# Disanalogies as a failure of De-idealization

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## Abstract

In this paper, the issue of analogical failure (or disanalogy) is dealt with, attempting to shed light on this phenomenon in terms of failure of a de-idealization procedure with respect to a relevant feature of the source model. First, the role that idealizations play in defusing negative analogies is investigated. Then, it is shown how a crucial negative analogy can be characterized as an instance of failure in de-idealization of one or more of said idealizations. To illustrate this process, the case of the now-abandoned atomic vortex theory by W. Thomson is analyzed in light of the proposed framework.

***Keywords:*** analogy, idealization, de-idealization, vortices

# 1 Introduction

In *Physics and Beyond*, W. Heisenberg recounts a conversation held with N. Bohr in 1933, attributing the following remark to the colleague: “We must be clear that when it comes to atoms, language can be used only as in poetry. The poet, too, is not nearly so concerned with describing facts as with creating images and establishing mental connections.” (Heisenberg 1970, 1969, p.41).

Now, there is little doubt that, however evocative the Danish physicist’s statement may sound, it can hardly be taken literally; after all, the job of a poet and that of a physicist are quite different. Indeed, the expression “as in” hides a crucial distinction between the two: while the former establishes creative mental connections mainly through the use of metaphors, the latter instead resorts to the help of *analogy* (Gentner and Jeziorski 1993).

Today, analogy stands at the forefront of an intense debate in philosophy of science and all its disciplinary branches (Bartha 2018, Hangleiter, Carolan, and Thébault 2022, Scott-Fordsmand and Suárez 2025, Knuuttila and Loettgers 2014, Nappo, Cangiotti, and Sisti 2024), a discussion that is most likely to be further enriched as a result of its relevance for new developments in computer science and AI (Mitchell 2021). Nevertheless, despite their crucial heuristic role, the precise features of analogical arguments remain elusive and resist to this day any attempt at outright formalization (Weitzenfeld 1984). In particular, the use of analogical reasoning seems to exhibit an almost paradoxical character: on the one hand, analogies play a heuristic and generative role that provides us with new (plausible) models for approaching the treatment of a phenomenon[1], while on the other, they can act as an important factor of inertia that stalls scientific progress, or lead us to fruitless parallels (Bunge 1968, North 1980, Sullivan-Clarke 2019)[2]. For the former, suffice it to mention the central role that analogical reasoning played in allowing Carnot to develop the foundations of thermodynamics (Norton 2022), while for the latter, one may think of the unfortunate analogy between the grooves observed on the surface of Mars by Schiapparelli and canals of artificial origin (Lowell 1895). Keynes (2013, 1921) addressed the issue of this dual nature of

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[1] For some considerations on the generative role of analogy in science and its relation to pursuitworthiness, see Nyrup 2020.

[2] E. Ippoliti attributes this contradictory nature of analogical arguments to their logical properties of *Ampliativity* and *Non-monotonicity* (Ippoliti 2008). The latter in particularly endows analogy with an unpredictable and time-sensitive character that makes it difficult to treat in a deterministic manner.

analogy by introducing a distinction, later developed by Hesse (1963), between *positive* analogies (those which establish a correspondence between features of the systems involved), and *negative* ones (which instead point to the differences between features of the target and the source).

However, while the topic of positive analogies has been widely discussed in the literature on the subject, that of negative analogies and their possible role in determining the failure of a given analogy has been relatively less emphasized (Scott-Fordsmand and Suárez 2025)[3]. The aim of this short essay is precisely to focus on this second, less-explored corner of the dual essence of analogy and, in particular, to argue that the failure of an analogical argument (or *disanalogy*, as I will call it) can be, at least in some relevant cases, characterized in terms of a failure in a *de-idealization procedure* applied to the description of the target with regard to a crucial feature of the source model. Under this perspective then, disanalogies emerge when the idealizations required to sustain an analogy cannot be consistently relaxed.

First, I will provide a brief overview of the relationship between idealization and analogy, highlighting the role that the former plays when constraining the space of possible models provided by the latter. Secondly, I will introduce the issue of analogical failure through a discussion of the vortex atomic theory as proposed by Helmholtz and Lord Kelvin. Finally, I will discuss the topic of failure in a de-idealization process as the driver behind an analogical failure endorsing the framework recently developed by (Luo and Chua forthcoming) and showing how it can be applied to the case for atomic vortex theory.

## 2 Idealizations and analogy

To quote Bartha, an analogy is “a comparison between two objects, or systems of objects, that highlights respects in which they are thought to be similar” (Bartha 2010, p.1)[4]. A minimal characterization of it, therefore is that of a relation of similarity, symmetric and reflexive[5] between objects, properties, relations, and functions of a source $S$ and those of a target system $T$. Bunge (1968) lists three main functions of analogy in science: *heuristic* (and exploratory), to find new laws , *computational*, to solve computation problems through the use of analogues, and *experimental*, to empirically test material analogues; in this context, we will focus exclusively on the first of these uses. Hesse (1953) further distinguishes two types of analogy: *substantive* and *formal*. The former occurs when a target system $T$ directly shares a set of (similar) properties with a source system $S$, whereas the latter occurs when there is a homomorphism between the higher-order relations among the properties of $S$ and those of $T$. This mapping between the source and target structures is thus characterized in terms of a shared mathematical model that describes the higher-order features of both domains. A substantial analogy implies a formal one, but the converse is not true. In general, analogies can be placed on a spectrum that ranges from perfect substantive analogy (identity) to a mapping between two maximally abstract objects. In physics, the distinction between these two types is not always clear-cut, and depending on the degree of idealization employed to provide a mathematical description of the systems considered, an analogy may at the same time be regarded as both formal and substantial (Chen 2022); however, it is still possible to identify the substantive and formal components of an analogy depending on the context and to characterize an analogical argument as an inference from the former to the latter.

Analogical reasoning ultimately relies on the condition that negative analogies concern only irrelevant features of the target (Bartha 2010), those that do not disrupt those causal factors that could be responsible for the two systems exhibiting a common behavior. However, given that short of identity, negative analogies are invariably present, how can the claim of their relative irrelevance be justified? To solve this dilemma, we may wish to consider the kind of relation holding between idealization and analogy, and draw upon the body of literature concerning the explanatory role of idealizations.

In recent years, the debate on idealizations has seen the emergence of an holistic account of the idealizing process, that instead of viewing idealizations as mere localized distortions of causally inert factors in the description

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[3] The problem of the absence of a satisfactory theory of error is a long standing issue in the history of philosophy, as already noted by Plato in the *Euthydemus* and *Theaetetus*.

[4] Note that the definition is in fact circular, as it assumes that one already possesses an idea of what the analogous properties in the two systems are; indeed, already Russell had noted the circularity inherent in appealing to similarity (2001).

[5] But not transitive, because it weakens through iteration.

of a phenomenon, holds that a mathematical model, *qua model*, is itself to be understood as an idealization (Rice 2019). To justify how a highly idealized mathematical model radically different from its target system, as in the case of minimal models (Batterman 2002, Batterman and Rice 2014), can provide explanations of a phenomenon, an appeal is often made to the notion of *universality* (Batterman 2000, Rice 2018, Batterman 2019). The reference to universality as an explanation for the introduction of idealizations is not uncontroversial (see Reutlinger 2014). For present purposes, however, the relevant question is not whether it can provide an exhaustive theory of de-idealization, but rather how it allows us to understand the characterization of negative analogies as irrelevant. The term has its origin in the physics of phase transitions, where it is used to characterize the sameness of behavior near a critical point of different systems that are described by reference to a given set of critical exponents and scaling functions. In philosophy, the notion of universality class has been employed more broadly to denote all those cases in which the microscopic constitution of a system can be taken to be relatively independent of its macroscopic behavior (Rodriguez 2021), and it is this second broader meaning that is relevant to the discussion at hand.

Applying the paradigm in question in the case of analogy in physics, we can say that the relevant positive analogies that enable us to support an argument of the kind described pertain solely to that minimal subset of parameters required for suggesting that both models belong to the same universality class; negative analogies, on the other hand, concern only those details that are neither necessary for this end nor sufficient to preclude the sharing of macroscopic behavior[6]. Thus, the universality account of idealization as applied to physical analogies achieves two distinct aims: on the one hand, it enables us to understand the role that idealizations[7] play in allowing a common mathematical treatment of two different systems, and on the other it further underscores the importance of analogy in providing a heuristic for the application of idealizations themselves. Indeed, one of the main points emphasized by Rice (2019) in opposition to what he calls the *standard view* of the justification of idealization, namely, the idea that idealizations can be introduced locally by virtue of their distorting effect affecting solely factors irrelevant to the explanation required, is that such a conception presupposes that scientists already know what kind of factors are allowed to be isolated and neutralized by idealizations. Yet, the very paradigm that justifies the explanatory power of models in terms of universality does not offer an interpretative key for understanding how, in the first place, a model comes to be selected to describe a physical phenomenon.

Analogical arguments can provide the key rationale behind a choice in the introduction of a package of idealizations (intended as a given mathematical model). Indeed, it is precisely on the basis of the similarities that form part of a substantive analogy that this package is introduced and directed, so as to obtain the same universal mathematical description of both ends of the analogical process[8]. This is the case, for instance, in the analogy underlying the application of the Ising model to financial market crashes developed by Johansen, Ledoit and Sornette and recently debated by Juhn et al. (Jhun, Palacios, and Weatherall 2018). Originally developed to describe phase transitions in ferromagnets, the model lends itself to this new domain by virtue of a set of similarities between the two phenomena: the parameter of spin values present in the source (+1 or –1), can be mapped in the target onto a trader's buy or sell attitude. Correspondingly, the system's global magnetization is taken to be analogous to net market sentiment, and finally, a market crash, amounts to a critical phenomenon, a swift and wholesale transformation of market structure, in analogy with the ferromagnetic phase transition.

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[6] Why then the sharing of fundamental causal factors (as described by Strevens 2011) should not be sufficient to defuse the problem of negative analogies? The idea is that the holistic account of idealizations allows us to focus on the specifically heuristic role of analogy: we need not to know which specific causal factors are relevant in order to allow for a common description, but the similarities between target and source are sufficient to warrant the application of a mathematical model from which, the causally relevant/irrelevant features can be deduced through a de-idealization process.

[7] In this case I am not making explicit any distinction between idealizations and *abstractions* (Levy 2021).

[8] This understanding of analogical reasoning as providing the positive grounds for the adoption of a formal description is close to the account provided by Campbell according to whom analogy is both indispensable for science and the same time once a theory has been established "and shown to lead by purely logical deduction to the laws to be explained, then certainly the analogy might be abandoned as having no further significance" (Campbell 2013, p.129).

## 3 Disanalogy: the case of Kelvin's atomic theory

I shall now introduce a specific case of disanalogy: William Thomson's now-abandoned vortex atomic theory, which was the result of an analogy between the behavior of vortices in a fluid and the nascent atomic theory of chemical elements. The idea that the study of vortices could prove useful in offering a purely geometric description of the properties of matter can be traced back to the insights of Descartes, but the earliest treatment of vortex theory in rigorous mathematical terms was presented only in 1858 with the publication of *Über Integrale der hydrodynamischen, welche den Wirbelbewegungen entsprechen* by H. von Helmoholz. The German scientist succeeded in demonstrating the following three theorems concerning an inviscid barotropic fluid (whose density depends only on pressure) subject only to conservative external forces:

- In the absence of external forces, the strength of a vortex does not change over time.
- The elements of the liquid lying on a vortex line at some point in time continue to lie on this vortex line, which either extends to infinity or has a closed shape.
- In the absence of external forces, vortex-free flow regions remain vortex-free.

In other words, Helmholtz theorems imply that given a vortex line $c$ and defining the circulation of a velocity field along the line as $\Gamma = \oint_c \vec{v} \cdot d\vec{c}$, in the case of an ideal fluid $\Gamma$ remains constant in time ($\frac{D\Gamma}{Dt} = 0$). This entails that vortex lines in the fluid are *material lines* in the sense that they are composed of the same set of material particles at all times; consequently, once a vortex is formed, it retains its identity over time without self-dissipating.

Around the same years, in 1861, J.C. Maxwell turned to vortices to develop his own mechanical ether model of magnetism and explore the mechanism of electromagnetic induction. These studies, combined with the publication in 1867 of the english translation of Helmholtz's text by P. Tait, had a significant influence on the development of mathematized physics in the British physicist community, affecting in particular the works of W. Thomson (Lord Kelvin). The latter, after having witnessed a series of experiments performed by Tait with smoke rings, and assessed their considerable persistence in the presence of turbulence, began seeing in Helmholtz's theory a gateway to develop a mathematical treatment of atoms (Silliman 1963). The underlying intuition by Kelvin was that vortex theory could deliver an account of molecular interactions capable of dispensing with the perfectly solid Lucretian atoms and substituting them with a purely kinematic description of vortices in an incompressible frictionless fluid, the electromagnetic ether. The first "atom" of this kind described by the scientist was a toroidal vortex (Kelvin 1867), and proceeding from the assumption of its stability, Kelvin and his colleagues sought to develop a full-fledged molecular theory based on this concept of ether vortices. Tait, in particular, developed in the subsequent years a comprehensive classification of possible molecular knots (Silver 2006).

With all its elegance and apparent simplicity, the theory quickly run into insurmountable difficulties, already known to Kelvin in 1887 (Silliman 1963), mainly owing to the instability of vortex rings[9]. Despite the remarkable stability shown by smoke rings in laboratory settings, ultimately atomic vortices proved too unstable to account for the kind of "lucretian" indestructibility that was required by chemical atoms. The numerous studies conducted by Kelvin to prove the stability of his atoms proved futile, and his initial hope that the kinetic energy for vortex would reveal itself to be at a minimum, proved false (Falconer 2019); in fact, even the single ring vortex in an ideal fluid "is almost always unstable." (Widnall and Sullivan 1973, p.335).

Despite this decisive setback, for more than a decade after Kelvin's realization the atomic vortex theory was considered a credible candidate for a description of the atomic structure of matter, mainly for two reasons. On the one hand, it was supported by a kind of *no-alternatives argument* (Dawid, Hartmann, and Sprenger 2015) given that at the time there were no credible competitors that could easily displace Kelvin's proposal. On the other hand, its high degree of mathematical formalization and the possibility of presenting it as a kind of theory of everything *ante litteram* exerted a particular fascination among the mathematical physicists of the time (Kragh 2002). Kelvin and Tait's knots were definitively superseded only toward the end of the 19th century with the discovery of the

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[9] Another reason for the ultimate downfall of the vortex atomic theory, proposed by H. Kragh, is that it lent itself too easily to a multiplicity of interpretations without offering real predictive constraints (2002).

electron in 1897, resulting from J. Thomson's studies with the cathode ray tube and his subsequent development of the so-called plum pudding model of the atom[10].

Today, Kelvin's theory has been completely abandoned and is in large part forgotten, except for its role in anticipating developments in knot theory and for its merit in having considerably advanced the study of vortices in fluid dynamics (Silliman 1963, Moffatt 2008).

## 4 De-idealization and analogical failure

In *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (1638) G. Galilei presents the reader his famous diagram of two analogous fictional bones, one belonging to a smaller animal and the other to a larger one. Despite the larger bone being shy of three times longer than the smaller one (as reported by Galilei) its cross section is shown to be much larger in order to bear the increased weight due to the scaling factor between length and volume (and thus mass). The point that Galilei tries to make with this illustration is that despite our inclination to believe that animal morphology between different species could be described through simple analogical reasoning, the mathematical model describing the ratio between length and thickness $\frac{l}{d}$ of animal bones breaks down when factoring other relevant conditions, as the limit in the flexural strength of bone tissue (so $\frac{l'}{d'} \neq k\frac{l}{d}$ for $l' >> l$). What Galilei is emphasizing thus is an elementary procedure of de-idealization with respect to a potential linear model of bone structure scaling, which should lead to our rejection of the proposed analogy between all bone structures regardless of animal size.

The failure of Kelvin's theory can be explained by the very mechanism underlying the disanalogy just outlined. It might be tempting to assume that the failure of an analogy in cases such as the one of the vortex atom theory can just be attributed to the fact that the resulting model is *empirically inadequate*. Yet, the value of an analogy is not solely tied to the truthfulness of the its model, but can also lie in its capacity to explore the causal constraints of a theory or deliver an approximate description of a phenomenon at a certain scale (Gelfert 2016). Unlike what happens, for instance, in the case of calculation errors or of a mistake in the application of a theorem, cases of formal analogy that fail to accurately represent their targets, such as the planetary model of the atom by Bohr and Sommerfeld, have been used to model a phenomenon under certain approximating conditions or retain a didactic value that often outlives their empirical failure. It is no coincidence that the model in question is widely quoted in the contemporary scientific literature on quantum mechanics and is still indirectly assumed, for instance, in the WKB method for finding approximate solutions to the Schrödinger equation.

In light of these considerations, it can be stated that the success of an analogy, and hence of the model resulting from it, depends on two fundamental desiderata:

- The *epistemic aim* of the model, i.e., those features of the target that the model intends to describe;
- The *degree of approximation* for which this description can be considered acceptable.

In the case of Kelvin's theory, as already introduced, the model's epistemic aim was to account for the stability of atoms and their interactions over an unbounded time interval.

But how is assessed, in practice, whether the mathematical model resulting from an analogical argument satisfies these two constraints? The example of the vortex theory of the atom shows us that this occurs *by attempting to de-idealize the model in relation to those features of it that are directly related to its epistemic aim*. A disanalogy therefore manifests itself when a particular limit, corresponding to a process of de-idealization, reveals a contradiction between the epistemic aim of the model and its relevant standard of admissible approximation.

Now, de-idealization, understood as the process of relaxing the idealizations or abstractions in a model is a research area that has started garnering increased scrutiny only recently (Knuuttila and Morgan 2019, Quack 2025, Luo and Chua forthcoming). The topic is somewhat contentious; in light of what was anticipated before, if models are to be intended as holistic and comprehensive distortions of their targets it is not clear how a full reversal

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[10] This was despite the fact that Thomson himself had been a supporter of Kelvin's theory, as evidenced by his master's thesis: *Treatise on the motion of vortex rings*. (1883)

of idealizations could be obtained[11]. Recently Luo & Chua have proposed a local approach to de-idealization processes based on a rejection of the assumption of a talk of full reversal of the idealizations in a model, by distinguishing between intra-model, inter-model (and across same and different domains) and measurement de-idealizations. In the case of Kelvin, too, the scientist was interested in establishing the effectiveness of his analogy under a specific respect and not through a wholesale de-idealization process. As stated, the theory was attempting to recover through the formalism of fluid dynamics the chemical theory of Dalton's atoms, which were supposedly endowed with arbitrary persistence, as the Atoms of Lucretius and Democritus. The process of de-idealization thus consisted in two nested de-idealization procedures. First, an inter-model de-idealization across different domains, that of the atomic theory of matter and of vortex theory in fluid dynamics, deriving the stability conditions of vortices relative to their possible modes and frequencies of oscillation, that were thought by the British physicist to provide the basis for an account of the frequency of empirically observed spectral lines of chemical atoms.

The task of determining the stability of the vibrating atoms was to begin with their simplest formulation, the toroidal vortex[12]. If we consider the case of a vortex line, the velocity $\vec{q}$ at a point $x_1$ is given by the Biot–Savart law:

$$\vec{q}(x_1) = \frac{\Gamma}{4\pi}\int_c \frac{\vec{y}-\vec{y_1}}{|\vec{y}-\vec{y_1}|^3} x d\vec{z}$$

From the law it can be derived that for sinusoidal displacements of the core the amplification rate is:

$$\alpha \simeq \left(\frac{\Gamma}{4\pi R^2}\right) n\sqrt{1-n^2}\, ln\left(\frac{a}{R}\right)$$

where $R$ is the radius of the ring, $n$ the wavenumber, $a$ is the radius of the vortex core and $a \to 0$ (very small core size). In the small core size limit, consistently with an approximation $O(ln\frac{a}{R})$, the ring has a neutrally stable configuration under the displacements (a result that was already known qualitatively to J.J. Thomson). However, as soon as we attempt to move away from said limit and de-idealize the model carrying it to $O(1)$, the ring ultimately proves almost always unstable even for small cores, as demonstrated by Widnall and Sullivan (1973). In the case of molecular configurations obtained through more complex vortex knots, the stability conditions become even more susceptible to such displacements. Ultimately, with the exception of the columnar vortex, none of the vortices envisaged by Kelvin were realistically capable of guaranteeing the permanence of his atoms and remain in accord with the existing experimental evidence about chemical atoms.

## 5 Conclusions

The lesson we can draw from the failure of Kelvin's theory is the following: his proposal for a description of atomic structure in terms of vortices was based on the formalism developed by Helmholtz for fluid dynamics and on an analogy with it. In fluid dynamics, the ideal vortices described by Helmholtz seemed to exhibit remarkable stability, for which there was abundant empirical evidence: the epistemic aim of Kelvin was thus to account for the stability of chemical atoms in terms of the stability of vortices. Yet the problem of the unbounded persistence of vortices proved insurmountable: their permanence could not reasonably be assumed in light of the potential perturbations to which they would have been realistically subjected in accordance with experimental data. The attempt to de-idealize the model by introducing sinusoidal displacements of the core could not lead to any smooth transition from the ideal model to a less ideal one, on pain of destroying the vortex itself[13]. This meant that the very epistemic aim of the model, accounting for the atom's stability, the explanatory standard set as a presupposition of the substantive analogy between atoms and vortices, had been violated by the de-idealization failure, sealing the fate of Kelvin's analogical argument.

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[11] As De Broglie observed: "We also could examine whether all idealizations are not that much less applicable to reality when they become more complete." (Broglie 1969, p. 219).

[12] Kelvin did not have direct access to a complete proof of the instability of his vortex atom. However, he became convinced of the instability of vortices through his conversations with Stokes around the end of the 1880s (Darrigol 2002).

[13] This is unlike what happens, for example, in the case of thermodynamic limits, that are arguably susceptible to controllable approximations (Emch, Liu, and Liu 2002,Palacios 2018).

I have thus shown the role that idealizations play within analogical reasoning and how the failure of a de-idealization process can determine the relative failure of an analogy. In doing so, I have adopted the diachronic and contextual approach to the process of de-idealization proposed by Luo & Chua. The hope for the framework presented is that it could sustain two directions for future work: first, the development of new formal tools capable of elucidating the structural link between idealizations and analogical reasoning; and second, a more systematic account of how analogical arguments must be monitored and constrained to prevent the kind of analogical inertia that, as the vortex atom case illustrates, can sustain a failing research program long past the point of its epistemic exhaustion.

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