Emergence of Classical Dynamics from a Random Matrix Schrödinger Model

Alexey A. Kryukov[1]

[1]*Department of Mathematics & Natural Sciences, University of Wisconsin-Milwaukee, USA**

The Newtonian motion of a macroscopic particle is derived from the linear Schrödinger equation with a Hamiltonian consisting of the free-particle term and a random Hamiltonian drawn from the Gaussian Unitary Ensemble. The random term models interaction with the environment. We show that the parameters governing the resulting state-space random walk, together with the treatment of experimentally indistinguishable states as equivalence classes, explain the contrasting behavior of microscopic and macroscopic systems. The analysis extends previous work deriving the Born rule for microscopic particles when the free-particle term is negligible.

It is widely asserted that linear quantum dynamics cannot produce state reduction, motivating nonlinear collapse models. These, however, must reconcile linear unitary evolution with nonlinear state reduction while remaining consistent with stringent laboratory and cosmological bounds on collapse-induced noise [1–9]. The resulting constraints rule out most parameter ranges and leave only a narrow window of admissible values.

Without contradicting the standard proofs excluding linear dynamics under certain assumptions, we show that these assumptions need not apply in a geometry based on equivalence classes of states. This yields a linear stochastic model of state reduction applicable to both classical and quantum measurements. In the classical regime, the model predicts a normal distribution for the position of a measured particle, while in the quantum regime it reproduces the Born rule. The model also accounts for the transition from the Schrödinger evolution of microscopic particles to the Newtonian dynamics of macroscopic bodies and sheds new light on the double-slit experiment.

The starting point in a derivation of these results is a geometric relation between Schrödinger and Newtonian dynamics. Let $M^{\sigma}_{3,3} \subset \mathbb{CP}^{L_2}$ denote the set of wave packets

$$\varphi(\mathbf{x}) = r_{\mathbf{a},\sigma}(\mathbf{x})\, e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}, \tag{1}$$

where

$$r_{\mathbf{a},\sigma}(\mathbf{x}) = \sigma^{-\frac{3}{2}}\, r\left(\frac{\mathbf{x}-\mathbf{a}}{\sigma}\right), \tag{2}$$

for a fixed, normalized, real-valued, C^2-function $r \in L_2(\mathbb{R}^3)$ with finite variance. For $\sigma \to 0$, $r^2_{\mathbf{a},\sigma}(\mathbf{x}) \to \delta^3(\mathbf{x}-\mathbf{a})$ [10]. Gaussian packets

$$g_{\mathbf{a},\sigma}(\mathbf{x}) = \left(\frac{1}{2\pi\sigma^2}\right)^{3/4} \exp\left[-\frac{(\mathbf{x}-\mathbf{a})^2}{4\sigma^2}\right], \tag{3}$$

provide a convenient example.

The inclusion of $M^{\sigma}_{3,3}$ into $\mathbb{CP}^{L_2}$, furnished with the Fubini-Study metric, induces the differentiable structure and the Riemannian metric on $M^{\sigma}_{3,3}$. With an appropriate choice of units, the map

$$\Omega : (\mathbf{a},\mathbf{p}) \longmapsto r_{\mathbf{a},\sigma}\, e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}$$

is an isometry between the Euclidean space $\mathbb{R}^3 \times \mathbb{R}^3$ and the Riemannian manifold $M^{\sigma}_{3,3}$. Moreover, a linear structure on $M^{\sigma}_{3,3}$ can be induced via Ω from the linear structure on $\mathbb{R}^3 \times \mathbb{R}^3$.

Variation of the action functional

$$S[\varphi] = \int \overline{\varphi}(\mathbf{x},t)\left[i\hbar\frac{\partial}{\partial t} - \widehat{h}\right]\varphi(\mathbf{x},t)\, d^3\mathbf{x}\, dt, \tag{4}$$

where the Hamiltonian $\widehat{h}$ is given by

$$\widehat{h} = -\frac{\hbar^2}{2m}\Delta + \widehat{V}(\mathbf{x},t) \tag{5}$$

with respect to φ, yields the Schrödinger equation for a particle. By constraining φ in (4) to lie on the manifold $M^{\sigma}_{3,3}$ with a sufficiently small σ, the action functional takes the classical form

$$S = \int \left[\mathbf{p}\cdot\frac{d\mathbf{a}}{dt} - h(\mathbf{p},\mathbf{a},t)\right] dt, \tag{6}$$

where

$$h(\mathbf{p},\mathbf{a},t) = \frac{\mathbf{p}^2}{2m} + V(\mathbf{a},t) \tag{7}$$

is the Hamiltonian function of the system. Constrained variation yields the Newton equations.

The construction extends to many-body systems. In particular, a two-particle system with Hamiltonian

$$\widehat{h} = -\frac{\hbar^2}{2m_1}\Delta_1 - \frac{\hbar^2}{2m_2}\Delta_2 + \widehat{V}(\mathbf{x}_1,\mathbf{x}_2,t) \tag{8}$$

whose state is constrained to the manifold $M^{\sigma}_{3,3} \otimes M^{\sigma}_{3,3}$ evolves according to Newtonian dynamics.

Writing a smooth wave packet as

$$\varphi(\mathbf{x}) = r_{\mathbf{a},\sigma}(\mathbf{x})\, e^{i\Theta(\mathbf{x})}, \tag{9}$$

and expanding $\Theta(\mathbf{x})$ around $\mathbf{a}$, quadratic terms can be neglected for small σ, since $r_{\mathbf{a},\sigma}$ is appreciable only near $\mathbf{x} = \mathbf{a}$. Thus (1) captures the general form of a sufficiently narrow packet. The parameter σ reflects the resolution of position-measuring devices, and $M^{\sigma}_{3,3}$ may be defined as the set of wave packets with spread $\leq \sigma$ and fixed expectation value of position. For such packets, the constrained dynamics reduces to Newtonian motion, consistent with the Ehrenfest theorem. For concreteness we begin with Gaussian representatives, setting $r_{\mathbf{a},\sigma} = g_{\mathbf{a},\sigma}$ in the definition of $M^{\sigma}_{3,3}$; we later extend the construction to equivalence classes of arbitrary localized states.

As shown in [11], the Schrödinger velocity of a state $\varphi \in M^{\sigma}_{3,3}$ decomposes into three orthogonal parts: two tangent to $M^{\sigma}_{3,3}$, corresponding to the classical velocity and acceleration, and one normal to the manifold that describes wave-packet spreading. Their squared norms add to

$$\left\|\frac{d\varphi}{dt}\right\|^2_{FS} = \frac{\mathbf{v}^2}{4\sigma^2} + \frac{m^2\mathbf{w}^2\sigma^2}{\hbar^2} + \frac{\hbar^2}{32\sigma^4 m^2}, \qquad (10)$$

where $\mathbf{v}$ is the classical velocity and $\mathbf{w} = -\nabla V/m$ the classical acceleration. Constraining the motion to $M^{\sigma}_{3,3}$ suppresses the normal (spreading) component of

$$\frac{d\varphi}{dt} = -\frac{i}{\hbar}\widehat{h}\varphi,$$

reducing commutators to Poisson brackets and yielding Newtonian dynamics [11].

The relation between the action functionals (4) and (6) identifies Newtonian particles with quantum systems whose states are constrained to the manifold $M^{\sigma}_{3,3}$ for suitable σ. The isometry Ω then provides a direct identification between the Euclidean phase space $\mathbb{R}^3 \times \mathbb{R}^3$ of a Newtonian particle and the manifold of states $M^{\sigma}_{3,3}$. Its restriction

$$\omega : \mathbf{a} \mapsto g_{\mathbf{a},\sigma}$$

is an isometry between the Euclidean position space $\mathbb{R}^3$ and the submanifold $M^{\sigma}_3 \subset \mathbb{CP}^{L_2}$ of states $g_{\mathbf{a},\sigma}$ [12–14]. This isometry is captured by the relation

$$e^{-\frac{(\mathbf{a}-\mathbf{b})^2}{4\sigma^2}} = \cos^2\rho(g_{\mathbf{a},\sigma}, g_{\mathbf{b},\sigma}), \qquad (11)$$

which relates the Euclidean distance $||\mathbf{a} - \mathbf{b}||$ with the Fubini-Study distance between the corresponding Gaussian states. For states including momenta, $\varphi = g_{\mathbf{a},\sigma}e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}$ and $\psi = g_{\mathbf{b},\sigma}e^{i\mathbf{q}\cdot\mathbf{x}/\hbar}$, the analogous relation is

$$e^{-\frac{(\mathbf{a}-\mathbf{b})^2}{4\sigma^2} - \frac{(\mathbf{p}-\mathbf{q})^2}{\hbar^2/\sigma^2}} = \cos^2\rho(\varphi, \psi). \qquad (12)$$

Embedding the Euclidean classical phase space into the curved state space $\mathbb{CP}^{L_2}$ leads naturally to nontrivial position-momentum commutators. For states $\varphi \in M^{\sigma}_{3,3}$, the position operator acts as the generator of momentum displacements,

$$\widehat{\mathbf{x}}\varphi = -i\hbar\nabla_{\mathbf{p}}\varphi. \qquad (13)$$

The vector fields $-\hbar\nabla_{\mathbf{p}}\varphi$ and $-\hbar\nabla_{\mathbf{x}}\varphi$ commute, and their integral curves furnish orthogonal coordinates on $M^{\sigma}_{3,3}$, confirming its identification with the Euclidean phase space $\mathbb{R}^3 \times \mathbb{R}^3$.

Because states in $M^{\sigma}_{3,3}$ form a complete set in the Hilbert space, the vector field $-\hbar\nabla_{\mathbf{p}}\varphi$ extends uniquely to a linear vector field on the full space, which by (13) is precisely $-i\widehat{\mathbf{x}}\varphi$. Although the Lie bracket of $-\hbar\nabla_{\mathbf{p}}\varphi$ and $-\hbar\nabla_{\mathbf{x}}\varphi$ vanishes on $M^{\sigma}_{3,3}$, the bracket of their extensions, $-i\widehat{\mathbf{p}}\varphi$ and $-i\widehat{\mathbf{x}}\varphi$, tangent to the unit sphere $\mathbb{S}^{L_2}$ or pushed down to $\mathbb{CP}^{L_2}$, yields

$$[\widehat{\mathbf{x}}, \widehat{\mathbf{p}}]\varphi = i\hbar\varphi. \qquad (14)$$

Unlike the submanifold $M^{\sigma}_{3,3}$, the full state space $\mathbb{CP}^{L_2}$ has nontrivial curvature. The sectional curvature in the plane spanned by the tangent vectors $-i\widehat{\mathbf{p}}\varphi$ and $-i\widehat{\mathbf{x}}\varphi$ can be expressed through their Lie bracket, i.e., through the commutator $[\widehat{\mathbf{x}}, \widehat{\mathbf{p}}]$. The curvature is independent of φ and corresponds to the sphere $\mathbb{S}^2 = \mathbb{CP}^1$ of radius $\hbar/2$ in that section [14]. Thus the classical phase space appears as a flat submanifold embedded in a Planck-scale curved state space. The "quantumness" of the microworld arises from extending classicality from the manifolds $M^{\sigma}_3 = \mathbb{R}^3$ and $M^{\sigma}_{3,3} = \mathbb{R}^3 \times \mathbb{R}^3$, which "wind" through the infinite dimensions of $\mathbb{CP}^{L_2}$, to the entire state space. The nontrivial canonical commutation relation between $\widehat{\mathbf{x}}$ and $\widehat{\mathbf{p}}$ reflects the nonvanishing sectional curvature of $\mathbb{CP}^{L_2}$.

We now apply this geometry to measurement. The familiar normal distribution of position outcomes for a classical particle arises by modeling measurement as a random walk of its position in $\mathbb{R}^3$ during the measurement interval, approximating Brownian motion. The following proposition defines an extension of this walk from $M^{\sigma}_3 = \mathbb{R}^3$ to the full state space $\mathbb{CP}^{L_2}$. It parallels Einstein's assumptions for Brownian motion [15] and provides a model of position measurement for microscopic particles:

> **(RM)** *The dynamics of a particle's state under position measurement can be modeled as a random walk in the space of states. In the absence of drift, each step is generated by the Schrödinger equation with a Hamiltonian drawn independently at each instant from the Gaussian Unitary Ensemble (GUE).*

Here **(RM)** denotes "random matrices." Physically, such Hamiltonians may arise from the complicated, rapidly fluctuating interaction between the particle and its measuring apparatus or environment, reminiscent of Wigner's approach to complex spectra [16] and of the Bohigas-Giannoni-Schmit conjecture [17], here applied in a time-dependent setting.

A small step in the state's random walk, driven by the Hamiltonian in **(RM)**, is represented by a random vector in the tangent space to $\mathbb{CP}^{L_2}$. The distribution of such steps is normal, homogeneous, and isotropic [11], implying that the transition probability between two states depends only on their Fubini-Study distance. When the steps are constrained to M_3^σ, the walk yields Brownian motion on $\mathbb{R}^3$ [11, 18], and the transition probability follows the normal distribution. Without this constraint, the same isotropic process yields the Born rule [11, 18, 19]. This establishes a dynamical link between classical and quantum measurements, placing them on an equal footing. The correspondence between the normal distribution and the Born rule also follows from (11).

Since Newtonian motion and the model for macroscopic position measurement were obtained by constraining the Schrödinger evolution and the random walk in **(RM)** to $M_{3,3}^\sigma$ and M_3^σ, a dynamical explanation of this constraint is required. We show that the choice of time-step and step-size parameters in **(RM)** enforces the constraint and thereby distinguishes microscopic from macroscopic behavior. Because measurement devices cannot distinguish sufficiently narrow states, the dynamical constraint must be formulated in terms of equivalence classes of such states rather than individual wave functions.

Restricting for simplicity to one spatial dimension with state spaces $L_2(\mathbb{R})$ and $\mathbb{CP}^{L_2}$, the classical Euclidean submanifold $M_1^\sigma = \mathbb{R}$ is represented, in particular, by the Gaussian states

$$g_{a,\sigma}(z) = \left(\frac{1}{2\pi\sigma^2}\right)^{1/4} \exp\left[-\frac{(z-a)^2}{4\sigma^2}\right]. \tag{15}$$

Among all functions in $L_2(\mathbb{R})$ with finite position expectation μ_z and standard deviation $\delta_z \leq \sigma$, the equivalence class $\{g_c\}$, interpreted as a physical eigenstate of position, consists of those with expectation value $\mu_z = c$.

The Fubini-Study distance between a state $\varphi \in L_2(\mathbb{R})$ and an equivalence class $\{g_c\}$ is defined by

$$\rho(\varphi, \{g_c\}) = \inf_{\psi \in \{g_c\}} \rho(\varphi, \psi), \tag{16}$$

where $\rho(\varphi, \psi)$ denotes the Fubini-Study distance between states. A state φ reaches the physical eigenstate $\{g_c\}$ precisely when $\rho(\varphi, \{g_c\}) = 0$. The distance between equivalence classes $\{g_c\}$ and $\{g_d\}$ is similarly defined:

$$\rho(\{g_c\}, \{g_d\}) = \inf_{\varphi \in \{g_c\}} \rho(\varphi, \{g_d\}). \tag{17}$$

This distance is realized by the Gaussian representatives g_c and g_d in (15), which minimize the Fubini–Study distance among all states in the corresponding equivalence classes. Thus the set $\widetilde{M}_1^\sigma$ of all classes $\{g_c\}$, $c \in \mathbb{R}$, may be identified with the manifold $M_1^\sigma = \mathbb{R}$. Each class $\{g_c\}$ is "large": it contains many mutually orthogonal states and thereby absorbs most of the dimensions of the Hilbert space.

The two-dimensional manifold $M_{1,1}^\sigma$ is defined analogously to $M_{3,3}^\sigma$. The manifold $\widetilde{M}_{1,1}^\sigma$ of equivalence classes is obtained by augmenting the equivalence classes of real-valued functions satisfying $\delta_z \leq \sigma$ with a factor $e^{ipz/\hbar}$. Distances between a state and an equivalence class of $\widetilde{M}_{1,1}^\sigma$, as well as those between two equivalence classes, are defined in the same way as in (16) and (17). The isometry between $M_{1,1}^\sigma$, $\widetilde{M}_{1,1}^\sigma$, and $\mathbb{R}^2$ is established using (12). See Supplemental Material for details. As before, Schrödinger evolution constrained to $\widetilde{M}_{1,1}^\sigma$ reproduces the Newtonian dynamics of the particle.

In this framework, the position of a classical particle is specified by an equivalence class of states labeled only by the expectation value μ_z and width δ_z; all remaining degrees of freedom are absorbed into the class. The sets of states satisfying $\mu_z = \tau$ and $\delta_z = \lambda$ are the level sets of the map

$$F(\varphi) = (\mu_z, \delta_z),$$

and form the leaves of a codimension-two foliation of state space. In the Supplemental Material we show that, by translating and scaling any suitable initial state φ, one obtains a two-dimensional submanifold $M_\varphi \subset \mathbb{CP}^{L_2}$, equipped with the induced metric, on which $\tau = \mu_z$ and $s = \ln \delta_z$ serve as orthogonal coordinates. Thus $M_\varphi = \mathbb{R}^2$, and points in M_φ parameterize the leaves of the foliation. State reduction is therefore fully described by a stochastic process on $\mathbb{R}^2$.

The random walk in **(RM)** on $\mathbb{CP}^{L_2}$ reduces to a Gaussian random walk on $\mathbb{R}^2$. The component along τ yields the normal distribution that extends uniquely to the Born rule on $\mathbb{CP}^{L_2}$, while the component along s gives probability 1/2 that the state lies on $\widetilde{M}_1^\sigma$, i.e., satisfies $\delta_z \leq \sigma$. This accounts for both the nonvanishing probability of reaching $\widetilde{M}_1^\sigma$ and the emergence of the Born rule, and it plays a direct role in deriving the Newtonian behavior of macroscopic bodies from **(RM)**.

Newtonian motion and the state of rest of macroscopic bodies presuppose experimentally verifiable knowledge of their positions and velocities at any given moment. Such information is obtained not only through explicit observation, e.g., illumination and detection of scattered light, but also through continual, unavoidable interaction with the environment, which records the body's position and momentum via scattering of ambient particles and radiation. For Newtonian motion to hold without referring explicitly to the environment, these interactions must be weak yet not absent; without them, neither position nor momentum would be well defined, and the notion of Newtonian motion would lose its operational meaning.

Moreover, this effective "continuous" measurement, together with the small diffusion coefficient of the associated Brownian motion, allows the body's Newtonian trajectory to remain well defined over long times without significant growth of positional uncertainty. Encounters with ambient particles or radiation cause the position distribution to spread, but it contracts again whenever these interactions supply new positional information. This alternating cycle of spreading and contraction maintains the stability of the observed Newtonian trajectory.

Although the normal distribution allows rare, large deviations from the Newtonian trajectory, such events are extremely unlikely and fall within the tolerance of classical measurements. We now show that, with appropriate parameter choices, the walk in **(RM)** combined with free Schrödinger evolution reproduces this behavior on the full state space, extending the classical dynamics defined on $M_3^\sigma = \mathbb{R}^3$ (or $M_1^\sigma = \mathbb{R}$) and simultaneously yielding Newtonian motion for macroscopic bodies.

Let us assume that the contributions of the free Hamiltonian $\widehat{h}$ and the random Hamiltonian $\widehat{h}_{\rm RM}$ in **(RM)** can be treated separately. For instance, during the instants when $\widehat{h}_{\rm RM}$ acts, the contribution from $\widehat{h}$ may be comparatively small, so the two operators effectively alternate in time and the evolution operator factorizes. The evolution of the state during measurement is then governed by the total Hamiltonian $\widehat{h}_{\rm tot} = \widehat{h} + \widehat{h}_{\rm RM}$.

Assume the initial state lies in $\widetilde{M}_{1,1}^\sigma$. Choose the time step dt and Gaussian step variance $(dz)^2$ in **(RM)** so that many steps N occur within a short interval $\Delta t = Ndt$, while the diffusion coefficient $D = (dz)^2/dt$ remains small. Then diffusion on $\mathbb{R}^2$ is suppressed, and Schrödinger spreading over Δt produces only a small change in s, keeping the state on or near $\widetilde{M}_{1,1}^\sigma$ and ensuring that the decomposition (10) applies to $\widehat{h}$. In this regime, the evolution of the state on the (τ, s)-plane during Δt has a simple structure: $\widehat{h}$ generates a Newtonian shift in τ together with a small drift in s, while $\widehat{h}_{\rm RM}$ contributes a suppressed Gaussian displacement in $\mathbb{R}^2$.

A large N also ensures that the condition $\delta_z \leq \sigma$ (i.e., $s \leq \ln \sigma$) is satisfied repeatedly with probability arbitrarily close to 1. Whenever this occurs, the state lies on $\widetilde{M}_{1,1}^\sigma$, and the particle exhibits classical behavior for as long as it remains within the corresponding equivalence class. If an environmental particle, such as a nitrogen or oxygen molecule in the surrounding air, is likewise represented by a narrow wave packet, which can be achieved by applying the same **(RM)** process to the pair, then their interaction reduces to classical scattering on $\widetilde{M}_{1,1}^\sigma \otimes \widetilde{M}_{1,1}^\sigma = \mathbb{R}^2 \times \mathbb{R}^2$, and a Hamiltonian such as (8), restricted to this manifold, becomes the classical Hamiltonian of the pair. Particles scattered in this Newtonian manner continually record the body's position and momentum in the environment.

Environmental monitoring repeatedly resets the evolution under $\widehat{h}_{\rm tot}$ whenever the state enters the region $s \leq \ln \sigma$. Because the Schrödinger drift in s is small, the random walk remains approximately symmetric in this direction, so the conditional probability of finding the state in $s \leq \ln \sigma$ a time Δt after each reset remains $\approx 1/2$. Consequently, the state occupies $\widetilde{M}_{1,1}^\sigma$ for roughly half the evolution, with stochastic returns continually reinforced by the environment.

This behavior is easily visualized on the (τ, s) plane: the state executes a random walk on $\mathbb{R}^2$ with a Newtonian drift along the τ-axis and a small drift in s. The environment acts as a detector array that registers a position whenever the walk enters $s \leq \ln \sigma$ (i.e., when the state lies on $\widetilde{M}_1^\sigma$), after which the walk restarts from the recorded value of τ. The sequence of recorded positions τ_m forms a set of narrow conditional distributions whose widths depend on the time since the previous detection. Note also that registering the state when it lies on $\widetilde{M}_1^\sigma$ does not constitute collapse: the state is already reduced, and the detector merely confirms this with certainty.

This reproduces the behavior of a macroscopic body, except that the random walk occurs in a neighborhood of $\widetilde{M}_{1,1}^\sigma$ within the full state space rather than being confined strictly to the classical space $\widetilde{M}_1^\sigma$ or the classical phase space $\widetilde{M}_{1,1}^\sigma$. Successive returns to $\widetilde{M}_1^\sigma$ then generate a sequence of recorded positions on $\mathbb{R}$ that are normally distributed around the Newtonian trajectory. The Newtonian dynamics of a macroscopic body under these conditions then follows, including the classical behavior of macroscopic measuring devices themselves.

Taken together, these results show that the same linear Schrödinger dynamics, supplemented by the random-matrix term **(RM)**, yields Born-rule state reduction for microscopic particles and Newtonian trajectories for macroscopic ones. The quantum-to-classical transition thus emerges as a change of regime within a single dynamical model rather than from different underlying laws. In particular, the effective confinement of a macroscopic particle's state to a classical phase-space submanifold is not an added postulate but an emergent feature of the combined drift-diffusion dynamics together with conditioning on detection events.

As in Brownian motion on $\mathbb{R}^3$, the time-step and variance parameters of the walk in **(RM)** may vary with the properties of the body, the environment, and their interaction. In deriving Newtonian motion, we assumed many frequent, sufficiently small steps so that the diffusion coefficient D is negligible. With appropriate tuning of these parameters, the motion of microscopic particles in various media can be described within the same dynamical framework. In particular, the observed similarity between the tracks of microscopic particles in a bubble chamber and the trajectories of macroscopic bodies in natural environments may reflect comparable parameter regimes.

Conversely, modeling the position measurement of a microscopic particle under typical laboratory conditions requires a very small time step dt but larger step sizes dz, and hence a larger diffusion coefficient D. The resulting random walk spreads into the full state space, yet after a short time the state still has probability $\approx 1/2$ of lying on $\widetilde{M}_1^\sigma$. At those moments, the interaction with the measuring device is effectively Newtonian, producing a classical measurement record. If the measurement interval is sufficiently short, the contribution of the free Hamiltonian

$\widehat{h}$ may be neglected, eliminating the associated drift.

A detailed analysis of the influence of these parameters on the motion of microscopic and macroscopic bodies will be presented in a subsequent work. The reader is referred to [18] for the analysis of the double-slit experiment with and without measurement by the slits in this framework.

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* kryukov@uwm.edu

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