Quantum physics and the Identity of Indiscernibles: a logical analysis

Décio Krause

Graduate Course in Logic and Metaphysics
Federal University of Rio de Janeiro
deciokrause@gmail.com
ORCID iD: 0000-0003-0463-2796

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Abstract

We consider the formulation of Leibniz's Principle of the Identity of Indiscernibles (PII) in a second-order language. We argue that it cannot hold in quantum physics because the notion of identity it encompasses, namely the standard identity of classical logic, does not hold in this realm. We provide some insightful examples to support this view.

Keywords: identity of indiscernibles, identity of quantum entities

> *"[I]n nature, there cannot be two individual things that differ in number alone."*
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> Leibniz, *Primary Truths, 1686*, in (1989, p.32).

> *"In the microworld, we need uniformity of the strongest kind: complete indistinguishability."*
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> Frank Wilczek and Betsy Devine (1987, p.135)

1 Introduction

THE ASSUMPTION that in nature there cannot be two completely indiscernible things is a strong metaphysical thesis. Its roots go back at least to the Stoics (Jammer, 1966, p.338),[1] passing through other philosophers of antiquity,

[1] Bochenski attributes the idea to Aristotle (Bocheński, 1961, p.93).

but it was solidified in Western culture by Leibniz through his Principle of the Identity of Indiscernibles (PII). Roughly speaking, PII states that if two entities share *all* their qualities or attributes, they are no longer two entities; they are *identical*, meaning that they are *the same* entity. However, quantum theory presents situations where PII seems to be questioned. The discussion about the validity of PII in the quantum domain is extensive and involves different viewpoints; some philosophers argue that PII is defensible, while others (the minority) argue that it is, in fact, not universally valid in the quantum domain. The question is: are there still other questions to ask about such a discussion that deserve a new paper on the subject? In our opinion, there are many reasons for doing so, the main one being the clarification of several points that have escaped most discussions. In particular, these are: (1) what is the range of the universal quantifier when we speak of *all* qualities? That is, if we restrict the range of this quantifier to some previously selected attributes only (such as considering only monadic properties), are we still dealing with PII? (2) what is the meaning of identity when we say that the entities are *the same*? (3) which logic is being used to derive such a conclusion? (4) *Where* is the discussion being performed? That is, what is the metamathematics? (5) Why is it important to consider the metamathematics? Due to problems of space, we shall restrict our account to these questions, hoping to contribute to a better understanding of PII and its relationships with quantum theory.

A remark about terminology: physicists call quantum entities of the same kind (such as electrons or neutrons) *identical*, and in certain situations, they cannot be discerned from one another; in such cases, they are *indistinguishable*. This terminology is strange for a mathematical and logical discussion; in logic and philosophy, *identical* means *the same*, *not more than one*, *sameness*, while indistinguishable means that they cannot be discerned in the considered situation, even though they can be distinguished by other means (e.g., photons by different polarisations). We use the terminology in this logical sense.

We start by discussing PII in different but equivalent formulations in the same logical language: a second order language. This aims to move from the purely informal philosophical discussion to a logically guided philosophical discussion. Secondly, we consider the involved concepts of identity and indistinguishability, which are mixed in the standard formulations of PII, along with the alleged *different versions* of the principle. We also consider the set-theoretical version of the principle and emphasise the mathematics that most discussions seem to presuppose; if something in that sense is considered (generally, the philosophical discussions do not specify either the involved logic or the assumed mathematics).

To argue about the relevance of considering the underlying logic, we present the idea that quantum entities should not be discerned *absolutely*; that is, by a monadic property. Notice why; in traditional metaphysics, *individuation* is closely linked to identity. So that, according to Gracia,

> "[I]ndividuation, in metaphysics, [is] a process whereby a universal, e.g. *cat*, becomes instantiated in an individual – also called a

particular ... (Gracia, 2015)

This means that we have to have a way to discern the particular thing that is being individuated; in other words, something like the standard notion of identity must hold in the domain. In a series of works, we have discerned among the concepts of *identity*, which we regard as a logical notion ascribed by your preferred logic, *individuation*, as in Gracia's sense of a metaphysical notion that provides a way to say that something is the thing it is, and *isolation*, which stands for the epistemological property of something being in complete isolation from other things of its species, such as a quantum object in a trap. In our opinion, these three notions must not be confounded; see (Krause and Arenhart, 2025, §2.3), (de Barros et al., 2026, §2.2.1).

Anyway, quantum things can be weakly distinguished by an adequate binary relation (see section 4.1 for the definitions), such as (in the quantum case) 'to have spin opposite to', which is irreflexive and symmetric. But we will show that if the assumed framework involves classical logic, this cannot be the only case: in the 'classical' domain, all entities are *individuals* endowed with identity conditions, with the consequence of being discerned *absolutely* as well; despite being weakly discerned, they can also be distinguished from one another by a monadic property coming from logic, which cannot be dismissed.[2]

Let me insist on this last point for clarity. In quantum physics, mainly from the point of quantum field theories, indistinguishability is not just an epistemological trait of quantum entities; it is a fundamental characteristic of them. For instance, a system with two electrons must be anti-symmetrical under the exchange of the electrons labels, showing that the labels do not represent individuation; the exchange makes the state signal change, but its square, which gives the relevant probabilities, is the same. To some physicists, like Leonard Susskind,[3] quantum entities are better treated as *patterns* (of energy and probabilities), charged densities, described by wave-functions that explain the material structures and their interactions. These patterns do not have separate identities, and there are no boundaries separating one electron from the next.

Summing up, our position is that *PII cannot hold in the quantum domain if the notion of identity it encompasses is described by the identity of classical logic, which, in our opinion, does not apply to quantum entities*. Furthermore, we defend the idea that whatever restriction one makes in the domain of the universal quantifier when PII speaks of *all* properties breaks down PII and transforms it into mere indistinguishability with respect to the selected properties.

2 Modern logic formulations of PII

Leibniz formulated PII in several of his works and in different styles, but he always mentions that any *two* things present an *inner* distinction: a quality that

[2]The theorems of the theory's underlying logic, though not made explicit, are theorems of the theory itself. Therefore, if the objects in the domain of the application of the theory can be discerned by some 'logical' property, such as their identities, this cannot be ignored by the theory.

[3]See the interesting video with him on YouTube: *Why electrons are not real objects*.

one has and the other does not; see (Rodriguez-Pereyra, 2014). PII involves two main concepts, namely, indiscernibility (or indistinguishability) and identity. We shall discuss both notions in what follows.

Rodriguez-Pereyra presents 36 different formulations of PII in Leibniz's texts, and he advises us that more can be found. The interesting fact is that practically all of them speak negatively, using phrases such as *it is impossible that*, *there cannot be*, and *it is not true that*, among several other ways to deny the existence of 'concrete things' that are completely similar. Perfect similarity, says Rodriguez-Pereyra, was accepted by Leibniz only with respect to *abstract notions*, such as two triangles or two lines (ib., p.21). But quantum entities, supposedly, are not abstract objects. So, Leibniz seems to reign here as well; let us consider logic to deal with such a situation and see if PII *can* be valid in this realm.

Consider a second-order language $\mathscr{L}$ with two kinds of variables: (1) individual variables, denoted by $x, y, z, \ldots$, and (2) predicate variables, denoted by $X, Y, Z, \ldots$. We use the symbol F to represent these predicate variables, as usual. Identity is defined by Leibniz Law, namely,[4]

$$x = y := \forall F(F(x) \leftrightarrow F(y)). \tag{LL}$$

The *definiendum* is *identity*, and in our 'semantic' interpretation, we say that if $x = y$, this means that x and y refer to the same entity; they are singular referential terms (proper names or definite descriptions that refer to *the same* entity). The *definiens* means *indiscernibility* or *indistinguishability*; things are indiscernible or indistinguishable when they share *all* their properties (notice the universal quantifier indicating that it is the class of *all* properties being considered, which will play an important role below).

In such a language, PII (in the 'negative' form) can be written as follows:

$$\forall x \forall y(x \neq y \to \exists F((F(x) \wedge \neg F(y)) \vee (F(y) \wedge \neg F(x)))), \tag{PIIn}$$

where '$x \neq y$' means that x and y are different, that is, they are *two*. In modern logic, this is equivalent to the 'positive' PII, which is the formulation most commonly used in the philosophical literature and is half of (LL):

$$\forall x \forall y(\forall F(F(x) \leftrightarrow F(y)) \to x = y). \tag{PII}$$

The other half is the Principle of the Indistinguishability of Identicals (II), namely,

$$\forall x \forall y(x = y \to \forall F(F(x) \leftrightarrow F(y))). \tag{II}$$

This formulation of PII, which we shall use from now on, presents two main points that need to be discussed; first, there is the universal quantifier ranging over the properties of the considered entities. Secondly, there is the symbol of

[4] This definition can be extended to higher-order types, as in Whitehead and Russell's *Principia Mathematica* (191013*).

equality, which serves to denote *identity*. Thus, we can ask: (i) which properties should we consider in the range of $\forall F$? (ii) what is the meaning of identity? Can it be merely *qualitative identity*, that is, indiscernibility? We address these questions in the next section. After discussing the notions involved in the above formulation of PII, we turn to the discussion of its validity in quantum physics.

This paper is organised as follows. In the next section, we analyse the two fundamental notions involved in the PII, namely, identity and indistinguishability. We consider these notions within present-day logic and mathematical frameworks; we shall see that in some formulations, PII is a theorem of classical logic (set-theory involved), and so it cannot be rejected within such a context.

3 Identity and indistinguishability

Leibniz was not clear about the meaning of the term *identity*. But from all we know, it is reasonable to say that he intended to mean *sameness*, *the same*, *not two*; that is, *numerical identity*. Indistinguishability, or indiscernibility, on the other hand, means *similarity*, sometimes also called (wrongly, in my opinion) *relative identity*. Two things can be said to be *similar* relative to some characteristic or a group of characteristics. In this case, we say that they are *indistinguishable* with respect to such a property or group of properties. If they share *all* their properties, they are said to be *completely indistinguishable*. Then comes the first question mentioned in the previous section: which properties may an object have? We shall address this question below, but first, let us focus on the mathematics used in the discussion.

4 Some theorems of ZFC and about ZFC

The mathematics in which the discussions are conducted matters. To see this, we present some theorems and metatheorems regarding the set theory that is most commonly used as the metamathematics in the discussions (see section 4.1 below). *When* philosophers say something in this respect, they refer to the ZFC system; that is, the Zermelo-Fraenkel set theory with the Axiom of Choice; here, we do not consider *Urelemente* but rather sets only. Statements that are consequences of the axioms of ZFC are called *theorems*. Statements *about* ZFC are called *metatheorems*.

To clarify the discussion, we recall some quite trivial theorems of ZFC and also an important metatheorem. They serve to emphasise some points we are defending.

Remember that the Standard Theory of Identity in ZFC encompasses the axioms of Reflexivity ($\forall x(x = x)$), Substitutivity (for any formula $\alpha(x)$ with the variable x free, we have that $\forall x \forall y(x = y \to (\alpha(x) \leftrightarrow \alpha(y)))$), and the Axiom of Extensionality ($\forall x \forall y(\forall z(z \in x \leftrightarrow z \in y) \to x = y)$).

A useful fact *about* ZFC is this: the cumulative hierarchy of sets (also called the *von Neumann universe*) is a collection of sets obtained by transfinite recursion on the class *On* of ordinal numbers. It provides a standard model (supposing ZFC is consistent) and an intuitive motivation for the axioms of Zermelo–Fraenkel set theory (ZFC).[5] Let $On = \{0, 1, 2, \ldots, \omega, \omega + 1, \ldots\}$ be the proper class of all ordinal numbers, defined *a la* von Neumann: $0 \coloneqq \emptyset$, $1 \coloneqq \{0\}$, $\ldots, n + 1 \coloneqq \{0, 1, \ldots, n\}$, $\omega \coloneqq \{0, 1, 2, \ldots\}$, $\omega + 1 \coloneqq \{0, 1, 2, \ldots, \omega\}$, etc.[6]

Then we put

[label=iv)]

$V_0 \coloneqq \emptyset$

$V_1 \coloneqq \mathcal{P}(V_0)$

$V_{n+1} \coloneqq \mathcal{P}(V_n)$

$V_\lambda \coloneqq \bigcup_{\beta<\lambda} V_\beta$ if λ is a limit ordinal,[7] and

$V \coloneqq \bigcup_{\alpha \in On} V_\alpha$

A *structure* in ZFC, or in a model of ZFC, is an ordered pair $\mathfrak{A} = \langle D, \{R_i\}\rangle_{i \in I}$ where D is a set (given by the axioms) and $\{R_i\}$ is a collection of n-ary relations over the elements of D; it suffices to consider only relations since individual constants and functional symbols can be expressed by means of relations. For instance, a group (really, a *model* of the axioms for groups) is a structure of the form $\mathfrak{G} = \langle G, * \rangle$, where G is a non-empty set and $*$ is a binary operation (a ternary relation) over G. All the relevant structures that model the most important mathematical and even physical structures are structures of this kind. A small remark is in order: in most cases, mainly involving theories of physics, the relations are not only relations over D, but may comprise more sophisticated relations such as derivatives, Hilbert spaces, and the like. They are *higher-order* structures, but the main results about them can be achieved with our simple description.[8]

An *automorphism* of $\mathfrak{A}$ is a bijective function $h : D \to D$ such that, for any n-ary relation R, we have

$$\mathfrak{A} \models R(x_1, \ldots, x_n) \quad \text{iff} \quad \mathfrak{A} \models R(h(x_1), \ldots, h(x_n)), \tag{1}$$

[5] A standard model of ZFC is a structure $\mathscr{M} = \langle M, E \rangle$ where M is a set and E is a binary relation over M that represents the *true membership* $\in$ in the model. If ZFC is consistent, no model can be proven to exist as a set of ZFC, necessitating that it be find in some stronger theory such as the KM (Kelley-Morse) system (Roitman, 2013; Halbeisen and Krapf, 2025).

[6] This collection is also not a *set* in ZFC, meaning it cannot be obtained from the axioms of this theory. Collections like *On* are called *proper classes*.

[7] An ordinal β is a *successor* if there exists an ordinal α such that $\beta = \alpha + 1$. For instance, any natural number (the finite ordinals) is a successor, so is $\omega + 1$. If the ordinal is not a successor, it is a *limit ordinal*; for example, ω is a limit ordinal.

[8] In the case of higher-order structures, it must be recalled that there is no 'model theory' as there is for first-order structures. The results must be looked for one by one.

that is, iff h 'preserves' all the relations of the structure. For instance, the operation of taking the conjugate is an automorphism of the structure of the field of complex numbers $\mathfrak{C} = \langle \mathbb{C}, +, \cdot, 0, 1\rangle$. A *rigid* structure has only the identity function as an automorphism. Otherwise, it is *non-rigid* or *deformable*.

Then we have:

Let V be the von Neumann cumulative hierarchy of sets. The structure $\mathcal{V} = \langle V, \in\rangle$ is rigid.

Proof: A structure is rigid iff its only automorphism is the identity function. Let $h : V \to V$ a bijection such that $x \in y \to h(x) \in h(y)$; such an h is called an $\in$-automorphism.[9] We shall prove (by a process called $\in$-induction) that, for all x, $h(x) = x$. Consequently, h is the identity function. Let $z \in x$, and let us assume that $h(z) = z$. Let $y = h(x)$; then $x \subseteq y$, since if $z \in x$, then $z = h(z) \in y = h(x)$. But we also have that $y \subseteq x$, since if $t \in y$, and since $y \subseteq V$, there exists $z \in V$ such that $h(z) = t$. But $h(z) \in y$, so $z \in x$ and then $t = h(z) = z$. Hence $t \in x$, hence $h(x) = x$. ∎

Notice that since V is not a set in ZFC, the structure of the theorem is not a structure *in* ZFC; this is the reason we have a *meta*theorem here.

The metatheorem implies that any element of the domain of V, that is, *any* set, is $\mathcal{V}$-indiscernible only from itself, since the identity function is the only automorphism. We say that they are *individuals*. Emphasising: in ZFC, there are no *complete indistinguishables*; that is, many objects sharing all their properties (belonging to the same sets and having the same elements). If they share all their properties, they are not two, but just one object. This is Leibniz's PII.[10]

Other simple yet useful results are as follows:

For any x and y, if the cardinality of the set $\{x, y\}$ is 2, then $x \neq y$.

Let us prove the contrapositive. If $x = y$, then $\{x, y\} = \{x\} = \{y\}$, whose cardinality is one; hence, it is distinct from 2. ∎

For any x and y. If $x \neq y$, then there exists a monadic predicate F such that $F(x)$ but $\neg F(y)$.
(Recall that by a 'monadic predicate' we understand, as usual, any formula with just one free variable).

Let us suppose that $x \neq y$. Let $X = \{x\}$ and $Y = \{y\}$ be their unitary sets, which exist by the Pairing Axiom; then $X \neq Y$ follows by extensionality. Define $F(z) := z \in \{x\}$. Then $F(x)$ and $\neg F(y)$. ∎

These theorems show that any *two* objects are *different*, and this implies that there exists a monadic property (absolute discernibility) 'to belong to the unitary set' for each object that only that object has, so PII holds. This conclusion is imposed by the underlying logic and should not be dismissed as not holding in the quantum domain, once we accept that quantum physics is grounded in classical logic.

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[9]Let us remark that since V is a proper class and not a set in ZFC, the notion of bijective function needs to be adapted for proper classes. But we can do it in our metamathematics.

[10]If the reader searches in the specialised literature, she will find notions such as *Silver indiscernibles*, and perhaps other kinds of 'indiscernibles' in the context of set theory (Jech, 2003, p.299ff,Chap.18). However, these are not what we are speaking about. Those 'indiscernibles' obey the axioms of ZFC; hence, they are *individuals* in our acceptance.

If we understand an *individual* to be an object that obeys the standard theory of identity, then the above theorems show that in ZFC, every object (every set) is an individual, so there are no completely indiscernible things. Elements of the universe V can be indiscernible only relative to 'some properties' (qualitative identity), say by the fact that they belong to the same sets of a certain collection of sets. But if this collection is the whole V, then the objects turn out to be identical.

The set-theoretical version of PII holds in V.

The ZFC-version of PII is exactly its Axiom of Extensionality, namely,

$$\forall x \forall y \big(\forall z (z \in x \leftrightarrow z \in y) \rightarrow x = y\big), \tag{2}$$

which is trivially true. ∎

In standard higher-order logics, since we can define identity (by Leibniz Law), to ensure that PII does not hold, we need to modify the logic, for instance, by means of Schrödinger Logics (da Costa and Krause, 1994).

The important consequence of this metacorollary is that if we have a ZFC formulation of quantum physics, its models are sets in V. Then the only way to consider indistinguishable objects (such as bosons in a bosonic condensate or electrons in a singlet state) is by confining them in a non-rigid structure that has other automorphisms besides the identity function. However, the reader should acknowledge that this is a mathematical trick *to make* some objects appear indiscernible when, in reality, they are not. This is what generally happens when we use standard logic in quantum discussions.

4.1 The relevance of the underlying logic

Any reasonably formulated scientific theory assumes some underlying logic, even if it is not clearly stated;[11] for instance, we do not know what the underlying logic of the Standard Model of Particle Physics is, but we can assume that there is one that guides the logical inferences of the theory (supposedly, it is classical logic).[12] The theorems of the underlying logic are theorems of the theory, and if the results of the specific part (say the 'physical part' in our case study) of the theory contradict what the logic determines, we are faced with a logical problem. This is the main reason why Birkhoff and von Neumann have proposed that 'the logic of quantum mechanics' should not be classical logic (Birkhoff and von Neumann, 1936). This is particularly problematic if the underlying logic is given explicitly, even if only by reference. Notwithstanding, some philosophers have made assumptions about the objects described by quantum physics that go against the results of the assumed underlying logic.

[11] For instance, Dalla Chiara and Toraldo di Francia say that a *formalized physical theory* – and we can assume that, even if only in principle, any physical theory can be described as a formal system endowed with some interpretation – is an ordered pair $T = \langle SF, \mathscr{K} \rangle$, where SF is a formal system and $\mathscr{K}$ is a class of structures, the *physical models* of SF (Dalla Chiara and Toraldo di Francia, 1981, p.60).

[12] We reaffirm that, to us, logic is *great* logic, involving also mathematics; that is, at least a set theory such as ZFC.

A good example is presented by F. A. Muller and S. Saunders, who gained prominence in the philosophical discussions on the philosophy of quantum physics (Muller and Saunders, 2008). They claim that quantum objects (they refer to fermions, but later the result was extended to bosons by Muller and Seevinck (2009)) can be at least *weakly discerned* (WD), but cannot be *absolutely discerned*. The entities are weakly discerned iff there exists a non-reflexive but symmetric relation that they satisfy, such as 'to have opposite spin'; they are absolutely discerned iff there exists a monadic property that holds for only one of the entities. However, as we have shown above, in a ZFC setting (Muller and Saunders explicitly claim that it is the ZFC system; see the quotation below), *every object* (set in ZFC) satisfies the ZFC axioms and, consequently, can be distinguished from any other object by a monadic property. The problem is that these philosophers, like many others, do not accept such a conclusion, which is imposed by the underlying logic, preferring to restrict the range of acceptable individuation properties that, for some of them, should not include either reference to individual objects (say by proper names) or the identity symbol (Bigaj, 2022, p.36). A look at this question will contribute to the discussion on the nature of the properties that can form the range of the universal quantifier in PII.

Muller and Saunders state that

> "we begin with some weak set theory sufficient to erect all the mathematics that ever will be needed in QM. The gold standard is Zermelo-Fraenkel set theory (ZFC)."

That is, they are *explicitly* grounded in the ZFC system. Let us follow Muller and Saunders by adapting the notation as follows; see (de Barros et al., 2026, Chap.8):

[label=1)]

PII-A (here, just 'A') is the *Principle of Absolute Indiscernibility*. It states that no two objects a and b can be absolutely indistinguishable. Hence, there is not a monadic predicate P such that $P(a) \wedge \neg P(b)$.

PII-R (simply 'R') is the *Principle of Relational Indiscernibility*. In words, no two objects a and b are relationally indistinguishable, that is, there exists a relation R such that RD holds.

PII is the Principle of the Identity of the Indiscernibles, discussed previously.

From Muller and Saunders (op.cit., p.504), we get the following:

[label=1)]

A $\rightarrow$ PII and R $\rightarrow$ PII , and therefore A $\vee$ R $\rightarrow$ PII .

Consequently, $\neg$PII $\rightarrow$ $\neg$A $\wedge$ $\neg$R.

Furthermore, they prove (ibid.) that A $\to$ R, or, in their words, "absolute discernibles are always weak discernibles," so, as they also say, "A is not necessary for PII." This enables them to dispense with PII-A and keep PII-R only.

Hence, R $\leftrightarrow$ PII (the proof is detailed in the mentioned paper).

Consequently, we have $\neg$(PII $\to$ A).

Hence PII $\wedge$ $\neg$A, that is, PII holds even if the things are not absolutely indistinguishable.

They conclude by assuming that similar (indistinguishable) quantum objects can be "nonidentical absolute indistinguishable"; hence $\exists P(P(a) \wedge \neg P(b))$. This culminates in their central thesis, namely that "absolute discernibles are always weakly discernibles." Consequently, we could retain only WD and dispense with A, and PII is preserved.

Muller and Saunders' views are quite interesting, and we partially agree with them. However, one should take into account that PII (in some of its formulations) is a theorem of classical logic.[13] Thus, in ZFC, we cannot refute PII, consequently, any discussion attempting to preserve PII, once established within a classical logic context such as ZFC, is redundant.

Then the question arises: is PII valid in the quantum domain? We claim that it is not, and the reasons are presented in what follows.

5 Selecting the properties

The first step to analysing the validity of PII in quantum physics. Remember that in the standard formulation (that is, in PII), there is a universal quantifier applied to a variable F that, in principle, ranges over the collection of *all* properties of the objects denoted by the individual variables x and y. The intuitive meaning of the universal quantifier is simple: it means *all*. As Joseph Melia has alerted us, "if the logician's '$\forall$' doesn't mean 'all', what does it mean?" (Melia, 1995). In the standard formulations, nothing is said about this collection of properties, so some philosophers consider different versions of PII, depending on the chosen properties. For instance, we have (French and Krause, 2006, p.10):

[label=v)]

(PII-I) Includes all properties; hence, in the negative form, we can state it this way: *it is not possible for two quantum objects to possess all their properties (relational and non-relational) in common.* Or, in the positive formulation, *if two quantum objects have all their properties in common, they are the same object; they are identical.* It is remarkable to emphasise that if for any given objects

[13] For instance, in the language of ZFC, we can state PII this way: $\forall x \forall y (\forall z (z \in x \leftrightarrow z \in y) \to x = y)$, which is the Axiom of Extensionality.

a and b, we include the property 'being identical with a' in the range of the variable F, then PII becomes a theorem of second-order logic. The proof is simple: by particularising PII to a and b, we have

$$\forall F(F(a) \leftrightarrow F(b)) \to a = b. \tag{3}$$

Now let us define 'being identical with a' as follows: $I_a(x) \coloneqq x = a$. Since (3) holds for any property F, it holds for I_a, that is, we have

$$(I_a(a) \leftrightarrow I_a(b)) \to a = b, \tag{4}$$

and then, to conclude that $a = b$, as demanded by PII, the antecedent of the conditional cannot be false; hence, once $I_a(a)$ is true, so must be $I_a(b)$; that is, $a = b$.

(PII-II) Excludes spatio-temporal properties. These are *extrinsic* properties of quantum systems. Consequently, with this formulation, we cannot use spatial location to differentiate between two quantum entities. A typical case where this cannot be made is when we have the two electrons of a neutron Helium atom. In this case, the wave-functions of both electrons are completely superposed in a way that makes it impossible to know which electron has a particular wave-function (Eisberg, 1964, p.360).

(PII-III) In this formulation, only monadic properties are enabled in the range of the variable F.

In our present-day opinion, only PII-I can be considered legitimate PII; all the others, which impose restrictions on the enabled properties, contribute only to different forms of *indistinguishability* (relative to the chosen properties). This is true even for the general case in second-order semantics. As is known, second-order logic has (essentially) two kinds of semantics: *full semantics* and *Henkin semantics*. In the former, all subsets of the domain of the variable F are taken as the extensions of the monadic predicate symbols, while in the latter, there is some restriction corresponding, for instance, to PII-II and PII-III. In such cases, we may have that PII is still a theorem of the logic, but we cannot conclude that, despite $a = b$, the individual constants a and b denote *the same* object of the domain; yet, they satisfy all the chosen properties; their corresponding elements in the domain may be distinct. A simple example is this: suppose that the language has two monadic predicate symbols; furthermore, there are two individual constants a and b. Let us denote the predicates respectively by F and G and define an interpretation as follows. The domain is the set $D = \{1, 2, 3, 4\}$; the individual constants a and b are interpreted respectively in 1 and 2, and the predicate symbols in the subsets $Ext(F) = \{1, 2\}$ and $Ext(G) = \{1, 2, 3\}$. Then it is clear that $F(a) \leftrightarrow F(b)$ since both belong to the extensions of the predicates. However, $1 \neq 2$ although we have $a = b$ from the point of view of the language.

This poses a fundamental question: when we discuss PII, are we speaking of the objects of an interpretation, or are we merely fearful of the language?

Things change depending on our choices. One example of a case where the language is considered is van Fraassen's approach. In discussing the PII in quantum mechanics, he stressed that

> ""identical particles [...] are certainly qualitatively the same, in all the respects represented in quantum-mechanical models—yet still numerically distinct." (van Fraassen, 1991, p.376)
>
> "[i]f two particles are of the same kind, and have the same state of motion, nothing in the quantum-mechanical description distinguishes them. Yet this is possible." (van Fraassen, 1998)

It looks like Skolem's paradox: first-order ZFC has denumerable models (due to the Downward Löwenheim-Skolem theorem, the set (in the model) that represents the set of real numbers must be denumerable, although it appears non denumerable to someone not in the model). The answer is that the bijection between the natural numbers and the real numbers can exist *outside* the denumerable model (Moore, 1982, p.252). In the same vein, we think that van Fraassen's distinction between *that* which can be described by quantum theory and *that* which cannot lies on the same principle: PII can be true *from the outside* (say, by some ad hoc supposition) but not from *inside* the theory. The former is a theoretical construct (hence, inside the theory), and part of the challenge involved in the construction of the theory depends upon a proper representation of these states. Value attributions, in turn, express values that an observable actually has (hence, something that demands more than the theory). So, van Fraassen 'saves' the PII by going *outside* the theory, making it hold in the metamathematics.

For more details on such a comparison between Skolem's paradox and van Fraassen's view, see (Krause and Bueno, 2007).

6 Restricting quantum predicates

Let us consider now the particular case of quantum physics. First, we generalise the notion of *relative indistinguishability* and then turn to quantum mechanics.

Suppose that we have a language $\mathscr{L}$ with predicate symbols in the desired quantity. Suppose further that we would like to classify some property as *physical*, whatever definition you use. How could we do it? The only syntactical way (that is, with the resources of the language) is to admit the existence of a third-order predicate variable F. Then, for X being a predicate variable of second-order, $F(X)$ states that X is a 'physical property.' If F can also be quantified, the language must be at least of third-order. However, if we take F as a predicate constant, we can remain in the second-order level, which is enough for the argumentation. We can impose the following definition (or axiom), which states that x and y are indistinguishable with respect to the F-properties:

[Relative indistinguishability]

$$x \equiv_F y := \forall X(F(X) \to (X(x) \leftrightarrow X(y))).$$

This is precisely the definition of the identity of elementary particles given by Joseph Jauch in his book (Jauch, 1968, p.275): elementary particles are identical iff they agree with respect to their intrinsic properties. It does not matter what 'intrinsic' means here (they are those properties that do not depend on the state of the system), but only that they are a kind of special properties of a quantum system. The first remark is that the above definition defines *identity* only in the jargon of the physicist, which is precisely what Jauch states. But in mathematics and philosophy, *identity* means *sameness*, *the same*, while according to Jauch's definition, two distinct particles of the same kind, say two electrons, can be 'identical.'[14] We call such a definition *indistinguishability with respect to the F-properties*. Consequently, whatever restriction is made in the range of the universal quantifier in PII, admitting that it holds, does not conduce to identity properly speaking (in the logical and mathematical sense), but to indistinguishability relative to the chosen properties. We shall see in the sequence that the failure of PII in the quantum domain is not due to the restriction of the properties; rather, it is due to the failure of the standard notion of identity.

Continuing to explore the syntax of $\mathscr{L}$, how can we distinguish the F properties from the remaining ones? Basically, we have two alternatives: (1) to introduce some postulates that hold for F properties only, or (2) to recur to semantics. The postulates, in the case of Jauch's definition, could be a group of characteristics specifying what intrinsic properties are; however, in general, a physical theory claims meaning, so we need to refer to semantics. In extensional contexts, that is, according to standard semantics for second order logics (Shapiro, 1991), monadic second order variables are associated with subsets of the domain of the interpretation, while F would designate a collection of such selected subsets, those that are the *extensions* of the F-properties. Then the problem can be formulated this way: how can we select those subsets that will be elements of the image of F? This is discussed next.

7 Why standard identity cannot hold for quantum objects

So far, we have stated that, in our opinion, PII should not accept restrictions in the range of the predicate variable F, and that PII must be false in quantum theory because the notion of standard identity is questionable; by the way, it is *highly questionable*. In this section, we reinforce this point of view.

By *standard identity*, let us recall, we mean that notion axiomatised by classical logic. In first-order languages (with identity as a primitive notion, symbol-

[14] By the way, all particles of the same kind are identical (in the physicists' sense).

ised by the binary predicate =), the postulates of the relation '=' are (i) reflexivity and (ii) substitutivity. In a higher-order language, identity can be defined by Leibniz Law (LL). In set theory such as the ZFC system, when axiomatised as a first-order theory, in addition to the postulates (i) and (ii), we add an Axiom of Extensionality, as has already been seen. As we have made clear, every object in the universe of sets, or in whatever model, obeys these axioms, which implies that if there are *two* objects, they differ by at least one property. It is this fact that prevents the application of standard identity to quantum objects. For such an application, we need to *interpret* quantum entities as sets (or as elements of sets), and hence standard identity applies to them, making any two of them *different* and distinguishable by means of some property. We do not think that this is the better option for considering *quantum semantics*.

One should acknowledge that quantum objects are different from 'classical' or standard objects described by classical mechanics, something that seems to be recognised today. As Schrödinger stressed in his fundamental paper from 1935,

> "When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought." (Schrödinger, 1935)

This characteristic trait is *entanglement*; two (or more) quantum systems can share a state such that there is no way to describe them as separate systems. As has been stressed by Robert Eisberg, "indistinguishability is a purely quantum mechanical situation" (Eisberg, 1964, p.361); they do not have 'separate' properties that would enable us to identify which system is which, since the experimental outcomes obtained for any measurable quantity are independent of the identification of the particles. For instance, a Bose-Einstein Condensate (BEC) may involve milliards of bosons, all of which share the same quantum state; as the Nobel laureate Wolfgang Ketterle stresses, the elements of a BEC become a "quantum soap" (his words) of indistinguishable particles (Ketterle, 2001). There is no way to discern among them, and this is not a defect of the theory; on the contrary, the theory leads precisely to this conclusion. Consequently, it seems reasonable to guess that standard identity cannot hold for such entities (there are several other examples that confirm this claim).[15] Despite the fact that Schrödinger did not speak of the standard identity as we have done, but just of 'identity', it is clear that he was referring to something along these lines. Then he made a strong claim:

[15] Another clear example is the Hong-Ou-Mandel (HOM) effect. Initially made with bosons, it requires complete indistinguishability to explain the observable phenomena; see (Brańczyk, 2017).

> “And I beg to emphasize this and I beg you to believe it: It is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of ’sameness’, of identity, really and truly has no meaning.” (Schrödinger, 1952)

The resume is that if standard identity holds for quantum objects, as we have emphasised before, any two of them, even in an entangled system (say in a BEC), would be different, and some property would exist to differentiate them, even if only in principle. Of course, this is not possible according to standard quantum mechanics.

Bohmian quantum mechanics (BQM) postulates that particles exist (are *real*) and have well-defined deterministic trajectories at all times; these trajectories provide their identities (Tumulka, 2022). So it seems that the last sentence of the previous paragraph is misleading. However, one must consider that to know the exact trajectory of a particle, we need to know its initial position exactly; nevertheless, that cannot be known in practice due to the *quantum equilibrium hypothesis*, which posits that it is the probability density $\rho = |\psi|^2$ that describes the particle configuration and provides us only with a probability density of the initial position, rather than the position strictly speaking. Being unknowable, the trajectories (hence the positions) of the particles are determined by *hidden variables* (Goldstein, 2024) and cannot be known. As Mahler has stated,

> “A consequence of [the Heisenberg uncertainty principle] is that the trajectory of a single Bohmian particle cannot be observed in an experiment on that particle; any measurement of a particle’s position changes the wave function and thus the guiding potential that the particle experiences.” (Mahler, 2016)

The alleged identity of the particles is a supposition that cannot be proven in experimental terms; yet, complete indistinguishability, which contradicts standard identity, is widely demonstrated by experiments such as the aforementioned BECs, the HOM effect, entangled systems, and so on; see (de Barros et al., 2026) for a more in-depth discussion.

8 Conclusions

The standard identity, which *is the identity* defined by classical logic, entails that whenever we have two or more things, they differ by at least one property. In a ‘classical setting’, such as by using standard mathematics and logic (grounded in a system like ZFC set theory), two distinct entities can *always* be discerned by a monadic property.

However, quantum physics presents entities that can be *completely* indistinguishable, contrasting with standard identity theory. Consequently, standard identity cannot hold for such entities. This is the main reason why PII cannot hold in the quantum domain. All attempts to ‘save’ PII in this realm rest on some restrictions in the range of the admissible properties, conducive not to identity but to some form of indiscernibility.

Today, we have freed ourselves from the ancient *principles of reason*, namely, the principles of Non-Contradiction, Excluded Middle, and others. But the very notion of standard identity remains a taboo to be superseded.

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