψ_P gauge-invariant erases the topological information needed for the AB effect. Hence, the electron cannot be described as dressed in Dirac's sense.

5.2 Comparison with the Madelung Formulation

It is instructive to compare the DFM with the Madelung hydrodynamic formulation. In Madelung's approach, one writes the wave function as $\psi = \sqrt{\rho}\, e^{iS}$ and obtains a gauge-invariant velocity field $\mathbf{v} = (\nabla S + e\mathbf{A})/m$ [7]. In the AB effect, Madelung reproduces the correct phase shift because it imposes an additional global quantization condition on the circulation of $\mathbf{v}$. The DFM, on the other hand, attempts to eliminate $\mathbf{A}$ entirely by absorbing it into a phase factor attached to the matter field. The result is a dressed field that satisfies a free equation but loses the memory of the flux. The comparison highlights that eliminating the gauge potential is not always harmless; the topological information encoded in the line integrals of $\mathbf{A}$ must be preserved to correctly describe interference in multiply-connected spaces.

5.3 Consequences for Practical Applications

Despite its failure in topologically non-trivial backgrounds, the Dirac DFM remains a useful technical tool in simply-connected spacetimes. It has been successfully applied to problems such as the elimination of infrared divergences in QED, the description of infraparticles, and the construction of gauge-invariant quark fields in QCD [6, 2, 5, 9, 3]. Our analysis implies that these applications should be restricted to situations where the topology is trivial (e.g., Minkowski space with no holes or defects). When dealing with systems that exhibit topological effects (e.g., the AB effect or systems with magnetic monopoles), the DFM must be either abandoned or supplemented by additional non-local information.

In summary, the failure of the Dirac DFM in the AB effect demonstrates that the method is not universally equivalent to standard QED. The equivalence holds only in simply-connected backgrounds. This result clarifies the range of validity of the DFM and warns against its uncritical use in topologically non-trivial settings.

6 Conclusion

We have provided a rigorous analysis of Dirac's DFM. We defined the dressed field and its associated dressed electromagnetic potential, examined their path-dependence and multi-valuedness in multiply-connected spaces, and derived the exact Dirac equation satisfied by the dressed field. In field-free regions with a suitably chosen reference path, the dressed field satisfies the free Dirac (and therefore free Schrödinger) equation. Applying the theory to the AB effect, we computed the interference pattern predicted for the dressed field. The result is independent of the magnetic flux Φ, in direct contradiction with the experimentally observed AB phase shift $e^{-ie\Phi}$. The mathematical origin of this failure is the exact cancellation of the topological phases between the dressing factor and the bare wave function, which follows from Stokes' theorem and the multi-valuedness of the Wilson line.

Thus, the Dirac DFM cannot serve as a complete physical description of the electron; it fails to account for the AB effect. This also suggests that the gauge potential $\mathbf{A}$ retains irreducible topological information that cannot be eliminated by a local dressing. Consequently, any claim that DFM provides an equivalent reformulation of QED must be restricted to simply-connected spacetimes, where all closed loops are contractible and holonomies are trivial.

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References

[1] Y. Aharonov and D. Bohm, "Significance of Electromagnetic Potentials in the Quantum Theory," Phys. Rev. 115, 485 (1959).

[2] J. Attard, J. François, S. Lazzarini, and T. Masson, "The dressing field method of gauge symmetry reduction, a review with examples," arXiv:1702.02753 (2017).

[3] P. Berghofer, J. François, S. Friederich, H. Gomes, G. Hetzroni, A. Maas, and R. Sondenheimer, *Gauge Symmetries, Symmetry Breaking, and Gauge-Invariant Approaches*, Cambridge University Press (2023).

[4] P. A. M. Dirac, "Gauge-Invariant Formulation of Quantum Electrodynamics," Can. J. Phys. 33, 650 (1955).

[5] H. Gomes and A. Riello, "Unified geometric framework for boundary charges and particle dressings," Phys. Rev. D 98, 025013 (2018).

[6] M. Lavelle and D. McMullan, "Constituent Quarks from QCD," Phys. Rep. 279, 1 (1997).

[7] E. Madelung, "Quantentheorie in hydrodynamischer Form," Z. Phys. 40, 322 (1927).

[8] S. Mandelstam, "Quantum Electrodynamics without Potentials," Ann. Phys. 19, 1 (1962).

[9] J. Mund, K.-H. Rehren, and B. Schroer, "Infraparticle quantum fields and the formation of photon clouds," JHEP 04 (2022) 083.

Appendix: Derivative of the Path-Dependent Phase

In this appendix we derive the identity

$$\partial_\mu \Theta(x) = A_\mu(x) + \int_{\Gamma_x} F_{\mu\nu}(y)\, dy^\nu, \tag{A1}$$

for the path-dependent phase

$$\Theta(x) = \int_{\Gamma_x} A_\rho(y)\, dy^\rho, \tag{A2}$$

where Γ_x is a path from a fixed reference point x_0 to the point x, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength.

Let $x \to x + \delta x$ be an infinitesimal displacement of the endpoint. The path Γ_x is deformed to a nearby path $\Gamma_{x+\delta x}$. The change in Θ is

$$\delta\Theta = \int_{\Gamma_{x+\delta x}} A - \int_{\Gamma_x} A. \tag{A3}$$

The difference between the two integrals can be represented as an integral over a closed contour formed by traversing Γ_x forward and $\Gamma_{x+\delta x}$ backward, together with a short segment connecting x to $x + \delta x$. This closed contour naturally decomposes into two contributions. The short segment at the endpoint gives

$$\delta\Theta\big|_{\text{endpoint}} = A_\mu(x)\,\delta x^\mu. \tag{A4}$$

In addition, the deformation of the path sweeps out a narrow strip between Γ_x and $\Gamma_{x+\delta x}$. By Stokes' theorem, this produces a surface contribution

$$\delta\Theta\big|_{\text{surface}} = \int_\Sigma F_{\mu\nu}(y)\,d\sigma^{\mu\nu}, \tag{A5}$$

where Σ denotes the infinitesimal surface between the two paths. To first order in δx, the surface element can be written as $d\sigma^{\mu\nu} = dy^\nu\,\delta x^\mu$ along the strip, yielding

$$\delta\Theta\big|_{\text{surface}} = \int_{\Gamma_x} F_{\mu\nu}(y)\,dy^\nu\,\delta x^\mu. \tag{A6}$$

Combining the endpoint and surface contributions, we obtain

$$\delta\Theta = \left[A_\mu(x) + \int_{\Gamma_x} F_{\mu\nu}(y)\,dy^\nu\right]\delta x^\mu. \tag{A7}$$

Since this holds for arbitrary δx^μ, it follows that

$$\partial_\mu\Theta(x) = A_\mu(x) + \int_{\Gamma_x} F_{\mu\nu}(y)\,dy^\nu, \tag{A8}$$

which establishes Eq. (A1).

The above derivation assumes that the deformation of the path is smooth and does not cross singular regions (for example, the interior of an idealized solenoid); in the presence of such regions, the result applies within a fixed homotopy class of paths.