A kinematic derivation of the Born rule

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Abstract

Physical theories assign values of quantities to physical systems, and they assign probabilities to the possible outcomes of measurements of those quantities. In quantum theory, probabilities are assigned using the Born rule. Here, I show that the Born rule is uniquely determined by the conditions under which quantum systems instantiate values of quantities as they are specified by the eigenstate-eigenvalue link, together with minimal theory-independent assumptions about how probabilities are sensitive to the structure of physical quantities.

1 Introduction

We learn about physical systems by measuring the quantities they possess. Physical theories help us to mathematically represent these quantities, and to predict the relative frequencies of outcomes we expect to observe when measuring them. In quantum theory, where possible system configurations are represented by density operators and quantities are represented by self-adjoint operators, the probabilities

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which estimate the relative frequencies of measurement outcomes are assigned using the Born rule:

> BORN RULE: The probability of observing the value λ of the quantity $\hat{A}$ upon measuring a system in state $\hat{\rho}$ is $\mathrm{tr}(\hat{\rho}\hat{\Pi}_\lambda^{(\hat{A})})$, where $\hat{\Pi}_\lambda^{(\hat{A})}$ is the projection onto the λ-eigenspace of $\hat{A}$.

As a recipe for making predictions, the Born rule is remarkably successful. Many alternative probability assignment schemes could have been adopted instead,[1] but these would not have been empirically adequate. Despite its widespread empirical success, however, the Born rule is often introduced as a postulate, lacking a physically elucidating justification of why it works.

Historically, this now-standard probability assignment scheme was introduced to facilitate prediction-making in the presence of indeterministic dynamics.[2] Any physical theory needs some sort of probability assignment scheme within its inferential architecture to make predictions about what will happen when quantities are measured for systems in particular states. When a system evolves deterministically, its state at one time uniquely determines its state at all other times, and so identically configured systems always yield the same outcomes upon measurement, rendering the relevant probability assignment scheme trivial. However, when a system evolves indeterministically, its state at one time does *not* uniquely determine its state at all other times, and so identically prepared systems need not yield the same outcomes upon measurement. When this happens, the relevant probability assignment scheme has serious inferential work to do. Consequently, the fact that quantum systems undergo indeterministic dynamical evolution suggests that quantum theory needs a non-trivial probability assignment scheme; the Born rule fills this role.

1. (Galley and Masanes 2017)
2. See (Born 1926). For historical discussion, see (Duncan and Janssen 2023, Ch. 16).

Since the need for a non-trivial probability assignment scheme in quantum theory was first realized as a consequence of dynamical considerations, one might think that a physically elucidating justification for adopting the Born rule, rather than some other scheme, could be found by appealing to the dynamics of quantum systems. Indeed, most efforts made to derive the Born rule begin with dynamical assumptions. Advocates of Everettian interpretations derive the Born rule by appealing to dynamical assumptions about how processes like decoherence fix the underlying branching structure of the Everettian multiverse, together with assumptions about how agents are situated within this branching structure.[3] Advocates of Bohmian interpretations, by contrast, derive the Born rule by appealing to dynamical assumptions about the evolution of ensembles of Bohmian particles and showing either that configurations of these ensembles which obey the Born rule are typical,[4] or else that configurations which do not obey the Born rule equilibrate towards configurations that do.[5] And advocates of objective collapse interpretations characterize the dynamics of quantum systems using stochastic modifications to the Schrödinger equation which are tuned to ensure that macroscopic measurement devices are very likely to collapse and yield outcomes with relative frequencies approximating Born rule probabilities.[6]

Dynamical derivations of the Born rule are often sensitive to how the measurement problem is resolved. There is, however, a way of asking why the Born rule is the right probability assignment scheme for quantum theory which is agnostic towards this, and which instead frames the Born rule kinematically.[7] When a quantum system occupies some state at a given moment, it instantiates some collection of quan-

3. (Deutsch 1999; Wallace 2010; Sebens and Carroll 2018; Saunders 2021)
4. (Dürr, Goldstein, and Zanghí 1992; Dürr and Teufel 2009; Norsen 2018)
5. (Valentini 1991a, 1991b; Norsen 2018)
6. Other dynamical derivations of the Born rule are given in (Zurek 2003, 2005; Wang 2020; Cicchella et al. 2025).
7. I do not mean to suggest that any existing dynamical derivations of the Born rule are defective or invalid; they surely provide much insight. Rather, I am proposing that there is a very different way to pursue the issue which is also informative.

tity values; its state at a given moment also encodes its probabilistic dispositions to instantiate other quantity values if measured in various way. It is natural to ask: how do the quantity values a system instantiates at a moment in time determine, perhaps probabilistically, the quantity values it is disposed to instantiating immediately after some measurement or another is implemented? This is a kinematic question about what the state of a system at a moment in time tells us about what properties that system has, and how that system will behave, and an answer to it would, in effect, explain why we should expect measurement outcomes are distributed in accordance with Born rule probabilities, rather than some alternative rule. Hence, such an answer would amount to a kinematic derivation of the Born rule in a manner that does not prejudge how the measurement problem is to be resolved.

By conceiving of the inferential role of the Born rule in this way, it seems that the way we make predictions about which quantity values we expect to observe systems taking on should be constrained and informed by the conditions under which these values are in fact instantiated. That is, how we use a physical theory to describe target systems should play a role in determining how we use that theory to predict what these target systems will do. Dynamical derivations of the Born rule do not make this is not apparent.

Here, the Born rule (for $\dim(\mathcal{H}) > 2$) is instead derived kinematically, rather than dynamically, from assumptions about the conditions under which quantum systems instantiate particular values of quantities, and how the instantiation conditions of these quantity values determine what we should expect to observe when we measure them. The project of deriving the Born rule, as I understand it here, is a very specific project of showing why the numerical values of quantum probabilities expressed by the Born rule are the only viable numerical values for making empirically adequate predictions about the outcomes of quantum measurements. It is worth emphasizing that, understood in this way, the project of deriving the Born rule can be separated from a variety of related, but ultimately distinct, projects; it does not require, for instance, explaining what sort of mechanism is responsible for producing outcomes of quantum measurements, explaining why the post-measurement state of a system is what it is, explaining what sorts of things quantum probabilities are (e.g. objective chances or subjective credences), or solving the measurement problem. Section 2 motivates the assumptions from which the Born rule is then derived in Section 3, and Section 4 concludes.

2 Assumptions

Physical theories characterize the physical quantities and quantity values systems instantiate when they occupy various states. To use a theory to describe the features of physical systems, one requires a scheme which specifies when systems in specific states take on particular values of these quantities. To use a theory to make empirical predictions, one further requires a scheme for assigning probabilities to expected outcomes of measurements of these quantities. These schemes play distinct theoretical roles, but they must nevertheless be compatible with each other in various ways in order to do their jobs properly. In particular, the conditions under which systems instantiate quantity values should, at least in part, dictate what one expects to see when these quantities are measured. This observation underwrites the general strategy by which the Born rule is derived in what follows. Specifically, the Born rule is here derived as a consequence of three theory-independent compatibility assumptions about how the quantity valuation scheme of a physical theory constrains its probability assignment scheme, together with an assumption about what the quantity valuation scheme is for quantum theory in particular.

It will be helpful to contrast this strategy with other existing views. Some view quantum probabilities as being determined epistemically by constraints on rational belief. On these views, Born rule probabilities reflect the credences rational agents ought to adopt towards outcomes of quantum measurements because any other credences would violate reasonable restrictions on rationality, e.g., by being susceptible to Dutch books or by being accuracy dominated.[8] In contrast with these views, I propose that quantum probabilities are determined metaphysically by features of the world, namely, the patterns of properties quantum systems instantiate when they are configured in particular states.

Before introducing the assumptions from which the Born rule is derived, I will first say more about what a probability assignment scheme for an arbitrary physical theory consists of. A probability assignment scheme provides an estimate of the relative frequencies of measurement outcomes one would expect to observe if one were to repeatedly measure values of quantities for systems occupying particular states. So, for every measurable physical quantity Q characterized by some physical theory, and for every physically possible state s of a system which instantiates Q, the probability assignment scheme of the theory in question specifies a probability measure $Pr_s^{(Q)}$ over (the Lebesgue-measurable subsets of) the possible values of Q, denoted $\mathrm{val}(Q) \subseteq \mathbb{R}$. These probabilities reflect how likely it is, according to the theory in question, that a measurement of Q performed on a system in the state s will yield an outcome in the specified subset of values. Now, physical theories often allow one to represent systems as occupying statistical mixtures of different pure states; the probabilities assigned to measurement outcomes for systems in such mixed states are determined by the probabilities assigned to these same measurement outcomes for systems in the pure states from which the mixture is composed. Specifically, if a state

8. See, e.g., (Caves, Fuchs, and Schack 2002; Pitowsky 2006; Steeger 2019; Meehan and Steeger 2023).

$s = \sum_i p_i s_i$ is a statistical mixture of pure states $\{s_i\}$ (with $0 \leq p_i \leq 1$ and $\sum_i p_i = 1$), then for every value v of every quantity Q, $Pr_s^{(Q)}(v) = \sum_i p_i Pr_{s_i}^{(Q)}(v)$.[9]

Given this understanding of what the probability assignment scheme of a physical theory consists of, I now propose three theory-independent compatibility assumptions about how the quantity valuation scheme of a physical theory constrains its probability assignment scheme.

First, whenever a system takes on a value of some quantity, any ideal measurement of that quantity should indicate the instantiated value with probability 1:[10]

> AGREEMENT: If a system in state s instantiates the value v of the quantity Q, then $Pr_s^{(Q)}(v) = 1$.

In this way, any process which can be called a measurement (in the absence of loss and noise, etc.) of some quantity just reveals which quantity value a system already instantiates when such a system does in fact already instantiate some such value.

Second, some quantities coarse-grain others. A quantity Q coarse-grains another quantity R when there exists some (perhaps non-injective) function $\phi_{RQ} : \mathrm{val}(R) \to \mathrm{val}(Q)$ such that a system has a value v of R only if it also has the value $\phi_{RQ}(v)$ of Q. When quantities are related via coarse-graining, the probability assigned to a value of the coarser quantity should equal the sum of the probabilities assigned to the values of the finer-grained quantity that value aggregates:

> COARSE-GRAINING: If Q coarse-grains R, then for every $u \in \mathrm{val}(Q)$, $Pr_s^{(Q)}(u) = \sum_{v \in \phi_{RQ}^{-1}(u)} Pr_s^{(R)}(v)$.[11]

9. These general features of probability assignment schemes for physical theories all have a strong precedent in broader discussions of generalized probabilistic theories; cf. (Jotta and Hinrichsen 2014; D'Ariano, Chiribella, and Perinotti 2017; Plávala 2023).

10. One might find the converse, inferring which quantity values a system instantiates from probabilities, problematic, but this is not needed.

11. When Q values aggregate infinite collections of R values, this is $Pr_s^{(Q)}(u) = \int_{\phi_{RQ}^{-1}(u)} Pr_s^{(R)}(v) dv$.

In this way, wholesale relations between distinct quantities constrain which probabilities should be assigned to observing their values upon measurement. Specifically, probabilities assigned to values of finer-grained quantities should refine of the probabilities assigned to coarser-grained quantities.

Third, sometimes distinct quantity values are instantiated under exactly the same conditions. That is, it can happen that the value v of a quantity Q is instantiated if and only if the value u of some other quantity R is also instantiated (even if other Q and R values are not coinstantiated in this way). When this happens, such values should be assigned the same probabilities, for no system can have one without the other:

> COINSTANTIATION: Suppose a system instantiates the value v of the quantity Q if and only if it instantiates the value u of the quantity R. Then for every state s, $Pr_s^{(Q)}(v) = Pr_s^{(R)}(u)$.

In this way, specific relations between distinct values of distinct quantities constrain which probabilities should be assigned to observing them upon measurement. Specifically, when one cannot distinguish the fact that a system has one value of one quantity from the fact that it has another value of another quantity, predictions made about whether a system will be found upon measurement to have one or another of these values should not come apart.

These compatibility assumptions are highly general and it is illustrative to see that they are all satisfied in classical mechanics. The state of a classical system is represented by a probability measure μ over a phase space X; its associated quantities are represented by real-valued measurable phase space functions. A classical system in state μ has the value v of the quantity $f : X \to \mathbb{R}$ just in case $f^{-1}(v)$ contains the support of μ. Probabilities are then assigned to outcomes of measurements of classical quantities by $Pr_\mu^{(f)}(v) = \int_{f^{-1}(v)} d\mu$. If a system in state μ takes on the value v of f, the support of μ is contained in $f^{-1}(v)$ and so $Pr_\mu^{(f)}(v) = 1$, satisfying AGREEMENT.

Moreover, if f is a coarse-graining of g (under ϕ_{gf}), then $f^{-1}(u) = g^{-1} \circ \phi_{gf}^{-1}(u)$, from which it follows that $Pr_{\mu}^{(f)}(u) = \int_{\phi_{gf}^{-1}(u)} Pr_{\mu}^{(g)}(v)dv$ and hence COARSE-GRAINING is also satisfied. And since the values v of f and u of g are have the same instantiation conditions if and only if $f^{-1}(v) = g^{-1}(u)$, it is clear that $Pr_{\mu}^{(f)}(v) = Pr_{\mu}^{(g)}(u)$ for such values, so COINSTANTIATION is satisfied as well.

These theory-independent compatibility assumptions relating kinematic features of quantities posited by a physical theory to the empirical predictions licensed by that theory involve nothing special about quantum theory. Yet, when they are enforced in the quantum setting, the Born rule follows (as shown below), provided one assumes that quantity values are assigned to quantum systems under the following scheme:

> EIGENSTATE-EIGENVALUE LINK: A quantum system in state $\hat{\rho}$ takes on the value λ of the quantity $\hat{A}$ just when $\hat{A}\hat{\rho} = \lambda\hat{\rho}$.

This is equivalent to requiring that $\hat{\rho}$ be a mixture of pure λ-eigenstates of $\hat{A}$, and entails that if a system in state $\hat{\rho}$ instantiates the value λ of the quantity $\hat{A}$, then $\hat{\rho}$ is invariant under Lüders updating following a measurement of $\hat{A}$ yielding outcome λ, thus satisfying the EPR reality criterion.[12] Observe that on this construal EIGENSTATE-EIGENVALUE LINK only assigns quantity values to systems, and not quantities themselves.

EIGENSTATE-EIGENVALUE LINK does not require that systems take on values of every quantity. For this and related reasons, some view EIGENSTATE-EIGENVALUE LINK as controversial.[13] Nevertheless, some such quantity valuation scheme is required for one to be able to assign quantity values to quantum systems. Moreover, EIGENSTATE-EIGENVALUE LINK has played a significant historical and conceptual role in framing many foundational issues in quantum theory, and many considered

12. (Einstein, Podolsky, and Rosen 1935)
13. (Leifer 2014; Wallace 2019)

alternatives to EIGENSTATE-EIGENVALUE LINK delineate instantiation conditions which are very well approximated by EIGENSTATE-EIGENVALUE LINK.[14] So, while perhaps contentious, it is an assumption that is frequently endorsed, and whose consequences are surely worth exploring.

I now show that when $\dim(\mathcal{H}) > 2$, BORN RULE follows from the preceding theory-independent compatibility assumptions—AGREEMENT, COARSE GRAINING, and COINSTANTIATION—when supplemented in the quantum setting with EIGENSTATE-EIGENVALUE LINK.

3 Derivation

To begin, suppose $\dim(\mathcal{H}) > 2$. Given arbitrary values λ and τ of quantum-mechanical quantities $\hat{A}$ and $\hat{B}$, respectively, the instantiation conditions for λ and τ are determined by EIGENSTATE-EIGENVALUE LINK: λ and τ have the same instantiation conditions when their eigenspaces are the same. Thus, COINSTANTIATION requires:

$$\hat{\Pi}_\lambda^{(\hat{A})} = \hat{\Pi}_\tau^{(\hat{B})} \quad \text{only if} \quad Pr_{\hat{\rho}}^{(\hat{A})}(\lambda) = Pr_{\hat{\rho}}^{(\hat{B})}(\tau). \tag{1}$$

So, letting $\mathcal{P}(\mathcal{H})$ denote the projection lattice, all quantum probabilities assigned to quantity values by a state $\hat{\rho}$ may be unambiguously encoded by one map $F_{\hat{\rho}} : \mathcal{P}(\mathcal{H}) \to [0,1]$ defined by $F_{\hat{\rho}}(\hat{\Pi}_\lambda^{(\hat{A})}) = Pr_{\hat{\rho}}^{(\hat{A})}(\lambda)$. Since every orthogonal projection is a spectral projection for a value of some quantum-mechanical quantity, the fact that $Pr_{\hat{\rho}}^{(\hat{A})}$ is well-defined for every state and every value of every quantity ensures $F_{\hat{\rho}}$ is a total function, while COINSTANTIATION ensures it is single-valued.

$F_{\hat{\rho}}$ has several properties. First, the identity $\hat{I}$ represents a quantity, and since $Pr_{\hat{\rho}}^{(\hat{I})}$ is a probability measure over $\mathrm{val}(\hat{I}) = \{1\}$, it follows that $F_{\hat{\rho}}(\hat{I}) = Pr_{\hat{\rho}}^{(\hat{I})}(1) = 1$.

14. (Gilton 2016; Lewis 2016)

Second, since the values of $F_{\hat{\rho}}$ are values of probability measures, $F_{\hat{\rho}}(\hat{\Pi}) \geq 0$ for all $\hat{\Pi} \in \mathcal{P}(\mathcal{H})$. Third, if $\hat{\Pi}_1, \hat{\Pi}_2 \in \mathcal{P}(\mathcal{H})$ are orthogonal, then $\hat{\Pi}_1 + \hat{\Pi}_2 \in \mathcal{P}(\mathcal{H})$ and we may define quantities $\hat{A} = \lambda_1\hat{\Pi}_1 + \lambda_2\hat{\Pi}_2$ and $\hat{B} = \lambda_3(\hat{\Pi}_1 + \hat{\Pi}_2)$, with each λ_i real and distinct. It is clear that $\hat{B}$ coarse-grains $\hat{A}$ (with $\phi_{\hat{A}\hat{B}}(0) = 0$ and $\phi_{\hat{A}\hat{B}}(\lambda_1) = \phi_{\hat{A}\hat{B}}(\lambda_2) = \lambda_3$) so COARSE-GRAINING requires that $Pr_{\hat{\rho}}^{(\hat{B})}(\tau) = Pr_{\hat{\rho}}^{(\hat{A})}(\lambda_1) + Pr_{\hat{\rho}}^{(\hat{A})}(\lambda_2)$. Hence $F_{\hat{\rho}}(\hat{\Pi}_1 + \hat{\Pi}_2) = F_{\hat{\rho}}(\hat{\Pi}_1) + F_{\hat{\rho}}(\hat{\Pi}_2)$. We may now utilize Gleason's theorem:

Theorem 1 (Gleason 1957)**.** If $\dim(\mathcal{H}) > 2$, then for every function $F_{\hat{\rho}} : \mathcal{P}(\mathcal{H}) \to [0, 1]$ such that *(i)* $F_{\hat{\rho}}(\hat{I}) = 1$, *(ii)* $F_{\hat{\rho}}(\hat{\Pi}) \geq 0$ for all $\hat{\Pi}$, and *(iii)* $F_{\hat{\rho}}(\hat{\Pi}_1 + \hat{\Pi}_2) = F_{\hat{\rho}}(\hat{\Pi}_1) + F_{\hat{\rho}}(\hat{\Pi}_2)$ for all $\hat{\Pi}_1, \hat{\Pi}_2 \in \mathcal{P}(\mathcal{H})$ orthogonal, there is a unique density operator $\hat{\sigma}$ such that $F_{\hat{\rho}}(\hat{\Pi}) = \mathrm{tr}(\hat{\sigma}\hat{\Pi})$.

AGREEMENT is then satisfied only if $\hat{\sigma} = \hat{\rho}$.[15] For suppose not. If $[\hat{\rho}, \hat{\sigma}] \neq 0$, then $\hat{\rho}$ and $\hat{\sigma}$ are not simultaneously diagonalizable so there will be some quantity $\hat{A}$ with $\hat{A}\hat{\rho} = \lambda\hat{\rho}$ and $Pr_{\hat{\rho}}^{(\hat{A})}(\lambda) = \mathrm{tr}(\hat{\sigma}\hat{\Pi}_\lambda^{(\hat{A})}) \neq 1$, violating AGREEMENT. If instead $[\hat{\rho}, \hat{\sigma}] = 0$, then there are decompositions $\hat{\sigma} = \sum_i p_i\hat{\omega}_i$ and $\hat{\rho} = \sum_i q_i\hat{\omega}_i$ into orthogonal pure states $\{\hat{\omega}_i\}$ with $p_k \neq q_k$ for some k. Since probability assignments must be compatible with state mixing, $Pr_{\hat{\rho}}^{(\hat{\omega}_k)}(1) = \sum_i q_i Pr_{\hat{\omega}_i}^{(\hat{\omega}_k)}(1)$. From AGREEMENT, $Pr_{\hat{\omega}_i}^{(\hat{\omega}_k)}(1) = \delta_{ik}$ so $Pr_{\hat{\rho}}^{(\hat{\omega}_k)}(1) = q_k$. But $Pr_{\hat{\rho}}^{(\hat{\omega}_k)}(1) = \mathrm{tr}(\hat{\sigma}\hat{\Pi}_1^{(\hat{\omega}_k)}) = p_k$, a contradiction.

So, if quantum-mechanical quantity values are assigned by EIGENSTATE-EIGENVALUE LINK, and the stated compatibility assumptions are satisfied, BORN RULE follows:

$$Pr_{\hat{\rho}}^{(\hat{A})}(\lambda) = F_{\hat{\rho}}(\hat{\Pi}_\lambda^{(\hat{A})}) = \mathrm{tr}(\hat{\rho}\hat{\Pi}_\lambda^{(\hat{A})}). \tag{2}$$

Two remarks are in order. First, this derivation required a judicious appeal to Gleason's theorem. Gleason's theorem is often associated with the quantum logic framework,[16] where propositions about quantum systems are presumed to have a

15. This resolves the coordination problem for Gleason's theorem discussed in (Steeger 2017).
16. See, for instance, (Gleason 1957; Pitowsky 2006; Steeger 2019).

non-classical logic,[17] and where basic linguistic operations like negation and disjunction are given revisionary interpretations. So, one might worry that the derivation provided presupposes quantum logic. This would be undesirable since there are many reasons one might want to reject the logical and semantic revisions required by the quantum logic framework.[18] However, despite invoking Gleason's theorem, no violation of classical logic was presupposed above. This is evident from the fact that none of the assumptions from which the Born rule was derived were semantic or logical in nature. The theory-independent compatibility assumptions are all satisfied in classical mechanics, where classical logic is unproblematic, and EIGENSTATE-EIGENVALUE LINK—at least as it is characterized here—is an assumption about the instantiation conditions of quantity values and not about the truth conditions of sentences.[19] $F_{\hat{\rho}}$ may be viewed as a compressed notation for keeping track of many distinct probability measures associated with different quantities; it need not be viewed as a single non-classical probability measure over a non-distributive lattice of quantum propositions, or anything of that sort.

Second, the only essentially 'quantum' ingredient required for this derivation is EIGENSTATE-EIGENVALUE LINK. This, however, requires the full kinematics of quantum theory—its state space and its collection of quantities—in order to be stated. If the kinematic representation of quantum-mechanical quantities were structurally different, the derivation would fail. So, while EIGENSTATE-EIGENVALUE LINK is viewed by some as contentious, if a different quantity valuation scheme were adopted instead, $F_{\hat{\rho}}$ would no longer be well-defined. It is therefore not obvious whether various alternative quantity valuation schemes could be adopts, given the proposed

17. (Birkhoff and Neumann 1936)

18. See, for instance, (Gardner 1971; Greechie and Gudder 1971; Dummett 1978; Hellman 1980; Maudlin 2005; Rumfitt 2015).

19. A more thorough discussion of the compatibility of the eigenstate-eigenvalue link with classical logic, which more carefully attends to the relation between truth-conditional semantics and the metaphysics of quantity, may be found in (Fraser, Miller, and Wilson 2026).

compatibility assumptions, without violating the Born rule; some replacements of EIGENSTATE-EIGENVALUE LINK would lead to empirical inadequacy. This, in turn, provides indirect support for EIGENSTATE-EIGENVALUE LINK.

4 Conclusions

The Born rule plays an important inferential role in quantum theory, regimenting the assignment of probabilities to measurement outcomes and thereby underwriting the empirical predictions of the theory. However, in standard presentations, the Born rule is merely postulated, lacking a more physically elucidating justification. Many efforts have been made to derive the Born rule from dynamical assumptions. Such derivations, however, are generally insensitive to the conditions under which values of quantum mechanical quantities are instantiated. This is peculiar, since the Born rule is centrally concerned with predicting which quantity values systems will to take on when measured. Here, the Born rule was instead derived kinematically from theory-independent assumptions about how the probability assignment scheme of a physical theory is constrained by the quantity valuation scheme of that theory, together with the assumption that the quantity valuation scheme of quantum theory in particular is given by the eigenstate-eigenvalue link. This shows how kinematic features of quantum-mechanical quantities suffice to fix the empirical predictions of quantum theory without requiring recourse to dynamical assumptions, and without modifying the quantum formalism.

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