An Old Look at Extended Wigner's Friend Scenarios

Abstract

We present a general formalism to formulate Bell's theorem, no-hidden variables proofs and extended Wigner's friend scenarios. This common mathematical framework allows to bridge the differences between an old (short-lived) research program (that of modal interpretations) and the more recent literature on perspectival anti-realist interpretations. We find that our unifying mathematical approach gives a clear content to the technical aspect of various no-go theorem, and provides new insights on their conceptual role by allowing comparison between different research programs.

1 Introduction

There is a rich and growing literature in quantum foundations about so-called extended Wigner's friend scenarios (henceforth EWFS). These works develop and discuss thought experiments, framed as no-go theorems, drawing on a combination of Wigner's original remarks about the collapse postulate and Bell's theorem (see Wigner (1961) and Bell (1964) respectively). This research area seems to exhibit some similarity with an older (short-lived) research program, that of *modal interpretations*. Both foundational investigations rely on unitary dynamics, yet postulate that some variables (encoding either measurement outcomes or intrinsic possessed properties) have determinate values. In particular, a recent review by Schmid et al. (2024) notices, but does not investigate, a resemblance between a class of EWFS and an older no-go theorem in the modal literature (Myrvold, 2002).

This paper aims to reveal the shared mathematical core between Myrvold's theorem and the EWFS. In particular, we will translate them into a common algebraic language, and derive within it Bell's theorem, as well as a variant of Bell's theorem and a corollary. We will then show that Myrvold's original theorem and two EWFS (a relativistic argument by Ormrod and Barrett (2022) and the Local friendliness theorem by Bong et al. (2020)) are particular instantiations of this corollary. The take away of our work is that both the EWFS and Myrvold's theorem can be viewed as different interpretive set ups for the underlying corollary. Therefore, both realist hidden-variable and perspectival anti-realist approaches can viewed through the same lens, that of constraints on value-assignment. Section 2 will introduce the relevant formalism, and derive Bell's theorem within it. Section 3 will derive Bell's variant and the corollary. We will draw some philosophical conclusions in the discussion.

2 Bell's Theorem

Bell's inequalities are a constraint ($\star$) on the statistical correlations between measurements performed at two sites A and B, or on two parts $\mathfrak{A}$ and $\mathfrak{B}$ of a bipartite system. Bell's theorem is the statement that if a correlation experiment can be modelled by a single classical probability measure, provided by a "local hidden-variable theory", then ($\star$) must be satisfied. However, it is violated by certain quantum mechanical predictions, and quantum mechanics has been experimentally vindicated.[1]

However, the mention of hidden-variable theories is irrelevant to the spirit of Bell's theorem. It has long been shown that the Bell inequalities are equivalent to the existence of a joint classical distribution for all measured observables (Fine, 1982, Pitowsky, 1989). The literature on EWFS sometimes reject talk of hidden-variable theories, for these "appeal to the classical notion of realism typically codified in the framework" (Schmid et al., 2024, 30). For this reason, it will be useful to derive Bell's theorem in an algebraic framework, very general and agnostic for what regards its interpretation. This will allow us to formalise EWFS as variants of Bell's theorem in that framework, where the single classical joint distribution ranges only over measurement outcomes actually observed in a single run of the experiment.

Following Summers and Werner (1987), we model a correlation experiment between parts $\mathfrak{A}$ and $\mathfrak{B}$ with a "*correlation duality*" $(p, \mathfrak{A}, \mathfrak{B})$, which is comprised of two real vector spaces $\mathfrak{A}$ and $\mathfrak{B}$, equipped with a partial order relation $\leq$ and a unit 1, as well as a bilinear function $p : \mathfrak{A} \times \mathfrak{B} \to \mathbb{R}$, such that $a \in \mathfrak{A}, b \in \mathfrak{B}$ and $a, b \geq 0$ imply $p(a, b) \geq 0$ and $p(1, 1) = 1$. The observables of one subsystem are represented by partitions $\{a_i\}_I$ of the unit in $\mathfrak{A}$: $\sum_I a_i = 1$ with $a_i \geq 0$ for all i, and similarly for $\mathfrak{B}$. We can interpret each $i \in I$ as a possible outcome of the measurement of the observable. The probability (relative frequency) of the joint occurrence of the outcomes $i \in I$ and $j \in J$ in the two subsystems respectively is given by $p(a_i, b_j)$. The probability of obtaining $i \in I$ is obtained as a marginal[2] of the correlation function: $p(a_i, 1) = \sum_J p(a_i, b_j)$. In the following, we specialize to the case where $\mathfrak{A}$ and $\mathfrak{B}$ are self-adjoint parts of C^*-algebras.

2.1 Bell's theorem for C^*-algebras

We take $\mathfrak{A}$ and $\mathfrak{B}$ to be self-adjoint parts of C^*-subalgebras of a larger C^*-algebra $\mathfrak{R}$.[3] Although an arbitrary C^*-algebra may not have an identity, one can always be adjoined to it (Bratteli and Robinson, 1987, Prop. 2.1.5.), so for convenience, all the C^*-algebras discussed here will be assumed to be unital. The set of positive elements $\mathfrak{R}_+ := \{s^*s : s \in \mathfrak{R}\}$ induces a partial order: $a \leq b$ whenever $b - a \in \mathfrak{R}_+$. Finally, as $\mathfrak{R}_{sa}$ can be thought of as the 'real' part of $\mathfrak{R} = \mathfrak{R}_{sa} + i\mathfrak{R}_{sa}$, it inherits a real vector space structure. From now on, we will denote by the index '*sa*' the self-adjoint parts of C^*-algebras. We will thus relabel $\mathfrak{A} := \mathfrak{A}_{sa}$ and $\mathfrak{B} := \mathfrak{B}_{sa}$.

A preparation is represented by a state ω on $\mathfrak{R}$, *i.e.* a (complex-valued) linear map $\omega : \mathfrak{R} \to \mathbb{C}$ that maps positive operators to nonnegative numbers (it is *positive*) and the identity to 1 (it is *normalized*). These are in one-to-one correspondence with states on $\mathfrak{R}_{sa}$ (positive normalized *real* valued linear functionals), so I will speak of them interchangeably.[4] $\omega(a_i)$ is the probability for obtaining a result i for a measurement $\{a_i\}_I$. In this framework, $\mathfrak{A}_{sa}$ and $\mathfrak{B}_{sa}$ are taken to be self-adjoint parts of commuting subalgebras of $\mathfrak{R}$, and $p(a, b) := \omega(ab)$.

Let us restrict ourselves to measurements admitting only two outcomes $\{a_i\}_I := \{a_+, a_-\}$ with $a_+, a_- \geq 0$ and $a_+ + a_- = 1$. Think for example of a photon having two possible linear polarization states, with respect to a given direction. Such pairs are in one-to-one correspondence with elements $a \in \mathfrak{A}_{sa}$ such that $-1 \leq a \leq 1$, through the bijection $a_\pm = \frac{1}{2}(1 \pm a)$. You can think of this as a mathematical trick, replacing a model of measurement where each polarization state is represented by a yes-no question ($a_\pm$, having value 0 or 1) by one model with a single operator a, with possible values 1 or -1, representing the two polarization states. Following Summers and Werner (1987), we will call *admissible* a quadruple (a_1, a_2, b_2, b_2) such that $a_1, a_2 \in \mathfrak{A}_{sa}, b_1, b_2 \in \mathfrak{B}_{sa}$ and $-1 \leq \{a_1, a_2\} \leq 1$ and $-1 \leq \{b_1, b_2\} \leq 1$. We are now almost set to present Bell's theorem. Let us first introduce some final definitions.

For ω a state on $\mathfrak{R}$, let $\mathfrak{D}^\omega$ be its definite set: $\mathfrak{D}^\omega := \{a \in \mathfrak{R} : \omega(ab) = \omega(a)\omega(b) \quad \forall b \in \mathfrak{R}\}$. Its self-adjoint part is the Kadison-Singer definite set $\mathfrak{D}^\omega_{sa} := \{a \in \mathfrak{R}_{sa} : \omega(a^2) = \omega^2(a)\}$. We will say that a state ω is *dispersion-free* on $a \in \mathfrak{R}$ whenever $a \in \mathfrak{D}^\omega$, and dispersion-free on $\mathfrak{A} \subseteq \mathfrak{R}$ whenever $\mathfrak{A} \subseteq \mathfrak{D}^\omega$. Following Halvorson and Clifton (1999), we say that $\mathfrak{A} \subseteq \mathfrak{R}$ is *beable* for ω whenever $\omega \mid_\mathfrak{A}$ is in the weak* closure of the convex hull of dispersion-free states on $\mathfrak{A}$. In other words, $\mathfrak{A}$ is beable for ω whenever there exists a set $\{\omega_x\}_X$ of dispersion-free states on $\mathfrak{A}$ and a measure μ on X such that ω is a mixture of $\{\omega_x\}_X$:

$$\omega(a) = \int_X \omega_x(a) d\mu(x), \quad \forall a \in \mathfrak{A} \tag{1}$$

We can finally state and prove an adaptation of Bell's theorem.

Theorem 1 (adapted from Summers&Werner 1987, Baez 1987, Fine 1982): Let $(p, \mathfrak{A}_{sa}, \mathfrak{B}_{sa})$ be a correlation duality for C^*-algebras, let ω be the state $\omega(a) = p(a, 1)$, and let $\underline{s} = (a_1, a_2, b_1, b_2)$ be an admissible quadruple. Setting $\chi = \frac{1}{2}|p(a_1, b_1 + b_2) + p(a_2, b_1 - b_2)|$, the following statements are equivalent:

($\star$) $\chi \leq 1$.

(ii) $C^*(\underline{s})$, the smallest unital C^*-algebra generated by the quadruple, is beable for

ω.

(*iii*) There exists a classical joint distribution $P : \sigma(\underline{s}) \to [0,1]$, yielding the correlation functions p as marginals.

Proof: (*ii*) $\to$ ($\star$). Let us suppose that $C^*(\underline{s})$ is beable for ω. Let $\{\omega_x\}_X$ be the set of dispersion-free states of $C^*(\underline{s})$. Then, $C^*(\underline{s}) \subseteq \cap_X \mathfrak{D}^{\omega_x}$, so for any $x \in X$, $\omega_x(ab) = \omega_x(ba) = \omega_x(a)\omega_x(b)$. Dispersion-free states are thus product states for $\mathfrak{A} \vee \mathfrak{B}$. ω is in the w^*-closure of the convex hull of product states. Baez (1987) called such states *decomposable*, and proved (theorem 1) that decomposability for an admissible quadruple (in particular, self-adjoint elements of norm ≤ 1) implies ($\star$). It suffices to prove ($\star$) for product states, then it holds for convex combinations and by taking the limit, because states are continuous (Bratteli and Robinson, 1987, Prop 2.2.11).
($\star$) $\leftrightarrow$ (*iii*). ($\star$) is a restatement of the Clauser-Horne inequalities, as given in Pitowsky, 1989, (2-15), when one introduces back the operators $a_i^{\pm}, b_j^{\pm}$ via $a_i = 2a_i^{\pm} \mp 1$, with the notation: $p_1 := p(a_1^+, 1), p_2 := p(a_2^+, 1), p_3 := p(1, b_1^+), p_4 := p(1, b_2^+)$, and p_{ij} for the correlations. These inequalities are satisfied if and only if the vector $\vec{p} = (p_1, p_2, p_3, p_4, p_{13}, p_{14}, p_{23}, p_{24})$ belongs to the Clauser-Horne polytope $c(4, S)$ with $S = \{\{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}\}$ (Pitowsky, 1989, theorem (2-5)). By theorem (2-3) of the same material, a vector $\vec{p}$ belongs to a convex polytope $c(n, S)$ if and only if there is a probability space (Ω, Σ, μ) and (not necessarily distinct) events $A_1, ..., A_n \in \Sigma$ such that $p_i = \mu(A_i), 1 \leq i \leq n$ and $p_{ij} = \mu(A_i \cap A_j), \{i,j\} \in S$. This amounts to the existence of a joint distribution P given by $P(a_1^+, a_2^+, b_1^+, b_2^+) := \mu(A_1 \cap A_2 \cap A_3 \cap A_4), P(a_1^-, a_2^+, b_1^+, b_2^+) := \mu((\Omega \backslash A_1) \cap A_2 \cap A_3 \cap A_4)$, etc. The Clauser-Horne inequalities are exactly the necessary and sufficient constraints for these sets of numbers to be positive.
(*iii*) $\to$ (*ii*). for $\underline{\lambda} \in \sigma(\underline{s})$, define $\omega_{\underline{\lambda}} : a_1 \to \lambda_1, a_2 \to \lambda_2, b_1 \to \lambda_3, b_2 \to \lambda_4$ such that $\omega_{\underline{\lambda}}(ss') = \omega_{\underline{\lambda}}(s)\omega_{\underline{\lambda}}(s')$ and $\omega_{\underline{\lambda}}(rs + s') = r\omega_{\underline{\lambda}}(s) + \omega_{\underline{\lambda}}(s')$ for any $r \in \mathbb{C}, s, s' \in \underline{s}$. Then, because $C^*(\underline{s})$ is generated by $\underline{s}$, $\omega_{\underline{\lambda}}$ is a dispersion-free state on it, and by construction we have $\omega(s) = \int_{\sigma(\underline{s})} \omega_{\underline{\lambda}}(s) P(\underline{\lambda}) d\underline{\lambda}$. □

2.2 On the interpretation

Dispersion-free states are the right mathematical tools to represent hidden-variables. Indeed, if $a \in \mathfrak{D}^\omega$, then $\omega(a) \in \sigma(a)$. Physically, this means that dispersion-free states are suitable to represent valuations on the space of observables, assigning to a self-adjoint operator a number in its spectrum. For the specific case where $\mathfrak{R} := \mathfrak{B}(\mathcal{H})$ for some Hilbert space $\mathcal{H}$, and ω is represented by state vector ψ, we recover a more familiar definition: ψ is dispersion-free on self-adjoint a whenever the statistical spread $\Delta_\psi(a)$ for the values of a in state ψ is null.[5]

Therefore, $\mathfrak{A}$ is beable for ω just in case the observables in $\mathfrak{A}$ can be taken to have determinate values statistically distributed in accordance with ω's expectation values. This justifies the name *beable*, for this recovers what John Bell had in mind when introducing the term:

> "Could one not just promote some of the 'observables' of the present quantum theory to the status of beables? (...) The values which they are allowed to be would be the eigenvalues of those operators. For the general state the probability of a beable being a particular value would be calculated just as was formerly calculated the probability of observing that value" (Bell, 1987, 41).

It thus seems as though we have just translated a discourse of hidden-variable theories in an algebraic framework. However, this is not so, for the framework presented above is more general. First, the dispersion-free states are indeed defined not on the whole algebra $\mathfrak{R}$, but on subalgebras. This allows to apply this formalism to measurement contexts, represented by abelian subalgebras of $\mathfrak{R}$. We can indeed prove that every abelian subalgebra of $\mathfrak{R}$ is beable for any state ω. Second, the determinate values assigned by dispersion-free states can be interpreted as representing the values obtained upon measurement, and not necessarily the possessed value of an intrinsic property. The fact that any individual measurement context is beable for any preparation (*i.e.* any state) therefore justifies the mundane fact that any measurement yields determinate outcomes, statistically distributed according to the quantum mechanical probabilities for that preparation. Theorem 1 therefore allows us to use classical probabilities for the correlations between the measurement outcomes in an individual measurement context.

The violation of ($\star$) by quantum frequencies tells us that a Bell experimental set up cannot be represented by a beable joint algebra (as is trivially possible for an individual measurement context). This might be due to the fact that, in order to consider the joint experimental context as beable for the state, we must assign values (dispersion-free states) *counterfactually* to some observables in any given run of the experiment, as *either* b_1 or b_2 is measured, but never both. Locality and separation assumptions in Bell's theorem precisely play the role of warranting such counterfactual value assignment, as such principles could be used to justify assigning a value to a_1 independently of whether b_1 or b_2 is measured. The violations of Bell's inequalities could thus be construed as a prohibition on such counterfactual reasoning, even for separated subsystems. We will see that Myrvold's theorem and the EWFS are variants of theorem 1, modifying the correlation set up in such a way as to make the assumption that the whole measurement context is beable motivated without counterfactual reasoning.

3 Bell's Variant

We consider again a correlation experiment $(p, \mathfrak{C}_{sa}, \mathfrak{D}_{sa})$, with $c \in \mathfrak{C}_{sa}$, $d \in \mathfrak{D}_{sa}$, $-1 \leq \{c, d\} \leq 1$ and a state ω yielding correlations p. $C^*(c, d)$ is beable for ω, for it is abelian. It is thus always possible to assume that c and d have simultaneous determinate values. Again, these may represent the outcome of a performed measurement, or the value of a physical magnitude for the subsystems. The EWFS take it to be the former, Myrvold takes it to be the latter. Let us call *admissible* an automorphism $\alpha : \mathfrak{R} \to \mathfrak{R}$ satisfying $\alpha(\mathfrak{C}_{sa}) \subseteq \mathfrak{C}_{sa}$ and $\alpha(\mathfrak{D}_{sa}) \subseteq \mathfrak{D}_{sa}$ and such that $\underline{s}^\alpha = (c, \alpha(c), d, \alpha(d))$ is an admissible quadruplet–note that c and d are *fixed* elements. Then, a variant of Bell's theorem, underlying Myrvold's theorem as well as the EWFS, can be stated as follows:

Theorem 2 (Bell's variant): Let $\mathfrak{R}$ be a C^*-algebra, and $c \in \mathfrak{C}_{sa} \subset \mathfrak{R}$, $d \in \mathfrak{D}_{sa} \subset \mathfrak{R}$ with $[\mathfrak{C}_{sa}, \mathfrak{D}_{sa}] = 0$. Then there exists a correlation duality $(p, \mathfrak{C}_{sa}, \mathfrak{D}_{sa})$ given by a state $\omega(a) = p(a, 1)$ over $\mathfrak{R}$, and an admissible map $\alpha : \mathfrak{R} \to \mathfrak{R}$, such that:

$$\chi = \frac{1}{2}|p(c, d + \alpha(d)) + p(\alpha(c), d - \alpha(d))| > 1$$

Corollary (Myrvold-Wigner): There exists some state ω such that the two following statements are incompatible:

1. $(\omega, \mathfrak{C}_{sa}, \mathfrak{D}_{sa})$ is a correlation duality.
2. $C^*(\underline{s}^\alpha)$ is beable for ω for all admissible α.

We will see that Myrvold's theorem as well as the EWFS are no-go theorems relying on the Myrvold-Wigner corollary. First of all, the corollary follows from theorem 2 because, by virtue of theorem 1, the violation of $(\star)$ is equivalent to failure of beable status for the particular ω and α at hand. Therefore, if there exists a pair (ω, α) such that $(\star)$ is violated, then, there exists some ω such that beable status does not hold for all admissible α. The condition 1. in the corollary in effect imposes that the correlations between compatible observables are given by the Born rule. Both Myrvold and the EWFS are going to present assumptions sufficient to imply condition 2. of the corollary, for an arbitrary state ω. They then select a particular ω and α such that assumption 1 fails, *i.e.* they derive using assumption 2 the fact that $p(s, s') \neq \omega(ss')$ for a pair of compatible elements. Viewed as a no-go theorem, this implies that among the assumptions establishing 1. and the Born rule assumption, at least one of them is false. Note that the particularity of each no-go theorem is going to be determined by the interpretation of the admissible maps.

3.1 Myrvold's theorem

The original targets of Myrvold's no-go theorem are the so-called *modal interpretations* of quantum theory, which postulate that the quantum state always evolves unitarily, but that *some* observables are beables for the state (in the sense of the above presentation)–see Myrvold (2002) for the original derivation, and Myrvold (2009) for an algebraic framing. Myrvold's idea is to view α as a dynamical map, in the Heisenberg picture, such that c and d represent the same observables as $\alpha(c)$ and $\alpha(d)$ in the Schrödinger picture. This is motivation enough to impose beable status on the whole algebra $C^*(\underline{s})$, and thus a single joint distribution over the joint spectrum $\sigma(\underline{s})$, recovering the Born rule correlations as marginals (theorem 1). However, if any admissible (suitably relativistic) dynamical transformation is allowed,[6] then "as long as the timelike separated pairs fail to commute, there will be some states that yield correlations between the spacelike separated pairs that violate a Bell inequality" (Myrvold, 2009, 639).

Myrvold's set up is the following: consider four spacetime points A, B, C, D. Our two subsystems are relatively well localized around C and D, and each one dynamically evolves so as to be relatively well localized around A and B. So, C and A are related by a timelike curve, so are B and D. Every other pair of points is spacelike separated. Let us chose three foliations of spacetime into spacelike hypersurfaces: a red foliation, a blue one and and a green one (see figure 1). Each foliation is associated with a different dynamical group[7] $\left\{t \to \alpha_t^{r,g,b}\right\}$, representing the dynamics (in the Heisenberg picture).

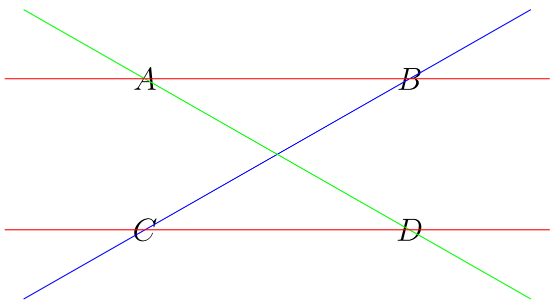

Figure 1: Four spacetime points, intersected by four spacelike hypersurfaces. Different colors represent different foliations.

By convention, c and d are taken to be simultaneously determinate valued in the red frame at $t = 0$, and we note a and b the value of α^r evaluated at time T on c and d respectively. Myrvold makes two assumptions to establish that $C^*(c, d, a, b)$ is beable for ω, one explicit he calls *locality*, one implicit I will call *objectivity*. By locality, Myrvold means that a subsystem's individual property is beable independently of the hypersurface containing the subsystem. In our formalism, this imposes the following constraint:

$$\begin{aligned}
\alpha_0^b(c) &= c \\
\alpha^g(c) &= a \\
\alpha_0^g(d) &= d \\
\alpha^b(d) &= b
\end{aligned} \tag{LOC}$$

In other words, *locality* ensures that all the automorphisms agree on their operator value when their underlying hypersurfaces intersect. This isolates an admissible map $\alpha := \alpha_T^r$. Each commuting pair is *beable* for ω, which means that for each pair $ij \in S$, there is a measure μ_{ij} on the space of dispersion-free states yielding correlations p. By *objectivity*, Myrvold imposes that μ_{ij} does *not* depend on the frame one choses, and thus enforces that there is a single classical distribution yielding correlations p. By theorem 1, this establishes beable status for ω on the whole algebra.

Then, Myrvold selects $\mathfrak{R} := \mathfrak{B}(\mathcal{H})$, where $\mathcal{H} \cong \mathbb{C}^2 \otimes \mathbb{C}^2$ is the Hilbert space of two qubits. ω is represented by a state vector $\psi = \frac{1}{2\sqrt{3}}(|11\rangle - |10\rangle - |01\rangle - 3\,|00\rangle)$, and $\alpha(\cdot) = (U \otimes U)^* \cdot (U \otimes U)$, with $U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$. He proves that, using the single joint distribution, one is forced to derive $p(s, s') \neq \omega(ss')$ for one of the commuting pairs.

3.2 A relativistic EWFS

An EWFS is a thought experiment aiming to combine a Bell correlation set up with Wigner (1961)'s original remarks on *supermeasurements*, *i.e.* measurements on a system which includes an observer who herself performed a measurement. Similarly to Myrvold's theorem, their ambition is to dispense with the counterfactual value-assignment in Bell's theorem. They also similarly assume unitary evolution of the state, and actual outcomes for some measured observables, *i.e.* some *beables* in our terminology.

However, whereas Myrvold views a map α as a dynamical transformation of the system (in some frame), the EWFS take them to be operations performed (by a superobserver) on a system. The subsystems $\mathfrak{C}_{sa}$ and $\mathfrak{D}_{sa}$ therefore represent agents in their lab with a qubit. As no-go theorems, the EWFS make explicit the assumption previously called *objectivity*, and this assumption of objective, or non-relative facts, is usually the one the authors prescribe to abandon. Ormrod and Barrett (2022) present such a no-go theorem, where they characterize this assumption as such: "if n rational, competent agents each believe they have witnessed the outcome of a measurement, then there is a single, non-relative fact about the set of outcomes $o = \{x_i\}_{i=1}^n \in \{X_i\}_{i=1}^n$ that were observed". By facts about outcomes, I will include probabilistic statements about the outcomes actually observed by some agent, such as their relative frequencies. In our formalism, this means that there exists a single measure space (X, μ) over the joint spectrum $\underline{\sigma}$ of *actually performed experiments*, specific to the whole joint measurement context. Because the superobservers do not chose between possible measurements to perform, the single distribution is defined on the joint spectrum of all the observables for the experiment. By virtue of theorem 1, this imposes beable status for the state.

Ormrod and Barrett's set up, as well as their assumptions, are exactly analogous to that of Myrvold's. Charlie and Daniela are spacelike separated agents and share an entangled pair of qubits. They each perform a measurement $-1 \leq c \leq 1$ and $-1 \leq d \leq 1$ respectively. Then Alice performs a supermeasurement on Charlie and its qubit, and Bob performs a supermeasurement on Daniela and her qubit.[8] They measure $a := \alpha(c)$ and $b := \alpha(d)$ respectively, with α a unitary transformation.

They then define a *Frame-independent theory* (definition 2 in their paper), which amounts to imposing that $(\omega, \mathfrak{C}_{sa}, \mathfrak{D}_{sa})$ is a correlation duality, *i.e.* that the Born rule is satisfied for all compatible pairs. Their theorem is then:

> **Theorem 3 (Ormrod and Barrett 2022):** Absoluteness of observed events [non-relative facts] and Frame-independent quantum theory are not both correct.

This is a restatement of the Myrvold-Wigner corollary. In their proof, Ormrod and Barrett select $\mathfrak{R} := \mathfrak{B}(\mathcal{H})$, where $\mathcal{H}$ is the Hilbert space of two qubits, and ω is represented by a vector state ψ on $\mathcal{H}$: the Hardy state $\psi := \frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$. α is a unitary map such that the Born rule correlations prescribe: $\omega(cd) = 0$; $\omega((1-a)(1-d)) = 0$; $\omega((1-c)(1-b)) = 0$. The beable status assumption gives us:

$$\begin{aligned} &\int_X \omega_x(c)\omega_x(d)d\mu(x) = 0 \\ &\int_X \omega_x(1-a)\omega_x(1-d)d\mu(x) = 0 \\ &\int_X \omega_x(1-c)\omega_x(1-b)d\mu(x) = 0 \end{aligned} \tag{2}$$

The three equalities 2 imply the following fact:[9] $\omega((1-a)(1-b)) = 0$, yet this is not the correlation prescribed by the Born rule for this pair in this state.

3.3 Local friendliness theorem

Another EWFS, presented in Bong et al. (2020), derives a Local Friendliness theorem, which the authors state as such:

> **Theorem 4 (Bong et al 2020):** If a superobserver can perform arbitrary quantum operations on an observer and its environment, then no physical theory can satisfy Local Friendliness.

By *physical theory* they mean any theory providing a correlation duality in accordance with the Born rule for outcomes observed by the superobservers Alice and Bob. This is the assumption 2 in the Myrvold-Wigner corollary. By "Local Friendliness" (LF), they mean the conjunction of three assumptions: absoluteness of observed events, no superdeterminism and locality, which is sufficient to imply assumption 2 (beable status) in the corollary.

Unlike in the relativistic EWFS, the superobservers Alice and Bob have a choice to make about the operation to perform:[10] each of them can either communicate with their friend and ask them about their result, or perform a unitary operation on the friend and the system. We can model the situation as follows: let $f : \Omega \to Aut^{ad}(\mathfrak{R})$ be a random variable (the *choice variable*), where $\Omega := \{0,1\} \times \{0,1\}$ and $Aut^{ad}(\mathfrak{R})$ is the set of admissible automorphisms. A probability distribution $p : \Omega \to [0,1]$ together with f determines a probability distribution p_f on the spectrum of f, *i.e.* on the set of four possible maps $\sigma(f) = \{\alpha : f = \alpha\} = \{f(t) : t \in \Omega\}$.

The choice variable is treated as an exogenous variable in the description of the theorem: it represents the choice by some agents to perform a certain operation. Depending on the outcome $\{f = \alpha\}$, the observed statistics may be represented by different states ω^α. These states encode the conditional probabilities $p(ab \mid xy)$, for $(x,y) \in \Omega$ used in Bong et al. (2020). Whereas Charlie and Daniela always measure observables $c \in \mathfrak{C}_{sa}, d \in \mathfrak{D}_{sa}$, Alice and Bob jointly measure observables $f(t)(cd)$. If the state of the joint system before measurement by Charlie of Daniela is ω, we require that $\omega^\alpha = \omega \circ \alpha$. Since the LF assumptions are constraints on the conditional probabilities, can now state these as constraints on the maps $\alpha \in \sigma(f)$.

$\{c, d\}$ is beable for ω, because these are *actually* measured by Charlie and Daniela. However, contrary to the previous EWFS, superobservers now have a choice to make. Absoluteness of observed events is therefore insufficient to impose a single distribution on the joint spectrum of all the observables, for it imposes that $\{\alpha(c), \alpha(d)\}$ is beable for ω *only if* $f = \alpha$. We now have four scenarios, one for each α, and a single measure μ^α for each scenario:

$$\begin{aligned} \omega(cd) &= \int_{X^\alpha} \omega_x(c)\omega_x(d)d\mu^\alpha(x) \\ \omega^\alpha(cd) &= \int_{X^\alpha} \omega^\alpha_x(c)\omega^\alpha_x(d)d\mu^\alpha(x) \end{aligned} \tag{3}$$

The choice $f = \alpha$ is made by the superobservers Alice and Bob *subsequently* to Charlie and Daniela's measurements. The assumption of No Superdeterminism (or of Free-Choice) ensures that the latter are uncorrelated with p_f, and thus that $\mu^\alpha := \mu$ is independent of f. The authors also implicitly rely on an assumption of *locality*: "the probability of an observable event e is unchanged by conditioning on a spacelike-separated free choice z". In our framework, this assumption becomes:

$$f(i,0)(c) = f(i,1)(c)$$
$$f(0,j)(d) = f(1,j)(d) \tag{LOC}$$

This assumption is the exact analogue of Myrvold's locality. They also rely on an assumption of faithful communication (an assumption called the tracking assumption in Schmid et al., 2024, 15), which consists in supplementing locality with the following:

$$f(0,0) = id \tag{COM}$$

Phrased in english, (COM) imposes that when Alice and Bob both chose to communicate, the observable they measure is the same as Charlie's and Daniela's respectively.[11] In one out of four scenarios, operation $\alpha = f(1,1)$ is performed (with probability $p_f(1,1)$). In this case, we recover the corollary.

4 Discussion

We have shown that EWFS (or at least two representative members of the family) and Myrvold's no-go theorem rely on the same mathematical theorem: a corollary to Bell's variant. Where previous no hidden-variables proofs derived an incompatibility between the existence of dispersion-free states (or equivalently beable status) and the preservation of functional relationships among operators, this theorem derives an incompatibility between the former assumption and the Born rule correlations for compatible operators. It is, like Bell's theorem, a theorem on the impossibility of classical joint distributions in quantum mechanics and–provided the quantum mechanical predictions are empirically vindicated–in nature. Where the conclusion of previous no-go theorems (Kochen and Specker, 1967, Gleason, 1957) was taken to be that value-assignments in quantum theory are necessarily contextual, this theorem seems to indicate that contextuality is not enough whenever the joint measurement context is comprised of multiple observers: value-assignments may need to be perspectival (modulo the acceptance of the other assumptions in the theorem). Let us now discuss how the same underlying mathematical can give rise to different no-go theorems through physical interpretation.

4.1 Mathematical theorem, physical assumptions

Pitowsky (1989), discussing Bell's theorem, states the following (p. 49-50):

> "[T]he physical aspects of the problems are intermingled with the purely mathematical character of the derivation of the inequalities. This is a source of prevailing confusion, as if Bell type inequalities have, in themselves something to do with physics. But they do not. I hope the reader is already convinced that these inequalities follow directly from the theory of probability (...). It is only their violation by quantum frequencies which makes them important for the foundations of physics."

Similarly, Bell's algebraic theorem, Bell's variant and the Myrvold-Wigner corollary are not *about* physics. They make no mention of Lorentz frames, or special relativity. They are mathematical results, stemming from probability theory and operator algebras. As mathematical results, they are equivalent (up to a relabelling and a particular choice of α).

However, each no-go theorem is entrenched in a *physical set up*, endowed with a physical interpretation. This is what makes them particular. Each physical set up motivates assumption 2 in the corollary, *i.e.* the assumption that the joint context should have beable status, in different ways.

4.2 Counterfactuality

Bell, Myrvold and the Local Friendliness theorem all use a sort of locality assumption. We have already mentioned that Bell's set up required values to be counterfactually assigned for imposing beable status on the joint algebra, containing both b_1 and b_2, whereas these are never measured simultaneously. Yet, this is precisely the sort of value-assignment that Bell himself denounced in von Neumann's infamous impossibility proof (see von Neumann (1932) for the original proof, Bell (1966) for a criticism, and Bub (2010) for a re-appraisal). However, in Bell's set up, such value-assignments are deemed acceptable only because of a physical motivation: a principle of locality. It is always possible to define valuations on $C^*(a_1, b_1)$ and on $C^*(a_1, b_2)$, because these are abelian. However, the remote choice to measure either b_1 or b_2 should not matter regarding their relative frequencies with the occurrence of a_1, if one imposes that any freely chosen measurement setting is uncorrelated with any set of relevant events – observed or not – outside its future light cone (the relevant events here are a_1 or a_2, only one of which is observed). Locality (in this sense) is the principle warranting counterfactual value-assignment. Weaker locality assumptions are used in the Local friendliness theorem, because all four observables represent *observed outcomes*. This means one can impose the weaker no-signaling assumption, where free-choice are uncorrelated only to observed events. However, it is only the interpretation of the operators as actually or potentially observed which changes the strength of the assumption.[12]

Some EWFS theorem not discussed here, like Frauchiger and Renner (2018), do not employ locality assumptions, but rather justify counterfactual value-assignment by the counterfactual reasoning of agents. This serves the same purpose.

4.3 (In)accessible correlations

All EWFS, as well as Myrvold's theorem, impose assumption 1 of the Myrvold-Wigner corollary: they impose Born rule correlations for all compatible operators. However, depending on the physical set up, this assumption does not play the same conceptual role. In the local friendliness theorem, the correlation duality can be associated to an external observer's perspective, or to that of a superobserver. Indeed, the correlations that end up being tested are those between the superobservers' outcomes, which can be tested. These correlations are thus accessible.

In Myrvold's set up and the relativistic EWFS, assumption 1 amounts to imposing the same correlations in *all* frames, although they can never be tested in a single frame. In that sense, they are *inaccessible*. The legitimacy of such an assumption was already discussed in the modal research program, and among others in Myrvold's paper, which built upon Bell's notion of "serious Lorentz invariance". Special relativity is indeed not enough to warrant inaccessible correlations, as frequencies are not observables, and only observable quantities are required to be frame-independent. However, a theory dispensing with this requirement in effect selects a privileged foliation of spacetime, within which the Born rule correlations are satisfied, although it may be empirically undetectable. This is what Bohm's theory does. Is the violation of serious Lorentz invariance in this sense compatible with the spirit of special relativity, and if not shouldn't this hypothesis need justification and not the other way around?

The discussion of serious Lorentz invariance, as opposed to empirical Lorentz invariance, meets the EWFS literature on the point of inaccessible correlations. Some other EWFS not discussed here, like the Pusey-Masanes theorem (see Schmid et al., 2024, VII), indeed make the same assumption, but do not justify it with relativity. They are framed as a set up with a choice variable, and make no mention of Lorentz frames. However, they impose Born rule correlations between spacelike separated measurements, one of which is eventually undone by a superobserver. Their relative frequencies is therefore inaccessible.

4.4 All admissible maps

As we have seen, each particular no-go theorem comes up with its own conjunction of assumptions to impose beable status on the joint context. Furthermore, the same mathematical conditions are endowed with different conceptual roles depending on the interpretive context. However, we have shown that *all* of the mentioned theorems rely on the *same mathematical corollary*. This unification might be insightful, as it suggests that there might be a common mathematical response to the no-go theorems.

In order to see why, let us entertain the idea to reject *inaccessible* Born rule correlations as a way out of some EWFS. This amounts to abandoning assumption 1 in the Myrvold-Wigner corollary. However, depending on the interpretive context, the correlation duality may represent *accessible* correlations (as in the Local Friendliness Theorem). In yet another interpretive context (that of Myrvold's theorem), the same mathematical way out would amount to choosing a privileged (yet undetectable) reference frame for the theory. One may not want to commit to these interpretive responses, for they may be harder to defend. Yet, this would require multiplying the failure of mathematical conditions in certain interpretive contexts, while they hold in others, to ensure a satisfactory way out for each no-go theorem. This is certainly possible, but a *simpler* possibility, suggested by the mathematical unification, is that all the different no-go theorems are answered by the failure of the same mathematical condition, which takes up a different (but equally defensible) conceptual role in different contexts.

One such condition, which seems to have gone undiscussed so far in the literature, is a restriction on the range of admissible maps α. The Myrvold-Wigner corollary indeed relies on the possibility to chose any admissible maps. It is an open question if one can find natural constraint (meaning a principled reason) selecting a proper subset of admissible maps, for which the corollary does not go through. In Myrvold's theorem, this would translate as a principled reason for why some dynamics are allowed or not in a relativistic setting, while in the EWFS it would mean a principled reason for why some operations may not be performable. I find such a route very much satisfying, at least as a heuristic guide for exploration, for the duality of interpretation relies on an already known duality between (realist) dynamical theories, and (operational) resource theory. As a matter of consistency, one should always be able to juggle between an operational lens on a theory, treating some variables as exogenous to the systems of interest, and a dynamical description including operations as ordinary interactions. Myrvold (2026) calls such a consistency requirement *perspective duality*.

In conclusion, we have shown that a large class of EWFS and Myrvold's theorem are in fact different interpretive framings of the same mathematical theorem. This old look at EWFS, inspired by the modal research program and carried through by unifying algebraic approach, hinted at a new undiscussed potential way out of the no-go theorems, and bridged two research programs that did not communicate until now.

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Notes

[1]See Bell (1964) for the original theorem, and Aspect et al. (1982) for the experimental results.

[2]The individual probability distribution is well-defined as a marginal because p is bilinear, which means that $p(a_i, 1) = \sum_J p(a_i, b_j)$ holds for *any* partition of the unit in $\mathfrak{B}$.

[3]A C^*-algebra $\mathfrak{R}$ is an associative algebra over $\mathbb{C}$, equipped with an involution (the adjoint operation), as well as a norm in which it is complete (it is a Banach space), such that the norm is sub-multiplicative (it is a Banach algebra: $\forall a, b \in \mathfrak{R}, |ab| \leq |a||b|$) and it satisfies the C^*-property: $|a^*a| = |a|^2$. See Landsman (2017) for more details.

[4]A state over $\mathfrak{R}$ defines a state over $\mathfrak{R}_{sa}$ by restriction. Conversely, the following decomposition is unique: $\forall c \in \mathfrak{R}, c = a + ib$, with $a = \frac{1}{2}(c + c^*) \in \mathfrak{R}_{sa}, b = -\frac{i}{2}(c - c^*) \in \mathfrak{R}_{sa}$. A state ω_{sa} then uniquely extend on $\mathfrak{R}$ by $\omega(c) = \omega_{sa}(a) + i\omega_{sa}(b)$.

[5]Just note that $\omega(a) = \langle a \rangle_\psi$, and that $\Delta_\psi(a) = \langle a^2 \rangle_\psi - \langle a \rangle_\psi^2$.

[6]Presented in this way, it is not clear why the admissible maps have to be "suitably relativistic transformations", and why the same theorem could not arise by allowing the admissible maps to range over Galilean transformations. The fact that it does was noticed and derived in Ruetsche (2005).

[7]By dynamical group, I mean a (strongly-continuous) one-parameter group of automorphisms acting on the algebra.

[8]There is an implicit assumption here: Alice and Bob can perform any supermeasurement, in particular it allows them to undo Charlie or Daniela's measurements and then measure the qubit in another basis. This is the equivalent in an operational context of letting α range over all admissible maps.

[9]$\omega((1-a)(1-b)) = \int_X \omega_x(1-a)\omega_x(1-b)d\mu(x)$. For all $x \in X$ such that $d\mu(x) \neq 0$, let us suppose that $\omega_x(1-a) \neq 0$ and prove that necessarily $\omega_x(1-b) = 0$. This will ensure that the integrand is zero for all x. Equation (2.2) implies $\omega_x(1-d) = 0$, thus $\omega_x(d) \neq 0$. Eq. (2.1) implies $\omega_x(1-c) \neq 0$, and Eq. (2.3) implies that $\omega_x(1-b) = 0$.

[10]The general set up is developed for a choice among N possible operations. I will consider here the case $N = 2$.

[11]Together with the assumption (LOC), faithful communication also implies this result when only one superobserver choses to communicate.

[12]The relative strength of these different assumptions is discussed in Schmid et al. (2024). We will not discuss it further because only the conjunction of multiple assumptions, some compensating for the relative weakness of others, is sufficient to impose beable status on the joint context.

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