Probability-Distribution Determinism: A Logical Reconstruction from Determination Relations to Causal Language

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Abstract

Classical determinism typically construes “determination” as “a single outcome being determined,” and consequently tends to regard probabilistic phenomena as challenges to determinism. Existing causal theories analyze causal relations respectively in terms of counterfactual dependence, interventional invariance, and structural models; this paper asks the further question of what form is taken by the determination relation presupposed by these analyses. To answer it, the paper proposes Probability-Distribution Determinism (PDD), which defines the general form of “determination” as follows: when the full determining conditions obtain, the resultant state of affairs is uniquely determined in the form of a probability distribution. The degenerate distribution in classical determinism, which assigns probability 1 to one outcome, is not abolished but repositioned as a special case of this more general form.

To avoid mistaking epistemic tools for metaphysical structures, this paper first distinguishes three levels: pure determination relations, physical ontology, and epistemic tools. Under this level distinction, PDD is situated primarily as a determination-relation structure at the metaphysical/logical level. Its theoretical value lies in the fact that this structure can also serve, at the epistemic level, as a regulative framework for understanding the empirical world and provide a conceptual perspective on relation types, background conditions, and causal language in empirical research.

The main contributions of this paper are threefold. First, PDD reconstructs the determined object from a single outcome into a resultant state of affairs expressed as a probability distribution, and on this basis analyzes the atemporality, multiple realizability, and unidirectionality of determination relations, together with the compositional forms of PDD relation networks. In chain structures with sustained probabilistic branching and no reconvergence, increasing logical distance is usually accompanied by dilution in the concentration of distal outcomes, while the complete probability distribution remains uniquely determined by the full determining conditions. Second, the paper distinguishes determination relations from whole-part constitutive relations and explains how conflating the two may produce misleading transitivity and category mismatches in graphical models. Third, it first analyzes how background-condition stability affects the epistemic representation of determination relations, then distinguishes general relations from retrospective explanation and responsibility attribution in terms of epistemic aim, and argues that Simpson’s paradox is more accurately described as the conflation, as one determination relation, of relation labels arising under different background conditions.

PDD thereby provides a framework for metaphysical relational analysis and conceptual audit, allowing debates about the nature of causation to be reformulated as analyses of determination-relation structures, background conditions, relation types, and the contextual adaptation of causal language. It does not replace specific methods of causal inference or statistics, but offers a conceptual perspective from which to understand the relational structures presupposed by those methods and the boundaries of their application.

Keywords: Probability-Distribution Determinism (PDD), determination relation, complete background conditions, constitutive relation, contextual adaptation of causal language, Simpson's paradox

1 Introduction

Causal theories seek to explain how certain conditions are related to outcomes, but different theories do not characterize exactly the same relational function. The Humean tradition emphasizes constant conjunction in experience (Hume, 1739); counterfactual theories focus on dependence across situations (Lewis, 1973); interventionism emphasizes invariance of response under intervention (Woodward, 2003); and structural causal models represent relations among variables through equations and directed graphs (Pearl, 2009).[1] Each captures important features of causal inquiry. Before these features can be compared, identified, or represented, however, a more basic question arises: in what form does a set of conditions determine an outcome? Unless this question is clarified separately, epistemological tools such as counterfactual comparison, interventional response, and graphical representation can easily be placed at the same level of analysis as the determination-relation structure on which they depend, while the different functions of the term "cause" in general relations, specific explanations, and responsibility attribution can likewise be conflated.

This paper argues that answering this basic question should begin neither by seeking a single essential definition of "causation" nor by merely adding probabilistic conditions to traditional determinism, but by first analyzing the concept of "determination" itself. The conventional view typically construes determination as a single outcome being determined, thereby making determinism and probability theory appear mutually incompatible. Probability-Distribution Determinism (PDD), as proposed here, instead holds that the more general form of a determination relation is the functional determination, by the full determining conditions, of a resultant state of affairs expressed as a probability distribution.

To clarify the theoretical position of this paper, a limited analogy from computer science may be useful. The counterfactual theory of Lewis (1973), the interventionism of Woodward (2003), and the SCM framework of Pearl (2009) characterize causal relations respectively in terms of counterfactual dependence, interventional invariance, and structural models. The PDD framework developed here asks a further question: what form is taken by the underlying determination relation presupposed by these analyses? This paper does not develop another technique of causal inference; it provides a framework for metaphysical relational analysis.

The core of this relational protocol is the completeness of the full determining conditions. This paper uses $[C]$ to denote the full determining conditions under which the resultant state of affairs is uniquely determined; formally, $[C]$ can be analyzed as a combination of the condition-side state of affairs and complete background conditions, i.e., $[C] \equiv [S(X), \Omega]$. Here and throughout the paper, "condition-side state of affairs" is a technical term for the state of affairs represented by $S(X)$ on the determining side of the mapping; Ω denotes the complete

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[1] For a comprehensive discussion of the principal approaches in contemporary theories of causation, see Beebee, Hitchcock, and Menzies (2009).

background conditions required for the determination relation to hold. The term “complete” does not refer to a list of observable, enumerable control variables in empirical research, but rather means that the full determining conditions jointly constituted by $S(X)$ and Ω are given in a metaphysical/logical sense. Covariates, stratification variables, or control terms in actual research can only be understood as the cognitive agent’s finite approximations of these background conditions, and cannot be equated with Ω itself.

To achieve this theoretical objective, the argument of this paper strictly follows a four-tier logical progression:

1. **Methodological distinction tier**: Distinguish between the ontological properties of the physical world, epistemic detection tools, and the purely formal determination-relation structure, thereby completing a preliminary conceptual clarification.
2. **Theoretical position and regulative application tier**: Explain that PDD is primarily a relational structure at the metaphysical/logical level, which can simultaneously serve as a regulative framework to organize the relationships among physical determinacy, empirical explanation, and non-PDD possibilities, rather than asserting rigid ontological claims.
3. **Metaphysical definition tier**: After completing the level distinctions, define the PDD relation at the purely formal level. Temporarily set aside “causal” intuitions, reducing them to functional mappings from full determining conditions to states of affairs expressed as probability distributions.
4. **Contextual language-adaptation tier**: After the formal definition is established, trace how the linguistic label “cause” is contextually adapted to the underlying relational structure: at the general-relation level, the adaptation is primarily to $S(X)/S(Y)$; at the responsibility-attribution level, the adaptation is primarily to $\Delta S(X)/\Delta S(Y)$.

This strategy of first distinguishing the logical level and then discussing physical and epistemological adaptation not only provides a new explanatory pathway for the traditional tension between determinism and probability theory, but also yields three specific contributions to contemporary causal science:

First, in terms of formal analysis, PDD provides relation-type criteria for distinguishing “determination relations” from “constitutive relations (part-whole).” This paper will argue that it is precisely the neglect of this distinction that makes current mainstream structural causal models (SCM) prone to relation-type misjudgments in transitivity analysis and effect-size calculation.

Second, in terms of philosophical methodology, PDD facilitates a shift from “seeking essential definitions” to “analyzing the functional differentiation and context-sensitive adaptation of causal language.” When causal theories encounter paradoxes or boundary cases, PDD provides an analytic framework for returning to the level of relational structure and examining relational slippage or category mismatch. This approach also resonates with existing discussions of the plurality of causal concepts and relations (Cartwright, 2004; Hall, 2004); Section 5 further develops the analysis of how different linguistic functions adapt to context.

Third, at the methodological level of causal modeling, PDD prompts us to examine the conceptual validity of graphical models, effect-size calculations, and causal discovery in terms of full determining conditions, complete background conditions, and relation-type distinctions. Accordingly, Simpson’s paradox, category mismatches in graphical models, and the condition-sensitivity of regularity/law language are no longer merely statistical or semantic concerns, but can be reformulated as mismatches among background placement, relation types, and the adaptation of causal labels.

It should be made clear that the task of this paper is limited to conceptual clarification and logical reconstruction at the metaphysical level. It does not aim to propose a causal-inference algorithm or statistical estimation formula that can be applied directly to a dataset. What it offers is a conceptual framework for explaining what form of association between states of affairs constitutes a determination relation. How cognitive agents capture and infer such a purely logical mapping from limited data is an epistemological and methodological question outside the paper's principal argument.

The structure of this paper is as follows. Section 2 sets out the methodological principle of the "three-level distinction" and explains the theoretical position of PDD. Section 3 derives the formal definition of PDD through an internal analysis of the concept of determination and explains why the determined state of affairs in PDD must be expressed as a probability distribution. Section 4 examines the structural features and boundaries of PDD relation networks, analyzing the temporal independence of determination, multiple realizability, unidirectionality, compositional operations, and the relation between logical distance and outcome concentration. It then distinguishes determination relations from constitutive relations and diagnoses interruptions of transitivity and category mismatches in graphical models. Section 5 presents the logic of contextual adaptation in causal language, first analyzing background-condition stability and then distinguishing the epistemic aims of general relational inquiry and retrospective explanation and responsibility attribution. Section 6 discusses existing causal theories, puzzles of responsibility attribution, and Simpson-style background-condition conflation, and explains the limits of graphical models and causal-discovery methods in its examination of Pearl/SCM. Section 7 concludes.

2 Level Distinction and Methodological Principles

2.1 The Three-Level Distinction: Pure Logic, Physical Ontology, and Epistemic Detection

In the analysis of causal concepts, three distinct levels of inquiry must be rigorously distinguished: first, the determination relation at the purely metaphysical-logical level (PDD vs. non-PDD); second, the ontological level of the physical world (how the world fundamentally operates); and third, the epistemological level at which agents identify knowledge (how we come to know). The persistent confusions in the Western history of causal concepts often originate from conflating these three levels—particularly from mistaking epistemological criteria for metaphysical or ontological claims.

The level-distinction strategy adopted in this paper is as follows: first delineate the structure of the determination relation at the purely logical level, and then distinguish the level-relationships between this structure, physical ontology, and epistemic detection. Under this strategy, the pure PDD relational structure is not an object that can be directly perceived; what human beings can observe at the epistemic level are typically only empirical traces left by data and shifts in probability distributions. Precisely for this reason, intervention, statistics, and counterfactual comparison should be understood as detection tools, rather than as the metaphysical definition of the determination relation itself.

To gain an intuitive understanding of this level distinction, one may draw on the example of a program mapping: in a virtual program system, relations among states of affairs can take the form of conditional mappings based on *if-then* functions. Even without appealing to physical mechanisms such as energy or momentum transfer, stable formal correspondences can obtain

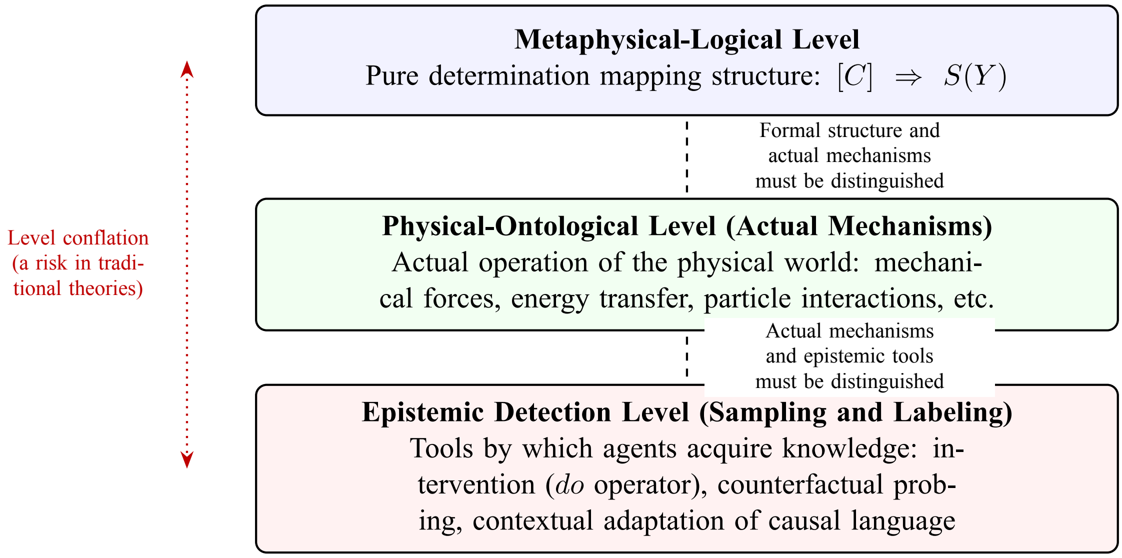

Figure 1: The level-distinction architecture of causal logic: the PDD framework distinguishes the pure determination mapping structure from physical ontology and epistemic detection tools. A major risk of traditional theories lies in level conflation—that is, mistaking epistemic detection tools for metaphysical definitions.

between states. This example shows only that the formal structure of a determination relation can be analyzed independently of a specific physical mechanism; it does not imply that the paper has established that real physical mechanisms are implementations of PDD structure.

2.2 The Theoretical Position of PDD: Intrinsic Structure and Cognitive Application

A clear distinction must be drawn between PDD's "intrinsic theoretical structure" and the "epistemic application of the theory." At the metaphysical-logical level, PDD itself is a formal relational system whose determined objects are states of affairs; at the epistemic level, cognitive agents employ PDD as an **Epistemic Regulative Model**.

What is meant by "regulative" is not a direct assertion that the ultimate ontological structure of the external world necessarily conforms entirely to PDD, nor that the determination relation itself depends on cognitive agents; rather, while suspending such a judgment, PDD is adopted as a framework assumption for organizing empirical understanding, conducting relation-type audits, and analyzing the contextual adaptation of causal language. Section 5 below will further demonstrate that this epistemic application encompasses at least two distinct functions: the general relation function of tracking stable mapping structures, and the function of responsibility attribution in specific contexts. Both presuppose the same underlying determination relation structure, but serve different linguistic and explanatory tasks.

In this sense, what PDD calls "determination" is neither the name of an empirical mechanism nor a mere syntactic logical necessity, but rather a formal constraint given the full determining conditions. When the condition-side state of affairs $S(X)$ and the complete background conditions Ω jointly constitute $[C] \equiv [S(X), \Omega]$, the probability distribution determined by the mapping possesses objective binding force; if Ω or the condition-side state of affairs is redefined, the full determining conditions and their mapping structure change accordingly.

2.3 The Methodological Predicament: Level Conflation

The core methodological contribution of the PDD framework lies in providing an analytical tool for diagnosing deep-seated problems in existing theories. Surveying the development of contemporary causal theories, many approaches readily fall into a methodological predicament: level conflation.

For example, Hume's analysis of constant conjunction emphasizes regularity in experience (Hume, 1739); counterfactual theory analyzes event causation through dependence across situations (Lewis, 1973); and interventionism characterizes causal relations and causal explanation through intervention and invariance (Woodward, 2003, 2016). These theories have each offered important analyses of causation. PDD asks a further question: what metaphysical/logical structure is taken by the determination relation presupposed by these regularities, dependencies, and invariances? Unless these two questions are distinguished, epistemological criteria, model representations, or linguistic labels may be treated directly as complete definitions of the determination relation itself.

This conflation gives rise to a systematic theoretical predicament: elevating epistemic detection tools to the status of metaphysical definitions readily encounters difficulties when facing uncertainty (e.g., the quantum domain) or complex constitutive relations (e.g., part-whole nesting), because the effectiveness of detection tools always depends on specific operational conditions, whereas the purely logical relational structure does not.

PDD's level-distinction strategy is designed precisely to correct this level misalignment: first establish the purely formal logical structure of the determination relation, and then employ it as a regulative framework for the contextual adaptation of causal language—rather than proceeding in the reverse direction. This strategy helps to restate the traditional tension between determinism and probability theory, and provides causal theory with a meta-diagnostic tool for returning to the relational-structural level to audit relational slippage or category mismatch.

3 The Metaphysical Definition of Probability-Distribution Determinism

3.1 Reconstructing the Essence of Determination: From Single Outcomes to States of Affairs Expressed as Probability Distributions

Traditional philosophy has long equated "determination" with "a single outcome being determined." This Laplacean form of classical determinism (Laplace, 1902) holds that, given the full determining conditions $[C]$, there must be one and only one definite state of affairs o that obtains. Symbolically:

$$[C] \Rightarrow \{o\} \tag{1}$$

This understanding captures an extremely clear-cut form of the determination relation, yet it should not be directly equated with the universal form of determination. The problem is not that "a single outcome being determined" is itself invalid, but that it is merely a restrictive case of the determination relation. With the development of modern physics (e.g., quantum mechanics) and complexity science, it has often been assumed that determinism fails in the face of fundamental probabilistic phenomena, and that "randomness" must be introduced to correct classical determinism. However, this judgment presupposes an unexamined premise: that genuine determination exists only when a single outcome is determined as the unique actuality.

To examine this premise, we need to distinguish the possible forms that the concept of "determination" may internally contain. The first form is **a single outcome being determined**: the full determining conditions determine a particular outcome with probability 1, i.e.,

$$[C] \Rightarrow \{y^*\}, \quad P(y^*|[C]) = 1, \quad P(y|[C]) = 0 \text{ for all } y \neq y^*. \tag{2}$$

Classical determinism operates primarily at this level. The ideal trajectory of a macroscopic classical particle or state transitions in a deterministic computer program serve as paradigmatic examples of this form.

The second form is **a probability value being determined**: the full determining conditions do not determine that a particular realizer necessarily obtains, yet they do determine the probability value of a particular outcome. For example:

$$[C] \Rightarrow P(Y = y) = p, \quad 0 < p < 1. \tag{3}$$

In this case, the single-trial outcome remains uncertain, but the probability value itself is neither floating nor arbitrary—it is determined by the full determining conditions. Radioactive decay probability provides an intuitive illustration: the relevant physical conditions as a whole can determine the half-life or decay probability, yet they do not determine that a particular atom necessarily decays at a particular moment.

The third form is **a state of affairs expressed as a probability distribution**: the full determining conditions determine neither a single outcome nor merely the probability value of a particular outcome, but rather the probability-distribution expression of the resultant state of affairs:

$$[C] \Rightarrow S(Y). \tag{4}$$

Here $S(Y)$ specifies the probabilities of the various possible outcomes of Y. Genetic inheritance probabilities, statistical mechanics predictions, and quantum measurement outcomes all belong more naturally to this level.

An individual probability value indicates only the probabilistic weight assigned to one possible outcome; it cannot simultaneously specify the other possible outcomes and their weights. This incompleteness is especially evident for multivalued discrete and continuous states of affairs. A probability distribution incorporates the range of possible outcomes and their corresponding weights into a single expression, and is therefore sufficient to serve as the complete formal expression of the resultant state of affairs.

These three forms are not coordinate categories but constitute an analytic sequence progressing from a restrictive form toward the general form. A single outcome being determined can be understood as the special case in which the probability distribution degenerates into a point distribution; a probability value being determined is a local expression manifest at the level of a particular outcome, a binary result, or a partial marginal probability. Only the third form yields the complete object of the determination relation: not an isolated outcome, nor a single probability value, but a resultant state of affairs expressed as a probability distribution. It follows that traditional determinism is not abolished by PDD but rather repositioned as the limiting case of probability-distribution expression.

Accordingly, the core argument of PDD does not consist in appending probabilistic conditions to classical determinism, but in analyzing the internal levels of the concept of determination itself to derive a more general form: **the fundamental object of a determination relation should be a state of affairs; within the theoretical context of PDD, that state of affairs has a probability distribution as its complete formal expression.** The general form of a determination relation is a functional mapping from the full determining conditions to a state of affairs expressed as a probability distribution. This requires us to abandon the intuition that "the presence of probability entails the absence of determination" and to raise the object of determination from a single outcome to a state of affairs expressible as a probability distribution. Before presenting the formal definition of PDD, it is necessary to explain why the determined state of affairs must be expressed as a probability distribution.

3.2 The Object of Determination in PDD: States of Affairs

In a PDD relation, what is genuinely determined is not an isolated entity, a variable name, or a single event, but a state of affairs. The resultant state of affairs does not refer only to a single state that has already been realized; it is the complete modal description constituted by the possible outcomes and their corresponding probabilistic weights. $S(Y)$ is the mathematical expression of this resultant state of affairs. Thus, within the PDD formal system, the resultant state of affairs is expressed not as a single outcome but as a probability distribution.[2]

This also explains why the outcome of PDD cannot be directly understood as a "thing," "entity," "variable," or "event." Isolated "things" or "entities/objects"—such as "a stone" or "a piece of glass"—are merely object names and cannot express configurations of properties and relations such as "impact," "fracture," "velocity," or "intensity." Variables are merely formal expressions and cannot be equated with the determined content in reality. The concept of "events," for its part, inherently carries a temporal-sequential connotation, which risks importing temporal succession into the atemporal functional mapping that PDD aims to characterize. By contrast, "states of affairs" can express the objects, properties, relations, and state configurations in which the outcome is situated, and thus serve more appropriately as the content expressed by $S(Y)$. The early Wittgenstein's insight that the world can be analyzed in terms of states of affairs rather than isolated things (Wittgenstein, 1922) is suggestive for this choice; however, in the present paper, the selection of states of affairs to explicate $S(Y)$ derives from PDD's own analytic requirements concerning probability distribution mapping, the completeness of full determining conditions, and type-checking.

It should be added that to say the state of affairs of outcome Y is expressed as $S(Y)$ does not imply that the world itself has already been naturally carved up in the variable-form presented in this paper. Whether a given object, process, or state configuration can be identified as a state of affairs has an epistemological dimension: cognitive agents always individuate the continuous complexity of reality under particular analytic purposes, discriminative capacities, and conceptual frameworks. Nevertheless, such individuation is not arbitrary naming. It is typically constrained by identifiable differences between the object and its environment, the relative stability of state configurations, and the discriminative standards shared by an epistemic community. For this reason, "state of affairs" avoids both the error of writing the determined outcome as an isolated entity and the error of prematurely temporalizing the determination relation into an event sequence; it indicates the state configuration that can be analyzed, compared, and redefined at different granularities. How different granularities of state-of-affairs individuation affect the boundary between determination relations and constitutive relations will be discussed in Section 4.

This allows us to state more clearly what PDD means by the "object" of determination.

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[2] This paper does not adjudicate among objective, subjective, or other interpretations of probability. PDD stipulates that, within its formal system, the full determining conditions uniquely determine the probability-distribution expression of the resultant state of affairs. Probability estimates formed by researchers under limited information are epistemic representations of the relevant relation and cannot be directly equated with the determined distribution at issue here.

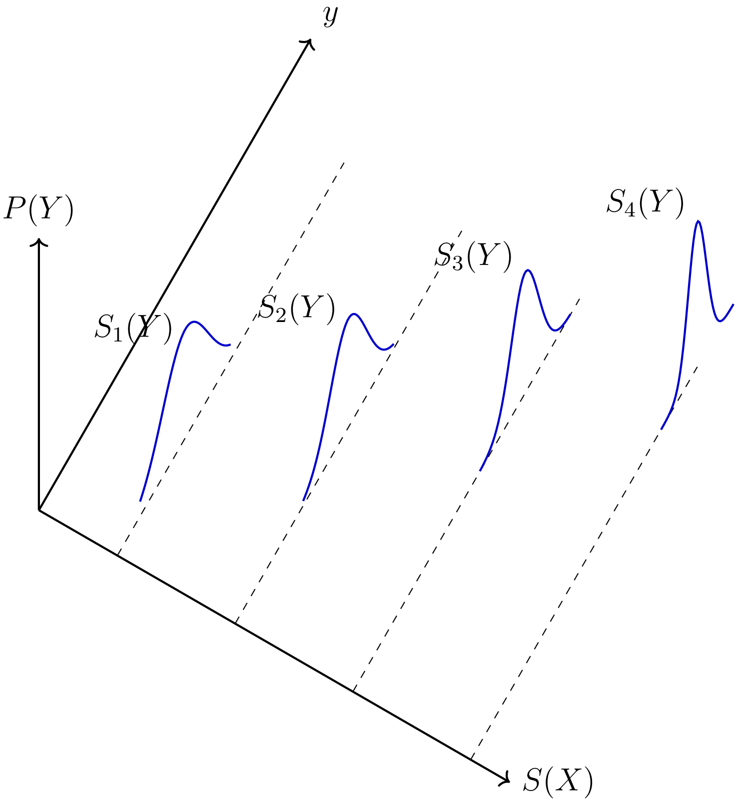

Figure 2: An intuitive representation of PDD: given the same complete background conditions Ω, different condition-side states of affairs $S_1(X) \ldots S_4(X)$ together with Ω constitute different full determining conditions $[C_1] \ldots [C_4]$, and determine different resultant states of affairs; $S_1(Y) \ldots S_4(Y)$ in the figure are the probability distribution expressions of these resultant states of affairs

Given the same complete background conditions Ω, different condition-side states of affairs $S_i(X)$ together with Ω constitute different full determining conditions $[C_i] \equiv [S_i(X), \Omega]$, and determine different resultant states of affairs; these resultant states of affairs can be expressed respectively as probability distributions $S_i(Y)$. The purpose of Figure 2 is to provide an intuitive illustration of PDD's basic idea: when the condition-side states of affairs differ, the full determining conditions they constitute differ, and the resultant states of affairs they determine differ accordingly.[3]

3.3 The Basic Definition of PDD

Having clarified the object of determination in PDD, we can now directly state the basic definition of PDD determinism. PDD stipulates a metaphysical/logical form: given the full determining conditions $[C]$, the state of affairs of outcome Y is not determined in the manner of a single outcome, but is uniquely determined in the form of a probability distribution $S(Y)$.

PDD inherits the formal requirement of classical determinism: when the full determining conditions obtain, the determined object is unique. However, it changes the type of the deter-

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[3] The foregoing analysis describes the relation, within the PDD formal system, between full determining conditions and resultant states of affairs, rather than our epistemological access to that relation. Here a probability distribution is the theoretical form in which a resultant state of affairs is expressed; the paper does not thereby assert that the ultimate ontological structure of the actual world must itself be a probability-distribution structure.

mined object—from a single outcome to a state of affairs expressed as a probability distribution.

Definition 3.1 (Probability-Distribution Determination)**.** Given the full determining conditions $[C]$, if when $[C]$ obtains, the state of affairs of outcome Y is uniquely determined in the form of a probability distribution $S(Y)$, then $[C]$ is said to determine $S(Y)$.

This definition formalizes the analytic result of the preceding subsection: the object of the determination relation is not a single fact, but a resultant state of affairs expressed as a probability distribution. The name *Probability-Distribution Determinism* refers specifically to the thesis that the full determining conditions uniquely determine a complete probability distribution. It should not be confused with the broader and variably used label *probabilistic determinism*, which may refer more generally to views that combine probabilistic laws or predictions with some notion of determinacy.[4] The classical definition becomes the special case in which $S(Y)$ assigns probability 1 to one outcome and probability 0 to all others. In subsequent relational analyses, the full determining conditions $[C]$ can be further analyzed as the combination of the condition-side state of affairs and the complete background conditions, i.e., $[C] \equiv [S(X), \Omega]$; but this decomposition pertains to the explication of the internal structure of $[C]$, and does not belong to the minimal definition of PDD determinism.

The full determining conditions may contain both the condition item singled out for analysis and background or supporting conditions required for the mapping to obtain. That a condition formally belongs to $[C]$ does not mean that it should be called a "cause" in every context of specific explanation or responsibility attribution. Its linguistic status must be determined in light of the background stability and epistemic aim discussed in Section 5.

Here, "full determining conditions" names the role played by $[C]$ in the formal relation; it does not mean that researchers have already identified an exhaustive list of variables through empirical procedures. How such conditions can be identified or approximated under finite epistemic conditions is an epistemological question distinct from the definitional task of this section.

To gain an intuitive understanding of this "probability distribution mapping," we may borrow the metaphor of a "loaded die": traditional determinism attempts to predict the specific number the die will land on (which is untenable in a probabilistic world); PDD, by contrast, is analogous to defining the physical structure of the die after it has been "loaded." Even though the outcome of any single throw remains random, the map describing "the probability of each number appearing" is fixed by the given full determining conditions. From this vantage point, what determinism determines is no longer an isolated "number," but the entire "structure of possibilities."

3.4 The Boundaries of PDD: Non-PDD States of Affairs and the Problem of Free Will

Having defined PDD relations, it is also necessary to specify the boundaries of applicability of this framework. PDD does not presuppose that all states of affairs in the universe necessarily

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[4] A minimal distinction from probabilistic theories of causation is also required. Suppes (1970) characterizes probabilistic causal relations through such concepts as probability raising and spurious causes; Eells (1991) further compares the probabilistic influence and causal significance of causes across different background contexts; and C. Hitchcock (2021); C. R. Hitchcock (1993) characterizes causal relevance through conditional-probability functions, so that the analysis is not restricted to comparisons of single probability values. PDD's distinctiveness does not lie in being the first theory to consider complete probabilistic information. It lies in specifying a state of affairs expressed as a probability distribution as the determined object and in defining the unique determination of that state of affairs by the full determining conditions as a metaphysical/logical relation. This paper does not develop an epistemological comparison among these theories.

fall within the PDD domain; what it primarily provides is a formal criterion for judging whether a given class of states of affairs can be determined by full determining conditions. Accordingly, one can conceptually distinguish PDD-type states of affairs from non-PDD-type states of affairs.

In this sense, states of affairs can in principle be divided into two classes:

1. **PDD-type states of affairs:** The state of affairs can be uniquely determined in the form $S(Y)$ by some full determining conditions $[C]$.
2. **Non-PDD-type states of affairs:** The state of affairs **cannot** be determined in the form $S(Y)$ by any full determining conditions $[C]$—that is, there exists no $[C]$ such that $[C] \Rightarrow S(Y)$ obtains.

This distinction has suggestive implications for the problem of free will, although this paper does not take free will as a primary object of argument. It merely indicates the following: if there exists a class of states of affairs whose state changes cannot be uniquely determined by any full determining conditions, then such states of affairs can be formally described as non-PDD-type states of affairs; if such states of affairs can moreover serve as antecedent conditions participating in determination mappings for other states of affairs, then they retain a possible position of autonomy outside the PDD framework. Free will can thus be reformulated as the question of whether the state evolution of a given agent or system can be reduced to PDD mappings. The subsequent discussion in this paper will remain focused on PDD relations themselves, without developing a full theory of free will.

4 Structural Features and Boundaries of PDD Relation Networks

Section 3 presented the basic definition of PDD: when the full determining conditions obtain, the resultant state of affairs is uniquely determined in the form of a probability distribution. This section further examines the relation-network structure that follows from that definition. First, a determination relation does not depend on temporal succession. Second, because the probability-distribution expression of the same resultant state of affairs can be realized by different full determining conditions, the determination relation is unidirectional. Third, multiple determination mappings can be composed into relation networks, and the continued nesting of probabilistic links usually distributes probability mass across a larger number of possible paths. Finally, such composition does not hold without restriction: once a proposed chain passes through a whole-part constitutive relation, the transitivity of the determination relation must be interrupted.

4.1 The Atemporality of the Determination Relation

First, the functional definition of PDD mapping entails that the logical direction of determination need not coincide with the direction of time. The determination relation structure $[C] \Rightarrow S(Y)$ defines a logical functional determination relation, not a physical relation of temporal production.

The clearest examples of non-temporally-ordered mappings come from programs or mathematical functions. Suppose a program's parameters, function rules, and inputs jointly constitute the full determining conditions $[C]$, and the program output is $S(Y)$. Even if the program has not yet actually run in time, as long as the rules and inputs have been given, the output structure is logically determined. Actual execution is merely the unfolding of this logical mapping in a temporal sequence, not the source of the determination relation itself.

Example 4.1 (Program Mapping Case)**.** Consider a simple program: given input x, parameters a, b, and function rule $f(x) = ax+b$, the output y is logically determined by $[x, a, b, f]$. We need not wait for the program to actually execute in order to derive the output from the rule structure; even though running the program on a physical machine requires time, the determination relation $[x, a, b, f] \Rightarrow y$ does not depend on this execution process. The same holds for probabilistic programs: given a random seed, probability rules, and parameters, what is determined may not be a single output value, but an output state expressed as a probability distribution $S(Y)$.

This shows that "determination" within the PDD framework is an atemporal logical relation. What possesses temporal order is the physical realization process, the cognitive agent's access to the relevant conditions, and the observations established over sequences of states of affairs—not the determination relation structure itself. The first structural property thereby obtained is: **the determination relation does not depend on temporal succession**. However, atemporality itself shows only that a determination relation does not depend on temporal properties. The direction of a determination relation and its irreversibility must be further explained by the ordered form and multiple realizability of PDD mappings.

4.2 Multiple Realizability and Unidirectionality

From the mapping form of PDD, a second core property of the determination relation can be further derived: multiple realizability. Since PDD mappings associate full determining conditions with resultant states of affairs expressed as probability distributions, the same resultant state of affairs expressed as a probability distribution $S(Y)$ can be realized by different full determining conditions. Formally, there may exist different $[C_1]$ and $[C_2]$ such that:

$$[C_1] \Rightarrow S(Y), \quad [C_2] \Rightarrow S(Y), \quad [C_1] \neq [C_2]. \tag{5}$$

This is not an additionally stipulated definition but a natural consequence of the functional mapping structure of PDD.

The direction of a determination relation arises first from the ordered form of a PDD mapping: the full determining conditions $[C]$ occupy the determining side, while the resultant state of affairs $S(Y)$ occupies the determined side. Multiple realizability further explains why this ordered relation usually cannot be reconstructed in reverse. Given any full determining conditions $[C]$, the resultant state of affairs is expressed by a unique probability distribution $S(Y)$; but because the same $S(Y)$ can be realized by different $[C]$, one ordinarily cannot uniquely infer the full determining conditions from $S(Y)$.

The second structural property is therefore: **PDD mappings are directional, and multiple realizability usually makes reverse derivation non-unique**. This irreversibility need not appeal to entropy increase or the passage of time; it follows from the combination of an ordered mapping and non-unique inverse images. Counterfactual analysis can help compare differences between conditions in a particular context, but it does not thereby become the definition of the determination relation itself. Having established the directionality of individual mappings, the next step is to explain how multiple PDD mappings compose into larger relational networks.

4.3 Combination Logic of the PDD Relational Network

PDD relations are not isolated arrows; once multiple mappings share complete background conditions or outcome items, they form a network. Where the relevant conditions are mutually compatible and relation types remain consistent, four schematic forms can illustrate how such a network is composed:

Operation 1: Irrelevant-item addition. If $[C] \Rightarrow S(Y)$ holds and item N is genuinely irrelevant to that mapping, incorporating N into the description does not change $S(Y)$. This form shows that a determination relation can accommodate redundant information, without treating irrelevance itself as an unexplained prior conclusion.

Operation 2: Parallel merging. If $M_1 : [C_1] \Rightarrow S(Y_1)$ and $M_2 : [C_2] \Rightarrow S(Y_2)$, and the two mappings are compatible and mutually independent under the relevant background conditions, they may be jointly represented as $[C_1, C_2] \Rightarrow S(Y_1, Y_2)$.

Two structures that are easily obscured by the same “parallel” diagram must also be distinguished. The first is **independent parallel determination**: $[C_A] \Rightarrow S^*(Y)$ and $[C_B] \Rightarrow S^*(Y)$ each hold, so that either conditional path is independently sufficient to determine the same target resultant state of affairs. The second is **joint determination**: only when A and B jointly enter the full determining conditions does $[A, B, \Omega] \Rightarrow S^*(Y)$ hold, while neither A nor B alone is sufficient to determine the target state of affairs. The former permits multiple independently sufficient paths to converge on one outcome; the latter treats multiple conditions as an inseparable joint condition. They must not be conflated in subsequent responsibility allocation or counterfactual comparison.

Operation 3: Serial nesting. If $M_1 : [S(X), \Omega_1] \Rightarrow S(Y)$ and $M_2 : [S(Y), \Omega_2] \Rightarrow S(Z)$, and the background conditions of the two mappings are compatible and no constitutive relation has been introduced, a composite representation from $S(X)$ to $S(Z)$ may be formed. This expresses the composability of relations; it does not commit the theory to the unconditional transitivity of every probabilistic association.

Operation 4: Redundant holization (classification downgrade). This operation is central to distinguishing determination relations from whole-part relations. If full determining conditions containing X determine $S(Y)$ while N is irrelevant to that outcome, a determination statement concerning $S(Y)$ can still be formed when (X, N) is described as a whole, but at a coarser analytic granularity. This provides an operational basis for the following demarcation between determination and constitution.

Together these four forms establish the third structural property: **PDD networks are composable when compatibility and relation-type requirements are satisfied**. They are not universal operational theorems detached from their conditions, but schematic explanations of how multiple determination mappings form larger relational structures. Analysis can thereby proceed from a single PDD mapping to a relation network. Before turning to the boundary between relation types, however, it is necessary to examine the relation between logical distance and the concentration of distal outcomes when probabilistic mappings are nested in sequence.

4.4 Logical Distance and Dilution of Outcome Concentration

Within a PDD relation network, the number of mapping links in the shortest determination chain between two states of affairs may be called their **logical distance**. Logical distance describes structural separation within a relation network, not temporal interval or spatial distance. When multiple probabilistic mappings are connected in sequence, each intermediate state of affairs may contain multiple possible outcomes, which in turn enter subsequent mappings and generate multiple possible paths toward distal outcomes.

Suppose there is a serial relation:

$$[C] \Rightarrow S(Y), \qquad [S(Y), \Omega_2] \Rightarrow S(Z). \tag{6}$$

For a possible distal outcome z, its probability receives contributions from paths passing through different intermediate outcomes y:

$$P(z \mid [C], \Omega_2) = \sum_y P(z \mid y, \Omega_2) P(y \mid [C]). \tag{7}$$

When $S(Y)$ distributes probability mass among multiple intermediate outcomes and those outcomes lead to different distal outcomes, the constraint exerted by the upstream full determining conditions on a particular distal outcome is usually dispersed across more possible paths. This paper calls the resulting reduction in probability concentration on a particular outcome the **dilution of outcome concentration**.

What is diluted is not the determination relation itself. Even when a chain contains multiple probabilistic links, the complete probability distribution of the distal resultant state of affairs remains uniquely determined once the full determining conditions are given; only the concentration of probability mass among possible outcomes changes. If every intermediate outcome in the chain is a point distribution, probability mass is not dispersed by branching, which is one sufficient condition under which dilution does not occur.

Nor is this tendency an unconditional monotonic theorem. Subsequent mappings may reconverge multiple intermediate outcomes on one distal outcome, while absorbing states or deterministic mappings may preserve or even increase the probability concentration of a particular outcome. Increasing logical distance therefore means increasing opportunities for probabilistic branching and path dispersion; it does not mean that the outcome space must expand monotonically or that the probability of every particular outcome must decline. The fourth structural property is thus: **in chain structures with sustained probabilistic branching and no reconvergence, increasing logical distance is usually accompanied by dilution in the concentration of distal outcomes**. How this structural property manifests as observable statistical association, and how it can be identified from finite data, are further epistemic questions not developed here.

4.5 The Distinction between Determination Relations and Constitutive Relations

In analyzing relations among things, we need to distinguish two different types of relations: determination relations and constitutive relations involving whole-part structures. Determination relations connect full determining conditions with the determined resultant state of affairs; constitutive relations connect a whole with its component parts, aspects of its properties, or levels of description, and ordinarily cannot be described in terms such as "causes" or analyzed by methods appropriate to determination relations. If constitutive relations are mistakenly treated as determination relations, analyses of determination chains, effect sizes, and responsibility attribution will all go astray.[5]

The problem of causal transitivity itself contains several different structures. McDonnell (2018) distinguishes relevant counterexamples into switching, short-circuit, and mismatch cases, showing that debates about transitivity cannot be resolved by a single principle. PDD does not attempt to solve every problem of transitivity here. It identifies one type of relation-type error: when an intermediate link actually expresses a constitutive relation but is incor-

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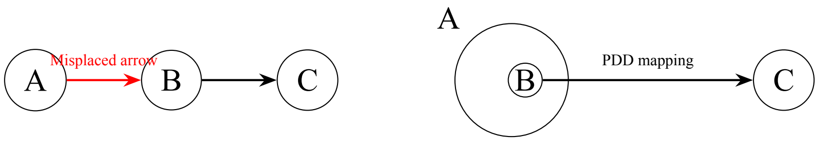
[5] The concern here is with whole-part constitution at the same analytic level and time. In the case of "diachronic constitution" in complex dynamical systems, some research argues that certain constitutive processes can be analyzed through continuous reciprocal causation (Kirchhoff & Kiverstein, 2024). Whether and how such relations are transformed over time lies outside the present distinction between relation types.



(a) SCM serial representation

(b) PDD nested representation

Figure 3: Graphical criterion comparison: (a) In SCM, the whole A and its component part B are drawn as serial causes, implying that both can enter the same determination chain as independent items; (b) In PDD, B is nested within A—the two stand in a whole-part constitutive relation, and no determination mapping arrow should be drawn between items that stand in a containment relation.

porated into a chain as if it were a determination relation, the resulting transitivity analysis is invalid from the outset.

The scurvy case illustrates this point intuitively. “Eating citrus fruits” can determine bodily physiological states or the probability distribution of scurvy—this is a determination relation between full determining conditions and a resultant state of affairs; but “citrus → vitamin C” is not a determination chain of the same type, because vitamin C is a component part of the whole that is citrus. If one chains “citrus determines vitamin C” onto “vitamin C determines the probability distribution of scurvy,” one misreads a whole-part constitutive relation as a transitive determination relation. The problem here is not that transitivity always fails in general, but that this chain mixes two different relations from the very start: a determination relation and a constitutive relation.

Pearl’s Structural Causal Model (SCM) uses arrows in directed graphs to represent the causal dependencies posited by a model (Pearl, 2009). This uniform arrow syntax does not by itself automatically provide a type distinction between determination relations and constitutive relations. If a modeler also represents the relation between a whole and its component part by the same kind of causal arrow, a category mismatch results. This does not mean that interventionist and causal-modeling research has never addressed non-causal dependence. For example, Hoffmann-Kolss (2022) explicitly discusses how conditions such as the non-overlap of minimal supervenience bases can distinguish causal relations from logical, conceptual, mathematical, and other non-causal dependencies. PDD addresses a related problem but proceeds through a relation-type audit: before formal modeling, it distinguishes the determination relation between full determining conditions and a resultant state of affairs from the constitutive relation between a whole and its component parts.

This paper proposes that, when graphical models are used as analytical aids and constitutive relations are involved, a nested representation should be adopted: the unit representing a component part should be placed inside the unit representing the whole. Figure 3 illustrates the difference.

This risk in SCM has two consequences. First, constitutive relations, such as citrus → vitamin C, may be treated as determination relations. Second, once such treatment enters path analysis, a model may treat whole-part containment as a transitive causal chain, thereby producing inflated effect sizes or misleading conclusions about transitivity. What PDD provides prior to graphical-model computation is therefore not another statistical estimation method, but a relation-type audit: which arrows express independent determination mappings, and which merely miswrite constitutive relations. This distinction also intersects with discussions in the new mechanist literature concerning mechanistic composition and causal influence (Craver, 2007; Machamer, Darden, & Craver, 2000). Recent work further argues that multilevel mechanistic models must represent causal connections and constitutive relations separately, rather than replacing constitutive information with purely causal chains (Kistler, 2025).

5 The Logic of Contextual Adaptation in Causal Language

Section 4 analyzed the structural features and constitutive boundaries of the PDD relational network. Building on this foundation, this section further explains how causal language is contextually adapted to the underlying relational structure. What is discussed here is not natural-language semantics per se, but rather how the same PDD structure acquires different model forms and linguistic labels in empirical cognition due to differences in background-condition stability and epistemic aim. Two levels must be distinguished: first, the stability of background conditions affects whether a given determination relation can be cognized in a general manner; second, given a specific epistemic aim, agents further distinguish between general universal relations and retrospective explanation or responsibility attribution. These two levels are not parallel classifications but two dimensions that act sequentially upon the contextual adaptation of causal language.

5.1 Background Condition Stability and Its Cognitive Representation

The basic form of a PDD relation is: the condition-side state of affairs $S(X)$ and the complete background conditions Ω jointly constitute the full determining conditions $[C] \equiv [S(X), \Omega]$; when $[C]$ obtains, the resultant state of affairs is determined in the form of a probability distribution. However, in empirical cognition, the background conditions contained in Ω do not always enter our explanations in the same way. Depending on the stability of the relevant background conditions, three situations can be roughly distinguished.

The first situation involves **background conditions that remain constant within the empirical range**. For example, cosmological constants, fundamental spacetime background, or certain auxiliary conditions that remain invariant throughout a particular analysis—although they participate in the obtaining of the determination relation, they are typically not repeatedly mentioned as explicit factors. Cognitive agents tend to treat these conditions as implicit background, even ignoring them in everyday explanations; in such situations, the relevant relations are more readily cognized and labeled as "laws" or "regularities." Scientific laws can be understood as the linguistic and theoretical representation of certain PDD relations whose background conditions are highly stable at empirical scales; they are not strong ontological entities that unconditionally govern the world, but rather relational manifestations under stable backgrounds.[6] This does not mean that these background conditions have no role, but rather that they do not provide variation items that can be compared and attributed within the empirical range.

The second situation involves **predictable and stable variation**. For example, conditions such as air pressure, temperature, ground roughness, and material states are not invariant, but their patterns of variation can be classified, measured, and repeatedly compared within a certain range. In such situations, background conditions are not entirely ignored but are treated as important influencing factors; the relevant relations are less often understood as unconditional

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[6] This point resonates with Cartwright's discussion of scientific laws "lying" and her "nomological machines": laws hold stably only within particular condition structures, rather than being unconditionally and universally valid independently of background conditions (Cartwright, 1983, 1999).

laws and more commonly manifest as empirical knowledge, scientific knowledge, or mechanistic explanations.

The third situation involves **highly dynamic background conditions that are difficult to predict**. For example, conditions such as local air currents, chance perturbations, and minute environmental changes at the scene—although they may also participate in the generation of a particular resultant state of affairs—are difficult to stably classify and repeatedly model within the general empirical range. In such situations, the relevant determination relations are typically difficult to grasp as general universal knowledge and more readily enter retrospective analysis: after an event has occurred, we trace how these dynamic conditions participated in the formation of a particular outcome.

These three situations are not fixed ontological categories of background conditions; they classify stability relative to an empirical scale, scope of analysis, and comparative situation. The same condition may be nearly constant within one scope of analysis but become a salient difference in another comparison. During the normal operation of a machine, for example, the state of a component may ordinarily be ignored as stable background. If that component fails and the failure situation is compared with normal operation, its state may become an important item in retrospective explanation and responsibility attribution.

It follows that what background-condition stability affects is whether a given PDD relation can be generalized, type-classified, and linguistically expressed by cognitive agents. The more constant the background conditions, the more readily the relation is abstracted as a law; when background states are classifiable and undergo repeatable variation, the relation more readily manifests as empirical or scientific knowledge; when the background is highly dynamic and contingent, the relation more commonly enters retrospective explanation. This level of analysis has not yet distinguished whether the cognitive agent is pursuing general structural knowledge or asking about the specific cause of an event that has already occurred; the latter belongs to the level of epistemic aim.

5.2 Epistemic Aim: General Relations and Retrospective Attribution

Beyond background-condition stability, epistemic aim must be further distinguished. When cognitive agents confront the same type of PDD relation, they may pose at least two different questions: one asks about general universal determination relations, while the other seeks retrospective explanation and responsibility attribution for events that have already occurred.

The general relation question (abbreviated as Q1) concerns: given a certain background, does there exist some generalizable determination or influence relation between $S(X)$ and $S(Y)$? This question is independent of whether any particular state of affairs has actually occurred; its goal is to obtain general structural knowledge. Scientific laws, empirical knowledge, general causal relations, and mechanistic characterizations belong primarily to this level, because they all require the cognitive agent to identify whether there exists a stable or repeatably comparable probability distribution mapping among certain types of condition-side states of affairs, complete background conditions, and resultant states of affairs.

The responsibility attribution question (abbreviated as Q2) concerns: between a particular situation and its relevant comparison situation, which condition-side change $\Delta S(X)$ can explain the resultant-side change $\Delta S(Y)$ of interest? This question typically takes an event that has already occurred as its starting point. Rather than tracking all the full determining conditions of a general relation, it identifies the change in the condition-side state of affairs that corresponds, within the current explanatory context, to the change in the resultant state of affairs. Here, "cause" or "responsible party" does not directly refer to variable X or to the absolute state of affairs $S(X)$, but to the explanatorily relevant $\Delta S(X)$ in the comparison at issue.

To analyze this relation further, $\Delta S(X)$ may represent the condition-side change obtained by comparing the actual situation with a relevant comparison situation, and $\Delta S(Y)$ the corresponding resultant-side change. Accordingly, in a specific-event context, the standard referents of cause/effect are not variables X/Y or absolute states of affairs $S(X)/S(Y)$, but changes in states of affairs, $\Delta S(X)/\Delta S(Y)$. Responsibility attribution marks the condition-side change that stands in an explanatory correspondence with the resultant-side change. "Change" here is an epistemic expression used in retrospective explanation and responsibility attribution; it does not imply that the PDD determination relation itself depends on temporal succession.

This distinction shows that "whether a determination relation stably obtains" and "which factor is attributed as the cause" are questions at two different levels. The general relation question concerns whether a determination or influence relation obtains in a universal sense; the responsibility attribution question concerns which state-of-affairs change quantity can explain a particular state-of-affairs change of special concern in a specific comparison context. An important limitation of traditional causal theories lies precisely in their attempt to answer these two functionally different questions simultaneously with a single definition or linguistic logic.

The different answers to the question "Can a fire be attributed to air?" clearly illustrate this distinction. At the level of general relations, air is one of the conditions under which combustion can occur. Yet in a comparison between a forest-fire situation and a relevant no-fire situation, the state of the air ordinarily remains the same, while the ignition source, fuel state, or local environmental conditions change. Air is therefore not ordinarily adapted as the responsible cause of that fire. Conversely, if gasoline is stored in a high-temperature vacuum container, the state of the air changes across the relevant comparison situations, and that change corresponds to a change in the combustion state of affairs, then the entry of air can be adapted as the cause of the fire. In retrospective explanation and responsibility attribution, therefore, the question is not merely whether a condition participates in a general determination relation, but which condition-side change corresponds to the resultant-side change in the relevant comparison.

According to the distinctions above, the contextual adaptation of causal language is constrained by at least two levels: background-condition stability affects whether a given relation is more readily cognized as a law, manifested as empirical or scientific knowledge, or enters retrospective explanation; epistemic aim then determines whether the language is primarily adapted to general relations or to specific responsibility attribution. Figure 4 summarizes the relationship between these two levels.

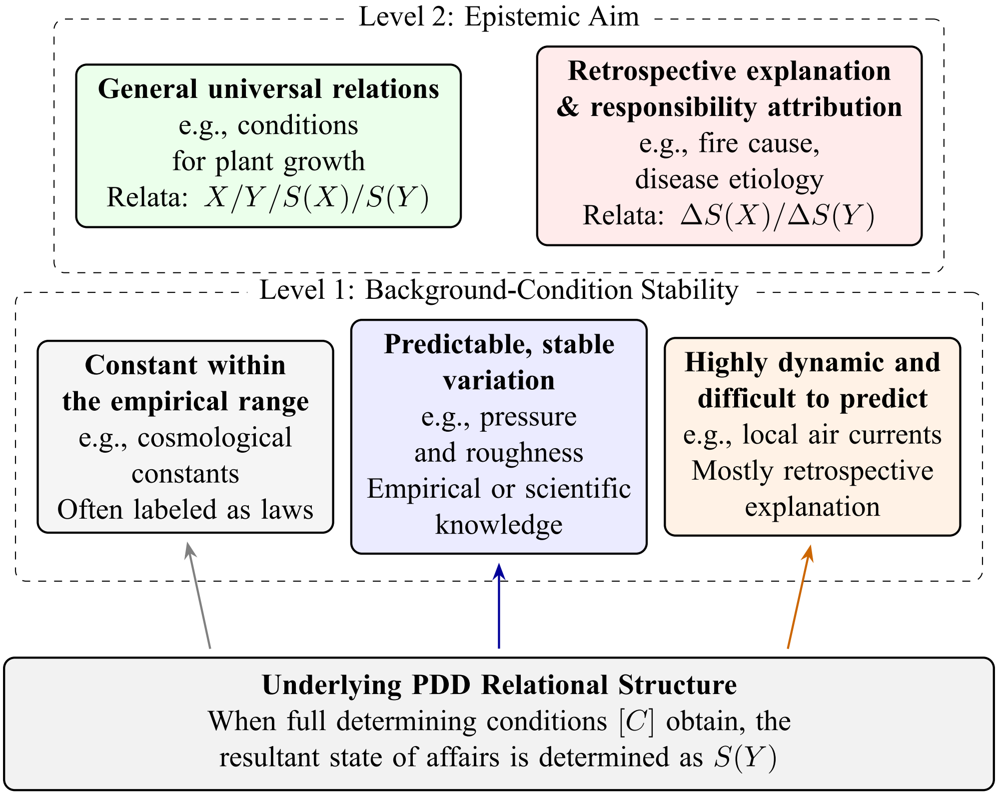

Figure 4: The two-level distinction between PDD background-condition stability and epistemic aim: background-condition stability affects whether a relation is commonly labeled a law, manifested as empirical or scientific knowledge, or mostly enters retrospective explanation; epistemic aim distinguishes general universal relations from retrospective explanation and responsibility attribution. Together they constrain the contextual adaptation of causal language, but do not constitute a one-to-one correspondence or derivation relation. General relations can be linguistically expressed using X/Y or $S(X)/S(Y)$, with $S(X)/S(Y)$ as the strict structural object; retrospective attribution primarily uses $\Delta S(X)/\Delta S(Y)$ from comparison contexts.

It follows that the polysemy of causal language does not arise from a set of parallel labels, but from the joint action of two different levels: background-condition stability affects whether a relation can be understood in a generalized, scientific, or retrospective manner, while epistemic aim determines whether the agent is tracking structural knowledge at the level of general relations or explaining the specific cause or responsible party of an event that has already occurred at the level of attribution. General relations can linguistically use X/Y or $S(X)/S(Y)$, but their strict structural object is $S(X)/S(Y)$; retrospective explanation and responsibility attribution primarily use $\Delta S(X)/\Delta S(Y)$ to express the explanatory relation between changes in states of affairs. Section 6 will, on the basis of the overall framework established in the preceding sections, comprehensively apply level distinction, the PDD definition, relational boundaries, and the logic of contextual adaptation in causal language to discuss existing causal theories, canonical puzzles, and theoretical boundaries in scientific modeling.

6 Discussion

Section 2 distinguished three levels—metaphysical structure, physical ontology, and epistemic detection; Section 3 provided the basic definition of PDD; Section 4 elucidated the structural features and constitutive boundaries of the PDD relational network; and Section 5 analyzed the logic of contextual adaptation in causal language. The present section introduces no new definitions or criteria; instead, it brings these prior analyses to bear on existing causal theories and canonical puzzles, examining their theoretical boundaries. The limits of graphical models and causal discovery methods will be addressed as part of the critical examination of Pearl/SCM.

6.1 Logical Examination of Existing Causal Frameworks

Existing causal theories typically capture one important dimension of causal inquiry: some emphasize empirical regularity, others stress interventionist detection, still others foreground graphical-model representation, and yet others highlight responsibility attribution in ordinary language. The role of PDD is not to refute these theories but, by drawing on the level distinctions, relational boundaries, and framework for the contextual adaptation of causal language established above, to clarify the task domain and limits of each.

This synthetic perspective can be brought into dialogue with Hall's (Hall, 2004) argument concerning "two concepts of causation." Hall distinguishes event causation into production and dependence: the former emphasizes generative or processual connections between events, while the latter emphasizes counterfactual dependence. PDD's distinction between general relations and responsibility attribution lies on a different analytic axis, one concerned with the epistemic functions served by causal language. It intersects with Hall's distinction but does not correspond to it one-to-one. A production relation, for example, may be used either to represent a general mechanism or in a specific attribution of responsibility; counterfactual dependence may likewise be used either in general relational analysis or in the explanation of an individual case. PDD therefore does not reduce Hall's two concepts respectively to general relation and responsibility attribution. It makes the further point that even where the relation type is the same, causal language may perform different functions because the epistemic aim differs.

The PDD mapping structure characterizes the formal conditions of determination relations, while existing theories respectively emphasize such causal features as constant conjunction, counterfactual dependence, interventional invariance, or structural modeling. From this metatheoretical perspective, the paper does not simply declare those theories mistaken; it specifies the level and boundary at which each is applicable.

First, regarding the **Humean tradition**. The "constant conjunction" that Hume observed can be understood as the stable correlations exhibited by underlying PDD mapping structures within finite empirical observation. The strength of the Humean tradition lies in characterizing repeatedly observable associations at the empirical level and their linguistic expression; its boundary is that it cannot directly yield the underlying determination-relation structure to which these empirical associations correspond. PDD therefore does not deny the importance of constant conjunction but relocates it to the level of epistemic observation and the contextual adaptation of causal language.

Second, regarding **interventionism**. An intervention in Woodward's sense is not an ordinary human operation but a strictly constrained idealized relation internal to a causal model. Interventionism uses invariance under intervention to characterize counterfactual dependence among variables and provides important criteria for causal explanation and inference (Woodward, 2003, 2016). PDD's disagreement is not a denial of the inferential value of these criteria. It is that interventional invariance cannot by itself constitute a complete metaphysical definition of a determination relation: a stable response to intervention may provide epistemological evidence for a determination relation without by itself revealing the direct path, hidden mediators, and complete relational structure.

Example 6.1 (Deceptive Monitor)**.** Suppose that in Room A, subject P_A repeatedly flips switch S_A. Monitor P_B in Room B sees the signal of this action through display D_B and manually operates control switch S_B, immediately illuminating lamp L_A in Room A. Within the subject's observable range, intervention on S_A and the illumination of L_A form a stable response. That response alone, however, does not establish a direct determination relation between S_A and L_A, because the actual structure includes the hidden mediating loop formed by the monitor and control switch.

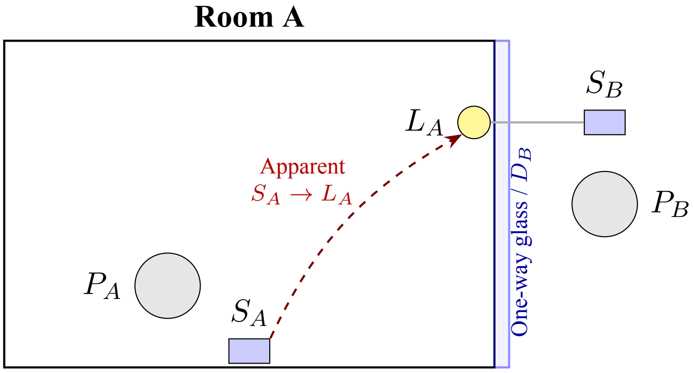

Figure 5: Deceptive monitor case: Subject P_A in Room A repeatedly operates S_A and observes L_A lighting up; monitor P_B outside the room observes the actions through one-way glass D_B and actually controls L_A via S_B. Thus, even if the state of L_A consistently covaries when S_A is intervened upon, this does not entail that the former bears a simple, direct determination relation to the latter.

Figure 5 shows that an interventional response by itself is insufficient to distinguish a direct relation from an indirect response formed through a hidden mediator. Interventional invariance can therefore serve as an important epistemological criterion for identifying and representing causal relations, but it cannot by itself bear the full burden of a metaphysical definition of determination.

Finally, regarding **Pearl's Structural Causal Models (SCM)**. SCM provides a powerful set of graphical computational tools. Given a structural model, the *do* operator represents an intervention and computes the corresponding distribution by replacing the relevant structural equation or cutting arrows directed into the intervened variable (Pearl, 2009).[7] PDD does not question the computational validity of do-calculus within a given model and its assumptions. It points out that SCM's uniform arrow syntax does not automatically perform relation-type checking and cannot by itself provide a complete metaphysical account of determination. As Section 4 argued, if a modeler represents constitutive relations with the same kind of causal arrow, the model may exhibit category mismatch and thereby affect analyses of effect size and transitivity. Before these tools are used, or when their results are interpreted, the relation type represented by each arrow and the compatibility of the relevant complete background conditions must therefore still be examined.

This point applies equally to causal discovery and model representation. Causal-discovery procedures typically begin from independence, correlation, or interventional response in data and attempt to recover directed structure among variables (Glymour, 2009; Spirtes, Glymour,

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[7] For a philosophical overview of the causal-modeling approaches associated respectively with Pearl and with Spirtes, Glymour, and Scheines, see C. Hitchcock (2009).

& Scheines, 2000). PDD does not replace SCM, do-calculus, statistical estimation, or causal-discovery algorithms. It offers a perspective from which to audit the conceptual validity of models and their results: whether arrows in a graph represent determination relations rather than whole-part constitution or relations between levels of description, and whether statistical relations placed in the same model arise under compatible complete background conditions. This audit does not require empirical researchers first to enumerate or verify all background conditions. It requires interpretations of a model to acknowledge that changes in relation type and background conditions may constitute boundaries on its application. PDD is thus not an empirical detection procedure, but a metaphysical audit framework concerning relation types, complete background conditions, and the functions served by the contextual adaptation of causal language.

6.2 Structural Diagnosis of Canonical Puzzles

These theoretical boundaries can also help us re-examine several canonical puzzles of responsibility attribution. Overdetermination, preemption, absent causation, and preventive judgment primarily involve responsibility attribution and comparative contexts, while also relying on the basic definition from Section 3 that "resultant states of affairs are determined in the form of probability distributions." Together they show that determination mappings at the general-relation level cannot be directly equated with responsibility attribution in specific contexts, and that the single outcome that has actually been realized cannot exhaust the probability-distribution changes of the resultant state of affairs.

First is the puzzle of **overdetermination**. In the case of "two fires simultaneously burning down a house," a classical dependence analysis centered on single counterfactual necessity encounters difficulty because Fire A is not necessary: without it, Fire B would still have destroyed the house (Lewis, 1973; Paul, 2009). This does not show that counterfactual analysis fails as a whole, but that the single comparison "without the cause, no effect" cannot exhaust independently parallel sufficient paths. In PDD, Fire A and Fire B each constitute an independent path sufficient for the state of affairs in which the house is destroyed; it is not only their conjunction that forms one inseparable sufficient condition. The key to responsibility attribution is therefore not to find a unique counterfactually necessary item, but to explain how multiple independently sufficient paths share or bear responsibility.

Second is the scenario of **causal preemption**. In the classic Suzy-and-Billy bottle-throwing case, Suzy's stone (A), thrown first, shatters the bottle (E), leaving Billy's subsequently thrown stone (B) to pass through empty air. Although at the level of general regularities both A $\Rightarrow$ E and B $\Rightarrow$ E can represent the stable relation whereby a stone striking a bottle causes the bottle to shatter, in the specific comparative context the action that actually enters the chain of resultant change is Suzy's; Billy's action is merely a backup path and does not form an actual change-correspondence with $\Delta S(E)$—the bottle's transition from intact to shattered. The crux of preemption, therefore, lies not in some mysterious competition of causal powers, but in the fact that multiple possible paths at the general-relation level cannot be directly equated with the actual change-correspondence at the level of specific responsibility attribution.

Third is the dilemma of **absent causation**. For the judgment "not watering caused the plant to wilt," counterfactual theory easily falls into the absurdity of infinite proliferation of cause-candidates (e.g., "Is the Queen's failure to water also a cause?"). PDD can treat "absence" as a value-difference of a certain state of affairs across comparative contexts. The gardener is labeled as the cause while the Queen is not, not because the former makes counterfactual construction logically easier, but because in the current responsibility-attribution context, whether or not the gardener waters constitutes a relevant state-of-affairs difference corresponding to the plant's $\Delta S(Y)$—the transition from alive to wilted. The Queen's failure to water, though it too can be constructed as some kind of counterfactual condition, does not belong to the relevant comparison class in this context. Thus, a potential condition that did not obtain does not automatically become a target of responsibility attribution; it must first enter the current comparative context and form a corresponding change-relation with the resultant state-of-affairs change under consideration.

The same distinction applies to **preventive causal judgment**. That an individual remains healthy after vaccination does not mean that the vaccine has no causal role. In the relevant comparison, vaccination and non-vaccination correspond to a change in the probability distribution of the resultant state of affairs, not necessarily to a change in the single outcome actually realized. This does not require denying the explanatory value of counterfactual comparison. PDD adds that the object of the comparison may be the complete probability distribution of the resultant state of affairs, rather than only one realized outcome. $\Delta S(Y)$ in responsibility attribution may therefore be a change at the level of probability distributions rather than a visible difference between realized outcomes.

6.3 Background-Condition Conflation in Simpson's Paradox

Simpson's paradox has long been regarded as a classic puzzle in statistics (Simpson, 1951). For PDD, if two statistical patterns are to be interpreted as different manifestations of the same determination relation, it must first be shown that they occur under compatible and stable complete background conditions. When the background conditions differ, so do the relevant full determining conditions $[C]$. In this sense, Simpson's paradox is more accurately described as background-condition conflation: the problem is not that marginal and stratified proportions cannot be compared statistically, but that statistical relations arising under different background descriptions are directly interpreted as the same PDD relation.

Consider the classic medical dilemma: aggregate population data suggest that a new drug lowers the recovery rate, yet after the population is divided into subgroups by sex, severity, or other relevant factors, the drug raises the recovery rate in every subgroup. What must be distinguished is not whether the determination relation between drug and recovery has mysteriously reversed, but whether the aggregate and stratified proportions express relations under the same background description. The drug/recovery relation in a subgroup is analyzed under a particular complete background condition Ω_{sub}; the aggregate proportion combines background conditions with different compositions, represented as Ω_{total}. The two can be compared as statistical objects, but they cannot, without qualification, be treated as the same determination relation under one and the same complete background.

From the PDD perspective, Simpson's paradox does not reveal a reversal in the determination relation itself. It reveals the conflation produced when the same causal label is retained after the background conditions have changed. If the stratified mapping under Ω_{sub} and the aggregate proportion under Ω_{total} are both directly interpreted as the same relation, statistical patterns arising under different background descriptions will be misread as a conflict within one PDD mapping. Simpson's paradox therefore reminds us that, before using the same causal label across backgrounds, we must ask whether the relevant complete background conditions are compatible and whether the relation under discussion is in fact the same object.

This diagnosis illustrates PDD's role as a framework of conceptual audit. Simpson's paradox is not a contradiction within the determination relation itself. Background-condition conflation occurs when an agent uses the same causal label for statistical relations under stratified and aggregate backgrounds without specifying the differences between their relational objects and background conditions. By requiring the relevant complete background conditions and relational objects to be made explicit, PDD makes this conflation easier to locate conceptually. How an empirical study should choose a stratification scheme and identify relevant background conditions remains a further epistemic question.

7 Conclusion

Through level distinction, the basic definition of PDD, the structure and boundaries of relation networks, and the contextual adaptation of causal language, this paper has constructed a formal analytic framework called Probability-Distribution Determinism (PDD). Counterfactual theory, interventionism, and structural causal models analyze causation respectively in terms of counterfactual dependence, interventional invariance, and structural models; PDD asks the further question of what form is taken by the underlying determination relation presupposed by those analyses. The framework understands the object of determination as a resultant state of affairs expressed as a probability distribution and analyzes determination relations under given full determining conditions and complete background conditions. The traditional opposition between determinism and probability can thereby be reconsidered. Logical distance in a PDD network can also be understood as a structural factor affecting the probability concentration of distal outcomes, without weakening the basic property that the complete probability distribution is uniquely determined. The polysemy of causal language concerning laws, general relations, and responsibility attribution can in turn be understood as the adaptation of the same underlying relational structure to different epistemic aims and contexts.

This reconceptualization yields a threefold clarification. The first is a metaphysical clarification: PDD shifts the determined object from a single outcome to a state of affairs expressed as a probability distribution, preserving the formal completeness of the determination relation while also reserving conceptual space for non-PDD-type states of affairs and the problem of free will. The second is a clarification of relation types: determination relations differ from part-whole constitutive relations, and constitutive relations must not be erroneously written as determination mappings or causal arrows in graphical models. The third is a clarification at the level of language and concepts: causal language does not correspond to an essential definition at a single level, but is contextually adapted among background-condition stability, general-relation tracking, and retrospective explanation and responsibility attribution.

The core conclusion is therefore that interpretations of causal inference, effect-size calculation, and graphical-model representation should remain conceptually sensitive to relation types and complete background conditions. A relation-type audit shows that incorporating constitutive relations into a determination chain may produce category mismatch, inflated effect sizes, and misleading transitivity. An audit of complete background conditions shows that Simpson's paradox is more accurately described as the conflation of relation labels arising under different background descriptions as if they expressed a single determination relation. PDD is thus first and foremost a metaphysical audit framework. It does not replace specific statistical methods or causal-discovery algorithms, but offers a conceptual perspective on their relation types, background conditions, and boundaries of contextual adaptation in causal language. Having laid this metaphysical foundation, the next task lies at the epistemic level: how these relational structures can be identified, approximated, or tested in finite data, statistical models, and interventional designs. That question is not part of the present definitional task, but it constitutes a central direction for the subsequent methodological development of PDD.

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